Exercise – 4A

1.

Sol:

(i) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we get:

$\frac{AD}{DB} = \frac{AE}{EC}$

∵ AD = 3.6 cm , AB = 10 cm, AE = 4.5cm

∴ DB = 10 – 3.6 = 6.4cm

Or, $\frac{3.6}{6.4} = \frac{4.5}{EC}$

Or, EC = $\frac{6.4 \times 4.5}{3.6}$

Or, EC= 8 cm

Thus, AC = AE + EC

= 4.5 + 8 = 12.5 cm

(ii) In Δ ABC, it is given that DE || BC.

Applying Thales' Theorem, we get :

$\frac{AD}{DB} = \frac{AE}{EC}$

Adding 1 to both sides, we get :

$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$

$\Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$

$\Rightarrow \frac{13.3}{DB} = \frac{11.9}{5.1}$

$\Rightarrow DB = \frac{13.3 \times 5.1}{11.9} = 5.7$ cm

Therefore, AD=AB-DB=13.5-5.7=7.6 cm

(iii) In Δ ABC, it is given that DE || BC.

Applying Thales' theorem, we get :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{7} = \frac{AE}{EC}$

Adding 1 to both the sides, we get :

$\frac{11}{7} = \frac{AC}{EC}$

$\Rightarrow EC = \frac{6.6 \times 7}{11} = 4.2$ cm

Therefore,

AE = AC – EC = 6.6 – 4.2 = 2.4 cm

(iv) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we get:

$\frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{8}{15} = \frac{AE}{AE+EC}$

$\Rightarrow \frac{8}{15} = \frac{AE}{AE+3.5}$

$\Rightarrow$ 8AE + 28 = 15AE

$\Rightarrow$ 7AE = 28

$\Rightarrow$ AE = 4cm

2. Sol:

(i) In Δ ABC, it is given that DE || BC.

Applying Thales' theorem, we have :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow$X(x-1) = (x-2) (x+2)

$\Rightarrow x^2 - x = x^2 - 4$

$\Rightarrow$x= 4 cm

(ii) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we have :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{x-4} = \frac{8}{3x-19}$

$\Rightarrow$ 4 (3x-19) = 8 (x-4)

$\Rightarrow$12x – 76 = 8x – 32

$\Rightarrow$4x = 44

$\Rightarrow$ x = 11 cm

(iii) In Δ ABC, it is given that DE || BC.

Applying Thales' theorem, we have :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$

$\Rightarrow$3x (7x-4) = (5x-2) (3x+4)

$\Rightarrow 21x^2 - 12x = 15x^2 + 14x - 8$
$\Rightarrow 6x^2 - 26x + 8 = 0$
$\Rightarrow (x-4)(6x-2) = 0$
$\Rightarrow x = 4, \frac{1}{3}$
$\because x \neq \frac{1}{3}$ (as if $x = \frac{1}{3}$ $then\ AE\ will\ become\ negative$)
$\therefore x = 4$ cm

3. **Sol**:

(i) We have:
$\frac{AD}{DE} = \frac{5.7}{9.5} = 0.6\ cm$
$\frac{AE}{EC} = \frac{4.8}{8} = 0.6\ cm$
Hence, $\frac{AD}{DB} = \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE || BC.

(ii) We have:
AB = 11.7 cm, DB = 6.5 cm
Therefore,
AD = 11.7 -6.5 = 5.2 cm
Similarly,
AC = 11.2 cm, AE = 4.2 cm
Therefore,
EC = 11.2 – 4.2 = 7 cm
Now,
$\frac{AD}{DB} = \frac{5.2}{6.5} = \frac{4}{5}$
$\frac{AE}{EC} = \frac{4.2}{7}$
Thus, $\frac{AD}{DB} \neq \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE is not parallel to BC.

(iii) We have:
AB = 10.8 cm, AD = 6.3 cm

Therefore,

DB = 10.8 – 6.3 = 4.5 cm
Similarly,
AC = 9.6 cm, EC = 4cm
Therefore,
AE = 9.6 – 4 = 5.6 cm
Now,
$\frac{AD}{DB} = \frac{6.3}{4.5} = \frac{7}{5}$
$\frac{AE}{EC} = \frac{5.6}{4} = \frac{7}{5}$
$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE || BC.

(iv) We have :
AD = 7.2 cm, AB = 12 cm
Therefore,
DB = 12 – 7.2 = 4.8 cm
Similarly,
AE = 6.4 cm, AC = 10 cm
Therefore,
EC = 10 – 6.4 = 3.6 cm
Now,
$\frac{AD}{DB} = \frac{7.2}{4.8} = \frac{3}{2}$
$\frac{AE}{EC} = \frac{6.4}{3.6} = \frac{16}{9}$
This, $\frac{AD}{DB} \neq \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE is not parallel to BC.

4.

Sol:

(i) It is give that AD bisects ∠A.
Applying angle – bisector theorem in Δ ABC, we get:
$\frac{BD}{DC} = \frac{AB}{AC}$
$\Rightarrow \frac{5.6}{DC} = \frac{6.4}{8}$
$\Rightarrow$DC $= \frac{8 \times 5.6}{6.4} = 7\ cm$

(ii) It is given that AD bisects $\angle A$.
Applying angle – bisector theorem in Δ ABC, we get:
$\frac{BD}{DC} = \frac{AB}{AC}$
Let BD be x cm.
Therefore, DC = (6- x) cm
$\Rightarrow \frac{x}{6-x} = \frac{10}{14}$
$\Rightarrow 14x = 60\text{-}10x$
$\Rightarrow 24x = 60$
$\Rightarrow x = 2.5$ cm
Thus, BD = 2.5 cm
DC = 6-2.5 = 3.5 cm

(iii) It is given that AD bisector $\angle A$.
Applying angle – bisector theorem in Δ ABC, we get:
$\frac{BD}{DC} = \frac{AB}{AC}$
BD = 3.2 cm, BC = 6 cm
Therefore, DC = 6- 3.2 = 2.8 cm
$\Rightarrow \frac{3.2}{2.8} = \frac{5.6}{AC}$
$\Rightarrow AC = \frac{5.6 \times 2.8}{3.2} = 4.9\ cm$

(iv) It is given that AD bisects $\angle A$.
Applying angle – bisector theorem in Δ ABC, we get:
$\frac{BD}{DC} = \frac{AB}{AC}$
$\Rightarrow \frac{BD}{3} = \frac{5.6}{4}$
$\Rightarrow BD = \frac{5.6 \times 3}{4}$
$\Rightarrow BD = 4.2$ cm
Hence, BC = 3+ 4.2 = 7.2 cm

5.

