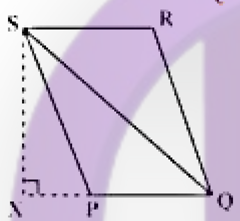
Quadrilaterals

- Diagonal of a parallelogram divides it into two congruent triangles.
  In the given figure, if ABCD is a parallelogram and AC is its diagonal then $\Delta ABC \cong \Delta CDA$.

Example: The area of the parallelogram PQRS is 120 cm^2. Find the distance between the parallel sides PQ and SR, if the length of the side PQ is 10 cm.

Solution: Let us draw a diagonal SQ of parallelogram PQRS and a perpendicular SX on the extended line PQ as shown in the figure.



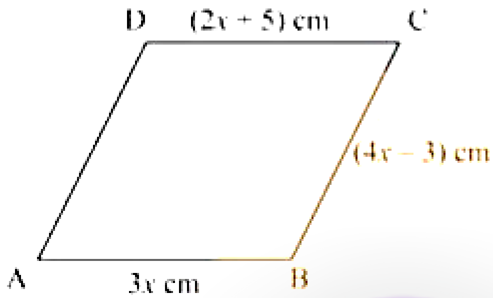
We know that a diagonal of a parallelogram divides it into two congruent triangles. Also, congruent figures are equal in area.

∴ area (ΔPQS) = area (ΔQRS)
Area of parallelogram PQRS = area (ΔPQS) + area (ΔQRS)
= 2 × area (ΔPQS)
$\Rightarrow$area (ΔPQS) = $\frac{1}{2}$(area of parallelogram PQRS) = $\frac{120}{2}$ cm^2 = 60 cm^2
Also, area (ΔPQS) = $\frac{1}{2}$ (PQ)(SX) = 60 cm^2
$\Rightarrow$ (PQ) (SX) = 120 cm^2
$\Rightarrow$ SX = $\frac{120}{10}$ cm^2
$\Rightarrow$ SX = 12 cm

Thus, the distance between the parallel sides PQ and SR is 12 cm.

- Opposite sides in a parallelogram are equal. Conversely, in a quadrilateral, if each pair of opposite sides are equal then the quadrilateral is a parallelogram.

Example: In the following figure, ABCD is a parallelogram. Find the length of each sides.



Solution: We know, the opposite sides of a parallelogram are equal in length.

Therefore, AB = CD

$3x = 2x + 5$

$\Rightarrow 3x - 2x = 5$

$\therefore x = 5$

Thus, AB = $3x = 3 \times 5 = 15$ cm

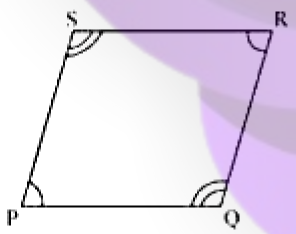
BC = $4x - 3 = 4 \times 5 - 3 = 17$ cm

CD = $2x + 5 = 2 \times 5 + 5 = 15$ cm

Also, BC = AD [opposite sides of parallelogram]

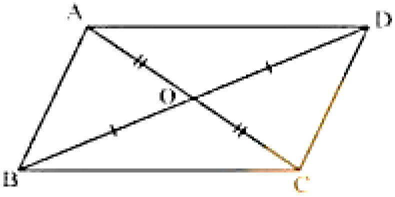
$\therefore$ AD = 17 cm

- In a parallelogram, opposite angles are equal. Conversely in a quadrilateral, if pair of opposite angles is equal, then the quadrilateral is a parallelogram.



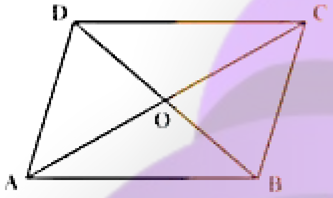
If in the quadrilateral PQRS, $\angle P = \angle R$ and $\angle Q = \angle S$ as shown in the above figure, then the quadrilateral is a parallelogram.

- The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect each other, then it is a parallelogram.
  Suppose ABCD is a quadrilateral. The diagonals of the quadrilateral intersect at O such that AO = OC and DO = OB



Therefore, ABCD is a parallelogram.

Example: In the given figure, ABCD is a parallelogram. If OD = $(3x - 2)$ cm and OB = $(2x + 3)$ cm, then find x and length of diagonal BD.



Solution: We know that the diagonals of a parallelogram bisect each other.

$\therefore$ OD = OB

$\Rightarrow 3x - 2 = 2x + 3$

$\Rightarrow 3x - 2x = 3 + 2$

$\Rightarrow x = 5$

Thus, the value of x is 5.

Length of BD = OD + OB

$= (3x - 2) + (2x + 3)$

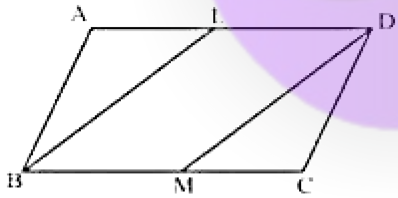
$= (3\times5 - 2) + (2\times5 + 3)$

$= 13 + 13$

= 26 cm

- A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.

Example: In the given figure, ABCD is a parallelogram and L and M are the mid-points of AD and BC respectively. Prove that BMDL is a parallelogram.



Solution: As L and M are the mid-points of AD and BC respectively.

Therefore, $BM = \frac{1}{2}BC \text{ and } LD = \frac{1}{2}AD$ ... (1)

As BC = AD (Since ABCD is a parallelogram)

$\Rightarrow \frac{1}{2}BC = \frac{1}{2}AD$

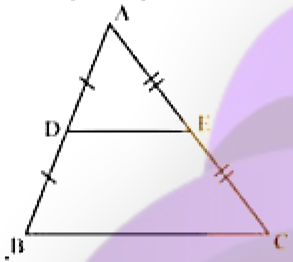
$\Rightarrow$ BM = LD ... (2) (From (1))

Also, BC || AD

$\Rightarrow$ BM || LD

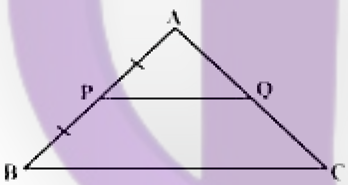
Hence, BMDL is a parallelogram.

- **Mid-point theorem** states that the line segment joining the mid-point of any two sides of a triangle is parallel to the third side and is half of it.



In ΔABC, if D and E are the mid-points of sides AB and AC respectively then by mid-point theorem DE ∥ BC and DE = $\frac{BC}{2}$

Converse of the mid-point theorem is also true, which states that a line through the mid-point of one side of a triangle and parallel to the other side bisects the third side.



In ΔABC, if AP = PB and PQ ∥ BC then PQ bisects AC i.e., Q is the mid-point of AC.