Chapter : 19. VOLUME AND SURFACE AREA OF SOLIDS

Exercise : 19A

Question: 1

Solution:

Let the length of the side of cube be 'a' cm.

Volume of each cube = 27 cm^3

Volume of cube = a^3

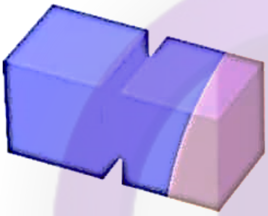
$\therefore a^3 = 27\ cm^3$

$\Rightarrow a = (27\ cm^3)^{1/3}$

$\Rightarrow a = 3$ cm

Length of a side of cube = 3 cm

Since, two cubes are joined and a cuboid is formed so,



Length of cuboid = l = 2a = 2 × 3 cm = 6 cm

Breadth of cuboid = b = a = 3 cm

Height of cuboid = h = a = 3 cm

Surface area of cuboid = 2 × (l × b + b × h + l × h)

$\therefore$ Surface area of resulting cuboid = 2 × (6 × 3 + 3 × 3 + 6 × 3) cm^2

= 2 × (18 + 9 + 18) cm^2

= 2 × 45 cm^2

= 90 cm^2

So, surface area of resulting cuboid is 90 cm^2

Question: 2

Solution:

Let the radius of hemisphere be r cm

Volume of hemisphere is given by $\frac{2}{3}\pi r^3$

Given, volume of hemisphere = 24251/2 cm^3

$\therefore \frac{2}{3}\pi r^3 = 24251/2$

$\Rightarrow r^3 = 4851 \times 1/2 \times 3/2 \times 7/22$

$\Rightarrow r^3 = 1157.625\ cm^3$

⇒ $r = (1157.625)^{1/3}$ cm

⇒ r = 10.5 cm

Curved Surface Area of hemisphere = $2\pi r^2$

Curved Surface Area of hemisphere = $2 \times 22/7 \times (10.5)^2$ cm^2

= 693 cm^2

∴ Curved surface area of hemisphere = 693 cm^2

Question: 3

Solution:

Let the radius of solid sphere be r cm

Total surface area of solid hemisphere = $3\pi r^2$

Given, total surface area of solid hemisphere = 462 cm^2

∴ $3\pi r^2 = 462$ cm^2

⇒ $3 \times 22/7 \times r^2 = 462$ cm^2

⇒ $r^2 = 462 \times 1/3 \times 7/22$ cm^2 = 49 cm^2

⇒ r = 7 cm

Volume of solid hemisphere = $\frac{4}{3}\pi r^3$

= $2/3 \times 22/7 \times 7^3$ cm^3

= 718.67 cm^3

∴ Volume of solid hemisphere is 718.67 cm^3

Question: 4

Solution:

Width of cloth used = 5 m

Diameter of conical tent to be made = 14 m

Let the radius of the conical tent be r m

Radius of conical tent = r = diameter ÷ 2 = 14/2 m = 7 m

Height of conical tent = h = 24 m

Let the slant height of conical tent be l

So, $l = \sqrt{r^2 + h^2}$

⇒ $l = \sqrt{7^2 + 24^2}$ m^2

⇒ l = 25 m

Area of cloth required to make a conical tent = Curved Surface area of conical tent

= πrl

= $22/7 \times 7 \times 25$ m^2

= 550 m^2

Length of cloth used = Area of cloth required ÷ width of cloth

= 550/5 m

= 110 m

∴ Length of cloth used = 110 m

Cost of cloth used = Rs 25 per meter

Total Cost of cloth required to make a conical tent = 110 × Rs 25

= Rs 2750

∴ Total cost of cloth required to make a conical tent = Rs 2750

Question: 5

Solution:

Let V_1 be the volume of first cone and V_2 be the volume of second cone.

Then, $V_1:V_2 = 1:4$

Let d_1 be the diameter of first cone and d_2 be the diameter of second cone.

Then $d_1:d_2 = 4:5$

Let h_1 be the height of first cone and h_2 be the height of second cone.

We know that volume of cone is given by $V = 1/3 \times \pi(d^2/4)h$

$$\frac{V_1}{V_2} = \frac{1}{4}$$

$$\therefore \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi\frac{d_1^2}{4}h_1}{\frac{1}{3}\pi\frac{d_2^2}{4}h_2}$$

$$\frac{V_1}{V_2} = \frac{d_1^2h_1}{d_2^2h_2}$$

$$\frac{d_1^2h_1}{d_2^2h_2} = \frac{1}{4}$$

$$\Rightarrow \left(\frac{d_1}{d_2}\right)^2 \times \frac{h_1}{h_2} = \frac{1}{4}$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 \times \frac{h_1}{h_2} = \frac{1}{4}$$

$$\Rightarrow \frac{16}{25} \times \frac{h_1}{h_2} = \frac{1}{4} \Rightarrow \frac{h_1}{h_2} = \frac{1}{4} \times \frac{25}{16}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{25}{64}$$

∴ $h_1:h_2 = 25:64$

∴ Ratio of height of two cones is 25:64.

Question: 6

Solution:

Let the radius of base be ‘r’ km and slant height be ‘l’ km

Slant height of conical mountain = 2.5 km

Area of its base = 1.54 km^2

Area of base is given by πr^2

$\therefore \pi r^2 = 1.54\ km^2$

$\Rightarrow 22/7 \times r^2 = 1.54\ km^2$

$\Rightarrow r^2 = 1.54 \times 7/22\ km^2 = .49\ km^2$

$\Rightarrow r = 0.7$ km

Let 'h' be the height of the mountain

We know,

$l^2 = r^2 + h^2$

Substituting the values of l and r in the above equation

$2.5^2 = 0.7^2 + h^2$

$h^2 = 2.5^2 - 0.7^2 = 6.25 - 0.49\ km^2$

$h^2 = 5.76\ km^2$

h = 2.4 km

∴ Height of the mountain = 2.4 km

Question: 7

Solution:

Let the Radius of the solid cylinder be 'r' m and its height be 'h' m.

Given,

Sum of radius and height of solid cylinder = 37 m

r + h = 37 m

r = 37 – h

Total surface area of solid cylinder = $1628\ m^2$

Total surface area of solid cylinder is given by $2\pi r (h + r)$

$\therefore 2\pi r (h + r) = 1628\ m^2$

Substituting the value of r + h in the above equation

$\Rightarrow 2\pi r \times 37 = 1628\ m^2$

$\Rightarrow r = 1628 \times 7/22 \times 1/2 \times 1/37$ m

$\Rightarrow r = 7$ m

Since, r + h = 37 m

h = 37 – r m

h = 37 – 7 m = 30 m

Volume of solid cylinder = $\pi r^2 h$

$= 22/7 \times 7^2 \times 30\ m^2$

$= 4620\ m^2$

Question: 8

Solution:

Let the radius of sphere be 'r' cm

Surface area of sphere = 2464 cm^2

Surface area of sphere is given by $4\pi r^2$

$\therefore 4\pi r^2 = 2464\ cm^2$

$\Rightarrow 4 \times 22/7 \times r^2 = 2464\ cm^2$

$\Rightarrow r^2 = 2464 \times 1/4 \times 7/22\ cm^2 = 196\ cm^2$

$\Rightarrow r = 14$ cm

Radius of new sphere is double the radius of given sphere.

Let the radius of new sphere be r' cm

$\therefore r' = 2r$

$r' = 2 \times 14$ cm = 28 cm

Surface area of new sphere = $4\pi r'^2$

$= 4 \times 22/7 \times 28^2\ cm^2$

$= 9856\ cm^2$

$\therefore$ Surface area of new sphere is 9856 cm^2.

Question: 9

Solution:

The military tent is made as a combination of right circular cylinder and right circular cone on top.

Total Height of tent = h = 8.25 m

Base diameter of tent = 30 m

Base radius of tent = r = 30/2 m = 15 m

Height of right circular cylinder = 5.5 m

Curved surface area of right circular cylindrical part of tent = $2\pi rh$

Height of conical part = total height of tent – height of cylindrical part

$h_{cone} = 8.25 - 5.5$ m = 2.75 m

Base radius of cone = 15 m

Let l be the slant height of cone

Then, $l^2 = h_{cone}^2 + r^2 = 2.75^2 + 15^2\ m^2$

$l^2 = 7.5625 + 225\ m^2 = 232.5625$

l = 15.25

Curved surface area of conical part of the tent = πrl

Total surface area of the tent = Curved surface area of cylindrical part + curved surface area of conical part

Total surface area of tent = $2\pi rh + \pi rl$

$= \pi r\ (2h + l)$

$= 22/7 \times 15 \times (2 \times 5.5 + 15.25)\ m^2$

$= 22/7 \times 15 \times (11 + 15.25)\ m^2$

= 22/7 × 15 × 26.25 m^2

= 1237.5 m^2

Breadth of canvas used = 1.5 m

Length of canvas used = Total surface area of tent ÷ breadth of canvas used

Length of canvas used = 1237.5\1.5 m = 825 m

∴ Length of canvas used is 825 m.

Question: 10

Solution:

The tent is made as a combination of right circular cylinder and right circular cone on top.

Height of cylindrical part of the tent = h = 3 m

Radius of its base = r = 14 m

Curved surface area of cylindrical part of tent = 2πrh

= 2 × 22/7 × 14 × 3 m^2

= 264 m^2

Total height of the tent = 13.5 m

Height of conical part of the tent = total height of tent – height of cylindrical part.

Height of conical part of the tent = 13.5 – 3 m =10.5 m

Let the slant height of the conical part be l

$l^2 = h_{cone}^2 + r^2$

$l^2 = 10.5^2 + 14^2 = 110.25 + 196\ m^2 = 306.25\ m^2$

l = 17.5 m

Curved surface area of conical part of tent = πrl

= 22/7 × 14 × 17.5 m^2

= 770 m^2

Total surface area of tent = Curved surface area of cylindrical part of tent + Curved surface area of conical part of tent

Total Surface area of tent = 264 m^2 + 770 m^2 = 1034 m^2

Cloth required = Total Surface area of tent = 1034 m^2

Cost of cloth = Rs 80/m^2

Total cost of cloth required = Total surface area of tent × Cost of cloth

= 1034 × Rs 80

= Rs 82720

Cost of cloth required to make the tent is Rs 82720

Question: 11

Solution:

The Circus tent is made as a combination of cylinder and cone on top.

Height of cylindrical part of tent = h = 3 m

Base radius of tent = r = 52.5 m

Area of canvas required for cylindrical part of tent = 2πrh

$= 2 \times 22/7 \times 52.5 \times 3 \text{ m}^2$

$= 990 \text{ m}^2$

Slant height of cone = l = 53 m

Area of canvas required for conical part of the tent = πrl

$= 22/7 \times 52.5 \times 53 \text{ m}^2$

$= 8745 \text{ m}^2$

Area of canvas required to make the tent = Area of canvas required for cylindrical part of tent + Area of canvas required for conical part of tent

Area of canvas required to make the tent = $990 + 8745 \text{ m}^2 = 9735 \text{ m}^2$

Question: 12

Solution:

The rocket is in the form of cylinder closed at the bottom and cone on top.

Height of cylindrical part rocket = h = 21 m

Base radius of rocket = r = 2.5 m

Surface Area of cylindrical part of rocket = $2\pi rh + \pi r^2$

$= 2 \times 22/7 \times 2.5 \times 21 + 22/7 \times 2.5 \times 2.5 \text{ m}^2$

$= 330 + 19.64 \text{ m}^2 = 349.64 \text{ m}^2$

Slant height of cone = l = 8 m

Surface Area of conical part of the rocket = πrl

$= 22/7 \times 2.5 \times 8 \text{ m}^2$

$= 62.86 \text{ m}^2$

Total surface area of the rocket = Surface Area of cylindrical part of rocket + Surface Area of conical part of rocket

Total surface area of the rocket = $349.64 + 62.86 \text{ m}^2 = 412.5 \text{ m}^2$

Question: 13

Solution:

The solid is in the form of a cone surmounted on a hemisphere.

Total height of solid = h = 9.5 m

Radius of Solid = r = 3.5 m

Volume of hemispherical part solid = $2/3 \times \pi r^3$

$= 2/3 \times 22/7 \times 3.5^3 \text{ m}^3$

$= 89.83 \text{ m}^3$

Height of conical part of solid = h_{cone} = Total height of solid – Radius of solid

Height of conical part of solid = $h_{cone} = 9.5 - 3.5 = 6$ m

Volume of conical part of solid = $1/3 \times \pi r^2 h_{cone}$

= $1/3 \times 22/7 \times 3.5^2 \times 6\ m^3$

= $77\ m^3$

Volume of solid = Volume of hemispherical part solid + Volume of conical part solid

Volume of solid = $89.83 + 77\ m^3 = 166.83\ m^3$

Question: 14

Solution:

The toy is in the form of a cone mounted on a hemisphere.

Total height of toy = h = 31 cm

Radius of toy = r = 7 cm

Surface area of hemispherical part toy = $2\pi r^2$

= $2 \times 22/7 \times 7^2\ cm^2$

= $308\ cm^2$

Height of conical part of toy = h_{cone} = Total height of toy – Radius of toy

Height of conical part of toy = $h_{cone} = 31 - 7 = 24$ cm

Let the slant height of the conical part be l

$l^2 = h_{cone}^2 + r^2$

$l^2 = 24^2 + 7^2 = 576 + 49\ cm^2 = 625\ cm^2$

l = 25 cm

Surface area of conical part of toy = πrl

= $22/7 \times 7 \times 25\ cm^2$

= $550\ cm^2$

Total surface area of toy = Surface area of hemispherical part of toy + Surface area of conical part of toy

Total Surface area of toy = $308\ cm^2 + 550\ cm^2 = 858\ cm^2$

Question: 15

Solution:

A toy is in the shape of a cone mounted on a hemisphere of same base radius.

Volume of Toy = $231\ cm^3$

Base Diameter of toy = 7 cm

Base radius of toy = 7/2 cm = 3.5 cm

Volume of hemisphere = $2/3\ \pi r^3$

= $2/3 \times 22/7 \times 3.5^3\ cm^3$

$$= \frac{2 \times 22 \times 35 \times 35 \times 35}{3 \times 7 \times 10 \times 10 \times 10}$$

$$= \frac{539}{6}$$

Volume of cone = Volume of hemisphere – Volume of toy

$= 231 - \frac{539}{6}$

$= \frac{1386 - 539}{6}$

$= \frac{847}{6}$

Volume of cone is given by 1/3 $\pi r^2 h$

Where h is the height of cone

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{847}{6}$

$\Rightarrow \frac{22}{7}\left(\frac{7}{2}\right)^2 h = \frac{847}{2}$

$\Rightarrow \frac{77}{2}h = \frac{747}{2}$

$\Rightarrow h = 11$

Height of cone = 11 cm

Height of toy = Height of cone + Height of hemisphere

= 11 cm + 3.5 cm = 14.5 cm

[Height of hemisphere = Radius of hemisphere]

∴ Height of toy is 14.5 cm.

Question: 16

Solution:

Radius of cylindrical container = r = 6 cm

Height of cylindrical container = h = 15 cm

Volume of cylindrical container = $\pi r^2 h$

= 22/7 × 6 × 6 × 15 cm^3

= 1697.14 cm^3

Whole ice-cream has to be distributed to 10 children in equal cones with hemispherical tops.

Let the radius of hemisphere and base of cone be r'

Height of cone = h = 4 times the radius of its base

h' = 4r'

Volume of Hemisphere = 2/3 $\pi(r')^3$

Volume of cone = 1/3 $\pi(r')^2h'$ = 1/3 $\pi(r')^2 \times 4r'$

= 2/3 $\pi(r')^3$

Volume of ice-cream = Volume of Hemisphere + Volume of cone

= 2/3 $\pi(r')^3$ + 4/3 $\pi(r')^3$ = 6/3 $\pi(r')^3$

Number of ice-creams = 10

∴ total volume of ice-cream = 10 × Volume of ice-cream

= 10 × 6/3 $\pi(r')^3$ = 60/3 $\pi(r')^3$

Also, total volume of ice-cream = Volume of cylindrical container

⇒ $60/3\ \pi(r')^3 = 1697.14\ cm^3$

⇒ $60/3 \times 22/7 \times (r')^3 = 1697.14\ cm^3$

⇒ $(r')^3 = 1697.14 \times 3/60 \times 7/22 = 27\ cm^3$

⇒ r = 3 cm

∴ Radius of ice-cream cone = 3 cm

Question: 17

Solution:

Vessel is in the form of a hemispherical bowl surmounted by a hallow cylinder.

Diameter of hemisphere = 21 cm

Radius of hemisphere = 10.5 cm

Volume of hemisphere = $2/3\ \pi r^3 = 2/3 \times 22/7 \times 10.5^3\ cm^3$

= $2425.5\ cm^3$

Total height of vessel = 14.5 cm

Height of cylinder = h = Total height of vessel - Radius of hemisphere

= 14.5 – 10.5 cm = 4 cm

Volume of cylinder = $\pi r^2 h = 22/7 \times 10.5 \times 10.5 \times 4\ cm^3$

= $1386\ cm^3$

Volume of vessel = Volume of hemisphere + Volume of cylinder

= $2425.5\ cm^3 + 1386\ cm^3$

= $3811.5\ cm^3$

∴ Capacity of vessel = $3811.5\ cm^3$

Question: 18

Solution:

Toy is in the form of a cylinder with hemisphere ends

Total length of toy = 90 cm

Diameter of toy = 42 cm

Radius of toy = r = 21 cm

Length of cylinder = l = Total length of toy – 2 × Radius of toy

= 90 – 2 × 21 cm = 48 cm

For cost of painting we need to find out the curved surface area of toy

Curved surface area of cylinder = $2\pi rl$

= $2 \times 22/7 \times 21 \times 48\ cm^2$

= $6336\ cm^2$

Curved surface area of hemispherical ends = $2 \times 2\pi r^2$

= $2 \times 2 \times 22/7 \times 21 \times 21\ cm^2$

$= 5544\ cm^2$

Surface area of toy = Curved surface area of cylinder + Curved surface area of hemispherical ends

Surface area of toy = $6336\ cm^2 + 5544\ cm^2 = 11880\ cm^2$

Cost of painting = Rs $0.70/cm^2$

Total Cost of painting = Surface area of toy × Cost of painting

$= 11880\ cm^2 \times$ Rs $0.70/cm^2$ = Rs 8316.00

Total cost of painting the toy = Rs 8316.00

Question: 19

Solution:

A medicine capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends.

