Polynomials

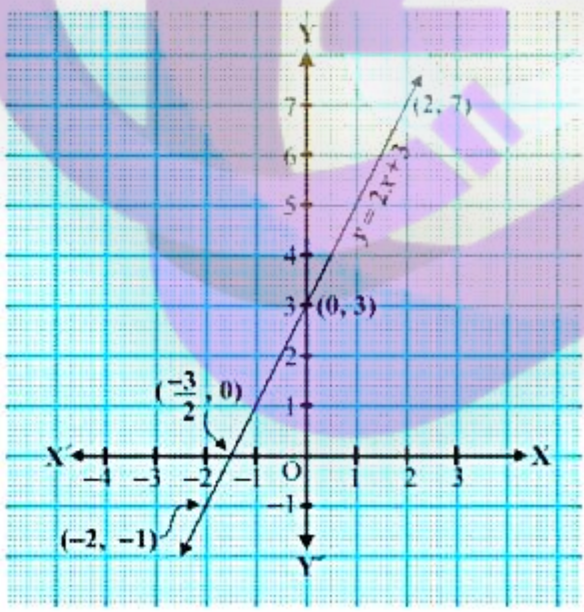
- If p(x) is a polynomial in x, the highest power of x in p(x) is called the degree of the polynomial p(x).

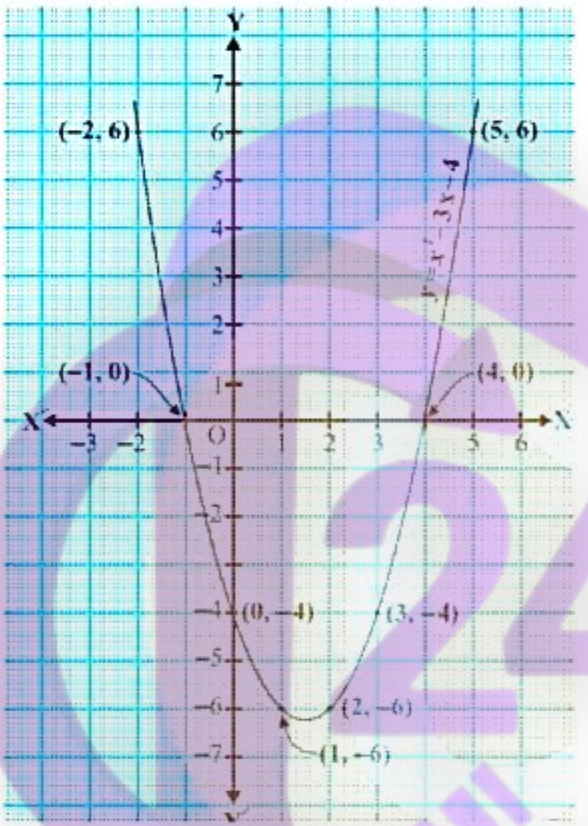
Types of Polynomials

- A polynomial of degree 1 is called a linear polynomial.
- A polynomial of degree 2 is called a quadratic polynomial.
- A polynomial of degree 3 is called a cubic polynomial.

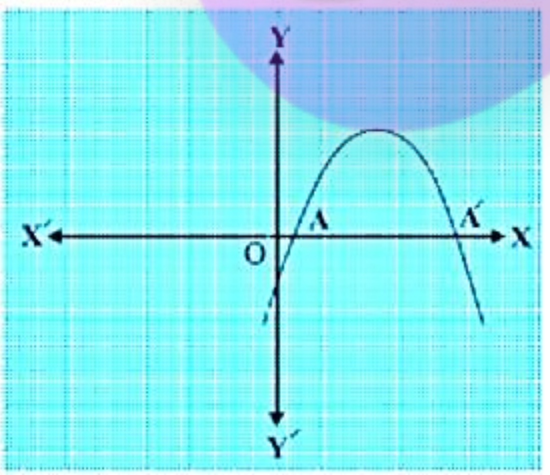
Zeroes of a Polynomial

- If p(x) is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in p(x), is called the value of p(x) at x = k, and is denoted by p(k).
- A real number k is said to be a zero of a polynomial p(x), if p(k) = 0.
- Geometrical Meaning of Zeroes of Polynomials

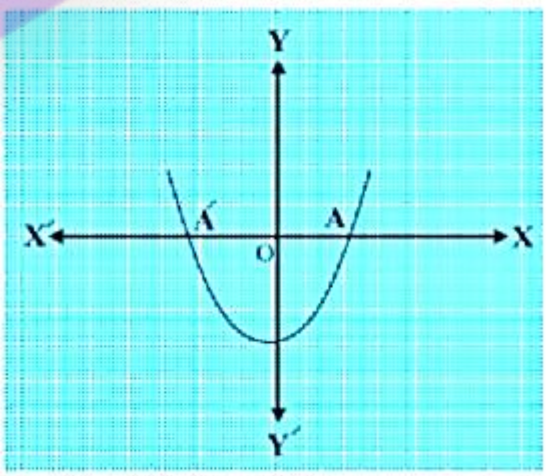




- The equation $ax^2 + bx + c$ can have three cases for the graphs
- Case (i): Here, the graph cuts x-axis at two distinct points A and A′.

