# Chapter : 12. CIRCLES

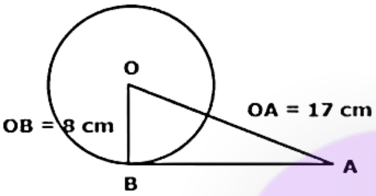
## Exercise : 12A

**Question: 1**

**Solution:**

Let us consider a circle with center O and radius 8 cm.

The diagram is given as:




Consider a point A 17 cm away from the center such that OA = 17 cm

A tangent is drawn at point A on the circle from point B such that OB = radius = 8 cm

To Find: Length of tangent AB = ?

As seen OB ⊥ AB

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

∴ In right - angled ΔAOB, By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$ ]

$(OA)^2 = (OB)^2 + (AB)^2$

$(17)^2 = (8)^2 + (AB)^2$

$289 = 64 + (AB)^2$

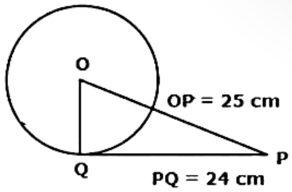
$(AB)^2 = 225$

AB = 15 cm

∴ The length of the tangent is 15 cm.

**Question: 2**

**Solution:**




Let us consider a circle with center O.

Consider a point P 25 cm away from the center such that OP = 25 cm

A tangent PQ is drawn at point Q on the circle from point P such that PQ = 24 cm

To Find : Length of radius OQ = ?

Now, OQ ⊥ PQ

[Tangent at any point on the circle is perpendicular to the radius through point c



∴ In right - angled △POQ,

By Pythagoras Theorem,

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$ ]

$(OP)^2 = (OQ)^2 + (PQ)^2$

$(25)^2 = (OQ)^2 + (24)^2$

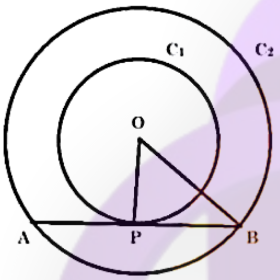
$625 = (OQ)^2 + 576$

$(OQ)^2 = 49$

OQ = 7 cm

**Question: 3**

**Solution:**




Given: Two concentric circles (say $C_1$ and $C_2$) with common center as O and radius $r_1 = 6.5$ cm and $r_2 = 2.5$ cm respectively.

To Find: Length of the chord of the larger circle which touches the circle $C_2$. i.e. Length of AB.

As AB is tangent to circle $C_2$ and we know that "Tangent at any point on the circle is perpendicular to the radius through point of contact"

So, we have,

$OP \perp AB$

∴ OPB is a right - angled triangle at P,

By Pythagoras Theorem in △OPB

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$ ]

We have,

$(OP)^2 + (PB)^2 = (OB)^2$

${r_2}^2 + (PB)^2 = {r_1}^2$

$(2.5)^2 + (PB)^2 = (6.5)^2$

$6.25 + (PB)^2 = 42.25$

$(PB)^2 = 36$

PB = 6 cm

Now, AP = PB ,

[ as perpendicular from center to chord bisects the chord and $OP \perp AB$ ]

So,

AB = AP + PB = PB + PB

= 2PB = 2(6)

= 12 cm

**Question: 4**

**Solution:**

Let AD = x cm, BE = y cm and CF = z cm

As we know that,

Tangents from an external point to a circle are equal,

In given Figure we have

AD = AF = x [Tangents from point A]

BD = BE = y [Tangents from point B]

CF = CE = z [Tangents from point C]

Now, Given: AB = 12 cm

AD + BD = 12

x + y = 12

y = 12 – x.... [1]

and BC = 8 cm

BE + EC = 8

y + z = 8

12 - x + z = 8 [From 1]

z = x – 4.... [2]

and

AC = 10 cm

AF + CF = 10

x + z = 10 [From 2]

x + x - 4 = 10

2x = 14

x = 7 cm

Putting value of x in [1] and [2]

y = 12 - 7 = 5 cm

z = 7 - 4 = 3 cm

So, we have AD = 7 cm, BE = 5 cm and CF = 3 cm

**Question: 5**

**Solution:**

Given: PA and PB are tangents to a circle with center O

To show : A, O, B and P are concyclic i.e. they lie on a circle i.e. AOBP is a cyclic quadrilateral.

