Exercise – 2A

1. Sol:

$x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow x(x+4) + 3(x+4) = 0$

$\Rightarrow (x+4)(x+3) = 0$

$\Rightarrow (x + 4) = 0$ or $(x + 3) = 0$

$\Rightarrow x = -4$ or $x = -3$

Sum of zeroes $= -4 + (-3) = \frac{-7}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= (-4)(-3) = \frac{12}{1} = \frac{constant\ \ term}{(coefficient\ of\ x^2)}$

2.

Sol:

$x^2 - 2x - 8 = 0$

$\Rightarrow x^2 - 4x + 2x - 8 = 0$

$\Rightarrow x(x - 4) + 2(x - 4) = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow (x - 4) = 0$ or $(x+2) = 0$

$\Rightarrow x = 4$ or $x = -2$

Sum of zeroes $= 4 + (-2) = 2 = \frac{2}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= (4)(-2) = \frac{-8}{1} = \frac{constant\ \ term}{(coefficient\ of\ x^2)}$

3.

Sol:

We have:

$f(x) = x^2 + 3x - 10$

$= x^2 + 5x - 2x - 10$

$= x(x + 5) - 2(x + 5)$

$= (x - 2)(x + 5)$

$\therefore f(x) = 0 \Rightarrow (x - 2)(x + 5) = 0$

$\Rightarrow x - 2 = 0$ or $x + 5 = 0$

$\Rightarrow x = 2$ or $x = -5$.

So, the zeroes of f(x) are 2 and -5.

Sum of zeroes = $2 + (-5) = -3 = \frac{-3}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $2 \times (-5) = -10 = \frac{-10}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

4.

Sol:

We have:

$f(x) = 4x^2 - 4x - 3$

$= 4x^2 - (6x - 2x) - 3$

$= 4x^2 - 6x + 2x - 3$

$= 2x (2x - 3) + 1(2x - 3)$

$= (2x + 1) (2x - 3)$

$\therefore f(x) = 0 \Rightarrow (2x + 1) (2x - 3)= 0$

$\Rightarrow 2x + 1= 0$ or $2x - 3 = 0$

$\Rightarrow x = \frac{-1}{2}$ or $x = \frac{3}{2}$

So, the zeroes of f(x) are $\frac{-1}{2}$ and $\frac{3}{2}$

Sum of zeroes = $\left(\frac{-1}{2}\right) + \left(\frac{3}{2}\right) = \frac{-1+3}{2} = \frac{2}{2} = 1 = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\left(\frac{-1}{2}\right) \times \left(\frac{3}{2}\right) = \frac{-3}{4} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

5.

Sol:

We have:

$f(x) = 5x^2 - 4 - 8x$

$= 5x^2 - 8x - 4$

$= 5x^2 - (10x - 2x) - 4$

$= 5x^2 - 10x + 2x - 4$

$= 5x (x - 2) + 2(x - 2)$

$= (5x + 2) (x - 2)$

$\therefore f(x) = 0 \Rightarrow (5x + 2) (x - 2) = 0$

$\Rightarrow 5x + 2= 0$ or $x - 2 = 0$

$\Rightarrow x = \frac{-2}{5}$ or $x = 2$

So, the zeroes of f(x) are $\frac{-2}{5}$ and 2.

Sum of zeroes = $\left(\frac{-2}{5}\right) + 2 = \frac{-2+10}{5} = \frac{8}{5} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\left(\frac{-2}{5}\right) \times 2 = \frac{-4}{5} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

6.

Sol:

$2\sqrt{3}x^2 - 5x + \sqrt{3}$

$\Rightarrow 2\sqrt{3}x^2 - 2x - 3x + \sqrt{3}$

$\Rightarrow 2x(\sqrt{3}x - 1) - \sqrt{3}(\sqrt{3}x - 1) = 0$

$\Rightarrow (\sqrt{3}x - 1)$ or $(2x - \sqrt{3}) = 0$

$\Rightarrow (\sqrt{3}x - 1) = 0$ or $(2x - \sqrt{3}) = 0$

$\Rightarrow x = \frac{1}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ or $x = \frac{\sqrt{3}}{2}$

Sum of zeroes $= \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{6} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{6} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

7.

Sol:

$f(x) = 2x^2 - 11x + 15$

$= 2x^2 - (6x + 5x) + 15$

$= 2x^2 - 6x - 5x + 15$

$= 2x(x - 3) - 5(x - 3)$

$= (2x - 5)(x - 3)$

$\therefore f(x) = 0 \Rightarrow (2x - 5)(x - 3) = 0$

$\Rightarrow 2x - 5 = 0$ or $x - 3 = 0$

$\Rightarrow x = \frac{5}{2}$ or $x = 3$

So, the zeroes of f(x) are $\frac{5}{2}$ and 3.

Sum of zeroes $= \frac{5}{2} + 3 = \frac{5+6}{2} = \frac{11}{2} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= \frac{5}{2} \times 3 = \frac{-15}{2} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

8. Sol:

$4x^2 - 4x + 1 = 0$

$\Rightarrow (2x)^2 - 2(2x)(1) + (1)^2 = 0$

$\Rightarrow (2x - 1)^2 = 0$ $[\because a^2 - 2ab + b^2 = (a-b)^2]$

$\Rightarrow (2x - 1)^2 = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = \frac{1}{2}$

Sum of zeroes $= \frac{1}{2} + \frac{1}{2} = 1 = \frac{1}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

9.

Sol:

We have:

$f(x) = x^2 - 5$

It can be written as $x^2 + 0x - 5$.

$= (x^2 - (\sqrt{5})^2)$

$= (x + \sqrt{5})(x - \sqrt{5})$

$\therefore f(x) = 0 \Rightarrow (x + \sqrt{5})(x - \sqrt{5}) = 0$

$\Rightarrow x + \sqrt{5} = 0$ or $x - \sqrt{5} = 0$

$\Rightarrow x = -\sqrt{5}$ or $x = \sqrt{5}$

So, the zeroes of f(x) are $-\sqrt{5}$ and $\sqrt{5}$.