Sol:
(i) Given: ABCD is a parallelogram

To prove:

(i) $\frac{DM}{MN} = \frac{DC}{BN}$

(ii) $\frac{DN}{DM} = \frac{AN}{DC}$

Proof: In Δ DMC and Δ NMB

∠DMC = ∠NMB (Vertically opposite angle)

∠DCM = ∠NBM (Alternate angles)

By AAA- Similarity

ΔDMC ~ ΔNMB

$\therefore \frac{DM}{MN} = \frac{DC}{BN}$

NOW, $\frac{MN}{DM} = \frac{BN}{DC}$

Adding 1 to both sides, we get

$\frac{MN}{DM} + 1 = \frac{BN}{DC} + 1$

$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+DC}{DC}$

$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+AB}{DC}$ [∵ ABCD is a parallelogram]

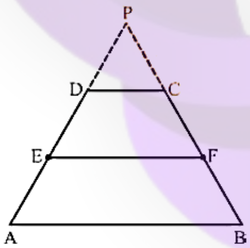
$\Rightarrow \frac{DN}{DM} = \frac{AN}{DC}$

6.

Sol:

(i)

Let the trapezium be ABCD with E and F as the mid Points of AD and BC, Respectively Produce AD and BC to Meet at P.



In Δ PAB, DC || AB.

Applying Thales' theorem, we get

$\frac{PD}{DA} = \frac{PC}{CB}$

Now, E and F are the midpoints of AD and BC, respectively.

$\Rightarrow \frac{PD}{2DE} = \frac{PC}{2CF}$

$\Rightarrow \frac{PD}{DE} = \frac{PC}{CF}$

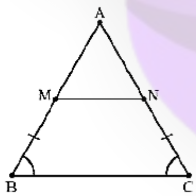
Applying the converse of Thales' theorem in Δ PEF, we get that DC
Hence, EF || AB.
Thus. EF is parallel to both AB and DC.
This completes the proof.

7.

Sol:
In trapezium ABCD, AB ∥ CD and the diagonals AC and BD intersect at O.
Therefore,
$\frac{AO}{OC} = \frac{BO}{OD}$
$\Rightarrow \frac{5x-7}{2x+1} = \frac{7x-5}{7x+1}$
$\Rightarrow (5x-7)(7x+1) = (7x-5)(2x+1)$
$\Rightarrow 35x^2 + 5x - 49x - 7 = 14x^2 - 10x + 7x - 5$
$\Rightarrow 21x^2 - 41x - 2 = 0$
$\Rightarrow 21x^2 - 42x + x - 2 = 0$
$\Rightarrow 21x(x-2) + 1(x-2) = 0$
$\Rightarrow (x-2)(21x + 1) = 0$
$\Rightarrow x = 2, -\frac{1}{21}$
$\because x \neq -\frac{1}{21}$
$\therefore\ x = 2$

8. I

Sol:



In ΔABC, ∠B = ∠C
∴ AB = AC (Sides opposite to equal angle are equal)
Subtracting BM from both sides, we get

AB – BM = AC – BM

⟹AB – BM = AC – CN (∵BM =CN)

⟹AM =AN

∴∠AMN =∠ ANM (Angles opposite to equal sides are equal)

Now, in ΔABC,

$\angle A + \angle B + \angle C = 180^0$ ------------------(1)

(Angle Sum Property of triangle)

Again In In ΔAMN,

$\angle A + \angle AMN + \angle ANM = 180^0$ ------- (2)

(Angle Sum Property of triangle)

From (1) and (2), we get

$\angle B + \angle C = \angle AMN + \angle ANM$

$\Rightarrow 2\angle B = 2\angle AMN$

$\Rightarrow \angle B = \angle AMN$

Since, ∠B and ∠ AMN are corresponding angles.

∴ MN || BC.

9.

Sol:

In Δ CAB, PQ || AB.

Applying Thales' theorem, we get:

$\frac{CP}{PB} = \frac{CQ}{QA}$ …(1)

Similarly, applying Thales theorem in ΔBDC, Where PR||DM we get:

$\frac{CP}{PB} = \frac{CR}{RD}$ …(2)

Hence, from (1) and (2), we have :

$\frac{CQ}{QA} = \frac{CR}{RD}$

Applying the converse of Thales' theorem, we conclude that QR ∥ AD in Δ ADC.

This completes the proof.

10.

Sol:

It is give that BC is bisected at D.

∴ BD = DC

It is also given that OD =OX

The diagonals OX and BC of quadrilateral BOCX bisect each other.

Therefore, BOCX is a parallelogram.

∴ BO || CX and BX || CO

BX || CF and CX || BE

BX || OF and CX || OE

Applying Thales' theorem in Δ ABX, we get:

$\frac{AO}{AX} = \frac{AF}{AB}$ ...(1)

Also, in Δ ACX, CX || OE.

Therefore by Thales' theorem, we get:

$\frac{AO}{AX} = \frac{AE}{AC}$ ...(2)

From (1) and (2), we have:

$\frac{AO}{AX} = \frac{AE}{AC}$

Applying the converse of Theorem in Δ ABC, EF || CB.

This completes the proof.

11.

Sol:

We know that the diagonals of a parallelogram bisect each other.

Therefore,

CS = $\frac{1}{2}$ AC ...(i)

Also, it is given that CQ = $\frac{1}{4}$ AC ...(ii)

Dividing equation (ii) by (i), we get:

$\frac{CQ}{CS} = \frac{\frac{1}{4}AC}{\frac{1}{2}AC}$

Or, CQ = $\frac{1}{2}$ CS

Hence, Q is the midpoint of CS.