Proof:

OB ⊥ PB and OA ⊥ AP

[Tangent at any point on the circle is perpendicular to the radius through point c

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∠OBP = ∠OAP = 90°

∠OBP + ∠OAP = 90 + 90 = 180°

AOBP is a cyclic quadrilateral i.e. A, O, B and P are concyclic.

[ As we know if the sum of opposite angles in a quadrilateral is 180° then quadrilateral is cyclic]

Hence Proved.

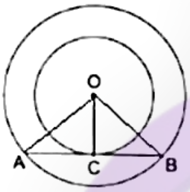
**Question: 6**

**Solution:**

Given: Two concentric circles with common center as O

To Prove: AC = CB

Construction: Join OC, OA and OB




Proof :

OC ⊥ AB

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

In △OAC and △OCB, we have

OA = OB

[∵ radii of same circle]

OC = OC

[∵ common]

∠OCA = ∠OCB

[∵ Both 90° as OC ⊥ AB]

△OAC ≅ △OCB

[By Right Angle - Hypotenuse - Side]

AC = CB

[Corresponding parts of congruent triangles are congruent]

Hence Proved.

**Question: 7**

**Solution:**

Given : From an external point P, two tangents, PA and PB are drawn to a circle with center O. At a point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. And PA = 14 cm

To Find : Perimeter of △PCD

As we know that, Tangents drawn from an external point to a circle are equal.

So we have

AC = CE ...[1] [Tangents from point C]

ED = DB ...[2] [Tangents from point D]

Now Perimeter of Triangle PCD

= PC + CD + DP

= PC + CE + ED + DP

= PC + AC + DB + DP [From 1 and 2]

= PA + PB

Now,

PA = PB = 14 cm as tangents drawn from an external point to a circle are equal

So we have

Perimeter = PA + PB = 14 + 14 = 28 cm

**Question: 8**

**Solution:**

As we know that tangents drawn from an external point to a circle are equal ,

In the Given image we have,

AP = AR = 7 cm ....[1]

[tangents from point A]

CR = QC = 5 cm ....[2]

[tangents from point C]

BQ = PB ...[3]

[tangents from point B]

Now,

AB = 10 cm [Given]

AP + PB = 10 cm

7 + PB = 10 [From 1]

PB = 3 cm

BQ = 3 cm .....[4]

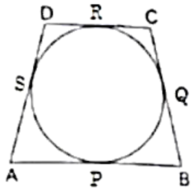
[From 3]

BC = BQ + QC = 5 + 3 = 8 cm [ From 2 and 4]

**Question: 9**

**Solution:**

Let sides AB, BC, CD, and AD touches circle at P, Q, R and S respectively.






As we know that tangents drawn from an external point to a circle are equal,

In the given image we have,

AP = AS = w (say) [Tangents from point A]

BP = BQ = x (say) [Tangents from point B]

CP = CR = y (say) [Tangents from point C]

DR = DS = z (say) [Tangents from point D]

Now,

Given,

AB = 6 cm

AP + BP = 6

w + x = 6 ...[1]

BC = 7 cm

BP + CP = 7

x + y = 7 ....[2]

CD = 4 cm

CR + DR = 4

y + z = 4 ....[3]

Also,

AD = AS + DS = w + z ....[4]

Add [1] and [3] and substracting [2] from the sum we get,

w + x + y + z - (x + y) = 6 + 4 - 7

w + z = 3 cm ; From [4]

AD = 3 cm

**Question: 10**

**Solution:**

As we know that tangents drawn from an external point to a circle are equal,

BR = BP [ Tangents from point B] [1]

QC = CP [ Tangents from point C] [2]

AR = AQ [ Tangents from point A] [3]

As ABC is an isosceles triangle,

AB = BC [Given] [4]

Now substract [3] from [4]

AB - AR = BC - AQ

BR = QC

BP = CP [ From 1 and 2]

∴ P bisects BC

Hence Proved.

