Chapter : 16. COORDINATE GEOMETRY

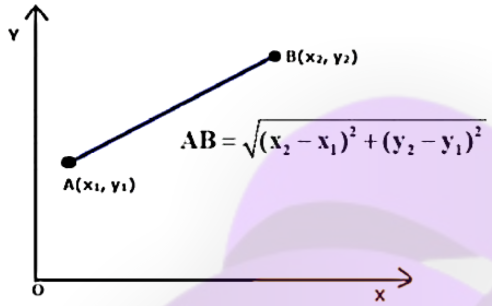
Exercise : 16A

Question: 1 A

Solution:

In this question, we have to use the distance formula to between two pointswhich is given by, say for points $P(x_1,x_2)$ and $Q(y_1,y_2)$ then

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$AB = \sqrt{\{(15 - 9)^2 + (11 - 3)^2\}}$

$= \sqrt{\{(6)^2 + (8)^2\}}$

$= \sqrt{\{36 + 64\}}$

$= \sqrt{100}$

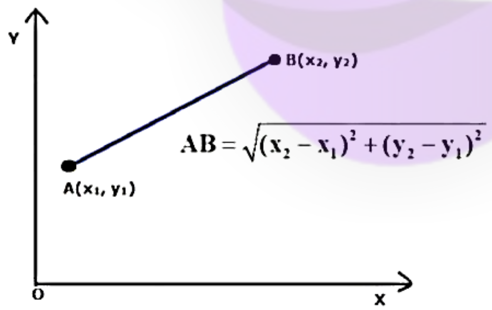
$\therefore$ AB = 10 units.

Question: 1 B

Solution:

In this question, we have to use the distance formula to between two pointswhich is given by, say for points $P(x_1,x_2)$ and $Q(y_1,y_2)$ then

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$AB = \sqrt{\{(-5 - 7)^2 + (1 - (-4))^2\}}$

$= \sqrt{\{(-12)^2 + (5)^2\}}$

$= \sqrt{\{144 + 25\}}$

$= \sqrt{169}$

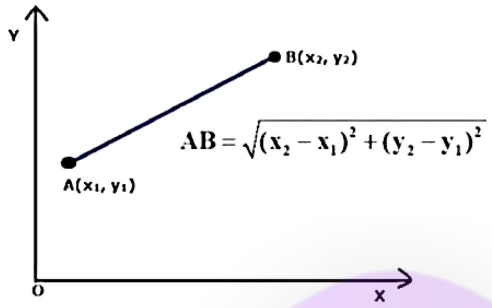
$\therefore$ AB = 13 units

Question: 1 C

Solution:

In this question, we have to use the distance formula to between two pointswhich is given by, say for points $P(x_1,x_2)$ and $Q(y_1,y_2)$ then

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$AB = \sqrt{\{(9 - (-6))^2 + (-12 - (-4))^2\}}$

$= \sqrt{\{(15)^2 + (-8)^2\}}$

$= \sqrt{\{225 + 64\}}$

$= \sqrt{289}$

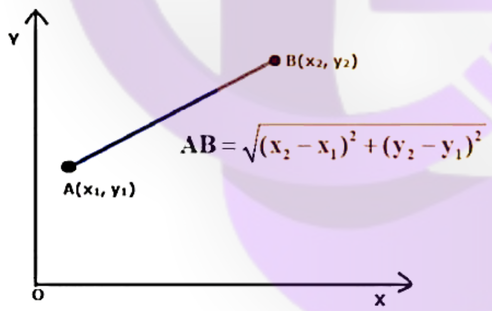
$\therefore$ AB = 17 units

Question: 1 D

Solution:

In this question, we have to use the distance formula to between two pointswhich is given by, say for points $P(x_1,x_2)$ and $Q(y_1,y_2)$ then

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$AB = \sqrt{\{(4 - 1)^2 + (-6 - (-3))^2\}}$

$= \sqrt{\{(3)^2 + (-3)^2\}}$

$= \sqrt{\{9 + 9\}}$

$= \sqrt{18}$

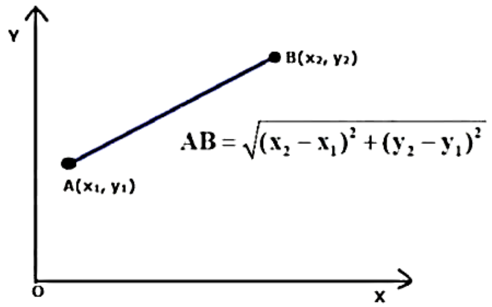
$\therefore AB = 3\sqrt{2}$ units

Question: 1 E

Solution:

In this question, we have to use the distance formula to between two pointswhi
say for points $P(x_1,x_2)$ and $Q(y_1,y_2)$ then

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$AB = \sqrt{\{((a - b) - (a + b))^2 + ((a + b) - (a - b))^2\}}$

$= \sqrt{\{(-2b)^2 + (2b)^2\}}$

$= \sqrt{\{4b^2 + 4b^2\}}$

$= \sqrt{8b^2}$

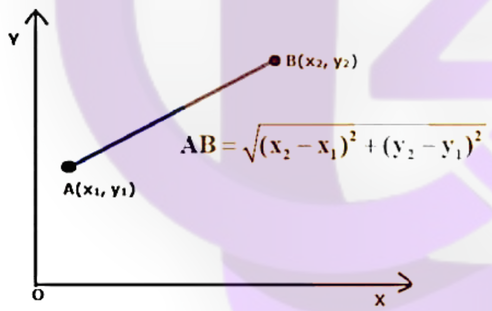
$\therefore AB = 2\sqrt{2}b$ units

Question: 1 F

Solution:

In this question, we have to use the distance formula to between two pointswhich is given by, say for points $P(x_1,x_2)$ and $Q(y_1,y_2)$ then

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$PQ = \sqrt{\{(a \cos a - a \sin a)^2 - (-a \sin a - a \cos a)^2\}}$

$= \sqrt{\{(a^2 \cos^2 a + a^2 \sin^2 a - 2a^2 \sin a.\cos a + a^2 \cos^2 a + a^2 \sin^2 a + 2a^2 \sin a.\cos a\}}$

$= \sqrt{\{a^2 (\cos^2 a + \sin^2 a) + (a^2 (\cos^2 a + \sin^2 a))\}}$

$= \sqrt{a^2(1) + a^2 (1)}$

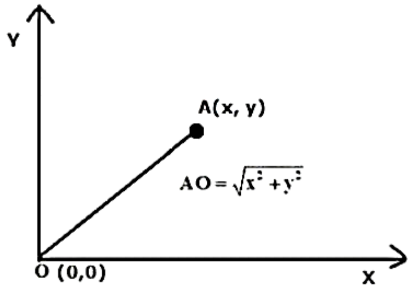
$= \sqrt{a^2(1 + 1)}$

$\therefore PQ = a\sqrt{2}$ units

Question: 2 A

Solution:

Since it is given that the distance is to be found from origin so in this question we have to use the distance formula keeping one – point fix i.e. O (0,0), as shown below:



OA = $\sqrt{(5 - 0)^2 + (-12 - 0)^2}$

= $\sqrt{(5)^2 + (-12)^2}$

= $\sqrt{25 + 144}$

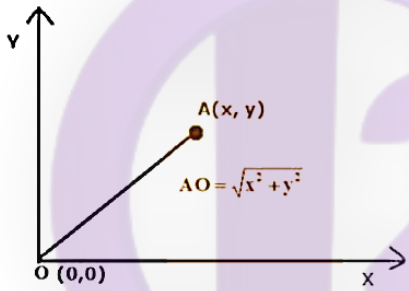
= $\sqrt{169}$

∴ OA = 13 units

Question: 2 B

Solution:

Since it is given that the distance is to be found from origin so in this question we have to use the distance formula keeping one – point fix i.e. O (0,0), as shown below:



OB = $\sqrt{(-5 - 0)^2 + (5 - 0)^2}$

= $\sqrt{(-5)^2 + (5)^2}$

= $\sqrt{25 + 25}$

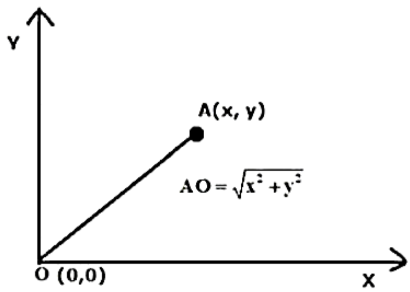
= $\sqrt{50}$

∴ OB = $5\sqrt{2}$ units

Question: 2 C

Solution:

Since it is given that the distance is to be found from origin so in this question we have to use the distance formula keeping one – point fix i.e. O (0,0), as shown below:



OC = $\sqrt{(-4-0)^2 + (-6-0)^2}$

= $\sqrt{(-4)^2 + (-6)^2}$

= $\sqrt{16 + 36}$

∴ OC = $\sqrt{52}$ units

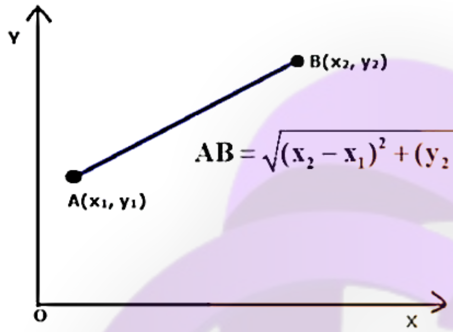
Question: 3

Solution:

Given:

Distance AB = 5 units

By distance formula, as shown below:



AB = $\sqrt{(5-x)^2 + (3-(-1))^2}$

5 = $\sqrt{(5-x)^2 + (4)^2}$

5 = $\sqrt{25 + x^2 - 10x + 16}$

5 = $\sqrt{41 + x^2 - 10x}$

Squaring both sides we get

25 = 41 + x^2 − 10x

⇒ 16 + x^2 − 10x = 0

⇒ (x − 8)(x − 2) = 0

⇒ x = 8 or x = 2

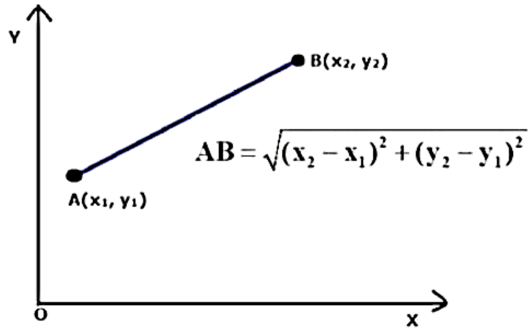
∴ The values of x can be 8 or 2

Question: 4

Solution:

Given, the distance AB = 10 units

By distance formula, as shown below:



$AB = \sqrt{\{(10 - 2)^2 + (y - (-3))^2\}}$

$10 = \sqrt{\{(8)^2 + (y + 3)^2\}}$

$10 = \sqrt{\{64 + y^2 + 6y + 9\}}$

$10 = \sqrt{\{73 + y^2 + 6y\}}$

Squaring both sides we get

$100 = 73 + y^2 + 6y$

On solving the equation, $100 = 73 + y^2 + 6y$

$\Rightarrow 27 + y^2 + 6y = 0$

$\Rightarrow y^2 + 6y + 27 = 0$

$\Rightarrow (y - 3)(y + 9) = 0$

$\Rightarrow y = 3$ or $y = -9$

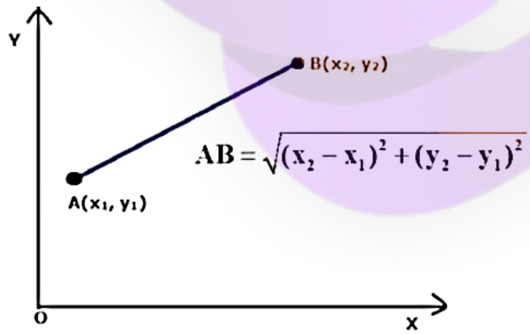
$\therefore$ The values of y can be 3 or – 9

Question: 5

Solution:

Given the distance PQ = 10 units

By distance formula, as shown below:



$PQ = \sqrt{\{(9 - x)^2 + (10 - 4)^2\}}$

$10 = \sqrt{\{(9 - x)^2 + (6)^2\}}$

$10 = \sqrt{\{81 + x^2 - 18x + 36\}}$

$10 = \sqrt{\{117 + x^2 - 18x\}}$

Squaring both sides we get

$\Rightarrow 100 = 117 + x^2 - 18x$

$\Rightarrow x^2 - 18x + 17x = 0$

$\Rightarrow (x - 1)(x - 17)$

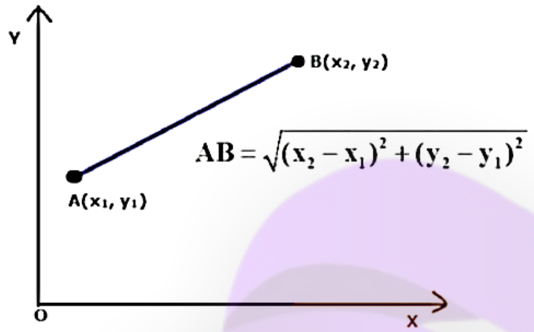
$\Rightarrow x = 1$ or $x = 17$

Question: 6

Solution:

Given that point A is equidistant from points B and C , so AB = AC

By distance formula, as shown below:



$AB = \sqrt{(8 - x)^2 + (-2 - 2)^2}$

$= \sqrt{(8 - x)^2 + (-4)^2}$

$= \sqrt{64 + x^2 - 16x + 16}$

$= \sqrt{80 + x^2 - 16x}$

$AC = \sqrt{(2 - x)^2 + (-2 - 2)^2}$

$= \sqrt{(2 - x)^2 + (4)^2}$

$= \sqrt{4 + x^2 - 4x + 16}$

$= \sqrt{20 + x^2 - 4x}$

Now, AB = AC

Squaring both sides, we get,

$(80 + x^2 - 16x) = (20 + x^2 - 4x)$

$60 = 12x$

$x = 5$

$\Rightarrow AB = \sqrt{80 + x^2 - 16x}$

$\Rightarrow AB = \sqrt{80 + 5^2 - 16 \times 5}$

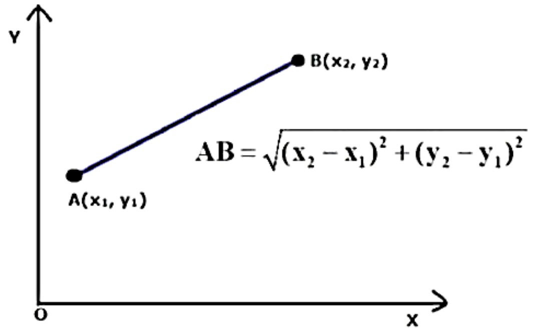
= 5 units

Question: 7

Solution:

Given that point A is equidistant from points B and C, so AB = AC

By distance formula, as shown below:



$AB = \sqrt{\{(3 - 0)^2 + (p - 2)^2\}}$

$= \sqrt{\{(3)^2 + (p - 2)^2\}}$

$= \sqrt{\{9 + p^2 - 4p + 4\}}$

$\Rightarrow AB = \sqrt{\{13 + p^2 - 4p\}}$

$AC = \sqrt{\{(p - 0)^2 + (5 - 2)^2\}}$

$= \sqrt{\{(p)^2 + (3)^2\}}$

$\Rightarrow AB = \sqrt{\{9 + p^2\}}$

Now, AB = AC

Squaring both sides, we get,

$(13 + p^2 - 4p) = (9 + p^2)$

$\Rightarrow 4 = 4p$

$\Rightarrow p = 1$

Now, $AB = \sqrt{\{13 + p^2 - 4p\}}$

$\Rightarrow AB = \sqrt{(13 + 1 - 4)}$

$= \sqrt{10}$ units

Therefore, the distance of $AB = \sqrt{10}$ units.