Therefore, according to midpoint theorem in ΔCSD
PQ || DS
If PQ || DS, we can say that QR || SB
In Δ CSB, Q is midpoint of CS and QR ∥ SB.
Applying converse of midpoint theorem , we conclude that R is the midpoint of CB.
This completes the proof.

12.

Sol:
Given:
AD = AE ...(i)
AB = AC ...(ii)
Subtracting AD from both sides, we get:
⟹ AB – AD = AC – AD
⟹ AB – AD = AC - AE (Since, AD = AE)
⟹ BD = EC ...(iii)
Dividing equation (i) by equation (iii), we get:
$\frac{AD}{DB} = \frac{AE}{EC}$
Applying the converse of Thales' theorem, DE∥BC
⟹ ∠DEC + ∠ECB = 180^0 (Sum of interior angles on the same side of a Transversal Line is 0^0 .)
⟹ ∠DEC + ∠CBD = 180^0 (Since, AB = AC ⟹ ∠B = ∠C)
Hence, quadrilateral BCED is cyclic.
Therefore, B,C,E and D are concylic points.

13.

Sol:
In triangle BQO, BR bisects angle B.
Applying angle bisector theorem, we get:
$\frac{QR}{PR} = \frac{BQ}{BP}$
⟹BP × QR = BQ × PR
This completes the proof.

Exercise – 4B

1.

Sol:

(i)

We have:

$\angle BAC = \angle PQR = 50^0$

$\angle ABC = \angle QPR = 60^0$

$\angle ACB = \angle PRQ = 70^0$

Therefore, by AAA similarity theorem, Δ ABC – QPR

(ii)

We have:

$\frac{AB}{DF} = \frac{3}{6} = \frac{1}{2}$ and $\frac{BC}{DE} = \frac{4.5}{9} = \frac{1}{2}$

But , $\angle ABC \neq \angle EDF$ (Included angles are not equal)

Thus, this triangles are not similar.

(iii)

We have:

$\frac{CA}{QR} = \frac{8}{6} = \frac{4}{3}$ and $\frac{CB}{PQ} = \frac{6}{4.5} = \frac{4}{3}$

$\Rightarrow \frac{CA}{QR} = \frac{CB}{PQ}$

Also, $\angle ACB = \angle PQR = 80^0$

Therefore, by SAS similarity theorem, Δ ACB - Δ RQP.

(iv)

We have

$\frac{DE}{QR} = \frac{2.5}{5} = \frac{1}{2}$

$\frac{EF}{PQ} = \frac{2}{4} = \frac{1}{2}$

$\frac{DF}{PR} = \frac{3}{6} = \frac{1}{2}$

$\Longrightarrow \frac{DE}{QR} = \frac{EF}{PQ} = \frac{DF}{PR}$

Therefore, by SSS similarity theorem, Δ FED- Δ PQR

(v)

In Δ ABC

$\angle A + \angle B + \angle C = 180^0$ (Angle Sum Property)

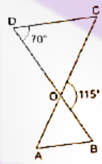
$\Longrightarrow 80^0 + \angle B + 70^0 = 180^0$

$\Longrightarrow \angle B = 30^0$

$\angle A = \angle M$ and $\angle B = \angle N$

Therefore, by AA similarity , Δ ABC - Δ MNR

2.



Sol:

(i)

It is given that DB is a straight line.

Therefore,

$\angle DOC + \angle COB = 180^0$

$\angle DOC = 180^0 - 115^0 = 65^0$

(ii)

In Δ DOC, we have:

$\angle ODC + \angle DCO + \angle DOC = 180^0$

Therefore,

$70^0 + \angle DCO + 65^0 = 180^0$

$\Longrightarrow \angle DCO = 180 - 70 - 65 = 45^0$

(iii)

It is given that Δ ODC - Δ OBA

Therefore,

$\angle OAB = \angle OCD = 45^0$

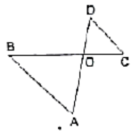
(iv)

Again, Δ ODC- Δ OBA

Therefore,

$\angle OBA = \angle ODC = 70^0$

3.



Sol:

(i) Let OA be X cm.

$\because \Delta\, OAB \sim \Delta\, OCD$

$\therefore \frac{OA}{OC} = \frac{AB}{CD}$

$\Rightarrow \frac{x}{3.5} = \frac{8}{5}$

$\Rightarrow x = \frac{8 \times 3.5}{5} = 5.6$

Hence, OA = 5.6 cm

(ii) Let OD be Y cm

$\because \Delta\, OAB \sim \Delta\, OCD$

$\therefore \frac{AB}{CD} = \frac{OB}{OD}$

$\Rightarrow \frac{8}{5} = \frac{6.4}{y}$

$\Rightarrow y = \frac{6.4 \times 5}{8} = 4$

Hence, DO = 4 cm

4.

Sol:

Given :

$\angle ADE = \angle ABC \text{ and } \angle A = \angle A$

Let DE be X cm

Therefore, by AA similarity theorem, $\Delta\, ADE \sim \Delta\, ABC$

$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \frac{3.8}{3.6+2.1} = \frac{x}{4.2}$

$\Rightarrow x = \frac{3.8 \times 4.2}{5.7} = 2.8$

Hence, DE = 2.8 cm

5.

Sol:

It is given that triangles ABC and PQR are similar.

Therefore,

$\frac{Perimeter\ (\Delta ABC)}{Perimeter\ (\Delta PQR)} = \frac{AB}{PQ}$

$\Rightarrow \frac{32}{24} = \frac{AB}{12}$

$\Rightarrow AB = \frac{32 \times 12}{24} = 16\ cm$

6.

Sol:
It is given that Δ ABC - Δ DEF.
Therefore, their corresponding sides will be proportional.
Also, the ratio of the perimeters of similar triangles is same as the ratio of their corresponding sides.

