CLASS24

Chapter : 10. QUADRATIC EQUATIONS

Exercise : 10A

Question: 1 A

Solution:

The given equation $x^2 - x + 3 = 0$ is a quadratic equation.

Explanation - It is of degree 2, it is in the form $ax^2 + bx + c = 0$ (a ≠ 0, a, b, c are real numbers) where a = 1, b = - 1, c = 3.

Question: 1 B

Solution:

The given equation $2x^2 + \frac{5}{2}x - \sqrt{3} = 0$ equation is a quadratic equation.

Explanation - It is of degree 2, it is in the form $ax^2 + bx + c = 0$ (a ≠ 0, a, b, c are real numbers) where a = 2, b = $\frac{5}{2}$, c = $-\sqrt{3}$

Question: 1 C

Solution:

The given equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ is a quadratic equation.

Explanation - It is of degree 2, it is in the form $ax^2 + bx + c = 0$ (a ≠ 0, a, b, c are real numbers) where a = √2, b = 7, c = 5√2.

Question: 1 D

Solution:

The given equation $\frac{1}{3}x^2 + \frac{1}{5}x - 2 = 0$ is a quadratic equation.

Explanation - It is of degree 2, it is in the form $ax^2 + bx + c = 0$ (a ≠ 0, a, b, c are real numbers) where a = 1/3 , b = 1/5, c = - 2.

Question: 1 E

Solution:

The given equation $x^2 - 3x - \sqrt{x} + 4 = 0$ is not a quadratic equation.

Explanation - It is not in the form of $ax^2 + bx + c = 0$ because it has an extra term - √x with power 1/2

Question: 1 F

Solution:

The given equation $x - \frac{6}{x} = 3$ is a quadratic equation.

Explanation - Given $x - \frac{6}{x} = 3$

On solving the equation it gets reduced to $x^2 - 6 = 3x; x^2 - 3x - 6 = 0$; It is of degree 2 and it is in the form $ax^2 + bx + c = 0$ (a ≠ 0, a, b, c are real numbers) where a = 1, b = - 3, c = - 6.

Question: 1 G

Solution:

The given equation $x + \frac{2}{x} = x^2$ is not a quadratic equation.

Explanation - Given $x + \frac{2}{x} = x^2$

On getting reduced it becomes$x^2 + 2 = x^3$, it has degree = 3, it is not in the form

$ax^2 + bx + c = 0$ (a ≠ 0, a, b, c are real numbers).

Question: 1 H

Solution:

The given equation $x^2 - \frac{1}{x^2} = 5$ is not a quadratic equation.

Explanation - Given $x^2 - \frac{1}{x^2} = 5$

On getting reduced it becomes $x^4 - 1 = 5x^2$; $x^4 - 5x^2 - 1 = 0$

It is not in the form $ax^2 + bx + c = 0$ (a ≠ 0, a, b, c are real numbers)

Question: 1 I

Solution:

The given equation $(x + 2)^3 = x^3 - 8$ is a quadratic equation.

Explanation Given $(x + 2)^3 = x^3 - 8$

On getting reduced it becomes $x^3 + 8 + 6x^2 + 12x = x^3 - 8$

$= 6x^2 + 12x + 16 = 0$

Now, using $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

where a = 6, b = 12, c = 16

It is in the form $ax^2 + bx + c = 0$ (a ≠ 0, a, b, c are real numbers)

Question: 1 J

Solution:

The given (2x + 3)(3x + 2) = 6(x - 1)(x - 2)equation is not a quadratic equation.

Explanation - Given (2x + 3)(3x + 2) = 6(x - 1)(x - 2)

On getting reduced it becomes $6x^2 + 4x + 9x + 6 = 6(x^2 - 2x - x + 2)$

$6x^2 + 13x + 6 = 6x^2 - 18x + 12$

$31x - 6 = 0$

It is not in the form $ax^2 + bx + c = 0$ ($a \neq 0$, a, b, c are real numbers)

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Question: 1 K

Solution:

The given equation $\left(x + \frac{1}{x}\right)^2 = 2\left(x + \frac{1}{x}\right) + 3$ is not a quadratic equation.

Explanation - Given $\left(x + \frac{1}{x}\right)^2 = 2\left(x + \frac{1}{x}\right) + 3$

On getting reduced it becomes - $\left(\frac{x^2 + 1}{x}\right)^2 = 2\left(\frac{x^2 + 1}{x}\right)^2 + 3$

$(x^2 + 1)^2 = 2x(x^2 + 1) + 3x^2$

$x^4 + 2x^2 + 1 = 2x^3 + 2x + 3x^2$

$x^4 - 2x^3 - x^2 - 2x + 1 = 0$

It is not in the form $ax^2 + bx + c = 0$ ($a \neq 0$, a, b, c are real numbers)

Question: 2

Solution:

(i) - 1 is the root of given equation.

Explanation - Substituting value - 1 in LHS

$= 3(-1)^2 + 2(-1) - 1$

= 3 - 2 - 1

= 3 - 3 = 0 = RHS

Value satisfies the equation or LHS = RHS.

(ii) $\frac{1}{3}$ is the root of the given euation $3x^2 + 2x - 1 = 0$

Explanation - Substituting value in LHS

$= 3\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\right) - 1$

$= \frac{1}{3} + \frac{2}{3} - 1$

= 1 - 1 = 0 = RHS

Value satisfies the equation or LHS = RHS.

(iii) $\frac{-1}{2}$ is not the root of given equation $3x^2 + 2x - 1 = 0$

Explanation - Substituting value in LHS

$= 3\left(\frac{-1}{2}\right)^2 + 2\left(\frac{-1}{2}\right) - 1 = 0$

$= \frac{3}{4} - 2$

$= \frac{-5}{4} \neq 0 \neq$ RHS

Value does not satisfy the equation or LHS ≠ RHS.

Question: 3
Solution:

Given x = 1 is a root of the equation $x^2 + kx + 3 = 0$ it means it satisfies the equation.

Substituting x = 1 in equation -

$1^2 + k(1) + 3 = 0$

k + k 4 = 0

Putting the value of k in the given equation : $x^2 + kx + 3 = 0$

This reduced to the quadratic equation $x^2 - 4x + 3 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation $a = 1$ $b = -4$ $c = 3$

= 1.3

= 3

And either of their sum or difference = b

= - 4

Thus the two terms are - 1 and - 3

Sum = - 1 - 3 = - 4

Product = - 1. - 3 = 3

$x^2 - 4x + 3 = 0$

$x^2 - x - 3x + 3 = 0$

x(x-1)-3(x-1) = 0

(x-1)(x-3) = 0

x = 1 or x = 3

Thus other root is 3.

Question: 4

Solution:

Given x = 3/4 or x = - 2 are the roots of the equation $ax^2 + bx - 6 = 0$

Putting $x = \frac{3}{4}$ in the equation gives -

$a\left(\frac{3}{4}\right)^2 + b\left(\frac{3}{4}\right) - 6 = 0$

$\frac{9a + 12b - 96}{16} = 0$;

9a + 12b-96 = 0

3a + 4b-32 = 0 -----------------------(1)

putting x = - 2 in equation gives

$a(-2)^2 + b(-2) - 6 = 0$

4a-2b-6 = 0

2a-b-3 = 0

2a-3 = b ------------------ (2)

Substituting (2) in (1)

3a + 4(2a-3)-32 = 0

⇒ 11a-44 = 0

⇒ a = 4

⇒ b = 2(4)-3 = 5

Thus for a = 4 or b = 5; $x = \frac{3}{4}$ or x = - 2 are the roots of the equation $ax^2 + bx - 6 = 0$

Question: 5

Solution:

(2x-3) (3x + 1) = 0

$6x^2 + 2x - 9x - 3 = 0$

2x(3x + 1)-3(3x + 1) = 0 taking common from first two terms and last two terms

(2x-3)(3x + 1) = 0

(2x-3) = 0 or (3x + 1) = 0

x = 3/2 or x = (-1)/3

Roots of equation are 3/2, (-1)/3

Question: 6

Solution:

$4x^2 + 5x = 0$

x(4x + 5) = 0 (On taking x common)

x = 0 or (4x + 5) = 0

x = (-5)/4

Roots of equation are 0, (-5)/4

Question: 7

Solution:

$3x^2 - 243 = 0$

$3x^2 = 243$

$x^2 = 81$

$x = \sqrt{81}$

$x = \pm 9$

Roots of equation are 9, - 9

Question: 8

Solution:

$2x^2 + x - 6 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + b$: divided in two such values that:

Product = a.c

For the given equation a = 2; b = 1; c = -6

= 2. - 6

= - 12

And either of their sum or difference = b

= 1

Thus the two terms are 4 and - 3

Difference = 4 - 3 = 1

Product = 4. - 3 = - 12

$2x^2 + x - 6 = 0$

$2x^2 + 4x - 3x - 6 = 0$

2x(x + 2)-3(x + 2) = 0

(2x-3)(x + 2) = 0

(2x-3) = 0 or (x + 2) = 0

x = 3/2, x = -2

Roots of equation are 3/2, - 2

Question: 9

Solution:

$x^2 + 6x + 5 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1, b = 6, c = 5

= 1.5 = 5

And either of their sum or difference = b

= 6

Thus the two terms are 1 and 5

Sum = 5 + 1 = 6

Product = 5.1 = 5

$x^2 + 6x + 5 = 0$

$x^2 + x + 5x + 5 = 0$

x(x + 1) + 5(x + 1) = 0

(x + 1)(x + 5) = 0

(x + 1) = 0 or (x + 5) = 0

x = -1, x = -5

Roots of equation are - 1, - 5

Question: 10

Solution:

$9x^2 - 3x - 2 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 9; b = -3; c = -2

= 9. - 2 = - 18

And either of their sum or difference = b

= - 3

Thus the two terms are - 6 and 3

Sum = - 6 + 3 = - 3

Product = - 6.3 = - 18

$9x^2 - 3x - 2 = 0$

$9x^2 - 6x + 3x - 2 = 0$

3x(3x-2) + 1(3x-2) = 0

(3x + 1)(3x-2) = 0

(3x + 1) = 0 or (3x-2) = 0

x = (-1)/3 or x = 2/3

Roots of equation are (-1)/3, 2/3

Question: 11

Solution:

$x^2 + 12x + 35 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1; b = 12; c = 35

= 1.35 = 35

And either of their sum or difference = b

= 12

Thus the two terms are 7 and 5

Sum = 7 + 5 = 12

Product = 7.5 = 35

$x^2 + 12x + 35 = 0$

$x^2 + 7x + 5x + 35 = 0$

x(x + 7) + 5(x + 7) = 0

$(x + 5)(x + 7) = 0$

$(x + 5) = 0$ or $(x + 7) = 0$

$x = -5$ or $x = -7$

Roots of equation are - 5, - 7

Question: 12

Solution:

$x^2 = 18x - 77$

$x^2 - 18x + 77 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1; b = -18; c = 77

= 1.77 = 77

And either of their sum or difference = b

= - 18

Thus the two terms are - 7 and - 11

Sum = - 7 - 11 = - 18

Product = - 7. - 11 = 77

$x^2 - 18x + 77 = 0$

$x^2 - 7x - 11x + 77 = 0$

$x(x-7)-11(x-7) = 0$

$(x-7)(x-11) = 0$

$(x-7) = 0$ or $(x-11) = 0$

$x = 7$ or $x = 11$

Roots of equation are 7, 11

Question: 13

Solution:

$6x^2 + 11x + 3 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 6; b = 11; c = 3

= 6.3 = 18

And either of their sum or difference = b

= 11

Thus the two terms are 9 and 2

Sum = 9 + 2 = 11

Product = 9.2 = 18

$6x^2 + 11x + 3 = 0$

$6x^2 + 9x + 2x + 3 = 0$

3x(2x + 3) + 1(2x + 3) = 0

(3x + 1)(2x + 3) = 0

(3x + 1) = 0 or (2x + 3) = 0

x = (-1)/3 or x = (-3)/2

Roots of equation are $\frac{-1}{3}, \frac{-3}{2}$

Question: 14

Solution:

$6x^2 + x - 12 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 6; b = 1; c = -12

= 6. - 12 = - 72

And either of their sum or difference = b

= 1

Thus the two terms are 9 and - 8

Difference = 9 - 8 = 1

Product = 9. - 8 = - 72

$6x^2 + x - 12 = 0$

$6x^2 + 9x - 8x - 12 = 0$

3x(2x + 3)-4(2x + 3) = 0

(2x + 3)(3x-4) = 0

(2x + 3) = 0 or (3x-4) = 0

x = (-3)/2 or x = 4/3

Roots of equation are $\frac{-3}{2}, \frac{4}{3}$

Question: 15

Solution:

$3x^2 - 2x - 1 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 3; b = -2; c = -1

= 3. - 1 = - 3

And either of their sum or difference = b

= - 2

Thus the two terms are - 3 and 1

Difference = - 3 + 1 = - 2

Product = - 3.1 = - 3

$3x^2 - 2x - 1 = 0$

$3x^2 - 3x + x - 1 = 0$

3x(x-1) + 1(x-1) = 0

(x-1)(3x + 1) = 0

(x-1) = 0 or (3x + 1) = 0

x = 1 or x = (-1)/3

Roots of equation are 1, (-1)/3

Question: 16

Solution:

$4x^2 - 9x = 100$

$4x^2 - 9x - 100 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 4; b = -9 ; c = -100

= 4. - 100 = - 400

And either of their sum or difference = b

= - 9

Thus the two terms are - 25 and 16

Difference = - 25 + 16 = - 9

Product = - 25.16 = - 400

$4x^2 - 9x - 100 = 0$

$4x^2 - 25x + 16x - 100 = 0$

x(4x-25) + 4(4x-25) = 0

(4x-25)(x + 4) = 0

(4x-25) = 0 or (x + 4) = 0

x = 25/4 or x = -4

Roots of equation are 25/4, - 4

Question: 17

Solution:

$15x^2 - 28 = x$

$15x^2 - x - 28 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + b$:
divided in two such values that:

Product = a.c

For the given equation a = 15 ; b = -1; c = -28

= 15. - 28 = - 420

And either of their sum or difference = b

= - 1

Thus the two terms are - 21 and 20

Difference = - 21 + 20 = - 1

Product = - 21.20 = - 420

$15x^2 - x - 28 = 0$

$15x^2 - 21x + 20x - 28 = 0$

3x(5x-7) + 4(5x-7) = 0

(5x-7)(3x + 4) = 0

(5x-7) = 0 or (3x + 4) = 0

x = 7/5 or x = (-4)/3

Roots of equation are 7/5, - 4/3

Question: 18

Solution:

$4 - 11x = 3x^2$

$3x^2 + 11x - 4 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 3; b = 11 ;c = -4

= 3. - 4 = - 12

And either of their sum or difference = b

= 11

Thus the two terms are 12 and - 1

Difference = 12 - 1 = 11

Product = 12. - 1 = - 12

$3x^2 + 11x - 4 = 0$

$3x^2 + 12x - 1x - 4 = 0$

3x(x + 4)-1(x + 4) = 0

(x + 4)(3x-1) = 0

(x + 4) = 0 or (3x-1) = 0

x = -4 or x = 1/3

Roots of equation are - 4, 1/3

Question: 19

Solution:

$48x^2 - 13x - 1 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 48; b = -13 ;c = -1

= 48× - 1 = - 48

And either of their sum or difference = b

= - 13

Thus the two terms are - 16 and 3

Difference = - 16 + 3 = - 13

Product = - 16.3 = - 48

$48x^2 - 13x - 1 = 0$

$48x^2 - 16x + 3x - 1 = 0$

16x(3x-1) + 1(3x-1) = 0

(16x + 1)(3x-1) = 0

(16x + 1) = 0 or (3x-1) = 0

x = (-1)/6 or x = 1/3

Roots of equation are $\frac{-1}{6}$ or $\frac{1}{3}$

Question: 20

Solution:

$x^2 + 2\sqrt{2}x - 6 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1; b = 2√2; c = -6

= 1. - 6 = - 6

And either of their sum or difference = b

= 2√2

Thus the two terms are 3√2 and -√2

Difference = 3√2-√2 = 2√2

Product = 3√2.-√2 = 3.-2 = -6

$x^2 + 2\sqrt{2}x - 6 = 0$

$x^2 + 3\sqrt{2}x - \sqrt{2}x - 3\sqrt{2}\sqrt{2} = 0$ using $2 = \sqrt{2}\sqrt{2}$

$x(x + 3\sqrt{2})-\sqrt{2}(x + 3\sqrt{2}) = 0$

$(x-\sqrt{2})(x + 3\sqrt{2}) = 0$

$(x-\sqrt{2}) = 0$ or $(x + 3\sqrt{2}) = 0$

$x = \sqrt{2}$ or $x = -3\sqrt{2}$

Roots of equation are $\sqrt{2}$ or $-3\sqrt{2}$

Question: 21

Solution:

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation $a = \sqrt{3}$;$b = 10$; $c = 7\sqrt{3}$

$= \sqrt{3}.7\sqrt{3} = 21$

(using $3 = \sqrt{3}\times\sqrt{3}$)

And either of their sum or difference = b

= 10

Thus, the two terms are 7 and 3

Sum = 7 + 3 = 10

Product = 7.3 = 21

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\sqrt{3}x^2 + 7x + 3x + 7\sqrt{3} = 0$ (using $3 = \sqrt{3}.\sqrt{3}$)

$x(\sqrt{3}x + 7) + \sqrt{3}(\sqrt{3}x + 7) = 0$

$(x + \sqrt{3})(\sqrt{3}x + 7) = 0$

$(x + \sqrt{3}) = 0$ or $(\sqrt{3}x + 7) = 0$

$x = -\sqrt{3}$ or $x = \frac{-7}{\sqrt{3}}$

Roots of equation are $-\sqrt{3}$ or $\frac{-7}{\sqrt{3}}$

Question: 22

Solution:

$\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation $a = \sqrt{(3;)}$ $b = 11$; $c = 6\sqrt{3}$

$= \sqrt{3}.6\sqrt{3} = 3.6 = 18$

(using $3 = \sqrt{3}.\sqrt{3}$

And either of their sum or difference = b

= 11

Thus the two terms are 9 and 2

Sum = 9 + 2 = 11

Product = 9.2 = 18

$\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$

$\sqrt{3}x^2 + 9x + 2x + 6\sqrt{3} = 0$

√3 x(x + 3√3) + 2(x + 3√3) = 0

(using 9 = 3.3 = 3√3 √3)

(√3 x + 2)(x + 3√3) = 0

(√3 x + 2)(x + 3√3) = 0

$x = -3\sqrt{3}$ or $x = \frac{-2}{\sqrt{3}}$

Roots of equation are $-3\sqrt{3}$ or $\frac{-2}{\sqrt{3}}$

Question: 23

Solution:

$3\sqrt{7}x^2 + 4x - \sqrt{7} = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 3√7 ;b = 4 ;c = -√7

= 3√7.-√7 = 3.-7 = -21

(using 7 = √7.√7)

And either of their sum or difference = b

= 4

Thus the two terms are 7 and - 3

Difference = 7 - 3 = 4

Product = 7× - 3 = - 21

$3\sqrt{7}x^2 + 4x - \sqrt{7} = 0$

$3\sqrt{7}x^2 + 7x - 3x - \sqrt{7} = 0$

(using 7 = √7.√7)

$\sqrt{7}x(3x + \sqrt{7}) - 1(3x + \sqrt{7}) = 0$

(√7 x-1)(3x + √7) = 0

(√7 x-1) = 0 or (3x + √7) = 0

x = 1/√7 or x = (-7)/√3

Roots of equation are $x = \frac{1}{\sqrt{7}}$ or $x = \frac{-7}{\sqrt{3}}$

Question: 24

Solve each of the

Solution:

$\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation $a = \sqrt{7};\ b = -6\ ;c = -13\sqrt{7}$

= √7.-13√7 = -13.7 = -91

And either of their sum or difference = b

= - 6

Thus the two terms are 7 and - 13

Difference = - 13 + 7 = - 6

Product = 7. - 13 = - 91

$\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0$

$\sqrt{7}x^2 - 13x + 7x - 13\sqrt{7} = 0$

$\sqrt{7}x^2 - 13x + 7x - 13\sqrt{7} = 0$

x(√7 x-13) + √7 (√7 x-13) = 0

(x + √7)(√7 x-13) = 0

(x + √7) = 0 or (√7 x-13) = 0

x = -√7 or x = 13/√7

Roots of equation are $-\sqrt{7}$ or $\frac{13}{\sqrt{7}}$

Question: 25

Solution:

$4\sqrt{6}x^2 - 13x - 2\sqrt{6} = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 4√6 ;b = -13 ;c = -2√6

= 4√6.-2√6 = -48

And either of their sum or difference = b

= - 13

Thus the two terms are - 16 and 3

Difference = - 16 + 3 = - 13

Product = - 16.3 = - 48

$4\sqrt{6}x^2 - 13x - 2\sqrt{6} = 0$

$4\sqrt{6}x^2 - 16x + 3x - 2\sqrt{6} = 0$

$4\sqrt{2}x(\sqrt{3}x - 2\sqrt{2}) + \sqrt{3}(\sqrt{3}x - 2\sqrt{2}) = 0$

(On using √6 = √3 √2 and 16 = 4.2.√2 √2)

⇒ (4√2 x + √3)(√3 x-2√2) = 0

⇒ (4√2 x + √3) = 0 or (√3 x-2√2) = 0

x = (-√3)/(4√2) or x = (2√2)/√3

Roots of equation are $\frac{-\sqrt{3}}{4\sqrt{2}}$ or $\frac{2\sqrt{2}}{\sqrt{3}}$

Question: 26

Solution:

$3x^2 - 2\sqrt{6}x + 2 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 3; b = -2√(6;) c = 2

= 3.2 = 6

And either of their sum or difference = b

= -2√6

Thus the two terms are -√6 and -√6

Sum = -√6 -√6 = -2√6

Product = -√6. -√6 = -6 6 = √6. √6

$3x^2 - 2\sqrt{6}x + 2 = 0$

$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

(On using 3 = √3.√3 and √6 = √3 √2)

$\sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$

$x = \frac{\sqrt{2}}{\sqrt{3}}$ or $x = \frac{\sqrt{2}}{\sqrt{3}}$

Equation has repeated roots $\frac{\sqrt{2}}{\sqrt{3}}$

Question: 27

Solution:

$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = √3 b = -2√2 c = -2√3

= √3.-2√3 = -2.3 = -6

And either of their sum or difference = b

= -2√2

Thus the two terms are -3√2 and √2

Difference = -3√2 + √2 = -2√2

Product = -3√2 × √2 = -3.2 = -6

$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x + 2\sqrt{3} = 0$

(On using3√2 = √3 √3 √2 = √3.√6)