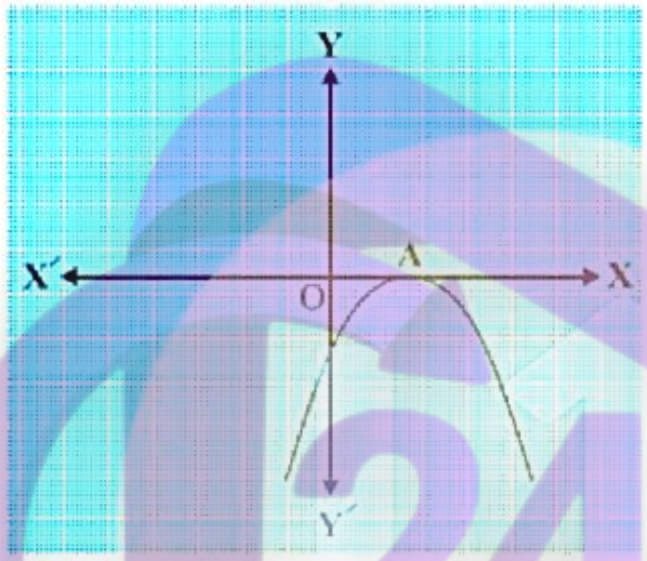


(i)

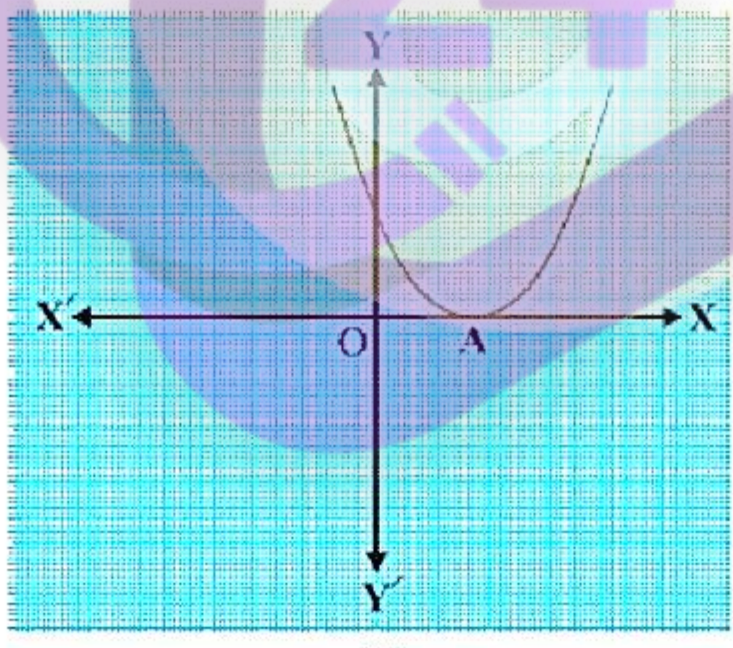


(ii)

Case (ii): Here, the graph cuts the x-axis at exactly one point

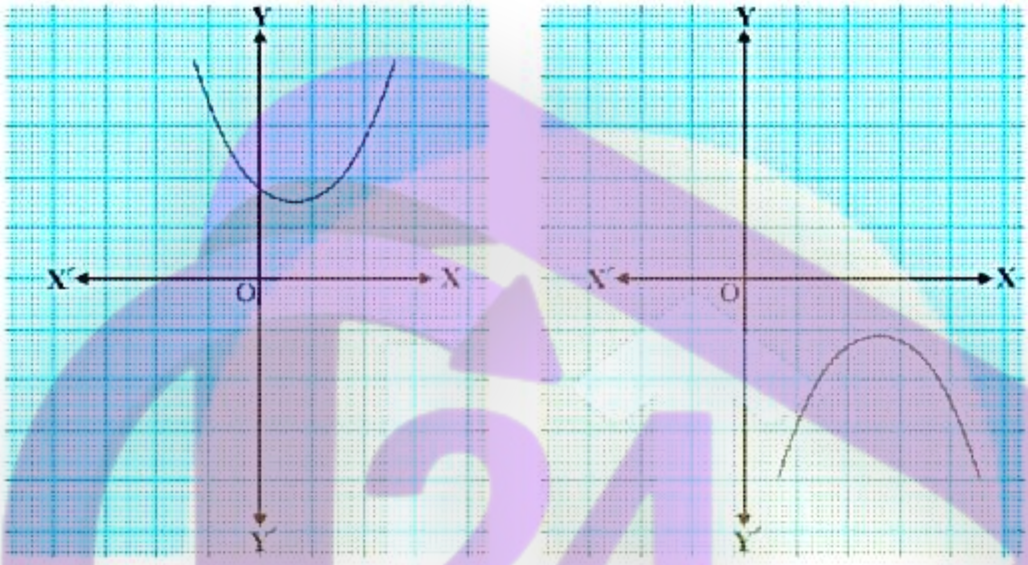


(i)



(ii)

Case (iii): Here, the graph is either completely above the x-axis or completely below the x-axis.



(i) (ii)

Relationship between Zeroes and Coefficients of a Polynomial

If α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$, then you know that $x - \alpha$ and $x - \beta$ are the factors of $p(x)$.

$A + \beta = - b/a$

$\alpha\beta = c/a$

Division Algorithm for Polynomials

- If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that
  $p(x) = g(x) \times q(x) + r(x)$,
  where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.
  This result is known as the Division Algorithm for polynomials.
- Consider the cubic polynomial $x^3 - 3x^2 - x + 3$.
  If one of its zeroes is 1, then $x - 1$ is a factor of $x^3 - 3x^2 - x + 3$.
  So, you can divide $x^3 - 3x^2 - x + 3$ by $x - 1$,
  Next, you could get the factors of $x^2 - 2x - 3$, by splitting the middle term, as:
  $(x + 1)(x - 3)$. This would give you:
  $x^3 - 3x^2 - x + 3 = (x - 1)(x^2 - 2x - 3)$
  $= (x - 1)(x + 1)(x - 3)$
  So, all the three zeroes of the cubic polynomial are now known to you as
  1, − 1, 3.