**Question: 11**

**Solution:**

In given Figure,

OA ⊥ AP

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

∴ In right - angled △OAP,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OP)^2 = (OA)^2 + (PA)^2$

Given, PA = 10 cm and OA = radius of outer circle = 6 cm

$(OP)^2 = (6)^2 + (100)^2$

$(OP)^2 = 36 + 100 = 136$ [1]

Also,

OB ⊥ BP

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

∴ In right - angled △OBP,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OP)^2 = (OB)^2 + (PB)^2$

Now, OB = radius of inner circle = 4 cm

And from [2]

$(OP)^2 = 136$

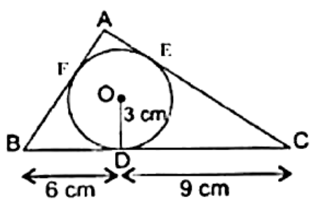
$136 = (4)^2 + (PB)^2$

$(PB)^2 = 136 - 16 = 120$

PB = 10.9 cm

**Question: 12**

**Solution:**



Given : △ABC that is drawn to circumscribe a circle with radius r = 3 cm and BI



Also, area(△ABC) = 54 $cm^2$

To Find : AB and AC

Now,

As we know tangents drawn from an external point to a circle are equal.

Then,

FB = BD = 6 cm [Tangents from same external point B]

DC = EC = 9 cm [Tangents from same external point C]

AF = EA = x (let) [Tangents from same external point A]

Using the above data, we get

AB = AF + FB = x + 6 cm

AC = AE + EC = x + 9 cm

BC = BD + DC = 6 + 9 = 15 cm

Now we have heron's formula for area of triangles if its three sides a, b and c are given

$$ar = \sqrt{s(s-a)(s-b)(s-c)}$$

Where,

$$\Rightarrow s = \frac{a+b+c}{2}$$

So for △ABC

a = AB = x + 6

b = AC = x + 9

c = BC = 15 cm

$$\Rightarrow s = \frac{x+6+x+9+15}{2} = x + 15$$

And

$$ar(\triangle ABC) = \sqrt{(x+15)(x+15-(x+6))(x+15-(x+9))(x+15-15)}$$

$$\Rightarrow 54 = \sqrt{(x+15)(9)(6)(x)}$$

Squaring both sides, we get,

54(54) = 54x(x + 15)

$x^2 + 15x - 54 = 0$

$x^2 + 18x - 3x - 54 = 0$

x(x + 18) - 3(x + 18) = 0

(x - 3)(x + 18) = 0

x = 3 or - 18

but x = - 18 is not possible as length can't be negative.

So

AB = x + 6 = 3 + 6 = 9 cm

AC = x + 9 = 3 + 9 = 12 cm

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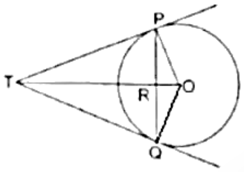
**Question: 13**

**Solution:**

Given : A circle with center O and radius 3 cm and PQ is a chord of length 4.8 cm. The tangents at P and Q intersect at point T

To Find : Length of TP

Construction : Join OQ




Now in △OPT and △OQT

OP = OQ [radii of same circle]

PT = PQ

[tangents drawn from an external point to a circle are equal]

OT = OT [Common]

△OPT ≅ △OQT [By Side - Side - Side Criterion]

∠POT = ∠OQT

[Corresponding parts of congruent triangles are congruent]

or ∠POR = ∠OQR

Now in △OPR and △OQR

OP = OQ [radii of same circle]

OR = OR [Common]

∠POR = ∠OQR [Proved Above]

△OPR ≅ △OQT [By Side - Angle - Side Criterion]

∠ORP = ∠ORQ

[Corresponding parts of congruent triangles are congruent]

Now,

∠ORP + ∠ORQ = 180° [Linear Pair]