$\sqrt{3}x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6}) = 0$

(∵2√3 = √2 √2 √3 = √2.√6)

$(x - \sqrt{6})(\sqrt{3}x + \sqrt{2}) = 0$

$x = \sqrt{6} \text{ or } x = -\frac{\sqrt{2}}{\sqrt{3}}$

Roots of equation are $\sqrt{6}$ or $-\frac{\sqrt{2}}{\sqrt{3}}$

Question: 28

Solution:

$x^2 - 3\sqrt{5}x + 10 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 ;b = -3√5 ;c = 10

= 1.10 = 10

And either of their sum or difference = b

= -3√5

Thus the two terms are -2√5 and -√5

Sum = -2√5-√5 = -3√5

Product = -2√5. -√5 = 2.5 = 10 using 5 = √5. √5

$x^2 - 3\sqrt{5}x + 10 = 0$

$x^2 - 2\sqrt{5}x - \sqrt{5}x + 10 = 0$

(On using: 10 = 2.5 = 2.√5 √5)

x(x-2√5)-√5 (x-2√5) = 0

(x-√5)(x-2√5) = 0

(x-√5) = 0 or (x-2√5) = 0

x = √5 or x = 2√5

Hence the roots of equation are √5 or 2√5

Question: 29

Solution:

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$x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

$x^2 - \sqrt{3}x - x + \sqrt{3} = 0$

On taking x common from first two terms and - 1 from last two

$x(x-\sqrt{3})-1(x-\sqrt{3}) = 0$

$(x-\sqrt{3})(x-1) = 0$

$(x-\sqrt{3}) = 0$ or $(x-1) = 0$

$x = \sqrt{3}$ or $x = 1$

Roots of equation are $\sqrt{3}$ or 1

Question: 30

Solution:

$x^2 + 3\sqrt{3}x - 30 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation $a = 1$; $b = 3\sqrt{3}$;$c = -30$

$= 1. - 30 = - 30$

And either of their sum or difference = b

$= 3\sqrt{3}$

Thus, the two terms are $5\sqrt{3}$ and $-2\sqrt{3}$

Difference = $5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$

Product = $5\sqrt{3}.-2\sqrt{3} = -10.3 = -30$

$x^2 + 3\sqrt{3}x - 30 = 0$

$x^2 + 5\sqrt{3}x - 2\sqrt{3}x - 30 = 0$

$x(x + 5\sqrt{3}) - 2\sqrt{3}(x + 5\sqrt{3}) = 0\ 3 = \sqrt{3}\sqrt{3}$

$(x + 5\sqrt{3})(x - 2\sqrt{3}) = 0$

$(x + 5\sqrt{3}) = 0$ or $(x - 2\sqrt{3}) = 0$

$x = -5\sqrt{3}$ or $x = 2\sqrt{3}$

Hence the roots of equation are $-5\sqrt{3}$ or $2\sqrt{3}$

Question: 31

Solution:

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation $a = \sqrt{2}$; $b = 7$; $c = 5\sqrt{2}$

$= \sqrt{2}.5\sqrt{2} = 2.5 = 10$

And either of their sum or difference = b

= 7

Thus the two terms are 5 and 2

Sum = 5 + 2 = 7

Product = 5.2 = 10

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$(\sqrt{2}x + 5) = 0$ or $(x + \sqrt{2}) = 0$

$x = \frac{-5}{\sqrt{2}}$ or $x = -\sqrt{2}$

Hence the roots of equation are $\frac{-5}{\sqrt{2}}$ or $-\sqrt{2}$

Question: 32

Solution:

$5x^2 + 13x + 8 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 5; b = 13; c = 8

= 5.8 = 40

And either of their sum or difference = b

= 13

Thus the two terms are 5 and 8

Sum = 5 + 8 = 13

Product = 5.8 = 40

$5x^2 + 5x + 8x + 8 = 0$

$5x(x + 1) + 8(x + 1) = 0$

$(x + 1)(5x + 8) = 0$

$(x + 1) = 0$ or $(5x + 8) = 0$

$x = -1$ or $x = \frac{-8}{5}$

Hence the roots of equation are -1 or $\frac{-8}{5}$

Question: 33

Solve each of the

Solution:

$x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$

$x^2 - x - \sqrt{2}x + \sqrt{2} = 0$

On taking x common from first two terms and - 1 from last two

$x(x - 1) - \sqrt{2}(x - 1) = 0$

$(x - \sqrt{2})(x - 1) = 0$

$(x - \sqrt{2}) = 0$ or $(x - 1) = 0$

$x = -1$ or $x = \sqrt{2}$

Hence the roots of equation are -1 or $\sqrt{2}$

Question: 34

Solution:

$9x^2 + 6x + 1 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 9; b = 6; c = 1

= 9.1 = 9

And either of their sum or difference = b

= 6

Thus the two terms are 3 and 3

Sum = 3 + 3 = 6

Product = 3.3 = 9

$9x^2 + 6x + 1 = 0$

$9x^2 + 3x + 3x + 1 = 0$

$3x(3x + 1) + 1(3x + 1) = 0$

$(3x + 1)(3x + 1) = 0$

$(3x + 1) = 0$ or $(3x + 1) = 0$

$x = \frac{-1}{3}$ or $x = \frac{-1}{3}$

Hence the equation has repeated roots $x = \frac{-1}{3}$

Question: 35

Solution:

$100x^2 - 20x + 1 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 100 ;b = - 20 ;c = 1

= 100.1 = 100

And either of their sum or difference = b

= - 20

Thus the two terms are - 10 and - 10

Sum = - 10 - 10 = - 20

Product = - 10. - 10 = 100

$100x^2 - 20x + 1 = 0$

$100x^2 - 10x - 10x + 1 = 0$

10x(10x-1)-1(10x-1) = 0

(10x-1)(10x-1) = 0

(10x-1) = 0 or (10x-1) = 0

$x = \frac{1}{10}$ or $x = \frac{1}{10}$

Roots of equation are repeated $\frac{1}{10}$

Question: 36

Solution:

$2x^2 - x + \frac{1}{8} = 0$

$16x^2 - 8x + 1 = 0$ (taking LCM)

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 16; b = - 8 ;c = 1

= 16.1 = 16

And either of their sum or difference = b

= - 8

Thus the two terms are - 4 and - 4

Sum = - 4 - 4 = - 8

Product = - 4. - 4 = 16

$16x^2 - 8x + 1 = 0$

$16x^2 - 4x - 4x + 1 = 0$

4x(4x-1)-1(4x-1) = 0

(4x-1)(4x-1) = 0

(4x-1) = 0 or (4x-1) = 0

$x = \frac{1}{4}$ or $x = \frac{1}{4}$

The equation has repeated roots $\frac{1}{4}$

Question: 37

Solution:

taking LCM

$10x - \frac{1}{x} = 3$

$10x^2 - 1 - 3x = 0$

$10x^2 - 3x - 1 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 10 ;b = - 3; c = - 1

= 10. - 1 = - 10

And either of their sum or difference = b

= - 3

Thus the two terms are - 5 and 2

Difference = - 5 + 2 = - 3

Product = - 5.2 = - 10

$10x^2 - 3x - 1 = 0$

$10x^2 - 5x + 2x - 1 = 0$

5x(2x-1) + 1(2x-1) = 0

(5x + 1)(2x-1) = 0

(5x + 1) = 0 or (2x-1) = 0

$x = \frac{-1}{5}$ or $x = \frac{1}{2}$

Question: 38

Solution:

$\frac{2}{x^2} - \frac{5}{x} + 2 = 0$

$2 - 5x + 2x^2 = 0$

$2x^2 - 5x + 2 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 2; b = - 5; c = 2

= 2.2 = 4

And either of their sum or difference = b

= - 5

Thus the two terms are - 4 and - 1

Difference = - 4 - 1 = - 5

Product = - 4. - 1 = 4

$2x^2 - 5x + 2 = 0$

$2x^2 - 4x - x + 2 = 0$

2x(x-2)-1(x-2) = 0

(2x-1)(x-2) = 0

(2x-1) = 0 or (x-2) = 0

$x = 2 \text{ or } x = \frac{1}{2}$

Hence the roots of equation are 2 or $\frac{1}{2}$

Question: 39

Solution:

$2x^2 + ax - a^2 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 2; b = a; c = - a^2

= - 2.a^2 = - 2 a^2

And either of their sum or difference = b

= a

Thus the two terms are 2a and - a

Difference = 2a - a = a

Product = 2a. - a = - $2a^2$

$2x^2 + ax - a^2 = 0$

$2x^2 + 2ax - ax - a^2 = 0$

2x (x + a)-a (x + a) = 0

(2x-a) (x + a) = 0

(2x-a) = 0 or (x + a) = 0

$x = \frac{a}{2} \text{ or } x = -a$

Hence the roots of equation are $\frac{a}{2}$ or $-a$

Question: 40

Solution:

$4x^2 + 4bx - (a^2 - b^2) = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 4 b = 4b c = - $(a^2 - b^2)$

= 4. - $(a^2 - b^2)$

= - $4a^2 + 4b^2$

And either of their sum or difference = b

= 4b

Thus the two terms are 2(a + b) and - 2(a - b)

Difference = 2a + 2b - 2a + 2b = 4b

Product = 2(a + b). - 2(a - b) = - $4(a^2 - b^2)$

using $a^2 - b^2 = (a + b)(a - b)$

$4x^2 + 4bx - (a^2 - b^2) = 0$

$\Rightarrow 4x^2 + 2(a + b)x - 2(a - b) - (a + b)(a - b) = 0$

$\Rightarrow 2x[2x + (a + b)]-(a-b)[2x + (a + b)] = 0$

$\Rightarrow [2x + (a + b)][2x-(a-b)] = 0$

$\Rightarrow [2x + (a + b)] = 0$ or $[2x-(a-b)] = 0$

$x = \frac{-(a + b)}{2}$ or $x = \frac{a - b}{2}$

Hence the roots of equation are $\frac{-(a + b)}{2}$ or $\frac{a-b}{2}$

Question: 41

Solution:

$4x^2 - 4a^2x + (a^4 - b^4) = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 4 ;b = - $4a^2$; c = $(a^4 - b^4)$

= 4. $(a^4 - b^4)$

= $4a^4 - 4b^4$

And either of their sum or difference = b

= - $4a^2$

Thus the two terms are - $2(a^2 + b^2)$ and - $2(a^2 - b^2)$

Difference = - $2(a^2 + b^2) - 2(a^2 - b^2)$

= - $2a^2 - 2b^2 - 2a^2 + 2b^2$

= - $4a^2$

Product = - $2(a^2 + b^2)$. - $2(a^2 - b^2)$

= $4(a^2 + b^2)(a^2 - b^2)$

= 4. $(a^4 - b^4)$

($\because$ using $a^2 - b^2 = (a + b)(a - b)$)

$\Rightarrow 4x^2 - 4a^2x + (a^4 - b^4) = 0$

$\Rightarrow 4x^2 - 4a^2x + ((a^2)^2 - (b^2)^2) = 0$

($\because$ using $a^2 - b^2 = (a + b)(a - b)$)

$\Rightarrow 4x^2 - 2(a^2 + b^2)x - 2(a^2 - b^2)x + (a^2 + b^2)(a^2 - b^2) = 0$

$\Rightarrow 2x[2x - (a^2 + b^2)] - (a^2 - b^2)[2x - (a^2 + b^2)] = 0$

$\Rightarrow [2x - (a^2 + b^2)][2x - (a^2 - b^2)] = 0$

$\Rightarrow [2x - (a^2 + b^2)] = 0$ or $[2x - (a^2 - b^2)] = 0$

$x = \frac{a^2 + b^2}{2}$ or $x = \frac{a^2 - b^2}{2}$

Hence the roots of given equation are $\frac{a^2 + b^2}{2}$ or $\frac{a^2 - b^2}{2}$

Question: 42

Solution:

$x^2 + 5x - (a^2 + a - 6) = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1; b = 5; c = - $(a^2 + a - 6)$

= 1. - $(a^2 + a - 6)$

= - $(a^2 + a - 6)$

And either of their sum or difference = b

= 5

Thus the two terms are (a + 3) and - (a - 2)

Difference = a + 3 –a + 2

= 5

Product = (a + 3). - (a - 2)

= - [(a + 3)(a - 2)]

= - $(a^2 + a - 6)$

$x^2 + 5x - (a^2 + a - 6) = 0$

$\Rightarrow x^2 + (a + 3)x - (a - 2)x - (a + 3)(a - 2) = 0$

$\Rightarrow x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$

$\Rightarrow [x + (a + 3)][x - (a - 2)] = 0$

$\Rightarrow [x + (a + 3)] = 0$ or $[x - (a - 2)] = 0$

$\Rightarrow x = -(a + 3)$ or $x = (a - 2)$

Hence the roots of given equation are - (a + 3) or (a - 2)

Question: 43

Solution:

$x^2 - 2ax - (4b^2 - a^2) = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + b$: divided in two such values that:

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Product = a.c

For the given equation a = 1 b = - 2a c = - $(4b^2 - a^2)$

= 1. - $(4b^2 - a^2)$

= - $(4b^2 - a^2)$

And either of their sum or difference = b

= - 2a

Thus the two terms are (2b - a) and - (2b + a)

Difference = 2b - a - 2b - a

= - 2a

Product = (2b - a) - (2b + a)

(∵ using $a^2 - b^2 = (a + b)(a - b)$)

= - $(4b^2 - a^2)$

$x^2 - 2ax - (4b^2 - a^2) = 0$

$\Rightarrow x^2 + (2b - a)x - (2b + a)x - (2b - a)(2b + a) = 0$

$\Rightarrow x[x + (2b - a)] - (2b + a)[x + (2b - a)] = 0$

$\Rightarrow [x + (2b - a)] [x - (2b + a)] = 0$

$\Rightarrow [x + (2b - a)] = 0$ or $[x - (2b + a)] = 0$

$\Rightarrow x = (a - 2b)$ or $x = (a + 2b)$

Hence the roots of given equation are (a - 2b) or x = (a + 2b)

Question: 44

Solution:

$x^2 - (2b - 1)x + (b^2 - b - 20) = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1; b = - (2b - 1); c = $b^2 - b - 20$

= $1(b^2 - b - 20)$

= $(b^2 - b - 20)$

And either of their sum or difference = b

= - (2b - 1)

Thus the two terms are - (b - 5) and - (b + 4)

Sum = - (b - 5) - (b + 4)

= - b + 5 - b - 4

= - 2b + 1

= - (2b - 1)

Product = - (b - 5) - (b + 4)

$= (b - 5)(b + 4)$

$= b^2 - b - 20$

$x^2 - (2b - 1)x + (b^2 - b - 20) = 0$

$\Rightarrow x^2 - (b - 5)x - (b + 4)x + (b - 5)(b + 4) = 0$

$\Rightarrow x[x - (b - 5)] - (b + 4)[x - (b - 5)] = 0$

$\Rightarrow [x - (b - 5)][x - (b + 4)] = 0$

$\Rightarrow [x - (b - 5)] = 0$ or $[x - (b + 4)] = 0$

$\Rightarrow x = (b - 5)$ or $x = (b + 4)$

Hence the roots of equation are $(b - 5)$ or $(b + 4)$

Question: 45

Solution:

$x^2 + 6x - (a^2 + 2a - 8) = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation $a = 1$; $b = 6$; $c = -(a^2 + 2a - 8)$

$= 1. -(a^2 + 2a - 8)$

$= -(a^2 + 2a - 8)$

And either of their sum or difference = b

$= 6$

Thus the two terms are $(a + 4)$ and $-(a - 2)$

Difference $= a + 4 - a + 2$

$= 6$

Product $= (a + 4) - (a - 2)$

$= -(a^2 + 2a - 8)$

$\Rightarrow x^2 + 6x - (a^2 + 2a - 8) = 0$

$\Rightarrow x^2 + (a + 4)x - (a - 2)x - (a + 4)(a - 2) = 0$

$\Rightarrow x[x + (a + 4)] - (a - 2)[x + (a + 4)] = 0$

$\Rightarrow [x + (a + 4)][x - (a - 2)] = 0$

$\Rightarrow [x + (a + 4)] = 0$ or $[x - (a - 2)] = 0$

$x = -(a + 4)$ or $x = (a - 2)$

Hence the roots of equation are $-(a + 4)$ or $(a - 2)$

Question: 46

Solution:

$abx^2 + (b^2 - ac)x - bc = 0$

$abx^2 + (b^2 - ac)x - bc = 0$

$abx^2 + b^2x - acx - bc = 0$

$bx(ax + b) - c(ax + b) = 0$ taking bx common from first two terms and $-c$ from las

$(ax + b)(bx - c) = 0$

$(ax + b) = 0$ or $(bx - c) = 0$

$x = \frac{-b}{a}$ or $x = \frac{c}{a}$

Hence the roots of equation are $\frac{-b}{a}$ or $\frac{c}{a}$

Question: 47

Solution:

$x^2 - 4ax - b^2 + 4a^2 = 0$

$x^2 - 4ax - ((b)^2 - (2a)^2) = 0$

{using $a^2 - b^2 = (a + b)(a - b)$}

$x^2 - (b + 2a)x + (b - 2a)x - (b + 2a)(b - 2a) = 0$

$\Rightarrow x[x - (b + 2a)] + (b - 2a)[x - (b + 2a)] = 0$

$\Rightarrow [x - (b + 2a)][x + (b - 2a)] = 0$

$\Rightarrow [x - (b + 2a)] = 0$ or $[x + (b - 2a)] = 0$

$\Rightarrow x = (b + 2a)$ or $x = -(b - 2a)$

$\Rightarrow x = (2a + b)$ or $x = (2a - b)$

Hence the roots of equation are $(2a + b)$ or $(2a - b)$

Question: 48

Solution:

$4x^2 - 2a^2x - 2b^2x + a^2b^2 = 0$

$2x(2x - a^2) - b^2(2x - a^2) = 0$

(On taking 2x common from first two terms and $-b^2$ from last two)

$\Rightarrow (2x - a^2)(2x - b^2) = 0$

$\Rightarrow (2x - a^2) = 0$ or $(2x - b^2) = 0$

$\Rightarrow x = \frac{a^2}{2}$ or $x = \frac{b^2}{2}$

Hence the roots of equation are $\frac{a^2}{2}$ or $\frac{b^2}{2}$

Question: 49

Solution:

$12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

$12abx^2 - 9a^2x + 8b^2x - 6ab = 0$

$3ax(4bx - 3a) + 2b(4bx - 3a) = 0$ taking 3ax common from first two terms and 2b from last two

$(4bx - 3a)(3ax + 2b) = 0$

$(4bx - 3a) = 0$ or $(3ax + 2b) = 0$

$$x = \frac{3a}{4b} \text{ or } x = \frac{-2b}{3a}$$

Hence the roots of equation are $x = \frac{3a}{4b}$ or $x = \frac{-2b}{3a}$

Question: 50

Solution:

$a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$b^2x(a^2x + 1) - 1(a^2x + 1) = 0$ taking b^2x common from first two terms and - 1 from last two

$(a^2x + 1)(b^2x - 1) = 0$

$(a^2x + 1) = 0$ or $(b^2x - 1) = 0$

$$x = \frac{-1}{a^2} \text{ or } x = \frac{1}{b^2}$$

Hence the roots of equation are $\frac{-1}{a^2}$ or $\frac{1}{b^2}$

Question: 51

Solution:

$9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Using the splitting middle term - the middle term of the general equation $Ax^2 + Bx + C$ is divided in two such values that:

Product = AC

For the given equation $A = 9$, $B = -9(a + b)$, $C = 2a^2 + 5ab + 2b^2$

$= 9(2a^2 + 5ab + 2b^2) = 9(2a^2 + 4ab + ab + 2b^2) = 9[2a(a + 2b) + b(a + 2b)] = 9(a + 2b)(2a + b) = 3(a + 2b)3(2a + b)$

Also, $3(a + 2b) + 3(2a + b) = 9(a + b)$Therefore, $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

$9x^2 - 3(2a + b)x - 3(a + 2b)x + (a + 2b)(2a + b) = 0$

$3x[3x - (2a + b)] - (a + 2b)[3x - (2a + b)] = 0$

$[3x - (2a + b)][3x - (a + 2b)] = 0$

$[3x - (a + 2b)] = 0$ or $[3x - (2a + b)] = 0$

$$x = \frac{a + 2b}{3} \text{ or } x = \frac{2a + b}{3}$$

Hence the roots of equation are $\frac{a + 2b}{3}$ or $\frac{2a + b}{3}$

Question: 52

Solution:

$$\frac{16}{x} - 1 = \frac{15}{x + 1}$$

$$\frac{16}{x} - \frac{15}{x + 1} = 1$$

$\frac{16x + 16 - 15x}{x(x+1)} = 1$ taking LCM

$\frac{x + 16}{x^2 + x} = 1$

$x^2 + x = x + 16$ cross multiplying

$x^2 - 16 = 0$

$x^2 - (4)^2 = 0$ using $a^2 - b^2 = (a + b)(a - b)$

$(x + 4)(x - 4) = 0$

$(x + 4) = 0$ or $(x - 4) = 0$

$x = 4$ or $x = -4$

Hence the roots of equation are 4, - 4.

Question: 53

Solution:

$\frac{4}{x} - 3 = \frac{5}{2x + 3}$

$\frac{4}{x} - \frac{5}{2x + 3} = 3$

$\frac{8x + 12 - 5x}{x(2x + 3)} = 3$ taking LCM

$\frac{3x + 12}{2x^2 + 3x} = 3$

$\frac{3(x + 4)}{2x^2 + 3x} = 3$

$\frac{x + 4}{2x^2 + 3x} = 1$

$x + 4 = 2x^2 + 3x$ cross multiplying

$2x^2 + 2x - 4 = 0$ taking 2 common

$x^2 + x - 2 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 1 c = - 2

= 1. - 2 = - 2

And either of their sum or difference = b

= 1

Thus the two terms are 2 and - 1

Difference = 2 - 1 = 1

Product = 2. - 1 = - 2

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x + 2) - (x + 2) = 0$

(x + 2) (x - 1) = 0

(x + 2) = 0 or (x - 1) = 0

x = - 2 or x = 1

Hence the roots of equation are - 2 or 1.