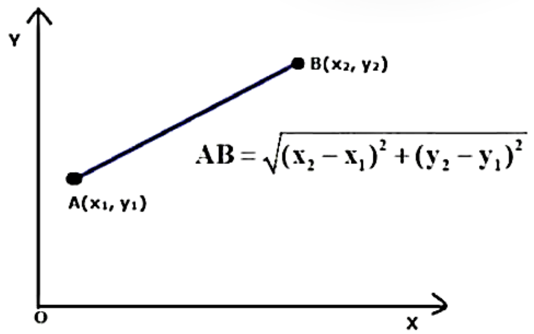
Question: 8

Solution:

Let the point be X(x,0) and the other two points are given as A(2, – 5) and B(– 2,9)

Given XA = XB

By distance formula, as shown below:



$XA = \sqrt{\{(2 - x)^2 + (-5 - 0)^2\}}$

$= \sqrt{(2 - x)^2 + (-5)^2}$

$= \sqrt{4 + x^2 - 4x + 25}$

$\Rightarrow XA = \sqrt{29 + x^2 - 4x}$

$XB = \sqrt{(-2 - x)^2 + (9 - 0)^2}$

$= \sqrt{(-2 - x)^2 + (9)^2}$

$= \sqrt{4 + x^2 + 4x + 81}$

$\Rightarrow XB = \sqrt{85 + x^2 + 4x}$

Now since

XA = XB

Squaring both sides, we get,

$(29 + x^2 - 4x) = (85 + x^2 + 4x)$

$56 = -8x$

$x = -7$

The point on × axis is (– 7, 0)

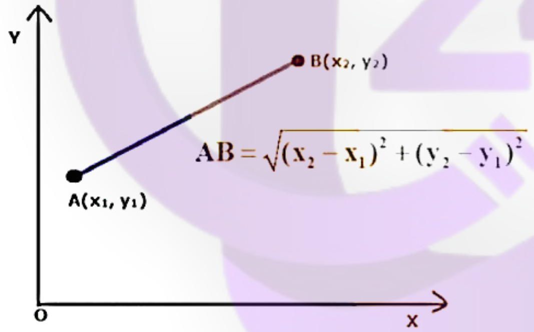
Question: 9

Solution:

Let the point be X(x,0)

XA = 10

By distance formula, as shown below:



$XA = \sqrt{(11 - x)^2 + (-8 - 0)^2}$

$10 = \sqrt{(11 - x)^2 + (-8)^2}$

$10 = \sqrt{121 + x^2 - 22x + 64}$

$10 = \sqrt{185 + x^2 - 22x}$

Squaring both sides we get

$100 = (185 + x^2 - 22x)$

$\Rightarrow 85 + x^2 - 22x = 0$

$\Rightarrow x^2 - 22x + 85 = 0$

$\Rightarrow (x - 5)(x - 17)$

$\Rightarrow x = 5$ or $x = 17$

The points are (5, 0) and (17, 0)

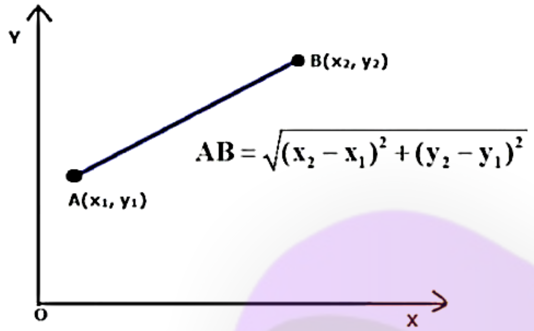
Question: 10

Solution:

Let the point be Y(0,y) and the other two points given as A(6,5) and B(– 4,3)

Given YA = YB

By distance formula, as shown below:



$YA = \sqrt{\{(6 - 0)^2 + (5 - y)^2\}}$

$= \sqrt{\{(6)^2 + (5 - y)^2\}}$

$= \sqrt{\{36 + 25 + y^2 - 10y\}}$

$\Rightarrow YA = \sqrt{\{61 + y^2 - 10y\}}$

$YB = \sqrt{\{(-4 - 0)^2 + (3 - y)^2\}}$

$= \sqrt{\{(-4)^2 + (9 + y^2 - 6y)\}}$

$= \sqrt{\{16 + 9 + y^2 - 6y\}}$

$\Rightarrow YB = \sqrt{\{25 + y^2 - 6y\}}$

Now, YA = YB

Squaring both sides, we get,

$(61 + y^2 - 10y) = (25 + y^2 - 6y)$

$36 = 4y$

$\Rightarrow y = 9$

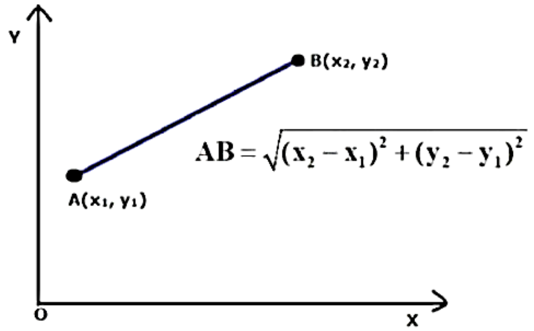
The point is (0, 9)

Question: 11

Solution:

The point P(x, y) is equidistant from the points A(5, 1) and B(– 1, 5), means PA = PB

By distance formula, as shown below:



PA = √{(5 − x)² + (1 − y)²}

= √{(25 + x^2 − 10x) + (1 + y^2 − 2y)}

⇒ PA = √{26 + x^2 − 10x + y^2 − 2y}

PB = √{(− 1 − x)^2 + (5 − y)^2}

= √{(1 + x^2 + 2x + 25 + y^2 − 10y)}

⇒ PB = √{(26 + x^2 + 2x + y^2 − 10y)}

Now, PA = PB

Squaring both sides, we get

26 + x^2 − 10x + y^2 − 2y = 26 + x^2 + 2x + y^2 − 10y

⇒ 12x = 8y

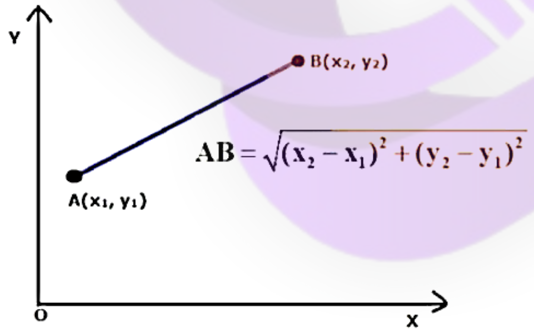
⇒3x = 2y

Hence proved.

Question: 12

Solution:

By distance formula, as shown below:



PA = √{(6 − x)^2 + (− 1 − y)^2}

= √{(36 + x^2 −12x) + (1 + y^2 + 2y)}

⇒ PA = √{37 + x^2 − 12x + y^2 + 2y}

PB = √{(2 − x)² + (3 − y)²}

= √{(4 + x^2 − 4x + 9 + y^2 − 6y)}

⇒ PB = √{(13 + x^2 − 4x + y^2 − 6y)}

Given: PA = PB

Squaring both sides, we get

$(37 + x^2 - 12x + y^2 + 2y) = (13 + x^2 - 4x + y^2 - 6y)$

$24 = 8x - 8y$

Dividing by 8

$x - y = 3$

Hence proved.

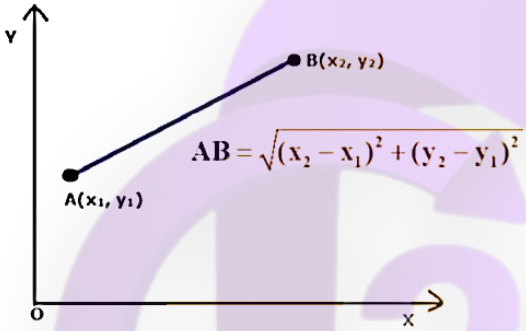
Question: 13

Solution:

Let the point be P(x,y), then since all three points are equidistant therefore

PA = PB = PC

By distance formula, as shown below:



We have, $PA = \sqrt{\{(5 - x)^2 + (3 - y)^2\}}$

$= \sqrt{\{25 + x^2 - 10x + 9 + y^2 - 6y\}}$

$\Rightarrow PA = \sqrt{\{34 + x^2 - 10x + y^2 - 6y\}}$

$PB = \sqrt{\{(5 - x)^2 + (-5 - y)^2\}}$

$= \sqrt{\{25 + x^2 - 10x + 25 + y^2 + 10y\}}$

$\Rightarrow PB = \sqrt{\{50 + x^2 - 10x + y^2 + 10y\}}$

$PC = \sqrt{\{(1 - x)^2 + (-5 - y)^2\}}$

$= \sqrt{\{1 + x^2 - 2x + 25 + y^2 + 10y\}}$

$\Rightarrow PC = \sqrt{\{26 + x^2 - 2x + y^2 + 10y\}}$

Squaring PA and PB we get

$\{34 + x^2 - 10x + y^2 - 6y\} = \{50 + x^2 - 10x + y^2 + 10y\}$

$\Rightarrow -16 = 16y$

$\Rightarrow y = -1$

Squaring PB and PC we get

$\{50 + x^2 - 2x + y^2 + 10y\} = \{26 + x^2 - 10x + y^2 + 10y\}$

$24 = -8x$

$x = -3$

$P(-3, -1)$

Question: 14

Solution:

$OA = \sqrt{(4 - 2)^2 + (3 - 3)^2}$

$= \sqrt{4}$

$= 2$

$OB = \sqrt{(x - 2)^2 + 4}$

$= \sqrt{x^2 + 4 - 4x + 4}$

$\sqrt{8 + x^2 - 4x}$

$OA^2 = OB^2$

$4 = 8 + x^2 - 4x$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow x^2 - 2x - 2x + 4 = 0$

$\Rightarrow x(x - 2) - 2(x - 2) = 0$

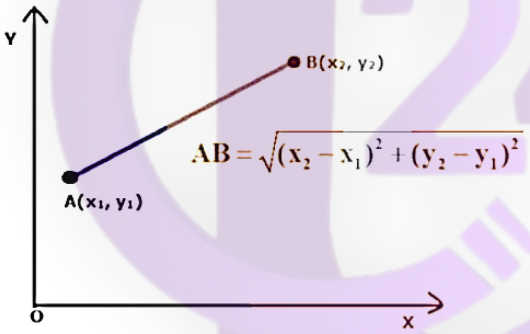
$\Rightarrow (x - 2)(x - 2) = 0$

$x = 2$

Question: 15

Solution:

By distance formula



$AC = \sqrt{(3 - (-2))^2 + (-1 - 3)^2}$

$= \sqrt{(5)^2 + (-4)^2}$

$= \sqrt{25 + 16}$

$= \sqrt{41}$

$BC = \sqrt{(x - (-2))^2 + (8 - 3)^2}$

$= \sqrt{(x + 2)^2 + 5^2}$

$= \sqrt{x^2 + 4 + 2x + 25}$

$= \sqrt{x^2 + 2x + 29}$

$AB = BC$

$\sqrt{x^2 + 2x + 29} = \sqrt{41}$

$x = 2$ or $x = -6$

Since, AB = BC

BC = $\sqrt{41}$ units

Question: 16

Solution:

AP = BP

AP = $\sqrt{\{(-2-2)^2 + (k-2)^2\}}$

= $\sqrt{\{16 + k^2 - 4k + 4\}}$

= $\sqrt{(k^2 - 2k + 20)}$

BP = $\sqrt{\{(-2k-2)^2 + (-3-2)^2\}}$

= $\sqrt{\{4k^2 + 8k + 4 + 25\}}$

= $\sqrt{(4k^2 + 8k + 29)}$

Squaring AP and BP and equating them we get

$k^2 - 4k + 20 = 4k^2 + 8k + 29$

$3k^2 + 12k + 9 = 0$

$(k + 3)(k + 1) = 0$

$\Rightarrow k = -3$

$\Rightarrow$ AP = $\sqrt{41}$units

Or $k = -1$

$\Rightarrow$ AP = 5 units

Question: 17

Solution:

Let point P(x,y) , A(a + b,a – b) , B(a – b,a + b)

Then AP = BP

AP = $\sqrt{\{((a+b)-x)^2 + ((a-b)-y)^2\}}$

= $\sqrt{\{(a+b)^2 + x^2 - 2(a+b)x + (a-b)^2 + y^2 - 2(a-b)y\}}$

= $\sqrt{(a^2 + b^2 + 2ab + x^2 - 2(a+b)x + b^2 + a^2 - 2ab + y^2 - 2(a-b)y)}$

BP = $\sqrt{\{((a-b)-x)^2 + ((a+b)-y)^2\}}$

= $\sqrt{\{(a-b)^2 + x^2 - 2(a-b)x + (a+b)^2 + y^2 - 2(a+b)y\}}$

= $\sqrt{(a^2 + b^2 - 2ab + x^2 - 2(a-b)x + b^2 + a^2 + 2ab + y^2 - 2(a+b)y)}$

Squaring and Equating both we get

$a^2 + b^2 + 2ab + x^2 - 2(a+b)x + b^2 + a^2 - 2ab + y^2 - 2(a-b)y = a^2 + b^2 - 2ab + x^2 - 2(a-b)x + b^2 + a^2 + 2ab + y^2 - 2(a+b)y$

$-2(a+b)x - 2(a-b)y = -2(a-b)x - 2(a+b)y$

$ax + bx + ay - by = ax - bx + ay + by$

Hence

$bx = ay$

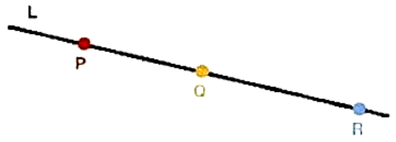
Question: 18

CLASS24

Solution:

Three or more points are collinear, if slope of any two pairs of points is same. With three points A, B and C if Slope of AB = slope of BC = slope of AC

then A, B and C are collinear points.