∠ORP + ∠ORP = 180°

∠ORP = 90°

⇒ OR ⊥ PQ

⇒ RT ⊥ PQ

As OR ⊥ PQ and Perpendicular from center to a chord bisects the chord we have

$$PR = QR = \frac{PQ}{2} = \frac{4.8}{2} = 2.4 \text{ cm}$$

∴ In right - angled △OPR,

By Pythagoras Theorem

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$]

$(OP)^2 = (OR)^2 + (PR)^2$

$(3)^2 = (OR)^2 + (2.4)^2$

$9 = (OR)^2 + 5.76$

$(OR)^2 = 3.24$

$OR = 1.8$ cm

Now,

In right angled $\Delta TPR$,

By Pythagoras Theorem

$(PT)^2 = (PR)^2 + (TR)^2$ ...[1]

Also, $OP \perp OT$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

In right angled $\Delta OPT$, By Pythagoras Theorem

$(PT)^2 + (OP)^2 = (OT)^2$

$(PR)^2 + (TR)^2 + (OP)^2 = (TR + OR)^2$ ...[From 1]

$(2.4)^2 + (TR)^2 + (3)^2 = (TR + 1.8)^2$

$4.76 + (TR)^2 + 9 = (TR)^2 + 2(1.8)TR + (1.8)^2$

$13.76 = 3.6TR + 3.24$

$3.6TR = 10.52$

$TR = 2.9$ cm [Appx]

Using this in [1]

$PT^2 = (2.4)^2 + (2.9)^2$

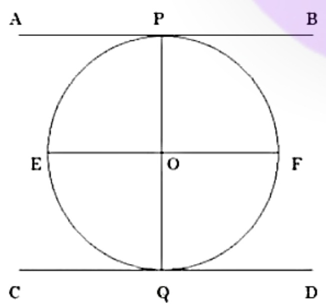
$PT^2 = 4.76 + 8.41$

$PT^2 = 13.17$

$PT = 3.63$ cm [Appx]

**Question: 14**

**Solution:**




Given: A circle with center O and AB and CD are two parallel tangents at points P and Q on the circle.

To Prove: PQ passes through O

Construction: Draw a line EF parallel to AB and CD and passing through O

Proof :

$\angle OPB = 90^\circ$



[Tangent at any point on the circle is perpendicular to the radius through point c

Now, AB || EF

$\angle OPB + \angle POF = 180^\circ$

$90^\circ + \angle POF = 180^\circ$

$\angle POF = 90^\circ$ ...[1]

Also,

$\angle OQD = 90^\circ$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

Now, CD || EF

$\angle OQD + \angle QOF = 180^\circ$

$90^\circ + \angle QOF = 180^\circ$

$\angle QOF = 90^\circ$ [2]

Now From [1] and [2]

$\angle POF + \angle QOF = 90 + 90 = 180^\circ$

So, By converse of linear pair POQ is a straight Line

i.e. O lies on PQ

Hence Proved.

**Question: 15**

**Solution:**

In quadrilateral POQB

$\angle OPB = 90^\circ$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\angle OQB = 90^\circ$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\angle PQB = 90^\circ$ [Given]

By angle sum property of quadrilateral PQOB

$\angle OPB + \angle OQB + \angle PBQ + \angle POQ = 360^\circ$

$90^\circ + 90^\circ + 90^\circ + \angle POQ = 360^\circ$

$\angle POQ = 90^\circ$

As all angles of this quadrilaterals are $90^\circ$ The quadrilateral is a rectangle

Also, OP = OQ = r

i.e. adjacent sides are equal, and we know that a rectangle with adjacent sides equal is a square

$\therefore$ POQB is a square

And OP = PB = BQ = OQ = r [1]

Now,

As we know that tangents drawn from an external point to a circle are equal

In given figure, We have

DS = DR = 5 cm

[Tangents from point D and DS = 5 cm is given]

AD = 23 cm [Given]

AR + DR = 23

AR + 5 = 23

AR = 18 cm

Now,

AR = AQ = 18 cm

[Tangents from point A]

AB = 29 cm [Given]

AQ + QB = 29

18 + QB = 29

QB = 11 cm

From [1]

QB = r = 11 cm

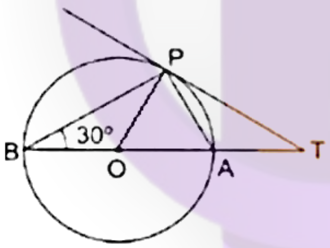
Hence Radius of circle is 11 cm.