$\Rightarrow \frac{Perimeter\ of\ \Delta ABC}{Perimeter\ of\ \Delta DEF} = \frac{BC}{EF}$

Let the perimeter of ΔABC be X cm
Therefore,

$\frac{x}{25} = \frac{9.1}{6.5}$

$\Rightarrow x = \frac{9.1 \times 25}{6.5} = 35$

Thus, the perimeter of ΔABC is 35 cm.

7.
Sol:
In Δ BDA and Δ BAC, we have :

$\angle BDA = \angle BAC = 90^0$

$\angle DBA = \angle CBA \quad (Common)$

Therefore, by AA similarity theorem, Δ BDA - Δ BAC

$\Rightarrow \frac{AD}{AC} = \frac{AB}{BC}$

$\Rightarrow \frac{AD}{0.75} = \frac{1}{1.25}$

$\Rightarrow$ AD = $\frac{0.75}{1.25}$

= 0.6 m or 60 cm

8.

Sol:
It is given that ABC is a right angled triangle and BD is the altitude drawn from the right angle to the hypotenuse.
In Δ BDC and Δ ABC, we have :

$\angle ABC = \angle BBC = 90^0\ (given)$

$\angle C = \angle C\ (common)$

By AA similarity theorem, we get :

Δ BDC- Δ ABC

$\frac{AB}{BD} = \frac{BC}{DC}$

$\Rightarrow \frac{5.7}{3.8} = \frac{BC}{5.4}$

$\Rightarrow BC = \frac{5.7}{3.8} \times 5.4$

$= 8.1$

Hence, BC = 8.1 cm

9.

Sol:

It is given that ABC is a right angled triangle

and BD is the altitude drawn from the right angle to the hypotenuse.

In Δ DBA and Δ DCB, we have :

$\angle BDA = \angle CDB$

$\angle DBA = \angle DCB = 90^0$

Therefore, by AA similarity theorem, we get :

ΔDBA - Δ DCB

$\Rightarrow \frac{BD}{CD} = \frac{AD}{BD}$

$\Rightarrow CD = \frac{BD^2}{AD}$

CD = $\frac{8\times8}{4} = 16\ cm$

10.

Sol:

We have :

$\frac{AP}{AB} = \frac{2}{6} = \frac{1}{3}\ and \frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$

In Δ APQ and Δ ABC, we have:

$\frac{AP}{AB} = \frac{AQ}{AC}$

$\angle A = \angle A$

Therefore, by AA similarity theorem, we get:

Δ APQ - Δ ABC

Hence, $\frac{PQ}{BC} = \frac{AQ}{AC} = \frac{1}{3}$

$\Rightarrow \frac{PQ}{BC} = \frac{1}{3}$

$\Rightarrow$ BC = 3PQ

This completes the proof.

11.

Sol:

We have:

$\angle AFD = \angle EFB$ (*Vertically Opposite angles*)

$\because$ DA || BC

$\therefore \angle DAF = \angle BEF$ (*Alternate angles*)

Δ DAF ~ Δ BEF (AA similarity theorem)

$\Rightarrow \frac{AF}{EF} = \frac{FD}{FB}$

Or, AF × FB = FD × EF

This completes the proof.

12. 1

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Sol:

In ΔBED and ΔACB, we have:

$\angle BED = \angle ACB = 90^0$

$\because \angle B + \angle C = 180^0$

$\therefore$ BD || AC

$\angle EBD = \angle CAB$ (*Alternate angles*)

Therefore, by AA similarity theorem, we get :

Δ BED ~ Δ ACB

$\Rightarrow \frac{BE}{AC} = \frac{DE}{BC}$

$\Rightarrow \frac{BE}{DE} = \frac{AC}{BC}$

This completes the proof.

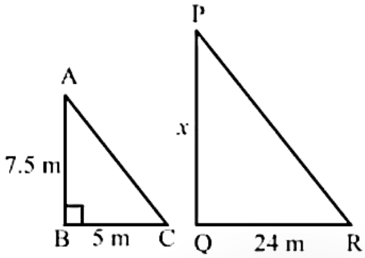
13.

Sol:

Let AB be the vertical stick and BC be its shadow.

Given:

AB = 7.5 m, BC = 5 m



Let PQ be the tower and QR be its shadow.

Given:

QR = 24 m

Let the length of PQ be x m.

In Δ ABC and Δ PQR, we have:

$\angle ABC = \angle PQR = 90^0$

$\angle ACB = \angle PRQ$ $(Angular\ elevation\ of\ the\ Sun\ at\ the\ same\ time)$

Therefore, by AA similarity theorem , we get :

Δ ABC ~ Δ PQR

$\Rightarrow \frac{AB}{BC} = \frac{PQ}{QR}$

$\Rightarrow \frac{7.5}{5} = \frac{x}{24}$

$x = \frac{7.5}{5} \times 24 = 36cm$

Therefore, PQ = 36 m

Hence, the height of the tower is 36 m.

14.

Sol:

Disclaimer: It should be ΔAPC ~ ΔBCQ instead of ΔACP ~ ΔBCQ

It is given that ΔABC is an isosceles triangle.

Therefore,

CA = CB

$\Rightarrow \angle CAB = \angle CBA$

$\Rightarrow 180^0 - \angle CAB = 180^0 - \angle CBA$

$\Rightarrow \angle CAP = \angle CBQ$

Also,

AP × BQ = AC^2

$\Rightarrow \frac{AP}{AC} = \frac{AC}{BQ}$

$\Rightarrow \frac{AP}{AC} = \frac{BC}{BQ}$ ($\because AC = BC$)

Thus, by SAS similarity theorem, we get

$\Delta APC \sim \Delta BCQ$

This completes the proof.

15.

Sol:

We have :

$\frac{AC}{BD} = \frac{CB}{CE}$

$\Rightarrow \frac{AC}{CB} = \frac{BD}{CE}$

$\Rightarrow \frac{AC}{CB} = \frac{CD}{CE}$ $(Since, BD = DC \ as \ \angle 1 = \angle 2)$

Also, $\angle 1 = \angle 2$

i.e, $\angle DBC = \angle ACB$

Therefore, by SAS similarity theorem, we get :

Δ ACB - Δ DCE

16.