Collinear points P, Q, and R.

Slope of any two points is given by:

$(y_2 - y_1)/(x_2 - x_1)$.

(i) Slope of AB = (2 – (– 1))/(5 – 1) = 3/4

Slope of BC = (5 – 2)/(9 – 5) = 3/4

Slope of AB = slope of BC

Hence collinear.

(ii) Slope of AB = (1 – 9)/(0 – 6) = 8/6 = 4/3

Slope of BC = (– 6 – 0)/(– 7 – 1) = 6/6 = 1

Slope of AC = (– 7 – 9)/(– 6 – 6) = – 16/ – 12 = 4/3

Slope of AB = slope of AC

Hence collinear.

(iii) Slope of AB = ((3 – (– 1))/((2 – (– 1)) = 4/3

Slope of BC = (11 – 2)/(8 – 3) = 9/5 = 1

Slope of AC = ((11 – (– 1))/((8 – (– 1)) = 12/9 = 4/3

Slope of AB = slope of AC

Hence collinear.

(iv) Slope of AB = (1 – 5)/((0 – (– 2)) = – 4/2 = – 2

Slope of BC = (– 3 – 1)/(2 – 0) = – 4/2 = – 2

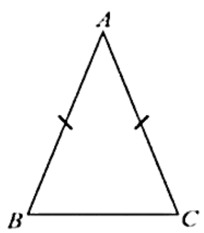
Slope of AB = slope of AB

Hence collinear.

Question: 19

Solution:

In an isosceles triangle any two sides are equal.



AB = $\sqrt{(-2-7)^2 + (5-10)^2}$

= $\sqrt{(-9)^2 + (-5)^2}$

= $\sqrt{81 + 25}$

= $\sqrt{106}$

BC = $\sqrt{(-4-5)^2 + (3-(-2))^2}$

= $\sqrt{(-9)^2 + (5)^2}$

= $\sqrt{81 + 25}$

= $\sqrt{106}$

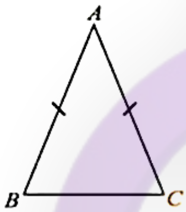
AB = BC

∴ It is an isosceles triangle.

Question: 20

Solution:

In an isosceles triangle any two sides are equal.



AB = $\sqrt{(6-3)^2 + (4-0)^2}$

= $\sqrt{(3)^2 + (4)^2}$

= $\sqrt{9 + 16}$

= $\sqrt{25}$ = 5 units

BC = $\sqrt{(-1-6)^2 + (3-4)^2}$

= $\sqrt{(-7)^2 + (-1)^2}$

= $\sqrt{49 + 1}$

= $\sqrt{50}$

AC = $\sqrt{(-1-3)^2 + (3-0)^2}$

= $\sqrt{(-4)^2 + (3)^2}$

= $\sqrt{16 + 9}$

= $\sqrt{25}$ = 5 units

AB = AC

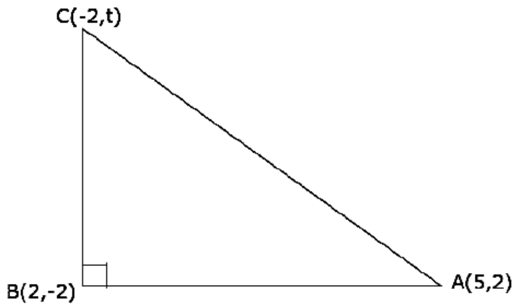
∴ It is an isosceles triangle.

Question: 21

Solution:

Given: A(5, 2), B(2, – 2) and C(– 2, t) are the vertices of a right triangle with ∠B = 90°**To**

find: The value of t.**Solution:** From the fig we



have $\angle B = 90°$, so by Pythagoras theorem we have $AC^2 = AB^2 + BC^2$

$AC^2 = (-2-5)^2 + (t-2)^2$

$= (-7)^2 + t^2 + 4 - 2t = 49 + t^2 + 4 - 2t = 53 + t^2 - 2t$

$AB^2 = (2-5)^2 + (-2-2)^2 = (-3)^2 + (-4)^2$

$= 9 + 16$

$= 25$

$BC^2 = (-2-2)^2 + (t+2)^2 = (-4)^2 + (t+2)^2$

$= 16 + t^2 + 4 + 2t$

$= 20 + t^2 + 2t$

$AB^2 + BC^2 = 25 + 20 + t^2 + 2t = 45 + t^2 + 2t$

$AC^2 = 53 + t^2 - 2t$

$\Rightarrow 53 + t^2 - 2t = 45 + t^2 + 2t$

$\Rightarrow 53 - 45 = 4t$

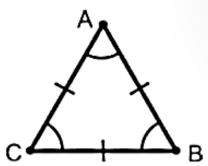
$\Rightarrow 8 = 4t \Rightarrow t = 2$

Question: 22

Solution:

For an equilateral triangle

AB = BC = AC



$AB = \sqrt{(6-4)^2 + (2-2)^2}$

$= \sqrt{(2)^2 + 0}$

$= \sqrt{4 + 0}$

$= \sqrt{4} = 2$ units

$BC = \sqrt{(2+\sqrt{3}-2)^2 + (5-6)^2}$

$= \sqrt{3 + (-1)^2}$

$= \sqrt{4} = 2$ units

$AC = \sqrt{(2+\sqrt{3}-2)^2 + (5-4)^2}$

$= \sqrt{3 + (-1)^2}$

$= \sqrt{4} = 2$ units

Hence, AB = BC = AC

∴ ABC is an equilateral triangle.

Question: 23

Solution:

Let the points be 3 (–3, –3), B (3, 3) and C $(-3\sqrt{3}, 3\sqrt{3})$

Then, $AB = \sqrt{(3 + 3)^2 + (3 + 3)^2}$

$= \sqrt{(-6)^2 + (6)^2}$

$= \sqrt{36+36}$

$= \sqrt{72}$

$= 3\sqrt{8}$

$BC = \sqrt{(-3\sqrt{3}+3)^2 + (3\sqrt{3}-3)^2}$

$= \sqrt{(1-\sqrt{3})^2 3^2 + (\sqrt{3}+1)^2 3^2}$

$= 3\sqrt{[1+3-2\sqrt{3}+3+1+2\sqrt{3}]}$

$= 3\sqrt{8}$

$CA = \sqrt{(-3\sqrt{3}-3)^2 + (3\sqrt{3}-3)^2}$

$= \sqrt{(-\sqrt{3}-1)^2 3^2 + (\sqrt{3}-1)^2 3^2}$

$= 3\sqrt{[3+1+2\sqrt{3}+3+1-2\sqrt{3}]}$

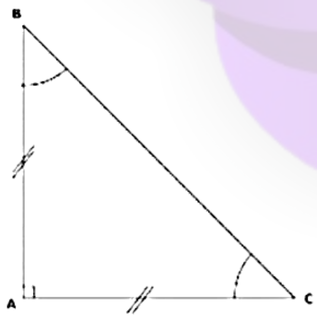
$= 3\sqrt{8}$

∵ AB = BC = CA

⇒ A, B, C are the vertices of an equilateral triangle.

Question: 24

Solution:



$AB = \sqrt{(0 - 6)^2 + (3 - (-5))^2}$

$= \sqrt{(-6)^2 + (8)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100} = 10$ units

$BC = \sqrt{(9 - 3)^2 + (8 - 0)^2}$

= $\sqrt{(6)^2 + (8)^2}$

= $\sqrt{36 + 64}$

= $\sqrt{100}$ = 10 units

AC = $\sqrt{(9 - (-5))^2 + (8 - 6)^2}$

= $\sqrt{(14)^2 + (2)^2}$

= $\sqrt{196 + 4}$

= $\sqrt{200}$

For the right angled triangle

$AC^2 = AB^2 + BC^2$

$AC^2 = 200$

$AB^2 + AC^2 = 100 + 100 = 200$

Since AB = BC

∴ ABC is an isosceles triangle.

Area = 1/2 (AB) (BC)

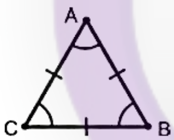
= 1/2 (10) (10)

= 1/2 (100)

= 50 sq units

Question: 25

Solution:



OA = $\sqrt{(\sqrt{3})^2 + (3 - 0)^2}$

= $\sqrt{(3) + (3)^2}$

= $\sqrt{3 + 9}$

= $\sqrt{12}$

AB = $\sqrt{(-\sqrt{3} - \sqrt{3})^2 + (3 - 3)^2}$

= $\sqrt{(-2\sqrt{3})^2}$

= $\sqrt{12}$

OB = $\sqrt{(3 - 0)^2 + (-\sqrt{3} - 0)^2}$

= $\sqrt{9 + 3}$

= $\sqrt{12}$

Since OA = AB = OB , ∴ equilateral triangle.

Area = 1/2 $[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

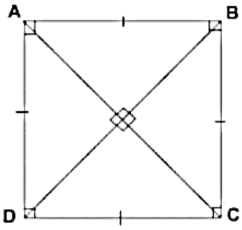
= 1/2[$-3\sqrt{3} - 3\sqrt{3}$]

= $-3\sqrt{3}$ sq units

Question: 26 A

CLASS24

Solution:



AB = √{(0 – 3)^2 + (5 – 2)^2} = √{9 + 9} = √18 units

BC = √{(– 3 – 0)^2 + (2 – 5)^2} = √{9 + 9} = √18 units

CD = √{(0 – (– 3))^2 + (– 1 – 2)^2} = √{9 + 9} = √18 units

DA = √{(0 – 3)^2 + (– 1 – 2)^2} = √{9 + 9} = √18 units

AC = √{(– 3 – 3)^2} = √36 = 6 units

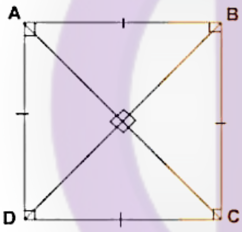
BD = √{(– 1 – 5)^2} = √36 = 6 units

Since AB = BC = CD = DA and AC = BD

∴ ABCD is a square.

Question: 26 B

Solution:



AB = √{(2 – 6)^2 + (1 – 2)^2} = √{16 + 1} = √17 units

BC = √{(1 – 2)^2 + (5 – 1)^2} = √{1 + 16} = √17 units

CD = √{(5 – 1)^2 + (6 – 5)^2} = √{16 + 1} = √17 units

DA = √{(5 – 6)^2 + (6 – 2)^2} = √{16 + 1} = √17 units

AC = √{(1 – 6)^2 + (5 – 2)^2} = √{25 + 9} = √34 units

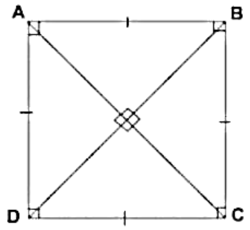
BD = √{(5 – 2)^2 + (6 – 1)^2} = √{25 + 9} = √34units

Since AB = BC = CD = DA and AC = BD

∴ ABCD is a square.

Question: 26 C

Solution:



AB = $\sqrt{\{(3-0)^2 + (1-(-2))^2\}} = \sqrt{\{9+9\}} = \sqrt{18}$ units

BC = $\sqrt{\{(0-3)^2 + (4-1)^2\}} = \sqrt{\{9+9\}} = \sqrt{18}$ units

CD = $\sqrt{\{(-3-0)^2 + (1-4)^2\}} = \sqrt{\{9+9\}} = \sqrt{18}$ units

DA = $\sqrt{\{(-3-0)^2 + (1-(-2))^2\}} = \sqrt{\{9+9\}} = \sqrt{18}$ units

AC = $\sqrt{\{(4-(-2))^2\}} = \sqrt{\{36\}} = 6$ units

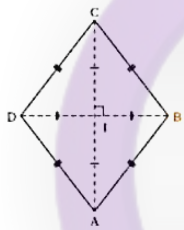
BD = $\sqrt{\{(-3-3)^2 + (1-1)^2\}} = \sqrt{\{36\}} = 6$units

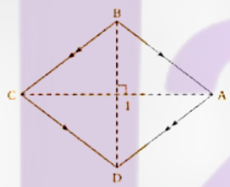
Since AB = BC = CD = DA and AC = BD

∴ ABCD is a square.

