Chapter : 11. ARITHMETIC PROGRESSION

Exercise : 11A

Question: 1 A

Solution:

Here, $T_2 - T_1 = 15 - 9 = 6$

$T_3 - T_2 = 21 - 15 = 6$

$T_4 - T_3 = 27 - 21 = 6$

Since the difference between each consecutive term is same, ∴ the progression is an AP.

So, first term = 9

Common difference = 15 - 9 = 6

Next term = $T_5 = T_4 + d = 27 + 6 = 33$

Question: 1 B

Solution:

Here, $T_2 - T_1 = 6 - 11 = -5$

$T_3 - T_2 = 1 - 6 = -5$

$T_4 - T_3 = -4 - 1 = -5$

Since the difference between each consecutive term is same, ∴ the progression is an AP.

So, first term = 11

Common difference = 6 - 11 = - 5

Next term = $T_5 = T_4 + d = -4 + (-5) = -9$

Question: 1 C

Solution:

Here, $T_2 - T_1 = (-5/6) - (-1) = 1/6$

$T_3 - T_2 = (-2/3) - (-5/6) = 1/6$

$T_4 - T_3 = (-1/2) - (-2/3) = 1/6$

Since the difference between each consecutive term is same, ∴ the progression is an AP.

So, first term = - 1

Common difference = (-5/6) - (-1) = 1/6

Next term = $T_5 = T_4 + d$

= (-1/2) + (1/6)

= (-2/6)

= (-1/3)

Question: 1 D

Solution:

Here, $T_2 - T_1 = \sqrt{8} - \sqrt{2}$

$= 2\sqrt{2} - \sqrt{2}$

$= \sqrt{2}$

$T_3 - T_2 = = \sqrt{18} - \sqrt{8}$

$= 3\sqrt{2} - 2\sqrt{2}$

$= \sqrt{2}$

$T_4 - T_3 = = \sqrt{32} - \sqrt{18}$

$= 4\sqrt{2} - 3\sqrt{2}$

$= \sqrt{2}$

Since the difference between each consecutive term is same, ∴ the progression is an AP.

So, first term = $\sqrt{2}$

Common difference = $\sqrt{8} - \sqrt{2} = \sqrt{2}$

Next term = $T_5 = T_4 + d$

$= \sqrt{32} + \sqrt{2}$

$= 4\sqrt{2} + \sqrt{2}$

$= 5\sqrt{2}$

$= \sqrt{50}$

Question: 1 E

Solution:

Here, $T_2 - T_1 = \sqrt{45} - \sqrt{20}$

$= 3\sqrt{5} - 2\sqrt{5}$

$= \sqrt{5}$

$T_3 - T_2 = = \sqrt{80} - \sqrt{45}$

$= 4\sqrt{5} - 3\sqrt{5}$

$= \sqrt{5}$

$T_4 - T_3 = = \sqrt{125} - \sqrt{80}$

$= 5\sqrt{5} - 4\sqrt{5}$

$= \sqrt{5}$

Since the difference between each consecutive term is same, ∴ the progression is an AP.

So, first term = $\sqrt{20}$

Common difference = $\sqrt{45} - \sqrt{20} = \sqrt{5}$

Next term = $T_5 = T_4 + d$

$= \sqrt{125} + \sqrt{5}$

$= 5\sqrt{5} + \sqrt{5}$

$= 6\sqrt{5}$

$= \sqrt{180}$

Question: 2 A

Solution:

Here, First term = a = 9

Common difference = d = 13 - 9 = 4

To find = 20^{th} term, ∴ n = 20

Using the formula for finding n^{th} term of an A.P.,

$a_n = a + (n - 1) \times d$

$\therefore a_n = 9 + (20 - 1) \times 4$

$\Rightarrow a_n = 9 + 19 \times 4 = 9 + 76 = 85$

∴ 20^{th} term of the given AP is 85.

Question: 2 B

Solution:

Here, First term = a = 20

Common difference = d = 17 - 20 = - 3

To find = 35^{th} term, ∴ n = 35

Using the formula for finding n^{th} term of an A.P.,

$a_n = a + (n - 1) \times d$

$\therefore a_n = 20 + (35 - 1) \times (-3)$

$\Rightarrow a_n = 20 + 34 \times (-3) = 20 - 102 = - 82$

∴ 20^{th} term of the given AP is - 82.

Question: 2 C

Solution:

The given AP can be rewritten as $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}$....

Here, First term = a = $\sqrt{2}$

Common difference = d = $3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

To find = 18^{th} term, ∴ n = 18

Using the formula for finding n^{th} term of an A.P.,

$a_n = a + (n - 1) \times d$

$\therefore a_n = \sqrt{2} + (18 - 1) \times 2\sqrt{2}$

$\Rightarrow a_n = \sqrt{2} + 17 \times 2\sqrt{2} = \sqrt{2} + 34\sqrt{2} = 35\sqrt{2}$

∴ 18^{th} term of the given AP is $35\sqrt{2}$.

Question: 2 D

Solution:

Here, First term = a = 3/4

Common difference = d = 5/4 - 3/4 = 2/4

To find = 9^{th} term, ∴ n = 9

Using the formula for finding n^{th} term of an A.P.,

$a_n = a + (n - 1) \times d$

∴ $a_n = (3/4) + (9 - 1) \times (2/4)$

⇒ $a_n = 3/4 + 8 \times (2/4) = 3/4 + 16/4 = 19/4$

∴ 9^{th} term of the given AP is 19/4.

Question: 2 E

Solution:

Here, First term = a = - 40

Common difference = d = - 15 - (-40) = 25

To find = 15^{th} term, ∴ n = 15

Using the formula for finding n^{th} term of an A.P.,

$a_n = a + (n - 1) \times d$

∴ $a_n = - 40 + (15 - 1) \times (25)$

⇒ $a_n = - 40 + 14 \times (25) = - 40 + 350 = 310$

∴ 15^{th} term of the given AP is 310.

Question: 3

Solution:

The given AP can be rewritten as 6, 31/4 ,19/2 , 45/4,...

Here, First term = a = 6

Common difference = d = (31/4) - 6 = 7/4

To find = 37^{th} term, ∴ n = 37

Using the formula for finding n^{th} term of an A.P.,

$a_n = a + (n - 1) \times d$

∴ $a_n = 6 + (37 - 1) \times (7/4)$

⇒ $a_n = 6 + 36 \times (7/4) = 6 + 63 = 69$

∴ 37^{th} term of the given AP is 69.

Question: 4

Solution:

Here, First term = a = 5

Common difference = d = 9/2 - 5 = - (1/2)

To find = 25^{th} term, ∴ n = 25

Using the formula for finding n^{th} term of an A.P.,

$a_n = a + (n - 1) \times d$

∴ $a_n = 5 + (25 - 1) \times (-1/2)$

⇒ $a_n = 5 + 24 \times (-1/2) = 5 - 12 = -7$

∴ 25^{th} term of the given AP is - 7.

Question: 5 A

Solution:

Here, First term = a = 5

Common difference = d = 11 - 5 = 6

To find = n^{th} term

Using the formula for finding n^{th} term of an A.P.,

$a_n = a + (n - 1) \times d$

∴ $a_n = 5 + (n - 1) \times 6$

⇒ $a_n = 5 + 6n - 6 = 6n - 1$

∴ n^{th} term of the given AP is (6n - 1).

Question: 5 B

Solution:

Here, First term = a = 16

Common difference = d = 9 - 16 = - 7

To find = n^{th} term

Using the formula for finding n^{th} term of an A.P.,

$a_n = a + (n - 1) \times d$

∴ $a_n = 16 + (n - 1) \times (-7)$

⇒ $a_n = 16 - 7n + 7 = 23 - 7n$

∴ n^{th} term of the given AP is (23 - 7n).

Question: 6

Solution:

n^{th} term of the AP is (4n - 10).

For n = 1, we have $a_1 = 4 - 10 = -6$

For n = 2, we have $a_2 = 8 - 10 = -2$

For n = 3, we have $a_3 = 12 - 10 = 2$

For n = 4, we have $a_4 = 16 - 10 = 6$, and so on.

$\therefore a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = 4$ = constant.

∴ the given progression is an AP.

Hence, (i) Its first term = a = - 6

(ii) common difference = 4

(iii) To find :16^{th} term

$\therefore a_{16} = a + (16 - 1)d$

$\Rightarrow a_{16} = -6 + 15 \times 4 = 54$

∴ 16^{th} term of the given AP is 54.

Question: 7

Solution:

In the given AP, the first term = a = 6

Common difference = d = 10 - 6 = 4

Last term = 174

To find: No. of terms in the AP.

Since, we know that

$a_n = a + (n - 1) \times d$

$\therefore 174 = 6 + (n - 1) \times 4$

$\Rightarrow 174 - 6 = 4n - 4$

$\Rightarrow 168 = 4n - 4$

$\Rightarrow 168 + 4 = 4n$

$\Rightarrow 4n = 172$

$\Rightarrow n = 172/4$

$\Rightarrow n = 43$

∴ Number of terms = 43.

Question: 8

Solution:

In the given AP, the first term = a =41

Common difference = d = 38 - 41 = - 3

Last term = 8

To find: No. of terms in the AP.

Since, we know that

$a_n = a + (n - 1) \times d$

$\therefore 8 = 41 + (n - 1) \times (-3)$

$\Rightarrow 8 - 41 = - 3n + 3$

$\Rightarrow - 33 = - 3n + 3$

$\Rightarrow - 33 - 3 = - 3n$

$\Rightarrow - 3n = - 36$

$\Rightarrow n = 36/3$

$\Rightarrow n = 12$

$\therefore$ Number of terms = 12.

Question: 9

Solution:

In the given AP, the first term = a = 18

Common difference = d = (31/2) - 18 = (-5/2)

Last term = - 47

To find: No. of terms in the AP.

Since, we know that

$a_n = a + (n - 1) \times d$

$\therefore - 47 = 18 + (n - 1) \times (-5/2)$

$\Rightarrow - 47 - 18 = (n - 1) \times (-5/2)$

$\Rightarrow - 65 = (n - 1) \times (-5/2)$

$\Rightarrow - 65 \times (-2/5) = n - 1$

$\Rightarrow n - 1 = 26$

$\Rightarrow n = 26 + 1$

$\Rightarrow n = 27$

$\therefore$ Number of terms = 27.

Question: 10

Solution:

In the given AP, the first term = a = 3

Common difference = d = 8 - 3 = 5

To find: place of the term 88.

So, let $a_n = 88$

Since, we know that

$a_n = a + (n - 1) \times d$

$\therefore 88 = 3 + (n - 1) \times 5$

$\Rightarrow 88 - 3 = 5n - 5$

$\Rightarrow 85 = 5n - 5$

$\Rightarrow 85 + 5 = 5n$

$\Rightarrow 5n = 90$

$\Rightarrow n = 90/5$

$\Rightarrow n = 18$

$\therefore 18^{th}$ term of the AP is 88.

Question: 11

Solution:

In the given AP, the first term = a = 72

Common difference = d = 68 - 72 = - 4

To find: place of the term 0.

So, let $a_n = 0$

Since, we know that

$a_n = a + (n - 1) \times d$

$\therefore 0 = 72 + (n - 1) \times (-4)$

$\Rightarrow 0 - 72 = - 4n + 4$

$\Rightarrow - 72 - 4 = - 4n$

$\Rightarrow - 76 = - 4n$

$\Rightarrow n = 76/4$

$\Rightarrow n = 19$

$\therefore 19^{th}$ term of the AP is 0.

Question: 12

Solution:

In the given AP, the first term = a = 5/6

Common difference = d = 1 - 5/6 = 1/6

To find: place of the term 3.

So, let $a_n = 3$

Since, we know that

$a_n = a + (n - 1) \times d$

$\therefore 3 = (5/6) + (n - 1) \times (1/6)$

$\Rightarrow 3 - (5/6) = (n - 1) \times (1/6)$

$\Rightarrow 13/6 = (n - 1) \times (1/6)$

$\Rightarrow 13 = n - 1$

$\Rightarrow n = 13 + 1$

$\Rightarrow n = 14$

$\therefore 14^{th}$ term of the AP is 3.

Question: 13

Solution:

In the given AP, the first term = a = 21

Common difference = d = 18 - 21 = - 3

To find: place of the term - 81.

So, let a_n = - 81

Since, we know that

$a_n = a + (n - 1) \times d$

$\therefore -81 = 21 + (n - 1) \times (-3)$

$\Rightarrow -81 - 21 = -3n + 3$

$\Rightarrow -102 = -3n + 3$

$\Rightarrow -102 - 3 = -3n$

$\Rightarrow -3n = -105$

$\Rightarrow n = 105/3$

$\Rightarrow n = 35$

$\therefore$ 35^{th} term of the AP is - 81.

Question: 14

Solution:

In the given AP, the first term = a = 3

Common difference = d = 8 - 3 = 5

To find: place of the term which is 55 more than its 20^{th} term.

So, we first find its 20^{th} term.

Since, we know that

$a_n = a + (n - 1) \times d$

$\therefore a_{20} = 3 + (20 - 1) \times 5$

$\Rightarrow a_{20} = 3 + 19 \times 5$

$\Rightarrow a_{20} = 3 + 95$

$\Rightarrow a_{20} = 98$

$\therefore$ 20^{th} term of the AP is 98.