**Question: 16**

**Solution:**

In Given Figure, we have a circle with center O let the radius of circle be r.

Construction : Join OP




Now, In △APB

∠ABP = 30°

∠APB = 90°

[Angle in a semicircle is a right angle]

By angle sum Property of triangle,

∠ABP + ∠APB + ∠PAB = 180

30° + 90° + ∠PAB = 180

∠PAB = 60°

OP = OA = r [radii]

∠PAB = ∠OPA = 60°

[Angles opposite to equal sides are equal]

By angle sum Property of triangle

$\angle OPA + \angle OAP + \angle AOP = 180°$

$60° + \angle PAB + \angle AOP = 180$

$60 + 60 + \angle AOP = 180$

$\angle AOP = 60°$

As all angles of $\triangle OPA$ equals to 60°, $\triangle OPA$ is an equilateral triangle

So, we have, OP = OA = PA = r [1]

$\angle OPT = 90°$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\angle OPA + \angle APT = 90$

$60 + \angle APT = 90$

$\angle APT = 30°$

Also,

$\angle PAB + \angle PAT = 180°$ [Linear pair]

$60° + \angle PAT = 180°$

$\angle PAT = 120°$

In $\triangle APT$

$\angle APT + \angle PAT + \angle PTA = 180°$

$30° + 120° + \angle PTA = 180°$

$\angle PTA = 30°$

So,

We have

$\angle APT = \angle PTA = 30°$

AT = PA

[Sides opposite to equal angles are equal]

AT = r [From 1] [2]

Now,

AB = OA + OB = r + r = 2r [3]

From [2] and [3]

AB : AT = 2r : r = 2 : 1

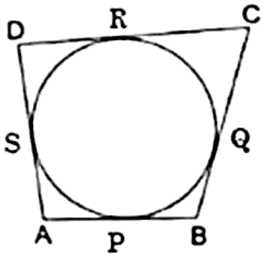
Hence Proved !

## Exercise : 12B

**Question: 1**

**Solution:**

Let sides AB, BC, CD, and AD touches circle at P, Q, R and S respectively.




As we know that tangents drawn from an external point to a circle are equal ,

In the given image we have,

AP = AS = w (say) [Tangents from point A]

BP = BQ = x (say) [Tangents from point B]

CP = CR = y (say) [Tangents from point C]

DR = DS = z (say) [Tangents from point D]

Now,

Given,

AB = 6 cm

AP + BP = 6

w + x = 6 [1]

BC = 9 cm

BP + CP = 9

x + y = 9 [2]

CD = 8 cm

CR + DR = 8

y + z = 8 [3]

Also,

AD = AS + DS = w + z [4]

Add [1] and [3] and substracting [2] from the sum we get,

w + x + y + z - (x + y) = 6 + 8 - 9

w + z = 5 cm

From [4]

AD = 5 cm

**Question: 2**

**Solution:**

In the given figure, PA and PB are two tangents from common point P

∴ PA = PB

[Tangents drawn from an external point are equal]

∠PBA = ∠PAB

[Angles opposite to equal angles are equal] [1]

By angle sum property of triangle in △APB

$\angle APB + \angle PBA + \angle PAB = 180°$

$50° + \angle PAB + \angle PAB = 180°$ [From 1]

$2\angle PAB = 130°$

$\angle PAB = 65°$ [2]

Now,

$\angle OAP = 90°$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

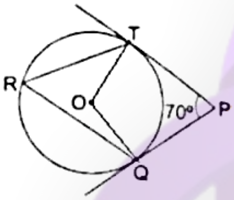
$\angle OAB + \angle PAB = 90°$

$\angle OAB + 65° = 90°$ [From 2]