Question: 54

Solution:

$\frac{3}{x+1} - \frac{2}{3x-1} = \frac{1}{2}, x \neq -1, \frac{1}{3}$

$\frac{3}{x + 1} - \frac{2}{3x - 1} = \frac{1}{2}$

$\frac{9x-3-2x-2}{(x+1)(3x-1)} = \frac{1}{2}$ taking LCM

$\frac{7x - 5}{3x^2 + 2x - 1} = \frac{1}{2}$

$3x^2 + 2x - 1 = 14x - 10$ cross multiplying

$3x^2 - 12x + 9 = 0$ taking 3 common

$x^2 - 4x + 3 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = - 4 c = 3

= 1.3 = 3

And either of their sum or difference = b

= - 4

Thus the two terms are - 3 and - 1

Sum = - 3 - 1 = - 4

Product = - 3. - 1 = 3

$x^2 - 4x + 3 = 0$

$x^2 - 3x - x + 3 = 0$

x(x - 3) - 1(x - 3) = 0

(x - 3) (x - 1) = 0

(x - 3) = 0 or (x - 1) = 0

x = 3 or x = 1

Hence the roots of equation are 3 or1.

Question: 55

Solution:

$\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$

$\frac{x+5-x+1}{(x-1)(x+5)} = \frac{6}{7}$ taking LCM

$$\frac{6}{(x-1)(x+5)} = \frac{6}{7}$$

$$\frac{6}{x^2+4x-5} = \frac{6}{7}$$

$x^2 + 4x - 5 = 7$ cross multiplying

$x^2 + 4x - 12 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 4 c = - 12

= 1. - 12 = - 12

And either of their sum or difference = b

= 4

Thus the two terms are 6 and - 2

Difference = 6 - 2 = 4

Product = 6. - 2 = - 12

$x^2 + 4x - 12 = 0$

$x^2 + 6x - 2x - 12 = 0$

x(x + 6) - 2(x + 6) = 0

(x + 6)(x - 2) = 0

(x + 6) = 0 or (x - 2) = 0

x = - 6 or x = 2

Hence the roots of equation are - 6 or 2.

Question: 56

Solution:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

$$\frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$$

$\frac{2x-2a-b-2x}{2x(2a+b+2x)} = \frac{1}{2a} + \frac{1}{b}$ taking LCM

$$\frac{-(2a+b)}{4x^2+4ax+2bx} = \frac{2a+b}{2ab}$$

$4x^2 + 4ax + 2bx = - 2ab$ cross multiplying

$4x^2 + 4ax + 2bx + 2ab = 0$

4x(x + a) + 2b(x + a) = 0 taking 4x common from first two terms and 2b from last two

(x + a) (4x + 2b) = 0

(x + a) = 0 or (4x + 2b) = 0

$x = -a$ or $x = \frac{-b}{2}$

Hence the roots of equation are $-a$ or $\frac{-b}{2}$

Question: 57

Solution:

$$\frac{x+3}{x-2} - \frac{1-x}{x} = 4\frac{1}{4}$$

$$\frac{x(x+3) - (1-x)(x-2)}{x(x-2)} = \frac{17}{4} \text{ taking LCM}$$

$$\frac{x^2 + 3x - (x - 2 - x^2 + 2x)}{x^2 - 2x} = \frac{17}{4}$$

$$\frac{x^2 + 3x + x^2 - 3x + 2}{x^2 - 2x} = \frac{17}{4}$$

$$\frac{2x^2 + 2}{x^2 - 2x} = \frac{17}{4}$$

$8x^2 + 8 = 17x^2 - 34x$ cross multiplying

$-9x^2 + 34x + 8 = 0$

$9x^2 - 34x - 8 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 9 b = - 34 c = - 8

= 9. - 8 = - 72

And either of their sum or difference = b

= - 34

Thus the two terms are - 36 and 2

Difference = - 36 + 2 = - 34

Product = - 36.2 = - 72

$9x^2 - 34x - 8 = 0$

$9x^2 - 36x + 2x - 8 = 0$

$9x(x - 4) + 2(x - 4) = 0$

$(9x + 2)(x - 4) = 0$

$$x = 4 \text{ or } x = \frac{-2}{9}$$

Hence the roots of equation are 4 or $\frac{-2}{9}$

Question: 58

Solution:

Given:

$$\frac{3x-4}{7} + \frac{7}{3x-4} = \frac{5}{2}, x \neq \frac{4}{3}$$

$$\frac{(3x-4)^2 + 49}{7(3x-4)} = \frac{5}{2} \text{ taking LCM}$$

$\frac{9x^2 - 24x + 16 + 49}{7(3x-4)} = \frac{5}{2}$ using $(a - b)^2 = a^2 + b^2 - 2ab$

$\frac{9x^2 - 24x + 65}{21x - 28} = \frac{5}{2}$ cross multiplying

$18x^2 - 48x + 130 = 105x - 140$

$18x^2 - 153x + 270 = 0$ taking 9 common

$2x^2 - 17x + 30 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 2 b = - 17 c = 30

= 2.30 = 60

And either of their sum or difference = b

= - 17

Thus the two terms are - 12 and - 5

Sum = - 12 - 5 = - 17

Product = - 12. - 5 = 60

$2x^2 - 17x + 30 = 0$

$2x^2 - 12x - 5x + 30 = 0$

$2x(x - 6) - 5(x - 6) = 0$

$(x - 6)(2x - 5) = 0$

$(x - 6) = 0$ or $(2x - 5) = 0$

$x = 6$ or $x = \frac{5}{2}$

Hence the roots of equation are 6 or $x = \frac{5}{2}$

Question: 59

Solution:

Given:

$\frac{x}{x-1} + \frac{x-1}{x} = 4\frac{1}{4}$

$\frac{(x-1)^2}{x(x-1)} = \frac{17}{4}$ taking LCM

$\frac{x^2 + x^2 - 2x + 1}{x(x-1)} = \frac{17}{4}$ using $(a - b)^2 = a^2 + b^2 - 2ab$

$\frac{2x^2 - 2x + 1}{x^2 - 1} = \frac{17}{4}$

$8x^2 - 8x + 4 = 17x^2 - 17x$ cross multiplying

$9x^2 - 9x - 4 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 9 b = - 9 c = - 4

= 9. - 4 = - 36

And either of their sum or difference = b

= - 9

Thus the two terms are - 12 and 3

Sum = - 12 + 3 = - 9

Product = - 12.3 = - 36

$9x^2 - 9x - 4 = 0$

$9x^2 - 12x + 3x - 4 = 0$

3x(3x - 4) + 1(3x - 4) = 0

(3x - 4) (3x + 1) = 0

(3x - 4) = 0 or (3x + 1) = 0

$x = \frac{4}{3}$ or $x = \frac{-1}{3}$

Hence the roots of equation are $\frac{4}{3}$ or $\frac{-1}{3}$

Question: 60

Solution:

Given: $\frac{x}{x+1} + \frac{x+1}{x} = 2\frac{4}{15}$ taking LCM

$$\frac{x^2 + x^2 + 2x + 1}{x(x+1)} = \frac{34}{15}$$

$$\frac{2x^2 + 2x + 1}{x^2 + x} = \frac{34}{15}$$

$30x^2 + 30x + 15 = 34x^2 + 34x$ cross multiplying

$4x^2 + 4x - 15 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 4 b = 4 c = - 15

= 4. - 15 = - 60

And either of their sum or difference = b

= 4

Thus the two terms are 10 and - 6

Difference = 10 - 6 = 4

Product = 10. - 6 = - 60

$4x^2 + 4x - 15 = 0$

$4x^2 + 10x - 6x - 15 = 0$

2x(2x + 5) - 3(2x + 5) = 0

(2x + 5) (2x - 3) = 0

(2x + 5) = 0 or (2x - 3) = 0

$x = \frac{-5}{2}$ or $x = \frac{3}{2}$

Hence the roots of equation are $\frac{-5}{2}$ or $\frac{3}{2}$

Question: 61

Solution:

Given:

$\frac{x-4}{x-5} + \frac{x-6}{x-7} = 3\frac{1}{3}$ $x \neq 5, 7$

$\frac{(x-7)(x-4) + (x-5)(x-6)}{(x-5)(x-7)} = \frac{10}{3}$ taking LCM

$\frac{x^2 - 11x + 28 + x^2 - 11x + 30}{x^2 - 12x + 35} = \frac{10}{3}$

$\frac{2x^2 - 22x + 58}{x^2 - 12x + 35} = \frac{10}{3}$

$\frac{x^2 - 11x + 29}{x^2 - 12x + 35} = \frac{5}{3}$

$3x^2 - 33x + 87 = 5x^2 - 60x + 175$ cross multiplying

$2x^2 - 27x + 88 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 2 b = - 27 c = 88

= 2.88 = 176

And either of their sum or difference = b

= - 27

Thus the two terms are - 16 and - 11

Sum = - 16 - 11 = - 27

Product = - 16. - 11 = 176

$2x^2 - 27x + 88 = 0$

$2x^2 - 16x - 11x + 88 = 0$

$2x(x - 8) - 11(x - 8) = 0$

$(x - 8)(2x - 11) = 0$

$(x - 8) = 0$ or $(2x - 11) = 0$

$x = 8$ or $x = \frac{11}{2} = 5\frac{1}{2}$

Hence the roots of equation are 8 or $5\frac{1}{2}$

Question: 62

Solution:

Given:

$\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

$\frac{(x-1)(x-4)+(x-2)(x-3)}{(x-2)(x-4)} = \frac{10}{3}$ taking LCM

$\frac{x^2-5x+4+x^2-5x+6}{x^2-6x+8} = \frac{10}{3}$

$\frac{2x^2-10x+10}{x^2-6x+8} = \frac{10}{3}$

$\frac{x^2-5x+5}{x^2-6x+8} = \frac{5}{3}$ cross multiplying

$3x^2 - 15x + 15 = 5x^2 - 30x + 40$

$2x^2 - 15x + 25 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 2 b = - 15 c = 25

= 2.25 = 50

And either of their sum or difference = b

= - 15

Thus the two terms are - 10 and - 5

Sum = - 10 - 5 = - 15

Product = - 10. - 5 = 50

$2x^2 - 15x + 25 = 0$

$2x^2 - 10x - 5x + 25 = 0$

$2x(x - 5) - 5(x - 5) = 0$

$(x - 5)(2x - 5) = 0$

$(x - 5) = 0$ or $(2x - 5) = 0$

$x = 5$ or $x = \frac{5}{2}$

Hence the roots of equation are 5 or $\frac{5}{2}$

Question: 63

Solution:

Given:

$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, x \neq 0, 1, 2$

$\frac{(x-1)+2(x-2)}{(x-2)(x-1)} = \frac{6}{x}$ taking LCM

$\frac{3x-5}{x^2-3x+2} = \frac{6}{x}$ cross multiplying

$3x^2 - 5x = 6x^2 - 18x + 12$

$3x^2 - 13x + 12 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 3 b = - 13 c = 12

= 3.12 = 36

And either of their sum or difference = b

= - 13

Thus the two terms are - 9 and - 4

Sum = - 9 - 4 = - 13

Product = - 9. - 4 = 36

$3x^2 - 13x + 12 = 0$

$3x^2 - 9x - 4x + 12 = 0$

3x (x - 3) - 4(x - 3) = 0

(x - 3) (3x - 4) = 0

$x = 3 \text{ or } x = \frac{4}{3}$

Hence the roots of equation are 3 or $\frac{4}{3}$

Question: 64

Solution:

Given:

$\frac{1}{x+1} + \frac{2}{x+2} = \frac{5}{x+4}$

$\frac{(x+2) + 2(x+1)}{(x+2)(x+1)} = \frac{5}{x+4}$ taking LCM

$$\frac{3x + 4}{x^2 + 3x + 2} = \frac{5}{x + 4}$$

$(3x + 4)(x + 4) = 5x^2 + 15x + 10$ cross multiplying

$3x^2 + 16x + 16 = 5x^2 + 15x + 10$

$2x^2 - x - 6 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 2 b = - 1 c = - 6

= 2. - 6 = - 12

And either of their sum or difference = b

= - 1

Thus the two terms are - 4 and 3

Difference = - 4 + 3 = - 1

Product = - 4.3 = 12

$2x^2 - x - 6 = 0$

$2x^2 - 4x + 3x - 6 = 0$

2x(x - 2) + 3(x - 2) = 0

(x - 2) (2x + 3) = 0

(x - 2) = 0 or (2x + 3) = 0

$x = 2 \text{ or } x = \frac{-3}{2}$

Hence the roots of equation are 2 or $\frac{-3}{2}$

Question: 65

Solution:

Given:

$3\left(\frac{3x-1}{2x+3}\right) - 2\left(\frac{2x+3}{3x-1}\right) = 5$

$\frac{3(3x-1)^2 - 2(2x+3)^2}{(2x+3)(3x-1)} = 5$ taking LCM

$\frac{3(9x^2-6x+1)-2(4x^2+12x+9)}{(2x+3)(3x-1)} = 5$ using $(a + b)^2 = a^2 + b^2 + 2ab$; $(a - b)^2 = a^2 + b^2 - 2ab$

$$\frac{27x^2 - 18x + 3 - 8x^2 - 24x - 18}{6x^2 + 7x - 3} = 5$$

$$\frac{19x^2 - 42x - 15}{6x^2 + 7x - 3} = 5$$

$19x^2 - 42x - 15 = 30x^2 + 35x - 15$ cross multiplying

$11x^2 + 77x = 0$

$11x(x + 7) = 0$ taking 11x common

$11x = 0$ or $(x + 7) = 0$

$x = 0$ or $x = -7$

Hence the roots of equation are 0, - 7

Question: 66

Solution:

Given:

$3\left(\frac{7x+1}{5x-3}\right) - 4\left(\frac{5x-3}{7x+1}\right) = 11$

$\frac{3(7x+1)^2 - 4(5x-3)^2}{(7x+1)(5x-3)} = 11$ taking LCM; using $(a + b)^2 = a^2 + b^2 + 2ab$

$$\frac{3(49x^2 + 14x + 1) - 4(25x^2 - 30x + 9)}{(7x + 1)(5x - 3)} = 11$$

$$\frac{147x^2 + 42x + 3 - 100x^2 + 120x - 36}{35x^2 - 16x - 3} = 11$$

$$\frac{47x^2 + 162x - 33}{35x^2 - 16x - 3} = 11$$

$47x^2 + 162x - 33 = 385x^2 - 176x - 33$ cross multiplying

$338x^2 - 338x = 0$

$338x(x - 1) = 0$ taking 338x common

$338x = 0$ or $(x - 1) = 0$

$x = 1$ or $x = 0$

Hence the roots of equation are 1, 0

Question: 67

Solve each of the

CLASS24

Solution:

Given: $\left(\frac{4x-3}{2x+1}\right) - 10\left(\frac{2x+1}{4x-3}\right) = 3$

$\frac{(4x-3)^2 - 10(2x+1)^2}{(2x+1)(4x-3)} = 3$ taking LCM; using $(a+b)^2 = a^2 + b^2 + 2ab$

$$\frac{(16x^2 - 24x + 9) - 10(4x^2 + 4x + 1)}{8x^2 - 6x + 4x - 3} = 3$$

$$\frac{16x^2 - 24x + 9 - 40x^2 - 40x - 10}{8x^2 - 6x + 4x - 3} = 3$$

$$\frac{-24x^2 - 64x - 1}{8x^2 - 6x + 4x - 3} = 3$$

$-24x^2 - 64x - 1 = 3(8x^2 - 2x - 3)$ cross multiplying

$-24x^2 - 64x - 1 = 24x^2 - 6x - 9$

$48x^2 + 58x - 8 = 0$ taking 2 common

$24x^2 + 29x - 4 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 24 b = 29 c = - 4

= 24. - 4 = - 96

And either of their sum or difference = b

= 29

Thus the two terms are 32 and - 3

Difference = 32 - 3 = 29

Product = 32. - 3 = - 96

$24x^2 + 29x - 4 = 0$

$24x^2 + 32x - 3x - 4 = 0$

$8x(3x + 4) - 1(3x + 4) = 0$

$(3x + 4)(8x - 1) = 0$

$(3x + 4) = 0$ or $(8x - 1) = 0$

$x = \frac{-4}{3}$ or $x = \frac{1}{8}$

Hence the roots of equation are $\frac{-4}{3}$ or $\frac{1}{8}$

Question: 68

Solution:

Given: $\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 6 = 0$ - - - - - - - - (1)

Let $\frac{x}{x+1} = y$

$y^2 - 5y + 6 = 0$ substituting value for y in (1)

Using the splitting middle term - the middle term of the general equation $ax^2 + b$: divided in two such values that:

CLASS24

Product = a.c

For the given equation a = 1 b = - 5 c = 6

= 1.6 = 6

And either of their sum or difference = b

= - 5

Thus the two terms are - 3 and - 2

Difference = - 3 - 2 = - 5

Product = - 3. - 2 = 6

$y^2 - 5y + 6 = 0$

$y^2 - 3y - 2y + 6 = 0$

$y(y - 3) - 2(y - 3) = 0$

$(y - 3)(y - 2) = 0$

$(y - 3) = 0$ or $(y - 2) = 0$

y = 3 or y = 2

Case I: if y = 3

$$\frac{x}{x+1} = 3$$

x = 3x + 3

2x + 3 = 0

x = - 3/2

Case II: if y = 2

$$\frac{x}{x+1} = 2$$

x = 2x + 2

x = - 2

$$x = \frac{-3}{2} \text{ or } = -2$$

Hence the roots of equation are $\frac{-3}{2}$ or -2

Question: 69

Solution:

Given: $\frac{a}{(x-b)} + \frac{b}{(x-a)} = 2$

$$\frac{a}{(x-b)} + \frac{b}{(x-a)} - 2 = 0$$

$$\left[\frac{a}{(x-b)} - 1\right] + \left[\frac{b}{(x-a)} - 1\right] = 0$$

taking - 1 with both terms

$$\frac{a-(x-b)}{(x-b)} + \frac{b-(x-a)}{(x-a)} = 0$$

taking LCM

$$(a-x+b)\left[\frac{1}{(x-b)} + \frac{1}{(x-a)}\right] = 0$$

taking common (a - x - b)

$$(a-x+b)\left[\frac{(x-a)+(x-b)}{(x-b)(x-a)}\right] = 0$$

taking LCM

(a - x + b)[2x - (a + b)] = 0

(a - x + b) = 0 or [2x - (a + b)] = 0

$$x = a + b \text{ or } x = \frac{a+b}{2}$$

Hence the roots of equation are $a + b$ or $\frac{a+b}{2}$

Question: 70

Solution:

Given: $\frac{a}{(ax-1)} + \frac{b}{(bx-1)} = (a + b)$

$$\frac{a}{(ax-1)} + \frac{b}{(bx-1)} - a - b = 0$$

$$\left[\frac{a}{(ax-1)} - b\right] + \left[\frac{b}{(bx-1)} - a\right] = 0$$

$$\frac{a-b(ax-1)}{(ax-1)} + \frac{b-a(bx-1)}{(bx-1)} = 0$$

taking LCM

$$\frac{a-bax+b}{(ax-1)} + \frac{b-abx+a}{(bx-1)} = 0$$

$$(a+b-abx)\left[\frac{1}{(ax-1)} + \frac{1}{(bx-1)}\right] = 0$$

taking common (a + b - abx)

$$(a+b-abx)\left[\frac{(bx-1)+(ax-1)}{(ax-1)(bx-1)}\right] = 0$$

taking LCM

$$(a+b-abx)\left[\frac{(a+b)x-2}{(ax-1)(bx-1)}\right] = 0$$

(a + b - abx)[(a + b)x - 2] = 0

(a + b - abx) = 0 or [(a + b)x - 2] = 0

$$x = \frac{a+b}{ab} \text{ or } x = \frac{2}{a+b}$$

Hence the roots of equation are $\frac{a+b}{ab}$ or $\frac{2}{a+b}$

Question: 71

Solution:

Given: $3^{(x+2)} + 3^{-x} = 10$

$3^x.3^2 + \frac{1}{3^x} = 10$ - - - - - - - - (1)

Let $3^x = y$ ---------------- (2)

$9y + \frac{1}{y} = 10$ substituting for y in (1)

$9y^2 - 10y + 1 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 9 b = - 10 c = 1

= 9.1 = 9

And either of their sum or difference = b

= - 10

Thus the two terms are - 9 and - 1

Sum = - 9 - 1 = - 10

Product = - 9. - 1 = 9

$9y^2 - 9y - 1y + 1 = 0$

$9y(y - 1) - 1(y - 1) = 0$

$(y - 1)(9y - 1) = 0$

$(y - 1) = 0$ or $(9y - 1) = 0$

y = 1 or y = 1/9

$3^x = 1$ or $3^x = 1/9$

On putting value of y in equation (2)

$3^x = 3^0$ or $3^x = 3^{-2}$

x = 0 or x = - 2

Hence the roots of equation are 0, - 2

Question: 72

Solution:

Given: $4^{(x+1)} + 4^{(1-x)} = 10$

$4^x.4 + 4.\frac{1}{4^x} = 10$ - - - - - - - (1)

Let $4^x = y$ ---------------- (2)

$4y + \frac{4}{y} = 10$ substituting for y in (1)

$4y^2 - 10y + 4 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 4 b = - 10 c = 4

= 4.4 = 16

And either of their sum or difference = b

= - 10

Thus the two terms are - 8 and - 2

Sum = - 8 - 2 = - 10

Product = - 8. - 2 = 16

$4y^2 - 10y + 4 = 0$

$4y^2 - 8y - 2y + 4 = 0$

4y(y - 2) - 2(y - 2) = 0

(y - 2) (4y - 2) = 0

(y - 2) = 0 or (4y - 2) = 0

y = 2 or y = 1/2

substituting the value of y in (2)

$4^x = 2$ or $4^x = 2^{-1}$

$2^{2x} = 2^1$ or $2^{2x} = 2^{-1}$

2x = 1 or 2x = - 1

$x = \frac{1}{2}$ or $x = \frac{-1}{2}$

Hence the roots of equation are $\frac{1}{2}$ or $\frac{-1}{2}$

Question: 73

Solution:

Given: $2^{2x} - 3.2^{(x+2)} + 32 = 0$

$(2^x)^2 - 3.2^x.2^2 + 32 = 0$ ------ (1)

Let $2^x = y$ ---------- (2)

substituting for y in (1)

$y^2 - 12y + 32 = 0$

Using the splitting middle term - the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = - 12 c = 32

= 1.32 = 32

And either of their sum or difference = b

= - 12

Thus the two terms are - 8 and - 4

Sum = - 8 - 4 = - 12

Product = - 8. - 4 = 32

$y^2 - 8y - 4y + 32 = 0$

$y(y - 8) - 4(y - 8) = 0$

$(y - 8)(y - 4) = 0$

$(y - 8) = 0$ or $(y - 4) = 0$

$y = 8$ or $y = 4$

$2^x = 8$ or $2^x = 4$

substituting the value of y in (2)

$2^x = 2^3$ or $2^x = 2^2$

$x = 2$ or $x = 3$

Hence the roots of equation are 2, 3

Exercise : 10B

Question: 1

Solution:

Given: $x^2 - 6x + 3 = 0$

$x^2 - 6x = - 3$

$x^2 - 2.x.3 + 3^2 = - 3 + 3^2$ (adding 3^2 on both sides)

$(x - 3)^2 = - 3 + 9 = 6$ using $a^2 - 2ab + b^2 = (a - b)^2$

$x - 3 = \pm\sqrt{6}$ (taking square root on both sides)

$x - 3 = \sqrt{6}$ or $x - 3 = -\sqrt{6}$

$x = 3 + \sqrt{6}$ or $x = 3 - \sqrt{6}$

Hence the roots of equation are $3 + \sqrt{6}$ or $3 - \sqrt{6}$

Question: 2

Solution:

Given: $x^2 - 4x + 1 = 0$

$x^2 - 4x = - 1$

$x^2 - 2.x.2 + 2^2 = - 1 + 2^2$ (adding 2^2 on both sides)

$(x - 2)^2 = - 1 + 4 = 3$ using $a^2 - 2ab + b^2 = (a - b)^2$

$x - 2 = \pm\sqrt{3}$ (taking square root on both sides)

$x - 2 = \sqrt{3}$ or $x - 2 = -\sqrt{3}$

$x = 2 + \sqrt{3}$ or $x = 2 - \sqrt{3}$

Hence the roots of equation are $2 + \sqrt{3}$ or $2 - \sqrt{3}$

Question: 3

Solve each of the

Solution:

Given: $x^2 + 8x - 2 = 0$

$x^2 + 8x = 2$

$x^2 + 2.x.4 + 4^2 = 2 + 4^2$ (adding 4^2 on both sides)

$(x + 4)^2 = 2 + 16 = 18$ using $a^2 + 2ab + b^2 = (a + b)^2$

$x + 4 = \pm\sqrt{18} = \pm 3\sqrt{2}$ (taking square root on both sides)

$x + 4 = 3\sqrt{2}$ or $x + 4 = -3\sqrt{2}$

$x = -4 + 3\sqrt{2}$ or $x = -4 - 3\sqrt{2}$

Hence the roots of equation are $-4 + 3\sqrt{2}$ or $-4 - 3\sqrt{2}$

Question: 4

Solution:

Given: $4x^2 + 4\sqrt{3}x + 3 = 0$

$4x^2 + 4\sqrt{3}x = -3$

$(2x)^2 + 2.2x.\sqrt{3} + (\sqrt{3})^2 = -3 + (\sqrt{3})^2$ (adding $(\sqrt{3})^2$ on both sides)

$(2x + \sqrt{3})^2 = -3 + 3$ using $a^2 + 2ab + b^2 = (a + b)^2$

$(2x + \sqrt{3})^2 = 0$

$(2x + \sqrt{3})(2x + \sqrt{3}) = 0$

$x = \frac{-\sqrt{3}}{2}$ or $x = \frac{-\sqrt{3}}{2}$

Hence the equation has repeated roots $\frac{-\sqrt{3}}{2}$

Question: 5

Solution:

Given: $2x^2 + 5x - 3 = 0$

$4x^2 + 10x - 6 = 0$ (multiplying both sides by 2)

$4x^2 + 10x = 6$

$(2x)^2 + 2.2x.\frac{5}{2} + \left(\frac{5}{2}\right)^2 = 6 + \left(\frac{5}{2}\right)^2$ (adding $\left(\frac{5}{2}\right)^2$ on both sides)

$\left(2x + \frac{5}{2}\right)^2 = 6 + \frac{25}{4}$ using $a^2 + 2ab + b^2 = (a + b)^2$

$\left(2x + \frac{5}{2}\right)^2 = \frac{25 + 24}{4} = \frac{49}{4} = \left(\frac{7}{2}\right)^2$

$2x + \frac{5}{2} = \pm\frac{7}{2}$ (taking square root on both sides)

$2x + \frac{5}{2} = \frac{7}{2}$ or $2x + \frac{5}{2} = -\frac{7}{2}$

$2x = \frac{7}{2} - \frac{5}{2}$ or $2x = -\frac{7}{2} - \frac{5}{2}$

$2x = 1$ or $2x = -6$

$x = \frac{1}{2}$ or $x = -3$

Hence the roots of equation are $x = \frac{1}{2}$ or $x = -3$

Question: 6

Solution:

Given: $3x^2 - x - 2 = 0$

$9x^2 - 3x - 6 = 0$ (multiplying both sides by 3)

$9x^2 - 3x = 6$

$(3x)^2 - 2.3x.\frac{1}{2} + \left(\frac{1}{2}\right)^2 = 6 + \left(\frac{1}{2}\right)^2$ (adding $\left(\frac{1}{2}\right)^2$ on both sides)

$\left(3x - \frac{1}{2}\right)^2 = 6 + \frac{1}{4} = \frac{25}{4} = \left(\frac{5}{2}\right)^2$ using $a^2 - 2ab + b^2 = (a - b)^2$

$3x - \frac{1}{2} = \pm\frac{5}{2}$ (taking square root on both sides)

$3x - \frac{1}{2} = \frac{5}{2}$ or $3x - \frac{1}{2} = -\frac{5}{2}$

$3x = \frac{5}{2} + \frac{1}{2} = \frac{6}{2} = 3$ or $3x = -\frac{5}{2} + \frac{1}{2} = -\frac{4}{2} = -2$

$x = 1$ or $x = \frac{-2}{3}$

Hence the roots of equation are 1 or $\frac{-2}{3}$

Question: 7

Solution:

Given: $8x^2 - 14x - 15 = 0$

$16x^2 - 28x - 30 = 0$ (multiplying both sides by 2)

$16x^2 - 28x = 30$

$(4x)^2 - 2.4x.\frac{7}{2} + \left(\frac{7}{2}\right)^2 = 30 + \left(\frac{7}{2}\right)^2$ (adding $\left(\frac{7}{2}\right)^2$ on both sides)

$\left(4x - \frac{7}{2}\right)^2 = 30 + \frac{49}{4} = \frac{169}{4} = \left(\frac{13}{2}\right)^2$ using $a^2 - 2ab + b^2 = (a - b)^2$

$4x - \frac{7}{2} = \pm\frac{13}{2}$ (taking square root on both sides)

$4x - \frac{7}{2} = \frac{13}{2}$ or $4x - \frac{7}{2} = -\frac{13}{2}$

$4x = \frac{13}{2} + \frac{7}{2} = \frac{20}{2} = 10$ or $4x = -\frac{13}{2} + \frac{7}{2} = -\frac{6}{2} = -3$

$x = \frac{5}{2}$ or $x = \frac{-3}{4}$

Hence the roots of equation are $\frac{5}{2}$ or $\frac{-3}{4}$

Question: 8

Solution:

Given: $7x^2 + 3x - 4 = 0$

$49x^2 + 21x - 28 = 0$ (multiplying both sides by 7)

$(7x)^2 + 2.7x.\frac{3}{2} + \left(\frac{3}{2}\right)^2 = 28 + \left(\frac{3}{2}\right)^2$ (adding $\left(\frac{3}{2}\right)^2$ on both sides)

$\left(7x + \frac{3}{2}\right)^2 = 28 + \frac{9}{4} = \frac{121}{4} = \left(\frac{11}{2}\right)^2$ using $a^2 + 2ab + b^2 = (a + b)^2$

$7x + \frac{3}{2} = \pm \frac{11}{2}$ (taking square root on both sides)

$7x + \frac{3}{2} = \frac{11}{2}$ or $7x + \frac{3}{2} = -\frac{11}{2}$

$7x = \frac{11}{2} - \frac{3}{2} = \frac{8}{2} = 4$ or $7x = -\frac{11}{2} - \frac{3}{2} = \frac{-14}{2} = -7$

$x = -1$ or $x = \frac{4}{7}$

Hence the roots of equation are -1 or $\frac{4}{7}$

Question: 9

Solution:

Given: $3x^2 - 2x - 1 = 0$

$9x^2 - 6x = 3$ (multiplying both sides by 3)

$(3x)^2 - 2.3x.1 + (1)^2 = 3 + (1)^2$ (adding $(1)^2$ on both sides)

$(3x - 1)^2 = 3 + 1 = 4 = (2)^2$ using $a^2 - 2ab + b^2 = (a - b)^2$

$3x - 1 = \pm 2$ (taking square root on both sides)

$3x - 1 = 2$ or $3x - 1 = -2$

$3x = 3$ or $3x = -1$

$x = -1$ or $x = \frac{-1}{3}$

Hence the roots of equation are -1 or $\frac{-1}{3}$

Question: 10

Solution:

Given: $5x^2 - 6x - 2 = 0$

$25x^2 - 30x - 10 = 0$ (multiplying both sides by 5)

$25x^2 - 30x = 10$

$(5x)^2 - 2.5x.3 + (3)^2 = 10 + (3)^2$ (adding $(3)^2$ on both sides)

$(5x - 2)^2 = 10 + 9 = 19$ using $a^2 - 2ab + b^2 = (a - b)^2$

$5x - 3 = \pm\sqrt{19}$ (taking square root on both sides)

$5x - 3 = \sqrt{19}$ or $5x - 3 = -\sqrt{19}$

$5x = 3 + \sqrt{19}$ or $5x = 3 - \sqrt{19}$

$x = \frac{3 + \sqrt{19}}{5}$ or $x = \frac{3 - \sqrt{19}}{5}$

Hence the roots of equation are $\frac{3 + \sqrt{19}}{5}$ or $\frac{3-\sqrt{19}}{5}$

Question: 11

Solution:

Given:

$\frac{2}{x^2} - \frac{5}{x} + 2 = 0$

$\frac{2 - 5x + 2x^2}{x^2} = 0$

$2x^2 - 5x + 2 = 0$

$4x^2 - 10x + 4 = 0$

$4x^2 - 10x = -4$ (multiplying both sides by 2)

$(2x)^2 - 2.2x.\frac{5}{2} + \left(\frac{5}{2}\right)^2 = -4 + \left(\frac{5}{2}\right)^2$ (adding $\left(\frac{5}{2}\right)^2$ on both sides)

$\left(2x - \frac{5}{2}\right)^2 = -4 + \frac{25}{4} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$ using $a^2 - 2ab + b^2 = (a - b)^2$

$2x - \frac{5}{2} = \pm\frac{3}{2}$ (taking square root on both sides)

$2x - \frac{5}{2} = \frac{3}{2}$ or $2x - \frac{5}{2} = -\frac{3}{2}$

$2x = \frac{3}{2} + \frac{5}{2} = \frac{8}{2} = 4$ or $2x = -\frac{3}{2} + \frac{5}{2} = \frac{2}{2} = 1$

$x = 2$ or $x = \frac{1}{2}$

Hence the roots of equation are 2 or $\frac{1}{2}$

Question: 12

Solution:

$4x^2 + 4bx = (a^2 - b^2)$

$(2x)^2 + 2.2x.b + b^2 = a^2 - b^2 + b^2$ (adding b^2 on both sides)

$(2x + b)^2 = a^2$ using $a^2 + 2ab + b^2 = (a + b)^2$

$2x + b = \pm a$ (taking square root on both sides)

$2x + b = -a$ or $2x + b = a$

$x = \frac{-(a + b)}{2}$ or $x = \frac{a - b}{2}$

Hence the roots of equation are $\frac{-(a + b)}{2}$ or $\frac{a-b}{2}$

Question: 13

Solution:

Given : $x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$

$x^2 - (\sqrt{2} + 1)x = -\sqrt{2}$

$x^2 - 2.x.\left(\frac{\sqrt{2}+1}{2}\right) + \left(\frac{\sqrt{2}+1}{2}\right)^2 = -\sqrt{2} + \left(\frac{\sqrt{2}+1}{2}\right)^2$ (adding $\left(\frac{\sqrt{2}+1}{2}\right)^2$ on both sides)

$\left(x - \left(\frac{\sqrt{2}+1}{2}\right)\right)^2 = \frac{-4\sqrt{2}+2+1+2\sqrt{2}}{4} = \frac{2-2\sqrt{2}+1}{4} = \left(\frac{\sqrt{2}-1}{2}\right)^2$ using $a^2 - 2ab + b^2 = (a - b)^2$

$x - \left(\frac{\sqrt{2}+1}{2}\right) = \left(\frac{\sqrt{2}+1}{2}\right) or\ x - \left(\frac{\sqrt{2}+1}{2}\right) = -\left(\frac{\sqrt{2}+1}{2}\right)$ taking square root on both sides

$x = \left(\frac{\sqrt{2}+1}{2}\right) + \left(\frac{\sqrt{2}-1}{2}\right) or\ x = \left(\frac{\sqrt{2}+1}{2}\right) - \left(\frac{\sqrt{2}-1}{2}\right)$

$x = \sqrt{2}\ or\ x = 1$

Hence the roots of equation are $\sqrt{2}\ or\ 1$

Question: 14

Solution:

Given:

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

$2x^2 - 3\sqrt{2}x - 4 = 0$ (multiplying both sides by $\sqrt{2}$)

$2x^2 - 3\sqrt{2}x = 4$

$(\sqrt{2}x)^2 - 2.\sqrt{2}x.\frac{3}{2} + \left(\frac{3}{2}\right)^2 = 4 + \left(\frac{3}{2}\right)^2$ [Adding $\left(\frac{3}{2}\right)^2$ on both sides]

$\left(\sqrt{2}x - \frac{3}{2}\right)^2 = 4 + \frac{9}{4} = \frac{25}{4} = \left(\frac{5}{2}\right)^2$ using $a^2 - 2ab + b^2 = (a - b)^2$

$\sqrt{2}x - \frac{3}{2} = \pm\frac{5}{2}$ (taking square root on both sides)

$\sqrt{2}x - \frac{3}{2} = \frac{5}{2}$ or $\sqrt{2}x - \frac{3}{2} = -\frac{5}{2}$

$\sqrt{2}x = \frac{5}{2} + \frac{3}{2} = \frac{8}{2} = 4$ or $\sqrt{2}x = -\frac{5}{2} + \frac{3}{2} = -1$

$\sqrt{2}x = 4$ or $\sqrt{2}x = -1$

$x = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ or $x = \frac{-1}{\sqrt{2}}$

Hence the roots of equation are $2\sqrt{2}$ or $\frac{-1}{\sqrt{2}}$

Question: 15

Solution:

Given: $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$3x^2 + 10\sqrt{3}x + 21 = 0$ (multiplying both sides with $\sqrt{3}$)

$3x^2 + 10\sqrt{3}x = -21$

$(\sqrt{3}x)^2 + 2.\sqrt{3}x.5 + 5^2 = -21 + 5^2$ [Adding 5^2 on both sides]

$\left(\sqrt{3}x + 5\right)^2 = -21 + 25 = 4 = 2^2$ using $a^2 + 2ab + b^2 = (a + b)^2$

$\sqrt{3}x + 5 = \pm 2$ (taking square root on both sides)

$\sqrt{3}x + 5 = 2$ or $\sqrt{3}x + 5 = -2$

$\sqrt{3}x = 2 - 5$ or $\sqrt{3}x = -2 - 5$

$\sqrt{3}x = -3$ or $\sqrt{3}x = -7$

$x = -\sqrt{3}$ or $x = \frac{-7}{\sqrt{3}}$

Hence the roots of equation are $-\sqrt{3}$ or $\frac{-7}{\sqrt{3}}$

Question: 16

Solution:

$2x^2 + x + 4 = 0$

$4x^2 + 2x + 8 = 0$ (multiplying both sides by 2)

$4x^2 + 2x = -8$

$(2x)^2 + 2.2x.\frac{1}{2} + \left(\frac{1}{2}\right)^2 = -8 + \left(\frac{1}{2}\right)^2$ [Adding $\left(\frac{1}{2}\right)^2$ on both sides]

$\left(2x + \frac{1}{2}\right)^2 = -8 + \frac{1}{4} = -\frac{31}{4} < 0$ using $a^2 + 2ab + b^2 = (a + b)^2$

But $\left(2x + \frac{1}{2}\right)^2$ cannot be negative for any real value of x

So there is no real value of x satisfying the given equation.

Hence the given equation has no real roots.

Exercise : 10C

Question: 1 A

Solution:

Given: $2x^2 - 7x + 6 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2, b = -7, c = 6$

Discriminant $D = b^2 - 4ac$

$= (-7)^2 - 4.2.6$

$= 49 - 48 = 1$

Question: 1 B

Solution:

Given: $3x^2 - 2x + 8 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 3, b = -2, c = 8$

Discriminant $D = b^2 - 4ac$

$= (-2)^2 - 4.3.8$

$= 4 - 96 = -92$

Question: 1 C

Solution:

Given: $2x^2 - 5\sqrt{2}x + 4 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2\ b = -5\sqrt{2}\ c = 4$

Discriminant $D = b^2 - 4ac$

$= (-5\sqrt{2})^2 - 4.2.4$

$= 25.2 - 32$

$= 50 - 32 = 18$

Question: 1 D

Solution:

Given: $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = \sqrt{3}\ b = 2\sqrt{2}\ c = -2\sqrt{3}$

Discriminant $D = b^2 - 4ac$

$= (2\sqrt{2})^2 - 4.\sqrt{3}.-2\sqrt{3}$

$= 8 + 24 = 32$

Question: 1 E

Solution:

Given: $(x - 1)(2x - 1) = 0$

$2x^2 - 3x + 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2, b = -3, c = -1$

Discriminant $D = b^2 - 4ac$

$= (-3)^2 - 4.2.1$

$= 9 - 8 = 1$

Question: 1 F

Solution:

Given: $1 - x = 2x^2$

$2x^2 + x - 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2, b = 1, c = -1$

Discriminant $D = b^2 - 4ac$

$= (1)^2 - 4.2. - 1$

$= 1 + 8 = 9$

Question: 2

Solution:

Given: $x^2 - 4x - 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 1, b = -4, c = -1$

Discriminant $D = b^2 - 4ac$

$= (-4)^2 - 4.1. - 1$

$= 16 + 4 = 20 > 0$

Hence the roots of equation are real.

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-4) + \sqrt{20}}{2 \times 1} = \frac{4 + 2\sqrt{5}}{2} = \frac{2(2 + \sqrt{5})}{2} = (2 + \sqrt{5})$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-4) - \sqrt{20}}{2 \times 1} = \frac{4 - 2\sqrt{5}}{2} = \frac{2(2 - \sqrt{5})}{2} = (2 - \sqrt{5})$$

$x = (2 + \sqrt{5})$ or $x = (2 - \sqrt{5})$

Hence the roots of equation are $(2 + \sqrt{5})$ or $(2 - \sqrt{5})$

Question: 3

Solution:

Given: $x^2 - 6x + 4 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 1, b = -6, c = 4$

Discriminant $D = b^2 - 4ac$

$= (6)^2 - 4.1.4$

$= 36 - 16 = 20 > 0$

Hence the roots of equation are real.

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-6) + \sqrt{20}}{2 \times 1} = \frac{6 + 2\sqrt{5}}{2} = \frac{2(3 + \sqrt{5})}{2} = (3 + \sqrt{5})$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-6) - \sqrt{20}}{2 \times 1} = \frac{6 - 2\sqrt{5}}{2} = \frac{2(3 - \sqrt{5})}{2} = (3 - \sqrt{5})$$

$x = (3 + \sqrt{5})$ or $x = (3 - \sqrt{5})$

Hence the roots of equation are $(3 + \sqrt{5})$ or $(3 - \sqrt{5})$

Question: 4

Solution:

Given: $2x^2 + x - 4 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2, b = 1, c = -4$

Discriminant $D = b^2 - 4ac$

$= (1)^2 - 4.2. - 4$

$= 1 + 32 = 33 > 0$

Hence the roots of equation are real.

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-1 + \sqrt{33}}{2 \times 2} = \frac{-1 + \sqrt{33}}{4}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-1 - \sqrt{33}}{2 \times 2} = \frac{-1 - \sqrt{33}}{4}$$

$x = \frac{-1 + \sqrt{33}}{4}$ or $x = \frac{-1 - \sqrt{33}}{4}$

Hence the roots of equation are $\frac{-1 + \sqrt{33}}{4}$ or $\frac{-1 - \sqrt{33}}{4}$

Question: 5

Solution:

Given: $25x^2 + 30x + 7 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 25, b = 30, c = 7$

Discriminant $D = b^2 - 4ac$

$= (30)^2 - 4.25.7$

$= 900 - 700 = 200 > 0$

Hence the roots of equation are real.

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-30 + \sqrt{200}}{2 \times 25} = \frac{-30 + 10\sqrt{2}}{50} = \frac{10(-3 + \sqrt{2})}{50} = \frac{(-3 + \sqrt{2})}{5}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-30 - \sqrt{200}}{2 \times 25} = \frac{-30 - 10\sqrt{2}}{50} = \frac{10(-3 - \sqrt{2})}{50}$$

$$= \frac{(-3 - \sqrt{2})}{5}$$

$$x = \frac{(-3 + \sqrt{2})}{5} \text{ or } x = \frac{(-3 - \sqrt{2})}{5}$$

Hence the roots of equation are $\frac{(-3 + \sqrt{2})}{5}$ or $\frac{(-3 - \sqrt{2})}{5}$

Question: 6

Solution:

Given: $16x^2 = 24x + 1$

$16x^2 - 24x - 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

a = 16, b = – 24, c = – 1

Discriminant $D = b^2 - 4ac$

$= (-24)^2 - 4.16. -1$

$= 576 + 64 = 640 > 0$

Hence the roots of equation are real.

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-24) + \sqrt{640}}{2 \times 16} = \frac{24 + 8\sqrt{10}}{32} = \frac{8(3 + \sqrt{10})}{32}$$

$$= \frac{(3 + \sqrt{10})}{4}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-24) - \sqrt{640}}{2 \times 16} = \frac{24 - 8\sqrt{10}}{32} = \frac{8(3 - \sqrt{10})}{32}$$

$$= \frac{(3 - \sqrt{10})}{4}$$

$$x = \frac{(3 + \sqrt{10})}{4} \text{ or } x = \frac{(3 - \sqrt{10})}{4}$$

Hence the roots of equation are $\frac{(3 + \sqrt{10})}{4}$ or $\frac{(3 - \sqrt{10})}{4}$

Question: 7

Solution:

Given: $15x^2 - 28 = x$

$15x^2 - x - 28 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

a = 15, b = – 1, c = – 28

Discriminant $D = b^2 - 4ac$

$= (-1)^2 - 4.15. -28$

$= 1 + 1680 = 1681 > 0$

Hence the roots of equation are real.

