Linear Equations in Two Variables

Linear equation in two variables:
An equation of the form, $ax + by + c = 0$, where a, b and c are constants, such that a and b are both not zero and x and y are variables is called a linear equation in two variables.
For example, $2x + 3y + 10 = 0$, $3x + 7y = 0$

- Real life situations can be expressed mathematically as linear equations in two variables.

Example: The age of Ram is 3 more than twice the age of Mohan. Write a linear equation in two variables to represent this statement.

Solution: Let the age of Mohan be x years and the age of Ram be y years.
Thus, the given condition can be expressed as $y = 2x + 3$

Solution of a linear equation in two variables:

The values of the variables in a linear equation, which satisfy the equation are the solutions of that linear equation.

- A linear equation in two variables has infinitely many solutions.
- Solution of linear equation in two variables can be found by substitution method.

Example: Find two different solutions of the equation $4x + 5y = 20$.

Solution: Given equation is $4x + 5y = 20$.
If we take $x = 0$, we obtain:
$4 \times 0 + 5y = 20$
$\Rightarrow 5y = 20$
$\Rightarrow y = 4$
So, (0, 4) is a solution of the given equation.

If we take $y = 0$, we obtain:
$4x + 5 \times 0 = 20$
$\Rightarrow 4x = 20$
$\Rightarrow x = 5$
So, (5, 0) is a solution of the given equation.

- The geometrical representation of the linear equation, $ax + by + c = 0$, is a straight line.
- In order to represent a linear equation in two variables graphically, its two or three different points are calculated and then the corresponding points are plotted and joined on

the coordinate plane.

Example: Represent $x + 3y = 6$ on a graph paper.

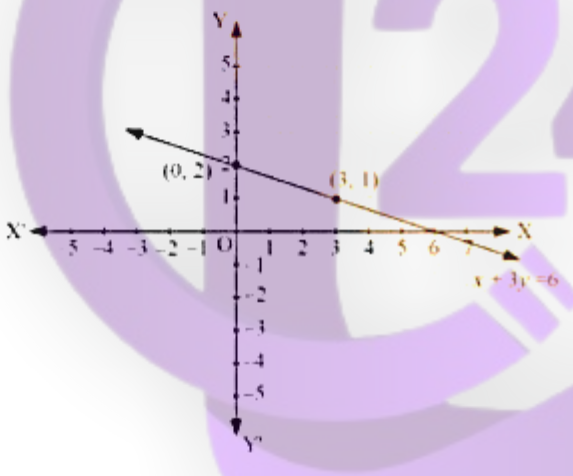
Solution: The given equation is $x + 3y = 6$

For $x = 0$, $3y = 6 \Rightarrow y = \frac{6}{3} = 2$

For $x = 3$, $3 + 3y = 6 \Rightarrow 3y = 3 \Rightarrow y = 1$

x	0	3
y	2	1

By plotting (0, 2) and (3, 1) on coordinate plane and then joining them, the given equation can be represented as:



- An equation of the form, $y = mx$, represents a line passing through the origin.
- **Graphical solution of linear equation in two variables:**

Every point on the graph of a linear equation in two variables is a solution of the linear equation and moreover, every solution of the linear equation is a point on the graph of the linear equation.