Now, 55 more than 20^{th} term of the AP is 55 + 98 = 153.

So, to find: place of the term 153.

So, let a_n = 153

Since, we know that

$a_n = a + (n - 1) \times d$

$\therefore 153 = 3 + (n - 1) \times 5$

$\Rightarrow 153 - 3 = 5n - 5$

$\Rightarrow 150 = 5n - 5$

$\Rightarrow 150 + 5 = 5n$

$\Rightarrow 5n = 155$

$\Rightarrow n = 155/5 = 31$

∴ 31^{st} term of the AP is the term which is 55 more than 20^{th} term.

Question: 15

Solution:

In the given AP, the first term = a = 5

Common difference = d = 15 - 5 = 10

To find: place of the term which is 130 more than its 31^{st} term.

So, we first find its 31^{st} term.

Since, we know that

$a_n = a + (n - 1) \times d$

$\therefore a_{31} = 5 + (31 - 1) \times 10$

$\Rightarrow a_{31} = 5 + 30 \times 10$

$\Rightarrow a_{31} = 5 + 300$

$\Rightarrow a_{31} = 305.$

∴ 31^{st} term of the AP is 305.

Now, 130 more than 31^{st} term of the AP is 130 + 305 = 435.

So, to find: place of the term 435.

So, let $a_n = 435$

Since, we know that

$a_n = a + (n - 1) \times d$

$\therefore 435 = 5 + (n - 1) \times 10$

$\Rightarrow 435 - 5 = 10n - 10$

$\Rightarrow 430 = 10n - 10$

$\Rightarrow 430 + 10 = 10n$

$\Rightarrow 10n = 440$

$\Rightarrow n = 440/10 = 44$

∴ 44^{th} term of the AP is the term which is 130 more than 31^{st} term.

Question: 16

Solution:

Given: 10^{th} term of the AP is 52.

17^{th} term is 20 more than the 13^{th} term.

Let the first term be a and the common difference be d.

Since,

$a_n = a + (n - 1) \times d$

therefore for 10^{th} term, we have,

$52 = a + (10 - 1) \times d$

$\Rightarrow 52 = a + 9d$............... (1)

Now, 17^{th} term is 20 more than the 13^{th} term.

$\therefore a_{17} = 20 + a_{13}$

$\Rightarrow a + (17 - 1)d = 20 + a + (13 - 1)d$

$\Rightarrow 16d = 20 + 12d$

$\Rightarrow 4d = 20$

$\Rightarrow d= 5$

$\therefore$ from equation (1), we have,

$52 = a + 9d$

$\Rightarrow 52 = a + 9 \times 5$

$\Rightarrow 52 = a + 45$

$\Rightarrow a = 52 - 45$

$\Rightarrow a = 7$

$\therefore$ AP is a, a + d, a + 2d, a + 3d,...

$\therefore$ AP is 7, 12, 17, 22....

Question: 17

Solution:

First term of the AP = 6

Common difference = d = 13 - 6 = 7

Last term = 216

Since

$a_n = a + (n - 1) \times d$

$\therefore 216 = 6 + (n - 1) \times 7$

$\Rightarrow 216 - 6 = 7n - 7$

$\Rightarrow 210 = 7n - 7$

$\Rightarrow 210 + 7 = 7n$

$\Rightarrow 7n = 217$

$\Rightarrow n = 217/7 = 31$

$\therefore$ Middle term is $(31 + 1)/2 = 16^{th}$

So, $a_{16} = a + (16 - 1) \times d$

$\therefore a_{16} = 6 + 15 \times 7$

$\Rightarrow a_{16} = 6 + 105 = 111$

$\therefore$ Middle term of the AP is 111.

Question: 18

Find the middle t

Solution:

First term of the AP = 10

Common difference = d = 7 - 10 = - 3

Last term = - 62

Since

$a_n = a + (n - 1) \times d$

$\therefore - 62 = 10 + (n - 1) \times (-3)$

$\Rightarrow - 62 - 10 = - 3n + 3$

$\Rightarrow - 72 = - 3n + 3$

$\Rightarrow - 72 - 3 = - 3n$

$\Rightarrow 3n = 75$

$\Rightarrow n = 75/3 = 25$

$\therefore$ Middle term is $(25 + 1)/2 = 13^{th}$

So, $a_{13} = a + (13 - 1) \times d$

$\therefore a_{13} = 10 + 12 \times (-3)$

$\Rightarrow a_{13} = 10 - 36 = - 26$

$\therefore$ Middle term of the AP is - 26.

Question: 19

Solution:

First term of the AP = - (4/3)

Common difference = $d = - 1 - (-4/3) = - 1 + (4/3) = 1/3$

Last term = 13/3

Since

$a_n = a + (n - 1) \times d$

$\therefore 13/3 = (-4/3) + (n - 1) \times (1/3)$

$\Rightarrow (13/3) + (4/3) = (n - 1) \times (1/3)$

$\Rightarrow 17/3 = (n - 1) \times (1/3)$

$\Rightarrow 17 = n - 1$

$\Rightarrow n = 17 + 1$

$\Rightarrow n = 18$

$\therefore$ Two middle most terms of the AP are 18/2 and (18/2) + 1 terms, i.e. 9^{th} and 10^{th} terms.

So, $a_9 = a + (9 - 1) \times d$

$\therefore a_9 = (-4/3) + [8 \times (1/3)]$

$\Rightarrow a_9 = (-4/3) + (8/3) = 4/3$

Also, $a_{10} = a_9 + d$

$= (4/3) + (1/3)$

$= 5/3$

Now, $a_{10} + a_9 = (4/3) + (5/3)$

$= 9/3$

$= 3$

∴ Sum of two middle most terms of the AP is 3.

Question: 20

Solution:

Here, First term = a = 7

Common difference = d = 10 - 7 = 3

Last term = l = 184

To find: 8^{th} term from end.

So, n^{th} term from end is given by:

$a_n = l - (n - 1)d$

∴ 8^{th} term from end is:

$a_8 = 184 - (8 - 1) \times 3$

= 184 - 21

= 163

Question: 21

Solution:

Here, First term = a = 17

Common difference = d = 14 - 17 = - 3

Last term = l = - 40

To find: 6^{th} term from end.

So, n^{th} term from end is given by:

$a_n = l - (n - 1)d$

∴ 6^{th} term from end is:

$a_6 = - 40 - (6 - 1) \times (-3)$

= - 40 + 15

= - 25

Question: 22

Solution:

Here, First term = a = 3

Common difference = d = 7 - 3 = 4

Now, to check: 184 is a term of the AP or not.

Since, n^{th} term of an AP is given by:

$a_n = a + (n - 1)d$

If 184 is a term of the AP, then it must satisfy this equation.

So, let $a_n = 184$

∴ $184 = 3 + (n - 1) \times 4$

⇒ 184 - 3 = 4n - 4

⇒ 181 = 4n - 4

⇒ 181 + 4 = 4n

⇒ 4n = 185

⇒ n = 185/4 = 46.25

But this is not possible because n is the number of terms which can't be a fraction.

Therefore, 184 is not a term of the given AP.

Question: 23

Solution:

Here, First term = a = 11

Common difference = d = 8 - 11 = - 3

Now, to check: - 150 is a term of the AP or not.

Since, n^{th} term of an AP is given by:

$a_n = a + (n - 1)d$

If - 150 is a term of the AP, then it must satisfy this equation.

So, let $a_n = -150$

∴ - 150 = 11 + (n - 1) × (-3)

⇒ - 150 - 11 = - 3n + 3

⇒ - 161 = - 3n + 3

⇒ - 161 - 3 = - 3n

⇒ 3n = 164

⇒ n = 164/3 = 54.66

But this is not possible because n is the number of terms which can't be a fraction.

Therefore, - 150 is not a term of the given AP.

Question: 24

Solution:

Here, First term = a = 121

Common difference = d = 117 - 121 = - 4

Let n^{th} term of the AP be its first negative term.

∴ $a_n < 0$

Since, n^{th} term of an AP is given by:

$a_n = a + (n - 1)d$

∴ a + (n - 1)d < 0

⇒ 121 + (n - 1) × (-4) < 0

⇒ - 4n + 125 < 0

⇒ - 4n < - 125

$\Rightarrow 4n > 125$

$\Rightarrow n > 31.25$

Since n is an integer, therefore n must be 32.

$\therefore 32^{nd}$ term will be the first negative term of the AP.

Question: 25

Solution:

Here, First term = a = 20

Common difference = d = (77/4) - 20 = (-3/4)

Let n^{th} term of the AP be its first negative term.

$\therefore a_n < 0$

Since, n^{th} term of an AP is given by:

$a_n = a + (n - 1)d$

$\therefore a + (n - 1)d < 0$

$\Rightarrow 20 + (n - 1) \times (-3/4) < 0$

$\Rightarrow 80 + (n - 1) \times (-3) < 0$ (multiplying both sides by 4)

$\Rightarrow 80 - 3n + 3 < 0$

$\Rightarrow -3n < -83$

$\Rightarrow 3n > 83$

$\Rightarrow n > 27.66$

Since n is an integer, therefore n must be 28.

$\therefore 28^{th}$ term will be the first negative term of the AP.

Question: 26

Solution:

Let a be the first term and d be the common difference.

Given: $a_7 = -4$

$a_{13} = -16$

Now, Consider $a_7 = -4$

$\Rightarrow a + (7 - 1)d = -4$

$\Rightarrow a + 6d = -4$ ....(1)

Consider $a_{13} = -16$

$\Rightarrow a + (13 - 1)d = -16$

$\Rightarrow a + 12d = -16$ .... (2)

Now, subtracting equation (1) from (2), we get,

$6d = -12$

$\Rightarrow d = -2$

$\therefore$ from equation (1), we get,

a = - 4 - 6d

⇒ a = - 4 - 6 × (-2)

⇒ a = - 4 + 12

⇒ a = 8

Thus the AP is a, a + d, a + 2d, a + 3d, a + 4d,....

Therefore the AP is 8, 6, 4, 2, 0,....

Question: 27

Solution:

Let a be the first term and d be the common difference.

Given: $a_4 = 0$

To prove: $a_{25} = 3 \times a_{11}$

Now, Consider $a_4 = 0$

⇒ a + (4 - 1)d = 0

⇒ a + 3d = 0

⇒ a = - 3d ....(1)

Consider a_{25} = a + (25 - 1)d

⇒ a_{25} = - 3d + 24d (from equation (1))

⇒ a_{25} = 21d........... (2)

Now, consider a_{11} = a + (11 - 1)d

⇒ a_{11} = - 3d + 10d (from equation (1))

⇒ a_{11} = 7d.......... (3)

From equation (2) and (3), we get,

$a_{25} = 3 \times a_{11}$

Hence, proved.

Question: 28

Solution:

Let a be the first term and d be the common difference.

Given: $a_8 = 0$

To prove: $a_{38} = 3 \times a_{18}$

Now, Consider $a_8 = 0$

⇒ a + (8 - 1)d = 0

⇒ a + 7d = 0

⇒ a = - 7d ....(1)

Consider a_{38} = a + (38 - 1)d

⇒ a_{38} = - 7d + 37d (from equation (1))

⇒ a_{38} = 30d........... (2)

Now, consider $a_{18} = a + (18 - 1)d$

$\Rightarrow a_{18} = - 7d + 17d$ (from equation (1))

$\Rightarrow a_{18} = 10d$ .... (3)

From equation (2) and (3), we get,

$a_{38} = 3 \times a_{18}$

Hence, proved.

Question: 29

Solution:

Let *a* be the first term and *d* be the common difference.

Given: $a_4 = 11$

$a_5 + a_7 = 34$

To find: common difference = d

Now, Consider $a_4 = 11$

$\Rightarrow a + (4 - 1)d = 11$

$\Rightarrow a + 3d = 11$............... (1)

Consider $a_5 + a_7 = 34$

$\Rightarrow a + (5 - 1)d + a + (7 - 1)d = 34$

$\Rightarrow 2a + 10d = 34$

$\Rightarrow a + 5d = 17$..........(2)

Subtracting equation (1) from equation (2), we get,

$2d = 6$

$\Rightarrow d = 3$

$\therefore$ Common difference = d = 3

Question: 30

Solution:

Let *a* be the first term and *d* be the common difference.

Given: $a_9 = - 32$

$a_{11} + a_{13} = - 94$

To find: common difference = d

Now, Consider $a_9 = - 32$

$\Rightarrow a + (9 - 1)d = - 32$

$\Rightarrow a + 8d = - 32$ .... (1)

Consider $a_{11} + a_{13} = - 94$

$\Rightarrow a + (11 - 1)d + a + (13 - 1)d = - 94$

$\Rightarrow 2a + 22d = - 94$

$\Rightarrow a + 11d = - 47$ .... (2)

Subtracting equation (1) from equation (2), we get,

3d = - 15

⇒ d = - 5

∴ Common difference = d = - 5

Question: 31

Solution:

Let a be the first term and d be the common difference.