Total length of entire capsule = 14 mm

Diameter of capsule = 5 mm

Radius of capsule = r = Diameter ÷ 2 = 5/2 mm = 2.5 mm

Length of cylindrical part of capsule = l = Total length of entire capsule – 2 × Radius of capsule

= 14 – 2 × 2.5 mm = 14 - 5 mm = 9 mm

Curved surface area of cylindrical part of capsule = $2\pi rl$

$= 2 \times 3.14 \times 9 \times 2.5\ mm^2$

$= 141.3\ mm^2$

Curved surface area of hemispherical ends = $2 \times 2\pi r^2$

$= 2 \times 2 \times 3.14 \times 2.5 \times 2.5\ cm^2$

$= 78.5\ mm^2$

Surface area of capsule = Curved surface area of cylindrical part of capsule + Curved surface area of hemispherical ends

Surface area of capsule = $141.3\ mm^2 + 78.5\ mm^2 = 219.8\ mm^2$

Question: 20

Solution:

The wooden article was made by scooting out a hemisphere from each end of a cylinder

Let the radius of cylinder be r cm and height be h cm.

Height of cylinder = h = 20 cm

Base diameter of cylinder = 7 cm

Base radius of cylinder = r = diameter ÷ 2 = 7/2 cm = 3.5 cm

Lateral Surface area of cylinder = $2\pi rh$

$= 2 \times 22/7 \times 3.5 \times 20\ cm^2$

$= 2 \times 22/7 \times 3.5 \times 20\ cm^2$

$= 440\ cm^2$

Since, the wooden article was made by scooting out a hemisphere from each end of a cylinder

∴ Two hemispheres are taken out in total

Radius of cylinder = radius of hemisphere

∴ Radius of hemisphere = 3.5 cm

Lateral Surface area of hemisphere = $2\pi r^2$

$= 2 \times 22/7 \times 3.5 \times 3.5\ cm^2$

$= 77\ cm^2$

Total Surface area of two hemispheres = $2 \times 77\ cm^2 = 154\ cm^2$

Total surface area of the article when it is ready = Lateral Surface area of cylinder + Lateral Surface area of hemisphere

Total surface area of the article when it is ready = $440\ cm^2 + 154\ cm^2$

$= 594\ cm^2$

Question: 21

Solution:

A solid is in the form of a right circular cone mounted on a hemisphere.

Let r be the radius of hemisphere and cone

Let h be the height of the cone

Radius of hemisphere = r = 2.1 cm

Volume of hemisphere = $2/3\ \pi r^3$

$= 2/3 \times 22/7 \times 2.1 \times 2.1 \times 2.1\ cm^3$

$= 19.404\ cm^3$

Height of cone = h = 4 cm

Radius of cone = r = 2.1 cm

Volume of cone = $1/3\ \pi r^2 h$

$= 1/3 \times 22/7 \times 2.1 \times 2.1 \times 4\ cm^3$

$= 18.48\ cm^3$

Volume of solid = Volume of hemisphere + Volume of cone

$= 19.404\ cm^3 + 18.48\ cm^3 = 37.884\ cm^3$

The solid is placed in a cylindrical tub full of water in such a way that the whole solid is submerged in water, so, to find the volume of water left in the tub we need to subtract volume of solid from cylindrical tub.

Radius of cylinder = r' = 5 cm

Height of cylinder = h' = 9.8 cm

Volume of cylindrical tub = $\pi r'^2 h' = 22/7 \times 5 \times 5 \times 9.8\ cm^3$

$= 770\ cm^3$

Volume of water left in the tub = Volume of cylindrical tub – Volume of solid

Volume of water left in the tub = $770\ cm^3 - 37.884\ cm^3 = 732.116\ cm^3$

∴ Volume of water left in the tub is $732.116\ cm^3$

Question: 22

Solution:

Height of solid cylinder = h = 8 cm

Radius of solid cylinder = r = 6 cm

Volume of solid cylinder = $\pi r^2 h$

= $3.14 \times 6 \times 6 \times 8$ cm^3

= 904.32 cm^3

Curved Surface area of solid cylinder = $2\pi rh$

Height of conical cavity = h = 8 cm

Radius conical cavity = r = 6 cm

Volume of conical cavity = $1/3\ \pi r^2 h$

= $1/3 \times 3.14 \times 6 \times 6 \times 8$ cm^3

= 301.44 cm^3

Let l be the slant height of conical cavity

$l^2 = r^2 + h^2$

$\Rightarrow l^2 = (6^2 + 8^2)$ cm^2

$\Rightarrow l^2 = (36 + 64)$ cm^2

$\Rightarrow l^2 = 100$ cm^2

$\Rightarrow l = 10$ cm

Curved Surface area of conical cavity = πrl

Since, conical cavity is hollowed out from solid cylinder, so, volume and total surface area of remaining solid will be found out by subtracting volume and total surface area of conical cavity from volume and total surface area of solid cylinder.

Volume of remaining solid = Volume of solid cylinder – Volume of conical cavity

Volume of remaining solid = 904.32 cm^3 – 301.44 cm^3

= 602.88 cm^3

Total surface area of remaining solid = Curved Surface area of solid cylinder + Curved Surface area of conical cavity + Area of circular base

Total surface area of remaining solid = $2\pi rh + \pi rl + \pi r^2$

= $\pi r \times (2h + l + r)$

= $3.14 \times 6 \times (2 \times 8 + 10 + 6)$ cm^2

= $3.14 \times 6 \times 32$ cm^2

= 602.88 cm^2

Question: 23

Solution:

Height of solid cylinder = h = 2.8 cm

Diameter of solid cylinder = 4.2 cm

Radius of solid cylinder = r = Diameter ÷ 2 = 2.1 cm

Curved Surface area of solid cylinder = 2πrh

= 2 × 22/7 × 2.1 × 2.8 cm^2

= 2 × 22/7 × 2.1 × 2.8 cm^2

= 36.96 cm^2

Height of conical cavity = h = 2.8 cm

Radius conical cavity = r = 2.1 cm

Let l be the slant height of conical cavity

$l^2 = r^2 + h^2$

$\Rightarrow l^2 = (2.8^2 + 2.1^2)\ cm^2$

$\Rightarrow l^2 = (7.84 + 4.41)\ cm^2$

$\Rightarrow l^2 = 12.25\ cm^2$

⇒ l = 3.5 cm

Curved Surface area of conical cavity = πrl

= 22/7 × 2.1 × 3.5

= 23.1 cm^2

Total surface area of remaining solid = Curved surface area of solid cylinder + Curved surface area of conical cavity + Area of circular base

Total surface area of remaining solid = (36.96 + 23.1 + 22/7 × 2.1^2) cm^2

= (36.96 + 23.1 + 13.86) cm^2

= 73.92 cm^2

Question: 24

Solution:

Height of solid cylinder = h = 14 cm

Diameter of solid cylinder = 7 cm

Radius of solid cylinder = r = Diameter ÷ 2 = 7/2 cm = 3.5 cm

Volume of solid cylinder = $\pi r^2 h$

= 22/7 × 3.5 × 3.5 × 14 cm^3

= 539 cm^3

Height of conical cavity = h' = 4 cm

Radius conical cavity = r' = 2.1 cm

Volume of conical cavity = 1/3 $\pi r'^2 h'$

= 1/3 × 22/7 × 2.1 × 2.1 × 4 cm^3

= 18.48 cm^3

Since, there are two conical cavities

∴ Volume of two conical cavities = 2 × 18.48 cm^3 = 36.96 cm^3

Volume of remaining solid = Volume of solid cylinder – Volume of two conical cavity

Volume of remaining solid = 539 cm^3 – 36.96 cm^3

$= 502.04\ cm^3$

Question: 25

Solution:

Height of metallic cylinder = h = 5 cm

Radius of metallic cylinder = r = 3 cm

Volume of solid cylinder = $\pi r^2 h$

$= \pi \times 3 \times 3 \times 5\ cm^3$

$= 45\pi\ cm^3$

Height of conical hole = h' = 8/9 cm

Radius conical hole = r' = 3/2 cm

Volume of conical hole = $1/3\ \pi r'^2 h'$

$= 1/3 \times \pi \times 3/2 \times 3/2 \times 8/9\ cm^3$

$= 2/3\ \pi\ cm^3$

Volume of metal left in cylinder = Volume of metallic cylinder – Volume of conical hole

Volume of metal left in cylinder = $45\pi - 2/3\ \pi = 133\pi/3$

Ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape = Volume of metal left in cylinder/ Volume of conical hole

Volume of metal left in cylinder : Volume of conical hole = $133\pi/3 : 2/3\ \pi$

Volume of metal left in cylinder: Volume of conical hole = 133: 2

Ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape is 339:4

Question: 26

Solution:

Length of cylindrical neck = l = 7 cm

Diameter of cylindrical neck = 4 cm

Radius of cylindrical neck = r = Diameter ÷ 2 = 4/2 cm = 2 cm

Volume of cylindrical neck = $\pi r^2 l$

$= 22/7 \times 2 \times 2 \times 7\ cm^3$

$= 88\ cm^3$

Diameter of spherical part = 21 cm

Radius of spherical part = r' = Diameter ÷ 2 = 21/2 cm = 10.5 cm

Volume of spherical part = $4/3\ \pi r'^3$

$= 4/3 \times 22/7 \times 10.5 \times 10.5 \times 10.5\ cm^3$

$= 4851\ cm^3$

Quantity of water spherical glass vessel with cylindrical neck can hold = Volume of spherical part + Volume of cylindrical neck

Quantity of water spherical glass vessel with cylindrical neck can hold = $4851\ cm^3 + 88\ cm^3 = 4939\ cm^3$

Quantity of water spherical glass vessel with cylindrical neck can hold is 4939 cm^3.

Question: 27

Solution:

The solid consisting of a cylinder surmounted by a cone at one end a hemisphere at the other.

Length of cylinder = l = 6.5 cm

Diameter of cylinder = 7 cm

Radius of cylinder = r = Diameter ÷ 2 = 7/2 cm = 3.5 cm

Volume of cylinder = $\pi r^2 l$

= 22/7 × 3.5 × 3.5 × 6.5 cm^3

= 250.25 cm^3

Length of cone = l' = 12.8 cm – 6.5 cm = 6.3 cm

Diameter of cone = 7 cm

Radius of cone = r = Diameter ÷ 2 = 7/2 cm = 3.5 cm

Volume of cone = 1/3 $\pi r^2 l'$

= 1/3 × 22/7 × 3.5 × 3.5 × 6.3 cm^3

= 80.85 cm^3

Diameter of hemisphere = 7 cm

Radius of hemisphere = r = Diameter ÷ 2 = 7/2 cm = 3.5 cm

Volume of hemisphere = 2/3 πr^3

= 2/3 × 22/7 × 3.5 × 3.5 × 3.5 cm^3

= 89.83 cm^3

Volume of the solid = Volume of cylinder + Volume of cone + Volume of hemisphere

Volume of solid = 250.25 cm^3 + 80.85 cm^3 + 89.83 cm^3

= 420.93 cm^3

Question: 28

Solution:

Length of cubical piece of wood = a = 21 cm

Volume of cubical piece of wood = a^3

= 21 × 21 × 21 cm^3

= 9261 cm^3

Surface area of cubical piece of wood = $6a^2$

= 6 × 21 × 21 cm^2

= 2646 cm^2

Since, a hemisphere is carved out in such a way that the diameter of the hemisphere is equal to the side of the cubical piece.

So, diameter of hemisphere = length of side of the cubical piece

Diameter of hemisphere = 21 cm

Radius of hemisphere = r = Diameter ÷ 2 = 21/2 cm = 10.5 cm

Volume of hemisphere = $2/3\ \pi r^3$

= $2/3 \times 22/7 \times 10.5 \times 10.5 \times 10.5\ cm^3$

= $2425.5\ cm^3$

Surface area of hemisphere = $2\pi r^2$

= $2 \times 22/7 \times 10.5 \times 10.5\ cm^2$

= $693\ cm^2$

A hemisphere is carved out from cubical piece of wood

Volume of remaining solid = Volume of cubical piece of wood – Volume of hemisphere

Volume of remaining solid = $9261 cm^3 - 2425.5\ cm^3 = 6835.5\ cm^3$

Surface area remaining piece of solid = surface area of cubical piece of wood – Area of circular base of hemisphere + Curved Surface area of hemisphere

Surface area remaining piece of solid = $6a^2 - \pi r^2 + 2\pi r^2$

= $(2646 - 22/7 \times 10.5^2 + 693)\ cm^2$

= $2992.5\ cm^2$

Question: 29

Solution:

Length of side of cubical block = a = 10 cm

Since, a cubical block is surmounted by a hemisphere, so, the largest diameter of hemisphere = 10 cm

Since, hemisphere will be touching the sides of cubical block.

Radius of hemisphere = r = Diameter ÷ 2 = 10/2 cm = 5 cm

Surface area of solid = Surface area of cube – Area of circular part of hemisphere + Curved surface area of hemisphere

Total Surface area of solid = $6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$

= $6 \times 10 \times 10\ cm^2 + 3.14 \times 5 \times 5\ cm^2$

= $678.5\ cm^2$

Rate of painting = Rs $5/100\ cm^2$

Cost of painting the total surface area of the solid so formed = Total Surface area of solid × Rate of painting

Cost of painting the total surface area of the solid = Rs 5/100 × 678.5

= Rs 33.925

Question: 30

Solution:

The toy is in the shape of a right circular cylinder surmounted by a cone at one end a hemisphere at the other.

Total height of toy = 30 cm

Height of cylinder = h = 13 cm

Radius of cylinder = r = 5 cm

Curved surface area of cylinder = $2\pi rh$

$= 2 \times 22/7 \times 5 \times 13\ cm^2$

Height of cone = h' = Total height of toy – Height of cylinder – Radius of hemisphere

Height of cone = h' = 30 cm – 13 cm – 5 cm = 12 cm

Radius of cone = r = Radius of cylinder

Radius of cone = r = 5 cm

Let the slant height of cone be l

$l^2 = h'^2 + r^2$

$\Rightarrow l^2 = 12^2 + 5^2\ cm^2 = 144 + 25\ cm^2 = 169\ cm^2$

$\Rightarrow l = 13$ cm

Curved surface area of cone = πrl

$= 22/7 \times 5 \times 13\ cm^2$

Radius of hemisphere = r = Radius of cylinder

Radius of hemisphere = r = 5 cm

Curved surface area of hemisphere = $2\pi r^2$

$= 2 \times 22/7 \times 5 \times 5\ cm^2$

Surface area of the toy = Surface area of cylinder + Surface area of cone + Surface area of hemisphere

Surface area of toy = $2\pi rh + \pi rl + 2\pi r^2$

$= \pi r\ (2h + l + 2r)$

$= 22/7 \times 5 \times (2 \times 13 + 13 + 2 \times 5)\ cm^2$

$= 22/7 \times 5 \times 49\ cm^2$

$= 770\ cm^2$

Surface area of toy is $770\ cm^2$

Question: 31

Solution:

Inner diameter of a glass = 7 cm

Inner radius of glass = r = 7/2 cm = 3.5 cm

Height of glass = h = 16 cm

Apparent capacity of glass = $\pi r^2 h$

$= 22/7 \times 3.5 \times 3.5 \times 16\ cm^3$

$= 616\ cm^3$

Volume of the hemisphere in the bottom = $2/3\ \pi r^3$

$= 2/3 \times 22/7 \times 3.5^3\ cm^3$

$= 89.83\ cm^3$

Actual capacity of the class = Apparent capacity of glass – Volume of the hemisphere

Actual capacity of the class = 616 cm^3 – 89.83 cm^3 = 526.17 cm^3

Question: 32

Solution:

The wooden toy is in the shape of a cone mounted on a cylinder

Total height of the toy = 26 cm

Height of conical part = H = 6 cm

Height of cylindrical part = Total height of the toy – Height of conical part

h = 26 cm – 6 cm = 20 cm

Diameter of conical part = 5 cm

Radius of conical part = R = Diameter/2 = 5/2 cm = 2.5 cm

Let L be the slant height of the cone

$L^2 = H^2 + R^2$

$\Rightarrow L^2 = 6^2 + 2.5^2\ cm^2 = 36 + 6.25\ cm^2 = 42.25\ cm^2$

$\Rightarrow$ L = 6.5 cm

Diameter of cylindrical part = 4 cm

Radius of cylindrical part = r = Diameter/2 = 4/2 cm = 2 cm

Area to be painted Red = Curved Surface area of cone + Base area of cone – base area of cylinder

Area to be painted Red = $\pi RL + \pi R^2 - \pi r^2 = \pi (RL + R^2 - r^2)$

= 22/7 × (2.5 × 6.5 + 2.5 × 2.5 – 2 × 2) cm^2

= 22/7 × (16.25 + 6.25 – 4) cm^2

= 22/7 × 18.5 cm^2

= 58.143 cm^2

Area to be painted White = Curved Surface area of cylinder + Base area of cylinder

Area to be painted White = $2\pi rh + \pi r^2 = \pi r (2h + r)$

= 22/7 × 2 × (2 × 20 + 2) cm^2

= 22/7 × 2 × (40 + 2) cm^2

= 22/7 × 2 × 42 cm^2 = 264 cm^2

∴ Area to be painted red is 58.143 cm^2 and area to be painted white is 264 cm^2.

