Quadratic Equations

A quadratic equation in the variable x is an equation of the form $ax^2 + bx + c = 0$, where a , b, c are real numbers, $a \neq 0$.

Roots of a Quadratic Equation:

- A real number α is called a root of the quadratic equation $ax^2 + bx + c = 0, a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$.
- $x = \alpha$ is a solution of the quadratic equation, or α satisfies the quadratic equation.
- The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic. Equation $ax^2 + bx + c = 0$ are the same.

Solution of Quadratic Equation by Factorisation:

- To factorise quadratic polynomials the middle term is split.
- By factorizing the equation into linear factors and equating each factor to zero the roots are determined.

Quadratic Equations - Method of Squares

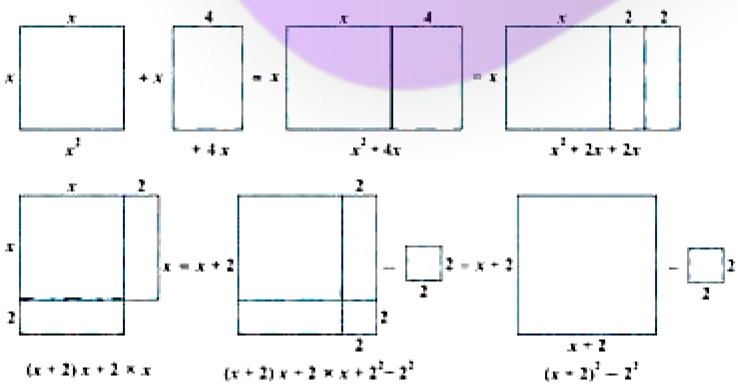
Solution of Quadratic Equation by method of Squares

- We can convert any quadratic equation to the form

  $(x + a)^2 - b^2 = 0$

  $x^2 + 4x$ is being converted to

  $(x + 2)^2 - 4. = (x + 2)^2 - 2^2$



The process is as follows:

$x^2 + 4x = \left(x^2 + \frac{4}{2}x\right) + \frac{4}{2}x$

$= x^2 + 2x + 2x$

$= (x + 2)\, x + 2 \times x$

$= (x + 2)\, x + 2 \times x + 2 \times 2 - 2 \times 2$

$= (x + 2)\, x + (x + 2) \times 2 - 2 \times 2$

$= (x + 2)\,(x + 2) - 2^2$

$= (x + 2)^2 - 4$

So, $x^2 + 4x - 5\ (x + 2)^2 - 4 - 5 = (x + 2)^2 - 9$

So, $x^2 + 4x - 5 = 0$ can be written as $(x + 2)^2 - 9 = 0$ by this process of completing the square. This is known as the **method of completing the square.**

Solution of Quadratic Equation by using Formula.

The formula is as follows:

The roots of $ax^2 + bx + c = 0$ *are* $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ *and* $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

If $b^2 - 4ac \geq 0$.

Thus, if $b^2 - 4ac \geq 0$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

This formula for finding the roots of a quadratic equation is known as the Quadratic formula.

Nature of Roots

We know that roots of the equation $ax^2 + bx + c = 0, a \neq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Where $b^2 - 4ac = \Delta$ is known as discriminant.

Nature of roots based on the discriminant value

1. If $\Delta = 0$, then the roots are real and equal.
2. If $\Delta > 0$, then the roots are real and distinct (unequal)
3. If $\Delta < 0$, then the roots are imaginary (not real)