Here, the coefficient of x is 0 and the coefficient of x^2 is 1.

Sum of zeroes $= -\sqrt{5} + \sqrt{5} = \frac{0}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= -\sqrt{5} \times \sqrt{5} = \frac{-5}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

10.

Sol:

We have:

$f(x) = 8x^2 - 4$

It can be written as $8x^2 + 0x - 4$

$= 4\{(\sqrt{2}x)^2 - (1)^2\}$

$= 4(\sqrt{2}x + 1)(\sqrt{2}x - 1)$

$\therefore f(x) = 0 \Rightarrow (\sqrt{2}x + 1)(\sqrt{2}x - 1) = 0$

$\Rightarrow (\sqrt{2}x + 1) = 0$ or $\sqrt{2}x - 1 = 0$

$\Rightarrow x = \frac{-1}{\sqrt{2}}$ or $x = \frac{1}{\sqrt{2}}$

So, the zeroes of f(x) are $\frac{-1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$

Here the coefficient of x is 0 and the coefficient of x^2 is $\sqrt{2}$

Sum of zeroes $= \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{-1+1}{\sqrt{2}} = \frac{0}{\sqrt{2}} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= \frac{-1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{-1 \times 4}{2 \times 4} = \frac{-4}{8} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

11.

Sol:

We have,

f (u) = $5u^2 + 10u$

It can be written as 5u (u+2)

$\therefore$ f (u) = 0 $\Rightarrow$ 5u = 0 or u + 2 = 0

$\Rightarrow$ u = 0 or u = -2

So, the zeroes of f (u) are -2 and 0.

Sum of the zeroes $= -2 + 0 = -2 = \frac{-2 \times 5}{1 \times 5} = \frac{-10}{5} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ u^2)}$

Product of zeroes $= -2 \times 0 = 0 = \frac{0 \times 5}{1 \times 5} = \frac{-0}{5} = \frac{constant\ term}{(coefficient\ of\ u^2)}$

12.

Sol:

$3x^2 - x - 4 = 0$

$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow x (3x - 4) + 1 (3x - 4) = 0$

$\Rightarrow (3x - 4) (x + 1) = 0$

$\Rightarrow$ (3x - 4) or (x + 1) = 0

$\Rightarrow$ x = $\frac{4}{3}$ or x = - 1

Sum of zeroes $= \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= \frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

13.

Sol:

Let $\alpha = 2$ and $\beta = -6$

Sum of the zeroes, $(\alpha + \beta) = 2 + (-6) = -4$

Product of the zeroes, $\alpha\beta = 2 \times (-6) = -12$

$\therefore$ Required polynomial $= x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (-4)x - 12$

$= x^2 + 4x - 12$

Sum of the zeroes $= -4 = \frac{-4}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= -12 = \frac{-12}{1} = \frac{constant\ \ term}{(coefficient\ of\ x^2)}$

14. Sol:

Let $\alpha = \frac{2}{3}$ and $\beta = \frac{-1}{4}$

Sum of the zeroes $= (\alpha + \beta) = \frac{2}{3} + (\frac{-1}{4}) = \frac{8-3}{12} = \frac{5}{12}$

Product of the zeroes, $\alpha\beta = \frac{2}{3} \times (\frac{-1}{4}) = \frac{-2}{12} = \frac{-1}{6}$

$\therefore$ Required polynomial $= x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - \frac{5}{12}x + (\frac{-1}{6})$

$= x^2 - \frac{5}{12}x - \frac{1}{6}$

Sum of the zeroes $= \frac{5}{12} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= \frac{-1}{6} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

15. Sol:

Let α and β be the zeroes of the required polynomial f(x).

Then $(\alpha + \beta) = 8$ and $\alpha\beta = 12$

$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow f(x) = x^2 - 8x + 12$

Hence, required polynomial $f(x) = x^2 - 8x + 12$

$\therefore f(x) = 0 \Rightarrow x^2 - 8x + 12 = 0$

$\Rightarrow x^2 - (6x + 2x) + 12 = 0$

$\Rightarrow x^2 - 6x - 2x + 12 = 0$

$\Rightarrow x(x - 6) - 2(x - 6) = 0$

$\Rightarrow (x - 2)(x - 6) = 0$

$\Rightarrow (x - 2) = 0$ or $(x - 6) = 0$

$\Rightarrow$ x = 2 or x = 6

So, the zeroes of f(x) are 2 and 6.

16.

Sol:

Let α and β be the zeroes of the required polynomial f(x).

Then $(\alpha + \beta) = 0$ and $\alpha\beta = -1$

$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow f(x) = x^2 - 0x + (-1)$

$\Rightarrow f(x) = x^2 - 1$

Hence, required polynomial $f(x) = x^2 - 1$.

$\therefore f(x) = 0 \Rightarrow x^2 - 1 = 0$

$\Rightarrow (x + 1)(x - 1) = 0$

$\Rightarrow (x + 1) = 0$ or $(x - 1) = 0$

$\Rightarrow x = -1$ or $x = 1$

So, the zeroes of f(x) are -1 and 1.

17. Sol:

Let α and β be the zeroes of the required polynomial f(x).

Then $(\alpha + \beta) = \frac{5}{2}$ and $\alpha\beta = 1$

$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow f(x) = x^2 - \frac{5}{2}x + 1$

$\Rightarrow f(x) = 2x^2 - 5x + 2$

Hence, the required polynomial is $f(x) = 2x^2 - 5x + 2$

$\therefore f(x) = 0 \Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow 2x^2 - (4x + x) + 2 = 0$

$\Rightarrow 2x^2 - 4x - x + 2 = 0$

$\Rightarrow 2x(x - 2) - 1(x - 2) = 0$

$\Rightarrow (2x - 1)(x - 2) = 0$

$\Rightarrow (2x - 1) = 0$ or $(x - 2) = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = 2$

So, the zeros of f(x) are $\frac{1}{2}$ and 2.