$\angle OAB = 25°$

**Question: 3**

**Solution:**




Given: In the figure, PT and PQ are two tangents and $\angle TPQ = 70°$

To Find: $\angle TRQ$

Construction: Join OT and OQ

In quadrilateral OTPQ

$\angle OTP = 90°$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle OQP = 90°$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle TPQ = 70°$ [Common]

By Angle sum of Quadrilaterals,

In quadrilateral OTPQ we have

$\angle OTP + \angle OQP + \angle TPQ + \angle TOQ = 360°$

$90° + 90° + 70° + \angle TOQ = 360°$

$250° + \angle TOQ = 360$

$\angle TQO = 110°$

Now,

As we Know the angle subtended by an arc at the center is double the angle subtended by it at any

point on the remaining part of the circle.

∴ we have

$\angle TOQ = 2\angle TRQ$

$110° = 2\ \angle TRQ$

∠TRQ = 55°

**Question: 4**

**Solution:**

Given: AB and CD are two tangents to two circles which intersects at E .

To Prove: AB = CD

Proof:

As

AE = CE ...[1]

[Tangents drawn from an external point to a circle are equal]

And

EB = ED ...[2]

[Tangents drawn from an external point to a circle are equal]

Adding [1] and [2]

AE + EB = CE + ED

AB = CD

Hence Proved.

**Question: 5**

**Solution:**

Given: PT is a tangent to a circle with center O and PQ is a chord of the circle such that ∠QPT = 70°

To Find: ∠POQ = ?

Now,

∠OPT = 90°

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

∠OPQ + ∠QPT = 90°

∠OPQ + 70° = 90°

∠OPQ = 20°

Also,

OP = OQ [Radii of same circle]

∠OQP = ∠OPQ = 20°

[Angles opposite to equal sides are equal]

In △OPQ By Angle sum property of triangles,

∠OPQ + ∠OQP + ∠POQ = 180°

20° + 20° + ∠POQ = 180°

∠POQ = 140°

**Question: 6**

**Solution:**

Given: $\triangle ABC$ that is drawn to circumscribe a circle with radius r = 2 cm and BD 3cm

Also, area($\triangle ABC$) = 21 $cm^2$

To Find: AB and AC

Now,

As we know tangents drawn from an external point to a circle are equal.

Then,

FB = BD = 4 cm [Tangents from same external point B]

DC = EC = 3 cm [Tangents from same external point C]

AF = EA = x (let) [Tangents from same external point A]

Using the above data, we get

AB = AF + FB = x + 4 cm

AC = AE + EC = x + 3 cm

BC = BD + DC = 4 + 3 = 7 cm

Now we have heron's formula for area of triangles if its three sides a, b and c are given

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Where, $s = \frac{a+b+c}{2}$

So, for $\triangle ABC$

a = AB = x + 4

b = AC = x + 3

c = BC = 7 cm

$$\Rightarrow s = \frac{x+4+x+3+7}{2} = x + 7$$

And

$$ar(\triangle ABC) = \sqrt{(x+7)(x+7-(x+4))(x+7-(x+3))(x+7-7)}$$

$$\Rightarrow 21 = \sqrt{(x+7)(3)(4)(x)}$$

Squaring both sides,

21(21) = 12x(x + 7)

$12x^2 + 84x - 441 = 0$

$4x^2 + 28x - 147 = 0$

As we know roots of a quadratic equation in the form $ax^2 + bx + c = 0$ are,

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

So roots of this equation are,

$$x = \frac{-28 \pm \sqrt{(28)^2-4(4)(-147)}}{2(4)}$$

$$\Rightarrow x = \frac{-28 \pm \sqrt{3136}}{8}$$

$\Rightarrow x = \frac{-28 \pm 56}{8} = 3.5 \text{ or } -10.5$



but x = - 10.5 is not possible as length can't be negative.