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-1) + \sqrt{1681}}{2 \times 15} = \frac{1 + 41}{30} = \frac{42}{30} = \frac{7}{5}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-1) - \sqrt{1681}}{2 \times 15} = \frac{1 - 41}{30} = \frac{-40}{30} = \frac{-4}{3}$$

$$x = \frac{7}{5} \text{ or } x = \frac{-4}{3}$$

Hence the roots of equation are $\frac{7}{5}$ or $\frac{-4}{3}$

Question: 8

Solution:

Given: $2x^2 - 2\sqrt{2}x + 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2\ b = -2\sqrt{2}\ c = 1$

Discriminant $D = b^2 - 4ac$

$= (-2\sqrt{2})^2 - 4.2.1$

$= 8 - 8 = 0$

Hence the roots of equation are real.

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-2\sqrt{2})}{2 \times 2} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-2\sqrt{2})}{2 \times 2} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$x = \frac{1}{\sqrt{2}} \text{ or } x = \frac{1}{\sqrt{2}}$$

Hence these are the repeated roots of the equation $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Question: 9

Solution:

Given: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = \sqrt{2}\ b = 7\ c = 5\sqrt{2}$

Discriminant $D = b^2 - 4ac$

$= (7)^2 - 4.\sqrt{2}.5\sqrt{2}$

$= 49 - 40 = 9 > 0$

Hence the roots of equation are real.

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-7 + \sqrt{9}}{2 \times \sqrt{2}} = \frac{-7 + 3}{2 \times \sqrt{2}} = \frac{-4}{2\sqrt{2}} = -\sqrt{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-7 - \sqrt{9}}{2 \times \sqrt{2}} = \frac{-7 - 3}{2 \times \sqrt{2}} = \frac{-10}{2\sqrt{2}} = \frac{-5\sqrt{2}}{2}$$

$$x = -\sqrt{2} \text{ or } x = \frac{-5\sqrt{2}}{2}$$

Hence the roots of equation are $-\sqrt{2}$ or $\frac{-5\sqrt{2}}{2}$

Question: 10

Solution:

Given: $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = \sqrt{3}$ $b = 10$ $c = -8\sqrt{3}$

Discriminant $D = b^2 - 4ac$

$= (10)^2 - 4.\sqrt{3}.-8\sqrt{3}$

$= 100 + 96 = 196 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{196} = 14$

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-10 + \sqrt{196}}{2 \times \sqrt{3}} = \frac{-10 + 14}{2 \times \sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{2\sqrt{3}}{3}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-10 - \sqrt{196}}{2 \times \sqrt{3}} = \frac{-10 - 14}{2 \times \sqrt{3}} = \frac{-24}{2\sqrt{3}} = \frac{-12}{\sqrt{3}}$$
$$= \frac{-12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-12\sqrt{3}}{3} = -4\sqrt{3}$$

$$x = \frac{2\sqrt{3}}{3} \text{ or } x = -4\sqrt{3}$$

Hence the roots of equation are $\frac{2\sqrt{3}}{3}$ or $-4\sqrt{3}$

Question: 11

Solution:

Given: $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = \sqrt{3}$ $b = -2\sqrt{2}$ $c = -2\sqrt{3}$

Discriminant $D = b^2 - 4ac$

$= (-2\sqrt{2})^2 - 4.\sqrt{3}.-2\sqrt{3}$

$= 8 + 24 = 32 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{32} = 4\sqrt{2}$

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-2\sqrt{2}) + 4\sqrt{2}}{2 \times \sqrt{3}} = \frac{6\sqrt{2}}{2 \times \sqrt{3}} = \frac{2\sqrt{3}\sqrt{3}\sqrt{2}}{2 \times \sqrt{3}} = \sqrt{6}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-2\sqrt{2}) - 4\sqrt{2}}{2 \times \sqrt{3}} = \frac{-2\sqrt{2}}{2 \times \sqrt{3}} = \frac{-\sqrt{2}}{\sqrt{3}}$$

$$x = \sqrt{6} \text{ or } x = \frac{-\sqrt{2}}{\sqrt{3}}$$

Hence the roots of equation are $\sqrt{6}$ or $\frac{-\sqrt{2}}{\sqrt{3}}$

Question: 12

Solution:

Given: $2x^2 + 6\sqrt{3}x - 60 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2$ $b = 6\sqrt{3}$ $c = -60$

Discriminant $D = b^2 - 4ac$

$= (6\sqrt{3})^2 - 4.2. -60$

$= 180 + 480 = 588 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{588} = 14\sqrt{3}$

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(6\sqrt{3}) + 14\sqrt{3}}{2 \times 2} = \frac{8\sqrt{3}}{4} = 2\sqrt{3}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(6\sqrt{3}) - 14\sqrt{3}}{2 \times 2} = \frac{-20\sqrt{3}}{4} = -5\sqrt{3}$$

$x = 2\sqrt{3}$ or $x = -5\sqrt{3}$

Hence the roots of equation are $2\sqrt{3}$ or $-5\sqrt{3}$

Question: 13

Solution:

Given $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 4\sqrt{3}$ $b = 5$ $c = -2\sqrt{3}$

Discriminant $D = b^2 - 4ac$

$= (5)^2 - 4.4\sqrt{3}. -2\sqrt{3}$

$= 25 + 96 = 121 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{121} = 11$

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-5 + 11}{2 \times 4\sqrt{3}} = \frac{6}{8 \times \sqrt{3}} = \frac{3}{4 \times \sqrt{3}} = \frac{\sqrt{3}}{4}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-5 - 11}{2 \times 4\sqrt{3}} = \frac{-16}{8 \times \sqrt{3}} = \frac{-2}{\sqrt{3}}$$

$$x = \frac{\sqrt{3}}{4} \text{ or } x = \frac{-2}{\sqrt{3}}$$

Hence the roots of equation are $\frac{\sqrt{3}}{4}$ or $\frac{-2}{\sqrt{3}}$

Question: 14

Solution:

Given: $3x^2 - 2\sqrt{6}x + 2 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 3$ $b = -2\sqrt{6}$ $c = 2$

Discriminant $D = b^2 - 4ac$

$= (-2\sqrt{6})^2 - 4.3.2$

$= 24 - 24 = 0$

$\sqrt{D} = 0$

Hence the roots of equation are real and repeated.

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-2\sqrt{6}) + 0}{2 \times 3} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3} = \frac{\sqrt{2}\sqrt{3}}{\sqrt{2}\sqrt{3}} = \sqrt{\frac{2}{3}}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-2\sqrt{6}) - 0}{2 \times 3} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3} = \frac{\sqrt{2}\sqrt{3}}{\sqrt{2}\sqrt{3}} = \sqrt{\frac{2}{3}}$$

Hence the roots of equation are $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$

Question: 15

Solution:

Given: $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2\sqrt{3}$ $b = -5$ $c = \sqrt{3}$

Discriminant $D = b^2 - 4ac$

$= (-5)^2 - 4.2\sqrt{3}.\sqrt{3}$

$= 25 - 24 = 1 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{1} = 1$

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-5) + 1}{2 \times 2\sqrt{3}} = \frac{6}{4 \times \sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-5) - 1}{2 \times 2\sqrt{3}} = \frac{4}{4 \times \sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$x = \frac{\sqrt{3}}{2} \text{ or } x = \frac{1}{\sqrt{3}}$$

Hence the roots of equation are $\frac{\sqrt{3}}{2}$ or $\frac{1}{\sqrt{3}}$

Question: 16

Solution:

Given: $x^2 + x + 2 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 1, b = 1, c = 2$

Discriminant $D = b^2 - 4ac$

$= (1)^2 - 4.1.2$

$= 1 - 8 = -7 < 0$

Hence the roots of equation do not exist

Question: 17

Solution:

Given: $2x^2 + ax - a^2 = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 2, B = a, C = -a^2$

Discriminant $D = B^2 - 4AC$

$= (a)^2 - 4.2. - a^2$

$= a^2 + 8a^2 = 9a^2 \geq 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{9a^2} = 3a$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-a + 3a}{2 \times 2} = \frac{2a}{4} = \frac{a}{2}$$

$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-a - 3a}{2 \times 2} = \frac{-4a}{4} = -a$

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$x = \frac{a}{2}$ or $x = -a$

Hence the roots of equation are $\frac{a}{2}$ or $-a$

Question: 18

Solution:

Given: $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 1$ $b = -(\sqrt{3} + 1)$ $c = \sqrt{3}$

Discriminant $D = b^2 - 4ac$

$D = [-(\sqrt{3} + 1)]^2 - 4.1.\sqrt{3} = 3 + 1 + 2\sqrt{3} - 4\sqrt{3} = 3 - 2\sqrt{3} + 1$

$D = (\sqrt{3} - 1)^2 > 0$

Using $a^2 - 2ab + b^2 = (a - b)^2$

Thus the roots of given equation are real.

$\sqrt{D} = \sqrt{3} - 1$

Roots α and β are given by

$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-[-(\sqrt{3} + 1)] + (\sqrt{3} - 1)}{2 \times 1} = \frac{\sqrt{3} + 1 + \sqrt{3} - 1}{2}$

$= \frac{2\sqrt{3}}{2} = \sqrt{3}$

$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-[-(\sqrt{3} + 1)] - (\sqrt{3} - 1)}{2 \times 1} = \frac{\sqrt{3} + 1 - \sqrt{3} + 1}{2} = \frac{2}{2}$

$= 1$

$x = 1$ or $x = \sqrt{3}$

Hence the roots of equation are $1, \sqrt{3}$

Question: 19

Solution:

Given: $2x^2 + 5\sqrt{3}x + 6 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2$ $b = 5\sqrt{3}$ $c = 6$

Discriminant $D = b^2 - 4ac$

$= (5\sqrt{3})^2 - 4.2.6$

$= 75 - 48 = 27 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{27} = 3\sqrt{3}$

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(5\sqrt{3}) + 3\sqrt{3}}{2 \times 2} = \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(5\sqrt{3}) - 3\sqrt{3}}{2 \times 2} = \frac{-8\sqrt{3}}{4} = -2\sqrt{3}$$

$$x = \frac{-\sqrt{3}}{2} \text{ or } x = -2\sqrt{3}$$

Hence the roots of equation are $\frac{-\sqrt{3}}{2}, -2\sqrt{3}$

Question: 20

Solution:

Given: $3x^2 - 2x + 2 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 3, b = -2, c = 2$

Discriminant $D = b^2 - 4ac$

$= (-2)^2 - 4.3.2$

$= 4 - 24 = -20 < 0$

Hence the roots of equation do not exist

Question: 21

Solution:

Given:

$x + \frac{1}{x} = 3$

taking LCM

$$\frac{x^2 + 1}{x} = 3$$

cross multiplying

$x^2 + 1 = 3x$

$x^2 - 3x + 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 1, b = -3, c = 1$

Discriminant $D = b^2 - 4ac$

$= (-3)^2 - 4.1.1$

$= 9 - 4 = 5 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{5}$

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-3) + \sqrt{5}}{2 \times 1} = \frac{3 + \sqrt{5}}{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-3) - \sqrt{5}}{2 \times 1} = \frac{3 - \sqrt{5}}{2}$$

$$x = \frac{3 + \sqrt{5}}{2} \text{ or } x = \frac{3 - \sqrt{5}}{2}$$

Hence the roots of equation are $\frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2}$

Question: 22

Solution:

Given:

$\frac{1}{x} - \frac{1}{x-2} = 3$

$\frac{x-2-x}{x(x-2)} = 3$ taking LCM

$$\frac{-2}{x^2 - 2x} = 3$$

$3x^2 - 6x + 2 = 0$ cross multiplying

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 3, b = -6, c = 2$

Discriminant $D = b^2 - 4ac$

$= (-6)^2 - 4.3.2$

$= 36 - 24 = 12 > 0$

Hence the roots of equation are real.

$$\sqrt{D} = \sqrt{12} = 2\sqrt{3}$$

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-6) + 2\sqrt{3}}{2 \times 3} = \frac{6 + 2\sqrt{3}}{6} = \frac{3 + \sqrt{3}}{3}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-6) - 2\sqrt{3}}{2 \times 3} = \frac{6 - 2\sqrt{3}}{6} = \frac{3 - \sqrt{3}}{3}$$

$$x = \frac{3 + \sqrt{3}}{3} \text{ or } x = \frac{3 - \sqrt{3}}{3}$$

Hence the roots of equation are $\frac{3 + \sqrt{3}}{3}$ or $\frac{3 - \sqrt{3}}{3}$

Question: 23

Solution:

Given:

$x - \frac{1}{x} = 3, x \neq 0$

$\frac{x^2 - 1}{x} = 3$ taking LCM

$x^2 - 3x - 1 = 0$ cross multiplying

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 1, b = -3, c = -1$

Discriminant $D = b^2 - 4ac$

$= (-3)^2 - 4.1. -1$

$= 9 + 4 = 13 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{13}$

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-3) + \sqrt{13}}{2 \times 1} = \frac{3 + \sqrt{13}}{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-3) - \sqrt{13}}{2 \times 1} = \frac{3 - \sqrt{13}}{2}$$

$$x = \frac{3 + \sqrt{13}}{2} \text{ or } \frac{3 - \sqrt{13}}{2}$$

Hence the roots of equation are $\frac{3 + \sqrt{13}}{2}$ or $\frac{3-\sqrt{13}}{2}$

Question: 24

Solution:

Given: $\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$

$$\frac{m^2x^2 + n^2}{mn} = 1 - 2x$$

taking LCM $m^2x + n^2 = mn - 2mnx$

On cross multiplying

$m^2x + 2mnx + n^2 - mn = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = m^2, b = 2mn, c = n^2 - mn$

Discriminant $D = b^2 - 4ac$

$= (2mn)^2 - 4.m^2. (n^2 - mn)$

$= 4m^2n^2 - 4m^2n^2 + 4m^3n > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{4m^3n} = 2m\sqrt{mn}$

Roots α and β are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(2mn) + 2m\sqrt{mn}}{2 \times m^2} = \frac{2m(-n + \sqrt{mn})}{2 \times m^2} = \frac{(-n + \sqrt{mn})}{m}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(2mn) - 2m\sqrt{mn}}{2 \times m^2} = \frac{2m(-n - \sqrt{mn})}{2 \times m^2} = \frac{(-n - \sqrt{mn})}{m}$$

$$x = \frac{(-n + \sqrt{mn})}{m} \text{ or } x = \frac{(-n - \sqrt{mn})}{m}$$

Hence the roots of equation are $\frac{(-n + \sqrt{mn})}{m}$ or $\frac{(-n-\sqrt{mn})}{m}$

Question: 25

Find the roots of

Solution:

Given: $36x^2 - 12ax + (a^2 - b^2) = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 36, B = -12a, C = a^2 - b^2$

Discriminant $D = B^2 - 4AC$

$= (-12a)^2 - 4.36.(a^2 - b^2)$

$= 144a^2 - 144a^2 + 144b^2 = 144b^2 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{144b^2} = 12b$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-(-12a) + 12b}{2 \times 36} = \frac{12(a + b)}{72} = \frac{(a + b)}{6}$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-(-12a) - 12b}{2 \times 36} = \frac{12(a - b)}{72} = \frac{(a - b)}{6}$$

$$x = \frac{(a + b)}{6} \text{ or } x = \frac{(a - b)}{6}$$

Hence the roots of equation are $\frac{(a + b)}{6}$ or $\frac{(a - b)}{6}$

Question: 26

Solution:

Given: $x^2 - 2ax + (a^2 - b^2) = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 1, B = -2a, C = a^2 - b^2$

Discriminant $D = B^2 - 4AC$

$= (-2a)^2 - 4.1.(a^2 - b^2)$

$= 4a^2 - 4a^2 + 4b^2 = 4b^2 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{4b^2} = 2b$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-(-2a) + 2b}{2 \times 1} = \frac{2(a + b)}{2} = a + b$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-(-2a) - 2b}{2 \times 1} = \frac{2(a - b)}{2} = a - b$$

$x = (a + b)$ or $x = (a - b)$

Hence the roots of equation are $(a + b)$ or $(a - b)$

Question: 27

Solution:

Given: $x^2 - 2ax - (4b^2 - a^2) = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 1, B = -2a, C = -(4b^2 - a^2)$

Discriminant $D = B^2 - 4AC$

$= (-2a)^2 - 4.1. - (4b^2 - a^2)$

$= 4a^2 - 4a^2 + 16b^2 = 16b^2 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{16b^2} = 4b$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-(-2a) + 4b}{2 \times 1} = \frac{2(a + 2b)}{2} = a + 2b$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-(-2a) - 4b}{2 \times 1} = \frac{2(a - 2b)}{2} = a - 2b$$

$x = (a + 2b)$ or $x = (a - 2b)$

Hence the roots of equation are $(a + 2b)$ or $(a - 2b)$

Question: 28

Solution:

Given: $x^2 + 6x - (a^2 + b^2 - 8) = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 1, B = 6, C = -(a^2 + b^2 - 8)$

Discriminant $D = B^2 - 4AC$

$= (6)^2 - 4.1. - (a^2 + b^2 - 8)$

$= 36 + 4a^2 + 8a - 32 = 4a^2 + 8a + 4$

$= 4(a^2 + 2a + 1)$

$= 4(a + 1)^2 > 0$ Using $a^2 + 2ab + b^2 = (a + b)^2$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{4(a + 1)^2}$

$= 2(a + 1)$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-6 + 2(a + 1)}{2 \times 1} = \frac{2a - 4}{2} = a - 2$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-6 - 2(a + 1)}{2 \times 1} = \frac{-2a - 8}{2} = -a - 4 = -(a + 4)$$

$x = (a - 2)$ or $x = -(4 + a)$

Hence the roots of equation are $(a - 2)$ or $-(4 + a)$

Question: 29

Find the roots of

Solution:

Given: $x^2 + 5x - (a^2 + a - 6) = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 1, B = 5, C = -(a^2 + a - 6)$

Discriminant $D = B^2 - 4AC$

$= (5)^2 - 4.1. - (a^2 + a - 6)$

$= 25 + 4a^2 + 4a - 24 = 4a^2 + 4a + 1$

$= (2a + 1)^2 > 0$ Using $a^2 + 2ab + b^2 = (a + b)^2$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{(2a + 1)^2}$

$= (2a + 1)$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-5 + (2a + 1)}{2 \times 1} = \frac{2a - 4}{2} = a - 2$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-5 - (2a + 1)}{2 \times 1} = \frac{-2a - 6}{2} = -a - 3 = -(a + 3)$$

$x = (a - 2)$ or $x = -(a + 3)$

Hence the roots of equation are $(a - 2)$ or $x = -(a + 3)$

Question: 30

Solution:

Given: $x^2 - 4ax - b^2 + 4a^2 = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 1, B = -4a, C = -b^2 + 4a^2$

Discriminant $D = B^2 - 4AC$

$= (-4a)^2 - 4.1. (-b^2 + 4a^2)$

$= 16a^2 + 4b^2 - 16a^2 = 4b^2 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{4b^2} = 2b$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-(-4a) + 2b}{2 \times 1} = \frac{4a + 2b}{2} = 2a + b$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-(-4a) - 2b}{2 \times 1} = \frac{4a - 2b}{2} = 2a - b$$

$x = (2a - b)$ or $x = (2a + b)$

Hence the roots of equation are $(2a - b)$ or $(2a + b)$

Question: 31

Find the roots of

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Solution:

Given: $4x^2 - 4a^2x + (a^4 - b^4) = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 4, B = -4a^2, C = (a^4 - b^4)$

Discriminant $D = B^2 - 4AC$

$= (-4a^2)^2 - 4.4.(a^4 - b^4)$

$= 16a^4 + 16b^4 - 16a^4 = 16b^4 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{16b^4} = 4b^2$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-(-4a^2) + 4b^2}{2 \times 4} = \frac{4(a^2 + b^2)}{8} = \frac{a^2 + b^2}{2}$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-(-4a^2) - 4b^2}{2 \times 4} = \frac{4(a^2 - b^2)}{8} = \frac{a^2 - b^2}{2}$$

$$x = \frac{a^2 + b^2}{2} \text{ or } x = \frac{a^2 - b^2}{2}$$

Hence the roots of equation are $\frac{a^2 + b^2}{2}, \frac{a^2 - b^2}{2}$

Question: 32

Solution:

Given: $4x^2 + 4bx - (a^2 - b^2) = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 4, B = 4b, C = -(a^2 - b^2)$

Discriminant $D = B^2 - 4AC$

$= (4b)^2 - 4.4. - (a^2 - b^2)$

$= 16b^2 + 16a^2 - 16b^2 = 16a^2 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{16a^2} = 4a$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-(4b) + 4a}{2 \times 4} = \frac{4(a - b)}{8} = \frac{a - b}{2}$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-(4b) - 4a}{2 \times 4} = \frac{-4(a + b)}{8} = \frac{-(a + b)}{2}$$

$$x = \frac{-(a + b)}{2} \text{ or } x = \frac{a - b}{2}$$

Hence the roots of equation are $\frac{-(a+b)}{2}$ or $\frac{a-b}{2}$

Question: 33

Find the roots of

Solution:

Given: $x^2 - (2b - 1)x + (b^2 - b - 20) = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 1, B = -(2b - 1), C = (b^2 - b - 20)$

Discriminant $D = B^2 - 4AC$

$= [-(2b - 1)^2] - 4.1.(b^2 - b - 20)$ Using $a^2 - 2ab + b^2 = (a - b)^2$

$= 4b^2 - 4b + 1 - 4b^2 + 4b + 80 = 81 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{81} = 9$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-[-(2b-1)] + 9}{2 \times 1} = \frac{2b + 8}{2} = b + 4$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-[-(2b - 1] - 9}{2 \times 1} = \frac{2b - 10}{2} = b - 5$$

$x = (b + 4)$ or $x = (b - 5)$

Hence the roots of equation are $(b + 4)$ or $(b - 5)$

Question: 34

Solution:

Given: $3a^2x^2 + 8abx + 4b^2 = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 3a^2, B = 8ab, C = 4b^2$

Discriminant $D = B^2 - 4AC$

$= (8ab)^2 - 4.3a^2 \cdot 4b^2$

$= 64a^2b^2 - 48a^2b^2 = 16a^2b^2 > 0$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{16a^2b^2}$

$= 4ab$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-8ab + 4ab}{2 \times 3a^2} = \frac{-4ab}{6a^2} = \frac{-2b}{3a}$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-8ab - 4ab}{2 \times 3a^2} = \frac{-12ab}{6a^2} = \frac{-2b}{a}$$

$$x = \frac{-2b}{3a} \text{ or } x = \frac{-2b}{a}$$

Hence the roots of equation are $\frac{-2b}{3a}$ or $x = \frac{-2b}{a}$

Question: 35

Solution:

Given: $a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = a^2b^2$, $B = -(4b^4 - 3a^4)$, $C = -12a^2b^2$

Discriminant $D = B^2 - 4AC$

$= [-(4b^4 - 3a^4)]^2 - 4a^2b^2. -12a^2b^2$

$= 16b^8 - 24a^4b^4 + 9a^8 + 48\ a^4b^4$

$= 16b^8 + 24a^4b^4 + 9a^8$

$= (4b^4 + 3a^4)^2 > 0$ Using $a^2 + 2ab + b^2 = (a + b)^2$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{(4b^4 + 3a^4)^2}$

$= 4b^4 + 3a^4$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-[-(4b^4 - 3a^4)] + (4b^4 + 3a^4)}{2 \times a^2b^2} = \frac{8b^4}{2a^2b^2} = \frac{4b^2}{a^2}$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-[-(4b^4 - 3a^4)] - (4b^4 + 3a^4)}{2 \times a^2b^2} = \frac{-6a^4}{2a^2b^2} = \frac{-3a^2}{b^2}$$

$$x = \frac{4b^2}{a^2} \text{ or } x = \frac{-3a^2}{b^2}$$

Hence the roots of equation are $\frac{4b^2}{a^2}$ or $\frac{-3a^2}{b^2}$

Question: 36

Solution:

Given: $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = 12ab$, $B = -(9a^2 - 8b^2)$, $C = -6ab$

Discriminant $D = B^2 - 4AC$

$= [-(9a^2 - 8b^2)]^2 - 4.12ab. -6ab$

$= 81a^4 - 144a^2b^2 + 64b^4 + 288\ a^2b^2$

$= 81a^4 + 144a^2b^2 + 64b^4$

$= (9a^2 + 8b^2)^2 > 0$ Using $a^2 + 2ab + b^2 = (a + b)^2$

Hence the roots of equation are real.