18.

Sol:

We can find the quadratic equation if we know the sum of the roots and product of the roots by using the formula

x^2 – (Sum of the roots)x + Product of roots = 0

$\Rightarrow x^2 - \sqrt{2}x + \frac{1}{3} = 0$

$\Rightarrow 3x^2 - 3\sqrt{2}x + 1 = 0$

19.

Sol:

Given: $ax^2 + 7x + b = 0$

Since, $x = \frac{2}{3}$ is the root of the above quadratic equation

Hence, it will satisfy the above equation.

Therefore, we will get

$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$

$\Rightarrow \frac{4}{9}a + \frac{14}{3} + b = 0$

$\Rightarrow 4a + 42 + 9b = 0$

$\Rightarrow 4a + 9b = -42$ …(1)

Since, $x = -3$ is the root of the above quadratic equation

Hence, It will satisfy the above equation.

Therefore, we will get

$a(-3)^2 + 7(-3) + b = 0$

$\Rightarrow 9a - 21 + b = 0$

$\Rightarrow 9a + b = 21$ …..(2)

From (1) and (2), we get

$a = 3, b = -6$

20.

Sol:

Given: $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$

So, we have

$x + a = 0$

$\Rightarrow x = -a$

Now, it will satisfy the above polynomial.

Therefore, we will get

$2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

$\Rightarrow 2a^2 - 2a^2 - 5a + 10 = 0$

$\Rightarrow -5a = -10$

$\Rightarrow a = 2$

21. Sol:

Given: $x = \frac{2}{3}$ is one of the zero of $3x^3 + 16x^2 + 15x - 18$

Now, we have

$x = \frac{2}{3}$

$\Rightarrow x - \frac{2}{3} = 0$

Now, we divide $3x^3 + 16x^2 + 15x - 18$ by $x - \frac{2}{3}$ to find the quotient

Quotient: $3x^2 + 18x + 27$

Divisor: $x - \frac{2}{3}$) $3x^3 + 16x^2 + 15x - 18$

$3x^3 - 2x^2$

– +

$18x^2 + 15x$

$18x^2 - 12x$

– +

$27x - 18$

$27x - 18$

– +

X

So, the quotient is $3x^2 + 18x + 27$

Now,

$3x^2 + 18x + 27 = 0$

$\Rightarrow 3x^2 + 9x + 9x + 27 = 0$

$\Rightarrow 3x(x + 3) + 9(x + 3) = 0$

⇒ (x + 3) (3x + 9) = 0
⇒ (x + 3) = 0 or (3x + 9) = 0
⇒ x = –3 or x = –3

CLASS24

Exercise – 2B

1.

Sol:

The given polynomial is $p(x) = (x^3 – 2x^2 – 5x + 6)$

$\therefore p(3) = (3^3 – 2 \times 3^2 – 5 \times 3 + 6) = (27 – 18 – 15 + 6) = 0$

$p(-2) = [(– 2^3) – 2 \times (– 2)^2 – 5 \times (– 2) + 6] = (–8 –8 + 10 + 6) = 0$

$p(1) = (1^3 – 2 \times 1^2 – 5 \times 1 + 6) = (1 – 2 – 5 + 6) = 0$

∴ 3, –2 and 1are the zeroes of p(x),

Let $\alpha = 3, \beta = –2$ and $\gamma = 1$. Then we have:

$(\alpha + \beta + \gamma) = (3 – 2 + 1) = 2 = \frac{-(coefficient\ of\ x^2)}{(coefficient\ of\ x^3)}$

$(\alpha\beta + \beta\gamma + \gamma\alpha) = (–6 –2 + 3) = \frac{-5}{1} = \frac{coefficient\ of\ x}{coefficient\ of\ x^3}$

$\alpha\beta\gamma = \{3 \times (-2) \times 1\} = \frac{-6}{1} = \frac{-(constant\ term)}{(coefficient\ of\ x^3)}$

2. **Sol:**

$p(x) = (3x^3 – 10x^2 – 27x + 10)$

$p(5) = (3 \times 5^3 – 10 \times 5^2 – 27 \times 5 + 10) = (375 – 250 – 135 + 10) = 0$

$p(–2) = [3 \times (–2^3) – 10 \times (–2^2) – 27 \times (–2) + 10] = (–24 – 40 + 54 + 10) = 0$

$p(\frac{1}{3}) = \{3 \times (\frac{1}{3})^3 - 10 \times (\frac{1}{3})^2 - 27 \times \frac{1}{3} + 10\} = (3 \times \frac{1}{27} - 10 \times \frac{1}{9} - 9 + 10)$

$= (\frac{1}{9} - \frac{10}{9} + 1) = (\frac{1-10-9}{9}) = (\frac{0}{9}) = 0$

∴ 5, –2 and $\frac{1}{3}$ are the zeroes of p(x).

Let $\alpha = 5, \beta = –2$ and $\gamma = \frac{1}{3}$. Then we have:

$(\alpha + \beta + \gamma) = (5 - 2 + \frac{1}{3}) = \frac{10}{3} = \frac{-(coefficient\ of\ x^2)}{(coefficient\ of\ x^3)}$

$(\alpha\beta + \beta\gamma + \gamma\alpha) = (-10 - \frac{2}{3} + \frac{5}{3}) = \frac{-27}{3} = \frac{coefficient\ of\ x}{coefficient\ of\ x^3}$

$\alpha\beta\gamma = \{5 \times (-2) \times \frac{1}{3}\} = \frac{-10}{3} = \frac{-(constant\ term)}{(coefficient\ of\ x^3)}$

3.