Sol:

In Δ ABC, P and Q are mid points of AB and AC respectively.

So, PQ || BC, and PQ = $\frac{1}{2} BC$ …(1)

Similarly, in ΔADC, …(2)

Now, in ΔBCD, SR $= \frac{1}{2} BC$ …(3)

Similarly, in ΔABD, PS = $\frac{1}{2} AD = \frac{1}{2} BC$ …(4)

Using (1), (2), (3), and (4).

PQ = QR = SR = PS

Since, all sides are equal

Hence, PQRS is a rhombus.

17. I

Sol:

Given : AB and CD are two chords

To Prove:

(a) Δ PAC ~ ΔPDB

(b) PA.PB = PC.PD

Proof: In Δ PAC and Δ PDB

$\angle APC = \angle DPB$ (*Vertically Opposite angles*)

$\angle CAP = \angle BDP$ (*Angles in the same segment are equal*)

by AA similarity criterion $\Delta PAC \sim PDB$

When two triangles are similar, then the ratios of lengths of their corresponding sides are proportional.

$\therefore \frac{PA}{PD} = \frac{PC}{PB}$

⟹ PA. PB = PC. PD

18. **Sol:**

Given : AB and CD are two chords

To Prove:

(a) Δ PAC - Δ PDB

(b) PA. PB = PC.PD

Proof: $\angle ABD + \angle ACD = 180^0$ …(1) (Opposite angles of a cyclic quadrilateral are supplementary)

$\angle PCA + \angle ACD = 180^0$ …(2) (Linear Pair Angles)

Using (1) and (2), we get

$\angle ABD = \angle PCA$

$\angle A = \angle A$ (Common)

By AA similarity-criterion Δ PAC - Δ PDB

When two triangles are similar, then the rations of the lengths of their co

sides are proportional.

$\therefore \frac{PA}{PD} = \frac{PC}{PB}$

$\Rightarrow$ PA.PB = PC.PD

19. 1

Sol:

We know that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then the triangles on the both sides of the perpendicular are similar to the whole triangle and also to each other.

(a) Now using the same property in In ΔBDC, we get

ΔCQD ~ ΔDQB

$\frac{CQ}{DQ} = \frac{DQ}{QB}$

$\Rightarrow DQ^2 = QB.CQ$

Now. Since all the angles in quadrilateral BQDP are right angles.

Hence, BQDP is a rectangle.

So, QB = DP and DQ = PB

$\therefore DQ^2 = DP.CQ$

(b)

Similarly, ΔAPD ~ ΔDPB

$\frac{AP}{DP} = \frac{PD}{PB}$

$\Rightarrow DP^2 = AP.PB$

$\Rightarrow DP^2 = AP.DQ$ $[\because DQ = PB]$

Exercise – 4C

1.

Sol:

It is given that Δ ABC ~ Δ DEF.

Therefore, ratio of the areas of these triangles will be equal to the ration of squares of their corresponding sides.

$\frac{ar\ (\Delta ABC)}{ar\ (\Delta DEF)} = \frac{BC^2}{EF^2}$

Let BC be X cm.

$\Rightarrow \frac{64}{121} = \frac{x^2}{(15.4)^2}$

$\Rightarrow x^2 = \frac{64 \times 15.4 \times 15.4}{121}$

$\Rightarrow x = \sqrt{\frac{(64 \times 15.4 \times 15.4)}{121}}$

$= \frac{8 \times 15.4}{11}$

$= 11.2$

Hence, BC = 11.2 cm

2.

Sol:

It is given that Δ ABC ~ Δ PQR

Therefore, the ration of the areas of triangles will be equal to the ratio of squares of their corresponding sides.

$\frac{ar\ (\Delta ABC)}{ar\ (\Delta PQR)} = \frac{BC^2}{QR^2}$

$\Rightarrow \frac{9}{16} = \frac{4^2}{QR^2}$

$\Rightarrow QR^2 = \frac{4.5 \times 4.5 \times 16}{9}$

$\Rightarrow QR = \sqrt{\frac{(4.5 \times 4.5 \times 16)}{9}}$

$= \frac{4.5 \times 4}{3}$

= 6 cm

Hence, QR = 6 cm

3.

Sol:

Given : $ar\ (\Delta ABC) = 4ar\ (\Delta PQR)$

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{4}{1}$

$\because \Delta ABC \sim \Delta PQR$

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$

$\therefore \frac{BC^2}{QR^2} = \frac{4}{1}$

$\Rightarrow QR^2 = \frac{12^2}{4}$

$\Rightarrow QR^2 = 36$

$\Rightarrow QR = 6\,cm$

Hence, QR = 6 cm

4.

Sol:

It is given that the triangles are similar.

Therefore, the ratio of the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Let the longest side of smaller triangle be X cm.

$\frac{ar\ (Larger\ triangle)}{ar\ (Smaller\ triangle)} = \frac{(Longest\ side\ of\ larger\ traingle)^2}{(\ Longest\ side\ of\ smaller\ traingle)^2}$

$\Rightarrow \frac{169}{121} = \frac{26^2}{x^2}$

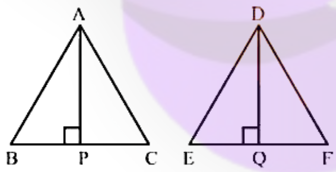
$\Rightarrow x = \sqrt{\frac{26 \times 26 \times 121}{169}}$

= 22

Hence, the longest side of the smaller triangle is 22 cm.

5.

Sol:



It is given that ΔABC ~ ΔDEF.

Therefore, the ration of the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Also, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

Let the altitude of ΔABC be AP, drawn from A to BC to meet BC at P and t

ΔDEF be DQ, drawn from D to meet EF at Q.