Exercise : 19B

Question: 1

Solution:

Given,

The dimensions of a metallic cuboid = 100 cm × 80 cm × 64 cm

Let's find out the volume of the cuboid first;

Volume of the cuboid = l × b × h

⇒ $100 \times 80 \times 64 = 512000 cm^3$

As cuboid is recast into a cube;

So,

Volume of cube = volume of cuboid

⇒ $l^3 = 512000 \Rightarrow l = \sqrt{512000}$

⇒ l = 80 cm

Now,

As the length of the side of the cube = 80 cm

The surface area of the cube = $6(l)^2$

= 6 x 80 x 80

= $38400 cm^2$

So, the surface area of the cube is $38400 cm^2$

Question: 2

Solution:

We have,

The radius of the cone (r) = 5cm and

The height of the cone (h) = 20cm

Let the radius of the sphere be R;

As,

Volume of sphere = Volume of cone

$\Rightarrow \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$

⇒ $4R^3 = 5 \times 5 \times 20$

⇒ $R^3 = \frac{5 \times 5 \times 20}{4} = 125$ cm

⇒ R = 5 cm

Diameter of the sphere = 2R = 2 x 5 = 10 cm

So, the diameter of the sphere is 10 cm

Question: 3

Solution:

Given,

The radius (r_1) of 1st sphere = 6 cm

Radius of 2nd sphere (r_2) = 8 cm and

Radius f third sphere (r_3) = 10 cm

Let the radius of the resulting sphere be R;

So now we have,

Volume of resulting sphere = Volume of three metallic spheres

$\Rightarrow \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(r_1^3 + r_2^3 + r_3^3)$

$\Rightarrow \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(6^3 + 8^3 + 10^3)$

$\Rightarrow R^3 = 216 + 512 + 1000$

$\Rightarrow R^3 = 1728$

$\Rightarrow R = \sqrt{1728}$

$R = 12$ cm

So, the radius of the resulting sphere is 12cm.

Question: 4

Solution:

Let the number of balls formed = n

Since the metal cone is melted to form spherical balls;

So we have;

Volume of metal cone = Total volume of n spherical balls

Volume of cone = n(Volume of 1 spherical ball)

$\frac{1}{3} \times \pi \times r^2 \times h = n \times \frac{4}{3} \times \pi \times r^3$

$\frac{1}{3} \times \pi \times (12)^2 \times 24 = n \times \frac{4}{3} \times \pi \times (3)^3$

$(12)^2 \times (24) = n \times 4 \times (3)^3$

$3456 = n \times 108$

$\Rightarrow \frac{3456}{108} = n$

$n = 32$

So, 32 spherical balls can be formed.

Question: 5

Solution:

We have,

The internal base radius of spherical shell, $r_1 = 3$ cm,

The external base radius of spherical shell, $r_2 = 5$ cm and

The base radius of solid cylinder, $r = 7$ cm

Let the height of the cylinder be h

As,

The hollow spherical shell is melted into a solid cylinder;

So,

Volume of solid cylinder = Volume of spherical shell

$\Rightarrow \pi r^2 h = \frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3$

$\Rightarrow \pi r^2 h = \frac{4}{3}\pi(r_2^3 - r_1^3)$

$\Rightarrow r^2 h = \frac{4}{3}(r_2^3 - r_1^3)$

$\Rightarrow 49 \times h = \frac{4}{3}(125-27)$

$\Rightarrow h = \frac{4}{3} \times 9849$

Therefore,

$h = \frac{8}{3}$

So, the height of the cylinder is $\frac{8}{3}$.

Question: 6

Solution:

Given,

Internal diameter of the hemisphere = 6 cm

External diameter of the hemisphere = 10 cm

Diameter of cone = 14 cm

So we have,

Internal radius(r) of the hemisphere = 3 cm

External radius(R) of the hemisphere = 5 cm

Radius of cone = 7 cm

Now,

Volume of the hollow hemisphere = Volume of the coneVolume of the hollow hemisphere = $\frac{2}{3} \times \pi \times (R^3 - r^3)$

Volume of the cone = $\frac{1}{3}\pi r^2 h$

So,

$$\Rightarrow \frac{2}{3} \times \pi \times (R^3 - r^3) = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \frac{2}{3} \times \frac{22}{7} \times (5^3 - 3^3) = \frac{1}{3} \times \frac{22}{7} \times 72 \times h$$

$$\Rightarrow \frac{2}{3} \times \frac{22}{7} \times 98 = \frac{1}{3} \times \frac{22}{7} \times 49 \times h$$

$= 2 \times 98 = 49 \times h$

$\Rightarrow 2 \times \frac{98}{49} = h$

So,

$h = \frac{196}{49} = 4$ cm

Thus, the height of the cone formed is 4 cm.

Question: 7

Solution:

Given,

Diameter of the copper Rod = 2cm

Length of the Rod = 10 cm

Length of the wire = 10 m = 1000cm

So here we have,

Radius of the copper rod = 1 cm

Let suppose the radius of the wire = r

Volume of the rod = volume of the wire

$\pi r^2 h = \pi r^2 h$

$1 \times 1 \times 10 = 1000 \times r$

$10 = 1000r$

$r = \sqrt{\frac{1}{100}} \Rightarrow r = \frac{1}{10} = 0.1$ cm

The diameter of the wire = 2r = 2 × 0.1 = 0.2 cm

So, the thickness of the wire is 0.2 cm or 2mm.

Question: 8

Solution:

Let the required bottles = n

Internal diameter of the hemispherical sphere = 30 cm

Internal Radius of the hemispherical sphere = 15 cm

Diameter of the cylindrical bottle = 5 cm

Radius of the cylindrical bottle = 2.5 cm

Now,

Volume of the hemispherical sphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi \times 15 \times 15 \times 15$

$= \pi 10 \times 15 \times 15$

$= 2{,}250\pi$ cm^3

And,

Volume of 1 cylindrical bottle $= \pi r^2 h = \pi \times 2.5 \times 2.5 \times 6 = 37.5\pi$ cm^3

Amount of water in n bottles = Amount of water in bowl

$n \times 37.5\pi = 2{,}250\pi$

$n = \frac{2250}{37.5} = 60$ bottles

So, 60 numbers of bottles are required to empty the bowl.

Question: 9
Solution:

Given,

Diameter of the sphere = 21 cm

Diameter of the cone = 3.5 cm = $\frac{7}{4}$ cm

Height of the cone = 3 cm

So here we have,

Radius of the sphere = 10.5 cm

Radius of the cone = 1.75 cm

Volume of the sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$

Volume of the cone = $\frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 3$

No of cones = $\frac{\text{volume of the sphere}}{\text{Volume of the cone}}$

$$= \frac{\frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}}{\frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 3}$$

$$= \frac{4}{3} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} \times 3 \times \frac{4}{7} \times \frac{4}{7} \times \frac{1}{3}$$

$$= 2 \times 7 \times 7 \times 21 \times \frac{2}{7} \times \frac{2}{7} \times 3$$

= 504

So,

504 numbers of cones are formed.

Question: 10

Solution:

Given,

Diameter of cannon ball = 28cm

So the Radius of cannon ball = 14 cm

Volume of cannon ball = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 14^3$

Radius of the cone = $\frac{35}{2}$ cm

Let the height of cone be h cm.

Volume of cone = $\frac{1}{3}\pi\left(\frac{35}{2}\right)^2 \times h$ cm^3

Here we have,

$= \frac{4}{3}\pi \times 14^3 = \frac{1}{3}\pi\left(\frac{35}{2}\right)^2 \times h$

$= h = \frac{4}{3}\pi \times 14 \times 14 \times 14 \times \frac{3}{\pi} \times \frac{35}{2} \times \frac{35}{2}$

= h = 35.84 cm

Hence, the height of the cone = 35.84 cm.

Question: 11

Solution:

Given,

Radius of the spherical ball = 3 cm

After recasting the ball into three spherical balls;

Radius of the first ball = r_1 = 1.5 cm

Radius of the first ball = r_2 = 2 cm

Let the radius of the third ball be r_3 cm.

Then,

Volume of third ball = Volume of spherical ball – volume of 2 small balls

Volume of the third ball = $\frac{4}{3}\pi \times 3^3 - \frac{4}{3}\pi\left(\frac{3}{2}\right)^2 - \frac{4}{3}\pi \times 2^3$

$36\pi - \frac{9\pi}{2} - 32\pi/3cm^3 = 125\pi/6cm^3$

$4/3\pi r^3 = 125\pi/6$

$r^3 = 125\pi \times 3/6 \times 4 \times \pi = 125/8$

$r = \frac{5}{2} = 2.5$ cm

Hence, the radius of the third ball is 2.5cm.

Question: 12

Solution:

External diameter of shell = 24cm and

Internal diameter of shell = 18cm

So,

External radius of shell = 12 cm and

Internal radius = 9 cm

Volume of lead in the shell = $\frac{4}{3}\pi[12^3 - 9^3]cm^3$

Let the radius of the cylinder be r cm

Height of the cylinder = 37 cm

Volume of the cylinder = $\pi r^2 h = \pi r^2(37)$

$\frac{4}{3}\pi[12^3 - 9^3] = \pi r^2 \times 37$

$\frac{4}{3}\pi \times 999 = \pi r^2 \times 37$

$r^2 = \frac{4}{3} \times \pi \times 999 \times \frac{1}{37}\pi = 36cm^2$

$r = \sqrt{36} = 6cm$

Hence, diameter of the base of the cylinder = 12 cm

Question: 13

Solution:

Volume of hemisphere of radius 9 cm

$= \frac{2}{3} \times \pi \times 9 \times 9 \times 9 cm^3$

Volume of circular cone (height = 72 cm)

$\frac{1}{3} \times \pi \times r^2 \times 72 cm$

Volume of cone = volume of hemisphere

$\frac{1}{3} \times \pi \times r^2 \times 72 = \frac{2}{3}\pi \times 9 \times 9 \times 9$

$r^2 = \frac{2}{3}\pi \times 9 \times 9 \times 9 \times \frac{1}{24}\pi = 20.25$

$r = \sqrt{20.25} = 4.5 cm$

Hence, radius of the base of the cone = 4.5 cm

Question: 14

Solution:

Diameter of sphere = 21 cm

Hence, radius of the sphere = $\frac{21}{2}$

Volume of sphere = $\frac{4}{3}\pi \times r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$

Volume of cube = $a^3 = 1^3$

Let the number of cubes formed be n

Volume of sphere = n(volume of cube)

$\frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} = n \times 1$

$4 \times 22 \times \frac{1}{2} \times \frac{21}{2} \times \frac{21}{2} = n \times 1$

$11 \times 21 \times 21 = n \times 1$

$4851 = n$

So,

Hence, the number of cubes is 4851

Question: 15

Solution:

Given,

Radius of the sphere = 8 cm

Radius of the ball we made = 1 cm

So,

Volume of sphere (when r = 1 cm) = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 1 \times 1 \times 1 \times \pi cm^3$

Volume of sphere (when r = 8 cm) = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 8 \times 8 \times 8 \times \pi \text{ cm}^3$

Let the number of balls be n;

Then we have;

$n \times \frac{4}{3} \times 1 \times 1 \times 1 \times \pi = \frac{4}{3} \times 8 \times 8 \times 8 \times \pi$

$n = \frac{4 \times 8 \times 8 \times 8 \times 3}{3 \times 4} = 512$

Hence, the number of lead balls can be made is 512.

Question: 16

Solution:

Given,

Radius of sphere = 3 cm

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 3 \times 3 \times 3 \text{cm}^3 = 36\pi \text{ cm}^3$

Radius of small sphere = $\frac{0.6}{2}$ cm = 0.3 cm

Volume of small sphere = $\frac{4}{3} \times \pi \times 0.3 \times 0.3 \times 0.3 \text{cm}^3$

$= \frac{4}{3}\pi \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \text{cm}^3$

Let the number of small balls be n;

$n \times \frac{4}{3}\pi \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{4}{3} \times \pi \times 3 \times 3 \times 3$

n = 1000

Hence, the number of small balls = 1000

Question: 17

Solution:

Given,

Diameter of sphere = 42 cm

Radius of sphere = 21 cm

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 21 \times 21 \times 21 \text{ cm}^3$

Diameter of cylindrical wire = 2.8 cm

Radius of cylindrical wire = 1.4 cm

Volume of cylindrical wire = $\pi r^2 h = \pi \times 1.4 \times 1.4 \times h \text{ cm}^3 = 1.96\pi h \text{ cm}^3$

Volume of cylindrical wire = volume of sphere

$1.96\pi h = \frac{4}{2} \times \pi \times 21 \times 21 \times 21$

$h = \frac{\frac{4}{3}\pi \times 21 \times 21 \times 21 \times 1}{1.96 \times 1\pi} \text{ cm}$

h = 6300

$h\left(\frac{6300}{100}\right)m = 63$ m

Hence, length of the wire = 63 m

Question: 18

Solution:

Given,

Diameter of sphere = 18 cm

Length of wire = 108 m = 10800 cm

Radius of copper sphere = $\frac{3600}{100}$ m = 36 m

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 9 \times 9 \times 9 cm^3 = 972\pi\ cm^3$

Let the radius of wire be r cm

$= \pi r^2 l = \pi r^2 \times 10800$

But the volume of wire = volume of sphere

$\Rightarrow \pi r^2 \times 10800 = 972\pi$

$r^2 = \frac{972\pi}{10800\pi} = 0.09\ cm^2$

$r = \sqrt{0.09}$ cm = 0.3

Hence, the diameter = 2r = 0.6 cm

Question: 19

Solution:

Given,

Internal radius of hemispherical bowl (r_1) = 9 cm

Internal radius of cyclical vessel (r_2) = 6 cm

Let the height of water in the cylindrical vessel be h.

So,

Volume of hemispherical bowl = volume of cylindrical vessel

$$\frac{2}{3}\pi r_1^3 = \pi r_2^3 h$$

$$h = \frac{2}{3}\left(\frac{r_1^3}{r_2^3}\right)$$

$$h = \frac{2}{3} \times \frac{(9)^3}{(6)^2}$$

$$h = \frac{2 \times 729}{3 \times 36}$$

h = 13.5 cm

Hence, height of water in the cylindrical vessel is 13.5 cm

Question: 20

Solution:

Given,

Diameter of the hemispherical tank = 3 m

Radius of hemispherical tank = $\frac{3}{2}$ m

Volume of tank = $\frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{99}{14}$ m³

$= \frac{99000}{14}$ L ... (1m³ = 1000L)

Half the tank = $\frac{99000}{14} \times \frac{1}{2} = \frac{99000}{28}$ L

Now,

In 1 sec = $\frac{25}{7}$ L of water is emptied

Required time

$= \frac{\frac{99000}{28}}{\frac{25}{7}} = \frac{99000}{28} \times \frac{7}{25} = \frac{693000}{700}$

= 990 sec

So,

$= \frac{990}{60}$ = 16.5 min

To empty half the tank 16.5 min are required.

Question: 21

Solution:

Length of the roof = 44 m

Breadth of the roof = 20 m

Diameter of the cylindrical tank = 4 m

Radius of the cylindrical tank = 2 m

Height of the cylindrical tank = 3.5 m

Volume of roof = l x b x h= 44X20 X hvolume of cylinder = $\frac{22}{7}$x2x2x$\frac{35}{10}$

Height of roof = height of rainfallAndVolume of water on roof = Volume of Water in cylindrical tank44X 20 X h = $\frac{22}{7}$ × 2 x 2 x$\frac{35}{10}$44 X 20 X h = 22 × 2 × 2 × $\frac{5}{10}$

880 × h = 44 h = $\frac{44}{880} = \frac{1}{20}$h = .05 mh = 5 cm

Question: 22

Solution:

Given,

Length of the roof = 22 m

Breadth of the roof = 20 m

Diameter of the cylindrical vessel = 2 m

Radius of the cylindrical vessel = 1 m

Height of the cylindrical vessel = 3.5 m

$\frac{4}{5}$ Volume of the rainfall = area of the roof × depth of the rainfall

$\frac{4}{5}\pi r^2 h$ = area of the roof × d

$\frac{4}{5} \times 3.14 \times 1 \times 3.5 = 22 \times 20 \times d$

$\frac{4}{5} \times 11 = 440d$

8.8 = 440d

0.02 = d

d = 0.02 m

So,

The rainfall is 2 cm.

Question: 23

Solution:

Height of a cone = 60 cm,

Radius of a cone = 30 cmvolume of cone = $\frac{1}{3}\pi r^2 h$Height of cylinder = 180 cm,

Radius of cylinder = 60 cmVolume of cylinder = $\pi r^2 h$Volume of the remaining water = volume of cylinder - volume of cone

$= \pi \times 60 \times 60 \times 180 - \frac{1}{3} \times \pi \times 30 \times 30 \times 60$

$= 648000\pi - 1800\pi$ (Taking 1800 as common)

$= 18000(36\pi - \pi)$

$= 18000 \times 35 \times \frac{22}{7} = 1.98\ m^3$

Question: 24

Solution:

Given,

Diameter of the cylindrical pipe = 2 cmRadius of the cylindrical pipe = 1 mHeight of the cylindrical pipe = 0.4m/s = 40 cm/s

Water flown in 1 sec = 40 cmWater flown in 30 minutes (30 × 60 seconds) = 40 × 60 × 30 m = 72000 cm

Radius of cylindrical tank = 40 cmLet Height be h,As we can see the volume of water which passes through the cylindrical pipe is equal to the volume of water present in the cylindrical tank after half an hour.So,

Volume of water which passes through the cylindrical = volume of water present in the cylindrical tank after half an hour$\pi r^2 h = \pi r^2 h (1)^2 \times 72000 = (40)^2 \times h 72000 = 1600 \times h h = 45$ cm

Hence,

The rise in level of water in tank in half an hour will be 45 cm.

Question: 25

Solution:

Given,

Speed of the water flowing through the pipe, H = 6 km/hr

$= \frac{600000 \text{ cm}}{3600 \text{ s}} = \frac{500}{3} \text{ cm/s}$

Diameter of the pipe = 14 cm

Radius (R) of pipe = $\frac{14}{2}$ = 7 cm

Length (l) of the rectangular tank = 60 m = 6000 cm,

Breadth (b) of the rectangular tank = 22 m = 2200 cm and

Height (h) or Rise in the level of water in the tank = 7 cm

Now,

Volume of the water in rectangular tank = l × b × h = 6000 x 2200 x 7 = 92400000cm3

Also,

Volume of the water flowing through the pipe in 1 s = $\pi R^2 H$

$= \frac{22}{7} \times 7 \times \frac{500}{3} = \frac{77000}{3} \text{cm}^3$

So,

The time taken to rise the water = $\frac{\text{Volume of the water in the rectangular tank}}{\text{Volume of the water flowing through the pipe in 1 sec}}$

$= \left(\frac{92400000}{\frac{7700}{3}}\right) \text{cm}^3$

= 92400 x $\frac{3}{77}$ = 3600s = 1 hr

So,

In 1 hour time the level of water in the tank will rise by 7cm.