Sol:

If the zeroes of the cubic polynomial are a, b and c then the cubic polynomial can be found as

$x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc$ ……(1)

Let a = 2, b = –3 and c = 4

Substituting the values in 1, we get

$x^3 - (2 - 3 + 4)x^2 + (-6 - 12 + 8)x - (-24)$

$\Rightarrow x^3 - 3x^2 - 10x + 24$

4.

Sol:

If the zeroes of the cubic polynomial are a, b and c then the cubic polynomial can be found as

$x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc$ ……(1)

Let $a = \frac{1}{2}$, b = 1 and c = –3

Substituting the values in (1), we get

$x^3 - (\frac{1}{2} + 1 - 3)x^2 + (\frac{1}{2} - 3 - \frac{3}{2})x - (\frac{-3}{2})$

$\Rightarrow x^3 - (\frac{-3}{2})x^2 - 4x + \frac{3}{2}$

$\Rightarrow 2x^3 + 3x^2 - 8x + 3$

5. **Sol:**

We know the sum, sum of the product of the zeroes taken two at a time and the product of the zeroes of a cubic polynomial then the cubic polynomial can be found as

x^3 – (sum of the zeroes)x^2 + (sum of the product of the zeroes taking two at a time)x – product of zeroes

Therefore, the required polynomial is

$x^3 - 5x^2 - 2x + 24$

6.

Sol:

$x - 3$

$x - 2 \;\big)\; x^3 - 3x^2 + 5x - 3$

$x^3 \quad - 2x$

$- \quad +$

$-3x^2 + 7x - 3$

$-3x^2 \quad + 6$

$+ \quad -$

$7x - 9$

Quotient q(x) = x – 3

Remainder r(x) = 7x – 9

7.

Sol:

$$
\begin{array}{r|l}
 & x^2 + x - 3 \\
\hline
x^2 - x + 1 & x^4 + 0x^3 - 3x^2 + 4x + 5 \\
 & x^4 - x^3 + x^2 \\
 & \quad - \quad + \quad - \\
\hline
 & x^3 - 4x^2 + 4x + 5 \\
 & x^3 - x^2 + x \\
 & - \quad + \quad - \\
\hline
 & -3x^2 + 3x + 5 \\
 & -3x^2 + 3x - 3 \\
 & + \quad - \quad + \\
\hline
 & 8 \\
\hline
\end{array}
$$

Quotient $q(x) = x^2 + x - 3$

Remainder $r(x) = 8$

8. Sol:

We can write

$f(x)$ as $x^4 + 0x^3 + 0x^2 - 5x + 6$ and $g(x)$ as $-x^2 + 2$

$$
\begin{array}{r|l}
 & -x^2 - 2 \\
\hline
-x^2 + 2 & x^4 + 0x^3 + 0x^2 - 5x + 6 \\
 & x^4 \qquad -2x^2 \\
 & - \qquad + \\
\hline
 & 2x^2 - 5x + 6 \\
 & 2x^2 \qquad 4 \\
 & - \qquad + \\
\hline
 & -5x + 10 \\
\hline
\end{array}
$$

Quotient $q(x) = -x^2 - 2$

Remainder $r(x) = -5x + 10$

9.

Sol:

Let $f(x) = 2x^4 + 3x^3 - 2x^2 - 9x - 12$ and $g(x)$ as $x^2 - 3$

Quotient: $2x^2 + 3x + 4$

Divisor: $x^2 - 3$

$2x^4 + 3x^3 - 2x^2 - 9x - 12$

$2x^4 \quad - 6x^2$

$- \quad +$

$3x^3 + 4x^2 - 9x - 12$

$3x^3 \quad - 9x$

$- \quad +$

$4x^2 - 12$

$4x^2 - 12$

$- \quad +$

x

Quotient $q(x) = 2x^2 + 3x + 4$

Remainder $r(x) = 0$

Since, the remainder is 0.

Hence, $x^2 - 3$ is a factor of $2x^4 + 3x^3 - 2x^2 - 9x - 12$

10.

Sol:

By using division rule, we have

Dividend = Quotient × Divisor + Remainder

$\therefore 3x^3 + x^2 + 2x + 5 = (3x - 5)g(x) + 9x + 10$

$\Rightarrow 3x^3 + x^2 + 2x + 5 - 9x - 10 = (3x - 5)g(x)$

$\Rightarrow 3x^3 + x^2 - 7x - 5 = (3x - 5)g(x)$

$\Rightarrow g(x) = \frac{3x^3 + x^2 - 7x - 5}{3x - 5}$

Quotient: $x^2 + 2x + 1$

Divisor: $3x - 5$

$3x^3 + x^2 - 7x - 5$

$3x^3 - 5x^2$

$- \quad +$

$6x^2 - 7x - 5$

$6x^2 - 10x$

$- \quad +$

$3x - 5$

$3x - 5$

$- \quad +$

X

$\therefore g(x) = x^2 + 2x + 1$

11. Sol:

We can write f(x) as $-6x^3 + x^2 + 20x + 8$ and g(x) as $-3x^2 + 5x + 2$

$x^2 + 2x + 1$

$-3x^2 + 5x + 2 \,\big)\, -6x^3 + x^2 + 20x + 8$

$-6x^3 + 10x^2 + 4x$

$+ \quad - \quad -$

$-9x^2 + 16x + 8$

$-9x^2 + 15x + 6$

$+ \quad - \quad -$

$x + 2$

Quotient = $2x + 3$

Remainder = $x + 2$

By using division rule, we have

Dividend = Quotient × Divisor + Remainder

$\therefore -6x^3 + x^2 + 20x + 8 = (-3x^2 + 5x + 2)(2x + 3) + x + 2$

$\Rightarrow -6x^3 + x^2 + 20x + 8 = -6x^3 + 10x^2 + 4x - 9x^2 + 15x + 6 + x + 2$

$\Rightarrow -6x^3 + x^2 + 20x + 8 = -6x^3 + x^2 + 20x + 8$

12. Sol:

Let $f(x) = x^3 + 2x^2 - 11x - 12$

Since -1 is a zero of f(x), (x+1) is a factor of f(x).