Given: $a_7 = -1$

$a_{16} = 17$

Now, Consider $a_7 = -1$

⇒ a + (7 - 1)d = - 1

⇒ a + 6d = - 1 ....(1)

Consider $a_{16} = 17$

⇒ a + (16 - 1)d = 17

⇒ a + 15d = 17 ....(2)

Now, subtracting equation (1) from (2), we get,

9d = 18

⇒ d = 2

∴ from equation (1), we get,

a = - 1 - 6d

⇒ a = - 1 - 6 × (2)

⇒ a = - 1 - 12

⇒ a = - 13

Now, the n^{th} term of the AP is given by:

$a_n = a + (n - 1)d$

∴ $a_n = -13 + (n - 1) \times 2$

⇒ $a_n = 2n - 15$

∴ n^{th} term of the AP is (2n - 15)

Question: 32

Solution:

Given: $4 \times a_4 = 18 \times a_{18}$

To find : a_{22}

Consider $4 \times a_4 = 18 \times a_{18}$

⇒ 4 [a + (4 - 1)d] = 18 [a + (18 - 1)d]

⇒ 4a + 12d = 18a + 306d

⇒ - 14 a = 294 d

⇒ $a = -21d$ ....(1)

Now, $a_{22} = a + (22 - 1)d$

⇒ $a_{22} = a + 21d$

⇒ $a_{22} = -21d + 21d$ (from equation 1)

⇒ $a_{22} = 0$

∴ $a_{22} = 0$

Question: 33

Solution:

Given: $10 \times a_{10} = 15 \times a_{15}$

To show : $a_{25} = 0$

Consider $10 \times a_{10} = 15 \times a_{15}$

⇒ $10 [a + (10 - 1)d] = 15 [a + (15 - 1)d]$

⇒ $10a + 90d = 15a + 210d$

⇒ $-5a = 120d$

⇒ $a = -24d$ ....(1)

Now, $a_{25} = a + (25 - 1)d$

⇒ $a_{25} = a + 24d$

⇒ $a_{25} = -24d + 24d$ (from equation 1)

⇒ $a_{25} = 0$

Hence, proved.

Question: 34

Solution:

Let a be the first term and d be the common difference of the AP.

Given: $a = 5$

Sum of first four terms = 1/2(sum of next four terms)

⇒ $a + (a + d) + (a + 2d) + (a + 3d) = 1/2 ((a + 4d) + (a + 5d) + (a + 6d) +$

$(a + 7d))$

⇒ $4a + 6d = 1/2(4a + 22d)$

⇒ $4a + 6d = 2a + 11d$

⇒ $2a = 5d$

⇒ $d = 2a/5$

As $a = 5$, therefore,

$d = 10/5 = 2$

Thus, Common difference = $d = 2$

Question: 35

Solution:

Let a be the first term and d be the common difference of the AP.

Given: $a_2 + a_7 = 30$

Also, $a_{15} = 2a_8 - 1$

Consider $a_2 + a_7 = 30$

$\Rightarrow (a + d) + (a + 6d) = 30$

$\Rightarrow 2a + 7d = 30$ .... (1)

Consider $a_{15} = 2a_8 - 1$

$\Rightarrow a + 14d = 2(a + 7d) - 1$

$\Rightarrow a + 14d = 2a + 14d - 1$

$\Rightarrow a = 1$

$\therefore$ First term = a = 1

Thus, from equation (1), we get,

$7d = 30 - 2a$

$\Rightarrow 7d = 30 - 2$

$\Rightarrow 7d = 28$

$\Rightarrow d = 4$

Thus, the AP is a, a + d, a + 2d, a + 3d,...

Therefore, the AP is 1, 5, 9, 13, 17,...

Question: 36

Solution:

Let a_1 and d_1 be the first term and common difference of the AP 63, 65, 67, 69,....

Let a_2 and d_2 be the first term and common difference of the AP 3, 10, 17,....

$\therefore a_1 = 63, d_1 = 2$

$a_2 = 3, d_2 = 7$

Let a_n be the n^{th} term of the first AP and b_n be the n^{th} term of the second AP.

So, $a_n = a_1 + (n - 1)d_1$

$\Rightarrow a_n = 63 + (n - 1)2$

$\Rightarrow a_n = 61 + 2n$

and, $b_n = a_2 + (n - 1)d_2$

$\Rightarrow b_n = 3 + (n - 1)7$

$\Rightarrow b_n = - 4 + 7n$

Since for nth terms of both the AP's to be same, $a_n = b_n$

$\Rightarrow 61 + 2n = - 4 + 7n$

$\Rightarrow 61 + 4 = 7n - 2n$

$\Rightarrow 65 = 5n$

$\Rightarrow n = 13$

Therefore, 13^{th} term of both the AP's will be same.

Question: 37

Solution:

Let a and d be the first term and common difference of the AP Given: $a_{17} = 2 \times a_8 + 5$

$a_{11} = 43$

To find: n^{th} term = a_n

Consider a11 = 43

$\Rightarrow a + (11 - 1)d = 43$

$\Rightarrow a + 10d = 43$ .... (1)

Consider $a_{17} = 2 \times a_8 + 5$

$\Rightarrow a + (17 - 1)d = 2[a + (8 - 1)d] + 5$

$\Rightarrow a + 16d = 2a + 14d + 5$

$\Rightarrow - a + 2d = 5$ ....(2)

Adding equation (1) and equation (2), we get

$12d = 48$

$\Rightarrow d = 4$

$\therefore$ from equation (1), we get,

$a = 43 - 10d$

$= 43 - 40$

$= 3$

Now, n^{th} term is given by:

$a_n = a + (n - 1)d$

$\Rightarrow a_n = 3 + (n - 1)4$

$\Rightarrow a_n = 4n - 1$

Therefore, n^{th} term is given by $(4n - 1)$.

Question: 38

Solution:

Let a be the first term and d be the common difference.

Given: $a_{24} = 2(a_{10})$

To prove: $a_{72} = 4 \times a_{15}$

Now, Consider $a_{24} = 2a_{10}$

$\Rightarrow a + 23d = 2[a + 9d]$

$\Rightarrow a + 23d = 2a + 18d$

$\Rightarrow a = 5d$.......................... (1)

Consider $a_{72} = a + (72 - 1)d$

$\Rightarrow a_{72} = 5d + 71d$ (from equation (1))

$\Rightarrow a_{72} = 76d$...................... (2)

Now, consider $a_{15} = a + (15 - 1)d$

$\Rightarrow a_{15} = 5d + 14d$ (from equation (1))

$\Rightarrow a_{18} = 19d$.........................(3)

From equation (2) and (3), we get,

$a_{72} = 4 \times a_{15}$

Hence, proved.

Question: 39

Solution:

Let a be the first term and d be the common difference.

Given: $a_9 = 19$

$a_{19} = 3\ a_6$

Now, Consider $a_9 = 19$

$\Rightarrow a + (9 - 1)d = 19$

$\Rightarrow a + 8d = 19$......................... (1)

Consider $a_{19} = 3\ a_6$

$\Rightarrow a + 18d = 3(a + 5d)$

$\Rightarrow a + 18d = 3a + 15d$

$\Rightarrow 2a - 3d = 0$ ....(2)

Now, subtracting twice of equation (1) from (2), we get,

$- 19d = - 38$

$\Rightarrow d = 2$

$\therefore$ from equation (1), we get,

$a = 19 - 8d$

$\Rightarrow a = 19 - 8 \times 2$

$\Rightarrow a = 19 - 16$

$\Rightarrow a = 3$

Thus the AP is a, a + d, a + 2d, a + 3d, a + 4d,....

Therefore the AP is 3, 5, 7, 9....

Question: 40

Solution:

Let a be the first term and d be common difference.

Given: $a_p = q$

$a_q = p$

To show: $a_{(p + q)} = 0$

We know, nth term of an AP is$a_n = a + (n - 1)d$where, a is first term and d is common differenceConsider $a_p = q$

$\Rightarrow a + (p - 1)d = q$ (1)

Consider $a_q = p$

$\Rightarrow a + (q - 1)d = p$ (2)

Now, subtracting equation (2) from equation (1), we get

$(p - q)d = (q - p)$

$\Rightarrow d = - 1$

$\therefore$ From equation (1), we get,

$a - p + 1 = q$

$\Rightarrow p + q = a + 1$..............................(3)

Consider $a_{(p + q)} = a + (p + q - 1)d$

$= a + (p + q - 1)(-1)$

$= a + (a + 1 - 1)(-1)$

(putting the value of p + q from equation 3)

$= a + (-a)$

$= 0$

$\therefore a_{(p + q)} = 0$

Hence, proved.

Question: 41

Solution:

Let d be the common difference of the AP.

First term = a

Last term = l = 1

n^{th} term from beginning of an AP is given by:

$a_n = a + (n - 1)d$ .... (1)

n^{th} term from the end of an AP is given by:

$T_n = l - (n - 1)d$

$= 1 - (n - 1)d$ .... (2)

Sum of the n^{th} term from the beginning and end is given by:

$a_n + T_n = a + (n - 1)d + 1 - (n - 1)d$

$= a + 1$

Hence, proved.

Question: 42

Solution:

The two digit numbers divisible by 6 are 12, 18, 24, 30,...96.

This forms an AP with first term $a = 12$

and common difference = d = 6

Last term is 96.

Now, number of terms in this AP are given as:

96 = a + (n - 1)d

⇒ 96 = 12 + (n - 1)6

⇒ 96 - 12 = 6n - 6

⇒ 84 + 6 = 6n

⇒ 90 = 6n

⇒ n = 15

There are 15 two - digit numbers that are divisible by 6.

Question: 43

Solution:

The two digit numbers divisible by 3 are 12, 15, 18, 21, …., 99.

This forms an AP with first term a = 12

and common difference = d = 3

Last term is 99.

Now, number of terms in this AP are given as:

99 = a + (n - 1)d

⇒ 99 = 12 + (n - 1)3

⇒ 99 - 12 = 3n - 3

⇒ 87 + 3 = 3n

⇒ 90 = 3n

⇒ n = 30

There are 30 two - digit numbers that are divisible by 3.

Question: 44

Solution:

The three digit numbers divisible by 9 are 108, 117, 126, …., 999.

This forms an AP with first term a = 108

and common difference = d = 9

Last term is 999.

Now, number of terms in this AP are given as:

999 = a + (n - 1)d

⇒ 999 = 108 + (n - 1)9

⇒ 999 - 108 = 9n - 9

⇒ 891 + 9 = 9n

⇒ 900 = 9n

⇒ n = 100

There are 100 three - digit numbers that are divisible by 9.

Question: 45

Solution:

The numbers between 101 and 999 that are divisible by both 2 and 5 are 110, 120, 130,..., 990.

This forms an AP with first term $a = 110$

and common difference = d = 10

Last term is 990.

Now, number of terms in this AP are given as:

$990 = a + (n - 1)d$

$\Rightarrow 990 = 110 + (n - 1)10$

$\Rightarrow 990 - 110 = 10n - 10$

$\Rightarrow 880 + 10 = 10n$

$\Rightarrow 890 = 10n$

$\Rightarrow n = 89$

There are 89 numbers between 101 and 999 that are divisible by both 2 and 5.

Question: 46

Solution:

The no of rose plants in each row can be arranged in the form of an AP as 43, 41, 39, ..., 11.

Here, First term = a = 43

Common difference = d = 41 - 43 = - 2

No of terms in the AP = No of rows in the flower bed.

$\therefore 11 = a + (n - 1)d$

$\Rightarrow 11 = 43 + (n - 1)(-2)$

$\Rightarrow 11 - 43 = - 2n + 2$

$\Rightarrow 11 - 43 - 2 = - 2n$

$\Rightarrow 2n = 34$

$\Rightarrow n = 17$

$\therefore$ No of rows in the flower bed = 17

Question: 47

Solution:

Let the first prize be Rs. x. Thus each succeeding prize is Rs. 200 less than the preceding prize.

$\therefore$ Second prize is Rs. $(x - 200)$

Third prize is Rs. $(x - 400)$

Fourth prize is Rs. $(x - 600)$

This forms an AP as $x, x - 200, x - 400, x - 600$.

Since, Total sum of prize amount = 2800.

$\therefore x + (x - 200) + (x - 400) + (x - 600) = 2800$

$\Rightarrow 4x - 1200 = 2800$

$\Rightarrow 4x = 2800 + 1200$

$\Rightarrow 4x = 4000$

$\Rightarrow x = 1000$

Thus, the first, second, third and fourth prizes are as Rs. 1000, Rs. 800, Rs. 600, Rs. 400.