$\sqrt{D} = \sqrt{(9a^2 + 8b^2)^2}$

$= 9a^2 + 8b^2$

Roots α and β are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-[-(9a^2 - 8b^2)] + (9a^2 + 8b^2)}{2 \times 12ab} = \frac{18a^2}{24ab} = \frac{3a}{4b}$$

$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-[-(9a^2 - 8b^2)] - (9a^2 + 8b^2)}{2 \times 12ab} = \frac{-16a^2}{24ab} = \frac{-2b}{3a}$

$x = \frac{3a}{4b}$ or $x = \frac{-2b}{3a}$

Hence the roots of equation are $\frac{3a}{4b}$ or $\frac{-2b}{3a}$

Exercise : 10D

Question: 1 A

Solution:

Given: $2x^2 - 8x + 5 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2, b = -8, c = 5$

Discriminant $D = b^2 - 4ac$

$= (-8)^2 - 4.2.5$

$= 64 - 40 = 24 > 0$

Hence the roots of equation are real and unequal.

Question: 1 B

Solution:

Given: $3x^2 - 2\sqrt{6}x + 2 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 3\ b = -2\sqrt{6}\ c = 2$

Discriminant $D = b^2 - 4ac$

$= (-2\sqrt{6})^2 - 4.3.2$

$= 24 - 24 = 0$

Hence the roots of equation are real and equal.

Question: 1 C

Solution:

Given: $5x^2 - 4x + 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 5, b = -4, c = 1$

Discriminant $D = b^2 - 4ac$

$= (-4)^2 - 4.5.1$

$= 16 - 20 = -4 < 0$

Hence the equation has no real roots.

Question: 1 D

Solution:

Given: $5x(x - 2) + 6 = 0$

$5x^2 - 10x + 6 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 5, b = -10, c = 6$

Discriminant $D = b^2 - 4ac$

$= (-10)^2 - 4.5.6$

$= 100 - 120 = -20 < 0$

Hence the equation has no real roots.

Question: 1 E

Solution:

Given: $12x^2 - 4\sqrt{15}x + 5 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 12, b = -4\sqrt{15}, c = 5$

Discriminant $D = b^2 - 4ac$

$= (-4\sqrt{15})^2 - 4.12.5$

$= 240 - 240 = 0$

Hence the equation has real and equal roots.

Question: 1 F

Solution:

Given: $x^2 - x + 2 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 1, b = -1, c = 2$

Discriminant $D = b^2 - 4ac$

$= (-1)^2 - 4.1.2$

$= 1 - 8 = -7 < 0$

Hence the equation has no real roots.

Question: 2

Solution:

Given: $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2(a^2 + b^2), b = 2(a + b), c = 1$

Discriminant $D = b^2 - 4ac$

$= [2(a + b)]^2 - 4.\ 2\ (a^2 + b^2).1$

$= 4(a^2 + b^2 + 2ab) - 8\ a^2 - 8b^2$

$= 4a^2 + 4b^2 + 8ab - 8a^2 - 8b^2$

$= -4a^2 - 4b^2 + 8ab$

$= -4(a^2 + b^2 - 2ab)$

$= -4(a - b)^2 < 0$

Hence the equation has no real roots.

Question: 3

Solution:

Given equation $x^2 + px - q^2 = 0$

$a = 1\ b = p\ x = -q^2$

Discriminant $D = b^2 - 4ac$

$= (p)^2 - 4.1. - q^2$

$= (p^2 + 4q^2) > 0$

Thus the roots of equation are real.

Question: 4

Solution:

Given: $3x^2 + 2kx + 27 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 3\ b = 2k\ c = 27$

Given that the roots of equation are real and equal

Thus $D = 0$

Discriminant $D = b^2 - 4ac = 0$

$(2k)^2 - 4.3.27 = 0$

$4k^2 - 324 = 0$

$4k^2 = 324$

$k^2 = 81$ taking square root on both sides

$k = 9$ or $k = -9$

The values of k are 9, – 9 for which roots of the quadratic equation are real and equal.

Question: 5

Solution:

Given equation is $kx(x - 2\sqrt{5}) + 10 = 0$

$kx^2 - 2\sqrt{5}kx + 10 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = k\ b = -2\sqrt{5}k\ c = 10$

Given that the roots of equation are real and equal

Thus D = 0

Discriminant $D = b^2 - 4ac = 0$

$(-k2\sqrt{5})^2 - 4.k.10 = 0$

$20k^2 - 40k = 0$

$20k(k - 2) = 0$

$20k = 0$ or $(k - 2) = 0$

$k = 0$ or $k = 2$

The values of k are 0, 2 for which roots of the quadratic equation are real and equal.

Question: 6

Solution:

Given equation is $4x^2 + px + 3 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 4\ b = p\ c = 3$

Given that the roots of equation are real and equal

Thus D = 0

Discriminant $D = b^2 - 4ac = 0$

$(p)^2 - 4.4.3 = 0$

$p^2 = 48$

$p = \pm 4\sqrt{3}$

$p = 4\sqrt{3}$ or $p = p = -4\sqrt{3}$

The values of p are $4\sqrt{3}, -4\sqrt{3}$ for which roots of the quadratic equation are real and equal.

Question: 7

Solution:

Given equation is $9x^2 - 3kx + k = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 9\ b = -3k\ c = k$

Given that the roots of equation are real and equal

Thus D = 0

Discriminant $D = b^2 - 4ac = 0$

$(-3k)^2 - 4.9.k = 0$

$9k^2 - 36k = 0$

$9k(k - 4) = 0$

$9k = 0$ or $(k - 4) = 0$

k = 0 or k = 4

But given k is non zero hence k = 4 for which roots of the quadratic equation are r

Question: 8

Solution:

Given equation is $(3k + 1)x^2 + 2(k + 1)x + 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = (3k + 1)$ $b = 2(k + 1)$ $c = 1$

Given that the roots of equation are real and equal

Thus D = 0

Discriminant $D = b^2 - 4ac = 0$

$(2k + 2)^2 - 4.(3k + 1).1 = 0$ using $(a + b)^2 = a^2 + 2ab + b^2$

$4k^2 + 8k + 4 - 12k - 4 = 0$

$4k^2 - 4k = 0$

$4k(k - 1) = 0$

$k = 0$ $(k - 1) = 0$

$k = 0$ $k = 1$

The values of k are 0, 1 for which roots of the quadratic equation are real and equal.

Question: 9

Solution:

Given equation is $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = (2p + 1)$ $b = -(7p + 2)$ $c = (7p - 3)$

Given that the roots of equation are real and equal

Thus D = 0

Discriminant $D = b^2 - 4ac = 0$

$[-(7p + 2)]^2 - 4.(2p + 1).(7p - 3) = 0$ using $(a + b)^2 = a^2 + 2ab + b^2$

$(49p^2 + 28p + 4) - 4(14p^2 + p - 3) = 0$

$49p^2 + 28p + 4 - 56p^2 - 4p + 12 = 0$

$-7p^2 + 24p + 16 = 0$

$7p^2 - 24p - 16 = 0$

$7p^2 - 28p + 4p - 16 = 0$

$7p(p - 4) + 4(p - 4) = 0$

$(7p + 4)(p - 4) = 0$

$(7p + 4) = 0$ or $(p - 4) = 0$

$p = \frac{-4}{7}$ or $p = 4$

The values of p are $\frac{-4}{7}$ or 4 for which roots of the quadratic equation are real and e

Question: 10

Solution:

Given equation is $(p + 1)x^2 - 6(p + 1)x + 3(p + 9) = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = (p + 1)$ $b = -6(p + 1)$ $c = 3(p + 9)$

Given that the roots of equation are equal

Thus D = 0

Discriminant $D = b^2 - 4ac = 0$

$[-6(p + 1)]^2 - 4.(p + 1).3(p + 9) = 0$

$36(p + 1)(p + 1) - 12(p + 1)(p + 9) = 0$

$12(p + 1)[3(p + 1) - (p + 9)] = 0$

$12(p + 1)[3p + 3 - p - 9] = 0$

$12(p + 1)[2p - 6] = 0$

$(p + 1) = 0$ or $[2p - 6] = 0$

$p = -1$ or $p = 3$

The values of p are – 1, 3 for which roots of the quadratic equation are real and equal.

Question: 11

Solution:

Given that – 5 is a root of the quadratic equation $2x^2 + px - 15 = 0$

$2(-5)^2 - 5p - 15 = 0$

$5p = 35$

$p = 7$

Given equation is $p(x^2 + x) + k = 0$

$px^2 + px + k = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = p$ $b = p$ $c = k$

Given that the roots of equation are equal

Thus D = 0

Discriminant $D = b^2 - 4ac = 0$

$[p]^2 - 4.p.k = 0$

$7^2 - 28k = 0$

$49 - 28k = 0$

$$k = \frac{49}{28} = \frac{7}{4}$$

The value of k is $\frac{7}{4}$ for which roots of the quadratic equation are equal.

Question: 12

Solution:

Given 3 is a root of the quadratic equation $x^2 - x + k = 0$

$(3)^2 - 3 + k = 0$

$k + 6 = 0$

$k = -6$

Given equation is $x^2 + k(2x + k + 2) + p = 0$

$x^2 + 2kx + (k^2 + 2k + p) = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 1$ $b = 2k$ $c = k^2 + 2k + p$

Given that the roots of equation are equal

Thus $D = 0$

Discriminant $D = b^2 - 4ac = 0$

$(2k)^2 - 4.1.(k^2 + 2k + p) = 0$

$4k^2 - 4k^2 - 8k - 4p = 0$

$-8k - 4p = 0$

$4p = -8k$

$p = -2k$

$p = -2. -6 = 12$

$p = 12$

The value of p is – 12 for which roots of the quadratic equation are equal.

Question: 13

Solution:

Given – 4 is a root of the equation $x^2 + 2x + 4p = 0$

$(-4)^2 + 2(-4) + 4p = 0$

$8 + 4p = 0$

$p = -2$

The quadratic equation $x^2 + px(1 + 3k) + 7(3 + 2k) = 0$ has equal roots

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 1$ $b = p(1 + 3k)$ $c = 7(3 + 2k)$

Thus $D = 0$

Discriminant $D = b^2 - 4ac = 0$

$[p(1 + 3k)]^2 - 4.1.7(3 + 2k) = 0$

$[-2(1 + 3k)]^2 - 4.1.7(3 + 2k) = 0$

$4(1 + 6k + 9k^2) - 4.7(3 + 2k) = 0$ using $(a + b)^2 = a^2 + 2ab + b^2$

$4(1 + 6k + 9k^2 - 21 - 14k) = 0$

$9k^2 - 8k - 20 = 0$

$9k^2 - 18k - 10k - 20 = 0$

$9k(k - 2) + 10(k - 2) = 0$

$(9k + 10)(k - 2) = 0$

$$k = \frac{-10}{9} \text{ or } k = 2$$

The value of k is $\frac{-10}{9}$ or 2 for which roots of the quadratic equation are equal.

Question: 14

Solution:

The quadratic equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = (1 + m^2)$ $b = 2mc$ $c = c^2 - a^2$

Thus D = 0

Discriminant $D = b^2 - 4ac = 0$

$(2mc)^2 - 4.(1 + m^2)(c^2 - a^2) = 0$

$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$-4c^2 + 4a^2 + 4m^2a^2 = 0$

$a^2 + m^2a^2 = c^2$

$c^2 = a^2(1 + m^2)$

Hence proved

Question: 15

Solution:

Given that the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are real and equal

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = (c^2 - ab)$ $b = -2(a^2 - bc)$ $c = (b^2 - ac)$

Thus D = 0

Discriminant $D = b^2 - 4ac = 0$

$[-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$4(a^4 - 2a^2bc + b^2c^2) - 4(b^2c^2 - ac^3 - ab^3 + a^2bc) = 0$

using $(a - b)^2 = a^2 - 2ab + b^2$

$a^4 - 2a^2bc + b^2c^2 - b^2c^2 + ac^3 + ab^3 - a^2bc = 0$

$a^4 - 3a^2bc + ac^3 + ab^3 = 0$

$a(a^3 - 3abc + c^3 + b^3) = 0$

$a = 0$ or $(a^3 - 3abc + c^3 + b^3) = 0$

Hence proved $a = 0$ or $a^3 + c^3 + b^3 = 3abc$

Question: 16

CLASS24

Solution:

Given that the quadratic equation $2x^2 + px + 8 = 0$ has real roots

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 2$ $b = p$ $c = 8$

Thus D = 0

Discriminant $D = b^2 - 4ac \geq 0$

$(p)^2 - 4.2.8 \geq 0$

$(p)^2 - 64 \geq 0$

$p^2 \geq 64$ taking square root on both sides

$p \geq 8$ or $p \leq -8$

The roots of equation are real for $p \geq 8$ or $p \leq -8$

Question: 17

Solution:

Given that the quadratic equation $(a - 12)x^2 + 2(a - 12)x + 2 = 0$ has equal roots

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = (a - 12)$ $B = 2(a - 12)$ $C = 2$

Thus D = 0

Discriminant $D = B^2 - 4AC \geq 0$

$[2(a - 12)]^2 - 4(a - 12)2 \geq 0$

$4(a^2 + 144 - 24a) - 8a + 96 = 0$ using $(a - b)^2 = a^2 - 2ab + b^2$

$4a^2 + 576 - 96a - 8a + 96 = 0$

$4a^2 - 104a + 672 = 0$

$a^2 - 26a + 168 = 0$

$a^2 - 14a - 12a + 168 = 0$

$a(a - 14) - 12(a - 14) = 0$

$(a - 14)(a - 12) = 0$

$a = 14$ or $a = 12$

for $a = 12$ the equation will become non quadratic – – $(a - 12)x^2 + 2(a - 12)x + 2 = 0$

A, B will become zero

Thus value of $a = 14$ for which the equation has equal roots.

Question: 18

Solution:

Given that the quadratic equation $9x^2 + 8kx + 16 = 0$ roots are real and equal.

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 9$ $b = 8k$ $c = 16$

Thus $D = 0$

Discriminant $D = b^2 - 4ac = 0$

$(8k)^2 - 4.9.16 = 0$

$64k^2 - 576 = 0$

$k^2 = 9$ taking square root both sides

$k = \pm 3$

Thus $k = 3$ or $k = -3$ for which the roots are real and equal.

Question: 19

Solution:

(i) Given: $kx^2 + 6x + 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = k$ $b = 6$ $c = 1$

For real and distinct roots: $D > 0$

Discriminant $D = b^2 - 4ac > 0$

$6^2 - 4k > 0$

$36 - 4k > 0$

$4k < 36$

$k < 9$

(ii) Given: $x^2 - kx + 9 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 1$ $b = -k$ $c = 9$

For real and distinct roots: $D > 0$

Discriminant $D = b^2 - 4ac > 0$

$(-k)^2 - 4.1.9 = k^2 - 36 > 0$

$k^2 > 36$

$k > 6$or $k < -6$ taking square root both sides

(iii) $9x^2 + 3kx + 4 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 9$ $b = 3k$ $c = 4$

For real and distinct roots: $D > 0$

Discriminant $D = b^2 - 4ac > 0$

$(3k)^2 - 4.4.9 = 9k^2 - 144 > 0$

$9k^2 > 144$

$k^2 > 16$

$k > 4$or < -4 taking square root both sides

(iv) $5x^2 - kx + 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = 5$ $b = -k$ $c = 1$

For real and distinct roots: $D > 0$

Discriminant $D = b^2 - 4ac > 0$

$(-k)^2 - 4.5.1 = k^2 - 20 > 0$

$k^2 > 20$

$k > 2\sqrt{5}$ or $k < -2\sqrt{5}$ taking square root both sides

Question: 20

Solution:

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = (a - b)$ $b = 5(a + b)$ $c = -2(a - b)$

Discriminant $D = b^2 - 4ac$

$= [5(a + b)]^2 - 4(a - b)(-2(a - b))$

$= 25(a + b)^2 + 8(a - b)^2$

Since a and b are real and $a \neq b$ then $(a + b)^2 > 0$ $(a - b)^2 > 0$

$8(a - b)^2 > 0$ – – – – (1) product of two positive numbers is always positive

$25(a + b)^2 > 0$ – – – – (2) product of two positive numbers is always positive

Adding (1) and (2) we get

$8(a - b)^2 + 25(a + b)^2 > 0$ (sum of two positive numbers is always positive)

$D > 0$

Hence the roots of given equation are real and unequal.

Question: 21

Solution:

Given the roots of the equation are equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal.

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$a = (a^2 + b^2)$ $b = -2(ac + bd)$ $c = (c^2 + d^2)$

For real and distinct roots: $D = 0$

Discriminant $D = b^2 - 4ac = 0$

$[-2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$

$4(a^2c^2 + b^2d^2 + 2abcd) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) = 0$

using $(a + b)^2 = a^2 + 2ab + b^2$

$4(a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2) = 0$

$2abcd - a^2d^2 - b^2c^2 = 0$

$-(2abcd + a^2d^2 + b^2c^2) = 0$

$(ad - bc)^2 = 0$

$ad = bc$

$\frac{a}{b} = \frac{c}{d}$

Hence proved.

Question: 22

Solution:

Given the roots of the equations $ax^2 + 2bx + c = 0$ are real.

Comparing with standard quadratic equation $Ax^2 + Bx + C = 0$

$A = a$ $B = 2b$ $C = c$

Discriminant $D_1 = B^2 - 4AC \geq 0$

$= (2b)^2 - 4.a.c \geq 0$

$= 4(b^2 - ac) \geq 0$

$= (b^2 - ac) \geq 0$ - - - - - (1)

For the equation $bx^2 - 2\sqrt{ac}x + b = 0$

Discriminant $D_2 = b^2 - 4ac \geq 0$

$= (-2\sqrt{ac})^2 - 4.b.b \geq 0$

$= 4(ac - b^2) \geq 0$

$= - 4(b^2 - ac) \geq 0$

$= (b^2 - ac) \geq 0$ - - - - - (2)

The roots of the are simultaneously real if (1) and (2) are true together

$b^2 - ac = 0$

$b^2 = ac$

Hence proved.