On dividing f(x) by (x+1), we get

$x + 1 \,\big)\, x^3 + 2x^2 - 11x - 12 \,\big(\, x^2 + x + 12$

$x^3 + x^2$

$- \quad -$

$x^2 - 11x - 12$

$x^2 + x$

$- \quad -$

$-12x - 12$

$-12x - 12$

$+ \quad +$

X

$f(x) = x^3 + 2x^2 - 11x - 12$
$= (x + 1)(x^2 + x - 12)$
$= (x + 1)\{x^2 + 4x - 3x - 12\}$
$= (x + 1)\{x(x+4) - 3(x+4)\}$
$= (x + 1)(x - 3)(x + 4)$

$\therefore f(x) = 0 \Rightarrow (x + 1)(x - 3)(x + 4) = 0$

$\Rightarrow (x + 1) = 0$ or $(x - 3) = 0$ or $(x + 4) = 0$

$\Rightarrow x = -1$ or $x = 3$ or $x = -4$

Thus, all the zeroes are – 1, 3 and – 4.

13.

Sol:

Let $f(x) = x^3 - 4x^2 - 7x + 10$

Since 1 and –2 are the zeroes of f(x), it follows that each one of (x–1) and (x+2) is a factor of f(x).

Consequently, $(x-1)(x+2) = (x^2 + x - 2)$ is a factor of f(x).

On dividing f(x) by $(x^2 + x - 2)$, we get:

$$
\begin{array}{r|l|l}
x^2 + x - 2 & x^3 - 4x^2 - 7x + 10 & x - 5 \\
 & x^3 + x^2 - 2x & \\
 & -\quad -\quad + & \\
\hline
 & -5x^2 - 5x + 10 & \\
 & -5x^2 - 5x + 10 & \\
 & +\quad +\quad - & \\
\hline
 & \times & \\
\hline
\end{array}
$$

$f(x) = 0 \Rightarrow (x^2 + x - 2)(x - 5) = 0$

$\Rightarrow (x - 1)(x + 2)(x - 5) = 0$

$\Rightarrow x = 1$ or $x = -2$ or $x = 5$

Hence, the third zero is 5.

14. Sol:

Let $x^4 + x^3 - 11x^2 - 9x + 18$

Since 3 and – 3 are the zeroes of f(x), it follows that each one of (x + 3) and (x – 3) is a factor of f(x).

Consequently, $(x - 3)(x + 3) = (x^2 - 9)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 9)$, we get:

$$
\begin{array}{r|l}
x^2 - 9 & x^4 + x^3 - 11x^2 - 9x + 18 \quad \left(x^2 + x - 2 \right. \\
 & x^4 \qquad - 9x^2 \\
 & - \qquad\quad + \\
\hline
 & x^3 - 2x^2 - 9x + 18 \\
 & x^3 \qquad - 9x \\
 & - \qquad\quad + \\
\hline
 & -2x^2 + 18 \\
 & -2x^2 + 18 \\
 & + \quad - \\
\hline
 & x \\
\hline
\end{array}
$$

$f(x) = 0 \Rightarrow (x^2 + x - 2)\,(x^2 - 9) = 0$

$\Rightarrow (x^2 + 2x - x - 2)\,(x - 3)\,(x + 3)$

$\Rightarrow (x - 1)\,(x + 2)\,(x - 3)\,(x + 3) = 0$

$\Rightarrow x = 1$ or $x = -2$ or $x = 3$ or $x = -3$

Hence, all the zeroes are 1, -2, 3 and -3.

15. Sol:

Let $f(x) = x^4 + x^3 - 34x^2 - 4x + 120$

Since 2 and -2 are the zeroes of f(x), it follows that each one of $(x - 2)$ and $(x + 2)$ is a factor of f(x).

Consequently, $(x - 2)\,(x + 2) = (x^2 - 4)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 4)$, we get:

$$
\begin{array}{r|l}
x^2 - 4 & x^4 + x^3 - 34x^2 - 4x + 120 \quad \left(x^2 + x - 2 \right. \\
 & x^4 \qquad - 4x^2 \\
 & - \qquad + \\
\hline
 & x^3 - 30x^2 - 4x + 120 \\
 & x^3 \qquad\quad -4x \\
 & - \qquad\quad + \\
\hline
 & -30x^2 + 120 \\
 & -30x^2 + 120 \\
 & + \qquad - \\
\hline
 & x \\
\hline
\end{array}
$$

$f(x) = 0$

$\Rightarrow (x^2 + x - 30)\,(x^2 - 4) = 0$

$\Rightarrow (x^2 + 6x - 5x - 30)(x - 2)(x + 2)$

$\Rightarrow [x(x + 6) - 5(x + 6)](x - 2)(x + 2)$

$\Rightarrow (x - 5)(x + 6)(x - 2)(x + 2) = 0$

$\Rightarrow x = 5$ or $x = -6$ or $x = 2$ or $x = -2$

Hence, all the zeroes are 2, -2, 5 and -6.

16. Sol:

Let $f(x) = x^4 + x^3 - 23x^2 - 3x + 60$

Since $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of f(x), it follows that each one of $(x - \sqrt{3})$ and $(x + \sqrt{3})$ is a factor of f(x).