Exercise : 11B

Question: 1

Solution:

If three terms are in AP, the difference between the terms should be equal, i.e. if a, b and c are in AP then,b - a = c - bSince, the terms are in an AP, therefore

$(4k - 6) - (3k - 2) = (k + 2) - (4k - 6)$

$\Rightarrow k - 4 = -3k + 8$

$\Rightarrow 4k = 12$

$\Rightarrow k = 3$

$\therefore k = 3$

Question: 2

Solution:

Given: The numbers (5x + 2), (4x - 1) and (x + 2) are in AP.**To find:** The value of x.**Solution:**Let $a_1 = (5x + 2)$ $a_2 = 4x - 1)$$a_3 = (x + 2)$ Since, the terms are in an AP, therefore common difference is same.$\Rightarrow a_2 - a_1 = a_3 - a_2 \Rightarrow (4x - 1) - (5x + 2) = (x + 2) - (4x - 1)$

$\Rightarrow 4x - 1 - 5x - 2 = x + 2 - 4x + 1$

$\Rightarrow -x - 3 = -3x + 3$

$\Rightarrow -x + 3x = 3 + 3$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3$

$\therefore x = 3$

Question: 3

Solution:

Since, the terms are in an AP, therefore

$(3y + 5) - (3y - 1) = (5y + 1) - (3y + 5)$

$\Rightarrow 6 = 2y - 4$

$\Rightarrow 2y = 10$

$\Rightarrow y = 5$

$\therefore y = 5$

Question: 4

Solution:

Given: (x + 2), 2x, (2x + 3) are three consecutive terms of an AP.**To find:** the value x **Solution:**Let $a_1 = x + 2$

$a_2 = 2x$

$a_3 = 2x + 3$

As, a_1, a_2 and a_3 are in AP, common difference will be equal

$\Rightarrow a_2 - a_1 = a_3 - a_2$

$\Rightarrow (2x) - (x + 2) = (2x + 3) - (2x) \Rightarrow 2x - x - 2 = 2x + 3 - 2x$

$\Rightarrow x - 2 = 3$

$\Rightarrow x = 5$

Question: 5

Solution:

Consider $(a^2 + b^2) - (a - b)^2$

$= (a^2 + b^2) - (a^2 + b^2 - 2ab)$

$= 2ab$

Consider $(a + b)^2 - (a^2 + b^2)$

$= (a^2 + b^2 + 2ab) - (a^2 + b^2)$

$= 2ab$

Since, the difference between consecutive terms is constant, therefore the terms are in AP.

Question: 6

Solution:

Let the numbers be (a - d), a, (a + d).

Now, sum of the numbers = 15

$\therefore (a - d) + a + (a + d) = 15$

$\Rightarrow 3a = 15$

$\Rightarrow a = 5$

Now, product of the numbers = 80

$\Rightarrow (a - d) \times a \times (a + d) = 80$

$\Rightarrow a^3 - ad^2 = 80$

Put the value of a, we get,

$125 - 5d^2 = 80$

$\Rightarrow 5d^2 = 125 - 80 = 45$

$d^2 = 9$

d = �3

$\therefore$ If d = 3, then the numbers are 2, 5, 8.

If d = - 3, then the numbers are 8, 5, 2.

Question: 7

Solution:

Let the numbers be (a - d), a, (a + d).

Now, sum of the numbers = 15

$\therefore (a - d) + a + (a + d) = 3$

$\Rightarrow 3a = 3$

$\Rightarrow a = 1$

Now, product of the numbers = - 35

$\Rightarrow (a - d) \times a \times (a + d) = -35$

$\Rightarrow a^3 - ad^2 = -35$

Put the value of a, we get,

$1 - d^2 = -35$

$\Rightarrow d^2 = 35 + 1 = 36$

$d^2 = 36$

$d = \pm 6$

∴ If d = 6, then the numbers are - 5, 1, 7.

If d = - 6, then the numbers are 7, 1, - 5.

Question: 8

Solution:

Let 24 be divided in numbers which are in AP as (a - d), a, (a + d).

Now, sum of the numbers = 24

$\therefore (a - d) + a + (a + d) = 24$

$\Rightarrow 3a = 24$

$\Rightarrow a = 8$

Now, product of the numbers = 440

$\Rightarrow (a - d) \times a \times (a + d) = 440$

$\Rightarrow a^3 - ad^2 = 440$

Put the value of a, we get,

$512 - 8d^2 = 440$

$\Rightarrow 8d^2 = 512 - 440 = 72$

$d^2 = 9$

d = �3

∴ If d = 3, then the numbers are 5, 8, 11.

If d = - 3, then the numbers are 11, 8, 5.

Question: 9

Solution:

Let the numbers be (a - d), a, (a + d).

Now, sum of the numbers = 21

$\therefore$ (a - d) + a + (a + d) = 21

$\Rightarrow$ 3a = 21

$\Rightarrow$ a = 7

Now, sum of the squares of the terms = 165

$\Rightarrow (a - d)^2 + a^2 + (a + d)^2 = 165$

$\Rightarrow a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 165$

$\Rightarrow 3a^2 + 2d^2 + a = 165$

Put the value of a = 7, we get,

$3(49) + 2d^2 = 165$

$\Rightarrow 2d^2 = 165 - 147$

$\Rightarrow 2d^2 = 18$

$\Rightarrow d^2 = 9$

$\Rightarrow d = \pm 3$

$\therefore$ If d = 3, then the numbers are 4, 7, 10.

If d = - 3, then the numbers are 10, 7, 4.

Question: 10

Solution:

Let these angles be x°, (x + 10)°, (x + 20)° and (x + 30)°.

Since, Sum of all angles of a quadrilateral = 360°.

$\Rightarrow$ x° + (x + 10)° + (x + 20)° + (x + 30)° = 360°

$\Rightarrow$ 4x + 60° = 360°

$\Rightarrow$ 4x = 300°

$\Rightarrow$ x = 75°

$\therefore$ the angles will be 75°, 85°, 95°, 105°.

Question: 11

Solution:

Let the numbers be (a - 3d), (a - d), (a + d), (a + 3d).

Now, sum of the numbers = 28

$\therefore$ (a - 3d) + (a - d) + (a + d) + (a + 3d) = 28

$\Rightarrow$ 4a = 28

$\Rightarrow$ a = 7

Now, sum of the squares of the terms = 216

$\Rightarrow (a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 216$

$\Rightarrow a^2 + 9d^2 - 6ad + a^2 + d^2 - 2ad + a^2 + d^2 + 2ad + a^2 + 9d^2 + 6ad = 216$

$\Rightarrow 4a^2 + 20d^2 = 216$

Put the value of a = 7, we get,

$4(49) + 20d^2 = 216$

$\Rightarrow 20d^2 = 216 - 196$

$\Rightarrow 20d^2 = 20$

$\Rightarrow d^2 = 1$

$\Rightarrow d = \pm 1$

∴ If d = 1, then the numbers are 4, 6, 8, 10.

If d = - 1, then the numbers are 10, 8, 6, 4.

Question: 12

Solution:

Let 32 be divided into parts as (a - 3d), (a - d), (a + d) and (a + 3d).

Now (a - 3d) + (a - d) + (a + d) + (a + 3d) = 32

$\Rightarrow 4a = 32$

$\Rightarrow a = 8$

Now, we are given that product of the first and the fourth terms is to the product of the second and the third terms as 7 : 15.

i.e. [(a - 3d) × (a + 3 d)] : [(a - d) × (a + d)] = 7 : 15

$\Rightarrow \frac{(a-3d)\times(a+3d)}{(a-d)\times(a+d)} = \frac{7}{15}$

$\Rightarrow 15[(a - 3d) \times (a + 3d)] = 7[(a - d) \times (a + d)]$

$\Rightarrow 15[a^2 - 9d^2] = 7[a^2 - d^2]$

$\Rightarrow 15a^2 - 135d^2 = 7a^2 - 7d^2$

$\Rightarrow 8a^2 - 128d^2 = 0$

$\Rightarrow 8a^2 = 128d^2$

Putting the value of a, we get,

$512 = 128d^2$

$\Rightarrow d^2 = 4$

$\Rightarrow d = \pm 2$

∴ If d = 2, then the numbers are 2, 6, 10, 14.

If d = - 2, then the numbers are 14, 10, 6, 2.

Question: 13

Solution:

Let the numbers be (a - d), a, (a + d).

Now, sum of the numbers = 48

∴ (a - d) + a + (a + d) = 48

⇒ $3a = 48$

⇒ $a = 16$

Now, we are given that,

Product of first and second terms exceeds 4 times the third term by 12.

⇒ $(a - d) \times a = 4(a + d) + 12$

⇒ $a^2 - ad = 4a + 4d + 12$

On putting the value of a in the above equation, we get,

$256 - 16d = 64 + 4d + 12$

⇒ $20d = 180$

⇒ $d = 9$

∴ The numbers are $a - d, a, a + d$

i.e. the numbers are 7, 16, 25.

Exercise : 11C

Question: 1

Solution:

Since, the terms are in an AP, therefore

$(3y + 5) - (3y - 1) = (5y + 1) - (3y + 5)$

⇒ $6 = 2y - 4$

⇒ $2y = 10$

⇒ $y = 5$

∴ $y = 5$

Question: 2

Solution:

Since, the terms are in an AP, therefore

$(2k - 1) - k = (2k + 1) - (2k - 1)$

⇒ $k - 1 = 2$

⇒ $k = 3$

∴ $k = 3$

Question: 3

Solution:

Since, the terms are in an AP, therefore

$a - 18 = (b - 3) - a$

⇒ $2a - b = -3 + 18$

⇒ $2a - b = 15$

∴ $2a - b = 15$

Question: 4

Solution:

Since, the terms are in an AP, therefore

9 - a = b - 9 = 25 - b

Consider b - 9 = 25 - b

⇒ 2b = 34

⇒ b = 17

Now, consider the first equality,

9 - a = b - 9

⇒ a = 18 - b

⇒ a = 18 - 17

⇒ a = 1

∴ a = 1, b = 17

Question: 5

Solution:

Since, the terms are in an AP, therefore

(3n + 2) - (2n - 1) = (6n - 1) - (3n + 2)

⇒ n + 3 = 3n - 3

⇒ 2n = 6

⇒ n = 3

∴ n= 3, and hence the numbers are 5, 11, 17.

Question: 6

Solution:

The three digit numbers divisible by 7 are 105, 112, 119, ..., 994.

This forms an AP with first term a = 105

and common difference = **d** = 7

Last term is 994.

Now, number of terms in this AP are given as:

994 = a + (n - 1)d

⇒ 994 = 105 + (n - 1)7

⇒ 994 - 105 = 7n - 7

⇒ 889 + 7 = 7n

⇒ 896 = 7n

⇒ n = 128

Therefore 994 is the 128^{th} term in the AP.

∴ There are 128 three - digit natural numbers that are divisible by 7.

Question: 7

Solution:

The three digit natural numbers divisible by 9 are 108, 117, 126, ..., 999.

This forms an AP with first term $a = 108$

and common difference = d = 9

Last term is 999.

Now, number of terms in this AP are given as:

$999 = a + (n - 1)d$

$\Rightarrow 999 = 108 + (n - 1)9$

$\Rightarrow 999 - 108 = 9n - 9$

$\Rightarrow 891 + 9 = 9n$

$\Rightarrow 900 = 9n$

$\Rightarrow n = 100$

Therefore 999 is the 100^{th} term in the AP.

∴ There are 100 three - digit natural numbers that are divisible by 9.

Question: 8

Solution:

Let S_n denotes the sum of first n terms of an AP.

Sum of first m terms = $S_m = 2m^2 + 3m$

Then n^{th} term is given by: $a_n = S_n - S_{n-1}$

We need to find the 2^{nd} term, so put n = 2, we get

$a_2 = S_2 - S_1$

$= (2(2)^2 + 3(2)) - (2(1)^2 + 3(1))$

$= 14 - 5$

$= 9$

∴ the second term of the AP is 9.

Question: 9

Solution:

Here, first term = a

Common difference = 3a - a = 2a

Now, Sum of first n terms of an AP is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

where a is the first term and d is the common difference.

∴ Sum of first n terms of given AP is given by:

$S_n = \frac{n}{2}[2a + (n - 1)2a]$

$= \frac{n}{2}[2a + 2an - 2a]$

$= \frac{n}{2}[2an]$

$= n^2a$

Question: 10

Solution:

Here, First term = a = 2

Common difference = d = 7 - 2 = 5

Last term = l = 47

To find: 5^{th} term from end.

So, n^{th} term from end is given by:

$a_n = l - (n - 1)d$

∴ 5^{th} term from end is:

$a_5 = 47 - (5 - 1) \times 5$

= 47 - 20

= 27

∴ 5^{th} term from the end is 27.

Question: 11

Solution:

Here, First term = a = 2

Common difference = d = 7 - 2 = 5

To find: $a_{30} - a_{20}$

So, n^{th} term is given by:

$a_n = a + (n - 1)d$

∴ 30^{th} term is:

$a_{30} = 2 + (30 - 1) \times 5$

= 2 + 145

= 147

Now, 20^{th} term is:

$a_{20} = 2 + (20 - 1) \times 5$

= 2 + 95

= 97

Now, $(a_{30} - a_{20}) = 147 - 97$

= 50

∴ $(a_{30} - a_{20}) = 50$

Question: 12

Solution:

n^{th} term of an AP = a_n = 3n + 5

Common difference (= d) of an AP is the difference between a term and its preceding term.

∴ d = a_n - a_{n-1}

= (3n + 5) - (3(n - 1) + 5)

= 3n + 5 - 3n + 3 - 5

= 3

∴ Common difference = 3

Question: 13

Solution:

n^{th} term of an AP = a_n = 7 - 4n

Common difference (= d) of an AP is the difference between a term and its preceding term.

∴ d = a_n - a_{n-1}

= (7 - 4n) - (7 - 4(n - 1))

= 7 - 4n - 7 + 4n - 4

= - 4

∴ Common difference = - 4.

Question: 14

Solution:

Here, first term = √8

Common difference = √18 - √8 = √2

Next term = T_4 = T_3 + d

= √32 + √2

= 4√2 + √2

= 5√2

= √50

Question: 15

Solution:

Here, first term = √2

Common difference = √8 - √2 = 2√2 - √2 = √2

Next term = T_4 = T_3 + d

= √18 + √2

= 3√2 + √2

= 4√2

$= \sqrt{32}$

Question: 16

Solution:

Here first term = 21

Common difference = 18 - 21 = - 3

Let a_n be the term which is zero.