Question: 27

Solution:





AC = $\sqrt{\{(2-(-3))^2 + (-32)^2\}} = \sqrt{\{25+25\}} = \sqrt{50}$ units

BD = $\sqrt{\{(4-(-5))^2 + (4-(-5))^2\}} = \sqrt{\{81+81\}} = \sqrt{162}$ units

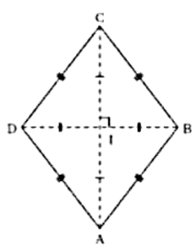
Area = 1/2× (product of diagonals)

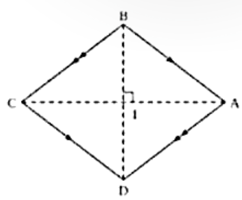
= $1/2 \times \sqrt{50} \times \sqrt{162}$

= 45 sq units

Question: 28

Solution:





AB = $\sqrt{\{(4-3)^2 + (5-0)^2\}} = \sqrt{\{1+25\}} = \sqrt{26}$ units

BC = $\sqrt{\{(-1-4)^2 + (4-5)^2\}} = \sqrt{\{25+1\}} = \sqrt{26}$ units

CD = $\sqrt{\{(-2-(-1))^2 + (-1-4)^2\}} = \sqrt{\{1+25\}} = \sqrt{26}$ units

$DA = \sqrt{(-2-3)^2 + (0-1)^2} = \sqrt{25+1} = \sqrt{26}$ units

$AC = \sqrt{(-1-3)^2 + (4-0)^2} = \sqrt{32}$

$BD = \sqrt{(-2-4)^2 + (-1-5)^2} = \sqrt{36+36} = 6\sqrt{2}$ units

Since AB = BC = CD = DA

Hence, ABCD is a rhombus

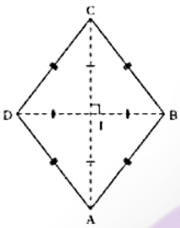
Area = 1/2 × (product of diagonals)

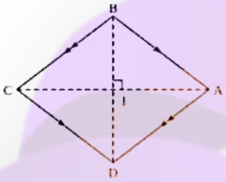
$= 1/2 \times 4\sqrt{2} \times 6\sqrt{2}$

= 24 sq units

Question: 29

Solution:





$AB = \sqrt{(8-6)^2 + (2-1)^2} = \sqrt{4+1} = \sqrt{5}$ units

$BC = \sqrt{(9-8)^2 + (4-2)^2} = \sqrt{1+4} = \sqrt{5}$ units

$CD = \sqrt{(7-9)^2 + (3-4)^2} = \sqrt{4+1} = \sqrt{5}$ units

$DA = \sqrt{(7-6)^2 + (3-1)^2} = \sqrt{1+4} = \sqrt{5}$ units

$AC = \sqrt{(9-6)^2 + (4-1)^2} = \sqrt{(9+9)} = 3\sqrt{2}$ units

$BD = \sqrt{(7-8)^2 + (3-2)^2} = \sqrt{1+1} = \sqrt{2}$ units

Since AB = BC = CD = DA

Hence, ABCD is a rhombus

Area = 1/2 × (product of diagonals)

$= 1/2 \times 3\sqrt{2} \times \sqrt{2}$

= 3 sq units

Question: 30

Solution:

$AB = \sqrt{(5-2)^2 + (2-1)^2} = \sqrt{9+1} = \sqrt{10}$ units

$BC = \sqrt{(6-5)^2 + (4-2)^2} = \sqrt{1+4} = \sqrt{5}$ units

$CD = \sqrt{(3-6)^2 + (3-4)^2} = \sqrt{9+1} = \sqrt{10}$ units

$DA = \sqrt{(3-2)^2 + (3-1)^2} = \sqrt{1+4} = \sqrt{5}$ units

Since AB = CD and BC = DA

∴ ABCD is Parallelogram

$AC = \sqrt{(6-2)^2 + (4-1)^2} = \sqrt{16+9} = 5$ units

For a Rectangle

$AC^2 = AB^2 + BC^2$

Here $AC^2 = 25$

But $AB^2 + BC^2 = 15$

∴ ABCD is not a rectangle

Question: 31

Solution:

$AB = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{9+1} = \sqrt{10}$ units

$BC = \sqrt{(6-4)^2 + (6-3)^2} = \sqrt{4+9} = \sqrt{13}$ units

$CD = \sqrt{(6-3)^2 + (5-6)^2} = \sqrt{9+1} = \sqrt{10}$ units

$DA = \sqrt{(3-1)^2 + (5-2)^2} = \sqrt{4+9} = \sqrt{13}$ units

AB = CD and BC = DA

∴ ABCD is a parallelogram ∴

$AC = \sqrt{(6-1)^2 + (6-2)^2} = \sqrt{25+16} = \sqrt{41}$ units

For a Rectangle

$AC^2 = AB^2 + BC^2$

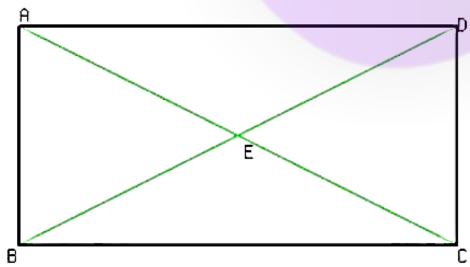
Here $AC^2 = 41$

But $AB^2 + BC^2 = 23$

∴ ABCD is not a rectangle

Question: 32 A

Solution:



A(– 4, – 1), B(– 2, – 4), C(4, 0) and D(2, 3)

$AB = \sqrt{(-2-(-4))^2 + (-4-(-1))^2}$

$= \sqrt{4+9} = \sqrt{13}$units

$BC = \sqrt{(4-(-2))^2 + (0-(-4))^2}$

$= \sqrt{36 + 16} = \sqrt{52}$units

$CD = \sqrt{(2 - 4)^2 + (3 - 0)^2}$

$= \sqrt{4 + 9} = \sqrt{13}$ units

$DA = \sqrt{(2 - (-4))^2 + (3 - (-1))^2}$

$= \sqrt{36 + 16} = \sqrt{52}$units

AB = CD and BC = DA

$AC = \sqrt{(4 - (-4))^2 + (0 - (-1))^2}$

$= \sqrt{64 + 1} = \sqrt{65}$ units

For a Rectangle

$AC^2 = AB^2 + BC^2$

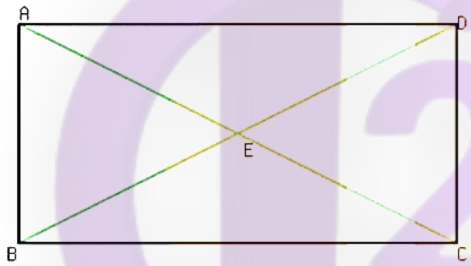
Here $AC^2 = 65$

But $AB^2 + BC^2 = 13 + 52 = 65$

∴ ABCD is a rectangle

Question: 32 B

Solution:



$AB = \sqrt{(14 - 2)^2 + (10 - (-2))^2}$

$= \sqrt{144 + 144} = \sqrt{288}$

$BC = \sqrt{(11 - 14)^2 + (10 - 13)^2}$

$= \sqrt{9 + 9} = \sqrt{18}$ units

$CD = \sqrt{(-1 - 11)^2 + (1 - 13)^2}$

$= \sqrt{144 + 144}$

$= \sqrt{288}$ units

$DA = \sqrt{(-1 - 2)^2 + (1 - (-2))^2}$

$= \sqrt{9 + 9} = \sqrt{18}$units

AB = CD and BC = DA

$AC = \sqrt{(11 - 2)^2 + (13 - (-2))^2}$

$= \sqrt{81 + 225}$

$= \sqrt{306}$ units

For a Rectangle

$AC^2 = AB^2 + BC^2$

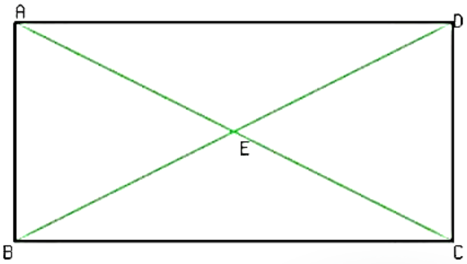
Here $AC^2 = 306$

But $AB^2 + BC^2 = 288 + 18 = 306$

∴ ABCD is a rectangle

Question: 32 C

Solution:



$AB = \sqrt{(6 - 0)^2 + (2 - (-4))^2}$

$= \sqrt{36 + 36}$

$= \sqrt{72}$ units

$BC = \sqrt{(3 - 6)^2 + (5 - 2)^2}$

$= \sqrt{9 + 9}$

$= \sqrt{18}$ units

$CD = \sqrt{(3 - (-3))^2 + (-1 - 5)^2}$

$= \sqrt{36 + 36}$

$= \sqrt{72}$ units

$DA = \sqrt{(-3 - 0)^2 + (-1 - (-4))^2}$

$= \sqrt{9 + 9}$

$= \sqrt{18}$ units

AB = CD and BC = DA

$AC = \sqrt{(3 - 0)^2 + (5 - (-4))^2}$

$= \sqrt{9 + 81}$

$= \sqrt{90}$ units

For a Rectangle

$AC^2 = AB^2 + BC^2$

Here $AC^2 = 90$

But $AB^2 + BC^2 = 72 + 18 = 90$

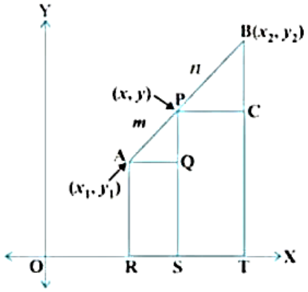
∴ ABCD is a rectangle

Exercise : 16B

Question: 1

Solution:

Let the point P(x,y) divides AB



Then

$X = (m_1x_2 + m_2x_1)/\ m_1 + m_2$

= (2 × 4 + 3 × (– 1))/2 + 3

= (8 – 3) /5

= 5/5 = 1

$Y = (m_1y_2 + m_2y_1)/\ m_1 + m_2$

= (2 × (– 3) + 3 × 7)/ 5

= (– 6 + 21)/5

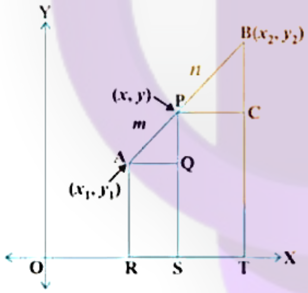
= 15 / 5 = 3

= (1, 3)

Question: 2

Solution:

Let the point P(x,y) divides AB



Then

$X = (m_1x_2 + m_2x_1)/\ m_1 + m_2$

= (7 × 4 + 2 × (– 5))/7 + 2

= (28 – 10) /9

= 18/9 = 2

$Y = (m_1y_2 + m_2y_1)/\ m_1 + m_2$

= (7 × (– 7) + 2 × 11)/ 9

= (– 49 + 22)/9

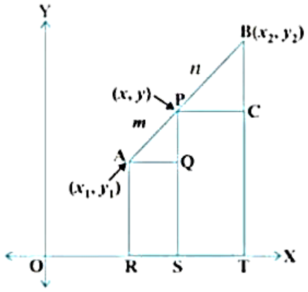
= – 27 / 9 = – 3

= (2, – 3)

Question: 3

Solution:

Let the point P(x,y) divides AB



Then

$X = (m_1x_2 + m_2x_1)/ m_1 + m_2$

= (3 × 2) + 4x (– 2))/ 3 + 4

= (6 – 8)/7

= – 2/7

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

= (3 × (– 4) + 4 × (– 2))/ 7

= (– 12 – 8)/ 7

= – 20 / 7

$P\left(\frac{-2}{7}, \frac{-20}{7}\right)$

Question: 4

Solution:

Let the point P(x,y) divides AB

Then

$X = (m_1x_2 + m_2x_1)/ m_1 + m_2$

= (2 × (– 4) + 3 × 6)/2 + 3

= (– 8 + 18) /5

= 10/5 = 2

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

= (2 × (– 1) + 3 x(– 6))/ 5

= (– 2 – 18)/5

= – 20 / 5 = – 4

If the point A also lies on the line 3x + k(y + 1) = 0

Then

3 × 2 + k(– 4 + 1) = 0

6 – 3k = 0

6 = 3k

k = 2

Question: 5

Solution:

P divides the segment AB in ratio 1:4

Q divides the segment AB in ratio 2:3

R divides the segment AB in ratio 3:2

For coordinates of P

$X = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$= (1 \times 6 + 4 \times 1)/1 + 4$

$= (6 + 4) /5$

$= 10/5 = 2$

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$= (1x\ 7 + 4 \times 2)/5$

$= (7 + 8)/5$

$= 15 / 5 = 3$

$= (2, 3)$

For coordinates of Q

$X = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$= (2x\ 6 + 3x\ 1)/5$

$= (12 + 3) /5$

$= 15/5 = 3$

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$= (2 \times 7 + 3 \times 2)/ 5$

$= (14 + 6)/5$

$= 20 / 5 = 4$

$= (3,4)$

For coordinates of R

$X = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$= (3 \times 6 + 2 \times 1)/5$

$= (18 + 2) /5$

$= 20/5 = 4$

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$= (3 \times 7 + 2 \times 2)/ 5$

$= (21 + 4)/5$

$= 25 / 5 = 5$

$= (4,5)$

Hence

P(2, 3), Q(3, 4), R(4, 5)

Question: 6

Solution:

P divides the segment AB in ratio 1:3

Q divides the segment AB in ratio 2:2

R divides the segment AB in ratio 3:1

For coordinates of P

$X = (m_1x_2 + m_2x_1)/\ m_1 + m_2$

$= (1 \times 5 + 3 \times 1)/1 + 3$

$= (5 + 3)\ /4$

$= 8/4 = 2$

$Y = (m_1y_2 + m_2y_1)/\ m_1 + m_2$

$= (1\times (-2) + 3 \times 6)/4$

$= (-2 + 18)/5$

$= 16\ /\ 4 = 4$

$= (2, 4)$

For coordinates of Q

$X = (m_1x_2 + m_2x_1)/\ m_1 + m_2$

$= (2x\ 5 + 2x\ 1)/4$

$= (10 + 2)\ /4$

$= 12/4 = 3$

$Y = (m_1y_2 + m_2y_1)/\ m_1 + m_2$

$= (2 \times (-2) + 2 \times 6)/\ 4$

$= (-4 + 12)/4$

$= 8\ /\ 4 = 2$

$= (3,2)$

For coordinates of R

$X = (m_1x_2 + m_2x_1)/\ m_1 + m_2$

$= (3x\ 5 + 1x\ 1)/4$

$= (15 + 1)\ /4$

$= 16/4 = 4$

$Y = (m_1y_2 + m_2y_1)/\ m_1 + m_2$

$= (3 \times (-2) + 1 \times 6)/\ 4$

$= (-6 + 6)/4$

$= 0/\ 4 = 0$

$= (4,0)$

$\therefore$ the coordinates are P(2, 4), Q(3, 2), R (4, 0)

Question: 7

Solution:

P divides the segment AB in ratio 1:2

Q divides the segment AB in ratio 2:1

For coordinates of P

$X = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$= (1 \times 1 + 2 \times 3)/1 + 2$

$= (1 + 6) /3$

$= 7/3 = p$

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$= (1x2 + 2 \times (-4))/3$

$= (2 - 8)/3$

$= -6/ 3 = -2$

For coordinates of Q

$X = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$= (2x\ 1 + 1x\ 3)/3$

$= (2 + 3) /3$

$= 5/3$

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$= (2 \times 2 + 1 \times (-4))/3$

$= (4 - 4)/3$

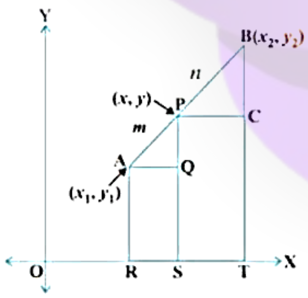
$= 0/ 3$

$= 0 = q$

$p = 7/3$, $q = 0$

Question: 8 A

Solution:



$X = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$= (1 \times (-5) + 1x\ 3)/1 + 1$

$= (-5 + 3) /2$

$= -2/2 = -1$

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$= (1x\ 4 + 1x\ 0)/2$

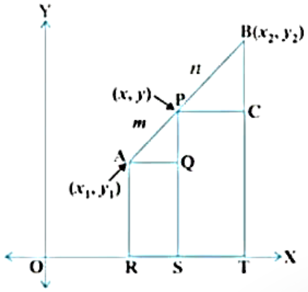
$= (4 + 0)/2$

= 4 / 2 = 2

(– 1, 2)

Question: 8 B

Solution:



X = $(m_1x_2 + m_2x_1)/\ m_1 + m_2$

= (1 × 8 + 1x (– 11)/1 + 1

= (8 – 11) /2

= – 3/2

Y = $(m_1y_2 + m_2y_1)/\ m_1 + m_2$

= (1x (– 2) + 1x – 8)/2

= (– 2 – 8)/2

= – 10 / 2 = – 5

$\left(\frac{-3}{2}, -5\right)$

Question: 9

Solution:

X = $(m_1x_2 + m_2x_1)/\ m_1 + m_2$

= (1 × (– 2) + 1x 6)/1 + 1

= (– 2 + 6) /2

= 4/2 = 2

Y = $(m_1y_2 + m_2y_1)/\ m_1 + m_2$

= (1x 11 + 1x (– 5))/2

= (11 – 5)/2

= 6 / 2 = 3

p = 3

Question: 10

Solution:

X = $(m_1x_2 + m_2x_1)/\ m_1 + m_2$

= (1 × (– 2) + 1× 2a)/1 + 1

$= (-2 + 2a)/2$

$(-2 + 2a)/2 = 1$

$-2 + 2a = 2$

$2a = 4$

$a = 2$

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$= (1 \times 3b + 1 \times 4)/2$

$= (3b + 4)/2$

$(3b + 4)/2 = 2a + 1$

$(3b + 4)/2 = 5$

$(3b + 4) = 10$

$3b = 6$

$b = 2$

$a = 2, b = 2$

Question: 11

Solution:

$X = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$= (1 \times 6 + 1x (-2)/1 + 1$

$= (6 - 2)/2$

$= 4/2 = 2$

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$= (1x\ 3 + 1x\ 9)/2$

$= (3 + 9)/2$

$= 12 / 2 = 6$

C(2,6)

Question: 12

Solution:

Let the coordinates of A be × & y. So A(X,Y) and B(1,4)

$2 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$2 = (1 \times 1 + 1 \times X)/1 + 1$

$2 = (1 + X)/2$

$1 + X = 4$

$x = 3$

$-3 = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$-3 = (1 \times 4 + 1 \times Y)/2$

$-3 = (4 + Y)/2$

$(4 + Y) = -6$

Y = – 10

A(3, – 10)

Question: 13

Solution:

$2 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$2 = (m_1 \times (-6) + m_2\ 8)/ m_1 + m_2$

$2 = (-6m_1 + 8m_2) / m_1 + m_2$

$-6m_1 + 8m_2 = 2(m_1 + m_2)$

$-8m_1 + 6m_2 = 0$

$5 = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$5 = (m_1 \times 9 + m_2\ 2)/ m_1 + m_2$

$5 = (9m_1 + 2m_2) / m_1 + m_2$

$9m_1 + 2m_2 = 5(m_1 + m_2)$

$4m_1 + 3m_2 = 0$

Solving for m_1 and m_2 we get

$m_1 = 3$

$m_2 = 4$

3:4

Question: 14

Solution:

$3/4 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$3/4 = (m_1 \times 2 + m_2 (1/2))/ m_1 + m_2$

$3/4 = (2m_1 + m_2 /2) / m_1 + m_2$

$6m_1 + 6m_2 = 16m_1 + 4m_2$

$6m_1 - 2m_2 = 0$

$5/12 = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$5/12 = (m_1 \times (-5) + m_2 (3/2))/ m_1 + m_2$

$5/12 = (-5m_1 + 3m_2 /2) / m_1 + m_2$

$-120m_1 + 36m_2 = 10(m_1 + m_2)$

$130m_1 - 26m_2 = 0$

Solving for m_1 and m_2 we get

$m_1 = 1$

$m_2 = 5$

1:5

Question: 15

Solution:

$6 = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$6 = (m_1 \times 8 + m_2\ 3)/ m_1 + m_2$

$6 = (8m_1 + 3m_2) / m_1 + m_2$

$8m_1 + 8m_2 = 6(m_1 + m_2)$

$2m_1 - 3m_2 = 0$

$m_1:m_2 = 3:2$

Now,

$m = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$m = (m_1 \times 2 + m_2 (-4))/ m_1 + m_2$

$m = (2m_1 - 4m_2) / m_1 + m_2$

$2m_1 - 4m_2 = m(m_1 + m_2)$

Putting the values of m_1 & m_2

$m = -2/5$

Hence, 3:2, $m = -2/5$

Question: 16

Solution:

$-3 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$-3 = (m_1 \times (-2) + m_2 (-5))/ m_1 + m_2$

$-3 = (-2m_1 - 5m_2) / m_1 + m_2$

$-2m_1 - 5m_2 = -3(m_1 + m_2)$

$2m_1 + 5m_2 = 3(m_1 + m_2)$

$m_1 - 2m_2 = 0$

$m_1:m_2 = 1:2$

Now,

$K = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$K = (m_1 \times 3 + m_2(-4))/ m_1 + m_2$

$K = (3m_1 - 4m_2) / m_1 + m_2$

$3m_1 - 4m_2 = k(m_1 + m_2)$

Putting the values of m_1 & m_2

$k = 2/3$

Hence,2:1, $k = 2/3$

Question: 17

Solution:

The segment is divided by x – axis i.e the coordinates are (x,0)

$x = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$x = (m_1 \times 5 + m_2\ 2)/ m_1 + m_2$

$x = (5m_1 + 2m_2\ ) / m_1 + m_2$

$5m_1 + 2m_2 = x(m_1 + m_2\ )$

$(5 - x)m_1 + (2 - x)m_2 = 0$

$0 = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$0 = (m_1 \times 6 + m_2(- 3))/ m_1 + m_2$

$0 = (6m_1 - 3m_2\ ) / m_1 + m_2$

$6m_1 - 3m_2 = 0$

Solving for m_1 and m_2 we get

$m_1 = 1$

$m_2 = 2$

(1 : 2),

Putting the values of m_1 and m_2

$x = 3$

Hence coordinates are (3,0)

Question: 18

Solution:

The segment is divided by y – axis i.e the coordinates are (0,y)

$0 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$0 = (m_1 \times 3 + m_2\ (- 2))/ m_1 + m_2$

$0 = (3m_1 - 2m_2\ ) / m_1 + m_2$

$3m_1 - 2m_2 = 0$

$m_1 = 2$

$m_2 = 3$

(2:3)

$y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$y = (m_1 \times 7 + m_2(- 3))/ m_1 + m_2$

$y = (7m_1 - 3m_2\ ) / m_1 + m_2$

$7m_1 - 3m_2 = y(m_1 + m_2)$

Putting the values of m_1 and m_2

$y = 1$

Question: 19

Solution:

The line segment joining any two points (x_1, y_1) and (x_2, y_2) y_2 is given as:

$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$

$\Rightarrow y - (-1) = \left(\frac{9-(-1)}{8-3}\right)(x - 3)$

$\Rightarrow y + 1 = 10/5\ (x-3)$

$\Rightarrow y + 1 = 2(x-3)$

$\Rightarrow y + 1 = 2x - 6 \Rightarrow 2x - y = 7$..eq(1) is the equation of line segment.

Now, we have to find the point of intersection of eq (1) & the given line: $x - y - 2 = 0$

$2x - y = 7$

& $x - y - 2 = 0$

$2x - 7 = x - 2$

$\Rightarrow x = 7 - 2$

$\Rightarrow x = 5$

And, $y = 3$

Let us say this point divides the line segment in the ratio of $k_1:k_2$

Then,

$5 = \frac{(8k_1 + 3k_2)}{k_1 + k_2}$

$\Rightarrow 5k_1 + 5k_2 = 8k_1 + 3k_2$

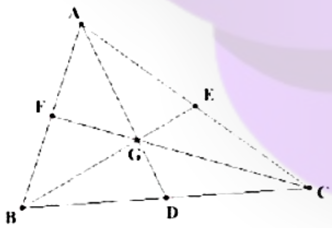
$\Rightarrow 5k_1 - 8k_1 + 5k_2 - 3k_2 = 0$

$\Rightarrow -3k_1 + 2k_2 = 0$

$\Rightarrow \frac{k_1}{k_2} = \frac{2}{3}$

Question: 20

Solution:



For coordinates of median AD segment BC will be taken

$X = (m_1x_2 + m_2x_1)/\ m_1 + m_2$

$= (1 \times 0 + 1x\ 2)/1 + 1$

$= (0 + 2)\ /2$

$= 2/2 = 1$

$Y = (m_1y_2 + m_2y_1)/\ m_1 + m_2$

$= (1x\ 3 + 1x\ 1)/2$

$= (3 + 1)/2$

= 4 / 2 = 2

D(1,2)

By distance Formula

AD = $\sqrt{(1-0)^2 + (2+1)^2}$

= $\sqrt{1+9}$

= $\sqrt{10}$

For coordinates of BE, segment AC will be taken

X = $(m_1x_2 + m_2x_1)/ m_1 + m_2$

= (1 × 0 + 1x 0)/1 + 1

= (0 + 0) /2

= 0/2 = 0

Y = $(m_1y_2 + m_2y_1)/ m_1 + m_2$

= (1x 3 + 1x (– 1))/2

= (3 – 1)/2

= 2 / 2 = 1

∴ E(0,1)

By distance Formula

BE = $\sqrt{(0-2)^2 + (1-1)^2}$

= $\sqrt{4+0}$

= $\sqrt{4}$ = 2

For coordinates of median CF segment AB will be taken

X = $(m_1x_2 + m_2x_1)/ m_1 + m_2$

= (1 × 2 + 1x 0)/1 + 1

= (2 + 0) /2

= 2/2 = 1

Y = $(m_1y_2 + m_2y_1)/ m_1 + m_2$

= (1x(– 1) + 1x 1)/2

= (– 1 + 1)/2

= 0 / 2 = 0

F(1,0)

By distance Formula

CF = $\sqrt{(1-0)^2 + (0-3)^2}$

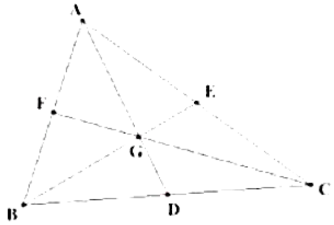
= $\sqrt{1+9}$

= $\sqrt{10}$

AD = $\sqrt{10}$ units, BE = 2 units, CF = $\sqrt{10}$ units

Question: 21

Solution:



First we need to calculate the coordinates of median

For coordinates of median AD segment BC will be taken

$X = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$= (1 \times 8 + 1x\ 5)/1 + 1$

$= (8 + 5)\ /2$

$= 13/2$

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$= (1x\ 2 + 1x\ (-\ 2))/2$

$= (0)/2$

$= 0\ /\ 2 = 0$

D(13/2,0)

The centroid of the triangle divides the median in the ratio 2:1

By section formula,

$X = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$= (2 \times 13/2 + 1x\ (-\ 1))/2 + 1$

$= (13 - 1)\ /3$

$= 12/3 = 4$

$Y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$= (2x\ 0 + 1x\ 0)/2 + 1$

$= 0/3$

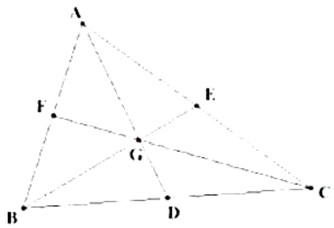
$= 0$

∴ G coordinate is (4, 0)

Question: 22

Solution:

The figure is shonw as:



$-\ 2 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$-\ 2 = (2 \times x + 1x\ 1)/2 + 1$

