Areas related to Circles

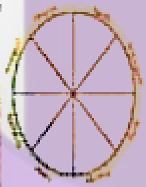
Perimeter and Area of a Circle - A Review Circle

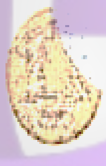
A circle is a path of a point which moves in such a way that its distance from a fixed point is a constant. The path of the point is called the locus of the point. The fixed point is called the centre of the circle. The constant distance is called the radius of the circle.

In our daily life we see a lot of things which are circular in shape.

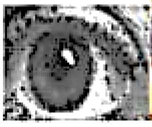
1. Bangle
2. Wheel
3. Coin
4. Chakra in the middle of the flag
5. Eyeball
6. Clock
7. Papad
8. Ring
9. Dosa


Bangle

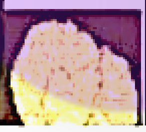

Wheel

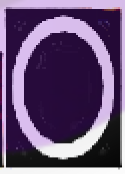

Coin


Flag


Eye ball


Clock


Papad


Ring


Dosa

Circumference

The perimeter of a circle is called the circumference of the circle. Using a rape or thread we measure the circumference of a wheel or a bangle. We can also measure the diameter. Diameter is the largest chord of a circle. In each case divide the circumference by the diameter. What do you notice? We find that in each case, the ratio of the circumference to the diameter is the same. This ratio is a constant called π (pi) which is a Greek letter.

$\pi = \frac{\text{Circumference of a circle}}{\text{Diameter}}$

Form this we get circumference of a circle is the product of π and its diameter.

$C = \pi d$

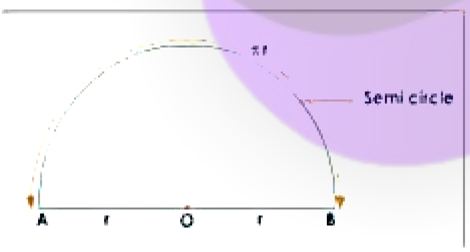
$= 2\pi r$ (since $d = 2r$)

Know about π

π is a Greek letter. It is a ratio of the circumference to diameter of a circle. Many mathematicians have given many values for this ratio. In the chapter Real numbers, we saw that 'π' is an irrational number. Aryabhatta gave the value of π as $\frac{62832}{20000}$ approximately, whereas Ramanujan found the value of π correct to a million places of decimals. However, for practical purposes we use π as $\frac{22}{7}$. But $\frac{22}{7}$ is a rational number. It is only an approximate value.

Semi Circle

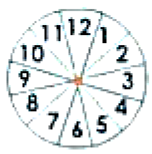
A diameter of a circle divides the circle into two halves called semi circles.



Perimeter of a semicircle is $\pi r + 2r = r(\pi + 2)$ units.

Area of Circle

Draw a circle. Divide it into equal (even) sectors and arrange them in line as shown below.



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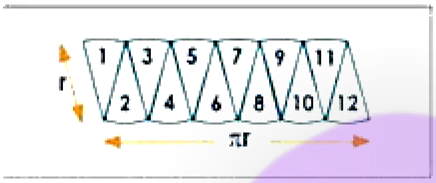


This shape looks like a rectangle of length 'πr' and breath 'r' units

Area of the circle = Area of the rectangle

= length x breadth

= πr x r

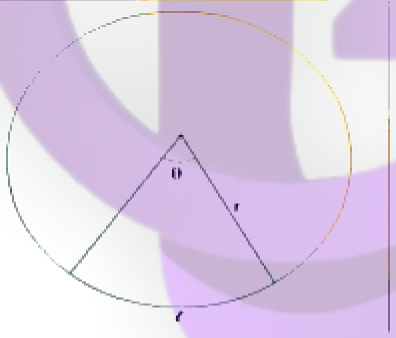
= πr^2 Sq. units

Area of a circle is πr^2 Sq. units

Area of a semicircle = $\frac{1}{2}\pi r^2$ Sq. units.