$\therefore a_n = 0$

$\Rightarrow a + (n - 1)d = 0$

$\Rightarrow 21 + (n - 1)(-3) = 0$

$\Rightarrow 21 - 3n + 3 = 0$

$\Rightarrow 3n = 24$

$\Rightarrow n = 8$

$\therefore 8^{th}$ term of the given AP will be zero.

Question: 17

Solution:

First n natural numbers are 1, 2, 3,..., n.

To find: sum of these n natural numbers.

The natural numbers forms an AP with first term 1 and common difference 1.

Now, Sum of first n terms of an AP is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

where a is the first term and d is the common difference.

∴ Sum of first n natural numbers is given by:

$S_n = \frac{n}{2}[2(1) + (n - 1)(1)]$

$= \frac{n}{2}[2 + n - 1]$

$= \frac{n}{2}[n + 1]$

∴ Sum of first n natural numbers is n(n + 1)/2.

Question: 18

Solution:

First n even natural numbers are 2, 4, 6,..., 2n.

To find: sum of these n even natural numbers.

The even natural numbers forms an AP with first term 2 and common difference 2.

Now, Sum of first n terms of an AP is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

where a is the first term and d is the common difference.

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∴ Sum of first n natural numbers is given by:

$S_n = \frac{n}{2}[2(2) + (n - 1)(2)]$

$= \frac{n}{2}[4 + 2n - 2]$

$= \frac{n}{2}[2n + 2]$

$= n(n + 1)$

∴ Sum of first n even natural numbers is $n(n + 1)$.

Question: 19

Solution:

Here, given: first term = p

Common difference = q

To find: a_{10}

$a_{10} = a + (10 - 1)d$

$\Rightarrow a_{10} = p + 9q$

∴ 10^{th} term of the given AP will be $p + 9q$.

Question: 20

Solution:

Since, the terms are in an AP, therefore

$a - (4/5) = 2 - a$

$\Rightarrow 2a = 2 + (4/5)$

$\Rightarrow 2a = 14/5$

$\Rightarrow a = 14/10$

$\Rightarrow a = 7/5$

∴ $a = 7/5$

Question: 21

Solution:

Since, the terms are in an AP, therefore

$13 - (2p + 1) = (5p - 3) - (13)$

$\Rightarrow 12 - 2p = 5p - 16$

$\Rightarrow 7p = 28$

$\Rightarrow p = 4$

∴ $p = 4$

Question: 22

Solution:

Since, the terms are in an AP, therefore

$7 - (2p - 1) = 3p - 7$

$\Rightarrow 8 - 2p = 3p - 7$

$\Rightarrow 5p = 15$

$\Rightarrow p = 3$

$\therefore p = 3$

Question: 23

Solution:

Let S_p denotes the sum of first p terms of an AP.

Sum of first p terms $= S_p = ap^2 + bp$

Then p^{th} term is given by: $a_p = S_p - S_{p-1}$

$\therefore a_p = (ap^2 + bp) - [a(p - 1)^2 + b(p - 1)]$

$= (ap^2 + bp) - [a(p^2 + 1 - 2p) + bp - b]$

$= ap^2 + bp - ap^2 - a + 2ap - bp + b$

$= b - a + 2ap$

Now, common difference $= d = a_p - a_{p-1}$

$= b - a + 2ap - [b - a + 2a(p - 1)]$

$= b - a + 2ap - b + a - 2ap + 2a$

$= 2a$

$\therefore$ common difference = 2a

ALITER: Let S_p denotes the sum of first p terms of an AP.

Sum of first p terms $= S_p = ap^2 + bp$

Put p = 1, we get $S_1 = a + b$

Put p = 2, we get $S_2 = 4a + 2b$

Now $S_1 = a_1$

$a_2 = S_2 - S_1$

$\therefore a_2 = 3a + b$

Now, $d = a_2 - a_1$

$= 3a + b - (a + b)$

$= 2a$

$\therefore$ Common difference = 2a

Question: 24

Solution:

Let S_n denotes the sum of first n terms of an AP.

Sum of first n terms $= S_n = 3n^2 + 5n$

Then n^{th} term is given by: $a_n = S_n - S_{n-1}$

$\therefore a_n = (3n^2 + 5n) - [3(n - 1)^2 + 5(n - 1)]$

$= (3n^2 + 5n) - [3(n^2 + 1 - 2n) + 5n - 5]$

$= 3n^2 + 5n - 3n^2 - 3 + 6n - 5n + 5$

$= 2 + 6n$

Now, common difference = $d = a_n - a_{n-1}$

$= 2 + 6n - [2 + 6(n - 1)]$

$= 2 + 6n - 2 - 6n + 6$

$= 6$

$\therefore$ Common difference = 6

ALITER: Let S_n denotes the sum of first n terms of an AP.

Sum of first n terms = $S_n = 3n^2 + 5n$

Put n = 1, we get $S_1 = 8$

Put n = 2, we get $S_2 = 22$

Now $S_1 = a_1$

$a_2 = S_2 - S_1$

$\therefore a_2 = 22 - 8 = 14$

Now, $d = a_2 - a_1$

$= 14 - 8$

$= 6$

$\therefore$ Common difference = 6

Question: 25

Solution:

Let a be the first term and d be the common difference.

Given: $a_4 = 9$

$a_6 + a_{13} = 40$

Now, Consider $a_4 = 9$

$\Rightarrow a + (4 - 1)d = 9$

$\Rightarrow a + 3d = 9$............(1)

Consider $a_6 + a_{13} = 40$

$\Rightarrow a + (6 - 1)d + a + (13 - 1)d = 40$

$\Rightarrow 2a + 17d = 40$ .... (2)

Subtracting twice of equation (1) from equation (2), we get,

$11d = 22$

$\Rightarrow d = 2$

$\therefore$ Common difference = d = 2

Now from equation (1),we get

$a = 9 - 3d$

= 9 - 6

= 3

∴ AP is a, a + d, a + 2d, a + 3d, ...

i.e. AP is 3, 5, 7,9, 11.....

Exercise : 11D

Question: 1 A

Solution:

Here, first term = 2

Common difference = 7 - 2 = 5

Sum of first n terms of an AP is

$S_n = \frac{n}{2}[2a + (n - 1)d]$

∴ $S_{19} = \frac{19}{2}[2(2) + (19 - 1)5]$

= (19)(4 + 90)/2

= (19 × 94)/2

= 893

Thus, sum of 19 terms of this AP is 893.

Question: 1 B

Solution:

Here, first term = 9

Common difference = 7 - 9 = - 2

Sum of first n terms of an AP is

$S_n = \frac{n}{2}[2a + (n - 1)d]$

∴ $S_{14} = \frac{14}{2}[2(9) + (14 - 1)(-2)]$

= (7)(18 - 26)

= (7) × (-8)

= - 56

Thus, sum of 14 terms of this AP is - 56.

Question: 1 C

Solution:

Here, first term = - 37

Common difference = (-33) - (-37) = 4

Sum of first n terms of an AP is

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{12} = \frac{12}{2}[2(-37) + (12 - 1)(4)]$

= (6)(-74 + 44)

= 6 × (-30)

= - 180

Thus, sum of 12 terms of this AP is - 180.

Question: 1 D

Solution:

Here, first term = 1/15

Common difference = (1/12) - (1/15) = 1/60

Sum of first n terms of an AP is

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{11} = \frac{11}{2}[2(1/15) + (11 - 1)(1/60)]$

= (11/2) × [(2/15) + (1/6)]

= (11/2) × [(3/10)]

= 33/20

Thus, sum of 11 terms of this AP is 33/20.

Question: 1 E

Solution:

Here, first term = 0.6

Common difference = 1.7 - 0.6 = 1.1

Sum of first n terms of an AP is

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{100} = \frac{100}{2}[2(0.6) + (100 - 1)(1.1)]$

= (50) × [1.2 + (99 × 1.1)]

= 50 × [1.2 + 108.9]

= 50 × 110.1

= 5505

Thus, sum of 100 terms of this AP is 5505.

Question: 2 A

Solution:

Here, First term = 7

Common difference = d = (21/2) - 7 = (7/2)

Last term = l = 84

Now, 84 = a + (n - 1)d

∴ 84 = 7 + (n - 1)(7/2)

⇒ 84 - 7 = (n - 1)(7/2)

⇒ 77 = (n - 1)(7/2)

⇒ 154 = 7n - 7 (multiplying both sides by 2)

⇒ 154 + 7 = 7n

⇒ 7n = 161

⇒ n = 23

∴ there are 23 terms in this Arithmetic series.

Now, Sum of these 23 terms is given by

∴ $S_{23} = \frac{23}{2}$ [2(7) + (23 - 1)(7/2)]

= (23/2) × [14 + (22)(7/2)]

= (23/2) × [14 + 77]

= (23/2) × [91]

= 2093/2

= 1046.5

Thus, sum of 23 terms of this AP is 1046.5.

Question: 2 B

Solution:

Here, First term = 34

Common difference = d = 34 - 32 = - 2

Last term = l = 10

Now, 10 = a + (n - 1)d

∴ 10 = 34 + (n - 1)(-2)

⇒ 10 - 34 = (n - 1)(-2)

⇒ - 24 = - 2n + 2

⇒ - 24 - 2 = - 2n

⇒ - 26 = - 2n

⇒ n = 13

⇒ n = 13

∴ there are 13 terms in this Arithmetic series.

Now, Sum of these 13 terms is given by

∴ $S_{13} = \frac{13}{2}$ [2(34) + (13 - 1)(-2)]

= (13/2) × [68 + (12)(-2)]

= (13/2) × [68 - 24]

= (13/2) × [44]

= 13 × 22

= 286

Thus, sum of 23 terms of this AP is 286.

Question: 2 C

Solution:

Here, First term = - 5

Common difference = d = - 8 - (-5) = - 3

Last term = l = - 230

Now, - 230 = a + (n - 1)d

∴ - 230 = - 5 + (n - 1)(-3)

⇒ - 230 + 5 = (n - 1)(-3)

⇒ - 225 = - 3n + 3

⇒ - 225 - 3 = - 3n

⇒ - 228 = - 3n

⇒ n = 76

∴ there are 76 terms in this Arithmetic series.

Now, Sum of these 76 terms is given by

∴ $S_{76} = \frac{76}{2}$ [2(-5) + (76 - 1)(-3)]

= 38 × [- 10 + (75)(-3)]

= 38 × [- 10 - 225]

= 38 × (-235)

= - 8930

Thus, sum of 23 terms of this AP - 8930.

Question: 3

Solution:

Since, nth term is given as (5 - 6n)

Put n = 1, we get a_1 = - 1 = first term

Put n = 2, we get a_2 = - 7 = second term

Now, d = a_2 - a_1 = - 7 - (-1) = - 6

Sum of first n terms = $S_n = \frac{n}{2}$ [2a + (n - 1)d]; where a is the first term and d is the common difference.

= $\frac{n}{2}$ [- 2 + (n - 1)(-6)]

= n[- 1 - 3n + 3]

= n(2 - 3n)

∴ sum of first 20 terms = S_{20}

= $\frac{20}{2}$ [2(-1) + (20 - 1)(-6)]

= 10 × [- 2 - 114]

= 10 × [- 116]

= - 1160

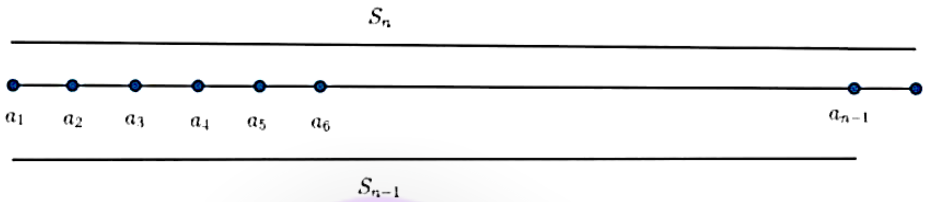
Question: 4

Solution:

Given: The sum of the first n terms of an AP is $(3n^2 + 6n)$. **To find:** the nth term and the 15th term of this AP.**Solution:**Sum of first n terms = $S_n = 3n^2 + 6n$

Now let a_n be the n^{th} term of the AP.

To find: a_n and a_{15}Since $a_n = S_n - S_{n-1}$



$\Rightarrow a_n =$

$(3n^2 + 6n) - (3(n - 1)^2 + 6(n - 1)) \Rightarrow a_n = (3n^2 + 6n) - (3(n^2 + 1 - 2n) + 6(n - 1))$

$\Rightarrow a_n = (3n^2 + 6n) - (3n^2 + 3 - 6n + 6n - 6)$

$\Rightarrow a_n = 3n^2 + 6n - 3n^2 - 3 + 6n - 6n + 6$

$\Rightarrow a_n = 6n + 3$

Now, $a_{15} = 6(15) + 3$

$\Rightarrow a_{15} = 93$

Question: 5

Solution:

(i) Let a_n be the n^{th} term of the AP.