Question: 26

Solution:

Given,

Width of the canal = 6 m

Depth of the canal, = 1.5 m

Length of the cuboid = 666.67 m

Water is flowing at a speed of 4 km/hr = 4000m/hr

Thus,

Area irrigated in 10 minutes = $\frac{1}{6}$ hour = $\frac{1}{6}$ x 4000 = 666.67 m

Hence,

Volume of the water flowing in $\frac{1}{6}$ hour = Volume of the canal

⇒ Volume of the water flowing in $\frac{1}{6}$ hour = 666.67 x 6 x 1.5 = 6000.03 = 60000 m^3

Let a m^2 is the area irrigated in $\frac{1}{6}$ hour,

Then,

$\Rightarrow a \times \frac{8}{100} = 6000$

$\Rightarrow a = \frac{600000}{8} m^2 = 75000 m^2$

Question: 27

Solution:

Given,

Internal diameter of the pipe = 25 cm

Radius (r) of the pipe = $\frac{25}{2}$ cm $= \frac{1}{8}$ m

Diameter of the cylindrical tank = 12 m

Radius (R) of the tank = 6 m

Height (h) of the tank = 2.5 m

In 1 hour water comes out of the pipe = 3.6 km = 3600 m

Let the total hrs = n

Volume of the water coming out of the pipe in n hrs = volume of the cylindrical tank

$n \times \frac{22}{7} \times \frac{1}{8} \times \frac{1}{8} \times 3600 = \frac{22}{7} \times 6 \times 6 \times 2.5$

$n \times \frac{1}{8} \times \frac{1}{8} \times 3600 = 6 \times 6 \times 2.5$

$3600\, n = 36 \times 2.5 \times 8 \times 8$

$100\, n = 8 \times 8 \times 2.5$

$1000\, n = 8 \times 8 \times 25$

$n = \frac{16}{10} = 1.6$ hrs

n = 1 hrs 36 minutes

Now calculate the cost;

Cost of the water = Volume of the cylindrical tank × 0.07

$= \frac{22}{7} \times 6 \times 6 \times 2.5 \times 0.07 = 19.80$ rs

So,

Total time required to fill the tank is 1 hr 36 minutes and the cost is 19.80 rs.

Question: 28

Solution:

Given,

Diameter of the cylindrical pipe = 7 cm

Radius of the pipe = 3.5 cm

Volumetric flow rate = 192.5 l/min

Flow rate = $\frac{\text{Volumetric flow rate}}{\text{Area}}$

Area = pi × r^2 = 3.14 × 3.5 × 3.5 = 38.5 cm^2

Since 1 l = 1 dm^3 = 1000 cm^3

Volumetric flow rate = 192.5 × 1000 cm^3/min

So,

Flow rate = $\frac{192.5 \times 1000\ cm^3/min}{38.5\ cm^2}$ = 5000 cm/min

Flow rate in km/h = 5000 cm/min = 5000 × 0.00001 km/($\frac{1}{60}$ h) = 3 km/h

Question: 29

Solution:

Given,

Diameter of marble = 1.4 cm

Radius of marble = 0.7 cm

Number of marbles = 150

Diameter of cylinder = 7cm

Radius of cylinder = 3.5cm

150 × Volume of spherical marbles = volume of cylindrical vessel⇒ 150 x $\frac{4}{3}$ × π × 0.7 × 0.7 × 0.7 = π × 3.5 × 3.5 × h

⇒ 50 x 4 x $(0.7)^3 = (3.5)^2$ x h

⇒ 200 x 0.343 = 12.25 x h

⇒ $\frac{68.6}{12.25}$ = 5.6 = h

So,

h = 5.6 cm

The rise in the level of water in the vessel (h) = 5.6 cm

Question: 30

Solution:

Given,

Diameter of marble = 1.4 cm

Diameter of cylinder = 7 cm

Radius of cylinder = 3.5 cm

Cylinder height = 5.6 cm

Radius of marble = 0.7 cm

Volume of 1 marble = $\frac{4}{3}\pi r^3$

= $\frac{4}{3} \times \frac{22}{7} \times \frac{22}{7} \times \frac{22}{7}$ = 1.4373

So,

Let the number of marbles be n,

Increase in the Volume of cylinder due to n marbles = $\pi r^2 h$

= 3.14 × 3.5 × 3.5 × 5.6 = 215.6 cm^3

Hence the total numbers of marble required = $\frac{215.6}{1.4373}$ = 150 marbles

Question: 31

Solution:

Given,

Diameter of the well = 10 m

Radius of the well = 5 mHeight of the well = 14 mWidth of the embankment = 5 mTherefore radius of the embankment = 5 + 5 = 10 mLet h be the height of the embankment,

Hence,

The volume of the embankment = volume of the well

$\pi(R - r)^2h = \pi r^2h(10^2 - 5^2)$ x h = 5^2 x 14(100 − 25) x h = 25 x 14h = $\frac{25 \times 14}{75} = \frac{14}{3}$Therefore,

The height of the embankment, h = 4.67 m

Question: 32

In a corner of a

Solution:

Given,

Diameter of the well = 14 m

Radius of the well = 7 m

Height of the well = 8 m

Now,

Volume of the earth dug out of the well = πr^2h

= $\frac{22}{7} \times 7 \times 7 \times 8 = 1{,}232\ m^3$

Area on which earth dug out is spread = $l \times b - r^2h$

= $35 \times 22 - \frac{22}{7} \times 7 \times 7$

= 770 - 154 = 616 m

Level of the earth raised = $\frac{1232}{616}$ = 2 m

So, the rise in the level of the field is 2 m.

Question: 33

Solution:

Given,

Diameter of the copper wire = 6 mm = 0.6 cm

Radius of the copper wire = 0.3 cm

Length of the cylinder = 18 cm

Diameter of the cylinder = 49 cm

Radius of the cylinder = $\frac{49}{2}$ cm

Density of the copper = 8.8g

Number of the rotations on the cylinder = $\frac{18}{0.6}$ = 30 cm

Base circumference of the cylinder = $2\pi r = 2 \times \frac{22}{7} \times \frac{49}{2} = 154$ cm

So,

Length of the wire = 154 × 30 = 4620 cm

Volume of the wire = $\pi r^2 h$

$= \frac{22}{7} \times 0.3 \times 0.3 \times 4620 = 1306.8$

Weight of the wire = volume × density

= 1306.8 × 8.8 = 11,499.84 gm

Question: 34

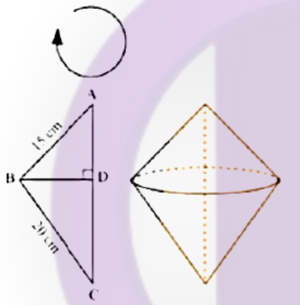
Solution:

Let name the triangle, ABC

Given,

Sides of the triangle are 15 and 20 cm,

BD ⊥ AC.



In the given case BD is the radius of the double cone generated by triangle by revolving.

Now by Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = (15)^2 + (20)^2$

$AC^2 = 225 + 400$

$AC^2 = 625 = (25)^2$

AC = 25 cm

Let AD be m cm;

∴ CD = (25 – m) cm

By using the Pythagoras theorem in ΔABD and ΔCBD;

ΔABD,

$AD^2 + BD^2 = AB^2$

$m^2 + BD^2 = 225$

$BD^2 = 225 - m^2$ ………… (i)

ΔCBD,

$BD^2 + CD^2 = BC^2$

$BD^2 + (25 - m)^2 = (20)^2$

$BD^2 = (20)^2 - (25 - m)^2$ .....(ii)

By putting both the equations (i) and (ii) together,

$225 - m^2 = (20)^2 - (625 - m)^2$

$225 - m^2 = 400 - (625 + m^2 - 50m)$ ....by $(a^2 - b^2)$

$225 - m^2 = - 225 - m^2 + 50m$

$225 - m^2 + 225 + m^2 = 50m$

$450 = 50m$

So,

$m = \frac{450}{50} = 9$ cm

$BD^2 = (15)^2 - (9)^2$

$BD^2 = 225 - 81 = 144$ cm

BD = 12 cm

Radius of the generated double cone = 12 cm

Now,

Volume of the cone generated = Volume of the upper cone + Volume of the lower cone

$\Rightarrow \frac{1}{3}\pi \times BD^2 \times AD + \frac{1}{3}\pi \times BD^2 \times CD$

$\Rightarrow \frac{1}{3}\pi \times BD^2 \times (AD + CD)$

$\Rightarrow \frac{1}{3}\pi\ (12)^2 \times (25)$

$= 1200\pi\ cm^3 = 3{,}771.42\ cm^3$

Surface area of the double cone formed;

= L.S.A of upper cone + L.S.A of the lower cone

$= \pi\ (BD) \times (AB) + \pi\ (BD) \times (BC)$

$= \pi \times 12cm \times 15\ cm + \pi \times 12\ cm \times 20\ cm$

$= 420\pi\ cm^2 = 1320\ cm^2$

So, the volume is $1200\pi\ cm^3$ and surface area is $420\pi\ cm^2$, of the double cone so formed.

Exercise : 19C

Question: 1

Solution:

Given: Height of glass = h = 14 cm

Diameter of lower circular end of glass = 12 cm

Diameter of upper circular end of glass = 16 cm

∴ Radius of lower circular end = r = 12/2 = 6 cm

∴ Radius of lower circular end = R = 16/2 = 8 cm

∴ Capacity of drinking glass $= \frac{1}{3} \times \frac{22}{7} \times 14 \times (8^2 + 6^2 + 8 \times 6)$ cm3

$= \frac{44}{3} \times (64 + 36 + 48)\ cm^3 = \frac{44 \times 148}{3}\ cm^3$

$= 44 \times 49.33\ cm^3$

$= 2170.52\ cm^3$

∴ Capacity of glass = $2170.52\ cm^3$

Question: 2

Solution:

Given: Radius of lower circular end = r = 12 cm

Radius of upper circular end = R = 18 cm

Height of frustum = h = 8 cm

Formula: Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l\ cm^2$

Where l = slant height

For slant height we have $l = \sqrt{(R-r)^2 + h^2}$ cm

$\therefore l = \sqrt{(18-12)^2 + 8^2} = \sqrt{36 + 64} = 10$ cm

∴ l = 10 cm

∴ total surface area of frustum = $\pi \times 12^2 + \pi \times 18^2 + \pi \times (18 + 12) \times 10\ cm^2$

$= 3.14 \times (144 + 324 + 300)\ cm^2$

$= 3.14 \times 768\ cm^2$

$= 2411.52\ cm^2$

∴ total surface area of frustum = $2411.52\ cm^2$

Question: 3

Solution:

Given: Height of bucket = h = 24 cm

Radius of lower circular end = r = 7 cm

Radius of upper circular end = R = 14 cm

Formula: Volume of frustum of cone $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)\ cm^3$

Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l\ cm^2$

Where l = slant height

For slant height we have $l = \sqrt{(R-r)^2 + h^2}$ cm

$\therefore l = \sqrt{(14-7)^2 + 24^2} = \sqrt{49 + 576} = 25$ cm

∴ l = 25 cm

(i) volume of water which will completely fill the bucket = volume of frustum

∴ Volume of frustum of cone $= \frac{1}{3}\pi \times 24 \times (14^2 + 7^2 + 14 \times 7)$

$= 8 \times \frac{22}{7} \times (343)$

= 8 × 22 × 49

= 8722 cm^3

∴ volume of water which will completely fill the bucket = 8722 cm^3

(ii) area of metal sheet used

Since the top is open we need to subtract the area of top/upper circle from total surface area of frustum because we don't require a metal plate for top.

Radius of top/upper circle = R

Area of upper circle = πR^2

∴ area of metal sheet used = (total surface area of frustum)-πR^2

∴ Area of metal sheet used = $\pi r^2 + \pi R^2 + \pi(R + r)l - \pi R^2 cm^2$

= $\pi r^2 + \pi(R + r)l$ cm^2

$= \frac{22}{7} \times (7^2 + (7 + 14) \times 25) = \frac{22}{7} \times (574)\ cm^2$

= 22 × 82 cm^2

= 1804 cm^2

∴ Area of metal sheet used to make bucket = 1804 cm^2

Question: 4

Solution:

Given: height of frustum container = h = 24 cm

Radius of lower circular end = r = 8 cm

Radius of upper circular end = R = 20 cm

Cost of 1 litre milk = 24 Rs

Formula: Volume of frustum of cone $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)\ cm^3$

Volume of milk completely fill the container = volume of frustum of cone

$= \frac{1}{3} \times \pi \times 24 \times (20^2 + 8^2 + 20 \times 8)\ cm^3$

= 8 × 3.14 × (400 + 64 + 160) cm^3

= 8 × 3.14 × 624 cm^3

= 15674.88 cm^3

Now, 1 litre is 1000 cm^3

∴ 15674.88 cm^3 = 15674.88/1000 = 15.67488 litres

∴ Cost of milk which can completely fill the container = 15.67488 × cost of 1 litre milk

= 15.67488 × 24 Rs

= 376.19712 Rs

∴ Cost of milk which can completely fill the container = 376.19712 Rs

Question: 5

Solution:

Given: height of container frustum = h = 16 cm

Diameter of lower circular end = 16 cm

Diameter of upper circular end = 14 cm

∴ Radius of lower circular end = r = 16/2 = 8 cm

∴ Radius of upper circular end = R = 14/2 = 7 cm

Cost of 100 cm^2 metal sheet = 10 Rs

∴ Cost of 1 cm^2 metal sheet = 10/100 = 0.1 Rs

Formula: Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l\ cm^2$

Where l = slant height

For slant height we have $l = \sqrt{(R-r)^2 + h^2}$ cm

$\therefore\ l = \sqrt{(7-8)^2 + 16^2} = \sqrt{1 + 256} = 16.0312$ cm

∴ l = 16.0312 cm

Since the top is open we need to subtract the area of top/upper circle from total surface area of frustum because we don't require a metal plate for top.

Radius of top/upper circle = R

Area of upper circle = πR^2

∴ area of metal sheet used = (total surface area of frustum)-πR^2

$= \pi r^2 + \pi R^2 + \pi(R + r)l - \pi R^2\ cm^2$

$= \pi r^2 + \pi(R + r)l\ cm^2$

$= \pi \times (8^2 + (7 + 8)16.0312)\ cm^2$

$= 3.14 \times 304.468\ cm^2$

$= 956.029\ cm^2$

∴ 956.029 cm^2 metal sheet is required to make the container.

∴ Cost of 956.029 cm^2 metal sheet = 956.029 × cost of 1 cm^2 metal sheet

= 956.029 × 0.1 Rs

= 95.6029 Rs

∴ Cost of metal sheet required to make container = 95.6029 Rs

Question: 6

Solution:

Given: Radius of lower circular end = r = 27 cm

Radius of upper circular end = R = 33 cm

Slant height = l = 10cm

Formula: Volume of frustum of cone $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)\ cm^3$

Total surface area of frustum $= \pi r^2 + \pi R^2 + \pi(R + r)l\ cm^2$

Here h height of frustum is not given and we need h to find the volume of frustum therefore we must first calculate the value of h as follows

slant height $= l = \sqrt{(R - r)^2 + h^2}$ cm

using formula for slant height and with the help of given data we get

$10 = \sqrt{(33 - 27)^2 + h^2}$

Squaring both sides

$\therefore 100 = 36 + h^2$

$\therefore h^2 = 64$

$\therefore h = \pm 8$

As length cannot be negative

$\therefore h = 8$ cm

Volume of frustum of cone $= \frac{1}{3} \times \frac{22}{7} \times 8 \times (33^2 + 27^2 + 33 \times 27)$

$= \frac{22}{21} \times 8 \times (1089 + 729 + 891)$

$= 22 \times 8 \times 129$

$= 22704\ cm^3$

$\therefore$ capacity = volume of frustum = 22704 cm^3

Total surface area of frustum $= \pi r^2 + \pi R^2 + \pi(R + r)l\ cm^2$

$= \frac{22}{7} \times (33^2 + 27^2 + (33 + 27) \times 10)$

$= (22/7) \times (1089 + 729 + 600)$

$= (22/7) \times 2418$

$= 7599.428\ cm^2$

$\therefore$ total surface area = 7599.428 cm^2

Question: 7

Solution:

Given: Depth of the bucket = height of frustum = h = 15 cm

Diameter of top of bucket = 56 cm

Diameter of bottom of bucket = 42 cm

$\therefore$ Radius of top = R = 56/2 = 28 cm

$\therefore$ Radius of bottom = r = 42/2 = 21 cm

Formula: Volume of frustum of cone $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)\ cm^3$

Volume of water bucket can hold = volume of bucket which is in form of frustum

∴ volume of water bucket can hold

$$= \frac{1}{3} \times \frac{22}{7} \times 15 \times (28^2 + 21^2 + 28 \times 21)\ cm^3$$

$= (22/7) \times 5 \times (784 + 441 + 588)\ cm^3$

$= (22/7) \times 5 \times 1813\ cm^3$

$= 22 \times 5 \times 259\ cm^3$

$= 28490\ cm^3$

Now 1 litre = $1000\ cm^3$

∴ $28490\ cm^3 = 28490/1000$ litres

= 28.49 litres

∴ bucket can hold 28.49 litres of water

Question: 8

Solution:

Given: height of container frustum = h = 16 cm

Radius of lower circular end = r = 8 cm

Radius of upper circular end = R = 20 cm

Cost of $100\ cm^2$ metal sheet = 15 Rs

∴ Cost of $1\ cm^2$ metal sheet = 15/100 = 0.15 Rs

Formula: Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l\ cm^2$

Where l = slant height

For slant height we have $l = \sqrt{(R - r)^2 + h^2}$ cm

∴ $l = \sqrt{(20 - 8)^2 + 16^2} = \sqrt{144 + 256} = 20$ cm

∴ l = 20 cm

Since it is given that a bucket is to be made hence the top is open we need to subtract the area of top/upper circle from total surface area of frustum because we don't require a metal plate for top.