$-2 = (2x + 1)/3$

$-6 = 2x + 1$

$-7 = 2x$

$\Rightarrow x = -7/2$

$1 = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$1 = (2x\ y + 1x\ (-6))/3$

$1 = (2y - 6)/2$

$2 = 2y - 6$

$8 = 2y$

$\Rightarrow y = 4$

D(– 7/2,4)

Now for BC

$-7/2 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$-7/2 = (1 \times x + 1x\ (-5))/1 + 1$

$-7/2 = (x - 5)/2$

$-7 = x - 5$

$-7 + 5 = x$

$\Rightarrow x = -2$

$4 = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$4 = (1 \times y + 1x\ 2)/2$

$4 = (y + 2)/2$

$8 = y + 2$

$\Rightarrow y = 6$

Hence, C(– 2, 6)

Question: 23

Solution:

Coordinate of D on median on BC

$x = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$x = (1 \times 0 + 1x\ (-3))/1 + 1$

$x = (0 - 3)/2$

$x = -3/2$

$y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$y = (1 \times (-2) + 1x\ 1)/2$

$y = (-2 + 1)/2$

$2y = -1$

$y = -1/2$

D(– 3/2, – 1/2)

Now for AD we have D(– 3/2, – 1/2) and Centroid C(0,0)

$0 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$0 = (2 \times (-3/2) + 1x\, x)/2 + 1$

$0 = (-3 + x) /3$

$-3 + x = 0$

$x = 3$

$0 = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$0 = (2 \times (-1/2) + 1x\, y)/2 + 1$

$0 = (-1 + y)/3$

$-1 + y = 0$

$y = 1$

Hence, A(3, 1)

Question: 24

Solution:

We know that if diagonals of a quadrilateral bisect each other, then the quadrilateral is parallelogram

Given, A(3, 1), B(0, -2), C(1, 1) and D(4, 4) are coordinates of a quadrilateral

So,If ABCD is a parallelogram, the coordinates of the mid-point of the AC = Coordinates of the mid-point of the BD

We know, midpoint formula that if P is mid point of $A(x_1, y_1)$ and $B(x_2, y_2)$ $P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Coordinates of mid-point of AC

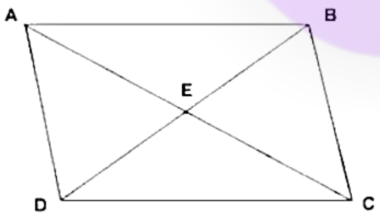
$= \left(\frac{3+1}{2}, \frac{1+1}{2}\right)$ = (2, 1)

Coordinates of mid-point of BD $= \left(\frac{0+4}{2}, \frac{-2+4}{2}\right)$ = (2, 1)

Hence, ABCD is a parallelogram.

Question: 25

Solution:



We know that the diagonals of a parallelogram bisect each other

So the coordinates of the mid – point of the PR = Coordinates of the mid – point of the QS

$\{(2 + a)/2, (15 - 11)/2\} = \{(5 + 1)/2, (b + 1)/2\}$

$2 + a = 6$

$a = 4$

$15 - 11 = b + 1$

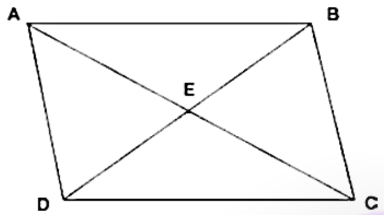
$4 = b + 1$

$b = 3$

Hence, $a = 4$, $b = 3$

Question: 26

Solution:



Coordinate of mid – point of AC = $\{(1 + 5)/2, (-2 + 10)/2\}$

implies $(3,4)$

This is equal to the coordinates of mid – point of BD

$3 = (3 + x)/2$

$6 = 3 + x$

$x = 3$

$4 = (6 + y)/2$

$8 = (6 + y)$

$y = 2$

Hence, D(3, 2)

Question: 27

Solution:

Let the coordinate of the point on y axis be (0,y)

$0 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$0 = (m_13 + m_2(-4))/ m_1 + m_2$

$0 = (3m_1 - 4m_2)/ m_1 + m_2$

$(3m_1 - 4m_2) = 0$

$3m_1 = 4m_2$

$m_1 : m_2 = 4:3$

Question: 28

Solution:

Given: The points P(1/2, y) lies on the line AB.

Then,

$1/2 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$1/2 = (m_1(-7) + m_2 3)/ m_1 + m_2$

$1/2 = (-7m_1 + 3m_2)/ m_1 + m_2$

$(m_1 + m_2) = -14 m_1 + 6 m_2$

$15m_1 = 5m_2$

$m_1: m_2 = 3:5$

$y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$y = (3 \times 9 + 5x (-5))/3 + 5$

$y = (27 - 25)/8$

$y = 2/8$

$y = 1/4$

Question: 29

Solution:

Let the coordinate of the point on x axis be (x,0)

$0 = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$0 = (m_1 7 + m2(-3)/ m_1 + m_2$

$0 = (7m_1 - 3m_2)/ m_1 + m_2$

$7m_1 - 3m_2 = 0$

$7 m_1 = 3m_2$

$m_1 : m_2 = 3:7$

$x = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$x = (3 x(-2) + 7 \times 3)/ 10$

$x = (-6 + 21)/ 10$

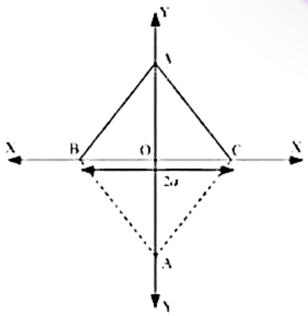
$x = 15/10$

$x = 3/2$

Hence the coordinate of the point be (3/2, 0)

Question: 30

Solution:



Let QR be the base

Since origin is mid – point O(0,0) of QR

Then the coordinates of R(x,y) is given by

$(-4 + x)/2 = 0$

$x = 4$

$(0 + y)/2 = 0$

$y = 0$

R(4,0)

Distance of QR = $\sqrt{(4 + 4)^2 + 0}$

QR = 8

∴ PR = 8

Let P(x,y)

$8 = \sqrt{(4 - x)^2 + (0 - y)^2}$

$64 = 16 + x^2 - 8x + y^2$

Since it will lie on x axis

∴ $x = 0$

$64 = 16 + y^2$

$48 = y^2$

$y = 4\sqrt{3}$ or $-4\sqrt{3}$

Hence,

P(0, $4\sqrt{3}$) or P(0, $-4\sqrt{3}$) and R(4, 0)

Question: 31

Solution:

Given: The base (BC) of the equilateral triangle ABC lies on y - axis, where, C has the coordinates: (0, - 3).The origin is the midpoint of the base.

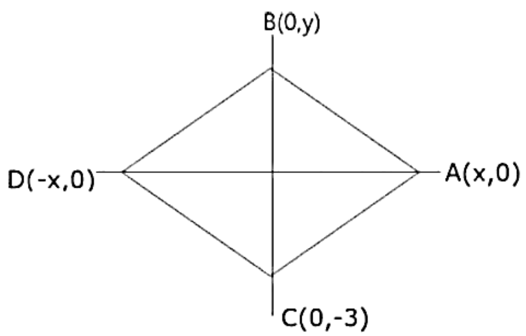
To find: The coordinates of the points A and B. Also, the coordinates of another point D such that ABCD is a rhombus.**Solution:**

Now, Δ ABC is an equilateral triangle

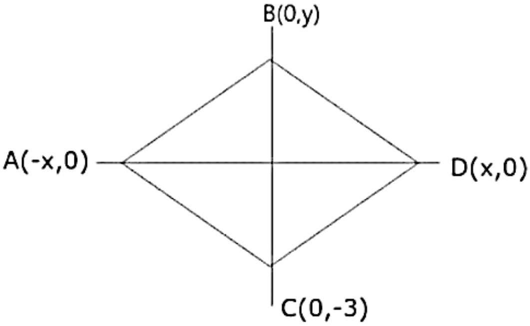
∴ AB = AC = BC ...(1)By symmetry the coordinate A lies on x axis. Also D is another point such that ABCD is rhombus and every side of rhombus is equal to each other.So For this condition to be possible D will also lie on x axis.Now,Let coordinates of A be (x,0),B be (0,y) and D be (-x,0).

or coordinates of A be (- x,0),B be (0,y) and D be (x,0).

The figures are shown below:



...... (a)



.... (b) From figure (a)

$BC = \sqrt{(0 - 0)^2 + (-3 - y)^2} \Rightarrow BC = \sqrt{0 + 9 + y^2 + 6y} \Rightarrow BC = \sqrt{9 + y^2 + 6y}$

Now, $AC = \sqrt{(0 - x)^2 + (-3 - 0)^2}$

$\Rightarrow AC = \sqrt{x^2 + (-3)^2} \Rightarrow AC = \sqrt{(x^2 + 9)}$

And

$AB = \sqrt{(0 - x)^2 + (y - 0)^2}$

$\Rightarrow AB = \sqrt{x^2 + y^2}$From (1)$AB = AC \Rightarrow \sqrt{x^2 + y^2} = \sqrt{x^2 + 9}$Taking square on both sides we get,$x^2 + y^2 = x^2 + 9 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3$Since B lies in positive y direction..∴. The coordinates of B are (0,3)Now from (1) $AB = BC \Rightarrow \sqrt{x^2 + y^2} = \sqrt{9 + y^2 + 6y}$Take square on both sides$\Rightarrow x^2 + y^2 = 9 + y^2 + 6y \Rightarrow x^2 = 9 + 6y$Put the value of y to get,$\Rightarrow x^2 = 9 + 6(3) \Rightarrow x^2 = 9 + 18 \Rightarrow x^2 = 27 \Rightarrow x = \pm 3\sqrt{3}$Hence the coordinates of A can be $(3\sqrt{3},0)$ or $(-3\sqrt{3},0)$ Also, ABCD is a rhombus.$\Rightarrow AB = BC = DC = BD$So coordinates of D will be $(-3\sqrt{3},0)$ or $(3\sqrt{3},0)$ Hence coordinates are $A(3\sqrt{3},0)$, $B(0,3)$, $D(-3\sqrt{3},0)$ Or coordinates are $A(-3\sqrt{3},0)$, $B(0,3)$, $D(3\sqrt{3},0)$

Question: 32

Solution:

$-1 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$-1 = (m_16 + m_2(-3))/ m_1 + m_2$

$-1 = (6m_1 - 3m_2)/ m_1 + m_2$

$(6m_1 - 3m_2) = - m_1 - m_2$

$7m_1 = 2 m_2$

$m_1 : m_2 = 2:7$

$y = (m_1y_2 + m_2y_1)/ m_1 + m_2$

$= (2x(-8) + 7 \times 10)/9$

$= (-16 + 70)/9$

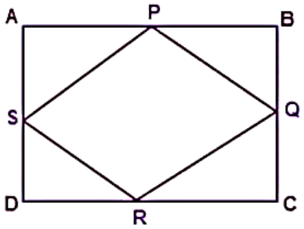
$= 54 / 9$

$y = 6$

Question: 33

Solution:

The figure is shown below:



P(x,y) = (– 1 – 1)/2 , (4 – 1)/2

= (– 1,3/2)

Q(x,y) = (5 – 1)/2 , (4 + 4)/2

= (2,4)

R(x,y) = (5 + 5)/2 , (– 1 + 4)/2

= (5,3/2)

S(x,y) = (5 – 1)/2 , (– 1 – 1)/2

= (2, – 1)

Coordinates of mid – point of PR = Coordinates of mid – point of QS

Coordinates of mid – point of PR = {(5 – 1)/2 ,(3/2 + 3/2)/2} = (2,3/2)

Coordinates of mid – point of QS = {(2 + 2)/2 , (– 1 + 4)/2 = (2,3/2)

Hence PQRS is a Rhombus.

Question: 34

Solution:

For P(x,y)

X = (– 10 – 2)/2 = – 6

Y = (4 + 0)/2 = 2

Thus, P(– 6,2)

Now

$-6 = (m_1x_2 + m_2x_1)/ m_1 + m_2$

$-6 = (m_1(-4) + m_2(-9))/ m_1 + m_2$

$-6 = (-4m_1 - 9m_2)/ m_1 + m_2$

$-6(m_1 + m_2) = -4m_1 - 9m_2$

$-2m_1 = -3m_2$

$m_1:m_2 = 3:2$,

$2 = (m_1y_2 + m_2y_1)/ m_1 + m_2$

2 = (3 × y + 2x (– 4))/5

2 = (3y – 8)/5

10 = 3y – 8

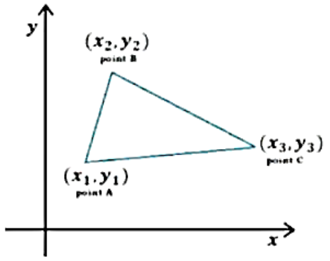
3y = 18

y = 6

Exercise : 16C

Question: 1 A

Solution:



Area of triangle

= $1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

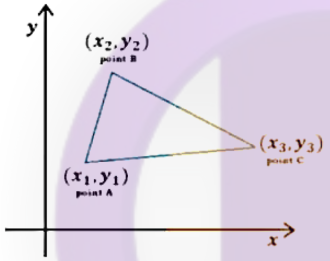
= 1/2(1(–2 + 3)–2(–4–2)–3(2–3))

= 1/2(1 + 12 + 3)

= 8 sq units

Question: 1 B

Solution:



Area of triangle

= $1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

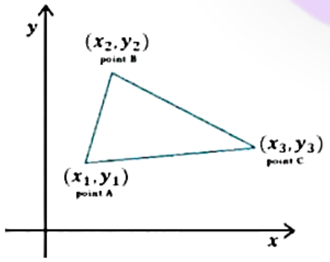
= 1/2(–5(–5–5)–4(5–7) + 4(7 + 5))

= 1/2(–50 + 8 + 48)

= 5 sq units

Question: 1 C

Solution:



Area of triangle

= $1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

= 1/2(3(2 + 1)−4(−1−8) + 5(8−2))

