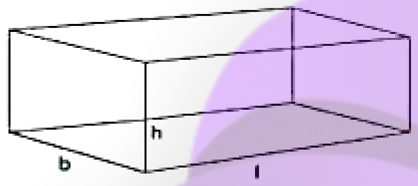
Surface Areas and Volumes

Cuboid

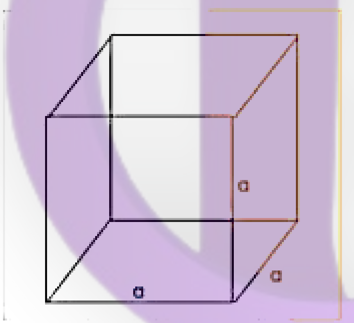


Total surface area = 2 (lb + bh + hl) Sq. units

Lateral surface area = Area of 4 walls = 2h (l + b) Sq. units

Volume = lbh Cu. units

Cube



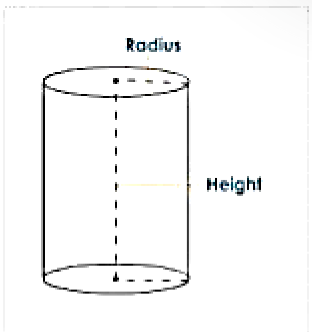
Let each side be 'a' units

Total surface area = $6a^2$ Sq. units

Lateral surface area = $4a^2$ Sq. units

Volume = a^3 Cu. units

Cylinder

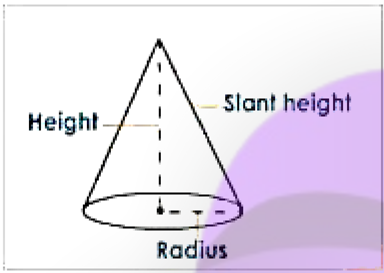


C.S.A = $2\pi rh$ Sq. units

T.S.A = $2\pi r(h+1)$ Sq. units

Volume = $\pi r^2 h$ Cu. units

Cone

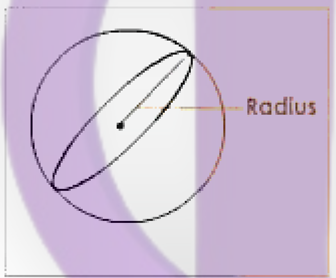


C.S.A = $\pi r l$ sq. units

T.S.A = $\pi r \ (l + r)$sq. units

Volume $= \frac{1}{3}\ \pi r^2 h$ cu. units

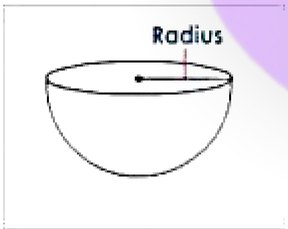
Sphere



Area = $4\pi r^2$Sq. units

Volume $= \frac{4}{3}\ \pi r^3$ Cu. units

Hemisphere

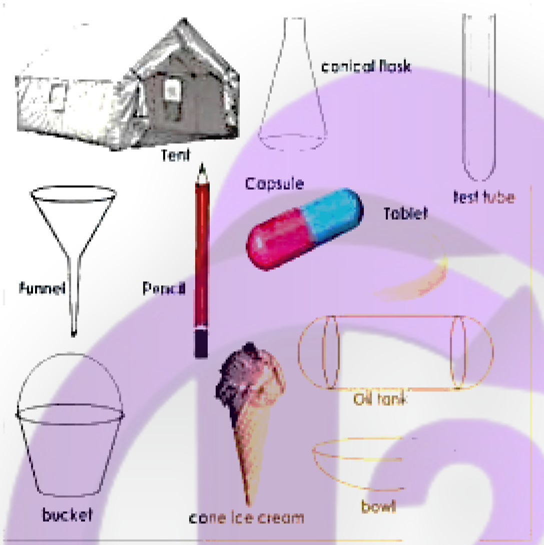


C.S.A = $2\pi r^2$ Sq. units

T.S.A = $3\pi r^2$Sq. units

Volume $= \frac{2}{3}\ \pi r^2$ Cu. units

Surface Area of a Combination of Solids



Total surface area of such a combined solid is found by adding the curved surface areas of the individual parts.