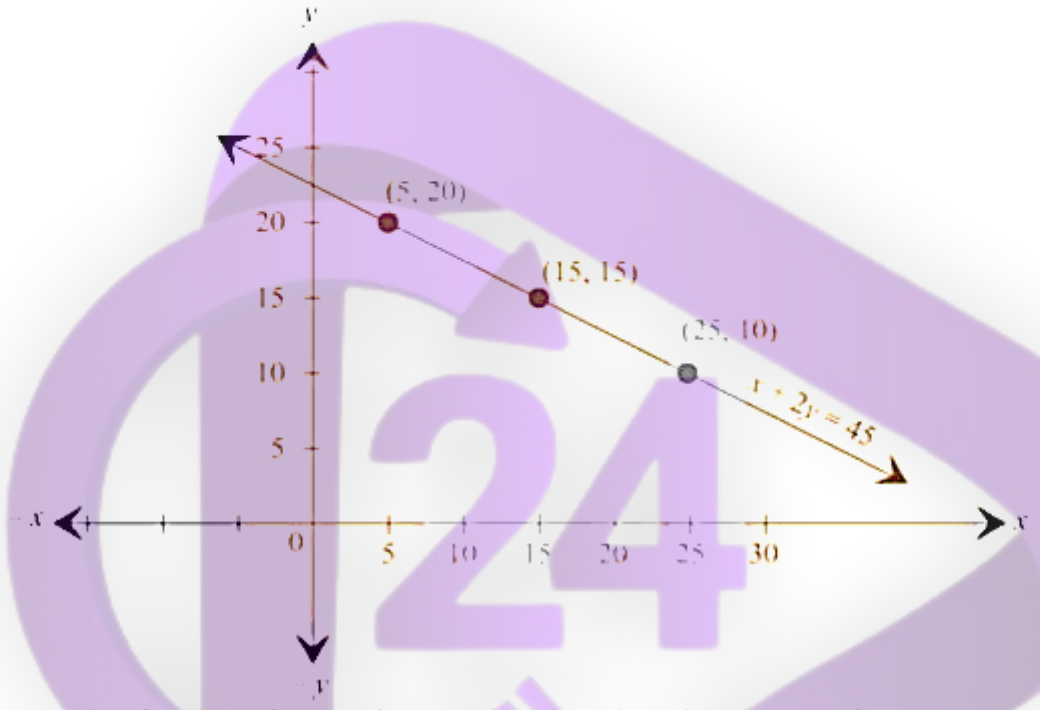
Example: A bag contains some Re 1 coins and some Rs 2 coins. The total worth of coins is Rs 45. Find the number of Re 1 coins, if there are 10 coins of Rs 2.

Solution: Let there be x coins of Re 1 and y coins of Rs 2.
Thus, $1x + 2y = 45$
$\Rightarrow x + 2y = 45$
This is the required linear equation of the given information. The three solutions of this equation have been given in the tabular form as follows:

x	5	15	25

y	20	15	10

By plotting the points (5, 20), (15, 15) and (25, 10), we obtain the following graph.



From the above graph, it can be seen that the value of x corresponding to $y = 10$ is 25. Therefore, there are 25 coins of Re 1, if there are 10 coins of Rs 2.

- The graph of $x = a$ is a straight line parallel to the y-axis, situated at a distance of a units from y-axis.
- The graph of $y = b$ is a straight line parallel to the x-axis, situated at a distance of b units from x-axis.

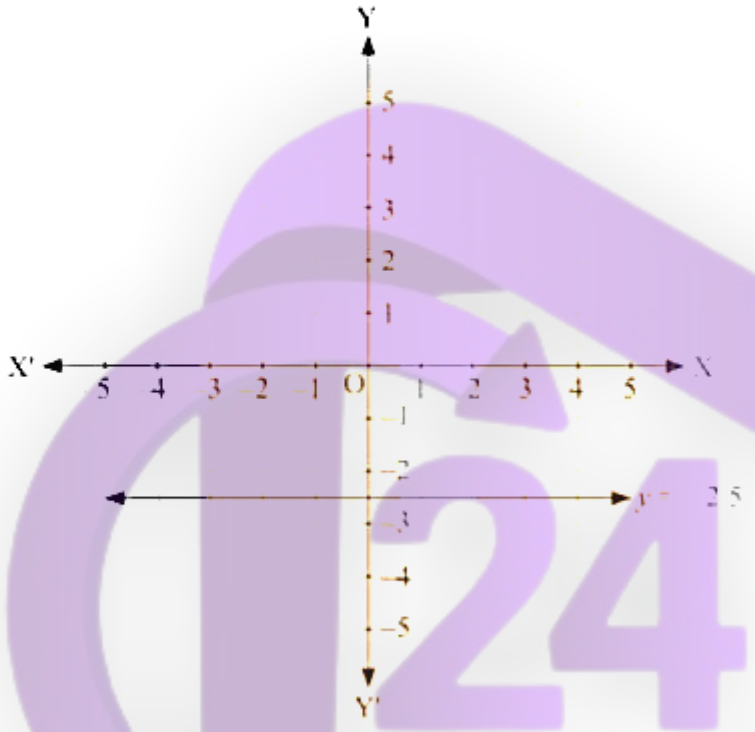
Example: Represent the equation $2y + 5 = 0$, on Cartesian plane.

Solution: $2y + 5 = 0$

$\Rightarrow 2y = -5$

$\Rightarrow y = \frac{-5}{2} = -2.5$, which is of the form $y = b$.

The graph of this equation can be drawn as follows:


Y
5
4
3
2
1
X'
X
−5 −4 −3 −2 −1 O 1 2 3 4 5
−1
−2
−3
−4
−5
Y'