To find: a_n

Then $a_n = S_n - S_{n-1}$

$= (3n^2 - n) - (3(n - 1)^2 - (n - 1))$

$= (3n^2 - n) - (3n^2 + 3 - 6n - n + 1)$

$= 6n - 4$

(ii) Since $a_n = 6n - 4$

$\therefore$ For first term, n = 1

By putting n = 1 in the nt^h term, we get,

$a_1 = 6(1) - 4$

$= 2$

$\therefore a = 2$

(iii) Put n = 2 in the nth term, we get

$a_2 = 6 \times (2) - 4$

$= 12 - 4$

= 8

Now common difference = d = a_2 - a_1

= 8 - 2

= 6

∴ Common difference = 6

Question: 6

Solution:

Let a_n be the n^{th} term of the AP.

To find: a_n and a_{20}

Since, $a_n = S_n - S_{n-1}$

$= \left(\frac{5n^2}{2} + \frac{3n}{2}\right) - \left(\frac{5(n-1)^2}{2} + \frac{3(n-1)}{2}\right)$

$= 1/2\ (5n^2 + 3n) - 1/2\ [5(n-1)^2 + 3(n-1)]$

$= 1/2\ (5n^2 + 3n - 5n^2 - 5 + 10n - 3n + 3)$

= 1/2 (10n - 2)

= 5n - 1

Since $a_n = 5n - 1$

∴ For 20^{th} term, put n = 20, we get,

$a_{20} = 5(20) - 1$

= 100 - 1

= 99

Question: 7

Solution:

Let a_n be the n^{th} term of the AP.

To find: a_n and a_{25}

Since, $a_n = S_n - S_{n-1}$

$= \left(\frac{3n^2}{2} + \frac{5n}{2}\right) - \left(\frac{3(n-1)^2}{2} + \frac{5(n-1)}{2}\right)$

$= 1/2\ (3n^2 + 5n) - 1/2\ [3(n-1)^2 + 5(n-1)]$

$= 1/2\ (3n^2 + 5n - 3n^2 - 3 + 6n - 5n + 5)$

= 1/2 (6n - 2)

= 3n + 1

Since $a_n = 5n - 1$

∴ For 25^{th} term, put n = 25, we get,

$a_{25} = 3(25) + 1$

= 75 + 1

= 76

Question: 8

Solution:

Here, first term = a = 21

Common difference = d = 18 - 21 = - 3

Let first n terms of the AP sums to zero.

∴ $S_n = 0$

To find: n

Now, $S_n = (n/2) \times [2a + (n - 1)d]$

Since, $S_n = 0$

∴ $(n/2) \times [2a + (n - 1)d] = 0$

⇒ $(n/2) \times [2(21) + (n - 1)(-3)] = 0$

⇒ $(n/2) \times [42 - 3n + 3)] = 0$

⇒ $(n/2) \times [45 - 3n] = 0$

⇒ $[45 - 3n] = 0$

⇒ $45 = 3n$

⇒ $n = 15$

∴ 15 terms of the given AP sums to zero.

Question: 9

Solution:

Here, first term = a = 9

Common difference = d = 17 - 9 = 8

Let first n terms of the AP sums to 636.

∴ $S_n = 636$

To find: n

Now, $S_n = (n/2) \times [2a + (n - 1)d]$

Since, $S_n = 636$

∴ $(n/2) \times [2a + (n - 1)d] = 636$

⇒ $(n/2) \times [2(9) + (n - 1)(8)] = 636$

⇒ $(n/2) \times [18 + 8n - 8)] = 636$

⇒ $(n/2) \times [10 + 8n] = 636$

⇒ $n[5 + 4n] = 636$

⇒ $4n^2 + 5n - 636 = 0$

⇒ $4n^2 + 5n - 636 = 0$

⇒ $(n - 12)(4n + 53) = 0$

⇒ $n = 12$ or $n = - 53/4$

But n can’t be negative and fraction.

$\therefore$ n= 12

$\therefore$ 12 terms of the given AP sums to 636.

Question: 10

Solution:

Here, first term = a =63

Common difference = d = 60 - 63 = - 3

Let first n terms of the AP sums to 693.

$\therefore S_n = 693$

To find: n

Now, $S_n = (n/2) \times [2a + (n - 1)d]$

Since, $S_n = 693$

$\therefore (n/2) \times [2a + (n - 1)d] = 693$

$\Rightarrow (n/2) \times [2(63) + (n - 1)(-3)] = 693$

$\Rightarrow (n/2) \times [126 - 3n + 3)] = 693$

$\Rightarrow (n/2) \times [129 - 3n] = 693$

$\Rightarrow n[129 - 3n] = 1386$

$\Rightarrow 129n - 3n^2 = 1386$

$\Rightarrow 3n^2 - 129n + 1386 = 0$

$\Rightarrow (n - 22)(n - 21)= 0$

$\Rightarrow n = 22$ or $n = 21$

$\therefore$ n= 22 or n = 21

Since, $a_{22} = a + 21d$

= 63 + 21(-3)

= 0

$\therefore$ Both the first 21 terms and 22 terms give the sum 693 because the 22^{nd} term is 0. So, the sum doesn't get affected.

Question: 11

Solution:

Here, first term = a =20

Common difference = d = 58/3 - 20 = - 2/3

Let first n terms of the AP sums to 300.

$\therefore S_n = 300$

To find: n

Now, $S_n = (n/2) \times [2a + (n - 1)d]$

Since, $S_n = 300$

$\therefore (n/2) \times [2a + (n - 1)d] = 300$

$\Rightarrow (n/2) \times [2(20) + (n - 1)(-2/3)] = 300$

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$\Rightarrow (n/2) \times [40 - (2/3)n + (2/3)] = 300$

$\Rightarrow (n/2) \times [(120 - 2n + 2)/3] = 300$

$\Rightarrow n[122 - 2n] = 1800$

$\Rightarrow 122n - 2n^2 = 1800$

$\Rightarrow 2n^2 - 122n + 1800 = 0$

$\Rightarrow n^2 - 61n + 900 = 0$

$\Rightarrow (n - 36)(n - 25)= 0$

$\Rightarrow n = 36$ or $n = 25$

$\therefore n= 36$ or $n = 25$

Now, $S_{36} = (36/2)[2a + 35d]$

$= 18(40 + 35(-2/3))$

$= 18(120 - 70)/3$

$= 6(50)$

$= 300$

Also, $S_{25} = (25/2)[2a + 24d]$

$= (25/2)(40 + 24(-2/3))$

$= (25/2)(40 - 16)$

$= (24 \times 25)/2$

$= 12 \times 25$

$= 300$

Now, sum of 11 terms from 26^{th} term to 36^{th} term $= S_{36} - S_{25} = 0$

$\therefore$ Both the first 25 terms and 36 terms give the sum 300 because the sum of last 11 terms is 0. So, the sum doesn't get affected.

Question: 12

Solution:

Odd numbers from 0 to 50 are 1, 3, 5, ..., 49

Sum of these numbers is 1 + 3 + 5 + ... + 49.

This forms an Arithmetic Series with first term = a = 1

and Common Difference = d = 3 - 1 = 2

There are 25 terms in this Arithmetic Series.

Now, sum of n terms is given as:

$S_n = (n/2)[2a + (n - 1)d]$

$S_{25} = (25/2)[2(1) + (25 - 1)2]$

$= (25/2)[2 + 48]$

$= (25 \times 50)/2$

$= 25 \times 25$

$= 625$

$\therefore$ Sum of odd numbers from 0 to 50 is 625.

Question: 13

Solution:

Natural numbers between 200 and 400 which are divisible by 7 are 203, 210, 217, ..., 399.

Sum of these numbers forms an arithmetic series 203 + 210 + 217 + ... + 399.

Here, first term = a = 203

Common difference = d = 7

$\therefore a_n = a + (n - 1)d$

$\Rightarrow 399 = 203 + (n - 1)7$

$\Rightarrow 399 = 7n + 196$

$\Rightarrow 7n = 203$

$\Rightarrow n = 29$

∴ there are 29 terms in the AP.

Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefore sum of 28 terms of this arithmetic series is given by:

$\therefore S_{29} = \frac{29}{2}[2(203) + (29 - 1)(7)]$

= (29/2) [406 + 196]

=(29/2) × 502

= 7279

Question: 14

Solution:

First 40 positive integers divisible by 6 are 6, 12, 18, ..., 240.

Sum of these numbers forms an arithmetic series 6 + 12 + 18 + ... + 240.

Here, first term = a = 6

Common difference = d = 6

Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefore sum of 40 terms of this arithmetic series is given by:

$\therefore S_{40} = \frac{40}{2}[2(6) + (40 - 1)(6)]$

= 20 [12 + 234]

=20 × 246

= 4920

Question: 15

Solution:

First 15 multiples of 8 are 8, 16, 24, ..., 120.

Sum of these numbers forms an arithmetic series 8 + 16 + 24 + ... + 120.

Here, first term = a = 8

Common difference = d = 8

Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefore sum of 15 terms of this arithmetic series is given by:

$\therefore S_{15} = \frac{15}{2}[2(8) + (15 - 1)(8)]$

= (15/2) [16 + 112]

=(15/2) × 128

= 15 × 64

= 960

Question: 16

Solution:

Multiples of 9 lying between 300 and 700 are 306, 315, 324, ..., 693.

Sum of these numbers forms an arithmetic series 306 + 315 + 324 + ... + 693.

Here, first term = a = 306

Common difference = d = 9

We first find the number of terms in the series.

Here, last term = l = 693

∴ 693 = a + (n - 1)d

⇒ 693 = 306 + (n - 1)9

⇒ 693 - 306 = 9n - 9

⇒ 387 = 9n - 9

⇒ 387 + 9 = 9n

⇒ 9n = 396

⇒ n = 44

Now, Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefore sum of 44 terms of this arithmetic series is given by:

$\therefore S_{44} = \frac{44}{2}[2(306) + (44 - 1)(9)]$

= 22 × [612 + 387]

= 22 × 999

= 21978

Question: 17

Solution:

Three - digit natural numbers which are divisible by 13 are 104, 117, 130, ..., 988.

Sum of these numbers forms an arithmetic series 104 + 117 + 130 + ... + 988.

Here, first term = a = 104

Common difference = d = 13

We first find the number of terms in the series.

Here, last term = l = 988

∴ 988 = a + (n - 1)d

⇒ 988 = 104 + (n - 1)13

⇒ 988 - 104 = 13n - 13

⇒ 884 = 13n - 13

⇒ 884 + 13 = 13n

⇒ 13n = 897

⇒ n = 69

Now, Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefore sum of 69 terms of this arithmetic series is given by:

$\therefore S_{69} = \frac{69}{2}[2(104) + (69 - 1)(13)]$

= (69/2) × [208 + 884]

= (69/2) × 1092

= 69 × 546

= 3767

Question: 18

Solution:

First 100 even natural numbers which are divisible by 5 are 10, 20, 30, ..., 1000

Here, first term = a = 10

Common difference = d = 10

Number of terms = 100

Now, Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefore sum of 100 terms of this arithmetic series is given by:

$\therefore S_{100} = \frac{100}{2}[2(10) + (100 - 1)(10)]$

= 50 × [20 + 990]

= 50 × 1010

= 50500

Question: 19

Solution:

The given sum can be written as (1 + 1 + 1 + ...) - (1/n, 2/n, 3/n, ...)

Sum of first series up to n terms = 1 + 1 + 1 + ... up to n terms

= n

Now, consider the second series:

Here, first term = a = 1/n

Common difference = d = (2/n) - (1/n) = (1/n)

Now, Sum of n terms of an arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefore sum of n terms of second arithmetic series is given by:

$\therefore S_n = \frac{n}{2}[2(1/n) + (n - 1)(1/n)]$

$= \frac{n}{2}[(2/n) + 1 - (1/n)]$

$= \frac{n}{2}[(1/n) + 1]$

$= = \frac{n}{2} \times \frac{n+1}{n} = (n + 1)/2$

Now, sum of n terms of the complete series = Sum of n terms of first series - Sum of n terms of second series

= n - (n + 1)/2

= (2n - n - 1)/2

= 1/2 (n - 1)

Question: 20

Solution:

Let the first term be a.

Let Common difference be d.

Given: $S_5 + S_7 = 167$

$S_{10} = 235$

Now, Sum of n terms of an arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

So, consider

$S_5 + S_7 = 167$

⇒ (5/2) [2a + (5 - 1)d] + (7/2) [2a + (7 - 1)d] = 167

⇒ (5/2) [2a + 4d] + (7/2) [2a + 6d] = 167

⇒ 5 × [a + 2d] + 7 × [a + 3d] = 167

⇒ 5a + 10d + 7a + 21d = 167

⇒ 12a + 31d = 167 .... (1)

Now, consider $S_{10} = 235$

⇒ (10/2) [2a + (10 - 1)d] = 235

⇒ 5 × [2a + 9d] = 235

⇒ 10a + 45d = 235

⇒ 2a + 9d = 47 ………… (2)

Subtracting equation (1) from 6 times of equation (2), we get,

⇒ 23d = 115

⇒ d = 5

So, from equation (2), we get,

a = 1/2 (47 - 9d)

⇒ a = 1/2 (47 - 45)

⇒ a = 1/2 (2)

⇒ a = 1

Therefore the AP is a, a + d, a + 2d, a + 3d,…

i.e. 1, 6, 11, 16, ….

Question: 21

Solution:

Here, first term = a = 2

Let the Common difference = d

Last term = l = 29

Sum of all terms = S_n = 155

Let there be n terms in the AP.