Radius of top/upper circle = R

Area of upper circle = πR^2

∴ area of metal sheet used = (total surface area of frustum)-πR^2

$= \pi r^2 + \pi R^2 + \pi(R + r)l - \pi R^2\ cm^2$

$= \pi r^2 + \pi(R + r)l\ cm^2$

$= \pi \times (8^2 + (20 + 8)20)\ cm^2$

$= 3.14 \times 624\ cm^2$

$= 1959.36\ cm^2$

∴ $1959.36\ cm^2$ metal sheet is required to make the container.

∴ Cost of $1959.36\ cm^2$ metal sheet = 1959.36 × cost of $1\ cm^2$ metal sheet

= 1959.36 × 0.15 Rs

= 293.904 Rs

∴ Cost of metal sheet required to make container = 293.904 Rs

Question: 9

Solution:

Given: depth of bucket = height of bucket/frustum = h = 24 cm

Diameter of lower circular end = 10 cm

Diameter of upper circular end = 30 cm

∴ Radius of lower circular end = r = 10/2 = 5 cm

∴ Radius of lower circular end = R = 30/2 = 15 cm

Cost of 100 cm^2 metal sheet = 10 Rs

∴ Cost of 1 cm^2 metal sheet = 10/100 = 0.1 Rs

Cost of 1 litre milk = 20 Rs

Formula: Volume of frustum of cone $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)\ cm^3$

Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l\ cm^2$

Where l = slant height

For slant height we have $l = \sqrt{(R - r)^2 + h^2}\ cm$

$\therefore\ l = \sqrt{(15 - 5)^2 + 24^2} = \sqrt{100 + 576} = 26\ cm$

∴ l = 26 cm

Since the top is open we need to subtract the area of top/upper circle from total surface area of frustum because we don't require a metal plate for top.

Radius of top/upper circle = R

Area of upper circle = πR^2

∴ area of metal sheet used = (total surface area of frustum)-πR^2

$= \pi r^2 + \pi R^2 + \pi(R + r)l - \pi R^2\ cm^2$

$= \pi r^2 + \pi(R + r)l\ cm^2$

$= \pi \times (5^2 + (15 + 5)26)\ cm^2$

$= 3.14 \times 545\ cm^2$

$= 1711.3\ cm^2$

∴ 1711.3 cm^2 metal sheet is required to make the container.

∴ Cost of 1711.3 cm^2 metal sheet = 1711.3 × cost of 1 cm^2 metal sheet

= 1711.3 × 0.1 Rs

= 171.13 Rs

∴ Cost of metal sheet required to make container = 171.13 Rs

Now,

Volume of milk which can completely fill the bucket = volume of frustum

$\therefore$ volume of milk $= \frac{1}{3} \times 3.14 \times 26 \times (15^2 + 5^2 + 15 \times 5)\ cm^3$

$= (1/3) \times 3.14 \times 26 \times 325\ cm^3$

= 26533/3 cm^3

= 8844.33 cm^3

Now 1 litre = 1000 cm^3

∴ 8844.33 cm^3 = 8844.33/1000 litres

= 8.84433 litres

∴ Volume of milk which can completely fill the bucket = 8.84433 litres

∴ Cost of milk which can completely fill the bucket = volume of milk which can completely fill the bucket × cost of 1 litre milk Rs

= 8.84433 × 20 Rs

= 176.8866 Rs

Cost of milk which can completely fill the bucket = 176.8866 Rs

Question: 10

Solution:

Given: height of container/frustum = h = 14 cm

Diameter of top of container = 35 cm

Diameter of bottom of container = 30 cm

∴ Radius of top = R = 35/2 = 17.5 cm

∴ Radius of bottom = r = 30/2 = 15 cm

1 cm^3 of oil = 1.2g of oil

Cost of 1 kg oil = 40 Rs

Formula: Volume of frustum of cone $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)\ cm^3$

Volume of oil in container = volume of container which is in form of frustum

∴ volume of oil in container

$$= \frac{1}{3} \times \frac{22}{7} \times 14 \times (17.5^2 + 15^2 + 17.5 \times 15)\ cm^3$$

= (22/3) × 2 × (306.25 + 225 + 262.5) cm^3

= (22/3) × 2 × 793.75 cm^3

= 34925/3 cm^3

= 11641.667 cm^3

∴ Volume of oil in container = 11641.667 cm^3

∴ 11641.667 cm^3 of oil = 11641.667 × 1.2 g

= 13970.0004 g

We know 1000 g = 1 kg

∴ 13970.0004 g = 13970.0004/1000 kg

= 13.970 kg

∴ Cost of 13.970 kg oil = 13.970 × cost of 1 kg oil Rs

= 13.970 × 40 Rs

= 558.8 Rs

∴ Cost of oil in container = 558.8 Rs

Question: 11

Solution:

Given: volume of bucket = 28.49 litres

1 litre = 1000 cm^3

∴ 28.49 litres = 28.49 × 1000 cm^3

∴ Volume of bucket = 28490 cm^3

Radius of upper circular end = R = 28 cm

Radius of lower circular end = r = 21 cm

Let 'h' be the height of the bucket

Formula: Volume of frustum of cone $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$ cm^3

Volume of bucket = volume of frustum of cone

$\therefore 28490 = \frac{1}{3} \times \frac{22}{7} \times h \times (28^2 + 21^2 + 28 \times 21)$

∴ 28490 × 21 = h × 22 × (784 + 441 + 588)

∴ h = 598290/39886 cm

∴ h = 15 cm

∴ Height of bucket = h = 15 cm

Question: 12

Solution:

Given: volume of bucket = 5390 cm^3

Radius of upper circular end = R = 14 cm

Radius of lower circular end = r cm & r is less than 14

Height of bucket = h = 15

Formula: Volume of frustum of cone $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$ cm^3

Volume of bucket = volume of frustum of cone

$\therefore 5390 = \frac{1}{3} \times \frac{22}{7} \times 15 \times (14^2 + r^2 + 14 \times r)$

∴ 5390 × 7 = 22 × 5 × (196 + r^2 + 14r)

∴ 37730/110 = 196 + r^2 + 14r

∴ 343 = 196 + r^2 + 14r

∴ r^2 + 14r-147 = 0

∴ r^2 + 21r-7r-147 = 0

∴ r(r + 21)-7(r + 21) = 0

∴ (r-7)(r + 21) = 0

∴ r = 7 or r = -21

Since we require r<14 ∴ r = 7 cm

Question: 13

Solution:

Given: Radius of lower circular end = r = 27 cm

Radius of upper circular end = R = 33 cm

Slant height = l = 10cm

Formula: Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l$ cm^2

$= 3.14 \times (33^2 + 27^2 + (33 + 27) \times 10)$ cm^2

$= 3.14 \times (1089 + 729 + 600)$ cm^2

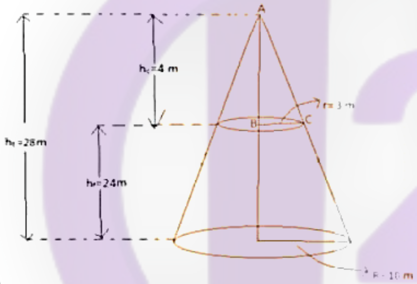
$= 3.14 \times 2418$ cm^2

$= 7592.52$ cm^2

∴ total surface area = 7592.52 cm^2

Question: 14

Solution:



Given: Diameter of base of frustum = 20 m

Diameter of top of frustum = 6 m

∴ Radius of base = R = 20/2 = 10 m

∴ Radius of top = r = 6/2 = 3 m

Height of frustum = h_f = 24 m

Height of tent = h_t = 28 m

∴ height of cone = $h_c = h_t - h_f$ = 28-24 = 4 m

Formula: Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l_f$ m^2

Total surface area of cone = $\pi r l_c$

Where l_f = slant height of frustum & l_c = slant height of cone

For slant height of frustum we have $l_f = \sqrt{(R - r)^2 + {h_f}^2}$ m

$\therefore l_f = \sqrt{(10 - 3)^2 + 24^2} = \sqrt{49 + 576} = 25$ m

∴ l_f = 25 m

For slant height of cone consider right angled ΔABC from figure

AB = h_c = 4 m ; BC = r = 3 m ; AC = l_c

By pythagoras theorm

$AB^2 + BC^2 = AC^2$

∴ $4^2 + 3^2 = l_c^2$

∴ $l_c = \pm 5$

Since length cannot be negative l_c = 5 m

Since for tent we don't require the upper circle of frustum and the lower circle of frustum hence we should subtract their area as we don't require canvas for that.

Surface area of upper circle = πr^2

Surface area of lower circle = πR^2

∴ Surface area of frustum for which canvas is required = $\pi r^2 + \pi R^2 + \pi(R + r)l_f - \pi r^2 - \pi R^2 cm^2$

= $\pi(R + r)l_f m^2$

= (22/7) × (10 + 3) × 25 m^2

= (22/7) × 325 m^2

= 1021.4285 m^2

Surface area of cone = $\pi r l_c$ m^2

= (22/7) × 3 × 5 m^2

= (22/7) × 15 m^2

= 47.1428 m^2

∴ Quantity of canvas required = surface area of frustum + surface area of cone

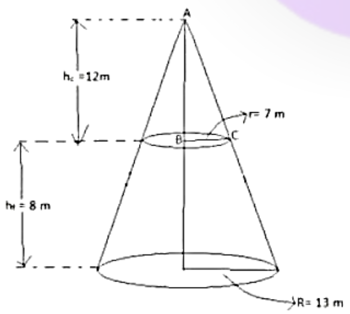
= 1021.4285 + 47.1428 m^2

= 1068.5713 m^2

∴ Quantity of canvas required = 1068.5713 m^2

Question: 15

Solution:



Given: Diameter of base of frustum = 26 m

Diameter of top of frustum = 14 m

∴ Radius of base = R = 26/2 = 13 m

∴ Radius of top = r = 14/2 = 7 m

Height of frustum = h_f = 8 m

∴ height of cone = h_c = 12 m

Formula: Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l_f\, m^2$

Total surface area of cone = $\pi r l_c$

Where l_f = slant height of frustum & l_c = slant height of cone

For slant height of frustum we have $l_f = \sqrt{(R - r)^2 + {h_f}^2}$ m

$\therefore\ l_f = \sqrt{(13 - 7)^2 + 8^2} = \sqrt{36 + 64} = 10$ m

∴ l_f = 10 m

Consider right angled ΔABC from figure

AB = h_c = 12 m ; BC = r = 7 m ; AC = l_c

By pythagoras theorm

$AB^2 + BC^2 = AC^2$

$\therefore\ 12^2 + 7^2 = {l_c}^2$

∴ l_c = ±13.892

Since length cannot be negative l_c = 13.892 m

Since for tent we don't require the upper circle of frustum and the lower circle of frustum hence we should subtract their area as we don't require canvas for that.

Surface area of upper circle = πr^2

Surface area of lower circle = πR^2

∴ Surface area of frustum for which canvas is required = $\pi r^2 + \pi R^2 + \pi(R + r)l_f - \pi r^2 - \pi R^2 cm^2$

$= \pi(R + r)l_f m^2$

= 3.14 × (13 + 7) × 10 m^2

= 3.14 × 200 m^2

= 628 m^2

Surface area of cone = $\pi r l_c\, m^2$

= 3.14 × 7 × 13.892 m^2

= 3.14 × 97.244 m^2

= 305.346 m^2

∴ Quantity of canvas required = surface area of frustum + surface area of cone

= 628 + 305.346 m^2

= 933.346 m^2

∴ Quantity of canvas required = 933.346 m^2

Question: 16

Solution:

Given: perimeter of upper circle = 36 cm

Perimeter of lower circle = 48 cm

Height of frustum = h = 11 cm

Let r: radius of upper circle & R: radius of lower circle

Formula: Volume of frustum of cone $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$ cm^3

Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l$ cm^2

Where l = slant height

For slant height we have $l = \sqrt{(R-r)^2 + h^2}$ cm

Now perimeter of circle = circumference of circle = 2π × radius

∴ Perimeter of upper circle = 2πr

∴ 36 = 2 × 3.14 × r

∴ r = 36/6.28 cm

∴ r = 5.732 cm

∴ Perimeter of lower circle = 2πR

∴ 48 = 2 × 3.14 × R

∴ R = 48/6.28 cm

∴ R = 7.643 cm

$\therefore l = \sqrt{(7.643 - 5.732)^2 + 11^2} = \sqrt{3.651 + 121} = 11.164$ cm

∴ l = 11.164 cm

∴ Volume of frustum = (1/3) × 3.14 × 11 × ($7.643^2 + 5.732^2$ + 7.643 × 5.732) cm^3

= (1/3) × 34.54 × (58.415 + 32.855 + 43.809) cm^3

= 11.513 × 135.079 cm^3

= 1555.164 cm^3

∴ Volume of frustum = 1555.164 cm^3

Now we have asked curved surface area so we should subtract the top and bottom surface areas which are flat circles.

Surface area of top = πr^2

Surface area of bottom = πR^2

∴ Curved surface area = total surface area - πr^2 - πR^2 cm^2

= $\pi r^2 + \pi R^2 + \pi(R + r)l - \pi r^2 - \pi R^2$ cm^2

= $\pi(R + r)l$ cm^2

= 3.14 × (7.643 + 5.732) × 11.164 cm^2

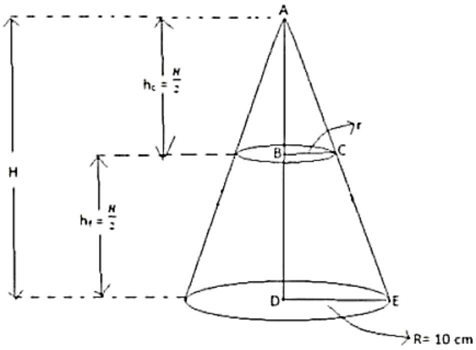
= 3.14 × 13.375 × 11.164 cm^2

= 468.86 cm^2

∴ curved surface area = 468.86 cm^2

Question: 17

Solution:



Let 'H' be the height of cone 'R' be the radius of base of cone.

R = 10 cm

When the cone is cut at midpoint of H by a plane parallel to its base we get a cone of height H/2 and a frustum also of height H/2

Let the radius of the base of the cone which we got after cutting and the radius of upper circle of frustum be 'r' as shown in figure

From figure consider ΔABC and ΔADE

∠ABC = ∠ADE = 90°

∠BAC = ∠DAE ...(common angle)

as two angles are equal by AA criteria we can say that

ΔABC~ΔADE

$$\therefore \frac{BC}{DE} = \frac{AB}{AD} \Rightarrow \frac{r}{10} = \frac{(H/2)}{(H)} \Rightarrow \frac{r}{10} = \frac{H}{2H}$$

∴ r = 5 cm

Let V_c be volume of the cone and V_f be the volume of frustum

Volume of cone = $(1/3)\pi(\text{radius})^2(\text{height})$ cm^3

$\therefore V_c = (1/3) \times \pi \times r^2 \times (H/2)$ cm^3

$= (1/3) \times \pi \times 5^2 \times (H/2)$ cm^3

$= (1/3) \times \pi \times 25 \times (H/2)$ cm^3

Volume of frustum = $(1/3)\pi h(R^2 + r^2 + Rr)$ cm^3

$\therefore V_f = (1/3) \times \pi \times (H/2)(10^2 + 5^2 + 10 \times 5)$ cm^3

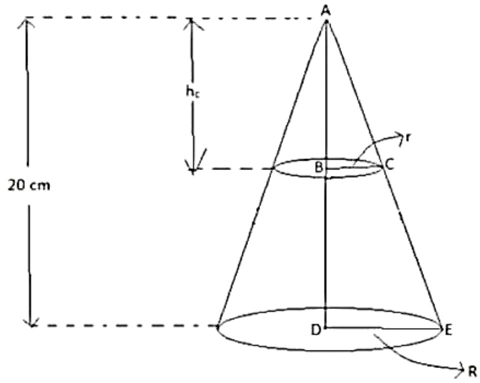
$= (1/3) \times \pi \times (H/2) \times 175$ cm^3

$$\text{Ratio} = \frac{V_c}{V_f} = \frac{(1/3) \times \pi \times 25 \times (H/2)}{(1/3) \times \pi \times (H/2) \times 175} = \frac{1}{7}$$

∴ The ratio of volumes of two parts after cutting = V_c:V_f = 1:7

Question: 18

Solution:



Let the cutting plane be passing through points B and C as shown

Height of cone = AD = H = 20 cm

Height of small cone which we get after cutting = AB = h_c

Let 'r' be the radius of small cone ∴ we have BC = r

'R' be radius of original cone which is to be cut ∴ we have DE = R

From figure consider ΔABC and ΔADE

∠ABC = ∠ADE = 90°

∠BAC = ∠DAE ...(common angle)

as two angles are equal by AA criteria we can say that

ΔABC~ΔADE

$$\therefore \frac{BC}{DE} = \frac{AB}{AD} \Rightarrow \frac{r}{R} = \frac{h_c}{20} \ldots (i)$$

Let V_1 be the volume of cone to be cut

Let V_2 be the volume of small cone which we get after cutting

Volume of cone = $(1/3)\pi(\text{radius})^2(\text{height})\ cm^3$

$\therefore V_1 = (1/3) \times \pi \times R^2 \times h_c$

$\therefore V_2 = (1/3) \times \pi \times r^2 \times 20$

Given is that the volume of small cone is (1/8) times the original cone

$\therefore V_2 = (1/8)\ V_1$

$\therefore (1/3) \times \pi \times r^2 \times 20 = (1/8) \times (1/3) \times \pi \times R^2 \times h_c$

$$\therefore \frac{r^2}{R^2} = \frac{(1/8) \times (1/3) \times \pi \times h_c}{(1/3) \times \pi \times 20}$$

Using equation (i) we get

$$\therefore \frac{{h_c}^2}{20^2} = \frac{(1/8) \times 20}{h_c}$$

$\therefore {h_c}^3 = 20^3/8$ cm

$\therefore h_c = 20/2$ cm

$\therefore h_c = 10$ cm

But we have to find the height from base i.e. we have to find BD from figure

∴ $20 = BD + h_c$

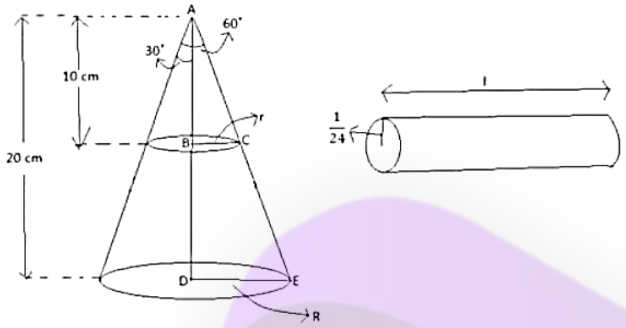
∴ $20 = BD + 10$

∴ BD = 10 cm

∴ 10 cm above base the section is made.