Consequently, $(x - \sqrt{3})(x + \sqrt{3}) = (x^2 - 3)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 3)$, we get:

$$
\begin{array}{r|l|l}
x^2 - 3 & x^4 + x^3 - 23x^2 - 3x + 60 & x^2 + x - 20 \\
 & x^4 \quad\quad - 3x^2 & \\
 & - \quad\quad\quad + & \\
\hline
 & x^3 - 20x^2 - 3x + 60 & \\
 & x^3 \quad\quad\quad -3x & \\
 & - \quad\quad\quad\quad + & \\
\hline
 & -20x^2 + 60 & \\
 & -20x^2 + 60 & \\
 & + \quad\quad - & \\
\hline
 & x & \\
\hline
\end{array}
$$

$f(x) = 0$

$\Rightarrow (x^2 + x - 20)(x^2 - 3) = 0$

$\Rightarrow (x^2 + 5x - 4x - 20)(x^2 - 3)$

$\Rightarrow [x(x + 5) - 4(x + 5)](x^2 - 3)$

$\Rightarrow (x - 4)(x + 5)(x - \sqrt{3})(x + \sqrt{3}) = 0$

$\Rightarrow x = 4$ or $x = -5$ or $x = \sqrt{3}$ or $x = -\sqrt{3}$

Hence, all the zeroes are $\sqrt{3}$, $-\sqrt{3}$, 4 and -5.

17. Sol:

The given polynomial is $f(x) = 2x^4 - 3x^3 - 5x^2 + 9x - 3$

Since $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of f(x), it follows that each one of $(x - \sqrt{3})$ and $(x + \sqrt{3})$ is a factor of f(x).

Consequently, $(x - \sqrt{3})(x + \sqrt{3}) = (x^2 - 3)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 3)$, we get:

$$
\begin{array}{r|l|l}
x^2 - 3 & 2x^4 - 3x^3 - 5x^2 + 9x - 3 & 2x^2 - 3x + 1 \\
 & 2x^4 \quad\quad - 6x^2 & \\
 & - \quad\quad\quad + & \\
\hline
 & -3x^3 + x^2 + 9x - 3 & \\
 & -3x^3 \quad\quad + 9x & \\
 & + \quad\quad\quad - & \\
\hline
 & x^2 - 3 & \\
 & x^2 - 3 & \\
 & - \quad + & \\
\hline
 & x & \\
\hline
\end{array}
$$

$f(x) = 0$

$\Rightarrow 2x^4 - 3x^3 - 5x^2 + 9x - 3 = 0$

$\Rightarrow (x^2 - 3)(2x^2 - 3x + 1) = 0$

$\Rightarrow (x^2 - 3)(2x^2 - 2x - x + 1) = 0$

$\Rightarrow (x - \sqrt{3})(x + \sqrt{3})(2x - 1)(x - 1) = 0$

$\Rightarrow x = \sqrt{3}$ or $x = -\sqrt{3}$ or $x = \frac{1}{2}$ or $x = 1$

Hence, all the zeroes are $\sqrt{3}$, $-\sqrt{3}$, $\frac{1}{2}$ and 1.

18.

Sol:

The given polynomial is $f(x) = x^4 + 4x^3 - 2x^2 - 20x - 15$.

Since $(x - \sqrt{5})$ and $(x + \sqrt{5})$ are the zeroes of f(x) it follows that each one of $(x - \sqrt{5})$ and $(x + \sqrt{5})$ is a factor of f(x).

Consequently, $(x - \sqrt{5})(x + \sqrt{5}) = (x^2 - 5)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 5)$, we get:

$x^2 - 5 \,\big)\, x^4 + 4x^3 - 2x^2 - 20x - 15 \,\big(\, 2x^2 - 3x + 1$

$x^4 \quad - 5x^2$

$- \quad +$

$4x^3 + 3x^2 - 20x - 15$

$4x^3 \quad - 20x$

$- \quad +$

$3x^2 - 15$

$3x^2 - 15$

$- \quad +$

x

f(x) = 0

$\Rightarrow x^4 + 4x^3 - 7x^2 - 20x - 15 = 0$

$\Rightarrow (x^2 - 5)(x^2 + 4x + 3) = 0$

$\Rightarrow (x - \sqrt{5})(x + \sqrt{5})(x + 1)(x + 3) = 0$

$\Rightarrow x = \sqrt{5}$ or $x = -\sqrt{5}$ or $x = -1$ or $x = -3$

Hence, all the zeroes are $\sqrt{5}$, $-\sqrt{5}$, -1 and -3.

19. Sol:

The given polynomial is $f(x) = 2x^4 - 11x^3 + 7x^2 + 13x - 7$.

Since $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$ are the zeroes of f(x) it follows that each one of $(x + 3 + \sqrt{2})$ and $(x + 3 - \sqrt{2})$ is a factor of f(x).

Consequently, $[(x - (3 + \sqrt{2})] [(x - (3 - \sqrt{2})] = [(x - 3) - \sqrt{2}] [(x - 3) + \sqrt{2}]$
$= [(x - 3)^2 - 2] = x^2 - 6x + 7$, which is a factor of f(x).

On dividing f(x) by $(x^2 - 6x + 7)$, we get:

$x^2 - 6x + 7 \,\big)\, 2x^4 - 11x^3 + 7x^2 + 13x - 7 \,\big(\, 2x^2 + x - 1$

$2x^4 - 12x^3 + 14x^2$

$- \quad + \quad -$

$x^3 - 7x^2 + 13x - 7$

$x^3 - 6x^2 + 7x$

$- \quad + \quad -$

$-x^2 + 6x - 7$

$-x^2 + 6x - 7$

+ – +

x

f(x) = 0

⇒ $2x^4 - 11x^3 + 7x^2 + 13x - 7 = 0$

⇒ $(x^2 - 6x + 7)(2x^2 + x - 7) = 0$

⇒ $(x + 3 + \sqrt{2})(x + 3 - \sqrt{2})(2x - 1)(x + 1) = 0$

⇒ $x = -3 - \sqrt{2}$ or $x = -3 + \sqrt{2}$ or $x = \frac{1}{2}$ or $x = -1$

Hence, all the zeroes are $(-3 - \sqrt{2})$, $(-3 + \sqrt{2})$, $\frac{1}{2}$ and -1.