Now, Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[a + a + (n - 1)d]$

$= \frac{n}{2}[a + l]$

Therefore sum of n terms of this arithmetic series is given by:

$\therefore S_n = \frac{n}{2}[2 + 29] = 155$

⇒ 31n = 310

⇒ n = 10

∴ there are 10 terms in the AP.

Thus 29 be the 10^{th} term of the AP.

∴ 29 = a + (10 - 1)d

⇒ 29 = 2 + 9d

⇒ 27 = 9d

⇒ d = 3

∴ common difference = d =3

Question: 22

Solution:

Here, first term = a = - 4

Let the Common difference = d

Last term = l = 29

Sum of all terms = S_n = 150

Let there be n terms in the AP.

Now, Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[a + a + (n - 1)d]$

$= \frac{n}{2}[a + l]$

Therefore sum of n terms of this arithmetic series is given by:

$\therefore S_n = \frac{n}{2}[-4 + 29] = 150$

$\Rightarrow 25n = 300$

$\Rightarrow n = 12$

$\therefore$ there are 12 terms in the AP.

Thus 29 is the 12^{th} term of the AP.

$\therefore 29 = a + (12 - 1)d$

$\Rightarrow 29 = -4 + 11d$

$\Rightarrow 29 + 4 = 11d$

$\Rightarrow 11d = 33$

$\Rightarrow d = 3$

$\therefore$ Common difference = d =3

Question: 23

Solution:

Here, first term = a = 17

Common difference = 9

Last term = l = 350

To find: number of terms and their sum.

Let there be n terms in the AP.

Since, l= 350

$\therefore 350 = 17 + (n - 1)9$

$\Rightarrow 350 - 17 = 9n - 9$

$\Rightarrow 333 = 9n - 9$

$\Rightarrow 333 + 9 = 9n$

$\Rightarrow 9n = 342$

$\Rightarrow n = 38$

Therefore number of terms = 38

Now, Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[a + a + (n - 1)d]$

$= \frac{n}{2}[a + l]$

Therefore sum of 38 terms of this arithmetic series is given by:

$\therefore S_{38} = \frac{38}{2}[17 + 350]$

$= 19 \times 367$

$= 6973$

$\therefore$ n= 38 and $S_n = 6973$

Question: 24

Solution:

Here, first term = a = 5

Let the Common difference = d

Last term = l = 45

Sum of all terms = $S_n = 400$

Let there be n terms in the AP.

Now, Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[a + a + (n - 1)d]$

$= \frac{n}{2}[a + l]$

Therefore sum of n terms of this arithmetic series is given by:

$\therefore S_n = \frac{n}{2}[5 + 45] = 400$

$\Rightarrow 50n = 800$

$\Rightarrow n = 16$

$\therefore$ there are 16 terms in the AP.

Thus 45 is the 16^{th} term of the AP.

$\therefore 45 = a + (16 - 1)d$

$\Rightarrow 45 = 5 + 15d$

$\Rightarrow 40 = 15d$

$\Rightarrow 15d = 40$

$\Rightarrow d = 8/3$

$\therefore$ Common difference = d = 8/3

Question: 25

Solution:

Here, first term = a = 22

Let the Common difference = d

n^{th} term = a_n = - 11

Sum of first n terms = S_n = 66

Let there be n terms in the AP.

Now, Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[a + a + (n - 1)d]$

$= \frac{n}{2}[a + a_n]$

Therefore sum of n terms of this arithmetic series is given by:

$\therefore S_n = \frac{n}{2}[22 + (-11)] = 66$

$\Rightarrow 11n = 132$

$\Rightarrow n = 12$

∴ there are 12 terms in the AP.

Thus n^{th} is the 12^{th} term of the AP.

$\therefore -11 = a + (12 - 1)d$

$\Rightarrow -11 = 22 + 11d$

$\Rightarrow -11 - 22 = 11d$

$\Rightarrow 11d = -33$

$\Rightarrow d = -3$

∴ Common difference = d = - 3

∴ n = 12, d = - 3

Question: 26

Solution:

Let a be the first term and d be the common difference.

Given: $a_{12} = -13$

$S_4 = 24$

To find: Sum of first 10 terms.

Consider $a_{12} = -13$

$\Rightarrow a + 11d = -13$ .... (1)

Also, $S_4 = 24$

$\Rightarrow (4/2) \times [2a + (4 - 1)d] = 24$

$\Rightarrow 2 \times [2a + 3d] = 24$

$\Rightarrow 2a + 3d = 12$ ....(2)

Subtracting equation (2) from twice of equation (1), we get,

$19d = -38$

$\Rightarrow d = -2$

Now, from equation (1), we get

a = - 13 - 11d

⇒ a = - 13 - 11(-2)

⇒ a = - 13 + 22

⇒ a = 9

Now, Sum of first n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefore sum of first 10 terms of this arithmetic series is given by:

$\therefore S_{10} = \frac{10}{2}[2(9) + (10 - 1)(-2)]$

= 5 × [18 - 18]

= 0

$\therefore S_{10} = 0$

Question: 27

Solution:

Let a be the first term and d be the common difference.

Given: $S_7 = 182$

4th and 17th terms are in the ratio 1 : 5.

i.e. [a + 3d] : [(a + 16d] = 1 : 5

$\Rightarrow \frac{(a+3d)}{(a+16d)} = \frac{1}{5}$

⇒ 5(a + 3 d) = (a + 16d)

⇒ 5a + 15d = a + 16d

⇒ 4a = d

Now, consider $S_7 = 182$

⇒ (7/2)[2a + (7 - 1)d] = 182

⇒ (7/2)[2a + 6(4a)] = 182

⇒ 7 × [26a] = 182 × 2

⇒ 182a = 364

⇒ a = 2

∴ d = 4a

⇒ d = 8

Thus the AP will be a, a + d, a + 2d,...

i.e. AP is 2, 10, 18, 26,....

Question: 28

Solution:

Let a be the first term and d be the common difference.

Given: $S_9 = 81$, $S_{20} = 400$

Now, consider $S_9 = 81$

$\Rightarrow (9/2)[2a + (9 - 1)d] = 81$

$\Rightarrow (9/2)[2a + 8d] = 81$

$\Rightarrow [2a + 8d] = 18$ ....(1)

Now, consider $S_{20} = 400$

$\Rightarrow (20/2)[2a + (20 - 1)d] = 400$

$\Rightarrow 10 \times [2a + 19d] = 400$

$\Rightarrow [2a + 19d] = 40$.............(2)

Now, on subtracting equation (2) from equation (1), we get,

$11d = 22$

$\Rightarrow d = 2$

$\therefore$ from equation (1), we get

$a = 1/2\ (18 - 8d)$

$\Rightarrow a = 9 - 4d$

$\Rightarrow a = 9 - 8$

$\Rightarrow a = 1$

$\therefore a = 1, d = 2$

Question: 29

Solution:

Let a be the first term and d be the common difference.

Given: $S_7 = 49$, $S_{17} = 289$

To find: sum of first n terms.

Now, consider $S_7 = 49$

$\Rightarrow (7/2)[2a + (7 - 1)d] = 49$

$\Rightarrow (7/2)[2a + 6d] = 49$

$\Rightarrow [a + 3d] = 7$ ....(1)

Now, consider $S_{17} = 289$

$\Rightarrow (17/2)[2a + (17 - 1)d] = 289$

$\Rightarrow (17/2) \times [2a + 16d] = 289$

$\Rightarrow [a + 8d] = 17$ ....(2)

Now, on subtracting equation (2) from equation (1), we get,

$5d = 10$

$\Rightarrow d = 2$

$\therefore$ from equation (1), we get

$a = (7 - 3d)$

$\Rightarrow a = 7 - 6$

$\Rightarrow a = 1$

$\therefore a = 1, d = 2$

Now, Sum of first n terms = S_n = (n/2)[2a + (n - 1)d]

= (n/2)[2 + (n - 1)2]

= (n/2)[2n]

= n^2

∴ $S_n = n^2$

Question: 30

Solution:

Let a_1 and a_2 be the first terms of the two APs

Let d_1 and d_2 be the common difference of the respective APs.

Given: $d_1 = d_2$ and $a_1 = 3$, $a_2 = 8$

To find: Difference between the sums of their first 50 terms.

i.e. to find: $(S_2)_{50} - (S_1)_{50}$

where $(S_1)_{50}$ denotes the sum of first 50 terms of first AP and $(S_2)_{50}$

denotes the sum of first 50 terms of second AP.

Now, consider $(S_1)_{50} = (50/2)[2a_1 + (50 - 1)d_1]$

= 25 × [2(3) + 49 × d_1]

= 25[6 + 49d_1]

= 150 + 1225d_1

Now, consider $(S_2)_{50} = (50/2)[2a_2 + (50 - 1)d_2]$

= 25 × [2(8) + 49 × d_2]

= 25[16 + 49d_1]

= 400 + 1225d_2

Now, $(S_2)_{50} - (S_1)_{50} = 400 + 1225d_2 - (150 + 1225d_2)$

= 400 - 150 (∵ $d_1 = d_2$)

= 250

∴ $(S_2)_{50} - (S_1)_{50} = 250$

Question: 31

Solution:

Let a be the first term and d be the common difference.

Given: Sum of first 10 terms = S_{10} = - 150

Sum of next 10 terms = - 550

i.e. $S_{20} - S_{10}$ = - 550

Consider S_{10} = - 150

⇒ (10/2)[2a + (10 - 1)d] = - 150

⇒ 5 × [2a + 9d] = - 150

⇒ [2a + 9d] = - 30......................... (1)

Now, consider S_{20} - S_{10} = - 550

⇒ (20/2)[2a + (20 - 1)d] - (10/2)[2a + (10 - 1)d] = - 550

⇒ 10 × [2a + 19d] - 5[2a + 9d] = - 550

⇒ 10a + 145d = - 550.......................... (2)

On subtracting equation (2) from 5 times of equation (2), we get,

- 100d = 400

⇒ d = - 4

∴ a = 1/2 (-30 - 9d)

⇒ a = 1/2 (-30 + 36)

⇒ a = 3

Therefore the AP is 3, - 1, - 5, - 9,....

Question: 32

Solution:

Let a be the first term and d be the common difference.

Given: a_5 = 16

a_{13} = 4 a_3

Now, Consider a_5 = 16

⇒ a + (5 - 1)d = 16

⇒ a + 4d = 16......................... (1)

Consider a_{13} = 4 a_3

⇒ a + 12d = 4(a + 2d)

⇒ a + 12d = 4a + 8d

⇒ 3a - 4d = 0 ....(2)

Now, adding equation (1) and (2), we get,

4a = 16

⇒ a = 4

∴ from equation (2), we get,

4d = 3a

⇒ 4d = 12

⇒ d = 3

Now, Sum of first n terms of an AP is

$S_n = \frac{n}{2}[2a + (n - 1)d]$

∴ Sum of first 10 terms is given by:

$S_{10} = \frac{10}{2}[2(4) + (10 - 1)(3)]$

= 5 × [8 + 27]

= 5 × 35

= 175

∴ S_{10}=175

Question: 33

Solution:

Let a be the first term and d be the common difference.

Given: $a_{10} = 41$

$a_{16} = 5\ a_3$

Now, Consider $a_{10} = 41$

⇒ a + (10 - 1)d = 41

⇒ a + 9d = 41……………………. (1)

Consider $a_{16} = 5\ a_3$

⇒ a + 15d = 5(a + 2d)

⇒ a + 15d = 5a + 10d

⇒ 4a - 5d = 0 ……………………..(2)

Now, subtracting equation (2) from 4 times of equation (1), we get,

41d = 164

⇒ d = 4

∴ from equation (2), we get,

4a= 5d

⇒ 4a = 20

⇒ a = 5

Now, Sum of first n terms of an AP is

$S_n = \frac{n}{2}[2a + (n - 1)d]$

∴ Sum of first 15 terms is given by:

$S_{15} = \frac{15}{2}[2(5) + (15 - 1)(4)]$

= (15/2) × [10 + 56]

= 15 × 33

= 495

∴ S_{15} = 495

Question: 34

Solution:

Here, First term = a = 5

Common difference = d = 12 - 5 = 7

No. of terms = 50

∴ last term will be 50^{th} term.

Using the formula for finding n^{th} term of an A.P.,

$l = a_{50} = a + (50 - 1) \times d$

$\therefore l = 5 + (50 - 1) \times 7$

$\Rightarrow l = 5 + 343 = 348$

Now, sum of last 15 terms = sum of first 50 terms - sum of first 35 terms

i.e. sum of last 15 terms = $S_{50} - S_{35}$

Now, Sum of first n terms of an AP is

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore$ Sum of first 50 terms is given by:

$S_{50} = \frac{50}{2}[2(5) + (50 - 1)(7)]$

$= 25 \times [10 + 343]$

$= 25 \times 353$

$= 8825$

Now, Sum of first 35 terms is given by:

$S_{35} = \frac{35}{2}[2(5) + (35 - 1)(7)]$

$= (35/2) \times [10 + 238]$

$= (35/2) \times 248$

$= 35 \times 124$

$= 4340$

Now, $S_{50} - S_{35} = 8825 - 4340$

$= 4485$

$\therefore$ last term = 348, sum of last 15 terms = 4485

Question: 35

Solution:

Here, First term = a = 8

Common difference = d = 10 - 8 = 2

No. of terms = 60

$\therefore$ last term will be 60^{th} term.