Question: 19

Solution:



Let 'R' be the radius of the base of the cone which is also the base of frustum i.e. lower circular end as shown in the figure

DE = R

Let 'r' be the radius of the upper circular end of frustum which we get after cutting the cone

BC = r

The height of the cone is 20 cm and we had cut the cone at midpoint therefore height of the frustum so obtained is 10 cm

Vertical angle as shown in the figure is 60°

Now a wire of diameter 1/12 (i.e. radius 1/24) is made out of the frustum let 'l' be the length of the wire

As we are using the full frustum to make wire therefore volumes of both the frustum and the wire must be equal.

∴ Volume of frustum = volume of wire made ...(i)

Consider ΔABC

∠BAC = 30° ; AB = 10 cm ; BC = r

$$\text{we have } \tan 30 = \frac{BC}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{r}{10}$$

∴ $r = 10/\sqrt{3}$ cm

Consider ΔADE

∠DAE = 30° ; AD = 20 cm ; DE = R

$$\text{we have } \tan 30 = \frac{DE}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{R}{20}$$

∴ $R = 20/\sqrt{3}$ cm

Now using equation (i)

$$\therefore \frac{1}{3} \times \pi \times h \times (R^2 + r^2 + Rr) = \pi \times \left(\frac{1}{24}\right)^2 \times l$$

$\therefore \frac{1}{3} \times 10 \times \left[\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{20}{\sqrt{3}} \times \frac{10}{\sqrt{3}}\right)\right] = \frac{1}{576} \times l$

$\therefore \frac{10}{9} \times (400 + 100 + 200) = \frac{l}{576}$

∴ 7000/9 = l/576

∴ 777.778 = l/576

∴ l = 448000 cm

Length of wire = 448000 cm

Question: 20

Solution:

Given: Radius of lower circular end = R = 10 cm

Radius of upper circular end = r = 4 cm

Slant height = l = 15cm

Formula: Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l$ cm^2

As the lower circular end is open we need to subtract the area of lower circular end from total surface area since we don't require material for lower circular end it should be open so that it is wearable.

∴ Total surface area = $\pi r^2 + \pi R^2 + \pi(R + r)l - \pi R^2$ cm^2

= (22/7) × (4^2 + (10 + 4) × 15) cm^2

= (22/7) × (16 + 210) cm^2

= 22 × 226/7 cm^2

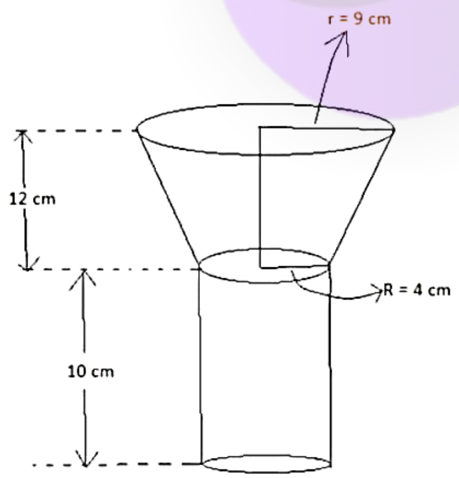
= 710.285 cm^2

∴ total surface area = 710.285 cm^2

∴ Area of material used = 710.285 cm^2

Question: 21

Solution:



Divide the funnel into two parts frustum and cylinder as shown in the figure

Parameters of frustum:

Diameter of upper circular end = 18 cm

∴ Radius of upper circular end = r = 18/2 = 9 cm

The radius of cylinder is equal to the radius of lower circular end of frustum

∴ radius of lower circular end = R = 4 cm

Height of frustum = total height – height of cylinder

= 22 – 10

= 12 cm

∴ height of frustum = h = 12 cm

Total surface area of frustum = $\pi r^2 + \pi R^2 + \pi(R + r)l$ cm^2

Where l = slant height

For slant height we have $l = \sqrt{(R - r)^2 + h^2}$ cm

$\therefore l = \sqrt{(4 - 9)^2 + 12^2} = \sqrt{25 + 144} = 13$ cm

∴ l = 13 cm

Since for the frustum part of the funnel we don't require the upper circular end and the lower circular end hence we need to subtract those areas from total surface area.

Area of upper circular end = πr^2

Area of lower circular end = πR^2

total surface area = $\pi r^2 + \pi R^2 + \pi(R + r)l - \pi r^2 - \pi R^2$

= $\pi(R + r)l$

= 3.14 × (9 + 4) × 13

= 530.66 cm²

∴ total surface area of frustum for which tin is required = 530.66 cm²

Parameters of cylinder:

Height of cylinder = 10 cm

Radius of cylinder = 4 cm

∴ Area of tin require to make cylinder = 2π × (radius) × (height)

= 2 × 3.14 × 4 × 10

= 251.2 cm²

∴ Area of tin required to make the funnel = area of frustum for which tin is required + area of tin require to make cylinder

∴ area of tin required to make funnel = 530.66 + 251.2

= 781.86 cm²

∴ Area of tin sheet require to make the funnel = 781.86 cm²

Exercise : 19D

Question: 1

Solution:

Given,

Depth of the river = 1.5 m

Width of the river = 36 m

Flow rate of river = 3.5 km/hr

Now first change the rate of flow of the water in meter/min

As we know,

1 km = 1000 m

1 hour = 60 minutes

So,

$$3.5 \text{ km/r} = \frac{3.5 \text{ km}}{1 \text{ hour}}$$

$$= \frac{3.5 \times 1000 \text{ m}}{1 \times 60 \text{ min}}$$

$$= \frac{350}{6} \text{ m/min}$$

River travel in a minute = 350/6 m

Now,

The amount of water that runs into sea per minute;

350/6 × 1.5 × 36 = 350 × 1.5 × 6 = 3150

So,

The amount of water that runs into the sea per minute is 3150 m^3

Question: 2

Solution:

Given,

Volume of the cube = 729 cm^3

Let the edge of the cube = a cm

So,

Volume (v) of the cube = $a^3a^3 = 729a^3 = (9cm)^3a = 9$ cm

Lateral surface area of cube = $4a^2$= 4 × 92= 4 × 81= 324cm^2Total surface area of the cube = $6a^2$= 6 × 9^2= 6 × 81= 486 cm^2

Question: 3

Solution:

Given,

Edge of the Cubical Box = 1 m

Volume of the Cubical Box = a^3

Edge of the cubes = 10 cm

Volume of the cube = a^3

Number of Cubes = Volume of box/volume of the cube= 100 × 100 × 100/10 × 10 × 10=

1000000/1000= 1000 cubes

Question: 4

Solution:

Given,

Edge of the first cube = 6 cm

Volume of the first cube = $a^3 = (6)^3$ cm

Edge of the second cube = 8 cm

Volume of the second cube = $a^3 = (8)^3$ cm

Edge of the third cube = 10 cm

Volume of the third cube = $a^3 = (10)^3$ cm

So,

Volume of the formed cube = Volume of the First + Second + Third Cube

Volume of the formed cube = v1 + v2 + v3= $6^3 + 8^3 + 10^3$= 216 + 512 + 1000= 1728Now volume of new cube = a^3= 1728Edge of new cube = a = $\sqrt[3]{1728}$a = 12Therefore surface area of new cube = $6a^2$= 6×12^2= 6 × 12 × 12= 864 cm^2

Question: 5

Solution:

Given,

Edge of the given Cube = 5 cm

Now,

Length (l) of the resulting cuboid = Edge × Number of cubes

= 5 x 5 = 25 cm

Breadth (b) of the resulting cuboid = 5 cm

Height (h) of the resulting cuboid = 5 cm

So,

Volume of the resulting cuboid = l × b × h

= 25 x 5 x 5

= 625 cm^3

Hence,

The volume of the resulting cube is 625 cm^3

Question: 6

Solution:

Given,

Ratio of two cube = 8:27

Let the edges of the cubes to x and y

As,

$\Rightarrow \frac{\text{Volume of the first cube}}{\text{Volume of the second cube}} = \frac{8}{27}$

$\Rightarrow \frac{x^3}{y^3} = \frac{8}{27}$

$\frac{x}{y} = \sqrt[3]{\frac{8}{27}}$

$\frac{x}{y} = \frac{2}{3}$ ……………(i)

Now,

The ratio of the surface areas of the cubes $= \frac{\text{Surface area of the first cube}}{\text{Surface area of the second cube}}$

$\Rightarrow 6x^2/6y^2$

$= (x/y)^2$

$= (2/3)^2$ ……………………………… [Using (i)]

$= 4/9$

$= 4 : 9$

So,

The ratio of the surface areas of the given cubes is 4 : 9.

Question: 7

Solution:

Given,

Radius of the right circular cylinder = 176/7 cm^3

Height of the right circular cylinder = Radius of the right circular cylinder

So,

$\Rightarrow h = r$

As,

Volume of the right circular Cylinder = 176/7 cm^3

$\Rightarrow \pi r^2 h = 176/7$

$\Rightarrow 22/7 \times h^2 \times h = 176/7$

$\Rightarrow h^3 = 176 \times 7/7 \times 22$

$\Rightarrow h^3 = 8$

$\Rightarrow h = 3\sqrt{8}$

Therefore,

h = 2 cm

So,

The height of the right circular cylinder is 2 cm

Question: 8

Solution:

Given,

Ratio of the base and the height of a cylinder is 2:3

So,

Let the radius of the base = r

And

The height of the cylinder = h

r : h = 2 : 3

That is,

r/h = 2/3

So,

h = 3r/2 – – – – – – – – – (i)

As,

Volume of the cylinder = 12936 cm^3

⇒ $\pi r^2 h=12936$

⇒ $22/7 \times r^2 \times 3r/2=12936$ [Using (i)]

⇒ $33/7 \times r^3=12936$

⇒ $r^3 = 12936 \times 7/33$

⇒ $r^3 = 2744$

⇒ $r = 3\sqrt{2744}$

Therefore,

r = 14 cm

So,

The radius of the base of the cylinder is 14 cm.

Question: 9

Solution:

Let the radus of the first cylinder = r_1

And the radus of the second cylinder = r_2;

Let the height of first cylinder = h_1

And the height of second cylinder = h_2

Given,

$r_1 : r_2 = 2 : 3$

r_1/r_2 = 23 ....(i)

And

$h_1 : h_2 = 5 : 3$

$h_1/h_2 = 5/3$ .... (ii)

Now,

The ratio of the volumes of the cylinders = $\frac{\text{Volume of first cylinder}}{\text{Volume of second cylinder}}$

$= \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$

$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$

$= \left(\frac{2}{3}\right)^2 \times \frac{5}{3}$ .... [Using (i) and (ii)]

= 20/27

= 20 : 27

So,

The ratio of the volumes of the given cylinders is 20 : 27

Question: 10

Solution:

Given,

Volume of the wire = 66 cm^3

Diameter of the wire = 1 mm

Radius (r) of wire = 12 = 0.5 mm = 0.05 cm

Let the length of the wire be l

As,

Volume of the wire = 66 cm^3

$\Rightarrow \pi r^2 l = 66$

⇒ 227 × 0.05 × 0.05 × l = 66

⇒ l = 66 × 722 × 0.05 × 0.05

Therefore,

l = 8400 cm = 84 m

So,

The length of the wire is 84 m.

Question: 11

Solution:

Given,

Area of the base of the right circular cone = 3850 cm^2

Height of the right circular cone = 84 cm

Let the radius of the cone = r

And the slant height of the cone = l

As,

Area of the base of the cone = 3850 cm^2

$\Rightarrow \pi r^2 = 3850$

$\Rightarrow 227 \times r^2 = 3850$

$\Rightarrow r^2 = 3850 \times 722$

$\Rightarrow r^2 = 1225$

$\Rightarrow r = \sqrt{1225}$

Therefore,

$r = 35$ cm

Now,

length $= \sqrt{h^2 + r^2}$

$= \sqrt{(84)^2 + (35)^2}$

$= \sqrt{7056 + 1225}$

$= \sqrt{8281}$

$= 91$ cm

So,

The slant height of the given cone is 91 cm.

Question: 12

Solution:

Given,

Base radius (r) of the cylinder = 8 cm

Height (h) of the cylinder = 2 cm and

Height (H) of the cone = 6 cm

Let the base radius of the cone = R

Now,

As the cylinder is melted to form the cone,

So,

Volume of the cone = Volume of the cylinder

$\Rightarrow 1/3\pi R^2 H = \pi r^2 h$

$\Rightarrow R^2 = 3r^2 hH$

$\Rightarrow R2=3 \times 8 \times 8 \times 26$

$\Rightarrow R^2 = 64$

$\Rightarrow R = \sqrt{64}$

Therefore,

R = 8 cm

So,

The radius of the base of the cone is 8 cm.

Question: 13

Solution:

Let suppose the radius of the cone = r

And height of the cone = h,

Then,

Radius of the cylindrical vessel = r and

Height of the cylindrical vessel = h

Now,

The required number of cones = Volume of the cylindrical vessel/Volume of a cone

$= \pi r^2 h/(1/3\pi r^2 h)$

= 3

So,

The number of the cones that is required to store the water is 3.

Question: 14

Solution:

The volume of the sphere = 4851 cm^3

Let the radius of the sphere = r

As,

Volume of the sphere = 4851 cm^3

$\Rightarrow 4/3\pi r^3 = 4851$

$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = 4851$

$\Rightarrow r^3 = 4851 \times \frac{3 \times 7}{4 \times 22}$

$\Rightarrow r^3 = 92618$

$\Rightarrow r = 3\sqrt{92618}$

$\Rightarrow r = 212cm$

Now,

The Curved surface area of the sphere = $4\pi r^2$

$= 4 \times 22/7 \times 21/2 \times 21/2$

= 1386 cm^2

So,

The curved surface area of the sphere is 1386 cm^2

Question: 15

Solution:

Given,

Curved surface area of the sphere = 5544 cm^3

Let the radius of the sphere = r

As we know,

Curved surface area of the sphere = $4\pi r^2$

So,

$\Rightarrow 4\pi r^2 = 5544$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 5544$

$\Rightarrow r^2 = 5544 \times \frac{7}{4 \times 22}$

$\Rightarrow r^2 = 441$

$\Rightarrow r = \sqrt{441}$

$\Rightarrow r = 21$ cm

Now,

Volume of the sphere = $4/3\pi r^3$

$= 4/3 \times 22/7 \times 21 \times 21 \times 21$

$= 38808$ cm³

So,

The volume of the sphere is 38808 cm³.

Question: 16

Solution:

Let suppose the radius of first spheres = r

And the radius of second spheres = R

As per the question,

Surface area of the first sphere /Surface area of the second sphere = 4/25

$\Rightarrow 4\pi r^2/4\pi R^2 = 4/25$

$\Rightarrow (r/R)^2 = 4/25$

$\Rightarrow r/R = \sqrt{4/25}$

$\Rightarrow r/R = 2/5$ .... (i)

Now,

The ratio of the volumes of the two sphere = $\frac{\text{Volume of the first sphere}}{\text{Volume of the second sphere}}$

$= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$

$= (r/R)^3$

$= (2/5)^3$ .... [Using (i)]

$= 8/125$

$= 8 : 125$

So,

The ratio of the volumes of the given spheres will be 8 : 125

Question: 17

Solution:

Given,

Radius (R) of the solid metallic sphere = 8 cm

Radius (r) of the spherical ball = 2 cm

Now,

The number of spherical balls obtained = $\frac{\text{Volume of the solid metallic sphere}}{\text{Volume of the a spherical ball}}$

$= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$

= $(R/r)^3$

= $(8/2)^3$

= 4^3

= 64

So,

The number of spherical balls obtained is 64.

Question: 18

Solution:

Given,

Diameter of the lead shot = 3 mm

Radius (r) of a lead shot = 3/2 = 1.5 mm = 0.15 cm and

Dimensions of the cuboid = 9 cm x 11 cm x 12 cm

Now,

The number of the lead shots = $\frac{\text{Volume of the cuboid}}{\text{Volume of a shot}}$

$= \frac{9 \times 12 \times 12}{\frac{4}{3}\pi r^3}$

$= \frac{9 \times 12 \times 12}{\frac{4}{3} \times \frac{22}{7} \times 0.15 \times 0.15 \times 0.15}$

= 84000

So,

84000 number of lead shots can be made from the cuboid.

Question: 19

Solution:

Cone:

Radius = 12cm

Height = 24 cm.

Sphere, radius = 2cm

Volume of cone is given as, $V = \frac{1}{3} \times \pi \times r^2 \times h$

$\Rightarrow V = \frac{1}{3} \times \frac{22}{7} \times 12^2 \times 24$

$\Rightarrow V = 3620\ cm^3$

Volume of sphere , $V = \frac{4}{3} \times \pi \times r^3$

$\Rightarrow V = \frac{4}{3} \times \pi \times 2^3$

$\Rightarrow V = 33.52\ cm^3$

∴ the number of squares formed will be: 3620/34 = 106

Question: 20

Solution:

Given,

Radius (R) of the hemisphere = 6 cm

Height (h) of the cone = 75 cm

Let the radius of the base of the cone = r

Now,

Volume of the cone = Volume of the hemisphere

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$

$\Rightarrow r^2 = \frac{2R^3}{h}$

$\Rightarrow r^2 = \frac{2 \times 6 \times 6 \times 6}{75}$

$\Rightarrow r^2 = 5.76$

$\Rightarrow r = \sqrt{5.76}$

Therefore,

r = 2.4 cm

So,

The radius of the base of the cone is 2.4 cm.