Then,

$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AP^2}{DQ^2}$

$\Rightarrow \frac{100}{49} = \frac{5^2}{DQ^2}$

$\Rightarrow \frac{100}{49} = \frac{25}{DQ^2}$

$\Rightarrow DQ^2 = \frac{49 \times 25}{100}$

$\Rightarrow DQ = \sqrt{\frac{49 \times 25}{100}}$

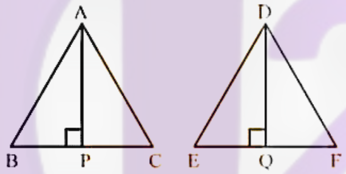
$\Rightarrow DQ = 3.5\ cm$

Hence, the altitude of ΔDEF is 3.5 cm

6.

Sol:

Let the two triangles be ABC and DEF with altitudes AP and DQ, respectively.



It is given that Δ ABC ~ Δ DEF.

We know that the ration of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AP)^2}{(DQ)^2}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(DEF)} = \frac{6^2}{9^2}$

$= \frac{36}{81}$

$= \frac{4}{9}$

Hence, the ratio of their areas is 4 : 9

7. **Sol:**

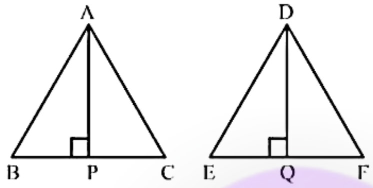
It is given that the triangles are similar.

Therefore, the areas of these triangles will be equal to the ratio of squares of their

corresponding sides.

Also, the ratio of areas of two similar triangles is equal to the ratio of square corresponding altitudes.

Let the two triangles be ABC and DEF with altitudes AP and DQ, respectively.



$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AP^2}{DQ^2}$

$\Rightarrow \frac{81}{49} = \frac{6.3^2}{DQ}$

$\Rightarrow DQ^2 = \frac{49}{81} \times 6.3^2$

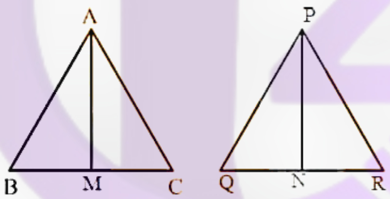
$\Rightarrow DQ^2 = \sqrt{\frac{49}{81} \times 6.3 \times 6.3}$

Hence, the altitude of the other triangle is 4.9 cm.

8.

Sol:

Let the two triangles be ABC and PQR with medians AM and PN, respectively.



Therefore, the ratio of areas of two similar triangles will be equal to the ratio of squares of their corresponding medians.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AM^2}{PN^2}$

$\Rightarrow \frac{64}{100} = \frac{5.6^2}{PN^2}$

$\Rightarrow PN^2 = \frac{64}{100} \times 5.6^2$

$\Rightarrow PN^2 = \sqrt{\frac{100}{64} \times 5.6 \times 5.6}$

= 7 cm

Hence, the median of the larger triangle is 7 cm.

9. 1

Sol:

We have :

$\frac{AP}{AB} = \frac{1}{1+3} = \frac{1}{4}$ and $\frac{AQ}{AC} = \frac{1.5}{1.5+4.5} = \frac{1.5}{6} = \frac{1}{4}$

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$

Also, $\angle A = \angle A$

By SAS similarity, we can conclude that ΔAPQ- ΔABC.

$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2} = \frac{1^2}{4^2} = \frac{1}{16}$

$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{16}$

$\Rightarrow ar(\Delta APQ) = \frac{1}{16} \times ar\ (\Delta ABC)$

Hence proved.

10.

Sol:

It is given that DE || BC

$\therefore \angle ADE = \angle ABC\ (Corresponding\ angles)$

$\angle AED = \angle ACB\ (Corresponding\ angles)$

By AA similarity, we can conclude that Δ ADE ~ Δ ABC

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \frac{15}{ar(\Delta ABC)} = \frac{3^2}{6^2}$

$\Rightarrow ar\ (\Delta ABC) = \frac{15 \times 36}{9}$

$= 60\ cm^2$

Hence, area of triangle ABC is 60 cm^2

11.

Sol:

In ΔABC and ΔADC, we have:

$\angle BAC = \angle ADC = 90^0$

$\angle ACB = \angle ACD\ (common)$

By AA similarity, we can conclude that Δ BAC~ Δ ADC.

Hence, the ratio of the areas of these triangles is equal to the ratio of squares of their corresponding sides.

$\therefore \frac{ar(\Delta BAC)}{ar(\Delta ADC)} = \frac{BC^2}{AC^2}$

$\Rightarrow \frac{ar(\Delta BAC)}{ar(\Delta ADC)} = \frac{13^2}{5^2}$

$= \frac{169}{25}$

Hence, the ratio of areas of both the triangles is 169:25

12.

Sol:

It is given that DE || BC.

$\therefore \angle ADE = \angle ABC\ (Corresponding\ angles)$

$\angle AED = \angle ACB\ (Corresponding\ angles)$

Applying AA similarity theorem, we can conclude that Δ ADE ~ ΔABC.

$\therefore \frac{ar(\Delta ABC)}{ar(ADE)} = \frac{BC^2}{DE^2}$

Subtracting 1 from both sides, we get:

$\frac{ar(\Delta ABC)}{ar(\Delta ADE)} - 1 = \frac{5^2}{3^2} - 1$

$\Rightarrow \frac{ar(\Delta ABC) - ar(\Delta ADE)}{ar(\Delta ADE)} = \frac{25-9}{9}$

$\Rightarrow \frac{ar(BCED)}{ar(\Delta ADE)} = \frac{16}{9}$

Or, $\frac{ar(\Delta ADE)}{ar(BCED)} = \frac{9}{16}$

13.

Sol:

It is given that D and E are midpoints of AB and AC.

Applying midpoint theorem, we can conclude that DE ∥ BC.

Hence, by B.P.T., we get :

$\frac{AD}{AB} = \frac{AE}{AC}$

Also, $\angle A = \angle A$

Applying SAS similarity theorem, we can conclude that Δ ADE~ Δ ABC.

Therefore, the ration of areas of these triangles will be equal to the ratio of squares of their corresponding sides.