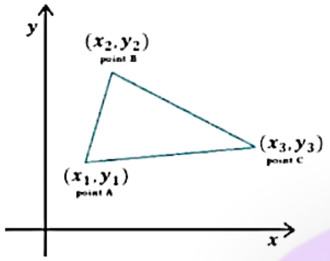
= 1/2(9 + 36 + 30)

= 1/2(75)

= 37.5 sq units

Question: 1 D

Solution:



Area of triangle

= $1/2(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

= 1/2(10(5−3) + 2(3 + 6)−1(−6−5))

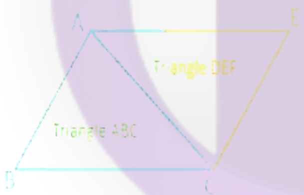
= 1/2(20 + 18 + 11)

= 1/2(49)

= 24.5 sq units

Question: 2

Solution:



For triangle ABC

Area of triangle

= $1/2(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

= 1/2(3(−5−0) + 9(0 + 1) + 14(−1 + 5))

= 1/2(−15 + 9 + 56)

= 1/2(50)

= 25

For triangle ACD

Area of triangle

= $1/2(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

= 1/2(3(0−19) + 14(19 + 1) + 9(−1−0))

= 1/2(−57 + 280−9)

$= 1/2(214)$

$= 107$

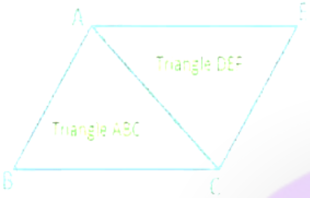
Area of ABCD = Area of ABC + Area of ACD

$= 25 + 107$

= 132 sq units

Question: 3

Solution:



For triangle PQR

Area of triangle

$= |1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))|$

$= 1/2(-5(-6 + 3)-4(-3 + 3) + 2(-3 + 6))$

$= 1/2(15 + 0 + 6)$

$= 1/2(21)$

For triangle PRS

Area of triangle

$= 1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

$= 1/2(-5(-3-2) + 2(2-(-3)) + 1(-3 + 3))$

$= 1/2(25 + 10 + 0)$

$= 1/2(35)$

Area of ABCD = Area of ABC + Area of ACD

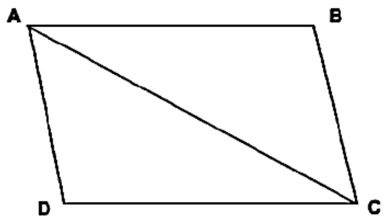
$= 21/2 + 35/ 2$

= 28 sq units

Question: 4

Solution:

We divide quadrilateral in two triangles, such thatArea of ABCD = Area of ΔABC + Area of ΔACD



Also,

We know area of a triangle, if it's coordinates are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is

$Area = \frac{1}{2}\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)$ Therefore, Area of ABC

$= \left|\frac{1}{2}[-3(-1+4)-2(-1+1)+4(-1+4)]\right|$

$= \left|\frac{1}{2}(-9-12)\right|$

$= \frac{21}{2}$

$= \frac{1}{2}[-3(-1-4)+4(4+1)+3(-1+1)]$

Area of ACD $= \frac{1}{2}(15+20)$

$= \frac{35}{2}$

Area of ABCD = Area of ABC + Area of ACD

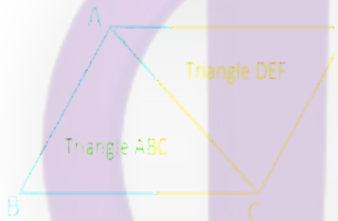
$= \frac{21}{2} + \frac{35}{2}$

$= \frac{56}{2}$

= 28 sq units

Question: 5

Solution:



For triangle ABC

Area of triangle

$= 1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

$= 1/2(-5(-5 + 6)-4(-6-7)-1(7 + 5))$

$= 1/2(-5 + 52-12)$

$= 1/2(35)$

For triangle ACD

Area of triangle

$= 1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

$= 1/2(-5(-6-5)-1(5-7) + 4(7 + 6))$

$= 1/2(-55 + 2 + 52)$

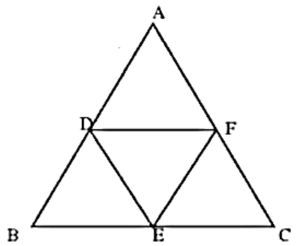
$= 1/2(1)$

Area of ABCD = Area of ABC + Area of ACD

= 18 sq units

Question: 6

Solution:



By applying section formula we get the coordinates of mid points of AB,BC and AC.

Mid point of AB = P = {(2 + 4)/2,(1 + 3)/2}

P = (3,2)

Mid point of BC = Q = {(4 + 2)/2,(3 + 5)/2}

Q = (3,4)

Mid point of AC = R = {(2 + 2)/2,(1 + 5)/2}

R = (2,3)

For triangle PQR

Area of triangle

$= 1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

= 1/2(3(4–3) + 3(3–2) + 2(2–4))

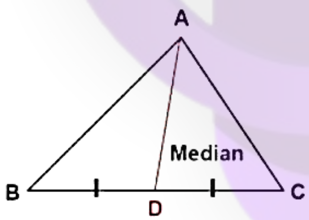
= 1/2(3 + 3–4)

= 1/2(2)

= 1 sq unit

Question: 7

Solution:



D = {(3 + 5)/2,(3–1)/2} = (4,1)

For triangle ABD

Area of triangle

$= 1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

= 1/2(7(3–1) + 5(1 + 3) + 4(–3–3))

= 1/2(14 + 20–24)

= 1/2(10)

= 5 sq unit

For triangle ACD

Area of triangle

$= 1/2(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

$= 1/2(7(-1-1) + 3(1+3) + 4(-3+1))$

$= 1/2(-14 + 12 - 8)$

$= 1/2(10)$

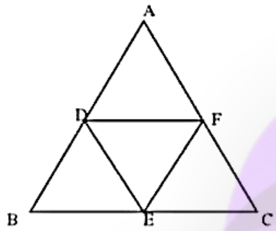
= 5 sq unit

Hence area of triangle ABD and ACD is equal.

Question: 8

Solution:

The diagram is given below:



Coordinates of B

$2 = (1 + x)/2$ [by section formula]

$4 = 1 + x$

$X = 3$

$-1 = (-4 + y)/2$

$-2 = (-4 + y)$

$Y = 2$

∴ the coordinates of B(3,2)

Coordinates of C [by section formula]

$0 = (1 + x)/2$

$0 = (1 + x)$

$x = -1$

$-1 = (-4 + y)/2$

$-2 = (-4 + y)$

$Y = 2$

∴ the coordinates of point C are (–1,2)

Now, Area of triangle ABC

$= 1/2(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

$= 1/2(1(2-2) + 3(2+4) - 1(-4-2))$

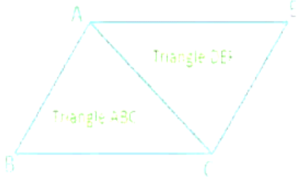
$= 1/2(0 + 18 + 6)$

$= 1/2(24)$

= 12 sq unit

Question: 9

Solution:



Let (x, y) be the coordinates of D and (x', y') be the coordinates of E. since the diagonals of a parallelogram bisect each other at the same point, therefore

(x + 8)/2 = (6 + 9)/2

X = 7

(y + 2)/2 = (1 + 4)/2

Y = 3

Thus, the coordinates of D are (7,3)

E is the midpoint of DC,

therefore

x' = (7 + 9)/2 = 8

y' = (3 + 4)/2 = 7/2

Thus, the coordinates of E are (8,7/ 2)

Let $A(x_1,y_1)$ = A(6,1), $E(x_2,y_2)$ = (8,7/2) and $D(x_3,y_3)$ = D(7,3)

Now Area

$= 1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

= 1/2(6(7/2−3) + 8(3−1) + 7(1−7/2))

= 1/2(3/2)

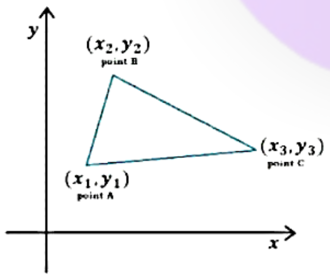
= 3/4 sq unit

Hence, the area of the triangle ΔADE is 3/4 sq. units.

Question: 10

Solution:

Area = 15



$\Rightarrow \Delta = 1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

$15 = 1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

15 = 1/2(1(p−7) + 4(7 + 3)−9(−3−p))

15 = 1/2(10p + 16)

|10p + 16| = 30

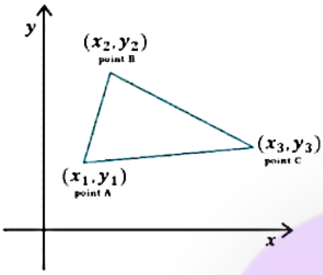
10p + 16 = 30 or −30

Hence, p = −9 or p = −3.

Question: 11

Solution:

Δ = 6



$\Rightarrow \Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

$6 = 1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

6 = 1/2(k + 1(−3 + k) + 4(−k−1) + 7(1 + 3))

$6 = 1/2(k^2-2k-3-4k-4 + 28)$

$k^2 - 6k + 9 = 0$

k = 3

Question: 12

Solution:

Given the area of triangle, Δ = 53

$\Rightarrow \Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

53 = 1/2{−2(−4−10) + k(10−5) + 2k + 1(5 + 4)}

53 = 1/2{28 + 5k + 9(2k + 1)}

106 = (28 + 5k + 18k + 9)

37 + 3k = 106

23k = 69

k = 3

Question: 13 A

Solution:

To show that the points are collinear, we show that the area of triangle is equilateral = 0

Given, the area of the triangle, Δ = 0

$\Rightarrow \Delta = 1/2(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))$

⇒ Δ = 1/2{2 (8− 4) + (−3) (4 + 2) −1 (2− 8)}

⇒ Δ = 1/2 {8−18 + 10}

$\Rightarrow \Delta = 0$

Hence the points A(2, – 2), B(−3, 8) and C(−1, 4) are collinear.

Question: 13 B

Solution:

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Rightarrow \Delta = 1/2 \{(x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2))\}$

$\Rightarrow \Delta = 1/2\{-5(5-7) + 5(7-1) + 10(1-5)\}$

$\Rightarrow \Delta = 1/2\{10 + 30-40\}$

$\Rightarrow \Delta = 0$

Hence collinear.

Question: 13 C

Show that the fol

Solution:

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Delta = 0$

$\Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

$\Rightarrow \Delta = 1/2\{5(-1-4) + 1(4-1) + 11(1+1)\}$

$\Rightarrow 1/2\{-25 + 3 + 22\}$

$= 0$

Hence collinear

Question: 13 D

Solution:

A(8, 1), B(3, −4) and C(2, −5)

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Delta = 0$

$\Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

$\Rightarrow 1/2\{8(-4+5) + 3(-5-1) + 2(1+4)\}$

$\Rightarrow 1/2\{8-18 + 10\}$

$= 0$

Hence collinear.

Question: 14

Solution:

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Delta = 0$

$\Delta = 1/2 \{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

$\Rightarrow \Delta = 1/2\{x(-4+5)-3(-5-2) + 7(2+4)\} = 0$

$\Rightarrow \Delta = 1/2\{x + 21 + 42\} = 0$

$x = -63$

Question: 15

Solution:

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Delta = 0$

$\Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

$\Rightarrow \Delta = 1/2\{-3(6-9) + 7\ (9 - 12) + x(12- 6)\} = 0$

$\Rightarrow (-3)(-3) + 7(-3) + 6x = 0$

$\Rightarrow 9-21 + 6x = 0$

$6x = 12$

$x = 2$

Question: 16

Solution:

Collinear points P, Q, and R.

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Delta = 0$

$\Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

$\Rightarrow \Delta = 1/2\{1(y-16) + 3\ (16-4)-3\ (4-y)\} = 0$

$\Rightarrow y-16 + 36-12 + 3y = 0$

$\Rightarrow 8 + 4y = 0$

$\Rightarrow 4y = -8$

$y = -2$

Question: 17

Solution:

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Delta = 0$

$\Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

$\Rightarrow \Delta = 1/2\{-3(y + 5) + 2\ (-5-9) + 4\ (9-y)\} = 0$

$\Rightarrow -3y-15-28 + 36-4y = 0$

$\Rightarrow 7y = 36-43$

$y = -1$

Question: 18

Solution:

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Delta = 0$

$\Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

$\Rightarrow \Delta = 1/2\{8(-2k + 5) + 3(-5-1) + k(1 + 2k)\} = 0$

$\Rightarrow -16k + 40-18 + k + 2k^2 = 0$

$\Rightarrow 2k^2 + 15k + 22 = 0$

$\Rightarrow 2k^2-11k-14k + 22 = 0$

$\Rightarrow K(2k-11)-2(2k-11) = 0$

$k = 2$ or $k = \frac{11}{2}$

Question: 19

Solution:

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Delta = 0$

$\Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

$\Rightarrow \Delta = 1/2\{2(y-5) + x(5-1) + 7(1-y)\}$

$\Rightarrow 2y-10 + 4x-7-7y = 0$

$\Rightarrow 4x - 5y - 3 = 0$

Question: 20

Solution:

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Delta = 0$

$\Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

$\Rightarrow \Delta = 1/2\{x(7-5) + (-5)(-5-y) - 4(y-7)\}$

$\Rightarrow 7x-5x-25 + 5y-4y + 28 = 0$

$\Rightarrow 2x + y + 3 = 0$

Question: 21

Solution:

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Delta = 0$

$\Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\} = 0$

$\Rightarrow \Delta = 1/2\{a(b-1) + 0(1-0) + 1(0-b)\} = 0$

$\Rightarrow (ab-a-b) = 0$

Dividing the equation by ab.

$1-1/b-1/a$

$1-(1/a+1/b)$

$1-1 = 0$

Hence collinear.

Question: 22

Solution:

Collinear points P, Q, and R.