Question: 21

Solution:

The basic concept required to solve any such question is that the volume of the two figures will be same, so here we will equate the volume of sphere to that of wire which is in shape of a cylinder and subsequently will find out the height of the cylinder.

Given diameter of copper sphere = D = 18 cm

∴ Radius of the sphere = R = d/2=18/2 = 9 cm

As we know the wire is cylindrical in shape so,

Let the height of the cylindrical wire be 'h' cm

Given diameter of cylindrical wire = d = 4 mm

⇒ Radius of the cylindrical wire = r = d/2= 4/2= 2 mm= 0.2 cm (∵ 1 mm = 0.1 cm)

Volume of a sphere= $\frac{4}{3}\pi R^3$ (where R=radius of sphere)→eqn1

$= \frac{4}{3}\pi(9^3)$ (putting value of R in eqn 1)

$= \frac{4}{3} \times \pi \times 729$

⇒ Volume of sphere = 4π × 243= 972π cm^3→eqn2

Volume of cylinder = $\pi r^2 h$

Where r = radius of base of cylinder and h = height of cylinder

⇒ Volume of cylindrical wire = π × $(0.2)^2$ × h (putting value of r)

= 0.04π × h cm^3→eqn3

Now on equating equation 2 and equation 3, we get,

Volume of sphere = Volume of cylindrical wire

⇒ 972π = 0.04πh

⇒ π(927) = π(0.04h) (taking π common on both sides)

⇒ 927 = 0.04h

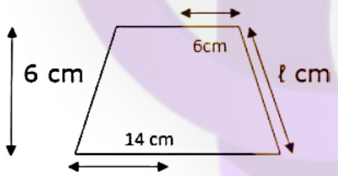
$\Rightarrow \frac{972}{0.04} = h$

∴ h = 24300 cm

The height of cylindrical wire is 24300 cm or 243 m.

Question: 22

Solution:



Given height of frustum = h =6 cm

Radius of top = r = 6 cm

Radius of bottom = R = 14 cm

Let the slant height of the frustum be 'ℓ' cm

We know in frustum

$(\text{Slant height})^2 = (\text{height})^2 + (R - r)^2$→eqn1

⇒ $\ell^2 = 6^2 + (14 - 6)^2$ (putting values of r, R and h in eqn1)

⇒ $\ell^2 = 36 + 8^2$

⇒ $\ell^2 = 36 + 64$

⇒ $\ell^2 = 100$

⇒ $\ell = \sqrt{100}$

$\therefore \ell = 10$ cm

Slant height of the frustum is 10 cm.

Question: 23

Solution:

Let the radius of the sphere be 'R' units

And the cube which will fit inside it be of edge 'a' units

Explanation: The longest diagonal of the cube that will fit inside the sphere will be the diameter of the of the sphere.

∴ The longest diagonal of cube = the diameter of the sphere

Consider ΔBCD, ∠BDC = 90°

BD = CD = a units (as they are the edges of cube)

$BC^2 = CD^2 + BD^2$ (Pythagoras theorem)

$\Rightarrow BC^2 = a^2 + a^2$ (putting value of BD and CD)

$\Rightarrow BC^2 = 2a^2$

$\Rightarrow BC = \sqrt{(2a^2)}$

$\therefore BC = a\sqrt{2}$ units →eqn1

Now consider ΔABC, ∠ABC = 90°

Here, AB = a units and BC = $a\sqrt{2}$ units

$AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\Rightarrow AC^2 = a^2 + (a\sqrt{2})^2$ (putting values of AB and BC)

$\Rightarrow AC^2 = a^2 + 2a^2$

$\Rightarrow AC^2 = 3a^2$

$\Rightarrow AC = \sqrt{(3a^2)}$

$\therefore AC = a\sqrt{3}$ units

∴ Diameter of sphere = D = $a\sqrt{3}$ units

And we know, D = 2 × R

⇒ R = D/2 (put value of D)

$$\therefore R = \frac{a\sqrt{3}}{2} \text{ units}$$

Also, Volume of a sphere= $\frac{4}{3}\pi R^3$→eqn2

Put value of R in eqn2

$$= \frac{4}{3}\pi\left(\frac{a\sqrt{3}}{2}\right)^2$$

$$= \frac{4 \times \pi \times 3a^2}{3 \times 4}$$

∴ Volume of sphere = πa^2 cubic units → eqn3

Volume of cube = $(\text{edge})^3$

∴ Volume of cube = a^3 cubic units →eqn4

Ratio of volume of cube to that of sphere= $\frac{\text{Volume of cube}}{\text{Volume of sphere}}$

= $\frac{a^3}{\pi(a)^2}$ (putting values from eqn3 and eqn4)

⇒Ratio of volume of cube to that of sphere= $\frac{a^3}{\pi \times a^2}$

= $\frac{a}{\pi}$

Ratio of volume of cube to that of sphere is a:π

Question: 24

Solution:

Let the radius of cylinder, cone and sphere be 'r' cm

Let the diameter of cylinder, sphere and cone be '2r' units

∴ Height of cylinder, cone, sphere = h = 2r units

Volume of cylinder = $\pi(r^2)h$

⇒ Volume of cylinder = $\pi \times r^2 \times 2r$ (putting value of h)

∴ Volume of cylinder = $2\pi \times r^3$→eqn1

Volume of sphere= $\frac{4}{3}\pi(r)^3$

= $\frac{4\pi \times r^3}{3}$ → eqn2

Volume of cone= $\frac{1}{3}\pi(r)^2h$ (put the value of h)

⇒Volume of cone= $\frac{\pi \times r^2 \times 2r}{3}$

∴ Volume of cone= $\frac{2\pi \times r^3}{3}$ → eqn3

Ratio of volume of cylinder to that of a cone to that of a sphere as:

⇒ Volume of cylinder : Volume of Cone : Volume of sphere

⇒ $2\pi \times r^3 : \frac{2\pi \times r^3}{3} : \frac{4\pi \times r^3}{3}$

⇒ $2 : \frac{2}{3} : \frac{4}{3}$ (dividing the above relation by (πr^3))

⇒ $1 : \frac{1}{3} : \frac{2}{3}$ (dividing the above ratio by 2)

⇒3 : 1 : 2 (multiplying the above ratio by 3)

Ratio of volume of cylinder to that of cone to that of sphere is 3 : 1 : 2

Question: 25

Solution:

Given volume of each cube = 125 cm^3

Let the edge of each cube be 'a' cm

So, Volume of cube = a^3

$\Rightarrow a^3 = 125$

$\Rightarrow a = \sqrt[3]{(125)}$

$\therefore$ a = 5 cm →eqn1

Now when we join the two cubes then resulting cuboid will be of length twice that of the cube and breadth and height of the resulting cuboid will be same as that of the cube

⇒ length of cuboid = L = 2 × a

⇒ L = 2 × 5 (putting value of a from eqn1)

$\therefore$ L = 10 cm

Now, Breadth of cuboid = B = a

⇒ B = 5 cm

Similarly, height of the cuboid = H = 5 cm

Note: Where ever in a question Surface Area is mentioned, it means Total surface area.

Surface area = Total surface area = 2(LB + BH + HL)

⇒ S.A = 2((10 × 5) + (5 × 5) + (10 × 5))

⇒ S.A = 2(50 + 25 + 50)

⇒ S.A = 2 × 125

$\therefore$ S.A = 250 cm^2

Surface Area of resulting cuboid is 250 cm^2.

Question: 26

Solution:

Let the edges of cubes be a_1, a_2 and a_3

So, a_1 = 3 cm, a_2 = 4 cm, and a_3 = 5 cm

Explanation: Here the sum of volumes of all three cubes will be equal to the volume of the resulting larger cube as the resulting cube is formed by melting the three cubes.

Volume of cube with edge $a_1 = v_1 = (a_1)^3$

$\Rightarrow v_1 = (3)^3$

$\therefore v_1 = 27\ cm^3$→eqn1

Similarly

Volume of cube with edge $a_2 = v_2 = (a_2)^3$

$\Rightarrow v_2 = (4)^3$

$\therefore v_2 = 64\ cm^3$→eqn2

Volume of cube with edge $a_3 = v_3 = (a_3)^3$

$\Rightarrow v_3 = (5)^3$

$\therefore v_3 = 125\ cm^3$→eqn3

Now let the volume of resulting cube be 'V' cm^3

So, $V = v_1 + v_2 + v_3$

⇒ V = 27 + 64 + 125 (from eqn1, eqn2 and eqn3)

∴ $V = 216\ cm^3 \rightarrow eqn4$

Let the edge of resulting cube be 'a' cm

So, volume of the resulting cube = $V = a^3 \rightarrow eqn5$

Equate equation 4 and 5,

$\Rightarrow a^3 = 216$

$\Rightarrow a = \sqrt[3]{(216)}$

∴ a = 6 cm

The edge of new cube formed is 6 cm.

Question: 27

Solution:

Let the diameter of sphere be 'D' and Radius of sphere be 'R'

∴ D = 8 m

Also, we know

R = D/2

⇒ R = 8/2

∴ R = 4 m

Explanation: Here the volume of sphere will be equal to the volume of the resulting cylinder as the resulting cylinder is formed by melting the sphere.

Volume of the sphere, $V_1 = \frac{4}{3}\pi R^3$ (put the value of R)

$$= \frac{4}{3}\pi(4^3)$$

$$= \frac{4\pi \times 64}{3}$$

$$= \frac{256\pi}{3}\ m^3 \rightarrow eqn1$$

Let the length/height of the cylinder be 'H' and let the radius of the cylinder be 'r' and volume of the cylinder be 'V_2'

∴ H = 12 m

Volume of the cylinder = $V_2 = \pi(r^2)H$

$\Rightarrow V_2 = \pi(r^2) \times 12$ (putting value of H)

$\Rightarrow V_2 = 12\pi \times r^2\ m^3 \rightarrow eqn2$

Now equate equation 1 and 2,

$\Rightarrow V_2 = V_1$

$$\Rightarrow 12\pi \times r^2 = \frac{256\pi}{3}$$

$$\Rightarrow r^2 = \frac{256\pi}{3 \times 12\pi}$$

$$\Rightarrow r^2 = \frac{256}{36}$$

$\Rightarrow r^2 = \frac{64}{9}$

$\Rightarrow r \sqrt{\frac{64}{9}}$

$\Rightarrow r = \frac{8}{3}$

∴ r = 2.66 m

Width of cylinder = diameter of cylinder = 2 × radius

⇒ Width of cylinder = 2 × r

⇒ Width of cylinder = 2 × 2.66

∴ Width of cylinder = 5.32 m

Width of the resulting cylinder is 5.32 m

Question: 28

Solution:

Let the length of the cloth used be 'L' cm

Area of cloth used = 5 × L →eqn1

Also, Given Diameter = d = 14 m and height = h = 24 m

∴ Radius = r = D/2

⇒ r = 14/2

∴ r = 7 m

Let the slant height of the cone be ℓ m

So, $(\text{Slant height})^2 = (\text{Height})^2 + (\text{Radius})^2$

Put the values in the above relation

$\Rightarrow \ell^2 = h^2 + r^2$

$\Rightarrow \ell^2 = 24^2 + 7^2$

$\Rightarrow \ell^2 = 576 + 49$

$\Rightarrow \ell^2 = 625$

$\Rightarrow \ell = \sqrt{(625)}$

∴ ℓ = 25 cm →eqn1

Also, we know Curved Surface Area of cone = $\pi r \ell$

Where r = radius of base, ℓ = slant height

C.S.A = π × 7 × 25

$\Rightarrow \text{C.S.A} = \frac{22}{7} \times 7 \times 25 \left(\pi = \frac{22}{7}\right)$

⇒ C.S.A = 22 × 25

⇒ C.S.A = 550 m^2→eqn2

Now the Curved surface area of conical tent will be equal to the area of the cloth used to make the tent

⇒ C.S.A = Area of cloth

⇒ 550 = 5 × L (from eqn1 and eqn2)

$\Rightarrow L = \frac{550}{5}$

∴ L = 110 m

So, cost of the cloth used = rate of cloth × Length of the cloth

⇒ Cost of cloth used = 25 × 110

⇒ Cost of cloth = Rs.2750

Cost of the cloth used is Rs. 2750

Question: 29

Solution:

Given height of cylinder = h = 10 cm

Radius of cylinder = r = 3.5 cm

Radius of hemisphere = R = 3.5 cm

Explanation: In this question the volume of wood in toy can be calculated by subtracting the volume of two hemisphere from the volume of cylinder.

So, volume of cylinder = $\pi r^2 h$

Where r = radius of cylinder and h = height of cylinder

⇒ Volume of cylinder = $\pi \times (3.5)^2 \times 10$ (from given values)

⇒ Volume of cylinder = π × 12.25 × 10

∴ Volume of cylinder = 122.5π cm^3→eqn1

Volume of a hemisphere= $\frac{2}{3}\pi R^3$ (where R is radius of hemisphere)

$= \frac{2}{3} \times \pi \times (3.5)^3$

$= \frac{2\pi \times 42.875}{3}$

$= \frac{85.75\pi}{3} cm^3$

$= 2 \times \frac{85.75\pi}{3}$

∴ Volume of two hemisphere= $\frac{171.5\pi}{3} cm^3 \rightarrow$ eqn2

Volume of wood in toy = eqn1 – eqn2

⇒Volume of wood in toy= $122.5\pi - \frac{171.5\pi}{3}$

$= \left(122.5 - \frac{171.5}{3}\right)\pi$ (taking π common)

$= \left(\frac{367.5 - 171.5}{3}\right)\pi$

$= \left(\frac{196}{3}\right)\pi \left(\text{put } \pi = \frac{22}{7}\right)$

$= \frac{196}{3} \times \frac{22}{7}$

$= \frac{28 \times 22}{3}$

∴ Volume of wood in toy = 205.333 cm^3

Volume of wood in toy is 205.333 cm^3.

Question: 30

Solution:

Let the edges of metal cubes be a_1, a_2 and a_3

And it is given that ratio of edges is 3:4:5

So, let $a_1 = 3x$, $a_2 = 4x$ and $a_3 = 5x$

Volume of cube with edge $a_1 = v_1 = (a_1)^3$

$\Rightarrow v_1 = (3x)^3$

∴ $v_1 = 27x^3$ cm^3→eqn1

Similarly

Volume of cube with edge $a_2 = v_2 = (a_2)^3$

$\Rightarrow v_2 = (4x)^3$

∴ $v_2 = 64x^3$ cm^3→eqn2

Volume of cube with edge $a_3 = v_3 = (a_3)^3$

$\Rightarrow v_3 = (5x)^3$

∴ $v_3 = 125x^3$ cm^3→eqn3

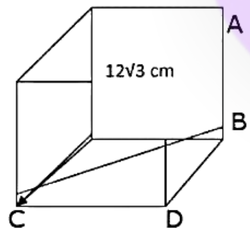
Now let the volume of resulting cube be 'V' cm^3

So, $V = v_1 + v_2 + v_3$

$\Rightarrow V = 27x^3 + 64x^3 + 125x^3$ (from eqn1, eqn2 and eqn3)

∴ $V = 216x^3$ cm^3→eqn4

It is given that the diagonal of the resulting cube is $12\sqrt{3}$ cm



Let the edge of resulting cube be 'a' cm

Consider ΔBCD, ∠BDC = 90°

BD = CD = a cm (as they are the edges of cube)

$BC^2 = CD^2 + BD^2$ (Pythagoras theorem)

$\Rightarrow BC^2 = a^2 + a^2$ (putting value of BD and CD)

$\Rightarrow BC^2 = 2a^2$

$\Rightarrow BC = \sqrt{(2a^2)}$

$\therefore$ BC = $a\sqrt{2}$ cm

Now consider ΔABC, ∠ABC = 90°

Here, AB = a cm and BC = $a\sqrt{2}$ cm and AC = $12\sqrt{3}$ cm

$AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\Rightarrow (12\sqrt{3})^2 = a^2 + (a\sqrt{2})^2$ (putting values of AB, AC and BC)

$\Rightarrow 144 \times 3 = a^2 + 2a^2$

$\Rightarrow 144 \times 3 = 3a^2$

$\Rightarrow 144 = a^2$

$\Rightarrow a = \sqrt{144}$

$\therefore$ a = 12 cm

So, volume of the resulting cube = $V = a^3$

$\Rightarrow V = 12^3$ (putting value of a)

$\therefore V = 1728$ cm^3 –eqn5

Equate equation 4 and 5

$\Rightarrow 216x^3 = 1728$

$\Rightarrow x^3 = 1728/216$

$\Rightarrow x^3 = 8$

$\Rightarrow x = \sqrt[3]{8}$

$\therefore x = 2$

So $a_1 = 3x = 3 \times 2$

$\Rightarrow a_1 = 6$ cm

Similarly $a_2 = 4x = 4 \times 2$

$\therefore a_2 = 8$ cm

Similarly $a_3 = 5x = 5 \times 2$

$\therefore a_3 = 10$ cm

The edges of three cubes are 6 cm, 8 cm and 10 cm.

Question: 31

Solution:

External diameter of hollow sphere = D = 8 cm

$\Rightarrow$ External radius of hollow sphere = R = D/2

$\Rightarrow$ R = 8/2

$\Rightarrow$ R = 4 cm

Internal diameter of hollow sphere = d = 4 cm

$\Rightarrow$ Internal radius of hollow sphere = r = d/2

$\Rightarrow$ r = 4/2

$\Rightarrow r = 2$ cm

Let the height of the resulting cone be 'h' cm

Let the volume of External sphere be V_1 and that of internal be V_2.

Explanation: Here the volume of hollow sphere will be equal to the volume of the resulting cone as the resulting cone is formed by melting the sphere.