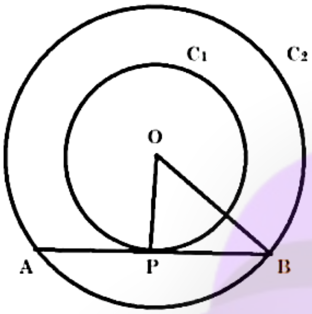
So

AB = x + 4 = 3.5 + 4 = 7.5 cm

AC = x + 3 = 3.5 + 3 = 6.5 cm

**Question: 7**

**Solution:**




Given : Two concentric circles (say $C_1$ and $C_2$) with common center as O and radius $r_1$ = 5 cm and $r_2$ = 3 cm respectively.

To Find : Length of the chord of the larger circle which touches the circle $C_2$. i.e. Length of AB.

As AB is tangent to circle $C_2$ and,

We know that "Tangent at any point on the circle is perpendicular to the radius through point of contact"

So, we have,

$OP \perp AB$

$\therefore$ OPB is a right - angled triangle at P,

By Pythagoras Theorem in $\triangle OPB$

[i.e. $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$ ]

We have,

$(OP)^2 + (PB)^2 = (OB)^2$

$r_2^2 + (PB)^2 = r_1^2$

$(3)^2 + (PB)^2 = (5)^2$

$9 + (PB)^2 = 25$

$(PB)^2 = 16$

PB = 4 cm

Now, AP = PB,

[ as perpendicular from center to chord bisects the chord and $OP \perp AB$ ]

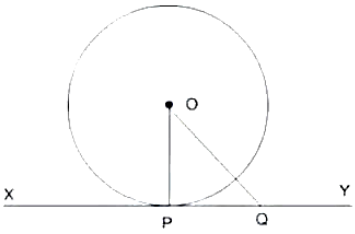
So,

AB = AP + PB = PB + PB

= 2PB = 2(4) = 8 cm

**Question: 8**

**Solution:**




Let us consider a circle with center O and XY be a tangent

To prove : Perpendicular at the point of contact of the tangent to a circle passes through the center i.e. the radius OP ⊥ XY

Proof :

Take a point Q on XY other than P and join OQ .

The point Q must lie outside the circle. (because if Q lies inside the circle, XY

will become a secant and not a tangent to the circle).

∴ OQ is longer than the radius OP of the circle. That is,

OQ > OP.

Since this happens for every point on the line XY except the point P, OP is the

shortest of all the distances of the point O to the points of XY.

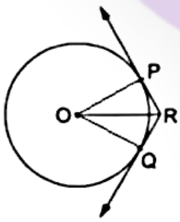
So OP is perpendicular to XY.

[As Out of all the line segments, drawn from a point to points of a line not passing through the point, the smallest is the perpendicular to the line.]

**Question: 9**

**Solution:**

Given : In the figure ,




Two tangents RQ and RP are drawn from an external point R to the circle with center O and ∠PRQ = 120°

To Prove: OR = PR + RQ

Construction: Join OP and OQ

Proof :

In ΔΔOPR and ΔOQR

OP = OQ [radii of same circle]

OR = OR [Common]

PR = PQ ...[1]

[Tangents drawn from an external point are equal]

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$\Delta OPR \cong \Delta OQR$

[By Side - Side - Side Criterion]

$\angle ORP = \angle ORQ$

[Corresponding parts of congruent triangles are congruent]

Also,

$\angle PRQ = 120°$

$\angle ORP + \angle ORQ = 120°$

$\angle ORP + \angle ORP = 120°$

$2\angle ORP = 120°$

$\angle ORP = 60°$

Also, $OP \perp PR$

[Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

So, In right angled triangle OPR,

$\cos\angle ORP = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{PR}{OR}$

$\cos 60° = \frac{PR}{OR} = \frac{1}{2}$

$\therefore OR = 2PR$

$OR = PR + PR$

$OR = PR + RQ$ [From 1]

Hence Proved.