Using the formula for finding n^{th} term of an A.P.,

$l = a_{60} = a + (60 - 1) \times d$

$\therefore l = 8 + (60 - 1) \times 2$

$\Rightarrow l = 8 + 118 = 126$

Now, sum of last 10 terms = sum of first 60 terms - sum of first 50 terms

i.e. sum of last 10 terms = $S_{60} - S_{50}$

Now, Sum of first n terms of an AP is

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore$ Sum of first 50 terms is given by:

$S_{50} = \frac{50}{2}[2(8) + (50 - 1)(2)]$

= 25 × [16 + 98]

= 25 × 114

= 2850

Now, Sum of first 60 terms is given by:

$S_{60} = \frac{60}{2}[2(8) + (60 - 1)(2)]$

= 30 × [16 + 118]

= 30 × 248

= 4020

Now, $S_{60} - S_{50}$ = 4020 - 2850

= 1170

∴ last term = 126, sum of last 10 terms = 1170

Question: 36

Solution:

Let a be the first term and d be the common difference.

Given: $a_4 + a_8 = 24$

and $a_6 + a_{10} = 44$

To find: S_{10}

Now, Consider $a_4 + a_8 = 24$

⇒ a + 3d + a + 7d= 24

⇒ 2a + 10d = 24 .... (1)

Consider $a_6 + a_{10} = 44$

⇒ a + 5d + a + 9d = 44

⇒ 2a + 14d = 44 .... (2)

Subtracting equation (1) from equation (2), we get,

4d = 20

⇒ d = 5

∴ Common difference = d = 5

Thus from equation (1), we get,

a = - 13

Now, Sum of first n terms of an AP is

$S_n = \frac{n}{2}[2a + (n - 1)d]$

∴ Sum of first 10 terms is given by:

$S_{10} = \frac{10}{2}[2(-13) + (10 - 1)(5)]$

= 5 × [- 26 + 45]

= 5 × 19

= 95

∴ $S_{10} = 95$

Question: 37

Solution:

Let a be the first term and d be the common difference.

Given: Sum of first m terms of an AP is given by:

$S_m = \frac{m}{2}[2a + (m - 1)d] = 4m^2 - m$

Now, n^{th} term is given by: $a_n = S_n - S_{n-1}$

∴ $a_n = (4n^2 - n) - [4(n - 1)^2 - (n - 1)]$

$= (4n^2 - n) - [4(n^2 + 1 - 2n) - n + 1]$

$= 4n^2 - n - 4n^2 - 4 + 8n + n - 1$

$= 8n - 5$ .... (1)

Now, given that $a_n = 107$

⇒ $8n - 5 = 107$

⇒ $8n = 112$

⇒ $n = 14$

For 21^{st} term of AP, put n = 21 in the value of the nth term in equation (1), we get

$a_{21} = 8 \times (21) - 5$

⇒ $a_{21} = 168 - 5$

= 163

∴ $a_{21} = 163$

Question: 38

Solution:

Let a be the first term and d be the common difference.

Given: Sum of first q terms of an AP is given by:

$S_q = \frac{q}{2}[2a + (q - 1)d] = 63q - 3q^2$

Now, p^{th} term is given by: $a_p = S_p - S_{p-1}$

∴ $a_p = (63p - 3p^2) - [63(p - 1) - 3(p - 1)^2]$

$= (63p - 3p^2) - [63p - 63 - 3p^2 - 3 + 6p]$

$= 63p - 3p^2 - 63p + 63 + 3p^2 + 3 - 6p$

$= 66 - 6p$ ....(1)

Now, given that $a_p = -60$

⇒ $66 - 6p = -60$

⇒ $6p = 126$

⇒ $p = 21$

For 11^{th} term of AP, put p = 11 in the value of the p^{th} term in equation (1), we get

$a_{11} = 66 - 6 \times (11)$

$\Rightarrow a_{11} = 66 - 66$

$= 0$

$\therefore a_{11} = 0$

Question: 39

Solution:

Here, first term = a = - 12

Common difference = d = - 9 - (-12) = 3

Last term is 21.

Now, number of terms in this AP are given as:

$21 = a + (n - 1)d$

$\Rightarrow 21 = -12 + (n - 1)3$

$\Rightarrow 21 + 12 = 3n - 3$

$\Rightarrow 33 + 3 = 3n$

$\Rightarrow 36 = 3n$

$\Rightarrow n = 12$

If 1 is added to each term, then the new AP will be - 11, - 8, - 5,..., 22.

Here, first term = a = - 11

Common difference = d = - 8 - (-11) = 3

Last term = l = 22.

Number of terms will be the same,

i.e, number of terms = n = 12

$\therefore$ Sum of 12 terms of the AP is given by:

$S_{12} = (12/2) \times [a + l]$

$= 6 \times [-11 + 22]$

$= 6 \times 11$

$= 66$

$\therefore$ Sum of 12 terms of the new AP will be 66.

Question: 40

Solution:

Here, first term = a = 10

Let the Common difference = d

Sum of first 14 terms = S_{14} = 1505

Now, Sum of n terms of this arithmetic series is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{14} = \frac{14}{2}[2(10) + (14 - 1)d] = 1505$

$\Rightarrow 7 \times [20 + 13d] = 1505$

$\Rightarrow [20 + 13d] = 215$

$\Rightarrow 13d = 195$

$\Rightarrow d = 15$

Now, n^{th} term is given by:

$\therefore a_n = a + (n - 1)d$

$\Rightarrow a_{25} = 10 + (25 - 1)15$

$= 10 + (24 \times 15)$

$= 10 + 360$

$= 370$

Question: 41

Solution:

Here, second term $= a_2 = 14$

Third term $= a_3 = 18$

$\therefore$ Common difference $= a_3 - a_2 = 18 - 14 = 4$

Thus first term $= a = a_2 - d = 14 - 4 = 10$

Now, Sum of first n terms of an AP is given by

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore$ Sum of first 51 terms is given by:

$S_{51} = \frac{51}{2}[2(10) + (51 - 1)(4)]$

$= (51/2) \times [20 + 200]$

$= (51/2) \times 220$

$= (51) \times 110$

$= 5610$

$\therefore S_{51} = 5610$

Question: 42

Solution:

Number of trees planted by one section of class $1^{st} = 2$

Now, there are 2 sections, $\therefore$ Number of trees planted by class $1^{st} = 4$

Number of trees planted by one section of class $2^{nd} = 4$

Now, there are 2 sections, $\therefore$ Number of trees planted by class $2^{nd} = 8$

This will follow up to class 12^{th} and we will obtain an AP as

4, 8, 12, ... upto 12 terms.

Now, Total number of trees planted by the students = 4 + 8 + 12 + ... upto 12 terms.

∴ In this Arithmetic series, first term = a = 4

Common difference = d = 4

Now, $S_{12} = (12/2)[2a + (12 - 1)d]$

$= 6[2(4) + 11(4)]$

$= 6 \times [8 + 44]$

$= 6 \times 52$

$= 312$

∴ Total number of trees planted by the students = 312

Values shown in the question are care and awareness about conservation of nature and environment.

Question: 43

Solution:

To pick the first potato, the competitor has to run 5 m to reach the potato and 5 m to run back to the bucket.

∴ Total distance covered by the competitor to pick first potato = 2 × (5) = 10 m

To pick the second potato, the competitor has to run (5 + 3) m to reach the potato and (5 + 3) m to run back to the bucket.

∴ Total distance covered by the competitor to pick second potato = 2 × (5 + 3) = 16 m

To pick the third potato, the competitor has to run (5 + 3 + 3) m to reach the potato and (5 + 3 + 3) m to run back to the bucket.

∴ Total distance covered by the competitor to pick third potato = 2 × (5 + 3 + 3) = 22 m

This will continue and we will get a sequence of distance as 10, 16, 22,... upto 10 terms (as there are 10 potatoes to pick).

Total distance covered by the competitor to pick all the 10 potatoes = 10 + 16 + 22 + ... upto 10 terms.

This forms an Arithmetic series with first term = a = 10

and Common difference = d = 6

Number of terms = n = 10

Now, $S_{10} = (10/2)[2a + (10 - 1)d]$

$= 5 \times [2(10) + 9(6)]$

$= 5 \times [20 + 54]$

$= 5 \times 74$

$= 370$

∴ Total distance covered by the competitor = 370 m

Question: 44

Solution:

To water the first tree, the gardener has to cover 10 m to reach the tree and 10 m to go back to the tank.

∴ Total distance covered by the gardener to water first tree = 2 × (10) = 20 m

To water the second tree, the gardener has to cover (10 + 5) m to reach the tree and (10 + 5) m to go back to the tank.

∴ Total distance covered by the gardener to water second tree = 2 × (10 + 5) = 30

To water the third tree, the gardener has to cover (10 + 5 + 5) m to reach the tree 5) m to go back to the tank.

∴ Total distance covered by the gardener to water third tree = 2 × (10 + 5 + 5) = 40 m

This will continue and we will get a sequence of distance as 20, 30, 40,... upto 25 terms (as there are 25 trees to be watered).

Total distance covered by the gardener to water all 25 trees = 20 + 30 + 40 + ... upto 25 terms.

This forms an Arithmetic series with first term = a = 20

and Common difference = d = 10

Number of terms = n = 25

Now, S_{25} = (25/2)[2a + (25 - 1)d]

= (25/2) × [2(20) + 24(10)]

= (25/2) × [40 + 240]

= (25/2) × 280

= 25 × 140

= 3500

∴ Total distance covered by the gardener = 3500 m

Question: 45

Solution:

Let the first prize be Rs. x. Thus each succeeding prize is Rs. 20 less than the preceding prize.

∴ Second prize, third prize, ..., seventh prize be Rs. (x - 20), (x - 40) , ..., (x - 120).

This forms an AP as x, x - 20, ..., x - 120.

Here, first term = x

Common difference = x - 20 - x = - 20

Total number of terms = 7

Since, Total sum of prize amount = 700.

∴ Sum of all the terms = 700

Now, sum of first n terms of an AP is given by:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

∴ Sum of 7 terms of an AP is given by:

$S_7 = \frac{7}{2}[2a + (7 - 1)d] = 700$

$\Rightarrow \frac{7}{2}[2x + (7 - 1)(-20)] = 700$

⇒ 7[2x - 120] = 1400

⇒ 2x - 120 = 200

⇒ x - 60 = 100

⇒ x = 160

Thus, the prizes are as Rs. 160, Rs.140, Rs.120, Rs. 100, Rs. 80, Rs. 60, Rs. 40.

Question: 46

Solution:

Let the amount of money the man saved in first month = Rs. x

Now, the amount of money he saved in second month = Rs.(x + 100)

The amount of money he saved in third month = Rs.(x + + 100 + 100)

This will continue for 10 months.

∴ We get a an AP as x , x + 100, x + 200,... up to 10 terms.

Here, first term = x

Common difference = d = 100

Number of terms = n = 10

Total amount of money saved by the man = x + (x + 100) + (x + 200) + ... up to 10 terms. = Rs. 33000 (given)

∴ Sum of 10 terms of the Arithmetic Series = 33000

⇒ S_{10} =33000

⇒ (10/2) × [2a + (10 - 1)d] =33000

⇒ (10/2) × [2(x) + 9(100)] =33000

⇒ 5 × [2x + 900] =33000

⇒ 2x + 900 =6600

⇒ 2x = 6600 - 900

⇒ 2x = 5700

⇒ x = 2850

∴ Amount of money saved by the man in first month = Rs. 2850

Question: 47

Solution:

Let the first installment = Rs. x

Since the instalments form an arithmetic series, therefore let the common difference = d

Now, amount paid in 30 installments = two - third of the amount = (2/3) × (36000) = Rs. 24000

∴ Total amount paid by the man in 30 installments = 24000

Let S_n be that amount paid in 30 installments.

∴ S_{30} = 24000

⇒ (30/2) × [2x + (30 - 1)d] = 24000

⇒ 15 × [2x + 29d] = 24000

⇒ 2x + 29d = 1600 ....(1)

Now, Total sum of the amount = 36000

∴ S_{40} = 36000

⇒ (40/2) × [2x + (40 - 1)d] = 36000

⇒ 20 × [2x + 39d] = 36000

⇒ 2x + 39d = 1800 .... (2)

Subtracting equation (1) from equation (2), we get:

10d = 200

⇒ d = 20

∴ from equation (1), we get

$x = 1/2(1600 - 29d)$

= 1/2 (1600 - 580)

= 1/2 (1020)

= 510

Therefore the amount of first installment = Rs. 510

Question: 48

Solution:

Penalty for delay for first day = Rs. 200

Penalty for delay for second day = Rs. 250

Penalty for delay for third day = Rs. 300

Penalty for each succeeding day is Rs. 50 more than for the preceding day.

∴ The amount of penalties are in AP with common difference

= d = Rs.50

Also, number of days in delay of the work = 30 days

Thus the penalties are 200, 250, 300, ... up to 30 terms

Thus the amount of money paid by the contractor is 200 + 250 + 300 + ... up to 30 terms

Here, first term = a = 200

Common difference = d = 50

Number of terms = n = 30

∴ The sum = S_{30} = (30/2) × [2(200) + (30 - 1)(50)]

= 15 × [400 + 1450]

= 15 × 1850

= 27750

Thus the total amount of money paid by the contractor = Rs. 27750