Exercise – 2C

1.

Sol:

Let the other zeroes of $x^2 - 4x + 1$ be a.

By using the relationship between the zeroes of the quadratic polynomial.

We have, sum of zeroes $= \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$

$\therefore 2 + \sqrt{3} + a = \frac{-(-4)}{1}$

⇒ $a = 2 - \sqrt{3}$

Hence, the other zeroes of $x^2 - 4x + 1$ is $2 - \sqrt{3}$.

2.

Sol:

$f(x) = x^2 + x - p(p + 1)$

By adding and subtracting px, we get

$f(x) = x^2 + px + x - px - p(p + 1)$

$= x^2 + (p + 1)x - px - p(p + 1)$

$= x[x + (p + 1)] - p[x + (p + 1)]$

$= [x + (p + 1)](x - p)$

f(x) = 0

⇒ $[x + (p + 1)](x - p) = 0$

⇒ $[x + (p + 1)] = 0$ or $(x - p) = 0$

⇒ $x = -(p + 1)$ or $x = p$

So, the zeroes of f(x) are $-(p + 1)$ and p.

3.

Sol:

$f(x) = x^2 - 3x - m(m + 3)$

By adding and subtracting mx, we get

$f(x) = x^2 - mx - 3x + mx - m(m + 3)$

$= x[x - (m + 3)] + m[x - (m + 3)]$

$= [x - (m + 3)](x + m)$

$f(x) = 0 \Rightarrow [x - (m + 3)](x + m) = 0$

$\Rightarrow [x - (m + 3)] = 0$ or $(x + m) = 0$

$\Rightarrow x = m + 3$ or $x = -m$

So, the zeroes of f(x) are –m and +3.

4.Sol:

If the zeroes of the quadratic polynomial are α and β then the quadratic polynomial can be found as $x^2 - (\alpha + \beta)x + \alpha\beta$ …..(1)

Substituting the values in (1), we get

$x^2 - 6x + 4$

5.

Sol:

Given: x = 2 is one zero of the quadratic polynomial $kx^2 + 3x + k$

Therefore, it will satisfy the above polynomial.

Now, we have

$k(2)^2 + 3(2) + k = 0$

$\Rightarrow 4k + 6 + k = 0$

$\Rightarrow 5k + 6 = 0$

$\Rightarrow k = -\frac{6}{5}$

6. Sol:

Given: x = 3 is one zero of the polynomial $2x^2 + x + k$

Therefore, it will satisfy the above polynomial.

Now, we have

$2(3)^2 + 3 + k = 0$

$\Rightarrow 21 + k = 0$

$\Rightarrow k = -21$

7.Sol:

Given: $x = -4$ is one zero of the polynomial $x^2 - x - (2k + 2)$
Therefore, it will satisfy the above polynomial.
Now, we have
$(-4)^2 - (-4) - (2k + 2) = 0$

$\Rightarrow 16 + 4 - 2k - 2 = 0$

$\Rightarrow 2k = -18$

$\Rightarrow k = 9$

8.

Sol:
Given: $x = 1$ is one zero of the polynomial $ax^2 - 3(a - 1)x - 1$
Therefore, it will satisfy the above polynomial.
Now, we have
$a(1)^2 - (a - 1)1 - 1 = 0$

$\Rightarrow a - 3a + 3 - 1 = 0$

$\Rightarrow -2a = -2$

$\Rightarrow a = 1$

9.

Sol:
Given: $x = -2$ is one zero of the polynomial $3x^2 + 4x + 2k$
Therefore, it will satisfy the above polynomial.
Now, we have
$3(-2)^2 + 4(-2)1 + 2k = 0$

$\Rightarrow 12 - 8 + 2k = 0$

$\Rightarrow k = -2$

10.

Sol:
$f(x) = x^2 - x - 6$
$= x^2 - 3x + 2x - 6$
$= x(x - 3) + 2(x - 3)$
$= (x - 3)(x + 2)$

$f(x) = 0 \Rightarrow (x - 3)(x + 2) = 0$

$\Rightarrow$ (x – 3) = 0 or (x + 2) = 0

$\Rightarrow$ x = 3 or x = –2

So, the zeroes of f(x) are 3 and –2.

11.

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes $= \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$

$\Rightarrow 1 = \frac{-(-3)}{k}$

$\Rightarrow$ k = 3

12.

Sol:

By using the relationship between the zeroes of he quadratic polynomial.

We have

Product of zeroes $= \frac{constant\ term}{coefficient\ of\ x^2}$

$\Rightarrow 3 = \frac{k}{1}$

$\Rightarrow$ k = 3

13.

Sol:

Given: (x + a) is a factor of $2x^2 + 2ax + 5x + 10$

We have

x + a = 0

$\Rightarrow$ x = –a

Since, (x + a) is a factor of $2x^2 + 2ax + 5x + 10$

Hence, It will satisfy the above polynomial

$\therefore 2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

$\Rightarrow -5a + 10 = 0$

$\Rightarrow$ a = 2

14.

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes $= \frac{-(coefficient\ of\ x^2)}{coefficient\ of\ x^3}$

$\Rightarrow$ a – b + a + a + b = $\frac{-(-6)}{2}$

$\Rightarrow$ 3a = 3

$\Rightarrow$ a = 1

15.

Sol:

Equating $x^2 - x$ to 0 to find the zeroes, we will get

x(x – 1) = 0

$\Rightarrow$ x = 0 or x – 1 = 0

$\Rightarrow$ x = 0 or x = 1

Since, $x^3 + x^2 - ax + b$ is divisible by $x^2 - x$.