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

$= \frac{(\frac{1}{2}BC)^2}{BC^2}$

$= \frac{1}{4}$

Exercise – 4D

1.

Sol:

For the given triangle to be right-angled, the sum of the two sides must be equal to the square of the third side.

Here, let the three sides of the triangle be a, b and c.

(i)

a = 9 cm, b = 16 cm and c = 18 cm

Then,

$a^2 + b^2 = 9^2 + 16^2$

$= 81 + 256$

$= 337$

$c^2 = 19^2$

$= 361$

$a^2 + b^2 \neq c^2$

Thus, the given triangle is not right-angled.

(ii)

A=7 cm, b = 24 cm and c = 25 cm

Then,

$a^2 + b^2 = 7^2 + 24^2$

$= 49 + 576$

$= 625$

$c^2 = 25^2$

$= 625$

$a^2 + b^2 = c^2$

Thus, the given triangle is a right-angled.

(iii)

A= 1.4 cm, b= 4.8 cm and c= 5 cm

Then,

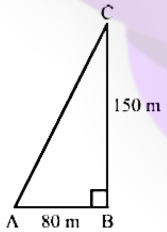
$a^2 + b^2 = (1.4)^2 + (4.8)^2$

$= 1.96 + 23.04$

$= 25$

$c^2 = 5^2$
$= 25$
$a^2 + b^2 = c^2$
Thus, the given triangle is right-angled.
(iii) A = 1.6 cm, b= 3.8 cm and c= 4 cm
Then
$a^2 + b^2 = (1.6)^2 + (3.8)^2$
$= 2.56 + 14.44$
$= 16$
$a^2 + b^2 \neq c^2$
Thus, the given triangle is not right-angled.
(v)
P = (a-1) cm, q = 2 $\sqrt{a}$ cm and $r = (a+1)cm$
Then,
$p^2 + q^2 = (a-1)^2 + (2\sqrt{a})^2$
$= a^2 + 1 - 2a + 4a$
$= a^2 + 1 + 2a$
$= (a+1)^2$
$r^2 = (a+1)^2$
$p^2 + q^2 = r^2$
Thus, the given triangle is right-angled.

2.
Sol:
Let the man starts from point A and goes 80 m due east to B.
Then, from B, he goes 150 m due north to c.



We need to find AC.
In right- angled triangle ABC, we have:
$AC^2 = AB^2 + BC^2$
$AC = \sqrt{80^2 + 150^2}$
$= \sqrt{6400 + 22500}$
$= \sqrt{28900}$
$= 170$ m

Hence, the man is 170 m away from the starting point.

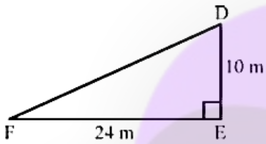
3.

Sol:

Let the man starts from point D and goes 10 m due south at E. He then goes 24 m due west at F.

In right ΔDEF, we have:

DE = 10 m, EF = 24 m



$DF^2 = EF^2 + DE^2$

$DF = \sqrt{10^2 + 24^2}$

$= \sqrt{100 + 576}$

$= \sqrt{676}$

= 26 m

Hence, the man is 26 m away from the starting point.

4.

Sol:

Let AB and AC be the ladder and height of the building.

It is given that :

AB = 13 m and AC = 12 m

We need to find distance of the foot of the ladder from the building, i.e, BC.

In right-angled triangle ABC, we have:

$AB^2 = AC^2 + BC^2$

$\Rightarrow BC = \sqrt{13^2 - 12^2}$

$= \sqrt{169 - 144}$

$= \sqrt{25}$

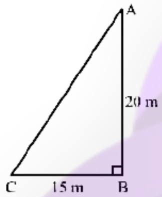
= 5 m

Hence, the distance of the foot ladder from the building is 5 m

CLASS24

5.

Sol:
Let the height of the window from the ground and the distance of the foot of the ladder from the wall be AB and BC, respectively.
We have :
AB = 20 m and BC = 15 m
Applying Pythagoras theorem in right-angled ABC, we get:



$AC^2 = AB^2 + BC^2$

$\Rightarrow AC = \sqrt{20^2 + 15^2}$

$= \sqrt{400 + 225}$

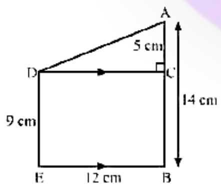
$= \sqrt{625}$

= 25 m

Hence, the length of the ladder is 25 m.

6.

Sol:
Let the two poles be DE and AB and the distance between their bases be BE.
We have:
DE = 9 m, AB = 14 m and BE = 12 m
Draw a line parallel to BE from D, meeting AB at C.
Then, DC = 12 m and AC = 5 m
We need to find AD, the distance between their tops.



Applying Pythagoras theorem in right-angled ACD, we have:

$AD^2 = AC^2 + DC^2$

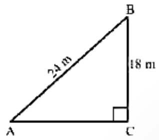
$AD^2 = 5^2 + 12^2 = 25 + 144 = 169$

$AD = \sqrt{169} = 13\ m$

Hence, the distance between the tops to the two poles is 13 m.

7.

Sol:



Let AB be a guy wire attached to a pole BC of height 18 m. Now, to keep the wire taut let it to be fixed at A.

Now, In right triangle ABC

By using Pythagoras theorem, we have

$AB^2 = BC^2 + CA^2$

$\Rightarrow 24^2 = 18^2 + CA^2$

$\Rightarrow CA^2 = 576 - 324$

$\Rightarrow CA^2 = 252$

$\Rightarrow CA = 6\sqrt{7}\,m$

Hence, the stake should be driven 6 $\sqrt{7}m$ far from the base of the pole.

8.

Sol:

Applying Pythagoras theorem in right-angled triangle POR, we have:

$PR^2 = PO^2 + OR^2$

$\Rightarrow PR^2 = 6^2 + 8^2 = 36 + 64 = 100$

$\Rightarrow PR = \sqrt{100} = 10\ cm$

IN Δ PQR,

$PQ^2 + PR^2 = 24^2 + 10^2 = 576 + 100 = 676$

And $QR^2 = 26^2 = 676$

$\therefore PQ^2 + PR^2 = QR^2$

Therefore, by applying Pythagoras theorem, we can say that ΔPQR is right-angled at P.

