Real Numbers

Euclid's Division Lemma

- An algorithm is a series of well defined steps which gives a procedure for solving a type of problem.
- A lemma is a proven statement used for proving another statement.
- Euclid's division algorithm is a technique to compute the Highest Common Factor (HCF) of two given positive integers.
- To obtain the HCF of two positive integers, say c and d, with $c > d$, follow the steps below:
  Step 1: Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \leq r < d$.
  Step 2: If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.
  Step 3: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

The Fundamental Theorem of Arithmetic

- Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

Rational and Irrational Numbers

- A number 's' is called rational if it can be written in the form p, q
  Where p and q are integers and $q \neq 0$.
- A number 's' is called irrational if it cannot be written in the form p, q
  Where p and q are integers and $q \neq 0$.

Irrationality of Square Roots of 2, 3 and 5

- Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.
- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are irrational

Decimal Expansions of Rational Numbers

- Let x be a rational number whose decimal expansion terminates. Then we can express x in the form pq, where p and q are coprime, and the prime factorization of q is of the form 2^n5^m, where n, m are non-negative integers.
- Let $x = pq$ be a rational number, such that the prime factorization of q is of the form 2^n5^m, where n, m are non-negative integers. Then x has a decimal expansion which terminates.
- Let $x = pq$ be a rational number, such that the prime factorization of q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).