Volume of the External sphere= $V_1 = \frac{4}{3}\pi R^3$ (put the value of R)

$= \frac{4}{3}\pi(4^3)$

$= \frac{4\pi \times 64}{3}$

$= \frac{256\pi}{3}\ \text{cm}^3 \rightarrow \text{eqn1}$

Volume of the Internal sphere $= V_2 = \frac{4}{3}\pi r^3$ (put the value of r)

$= \frac{4}{3}\pi(2^3)$

$= \frac{4\pi \times 8}{3}$

$\therefore$ Volume of the Internal sphere $= V_2 = \frac{32\pi}{3}\ \text{cm}^3 \rightarrow \text{eqn2}$

Volume of sphere = V = External volume – Internal volume

$\Rightarrow V = \frac{256\pi}{3} - \frac{32\pi}{3}$

$\Rightarrow V = \frac{(256 - 32)\pi}{3}$

$\Rightarrow V = \frac{224\pi}{3}\ \text{cm}^3 \rightarrow \text{eqn3}$

Given base radius of the resulting cone = r' = 8 cm

Le the height be 'h' and volume of the resulting cone be V'

$\Rightarrow V' = \frac{1}{3}\pi(r')^2 h$

$\Rightarrow V' = \frac{1}{3}\pi(8^2)h$ (putting value of r')

$\Rightarrow V' = \frac{64\pi h}{3}\ \text{cm}^3 \rightarrow \text{eqn4}$

Equate equation 3 and 4,

$\Rightarrow V = V'$

$\Rightarrow \frac{224\pi}{3} = \frac{64\pi h}{3}$

$\Rightarrow 224 = 64h$

$\Rightarrow h = 224/64$

$\therefore h = 3.5$ cm

The height of resulting cone is 3.5 cm.

Question: 32

Solution:

Upper end radius of frustum = d/2= 28/2 = 14 cm

Lower end radius of frustum = D/2 = 42/2 =21 cm

Height of the frustum = 24 cm

And we know,

The amount of milk that bucket can hold = Volume of the bucket

And, Volume of bucket = Volume of Frustum

∴ Amount of milk that bucket can hold = Volume of frustum

⇒Volume of frustum$= \frac{1}{3} \times \pi \times h \times (r^2 + R^2 + (r \times R))$

Where R = Radius of larger or lower end and r = Radius of smaller or upper end and h = height of frustum π = 22/7

⇒Volume of frustum$= \frac{1}{3} \times \frac{22}{7} \times 24 \times \left(14^2 + 21^2 + (14 \times 21)\right)$

$= \frac{22}{7} \times 8 \times (196 + 441 + 294)$

$= \frac{22}{7} \times 8 \times 931$

$= 22 \times 8 \times 133$

∴ Volume of frustum = 23408 cm^3

Also we know that 1 litre = 1000 cm^3

⇒ Volume of frustum in litre = 23408/1000= 23.408 litre

⇒ Amount of milk that the bucket hold = 23.408 litre

⇒ The cost of milk = Rate of milk × amount of milk bucket holds

⇒ The cost of milk = 30 × 23.408

∴ = Rs. 702.24

The cost of milk is Rs.702.24

Question: 33

Solution:

Explanation: Here in order to find th outer surface area of building we need to simply add the curved surface areas of cone and cylinder.

Given height of cylinder = h = 4 m

Height of cone = h’ = 2.8 m

Diameter of cylinder = diameter of cone = d = 4.2 m

⇒ Radius of cone = Radius of cylinder = d/2 =4.2m/2 = 2.1 m

Outer surface area of building = C.S.A of cylinder + C.S.A of cone

Now, C.S.A of cylinder = 2πrh →eqn1

Where r = radius of base of cylinder, h = height of cylinder

And C.S.A of cone = πrℓ →eqn2

Where r = radius of base of cone, ℓ = Slant height of cone

We know in a cone

$(\text{Slant height})^2 = (\text{height})^2 + (\text{radius})^2$ (put the given values)

$(\ell)^2 = (2.8)^2 + (2.1)^2$

$\Rightarrow (\ell)^2 = 7.84 + 4.41$

$\Rightarrow (\ell)^2 = 12.25$

$\Rightarrow \ell = \sqrt{(12.25)}$

$\therefore \ell = 3.5$ m

Now, C.S.A of cone = $\pi \times 2.1 \times 3.5$ (putting the values in eqn2)

$\Rightarrow$ C.S.A of cone = 7.35π $m^2 \rightarrow$ eqn3

C.S.A of cylinder = $2 \times \pi \times 2.1 \times 4$ (putting the values in eqn1)

$\Rightarrow$ C.S.A of cylinder = $2 \times \pi \times 4.41 \times 4$

$\therefore$ C.S.A of cylinder = 16.8π $m^2 \rightarrow$ eqn4

Outer surface area of building = eqn3 + eqn4

$\Rightarrow$ Outer surface area = $7.35\pi + 16.8\pi$

$= 24.15\pi$

$\therefore$ Outer surface area $= 75.9$ m^2

The outer surface area of building is 75.9 m^2.

Question: 34

Solution:

Let the Radius of cone be 'r' and height of cone be 'h'

$\therefore$ r = 21 cm and h = 84 cm

Explanation: Here the volume of cone will be equal to the volume of the resulting sphere as the resulting sphere is formed by melting the cone.

Volume of cone, $V_1 = \frac{1}{3}\pi(r^2)h$

$\Rightarrow V_1 = \frac{1}{3} \times \pi \times (21)^2 \times 84$

$\Rightarrow V_1 = \pi \times 441 \times 28$

$\therefore V_1 = 12348\pi$ $m^3 \rightarrow$ eqn1

Let the Radius of resulting sphere be 'R' cm

Volume of the sphere= $V_1 = \frac{4}{3}\pi(R)^3 \rightarrow$ eqn2 (put the value of R)

Now equate equation 1 and 2,

$\Rightarrow V_2 = V_1$

$\Rightarrow \frac{4}{3}\pi(R)^3 = 12348\pi$

$\Rightarrow \frac{4 \times R^3}{3} = 12348$

$\Rightarrow R^3 = \frac{12348 \times 3}{4}$

⇒ $R^3 = 3087 \times 3$

⇒ $R^3 = 9261$

⇒ $R = \sqrt[3]{9261}$

∴ R = 21 cm

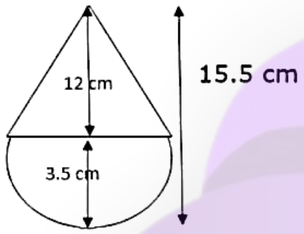
Diameter of Sphere = 2 × radius

⇒ Diameter of Sphere = 2 × R = 42 m

Diameter of the resulting Sphere is 42 m

Question: 35

Solution:



Explanation: The total surface area of the toy can be calculated by taking the sum of the Curved surface area of cone and that of hemisphere of same radius.

Given total height of toy = H = 15.5 cm

Radius of hemisphere = Radius of cone = r = 3.5 cm

Now the height of cone = h = total height – radius of hemisphere

⇒ Height of cone = h = 15.5 – 3.5

∴ Height of cone = h = 12 cm

Let the slant height of the cone be 'ℓ' cm

Also we know that in a cone,

$(\text{Slant height})^2 = (\text{Height})^2 + (\text{Radius})^2$

⇒ $\ell^2 = 12^2 + 3.5^2$ (putting the values)

⇒ $\ell^2 = 144 + 12.25$

⇒ $\ell^2 = 156.25$

⇒ $\ell = \sqrt{156.25}$

∴ ℓ = 12.5 cm

C.S.A of cone = $\pi r \ell$

⇒ C.S.A of cone = $\pi \times 3.5 \times 12.5$ (putting the given values)

∴ C.S.A of cone = 43.75π m^2 →eqn1

C.S.A of hemisphere = $2\pi r^2$

⇒ C.S.A of hemisphere = $2 \times \pi \times 3.5^2$

= $2 \times \pi \times 12.25$

= 24.5π m^2 →eqn2

Now total surface area of toy = eqn1 + eqn2

⇒ Total surface area of toy = $43.75\pi + 24.5\pi$

= 68.25π

⇒The total surface area of toy= $68.25 \times \frac{22}{7}$ (putting $\pi = 22/7$)

= 9.75×22

= $214.5\ m^2$

The total surface area of the toy is $214.5\ m^2$.

Question: 36

Solution:

Explanation: Here the bucket is in the shape of a frustum. So capacity of bucket will be equal to the volume of the frustum and in order to calculate the total surface area of the bucket we will subtract the top end circular area from the total surface area of the frustum as the bucket is open on top.

Upper end radius of frustum/bucket = R = 28 cm

Lower end radius of frustum/bucket = r = 7 cm

Height of the frustum/bucket = 28 cm

And we know,

The capacity of bucket = Volume of the bucket

And, Volume of bucket = Volume of Frustum

∴ Capacity of bucket = Volume of frustum

⇒Volume of frustum= $\frac{1}{3} \times \pi \times h \times (r^2 + R^2 + (r \times R))$

Where R = Radius of larger or upper end and r = Radius of smaller or lower end and h = height of frustum $\pi = 22/7$

⇒Volume of frustum= $\frac{1}{3} \times \frac{22}{7} \times 28 \times (7^2 + 28^2 + (7 \times 28))$

$= \frac{22}{3} \times 4 \times (49 + 784 + 196)$

$= \frac{22}{3} \times 4 \times 1029$

= $22 \times 4 \times 343$

∴ Volume of frustum = $30184\ cm^3$

∴ Capacity of bucket = $30184\ cm^3$

T.S.A of bucket = T.S.A of frustum – Area of upper circle →eqn1

Let the slant height of the frustum be 'ℓ' cm

So, $\ell^2 = h^2 + (R - r)^2$

⇒ $\ell^2 = 28^2 + (28 - 7)^2$

⇒ $\ell^2 = 784 + (21)^2$

⇒ $\ell^2 = 784 + 441$

⇒ $\ell^2 = 1225$

⇒ $\ell = \sqrt{(1225)}$

∴ $\ell = 35$ cm

$\Rightarrow$ T.S.A of frustum = $\pi(R + r)\ell + \pi R^2 + \pi r^2$

$= \pi(28 + 7) \times 35 + \pi(28)^2 + \pi(7)^2$

$= 35 \times 35\pi + 784\pi + 49\pi$

$= 1225\pi + 784\pi + 49\pi \rightarrow$eqn2

Area of upper circle = πR^2

$= \pi(28)^2$

$= 784\pi \rightarrow$eqn3

T.S.A of bucket = $1225\pi + 784\pi + 49\pi - 784\pi$ (from eqn2 and 3)

$\Rightarrow$ T.S.A of bucket = 1274π

$\Rightarrow$T.S.A of bucket=$1274 \times \frac{22}{7}$ $\left(\text{putting } \pi = \frac{22}{7}\right)$

$\Rightarrow$ T.S.A of bucket = 182×22

$\therefore$ T.S.A of bucket = 4004 cm^2

The capacity and total surface area of the bucket is 30184 cm^3 and 4004 cm^2.

Question: 37

Solution:

Upper end radius of frustum/bucket = R = 20 cm

Lower end radius of frustum/bucket =r = 12 cm

Height of the frustum/bucket be 'h' cm

And we know,

The capacity of bucket = Volume of the bucket

And, Volume of bucket = Volume of Frustum

Volume of frustum/bucket = V = 12308.8 cm^3

$\therefore$ Capacity of bucket = Volume of frustum

$\Rightarrow$Volume of frustum= $V = \frac{1}{3} \times \pi \times h \times (r^2 + R^2 + (r \times R))$

Where R = Radius of larger or upper end and r = Radius of smaller or lower end and h = height of frustum π = 3.14

$\Rightarrow \frac{1}{3} \times \pi \times h \times (r^2 + R^2 + (r \times R)) = V$

$\Rightarrow \frac{1}{3} \times 3.14 \times h \times \left(12^2 + 20^2 + (12 \times 20)\right) = 12308.8$ (putting the values)

$\Rightarrow 3.14 \times h \times (144 + 400 + 240) = 12308.8 \times 3$

$\Rightarrow 3.14 \times h \times 784 = 36926.4$

$\Rightarrow 2461.76 \times h = 36926.4$

$\Rightarrow h = 36926.4/2461.76$

$\therefore$ h = 15 cm

The height of the bucket is 15 cm.

Question: 38

Solution:

Upper end radius of container = R = 20 cm

Lower end radius of container = r = 8 cm

Height of the container be 'h' cm

As container is in shape of frustum

Volume of container = Volume of Frustum

$V = 10459\frac{3}{7}\ cm^3$

$\Rightarrow V = \frac{73216}{7}\ cm^3$

$\Rightarrow$ Volume of frustum $= V = \frac{1}{3} \times \pi \times h \times (r^2 + R^2 + (r \times R))$

Where R = Radius of larger or upper end and r = Radius of smaller or lower end and h = height of frustum π = 22/7

$\Rightarrow \frac{1}{3} \times \pi \times h \times (r^2 + R^2 + (r \times R)) = V$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times h \times \left(8^2 + 20^2 + (8 \times 20)\right) = \frac{73216}{7}$ (putting the values)

$\Rightarrow \frac{22}{3} \times h \times (64 + 400 + 160) = 73216$

$\Rightarrow \frac{22}{3} \times h \times 624 = 73216$

$\Rightarrow 22 \times h \times 208 = 73216$

$\Rightarrow 4576 \times h = 73216$

$\Rightarrow h = 73216/4576$

$\therefore h = 16$ cm

Let the slant height of the frustum be 'ℓ' cm

So, $\ell^2 = h^2 + (R - r)^2$

$\Rightarrow \ell^2 = 16^2 + (20 - 8)^2$

$\Rightarrow \ell^2 = 256 + (12)^2$

$\Rightarrow \ell^2 = 256 + 144$

$\Rightarrow \ell^2 = 400$

$\Rightarrow \ell = \sqrt{(400)}$

$\therefore \ell = 20$ cm

Now the Area of sheet used in making the container can be calculated by simply subtracting the area of the upper circular end from the T.S.A of frustum

$\Rightarrow$ T.S.A of frustum $= \pi(R + r)\ell + \pi R^2 + \pi r^2$

$= \pi(20 + 8) \times 20 + \pi(20)^2 + \pi(8)^2$

$= 28 \times 20\pi + 400\pi + 64\pi$

$= 560\pi + 464\pi$

$= 1024\pi \rightarrow$ eqn1

Area of upper circular end $= \pi R^2$

$= \pi(20)^2$

$= 400\pi \rightarrow$ eqn2

T.S.A of container = 1024π – 400π (from eqn2 and 1)

⇒ T.S.A of container = 624π

$= \frac{13728}{7}\ cm^2$

⇒ Cost of metal sheet used = T.S.A of container × Rate per cm^2

⇒ Cost of mtal sheet used= $\frac{13728}{7} \times 1.40$

= 13728 × 0.2

= Rs. 2745.6

The cost of the metal sheet used is Rs. 2745.6

Question: 39

Solution:

Explanation: Here the volume of all the resulting cones will be exactly equal to the volume of the sphere from which they are formed. Son we would find the volume of sphere and then divide the volume of sphere with the volume of one cone to find the number of cones formed.

Diameter of the sphere = D = 28 cm

Radius of the sphere = 28/2

Radius of the sphere = R = 14 cm Volume of the sphere= $V_1 = \frac{4}{3}\pi R^3$ (put the value of R)

⇒Volume of the sphere = $V_1 = \frac{4}{3}\pi(14^3)$

$= \frac{4\pi \times 2744}{3}$

$= \frac{10976\pi}{3}\ m^3 \rightarrow$ eqn1

Let the number of cones formed out of the sphere be 'x'

Diameter of each cone= $4\frac{2}{3}$ cm $= \frac{14}{3}$ cm

Given the height of each cone = h = 3 cm

Then, radius

$$r = \frac{\frac{14}{3}}{2}$$

$$r = \frac{14}{3 \times 2}$$

$$r = \frac{7}{3}\ cm$$

Volume of one cone $= V_2 = \frac{1}{3}\pi(r^2)h$

$$\Rightarrow V_2 = \frac{1}{3} \times \pi \times \left(\frac{7}{3}\right)^2 \times 3$$

$$\Rightarrow V_2 = \frac{49\pi}{9}\ cm^3 \rightarrow \text{eqn2}$$

Volume of 'n' number of cones = n × volume of one cone

Volumes of 'm' number of cones = volume of sphere

$$m \times \frac{49\pi}{9} = \frac{10976\pi}{3}$$

$$\Rightarrow m \times \frac{49}{3} = 10976$$

$$\Rightarrow m = \frac{10976 \times 3}{49}$$

$\Rightarrow m = 224 \times 3$

$\therefore m = 672$

The number of cones formed out of the sphere is 672

Question: 40

Solution:

Given internal diameter of cylinder = D = 10 cm

Internal radius of cylinder = R = D/2 = 10/2

Internal radius of cylinder = R = 5 cm

Height of cylinder = H = 10.5 cm

Diameter of solid cone = d = 7 cm

Radius of solid cone = r = d/2 = 7/2

Radius of solid cone = r = 3.5 cm

Height of cone = h = 6 cm

(i)Volume displaced out of cylinder

By Archimedes principle we can easily say that,

Volume displaced out of cylinder = Volume of the solid cone

Volume of cone $= V_2 = \frac{1}{3}\pi(r^2)h$

$$\Rightarrow V_2 = \frac{1}{3} \times \pi \times (3.5)^2 \times 6$$

$$\Rightarrow V_2 = \frac{22}{7} \times 12.25 \times 2$$

$\Rightarrow V = 22 \times 1.75 \times 2$

$\therefore V = 77\ cm^3$

The volume displaced out of cylinder is 77 cm^3.

(ii)Volume left in cylinder

Volume left in cylinder = Volume of cylinder – Volume displaced out

Volume of cylinder = $\pi(R)^2H$

$\Rightarrow$ Volume of cylinder = $\pi \times (5)^2 \times 10.5$ (putting the given values)

$\Rightarrow$ Volume of cylinder = $\pi \times 25 \times 10.5$

$\Rightarrow$ Volume of cylinder = $\pi \times 262.5$

$\Rightarrow$ Volume of cylinder $= \frac{22}{7} \times 262.5$

$\Rightarrow$ Volume of cylinder = 22×37.5

$\therefore$ Volume of cylinder = $825\ cm^3 \rightarrow$ eqn1

Volume left in cylinder = $825 - 77$ (from eqn1 and (i))

$\therefore$ Volume left in cylinder = $748\ cm^3$

The volume left in the cylinder is $748\ cm^3$.