**Question: 10**

**Solution:**

Let AD = x cm, BE = y cm and CF = z cm

As we know that,

Tangents from an external point to a circle are equal,

In given Figure we have

$AD = AF = x$

[Tangents from point A]

$BD = BE = y$

[Tangents from point B]6CF = CE = z [Tangents from point C]

Now, Given: AB = 14 cm

$AD + BD = 14$

$x + y = 14$

$y = 14 - x$ ... [1]

and BC = 8 cm

$BE + EC = 8$

$y + z = 8$

$14 - x + z = 8$ ... [From 1]

$z = x - 6$ [2]

and

CA = 12 cm

AF + CF = 12

$x + z = 12$ [From 2]

$x + x - 6 = 12$

$2x = 18$

$x = 9$ cm

Putting value of x in [1] and [2]

$y = 14 - 9 = 5$ cm

$z = 9 - 6 = 3$ cm

So, we have AD = 9 cm, BE = 5 cm and CF = 3 cm

**Question: 11**

**Solution:**

Given : PA and PB are tangents to a circle with center O

To show : AOBP is a cyclic quadrilateral.

Proof :

$OB \perp PB$ and $OA \perp AP$

[Tangent at any point on the circle is perpendicular to the radius through point of contact]

$\angle OBP = \angle OAP = 90°$

$\angle OBP + \angle OAP = 90 + 90 = 180°$

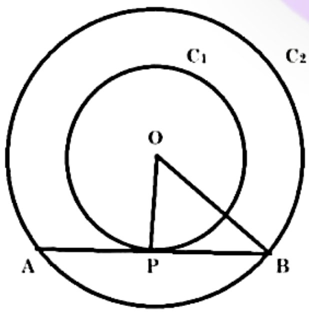
AOBP is a cyclic quadrilateral

[ As we know if the sum of opposite angles in a quadrilateral is 180° then quadrilateral is cyclic ]

Hence Proved.

**Question: 12**

**Solution:**




Let us consider circles $C_1$ and $C_2$ with common center as O. Let AB be a tangent to circle $C_1$ at point P and chord in circle $C_2$. Join OB

In $\triangle OPB$

OP ⊥ AB



[Tangents drawn at a point on circle is perpendicular to the radius through point

∴ OPB is a right - angled triangle at P,

By Pythagoras Theorem,

[i.e. $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$]

$(OB)^2 = (OP)^2 + (PB)^2$

Now, 2PB = AB

[As we have proved above that OP ⊥ AB and Perpendicular drawn from center to a chord bisects the chord]

2PB = 8 cm

PB = 4 cm

$(OB)^2 = (5)^2 + (4)^2$

[As OP = 5 cm, radius of inner circle]

$(OB)^2 = 41$

⇒ OB = $\sqrt{41}$ cm

**Question: 13**

**Solution:**

Given : , PQ is a chord of a circle with center 0 and PT is a tangent and ∠QPT = 60°.

To Find : ∠PRQ

∠OPT = 90°

∠OPQ + ∠QPT = 90°

∠OPQ + 60° = 90°

∠OPQ = 30° ... [1]

Also.

OP = OQ [radii of same circle]

∠OQP = ∠OPQ [Angles opposite to equal sides are equal]

From [1], ∠OQP = ∠OQP = 30°

In ΔOPQ , By angle sum property

∠OQP + ∠OPQ + ∠POQ = 180°

30° + 30° + ∠POQ = 180°

∠POQ = 120°

As we know, the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

So, we have

2∠PRQ = reflex ∠POQ

2∠PRQ = 360° - ∠POQ

2∠PRQ = 360° - 120° = 240°

∠PRQ = 120°

**Question: 14**

**Solution:**



In the given figure, PA and PB are two tangents from common point P

∴ PA = PB

[∵ Tangents drawn from an external point are equal]

$\angle PBA = \angle PAB$

[∵ Angles opposite to equal angles are equal] ...[1]

By angle sum property of triangle in $\triangle APB$

$\angle APB + \angle PBA + \angle PAB = 180°$

$60° + \angle PAB + \angle PAB = 180°$ [From 1]

$2\angle PAB = 120°$

$\angle PAB = 60°$ ...[2]

Now,

$\angle OAP = 90°$ [Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle OAB + \angle PAB = 90°$

$\angle OAB + 60° = 90°$ [From 2]

$\angle OAB = 30°$