Area of a quadrant = $\frac{1}{4}\pi r^2$ Sq. units.

Area of Sector of a Circle



Sector of a circle is a portion of a circular region enclosed between 2 radii and the corresponding arc. θ is the central angle, l is the length of the arc. When the central angle is 360°, area of the circle is πr^2. When the central angle is θ,

Area of sector = $\frac{\theta}{360} \times \pi r^2$

When central angle is 360°,

Length of arc = $2\pi r$

When central angle is θ°,

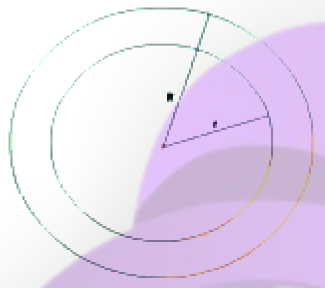
Length of arc = $\frac{\theta}{360} \times 2\pi r$

∴ Length of the arc = $\frac{\theta}{360} \times 2\pi r$

Perimeter of a sector = $2r + l$

Area of a Ring

Let 'R' and 'r' be the radii of the outer and inner circles respectively.

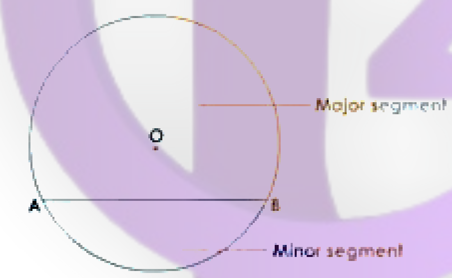


Area of ring = Area of outer circle - Area of inner circle

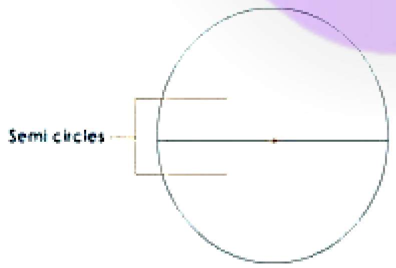
i.e. $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

Segment of a Circle

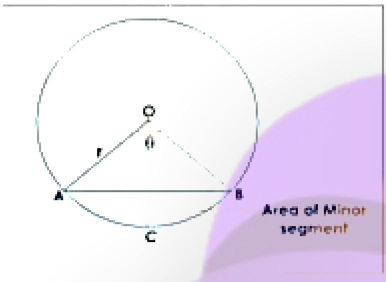
A segment of a circle is a portion of the circular region enclosed between a chord and the corresponding arc. A chord divides a circle into two portions called segments.



The segment which contains the centre of the circle is called the major segment. The other one is called the minor segment. The diameter divides the circle into two equal halves called semicircles.



Area of a Segment

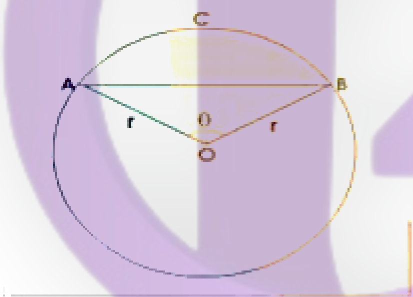


Area of the minor segment = Area of the sector OACB – Area of the ΔAOB.

Area of a Major Segment

Area of a major segment = Area of circle - Area of minor segment

Formula for finding the area of a segment of a circle



Draw a circle of radius 'r'. Let the chord AB cut the circle into two segments. We want to find the area of the minor segment (Coloured portion). Join O to A, O to B.

Let ∠AOB = θ. We get a sector; part of it is the segment whose area is to be found.

Area of the sector OACB = area of segment ACB + area of ΔAOB.

Area of segment ACB = Area of sector OACB - Area of ΔAOB.

Area of ΔAOB = $\frac{1}{2} \times r \sin\theta \times r = \frac{1}{2} r^2 \sin\theta$

Area of segment ACB = $\frac{\theta}{360}\pi r^2 - \frac{1}{2} r^2 \sin\theta$

$= r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$