Exercise : 10E

Question: 1

Solution:

Let the required number be x

According to given condition,

$x + x^2 = 156$

$x^2 + x - 156 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 1 c = – 156

$= 1. - 156 = - 156$

And either of their sum or difference = b

= 1

Thus the two terms are 13 and – 12

Sum = 13 – 12 = 1

Product = 13. – 12 = – 156

$x^2 + x - 156 = 0$

$x^2 + 13x - 12x - 156 = 0$

$x(x + 13) - 12(x + 13) = 0$

$(x - 12)(x + 13) = 0$

x = 12 or x = – 13

x cannot be negative

Hence the required natural number is 12

Question: 2

Solution:

Let the required number be x

According to given condition,

$x + \sqrt{x} = 132$

putting $\sqrt{x} = y$ or $x = y^2$ we get

$y^2 + y = 132$

$y^2 + y - 132 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 1 c = – 132

= 1. – 132 = – 132

And either of their sum or difference = b

= 1

Thus the two terms are 12 and – 11

Difference = 12 – 11 = 1

Product = 12. – 11 = – 132

$y^2 + y - 132 = 0$

$y^2 + 12y - 11y - 132 = 0$

$y(y + 12) - 11(y + 12) = 0$

$(y + 12)(y - 11) = 0$

$(y + 12) = 0$ or $(y - 11) = 0$

y = – 12 or y = 11 but y cannot be negative

Thus y = 11

Now $\sqrt{x} = y$

x = y squaring both sides

$x = (11)^2 = 121$

Hence the required number is 121

Question: 3

Solution:

Let the required number be x and 28 – x

According to given condition,

$x(28 - x) = 192$

$x^2 - 28x + 192 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 28 c = 192

= 1.192 = 192

And either of their sum or difference = b

= – 28

Thus the two terms are – 16 and – 12

Sum = – 16 – 12 = – 28

Product = – 16. – 12 = 192

$x^2 - 28x + 192 = 0$

$x^2 - 16x - 12x + 192 = 0$

$x(x - 16) - 12(x - 16) = 0$

$(x - 16)(x - 12) = 0$

$(x - 16) = 0$ or $(x - 12) = 0$

x = 16 or x = 12

Hence the required numbers are 16, 12

Question: 4

Solution:

Let the required two consecutive positive integers be x and x + 1

According to given condition,

$x^2 + (x + 1)^2 = 365$

$x^2 + x^2 + 2x + 1 = 365$ using $(a + b)^2 = a^2 + 2ab + b^2$

$2x^2 + 2x - 364 = 0$

$x^2 + x - 182 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 1 c = – 182

= 1. – 182 = – 182

And either of their sum or difference = b

= 1

Thus the two terms are 14 and – 13

Difference = 14 – 13 = 1

Product = 14. – 13 = – 182

$x^2 + x - 182 = 0$

$x^2 + 14x - 13x - 182 = 0$

$x(x + 14) - 13(x + 14) = 0$

$(x + 14)(x - 13) = 0$

$(x + 14) = 0$ or $(x - 13) = 0$

$x = -14$ or $x = 13$

x = 13 (x is a positive integer)

$x + 1 = 13 + 1 = 14$

Thus the required two consecutive positive integers are 13, 14

Question: 5

Solution:

Let the two consecutive positive odd numbers be x and x + 2

According to given condition,

$x^2 + (x + 2)^2 = 514$

$x^2 + x^2 + 4x + 4 = 514$ using $(a + b)^2 = a^2 + 2ab + b^2$

$2x^2 + 4x - 510 = 0$

$x^2 + 2x - 255 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 2 c = – 255

= 1. – 255 = – 255

And either of their sum or difference = b

= 2

Thus the two terms are 17 and – 15

Difference = 17 – 15 = 2

Product = 17. – 15 = – 255

$x^2 + 2x - 255 = 0$

$x^2 + 17x - 15x - 255 = 0$

$x(x + 17) - 15(x + 17) = 0$

(x + 17) (x – 15) = 0

(x + 17) = 0 or (x – 15) = 0

x = – 17 or x = 15

x = 15 (x is positive odd number)

x + 2 = 15 + 2 = 17

Thus the two consecutive positive odd numbers are 15 and 17

Question: 6

Solution:

Let the two consecutive positive even numbers be x and (x + 2)

According to given condition,

$x^2 + (x + 2)^2 = 452$

$x^2 + x^2 + 4x + 4 = 452$ using $(a + b)^2 = a^2 + 2ab + b^2$

$2x^2 + 4x - 448 = 0$

$x^2 + 2x - 224 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 2 c = – 224

= 1. – 224 = – 224

And either of their sum or difference = b

= 2

Thus the two terms are 16 and – 14

Difference = 16 – 14 = 2

Product = 16. – 14 = – 224

$x^2 + 2x - 224 = 0$

$x^2 + 16x - 14x - 224 = 0$

x(x + 16) – 14(x + 16) = 0

(x + 16) (x – 14) = 0

(x + 16) = 0 or (x – 14) = 0

x = – 16 or x = 14

x = 14 (x is positive odd number)

x + 2 = 14 + 2 = 16

Thus the two consecutive positive even numbers are 14 and 16

Question: 7

Solution:

Let the two consecutive positive integers be x and (x + 1)

According to given condition,

$x(x + 1) = 306$

$x^2 + x - 306 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 1 c = – 306

= 1. – 306 = – 306

And either of their sum or difference = b

= 1

Thus the two terms are 18 and – 17

Difference = 18 – 17 = 1

Product = 18. – 17 = – 306

$x^2 + x - 306 = 0$

$x^2 + 18x - 17x - 306 = 0$

$x(x + 18) - 17(x + 18) = 0$

$(x + 18)(x - 17) = 0$

$(x + 18) = 0$ or $(x - 17) = 0$

$x = -18$ or $x = 17$

but x = 17 (x is a positive integers)

$x + 1 = 17 + 1 = 18$

Thus the two consecutive positive integers are 17 and 18

Question: 8

Solution:

Let the two natural numbers be x and (x + 3)

According to given condition,

$x(x + 3) = 504$

$x^2 + 3x - 504 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 3 c = – 504

= 1. – 504 = – 504

And either of their sum or difference = b

= 3

Thus the two terms are 24 and – 21

Difference = 24 – 21 = 3

Product = 24. – 21 = – 504

$x^2 + 3x - 504 = 0$

$x^2 + 24x - 21x - 504 = 0$

$x(x + 24) - 21(x + 24) = 0$

$(x + 24)(x - 21) = 0$

$(x + 24) = 0$ or $(x - 21) = 0$

$x = -24$ or $x = 21$

Case I: $x = 21$

$x + 3 = 21 + 3 = 24$

The numbers are (21, 24)

Case I: $x = -24$

$x + 3 = -24 + 3 = -21$

The numbers are (– 24, – 21)

Question: 9

Solution:

Let the required consecutive multiples of 3 be 3x and 3(x + 1)

According to given condition,

$3x.3(x + 1) = 648$

$9(x^2 + x) = 648$

$x^2 + x = 72$

$x^2 + x - 72 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 1 c = – 72

= 1. – 72 = – 72

And either of their sum or difference = b

= 1

Thus the two terms are 9 and – 8

Difference = 9 – 8 = 1

Product = 9. – 8 = – 72

$x^2 + 9x - 8x - 72 = 0$

$x(x + 9) - 8(x + 9) = 0$

$(x + 9)(x - 8) = 0$

$(x + 9) = 0$ or $(x - 8) = 0$

$x = -9$ or $x = 8$

x = 8 (rejecting the negative values)

$3x = 3.8 = 24$

$3(x + 1) = 3(8 + 9) = 3.9 = 27$

Hence, the required numbers are 24 and 27

Question: 10

Solution:

Let the required consecutive positive odd integers be x and (x + 2)

According to given condition,

$x(x + 2) = 483$

$x^2 + 2x - 483 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 2 c = – 483

= 1. – 483 = – 483

And either of their sum or difference = b

= 2

Thus the two terms are 23 and – 21

Difference = 23 – 21 = 2

Product = 23. – 21 = – 483

$x^2 + 2x - 483 = 0$

$x^2 + 23x - 21x - 483 = 0$

$x(x + 23) - 21(x + 23) = 0$

$(x + 23)(x - 21) = 0$

$(x + 23) = 0$ or $(x - 21) = 0$

$x = -23$ or $x = 21$

x = 21 (x is a positive odd integer)

$x + 2 = 21 + 2 = 23$

Hence, the required integers are 21 and 23

Question: 11

Solution:

Let the two consecutive positive even integers be x and (x + 2)

According to given condition,

$x(x + 2) = 288$

$x^2 + 2x - 288 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 2 c = – 288

= 1. – 288 = – 288

And either of their sum or difference = b

= 2

Thus the two terms are 18 and – 16

Difference = 18 – 16 = 2

Product = 18. – 16 = – 288

$x^2 + 18x - 16x - 288 = 0$

x (x + 18) – 16(x + 18) = 0

(x + 18) (x – 16) = 0

(x + 18) = 0 or (x – 16) = 0

x = – 18 or x = 16

x = 16 (x is a positive odd integer)

x + 2 = 16 + 2 = 18

Hence, the required integers are 16 and 18

Question: 12

Solution:

Let the required natural numbers x and (9 – x)

According to given condition,

$$\frac{1}{x} + \frac{1}{9-x} = \frac{1}{2}$$

$\frac{9-x+x}{x(9-x)} = \frac{1}{2}$ taking LCM

$$\frac{9}{9x - x^2} = \frac{1}{2}$$

$9x - x^2 = 18$ cross multiplying

$$x^2 - 9x + 18 = 0$$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 9 c = 18

= 1.18 = 18

And either of their sum or difference = b

= – 9

Thus the two terms are – 6 and – 3

Sum = – 6 – 3 = – 9

Product = – 6. – 3 = 18

$$x^2 - 9x + 18 = 0$$

$$x^2 - 6x - 3x + 18 = 0$$

x(x – 6) – 3(x – 6) = 0

(x – 6) (x – 3) = 0

(x – 6) = 0 or (x – 3) = 0

x = 6 or x = 3

Case I: when x = 6

9 – x = 9 – 6 = 3

Case II: when x = 3

9 – x = 9 – 3 = 6

Hence required numbers are 3 and 6.

Question: 13

Solution:

Let the required natural numbers x and (15 – x)

According to given condition,

$$\frac{1}{x} + \frac{1}{15 - x} = \frac{3}{10}$$

taking LCM

$$\frac{15 - x + x}{x(15 - x)} = \frac{3}{10}$$

cross multiplying

$$\frac{15}{15x - x^2} = \frac{3}{10}$$

$$15x - x^2 = 50$$

$$x^2 - 15x + 50 = 0$$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 15 c = 50

= 1.50 = 50

And either of their sum or difference = b

= – 15

Thus the two terms are – 10 and – 5

Sum = – 10 – 5 = – 15

Product = – 10. – 5 = 50

$$x^2 - 10x - 5x + 50 = 0$$

x(x – 10) – 5(x – 10) = 0

(x – 5) (x – 10) = 0

(x – 5) = 0 or (x – 10) = 0

x = 5 or x = 10

Case I: when x = 5

15 – x = 15 – 5 = 10

Case II: when x = 10

15 – x = 15 – 10 = 5

Hence required numbers are 5 and 10.

Question: 14

Solution:

Let the required natural numbers x and (x + 3)

$x < x + 3$

Thus $\frac{1}{x} > \frac{1}{x+3}$

According to given condition,

$$\frac{1}{x} - \frac{1}{x+3} = \frac{3}{28}$$

taking LCM

$$\frac{x+3-x}{x(x+3)} = \frac{3}{28}$$

$$\frac{3}{x^2+3x} = \frac{3}{28}$$

cross multiplying

$x^2 + 3x = 28$

$x^2 + 3x - 28 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 3 c = – 28

= 1. – 28 = – 28

And either of their sum or difference = b

= 3

Thus the two terms are 7 and – 4

Difference = 7 – 4 = 3

Product = 7. – 4 = – 28

$x^2 + 3x - 28 = 0$

$x^2 + 7x - 4x - 28 = 0$

x(x + 7) – 4(x + 7) = 0

(x – 4) (x + 7) = 0

(x – 4) = 0 or (x + 7) = 0

x = 4 or x = – 7

x = 4 (x < x + 3)

x + 3 = 4 + 3 = 7

Hence required numbers are 4 and 7.

Question: 15

Solution:

Let the required natural numbers x and (x + 5)

$x < x + 5$

Thus $\frac{1}{x} > \frac{1}{x+5}$

According to given condition,

$$\frac{1}{x} - \frac{1}{x+5} = \frac{5}{14}$$

taking LCM

$$\frac{x + 5 - x}{x(x+5)} = \frac{5}{14}$$

$$\frac{5}{x^2 + 5x} = \frac{5}{14}$$

cross multiplying

$x^2 + 5x = 14$

$x^2 + 5x - 14 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 5 c = – 14

= 1. – 14 = – 14

And either of their sum or difference = b

= 5

Thus the two terms are 7 and – 2

Difference = 7 – 2 = 5

Product = 7. – 2 = – 14

$x^2 + 7x - 2x - 14 = 0$

x (x + 7) – 2(x + 7) = 0

(x – 2) (x + 7) = 0

(x – 2) = 0 or (x + 7) = 0

x = 2 or x = – 7

x = 2 (x < x + 3)

x + 5 = 2 + 5 = 7

Hence required natural numbers are 2 and 7.

Question: 16

Solution:

Let the required consecutive multiples of 7 be 7x and 7(x + 1)

According to given condition,

$(7x)^2 + [7(x + 1)]^2 = 1225$

$49x^2 + 49(x^2 + 2x + 1) = 1225$ using $(a + b)^2 = a^2 + 2ab + b^2$

$49x^2 + 49x^2 + 98x + 49 = 1225$

$98x^2 + 98x - 1176 = 0$

$x^2 + x - 12 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 1 c = – 12

= 1. – 12 = – 12

And either of their sum or difference = b

= 1

Thus the two terms are 4 and – 3

Difference = 4 – 3 = 1

Product = 4. – 3 = – 12

$x^2 + 4x - 3x - 12 = 0$

$x(x + 4) - 3(x + 4) = 0$

$(x - 3)(x + 4) = 0$

$(x - 3) = 0$ or $(x + 4) = 0$

$x = 3$ or $x = -4$

when x = 3,

$7x = 7.3 = 21$

$7(x + 1) = 7(3 + 1) = 7.4 = 28$

Hence required multiples are 21, 28.

Question: 17

Solution:

Let the required natural numbers x

According to given condition,

$$x + \frac{1}{x} = \frac{65}{8}$$

$$\frac{x^2 + 1}{x} = \frac{65}{8}$$

$$8x^2 + 8 = 65x$$

$$8x^2 - 65x + 8 = 0$$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 8 b = – 65 c = 8

= 8.8 = 64

And either of their sum or difference = b

= – 65

Thus the two terms are – 64 and – 1

Difference = – 64 – 1 = – 65

Product = – 64. – 1 = 64

$8x^2 - 64x - x + 8 = 0$

8x (x – 8) – 1(x – 8) = 0

(x – 8) (8x – 1) = 0

(x – 8) = 0 or (8x – 1) = 0

x = 8 or x = 1/8

x = 8 (x is a natural number)

Hence the required number is 8.

Question: 18

Solution:

Let the two consecutive positive even integers be x and (57 – x)

According to given condition,

x (57 – x) = 680

$57x - x^2 = 680$

$x^2 - 57x - 680 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 57 c = – 680

= 1. – 680 = – 680

And either of their sum or difference = b

= – 57

Thus the two terms are – 40 and – 17

Sum = – 40 – 17 = – 57

Product = – 40. – 17 = – 680

$x^2 - 57x - 680 = 0$

$x^2 - 40x - 17x - 680 = 0$

x (x – 40) – 17(x – 40) = 0

(x – 40) (x – 17) = 0

(x – 40) = 0 or (x – 17) = 0

x = 40 or x = 17

When x = 40

57 – x = 57 – 40 = 17

When x = 17

57 – x = 57 – 17 = 40

Hence the required parts are 17 and 40.

CLASS24

Question: 19

Solution:

Let the two parts be x and (27 – x)

According to given condition,

$$\frac{1}{x} + \frac{1}{27 - x} = \frac{3}{20}$$

$$\frac{27 - x + x}{x(27 - x)} = \frac{3}{20}$$

On taking the LCM

$$\frac{27}{27x - x^2} = \frac{3}{20}$$

$$27x - x^2 = 180$$

On Cross multiplying

$$x^2 - 27x + 180 = 0$$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 27 c = 180

= 1. – 180 = – 180

And either of their sum or difference = b

= – 27

Thus the two terms are – 15 and – 12

Sum = – 15 – 12 = – 27

Product = – 15. – 12 = 180

$$x^2 - 15x - 12x + 180 = 0$$

x (x – 15) – 12(x – 15) = 0

(x – 15) (x – 12) = 0

(x – 15) = 0 or (x – 12) = 0

x = 15 or x = 12

Case I: when x = 12

27 – x = 27 – 12 = 15

Case II: when x = 15

27 – x = 27 – 15 = 12

Hence required numbers are 12 and 15.

Question: 20

Solution:

Let the larger and the smaller parts be x and y respectively.

According to the question

$x + y = 16$ – – – – – (1)

$2x^2 = y^2 + 164$ – – – (2)

From (1) $x = 16 - y$ – – – (3)

From (2) and (3) we get

$2(16 - y)^2 = y^2 + 164$

$2(256 - 32y + y^2) = y^2 + 164$ using $(a + b)^2 = a^2 + 2ab + b^2$

$512 - 64y + 2y^2 = y^2 + 164$

$y^2 - 64y + 348 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation $a = 1$ $b = -64$ $c = 348$

$= 1.348 = 348$

And either of their sum or difference = b

$= -64$

Thus the two terms are – 58 and – 6

Sum $= -58 - 6 = -64$

Product $= -58. -6 = 348$

$y^2 - 64y + 348 = 0$

$y^2 - 58y - 6y + 348 = 0$

$y(y - 58) - 6(y - 58) = 0$

$(y - 58)(y - 6) = 0$

$(y - 58) = 0$ or $(y - 6) = 0$

$y = 6$ $(y < 16)$

putting the value of y in (3), we get

$x = 16 - 6$

$= 10$

Hence the two natural numbers are 6 and 10.

Question: 21

Solution:

Let the two natural numbers be x and y.

According to the question

$x^2 + y^2 = 25(x + y)$ – – – – – (1)

$x^2 + y^2 = 50(x - y)$ – – – – (2)

From (1) and (2) we get

$25(x + y) = 50(x - y)$

$x + y = 2(x - y)$

x + y = 2x – 2y

y + 2y = 2x – x

3y = x – – – – – (3)

From (2) and (3) we get

$(3y)^2 + y^2 = 50(3y – y)$

$9y^2 + y^2 = 50(3y – y)$

$10 y^2 = 100y$

y = 10

From (3) we have,

x = 3y = 3.10 = 30

Hence the two natural numbers are 30 and 10.

Question: 22

Solution:

Let the larger number be x and smaller number be y.

According to the question

$x^2 – y^2 = 45$ – – – – – (1)

$y^2 = 4x$ – – – – – – (2)

From (1) and (2) we get

$x^2 – 4x = 45$

$x^2 – 4x – 45 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 4 c = – 45

= 1. – 45 = – 45

And either of their sum or difference = b

= – 4

Thus the two terms are – 9 and 5

Sum = – 9 + 5 = – 4

Product = – 9.5 = – 45

$x^2 – 9x + 5x – 45 = 0$

x(x – 9) + 5(x – 9) = 0

(x + 5) (x – 9) = 0

(x + 5) = 0 or (x – 9) = 0

x = – 5 or x = 9

x = 9

putting the value of x in equation (2), we get

$y^2 = 4.9 = 36$

taking square root

$y = 6$

Hence the two numbers are 9 and 6

Question: 23

Solution:

Let the three consecutive positive integers be x, x + 1, x + 2

According to the given condition,

$x^2 + (x + 1)(x + 2) = 46$

$x^2 + x^2 + 3x + 2 = 46$

$2x^2 + 3x - 44 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 2 b = 3 c = – 44

= 2. – 44 = – 88

And either of their sum or difference = b

= 3

Thus the two terms are 11 and – 8

Sum = 11 – 8 = 3

Product = 11. – 8 = – 88

$2x^2 + 3x - 44 = 0$

$2x^2 + 11x - 8x - 44 = 0$

$x(2x + 11) - 4(2x + 11) = 0$

$(2x + 11)(x - 4) = 0$

$x = 4$ or $- 11/2$

x = 4 (x is a positive integers)

When x = 4

$x + 1 = 4 + 1 = 5$

$x + 2 = 4 + 2 = 6$

Hence the required integers are 4, 5, 6

Question: 24

Solution:

Let the digits at units and tens places be x and y respectively.

Original number = 10y + x

According to the question

$10y + x = 4(x + y)$

$10y + x = 4x + 4y$

$3x - 6y = 0$

$x = 2y$ - - - - (1)

also,

$10y + x = 2xy$

Using (1)

$10y + 2y = 2.2y.y$

$12y = 4y^2$

$y = 3$

From (1) we get

$x = 2.3 = 6$

Original number = $10y + x$

= $(10.3) + 6 = 36$

Question: 25

Solution:

Let the digits at units and tens place be x and y respectively

$xy = 14$

$y = \frac{14}{x}$ - - - - (1)

According to the question

$(10y + x) + 45 = 10x + y$

$9y - 9x = -45$

$y - x = -5$ - - - - - (2)

From (1) and (2) we get

$$\frac{14}{x} - x = -5$$

$$\frac{14 - x^2}{x} = -5$$

$14 - x^2 = -5x$

$x^2 - 5x - 14 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 5 c = – 14

= 1. – 14 = – 14

And either of their sum or difference = b

= – 5

Thus the two terms are – 7 and 2

Difference = – 7 + 2 = – 5

Product = – 7.2 = – 14

$x^2 - 5x - 14 = 0$

$x^2 - 7x + 2x - 14 = 0$

$x(x - 7) + 2(x - 7) = 0$

$(x + 2)(x - 7) = 0$

$x = 7$ or $x = -2$

$x = 7$ (neglecting the negative part)

Putting $x = 7$ in equation (1) we get

$y = 2$

Required number = 10.2 + 7 = 27

Question: 26

Solution:

Let the numerator be x

Denominator = x + 3

Original number = $\frac{x}{x+3}$

$$\frac{x}{x+3} + \frac{1}{\frac{x}{x+3}} = 2\frac{9}{10}$$

On taking the LCM

$$\frac{x}{x+3} + \frac{x+3}{x} = \frac{29}{10}$$

$$\frac{x^2 + (x+3)^2}{x(x+3)} = \frac{29}{10}$$

$$\frac{x^2 + x^2 + 6x + 9}{x^2 + 3x} = \frac{29}{10} \{\text{using } (a+b)^2 = a^2 + 2ab + b^2\}$$

$$\frac{2x^2 + 6x + 9}{x^2 + 3x} = \frac{29}{10}$$

$29x^2 + 87x = 20x^2 + 60x + 90$

$9x^2 + 27x - 90 = 0$

$9(x^2 + 3x - 10) = 0$

$x^2 + 3x - 10 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 3 c = – 10

= 1. – 10 = – 10

And either of their sum or difference = b

= 3

Thus the two terms are 5 and – 2

Difference = 5 – 2 = 3

Product = 5. – 2 = – 10

$x^2 + 5x - 2x - 10 = 0$

$x(x + 5) - 2(x + 5) = 0$

$(x + 5)(x - 2) = 0$

$(x + 5) = 0$ or $(x - 2) = 0$

$x = 2$ or $x = -5$

$x = 2$ (rejecting the negative value)

So numerator is 2

Denominator $= x + 3 = 2 + 3 = 5$

So required fraction is 2/5

Question: 27

Solution:

Let the denominator of required fraction be x

Numerator of required fraction be $= x - 3$

Original number $= \frac{x-3}{x}$

If 1 is added to the denominator, then the new fraction will become $\frac{x-3}{x+1}$

According to the given condition,

$$\frac{x-3}{x+1} = \frac{x-3}{x} - \frac{1}{15}$$

$$\frac{x-3}{x+1} - \frac{x-3}{x} = \frac{1}{15}$$

$$\frac{(x-3)(x+1) - x(x-3)}{x(x+1)} = \frac{1}{15}$$

$$\frac{x^2 - 2x - 3 - x^2 + 3x}{x^2 + x} = \frac{1}{15}$$

$$\frac{x-3}{x^2 + x} = \frac{1}{15}$$

$x^2 + x = 15x - 45$

$x^2 - 14x + 45 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation $a = 1$ $b = -14$ $c = 45$

$= 1.45 = 45$

And either of their sum or difference = b

$= -14$

Thus the two terms are -9 and -5

Sum $= -9 - 5 = -14$

Product $= -9. -5 = -45$

$x^2 - 14x + 45 = 0$

$x^2 - 9x - 5x + 45 = 0$

$x(x - 9) - 5(x - 9) = 0$

$(x - 9)(x - 5) = 0$

x = 9 or x = 5

Case I: x = 5

$\frac{x-3}{x} = \frac{5-3}{5} = \frac{2}{5}$

Case II: x = 9

$\frac{x-3}{x} = \frac{9-3}{9} = \frac{6}{9} = \frac{2}{3}$ (Rejected because this does not satisfy the condition given)

Hence the required fraction is $\frac{2}{5}$

Question: 28

Solution:

Let the required number be x.