To show that the points are collinear, we show that the area of triangle is equilateral = 0

$\Delta = 0$

$\Delta = 1/2\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

$\Rightarrow \Delta = 1/2\{-3\ (b+5) + a\ (-5-9) + 4\ (9-b)\} = 0$

$\Rightarrow -3b-150-14a+36-4b = 0$

$2a + b = 3$

Now solving a + b = 1 and 2a + b = 3 we get a = 2 and b = −1.

Hence a = 2, b = −1

Exercise : 16D

Question: 1

Solution:

The distance of any point which lies on the circumference of the circle from the centre of the circle is called radius.

$\therefore$ OA = OB = Radius of given Circle

taking square on both sides, we get-

$OA^2 = OB^2$

$\Rightarrow (-1-2)^2 + [y-(-3y)]^2 = (5-2)^2 + [7-(-3y)]^2$

[using distance formula, the distance between points (x_1,y_1) and (x_2,y_2) is equal to $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$ units.]

$\Rightarrow 9 + 16y^2 = 9 + (7+3y)^2$

$\Rightarrow 16y^2 = 49 + 42y + 9y^2$

$\Rightarrow 7y^2 - 42y - 49 = 0$

$\Rightarrow 7(y^2-6y-7) = 0$

$\Rightarrow y^2 - 7y + y - 7 = 0$

$\Rightarrow y(y-7) + 1(y-7) = 0$

$\Rightarrow (y + 1)(y-7) = 0$

$\therefore y = 7$ or $y = -1$

Thus, possible values of y are 7 or -1.

Question: 2

Solution:

According to question-

AB = AC

taking square on both sides, we get-

$AB^2 = AC^2$

$\Rightarrow (0-3)^2 + (2-p)^2 = (0-p)^2 + (2-5)^2$

[using distance formula, the distance between points (x_1,y_1) and (x_2,y_2) is equal to $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ units.]

$\Rightarrow 9 + 4 + p^2 - 4p = p^2 + 9$

$\Rightarrow 4p-4 = 0$

$\Rightarrow 4p = 4$

$\therefore p = 1$

Thus, the value of p is 1.

Question: 3

Solution:

fig.1

Clearly from fig.1, One of the diagonals of the rectangle ABCD is BD.

Length of diagonal BD is given by-

$BD = \sqrt{(4-0)^2 + (0-3)^2}$

$= \sqrt{4^2 + (-3)^2}$

$= \sqrt{(16+9)}$

$= \sqrt{25}$

= 5 units

Question: 4

Solution:

According to question-

AP = BP

taking square on both sides, we get-

$AP^2 = BP^2$

$\Rightarrow (k-4)^2 + (2-k)^2 = (-1)^2 + (2-5)^2$

[using distance formula, the distance between points (x_1,y_1) and (x_2,y_2) is equal to $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ units.]

$\Rightarrow k^2 - 8k + 16 + 4 + k^2 - 4k = 1 + 9$

$\Rightarrow 2k^2 - 12k + 20 = 10$

$\Rightarrow 2k^2 - 12k + 10 = 0$

$\Rightarrow 2(k^2-6k + 5) = 0$

$\Rightarrow (k^2-5k-k + 5) = 0$

$\Rightarrow k(k-5)-1(k-5) = 0$

$\Rightarrow (k-1)(k-5) = 0$

$\therefore k = 1$ or $k = 5$

Thus, the value of k is 1 or 5.

Question: 5

Solution:

Let the point P(x, 2) divides the join of A(12, 5) and B(4, -3) in the ratio of m:n.

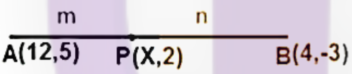


fig.2

Recall that if $(x,y) \equiv (a,b)$ then $x = a$ and $y = b$

$\therefore$ assume that

$(x,y) \equiv (x,2)$

$(x_1,y_1) \equiv (12,5)$

and, $(x_2,y_2) \equiv (4,-3)$

Now, Using Section Formula-

$$y = \frac{my_2 + ny_1}{m + n}$$

$$\Rightarrow 2 = \frac{m \times (-3) + n \times (5)}{m + n}$$

$\Rightarrow 2m + 2n = -3m + 5n$

$\Rightarrow 5m = 3n$

$\therefore m:n = 3:5$

Thus, the required ratio is 3:5.

Question: 6

Solution:

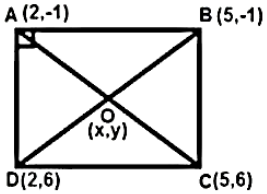


fig.3

Length of diagonal AC is given by-

$AC = \sqrt{(2-5)^2 + (-1-6)^2}$

$= \sqrt{(-3)^2 + (-7)^2}$

$= \sqrt{(9+49)}$

$= \sqrt{58}$ units

Length of diagonal BD is given by-

$BD = \sqrt{(5-2)^2 + (-1-6)^2}$

$= \sqrt{3^2 + (-7)^2}$

$= \sqrt{(9+49)}$

$= \sqrt{58}$ units

Clearly, the length of the diagonals of the rectangle ABCD are equal.

Mid-point of Diagonal AC is given by

$= \left(\frac{2+5}{2}, \frac{-1+6}{2}\right)$

$= \left(\frac{7}{2}, \frac{5}{2}\right)$

Similarly, Mid-point of Diagonal BD is given by

$= \left(\frac{5+2}{2}, \frac{-1+6}{2}\right)$

$= \left(\frac{7}{2}, \frac{5}{2}\right)$

Clearly, the coordinates of mid-point of both the diagonals coincide i.e. diagonals of the rectangle bisect each other.

Question: 7

Solution:

A **median of a triangle** is a line segment joining a vertex to the midpoint of the opposing side, bisecting it.

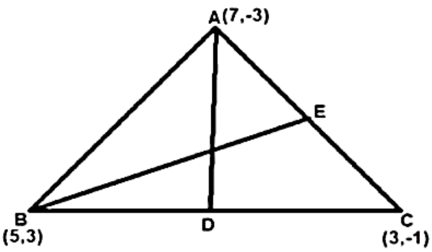


fig.4

Mid-point of side BC opposite to vertex A i.e. coordinates of point D is given by-

$$= \left(\frac{5\ +\ 3}{2}, \frac{3-1}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{2}{2}\right)$$

= (4,1)

Mid-point of side AC opposite to vertex B i.e. coordinates of point E is given by-

$$= \left(\frac{7\ +\ 3}{2}, \frac{-3-1}{2}\right)$$

$$= \left(\frac{10}{2}, \frac{-4}{2}\right)$$

= (5,-2)

Length of Median AD is given by-

$$AD = \sqrt{(7-4)^2 + (-3-1)^2}$$

$$= \sqrt{(3)^2 + (-4)^2}$$

$= \sqrt{(9 + 16)}$

$= \sqrt{25}$

= 5 units

Length of Median BE is given by-

$$BD = \sqrt{(5-5)^2 + (3-(-2))^2}$$

$$= \sqrt{0^2 + (3\ +\ 2)^2}$$

$= \sqrt{(0 + 5^2)}$

$= \sqrt{25}$

= 5 units

Thus, Length of Medians AD and BE are same which is equal to 5 units.

Question: 8

Solution:

Given that point C(k, 4) divides the join of A(2, 6) and B(5, 1) in the ratio 2 : 3.

$\therefore$ m:n = 2:3

Recall that if (x,y) ≡ (a,b) then x = a and y = b

Let (x,y) ≡ (k,4)

$(x_1,y_1) \equiv (2,6)$

and, $(x_2,y_2) \equiv (5,1)$

Now, Using Section Formula-

$$x = \frac{mx_2\ +\ nx_1}{m\ +\ n}$$

On dividing numerator and denominator of R.H.S by n, we get-

$$x = \frac{\frac{m}{n}x_2 + 1x_1}{\frac{m}{n} + 1}$$

$$\Rightarrow k = \frac{\frac{2}{3} \times (5) + 1 \times (2)}{\frac{2}{3} + 1}$$

$$\Rightarrow k = \frac{\frac{10 + 6}{3}}{\frac{5}{3}}$$

$\therefore$ k = (16/5)

Thus the value of k is (16/5).

Question: 9

Solution:

Let the point on the x-axis which is equidistant from points A(-1,0) and B(5,0) i.e. the point which divides the line segment AB in the ratio 1:1 be C(x,0).

$\therefore$ m:n = 1:1

Recall that if (x,y) ≡ (a,b) then x = a and y = b

Let (x,y) ≡ (x,0)

$(x_1,y_1) \equiv (-1,0)$

and $(x_2,y_2) \equiv (5,0)$

Using Section Formula,

$$x = \frac{1 \times (5) + 1 \times (-1)}{1 + 1}$$

$$\Rightarrow x = \frac{5 - 1}{2}$$

$\Rightarrow$ x = (4/2) = 2

Thus, the point on the x-axis which is equidistant from points A(-1,0) and B(5,0) is P(2,0).

Question: 10

Solution:

The distance between the points $\left(\frac{-8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$ is given by- $= \sqrt{\left(\frac{-8}{5} - \frac{2}{5}\right)^2 + (2-2)^2}$ [using distance formula, the distance between points (x_1,y_1) and (x_2,y_2) is equal to $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ units.]

$$= \sqrt{\left(\frac{-10}{5}\right)^2 + (0)^2}$$

$$= \sqrt{(-2)^2 + 0}$$

$$= \sqrt{4}$$

= 2 units

Question: 11

Solution:

Since the point (3, a) lies on the line represented by $2x - 3y = 5$

Thus, the point (3,a) will satisfy the above linear equation

$\therefore 2\times(3) - 3\times(a) = 5$

$\Rightarrow 3a = 6-5$

$\Rightarrow 3a = 1$

$\therefore a = (1/3)$

Thus, the value of a is (1/3).

Question: 12

Solution:

The distance of any point which lies on the circumference of the circle from the centre of the circle is called radius.

$\therefore$ OA = OB = Radius of given Circle

taking square on both sides, we get-

$OA^2 = OB^2$

$\Rightarrow (2-4)^2 + (3-3)^2 = (2-x)^2 + (3-5)^2$

[using distance formula, the distance between points (x_1,y_1) and (x_2,y_2) is equal to $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ units.]

$\Rightarrow (-2)^2 + 0 = x^2 - 4x + 4 + (-2)^2$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow (x-2)^2 = 0$

$\therefore x = 2$

Thus, the value of x is 2.

Question: 13

Solution:

According to question-

AP = BP

taking square on both sides, we get-

$AP^2 = BP^2$

$\Rightarrow (7-x)^2 + (1-y)^2 = (3-x)^2 + (5-y)^2$

[using distance formula, the distance between points (x_1,y_1) and (x_2,y_2) is equal to $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ units.]

$\Rightarrow x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$

$\Rightarrow -8x + 8y + 16 = 0$

$\Rightarrow -8(x-y-2) = 0$

$\Rightarrow x-y-2 = 0$

∴ x-y = 2

This is the required relation between x and y.

Question: 14

Solution:

Every **triangle** has exactly three **medians**, one from each vertex, and they all intersect each other at a common point which is called **centroid.**

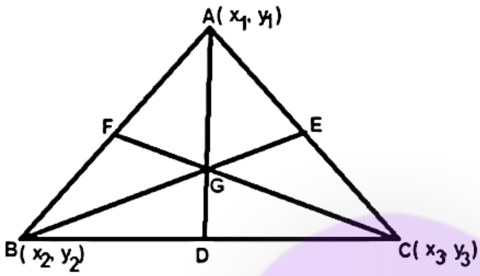


fig.5

In the fig.5, Let AD, BE and CF be the medians of ΔABC and point G be the centroid.

We know that-

Centroid of a Δ divides the medians of the Δ in the ratio 2:1.

Mid-point of side BC i.e. coordinates of point D is given by

$$= \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

Let the coordinates of the centroid G be (x,y).

Since centroid G divides the median AD in the ratio 2:1 i.e.

AG:GD = 2:1

∴ using section-formula, the coordinates of centroid is given by-

$$(x,y) \equiv \left(\frac{2\left(\frac{x_2 + x_3}{2}\right) + 1(x_1)}{2 + 1}, \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1(y_1)}{2 + 1}\right)$$

$$\therefore (x,y) \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Now, according to question-

Centroid of ΔABC having vertices A(a, b), B(b, c) and C(c, a) is the origin.

$$\therefore \left(\frac{a + b + c}{3}, \frac{b + c + a}{3}\right) \equiv (0,0)$$

Thus, the value of a + b + c is 0.

Question: 15

Solution:

The centroid of a Δ whose vertices are $(x_1 y_1)$, (x_2, y_2) and $(x_3 y_3)$ is given by-

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

∴ centroid of the given ΔABC ≡ [(2-4 + 5)/3 , (2-4-8)/3]

$\equiv (1,-10/3)$

Thus, the centroid of the given triangle ABC is (1,-10/3).

Question: 16

Solution:

Let the ratio in which the point C(4, 5) divide the join of A(2, 3) and B(7, 8) be m:n.

Recall that if $(x,y) \equiv (a,b)$ then x = a and y = b

Let $(x,y) \equiv (4,5)$

$(x_1,y_1) \equiv (2,3)$

and, $(x_2,y_2) \equiv (7,8)$

Now, Using Section Formula-

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$\Rightarrow 4 = \frac{m(7) + n(2)}{m + n}$$

$\Rightarrow 4m + 4n = 7m + 2n$

$\Rightarrow 3m = 2n$

$\therefore$ m:n = 2:3

Thus, the required ratio is 2:3.

Question: 17

Solution:

If the three points are collinear then the area of the triangle formed by them will be zero.

Area of a Δ ABC whose vertices are $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ is given by-

$$\sqrt{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)}\ \text{units}^2$$

$\therefore$ Area of given Δ ABC = 0

$\Rightarrow \sqrt{(2(k-(-3)) + 4(-3-3) + 6(3-k))} = 0$

squaring both sides, we get-

$2(k + 3) + 4(-6) + 6(3-k) = 0$

$\Rightarrow 2k + 6-24 + 18-6k = 0$

$\Rightarrow -4k + 24-24 = 0$

$\therefore k = 0$

Thus, the value of k is zero.