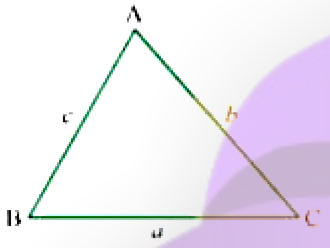
CLASS24

Heron's Formula

- **Perimeter** is the length of the boundary of a closed figure.

- The perimeter of a polygon is the sum of the lengths of all its sides.
  In case of a triangle ABC, with sides of lengths a, b and c units:



  Perimeter of ABC = AB + BC + AC = $a + b + c$

- The **semi-perimeter** of a triangle is half the perimeter of the triangle.

  The semi-perimeter (s) of a triangle with sides a, b and c is $\frac{a+b+c}{2}$.

- The semi-perimeter of a triangle is used for calculating its area when the length of altitude is not known.

- **Area of triangle using Heron's formula:**

  When all the three sides of a triangle are given, its area can be calculated using Heron's formula, which is given by:
  Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
  Here, s is the semi-perimeter of the triangle and is given by, $s = \frac{a+b+c}{2}$

  Example: Find the area of a triangle whose sides are 9 cm, 28 cm and 35 cm.

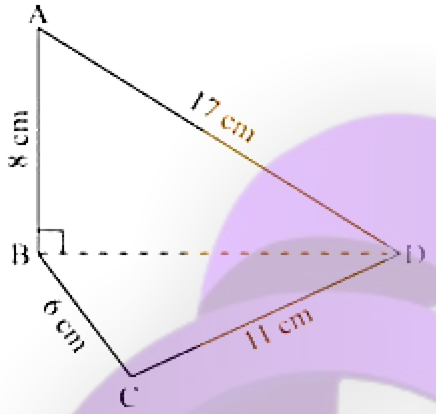
  Solution: Let a = 9 cm, b = 28 cm and c = 35 cm
  Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{9+28+35}{2}$ cm $=$ 36 cm
  Area of triangle = $\sqrt{36(36-9)(36-28)(36-35)}$ cm^2

  $= \sqrt{36 \times 27 \times 8 \times 1}$ cm^2
  $= 36\sqrt{6}$ cm^2

- **Area of quadrilaterals using Heron's formula:**

Area of a quadrilateral can also be calculated using Heron's formula. In this, the quadrilateral is divided into two triangles and then the area of each triangle is calculated using Heron's formula.

Example: What is the area of the given quadrilateral?



Solution: ΔABD is a right-angled triangle.

Using Pythagoras Theorem, we get

$BD = \sqrt{(AD)^2 - (AB)^2} = \left(\sqrt{(17)^2 - (8)^2}\right) cm = 15\ cm$

Area $(\Delta ABD) = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 15 \times 8 = 60\ cm^2$

For ΔBCD, let $a = 6$ cm, $b = 11$ cm and $c = 15$ cm

Semi-perimeter, $s = \frac{a+b+c}{2} = \left(\frac{6+11+15}{2}\right) cm = 16\ cm$

Area $(\Delta BCD) = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-6)(16-11)(16-15)}\ cm^2$

$= \sqrt{16 \times 10 \times 5 \times 1}\ cm^2$

$= 20\sqrt{2}\ cm^2$

Area of quadrilateral ABCD $= \left(60 + 20\sqrt{2}\right) cm^2 = 20\left(3 + \sqrt{2}\right)\ cm^2$