According to the given condition,

$x + \frac{1}{x} = 2\frac{1}{30}$

$\frac{x^2 + 1}{x} = \frac{61}{30}$

$30x^2 + 30 = 61x$

$30x^2 - 61x + 30 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 30 b = – 61 c = 30

= 30.30 = 900

And either of their sum or difference = b

= – 61

Thus the two terms are – 36 and – 25

Sum = – 36 – 25 = – 61

Product = – 36. – 25 = 900

$30x^2 - 36x - 25x + 30 = 0$

$6x(5x - 6) - 5(5x - 6) = 0$

$(5x - 6)(6x - 5) = 0$

$(5x - 6) = 0$ or $(6x - 5) = 0$

$x = \frac{5}{6}$ or $x = \frac{6}{5}$

Hence the required number is $\frac{5}{6}$ or $\frac{6}{5}$

Question: 29

CLASS24

Solution:

Let there be x rows

Then the number of students in each row will also be x

Total number of students $x^2 + 24$

According to the question,

$(x + 1)^2 - 25 = x^2 + 24$ using $(a + b)^2 = a^2 + 2ab + b^2$

$x^2 + 2x + 1 - 25 - x^2 - 24 = 0$

$2x - 48 = 0$

$x = 24$

Total number of students $= 24^2 + 24 = 576 + 24 = 600$

Question: 30

Solution:

Let the total number of students be x

According to the question

$$\frac{300}{x} - \frac{300}{x + 10} = 1$$

$\frac{300(x + 10) - 300x}{x(x + 10)} = 1$ taking LCM

$$\frac{300x + 3000 - 300x}{x^2 + 10x} = 1$$

$3000 = x^2 + 10x$ cross multiplying

$x^2 + 10x - 3000 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 10 c = – 3000

= 1. – 3000 = – 3000

And either of their sum or difference = b

= 10

Thus the two terms are 60 and – 50

Difference = 60 – 50 = 10

Product = 60. – 50 = – 3000

$x^2 + 60x - 50x - 3000 = 0$

$x(x + 60) - 50(x + 60) = 0$

$(x + 60)(x - 50) = 0$

$(x - 50) = 0$ or $(x + 60) = 0$

$x = 50$ or $x = -60$

x cannot be negative thus total number of students = 50

Question: 31

Solution:

Let Kamal's marks in mathematics and English be x and y, respectively

According to the question

$x + y = 40$ – – – – – – – (1)

Also $(x + 3)(y - 4) = 360$

$(x + 3)(40 - x - 4) = 360$ from (1)

$(x + 3)(36 - x) = 360$

$36x - x^2 + 108 - 3x = 360$

$33x - x^2 - 252 = 0$

$x^2 - 33x + 252 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 33 c = 252

= 1. – 252 = 252

And either of their sum or difference = b

= – 33

Thus the two terms are – 21 and – 12

Sum = – 21 – 12 = – 33

Product = – 21. – 12 = 252

$x^2 - 33x + 252 = 0$

$x^2 - 21x - 12x + 252 = 0$

$x(x - 21) - 12(x - 21) = 0$

$(x - 21)(x - 12) = 0$

$(x - 21) = 0$ or $(x - 12) = 0$

x = 21 or x = 12

if x = 21

y = 40 – 21 = 19

Kamal's marks in mathematics and English are 21 and 19

if x = 12

y = 40 – 12 = 28

Kamal's marks in mathematics and English are 12 and 28

Question: 32

Solution:

Let x be the number of students who planned picnic

Original cost of food for each member = Rs. $\frac{2000}{x}$

5 students failed to attend the picnic, so (x – 5) students attended the picnic

New cost of food for each member = Rs. $\frac{2000}{x-5}$

According to the question

$$\frac{2000}{x-5} - \frac{2000}{x} = 20$$

$\frac{2000x-2000x+10000}{x(x-5)} = 20$ taking LCM

$$\frac{10000}{x^2-5x} = 20$$

$x^2 - 5x = 500$ cross multiplying

$x^2 - 5x - 500 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 5 c = – 500

= 1. – 500 = – 500

And either of their sum or difference = b

= – 5

Thus the two terms are – 25 and 20

Sum = – 25 + 20 = – 5

Product = – 25.20 = – 500

$x^2 - 5x - 500 = 0$

$x^2 - 25x + 20x - 500 = 0$

x(x – 25) + 20(x – 25) = 0

(x + 20) (x – 25) = 0

(x + 20) = 0 or (x – 25) = 0

x = – 20 or x = 25

x cannot be negative thus x = 25

The number of students who planned picnic = x – 5 = 25 – 5 = 20

Cost of food for each member = Rs. $\frac{2000}{25-5}$ = Rs. $\frac{2000}{20}$ = Rs. 100

Question: 33

Solution:

Let the original price of the book be Rs x

Number of books bought at original price for 600 = $\frac{600}{x}$

If the price of a book is reduced by Rs. 5, then new price of book is Rs (x – 5)

Number of books bought at reduced price = $\frac{600}{x-5}$

According to the question – –

$$\frac{600}{x-5} - \frac{600}{x} = 4$$

$$\frac{600x - 600x + 3000}{x(x-5)} = 4$$

$$\frac{3000}{x^2 - 5x} = 4$$

$x^2 - 5x = 750$

$x^2 - 5x - 750 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 5 c = – 750

= 1. – 750 = – 750

And either of their sum or difference = b

= – 5

Thus the two terms are – 30 and 25

Difference = – 30 + 25 = – 5

Product = – 30.25 = – 750

$x^2 - 5x - 750 = 0$

$x^2 - 30x + 25x - 750 = 0$

$x(x - 30) + 25(x - 30) = 0$

$(x + 25)(x - 30) = 0$

$(x + 25) = 0$ or $(x - 30) = 0$

$x = -25$, $x = 30$

x = 30 (Price cannot be negative)

Hence the original price of the book is Rs 30.

Question: 34

Solution:

Let the original duration of the tour be x days

Original daily expenses = Rs. $\frac{10800}{x}$

If he extends his tour by 4 days his daily expenses = Rs. $\frac{10800}{x+4}$

According to the question – –

$$\frac{10800}{x} - \frac{1080}{x+4} = 90$$

$$\frac{10800x + 43200 - 10800x}{x(x+4)} = 90 \text{ taking LCM}$$

$$\frac{43200}{x^2 + 4x} = 90$$

$x^2 + 4x = 480$ cross multiplying

$x^2 + 4x - 480 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + b$ divided in two such values that:

Product = a.c

For the given equation a = 1 b = 4 c = – 480

= 1. – 480 = – 480

And either of their sum or difference = b

= 4

Thus the two terms are 24 and – 20

Difference = 24 – 20 = 4

Product = 24. – 20 = – 480

$x^2 + 24x - 20x - 480 = 0$

$x(x + 24) - 20(x + 24) = 0$

$(x + 24)(x - 20) = 0$

$(x + 24) = 0$ or $(x - 20) = 0$

$x = -24, x = 20$

x = 20 (number of days cannot be negative)

Hence the original price of tour is 20 days

Question: 35

Solution:

Let the marks obtained by P in mathematics and science be x and (28 – x) respectively

According to the given condition,

$(x + 3)(28 - x - 4) = 180$

$(x + 3)(24 - x) = 180$

$-x^2 + 21x + 72 = 180$

$x^2 - 21x + 108 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 21 c = 108

= 1.108 = 108

And either of their sum or difference = b

= – 21

Thus the two terms are – 12 and – 9

Difference = – 12 – 9 = – 21

Product = – 12. – 9 = 108

$x^2 - 12x - 9x + 108 = 0$

$x(x - 12) - 9(x - 12) = 0$

(x – 12) (x – 9) = 0

(x – 12) = 0 or (x – 9) = 0

x = 12, x = 9

When x = 12,

28 – x = 28 – 12 = 16

When x = 9,

28 – x = 28 – 9 = 19

Hence he obtained 12 marks in mathematics and 16 science or

He obtained 9 marks in mathematics and 19 science.

Question: 36

Solution:

Let the total number of pens be x

According to the question – –

$$\frac{180}{x} - \frac{180}{x+3} = 3$$

$\frac{180(x+3)-180x}{x(x+3)} = 3$ taking LCM

$$\frac{180x + 540 - 180x}{x^2 + 3x} = 3$$

$540 = 3x^2 + 9x$ cross multiplying

$3x^2 + 9x - 540 = 0$

$x^2 + 3x - 180 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 3 c = – 180

= 1. – 108 = – 180

And either of their sum or difference = b

= 3

Thus the two terms are 15 and – 12

Difference = 15 – 12 = 3

Product = 15. – 12 = – 180

$x^2 + 15x - 12x - 180 = 0$

x(x + 15) – 12(x + 15) = 0

(x + 15)(x – 12) = 0

(x + 15) = 0 or (x – 12) = 0

x = – 15, x = 12

x = 12 (Total number of pens cannot be negative)

Hence the Total number of pens is 12

Question: 37

Solution:

Let the cost price of the article be x

Gain percent x%

According to the given condition,

$x + \frac{x}{100}x = 75$ (cost price + gain = selling price)

$\frac{100x + x^2}{100} = 75$ taking LCM

by cross multiplying

$x^2 + 100x = 7500$

$x^2 + 100x - 7500 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 100 c = – 7500

= 1. – 7500 = – 7500

And either of their sum or difference = b

= 100

Thus the two terms are 150 and – 50

Difference = 150 – 50 = 100

Product = 150. – 50 = – 7500

$x^2 + 150x - 50x - 7500 = 0$

$x(x + 150) - 50(x + 150) = 0$

$(x + 150)(x - 50) = 0$

$(x + 150) = 0$ or $(x - 50) = 0$

$x = 50$ ($x \neq -150$ as price cannot be negative)

Hence the cost price of the article is Rs 50

Question: 38

Solution:

Let the present age of son be x years

The present age of man = x^2 years

One year ago age of son = $(x - 1)$years

age of man = $(x^2 - 1)$years

According to given question, One year ago, a man was 8 times as old as his son

$x^2 - 1 = 8(x - 1)$

$x^2 - 1 = 8x - 8$

$x^2 - 8x + 7 = 0$

$x^2 - 7x - x + 7 = 0$

$x(x - 7) - 1(x - 7) = 0$

$(x - 7)(x - 1) = 0$

$x = 1$ or $x = 7$

Man's age cannot be 1 year

Thus $x = 7$

Thus the present age of son is 7 years

The present age of man is $7^2 = 49$ years

Question: 39

Solution:

Let the present age of Meena be x years

Meena's age three years ago = $(x - 3)$ years

Meena's age five years hence = $(x + 5)$ years

According to given question

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$$

$$\frac{2x+2}{(x^2+2x-15)} = \frac{1}{3}$$

$x^2 + 2x - 15 = 6x + 6$

$x^2 - 4x - 21 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation $a = 1$ $b = -4$ $c = -21$

$= 1. - 21 = - 21$

And either of their sum or difference = b

$= - 4$

Thus the two terms are $- 7$ and 3

Sum $= - 7 + 3 = - 4$

Product $= - 7.3 = - 21$

$x^2 - 7x + 3x - 21 = 0$

$x(x - 7) + 3(x - 7) = 0$

$(x - 7)(x + 3) = 0$

$x = - 3$ or $x = 7$

$x = 7$ age cannot be negative

Hence the present age of Meena is 7 years

Question: 40

Solution:

Let the present age of boy and his brother be x years and (25 – x) years

According to given question

$x(25 - x) = 126$

$25x - x^2 = 126$

$x^2 - 25x + 126 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 25 c = 126

= 1.126 = 126

And either of their sum or difference = b

= – 25

Thus the two terms are – 18 and – 7

Sum = – 18 – 7 = – 25

Product = – 18. – 7 = 126

$x^2 - 18x - 7x + 126 = 0$

$x(x - 18) - 7(x - 18) = 0$

$(x - 18)(x - 7) = 0$

x = 18 or x = 7

x = 18 (Present age of boy cannot be less than his brother)

if x = 18

The present age of boy is 18 years and his brother is (25 – 18) = 7years

Question: 41

Solution:

Let the present age of Tanvy be x years

Tanvy's age five years ago = (x – 5) years

Tanvy's age eight years from now = (x + 8) years

$(x - 5)(x + 8) = 30$

$x^2 + 3x - 40 = 30$

$x^2 + 3x - 70 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 3 c = – 70

= 1. – 70 = – 70

And either of their sum or difference = b

= 3

Thus the two terms are 10 and – 7

Difference = 10 – 7 = 3

Product = 10. – 7 = – 70

$x^2 + 10x - 7x - 70 = 0$

$x(x + 10) - 7(x + 10) = 0$

$(x + 10)(x - 7) = 0$

x = – 10 or x = 7 (age cannot be negative)

x = 7

The present age of Tanvy is 7 years

Question: 42

Solution:

Let son's age 2 years ago be x years, Then

man's age 2 years ago be $3x^2$ years

son's present age = (x + 2) years

man's present age = $(3x^2 + 2)$years

In three years' time :

son's age = (x + 2 + 3) = (x + 5) years

man's age = $(3x^2 + 2 + 3)$years = $(3x^2 + 5)$ years

According to question

Man's age = 4 son's age

$3x^2 + 5 = 4(x + 5)$

$3x^2 + 5 = 4x + 20$

$3x^2 - 4x - 15 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 3 b = – 4 c = – 15

= 3. – 15 = – 45

And either of their sum or difference = b

= – 4

Thus the two terms are – 9 and 5

Difference = – 9 + 5 = – 4

Product = – 9.5 = – 45

$3x^2 - 9x + 5x - 15 = 0$

$3x(x - 3) + 5(x - 3) = 0$

$(x - 3)(3x + 5) = 0$

$(x - 3) = 0$ or $(3x + 5) = 0$

$x = 3$ or $x = -5/3$ (age cannot be negative)

$x = 3$

son's present age = (3 + 2) = 5years

man's present age = $(3.3^2 + 2)$ = 29years

Question: 43

Solution:

Let the first speed of the truck be x km/h

Time taken to cover 150 km = $\frac{150}{x}$ h

New speed of truck = x + 20 km/h

Time taken to cover 200 km = $\frac{200}{x+20}$ h

According to given question

$$\frac{150}{x} + \frac{200}{x + 20} = 5$$

$$\frac{150x + 3000 + 200x}{x(x + 20)} = 5$$

$$\frac{350x + 3000}{x(x + 20)} = 5$$

$350x + 3000 = 5(x^2 + 20x)$

$350x + 3000 = 5x^2 + 100x$

$5x^2 - 250x - 3000 = 0$

$x^2 - 50x - 600 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 50 c = – 600

= 1. – 600 = – 600

And either of their sum or difference = b

= – 50

Thus the two terms are – 60 and 10

Difference = – 60 + 10 = – 50

Product = – 60.10 = – 600

$x^2 - 60x + 10x - 600 = 0$

$x(x - 60) + 10(x - 60) = 0$

$(x - 60)(x + 10) = 0$

$x = 60$ or $x = -10$

x = 60 (speed cannot be negative)

Hence the first speed of the truck is 60 km/hr

Question: 44

Solution:

Let the original speed of the plane be x km/h

Actual speed of the plane = (x + 100) km/h

Distance of journey = 1500km

Time taken to reach destination at original speed = $\frac{1500}{x}$h

Time taken to reach destination at actual speed = $\frac{1500}{x + 100}$h

According to given question

30 mins = 1/2 hr

$$\frac{1500}{x} = \frac{1500}{x + 100} + \frac{1}{2}$$

$$\frac{1500}{x} - \frac{1500}{x + 100} = \frac{1}{2}$$

$$\frac{1500x + 150000 - 1500x}{x(x + 100)} = \frac{1}{2}$$

$$\frac{150000}{x(x + 100)} = \frac{1}{2}$$

$x^2 + 100x = 300000$

$x^2 + 100x - 300000 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 100 c = – 300000

= 1. – 300000 = – 300000

And either of their sum or difference = b

= 100

Thus the two terms are 600 and – 500

Difference = 600 – 500 = 100

Product = 600. – 500 = – 300000

$x^2 + 600x - 500x + 300000 = 0$

x (x + 600) – 500(x + 600) = 0

(x + 600)(x – 500) = 0

x = – 600 or x = 500

x = 500 (speed cannot be negative)

Hence the original speed of the plane is 500 km/hr

Question: 45

Solution:

Let the usual speed of the train be x km/h

Reduced speed of the train = (x – 8) km/h

Distance of journey = 480km

Time taken to reach destination at usual speed = $\frac{480}{x}$ h

Time taken to reach destination at reduced speed = $\frac{480}{x-8}$ h

According to given question

$$\frac{480}{x-8} = \frac{480}{x} + 3$$

$$\Rightarrow \frac{480}{x-8} - \frac{480}{x} = 3$$

$$\Rightarrow \frac{480x - 480x + 3840}{x(x-8)} = 3$$

$$\Rightarrow \frac{3840}{x(x-8)} = 3$$

$\Rightarrow x^2 - 8x = 1280$

$\Rightarrow x^2 - 8x - 1280 = 0$

$\Rightarrow x^2 - 40x + 32x - 1280 = 0$

$\Rightarrow x(x - 40) + 32(x - 40) = 0$

$\Rightarrow (x - 40)(x + 32) = 0$

$\Rightarrow x = 40$ or $x = -32$

$\Rightarrow x = 40$ (speed cannot be negative)

Hence the usual speed of the train is 40 km/h

Question: 46

Solution:

Let the first speed of the train be x km/h

Time taken to cover 54 km = $\frac{54}{x}$h

New speed of train = x + 6 km/h

Time taken to cover 63 km = $\frac{63}{x+6}$h

According to given question

$$\Rightarrow \frac{54}{x} + \frac{63}{x+6} = 3$$

$\Rightarrow \frac{54x + 324 + 63x}{x(x+6)} = 3$Taking LCM

$\Rightarrow 117x + 324 = 3(x^2 + 6x)$

$\Rightarrow 117x + 324 = 3x^2 + 18x$

$\Rightarrow 3x^2 - 99x - 324 = 0$

$\Rightarrow x^2 - 33x - 108 = 0$

$\Rightarrow x^2 - 36x + 3x - 108 = 0$

$\Rightarrow x (x - 36) + 3(x - 36) = 0$

⇒ (x – 36) (x + 3) = 0

⇒ x = 36 or x = – 3

⇒ x = 36 (speed cannot be negative)

Hence the first speed of the train is 36 km/hr

Question: 47

Solution:

Let the usual speed of the train be x km/h

Time taken to cover 180 km = $\frac{180}{x}$h

New speed of train = x + 9 km/h

Time taken to cover 180 km = $\frac{180}{x+9}$h

According to the question

$$\frac{180}{x} - \frac{180}{x + 9} = 1$$

$$\frac{180(x + 9 - x)}{x(x + 9)} = 1$$

$$\frac{180.9}{x(x + 9)} = 1$$

$$\frac{1620}{x(x + 9)} = 1$$

$1620 = x^2 + 9x$

$x^2 + 9x - 1620 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 9 c = – 1620

= 1. – 1620 = – 1620

And either of their sum or difference = b

= 9

Thus the two terms are 45 and – 36

Difference = – 36 + 45 = 9

Product = – 36.45 = – 1620

$x^2 + 45x - 36x + 1620 = 0$

x(x + 45) – 36(x + 45) = 0

(x + 45) (x – 36) = 0

x = – 45 or x = 36 (but x cannot be negative)

x = 36

Hence the usual speed of the train is 36 km/h

Question: 48

Solution:

Let the original speed of the train be x km/h

Time taken to cover 90 km = $\frac{90}{x}$ h

New speed of train = x + 15 km/h

Time taken to cover 90 km = $\frac{90}{x + 15}$ h

According to the question

$$\frac{90}{x} - \frac{90}{x + 15} = \frac{1}{2}$$

$$\frac{90(x + 15) - 90x}{x(x + 15)} = \frac{1}{2}$$

$$\frac{90x + 1350 - 90x}{x(x + 15)} = \frac{1}{2}$$

$$\frac{1350}{x(x + 15)} = \frac{1}{2}$$

$2700 = x^2 + 15x$

$x^2 + 15x - 2700 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 15 c = – 2700

= 1. – 2700 = – 2700

And either of their sum or difference = b

= 15

Thus the two terms are – 45 and 60

Difference = 60 – 45 = 15

Product = 60. – 45 = – 2700

$x^2 + 60x - 45x - 2700 = 0$

$x(x + 60) - 45(x + 60) = 0$

$(x + 60)(x - 45) = 0$

x = – 60 or x = 45 (but x cannot be negative)

x = 45

Hence the original speed of the train is 45 km/h

Question: 49

Solution:

Let the usual speed of the train be x km/h

Time taken to cover 300 km = $\frac{300}{x}$h

New speed of train = x + 5 km/h

Time taken to cover 90 km = $\frac{300}{x+5}$h

According to the question

$$\frac{300}{x} - \frac{300}{x+5} = 2$$

$$\frac{300(x+5) - 300x}{x(x+5)} = 2$$

$$\frac{300x + 1550 - 300x}{x(x+5)} = 2$$

$$\frac{1550}{x(x+5)} = 2$$

$750 = x^2 + 5x$

$x^2 + 5x - 750 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = 5 c = – 750

= 1. – 750 = – 750

And either of their sum or difference = b

= 5

Thus the two terms are – 25 and 30

Difference = 30 – 25 = 5

Product = 30. – 25 = – 750

$x^2 + 30x - 25x - 750 = 0$

$x(x + 30) - 25(x + 30) = 0$

$(x + 30)(x - 25) = 0$

x = – 30 or x = 25 (but x cannot be negative)

x = 25

Hence the usual speed of the train is 25 km/h

Question: 50

Solution:

Let the speed of Deccan Queen be x km/h

Speed of another train = (x – 20)km/h

According to the question

$$\frac{192}{x-20} - \frac{192}{x} = \frac{48}{60}$$

$$\frac{4}{x-20} - \frac{4}{x} = \frac{1}{60}$$

$\frac{4x-4(x-20)}{x(x-20)} = \frac{1}{60}$ taking LCM

$$\frac{4x - 4x + 80}{x(x-20)} = \frac{1}{60}$$

$$\frac{80}{x(x-20)} = \frac{1}{60}$$

$4800 = x^2 - 20x$ cross multiplying

$x^2 - 20x - 4800 = 0$

Using the splitting middle term – the middle term of the general equation $ax^2 + bx + c = 0$ is divided in two such values that:

Product = a.c

For the given equation a = 1 b = – 20 c = – 4800

= 1. – 4800 = – 4800

And either of their sum or difference = b

= – 20

Thus the two terms are – 80 and 60

Difference = – 80 + 60 = – 20

Product = – 80.60 = – 4800

$x^2 - 80x + 60x - 4800 = 0$

$x(x - 80) + 60(x - 80) = 0$

$(x - 80)(x + 60) = 0$

x = 80 or x = – 60 (but x cannot be negative)

Hence the speed of Deccan Queen is 80 km/hr