Hence, the zeroes of $x^2 - x$ will satisfy $x^3 + x^2 - ax + b$

$\therefore (0)^3 + 0^2 - a(0) + b = 0$

$\Rightarrow$ b = 0

And

$(1)^3 + 1^2 - a(1) + 0 = 0 \quad [\because b = 0]$

$\Rightarrow$ a = 2

16. Sol:

By using the relationship between the zeroes of he quadratic polynomial.

We have

Sum of zeroes $= \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$ and Product of zeroes $= \frac{constant\ term}{coefficient\ of\ x^2}$

$\therefore \alpha + \beta = \frac{-7}{2}$ and $\alpha\beta = \frac{5}{2}$

Now, $\alpha + \beta + \alpha\beta = \frac{-7}{2} + \frac{5}{2} = -1$

17.

Sol:

"If f(x) and g(x) are two polynomials such that degree of f(x) is greater than degree of g(x) where g(x) ≠ 0, there exists unique polynomials q(x) and r(x) such that

$$f(x) = g(x) \times q(x) + r(x),$$

where r(x) = 0 or degree of r(x) < degree of g(x).

18. Sol:

We can find the quadratic polynomial if we know the sum of the roots and product of the roots by using the formula

x^2 – (sum of the zeroes)x + product of zeroes

$\Rightarrow x^2 - \left(-\frac{1}{2}\right)x + (-3)$

$\Rightarrow x^2 + \frac{1}{2}x - 3$

Hence, the required polynomial is $x^2 + \frac{1}{2}x - 3$.

19.

Sol:

To find the zeroes of the quadratic polynomial we will equate f(x) to 0

$\therefore f(x) = 0$

$\Rightarrow 6x^2 - 3 = 0$

$\Rightarrow 3(2x^2 - 1) = 0$

$\Rightarrow 2x^2 - 1 = 0$

$\Rightarrow 2x^2 = 1$

$\Rightarrow x^2 = \frac{1}{2}$

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$

Hence, the zeroes of the quadratic polynomial $f(x) = 6x^2 - 3$ are $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$.

20.

Sol:

To find the zeroes of the quadratic polynomial we will equate f(x) to 0

$\therefore f(x) = 0$

$\Rightarrow 4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$\Rightarrow (\sqrt{3}x + 2) = 0$ or $(4x - \sqrt{3}) = 0$

$\Rightarrow x = -\frac{2}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{4}$

Hence, the zeroes of the quadratic polynomial $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are $-\frac{2}{\sqrt{3}}$ or $\frac{\sqrt{3}}{4}$

21.

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes $= \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$ and Product of zeroes $= \frac{constant\ term}{coefficient\ of\ x^2}$

$\therefore \alpha + \beta = \frac{-(-5)}{1}$ and $\alpha\beta = \frac{k}{1}$

$\Rightarrow \alpha + \beta = 5$ and $\alpha\beta = \frac{k}{1}$

Solving $\alpha - \beta = 1$ and $\alpha + \beta = 5$, we will get

$\alpha = 3$ and $\beta = 2$

Substituting these values in $\alpha\beta = \frac{k}{1}$, we will get

k = 6

22.

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes $= \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$ and Product of zeroes $= \frac{constant\ term}{coefficient\ of\ x^2}$

$\therefore \alpha + \beta = \frac{-1}{6}$ and $\alpha\beta = -\frac{1}{3}$

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{-1}{6}\right)^2 - 2\left(-\frac{1}{3}\right)}{-\frac{1}{3}}$

$= \frac{\frac{1}{36} + \frac{2}{3}}{-\frac{1}{3}}$

$= -\frac{25}{12}$

23. 1 —

Sol:

By using the relationship between the zeroes of he quadratic polynomial.

We have

Sum of zeroes = $\frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$ and Product of zeroes = $\frac{constant\ term}{coefficient\ of\ x^2}$

$\therefore \alpha + \beta = \frac{-(-7)}{5}$ and $\alpha\beta = \frac{1}{5}$

$\Rightarrow \alpha + \beta = \frac{7}{5}$ and $\alpha\beta = \frac{1}{5}$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{\frac{7}{5}}{\frac{1}{5}}$

$= 7$

24. Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes = $\frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$ and Product of zeroes = $\frac{constant\ term}{coefficient\ of\ x^2}$

$\therefore \alpha + \beta = \frac{-1}{1}$ and $\alpha\beta = \frac{-2}{1}$

$\Rightarrow \alpha + \beta = -1$ and $\alpha\beta = -2$

Now, $\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 = \left(\frac{\beta-\alpha}{\alpha\beta}\right)^2$

$= \frac{(\alpha+\beta)^2 - 4\alpha}{(\alpha\beta)^2}$ $\quad [\because (\beta-\alpha)^2 = (\alpha+\beta)^2 - 4\alpha\beta]$

$= \frac{(-1)^2 - 4(-2)}{(-2)^2}$ $\quad [\because \alpha + \beta = -1$ and $\alpha\beta = -2]$

$= \frac{(-1)^2 - 4(-2)}{4}$

$= \frac{9}{4}$

$\because \left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 = \frac{9}{4}$

$\Rightarrow \frac{1}{\alpha} - \frac{1}{\beta} = \pm\frac{3}{2}$

25.

Sol:

By using the relationship between the zeroes of he quadratic polynomial.

We have, Sum of zeroes = $\frac{-(coefficient\ of\ x^2)}{coefficient\ of\ x^3}$

$\therefore$ a – b + a + a + b = $\frac{-(-3)}{1}$

$\Rightarrow$ 3a = 3

$\Rightarrow a = 1$

Now, Product of zeroes $= \frac{-(constant\ term)}{coefficient\ of\ x^3}$

$\therefore (a - b)(a)(a + b) = \frac{-1}{1}$

$\Rightarrow (1 - b)(1)(1 + b) = -1 \quad [\because a = 1]$

$\Rightarrow 1 - b^2 = -1$

$\Rightarrow b^2 = 2$

$\Rightarrow b = \pm\sqrt{2}$