Volume of a Combination of Solids

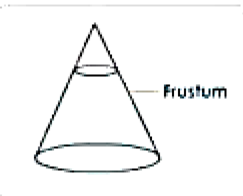
Volume of the new solid formed by the combination of solids is the sum of the volumes of the individual solids.

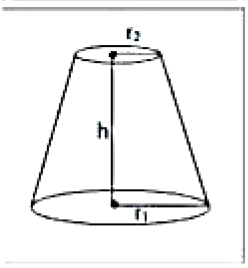
Conversion of solid from one Shape to another

When one solid is converted into other solid, then their volumes are the same.

Frustum of a Cone

Frustum of a cone is a solid obtained from a cone. It is a part of a cone. When we cut the cone by a plane parallel to its base and remove the top portion of the cone, the portion left over is called frustum of a cone.

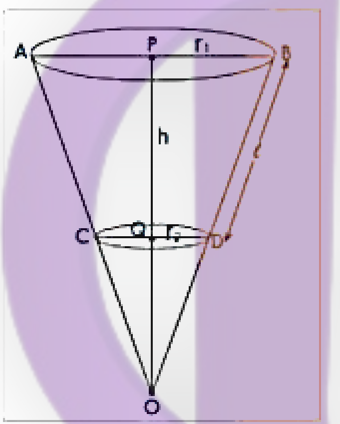




' r_1' is the radius of the bigger circular portion. 'r_2' is the radius of the smaller circular portion. 'h' is the perpendicular distance between two centres. It is height of the frustum.

1. Volume of a frustum $= \frac{1}{3}\pi h[r_1{}^2 + r_2{}^2 + r_1 r_2]$
2. Curved surface area of a frustum
   $= \pi(r_1 + r_2)l$
   Where $l = \sqrt{h^2 + (r_1 - r_2)^2}$
3. Total surface area of a frustum $= \pi(r_1 + r_2)\, l + \pi r_1{}^2 + \pi r_2{}^2$

Derivation of a formula to find the volume and surface area of a frustum of a cone



Let 'h' be the height, 'l' the slant height and r_1 and r_2 be the radii of the bases of the frustum of a cone and $r_1 > r_2$. Now complete the conical portion OCD. Volume of frustum of the right circular cone is the difference in the volumes of the two right circular cones OAB and OCD.

Let the height of the cone OAB be 'h_1' and its slant height be 'l_1'

OP = h_1, OA = OB = l_1

The height of the cone OCD = $h_1 - h$

ΔOQD ~ ΔOPB (AA similarity)

$$\frac{h_1 - h}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow 1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$

$$\frac{h}{h_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$$

$$h_1 = \frac{hr_1}{r_1 - r_2} \ldots\ldots (1)$$

Height of the cone OCD = $h_1 - h = \frac{hr_1}{r_1 - r_2} - h$

$$= \frac{hr_2}{r_1 - r_2} \ldots\ldots (2)$$

Volume of the frustum of cone = volume of the cone OAB − Volume of the cone OCD

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$$

$=\frac{\pi}{3}\left[r_1^2 \cdot \frac{hr_1}{r_1-r_2} - r_2^2 \cdot \frac{hr_2}{r_1-r_2}\right]$

From (1) and (2)

$= \frac{\pi h}{3}\left[\frac{r_1^3-r_2^3}{r_1-r_2}\right] = \frac{\pi h}{3}\left\{\frac{(r_1-r_2)(r_1^2+r_2^2+r_1.r_2)}{r_1-r_2}\right\}$

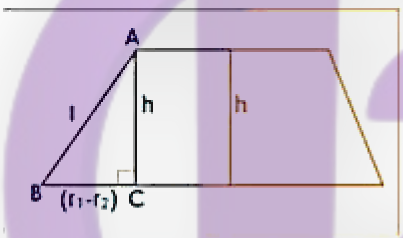
Volume $= \frac{\pi h}{3}\left[r_1^2 + r_2^2 + r_1.r_2\right]$

In the same way we can find the C.S.A and T.S.A of a frustum

C.S.A = $\pi l\,(r_1 + r_2)$, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

T.S.A = $\pi l\,(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

Relations among l, h, r_1 and r_2



In a right angled triangle ABC,

$AB^2 = AC^2 + BC^2$

$l^2 = h^2 + (r_1 - r_2)^2$