9.

Sol:

It is given that Δ ABC is an isosceles triangle.

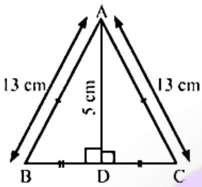
Also, AB = AC = 13 cm

Suppose the altitude from A on BC meets BC at D. Therefore, D is the mid

AD =5 cm

$\Delta\ ADB\ and\ \Delta\ ADC$ are right-angled triangles.

Applying Pythagoras theorem, we have;



$AB^2 = AD^2 + BD^2$

$BD^2 = AB^2 - AD^2 = 13^2 - 5^2$

$BD^2 = 169 - 25 = 144$

$BD = \sqrt{144} = 12$

Hence,

BC =2(BD) = 2 × 12 = 24 cm

10.

Sol:

In isosceles Δ ABC, we have:

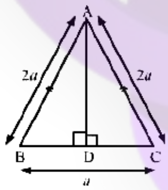
AB = AC = 2a units and BC = a units

Let AD be the altitude drawn from A that meets BC at D.

Then, D is the midpoint of BC.

BD = BC = $\frac{a}{2}\ units$

Applying Pythagoras theorem in right-angled ΔABD, we have:



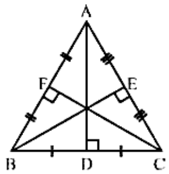
$AB^2 = AD^2 + BD^2$

$AD^2 = AB^2 - BD^2 = (2a)^2 - \left(\frac{a}{2}\right)^2$

$AD^2 = 4a^2 - \frac{a^2}{4} = \frac{15a^2}{4}$

$AD = \sqrt{\frac{15a^2}{4}} = \frac{a\sqrt{15}}{2}\ units.$

11. **Sol:**



Let AD, BE and CF be the altitudes of ΔABC meeting BC, AC and AB at D, E and F, respectively.
Then, D, E and F are the midpoint of BC, AC and AB, respectively.
In right-angled ΔABD, we have:
AB = 2a and BD = a
Applying Pythagoras theorem, we get:

$AB^2 = AD^2 + BD^2$

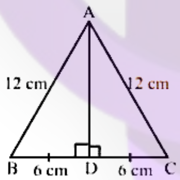
$AD^2 = AB^2 - BD^2 = (2a)^2 - a^2$

$AD^2 = 4a^2 - a^2 = 3a^2$

$AD = \sqrt{3}a\ units$

Similarly,
BE = $a\sqrt{3}\ units\ and\ CF = a\sqrt{3}\ units$

12.
Sol:
Let ABC be the equilateral triangle with AD as an altitude from A meeting BC at D. Then, D will be the midpoint of BC.
Applying Pythagoras theorem in right-angled triangle ABD, we get:



$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = 12^2 - 6^2\ (\because BD = \frac{1}{2}BC = 6)$

$\Rightarrow AD^2 = 144 - 36 = 108$

$\Rightarrow AD = \sqrt{108} = 6\sqrt{3}\ cm.$

Hence, the height of the given triangle is $6\sqrt{3}\ cm$.

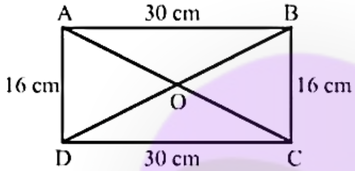
13.

Sol:

Let ABCD be the rectangle with diagonals AC and BD meeting at O.

According to the question:

AB = CD = 30 cm and BC = AD = 16 cm



Applying Pythagoras theorem in right-angled triangle ABC, we get:

$AC^2 = AB^2 + BC^2 = 30^2 + 16^2 = 900 + 256 = 1156$

$AC = \sqrt{1156} = 34\ cm$

Diagonals of a rectangle are equal.

Therefore, AC = BD = 34 cm

14.

Sol:

Let ABCD be the rhombus with diagonals (AC = 24 cm and BD = 10 cm) meeting at O.

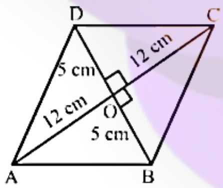
We know that the diagonals of a rhombus bisect each other at angles.

Applying Pythagoras theorem in right-angled AOB, we get:

$AB^2 = AO^2 + BO^2 = 12^2 + 5^2$

$AB^2 = 144 + 25 = 169$

$AB = \sqrt{169} = 13\ cm$



Hence, the length of each side of the rhombus is 13 cm.

–

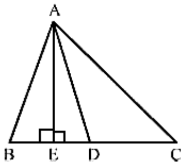
15.

Sol:

In right-angled triangle AED, applying Pythagoras theorem, we have:

$AB^2 = AE^2 + ED^2$ ... (i)

In right-angled triangle AED, applying Pythagoras theorem, we have:



$AD^2 = AE^2 + ED^2$

$\Rightarrow AE^2 = AD^2 - ED^2 \quad ...(ii)$

Therefore,

$AB^2 = AD^2 - ED^2 + EB^2 \; (from\ (i) and\ (ii))$

$AB^2 = AD^2 - ED^2 + (BD - DE)^2$

$= AD^2 - ED^2 + (\frac{1}{2} BC - DE)^2$

$= AD^2 - DE^2 + \frac{1}{4} BC^2 + DE^2 - BC.DE$

$= AD^2 + \frac{1}{4} BC^2 - BC.DE$

This completes the proof.

16. =

Sol:

Given: $\angle ACB = 90^0\ and\ CD \perp AB$

To Prove; $\frac{BC^2}{AC^2} = \frac{BD}{AD}$

Proof: In Δ ACB and Δ CDB

$\angle ACB = \angle CDB = 90^0\ (Given)$

$\angle ABC = \angle CBD\ (Common)$

By AA similarity-criterion Δ ACB ~ ΔCDB

When two triangles are similar, then the ratios of the lengths of their corresponding sides are proportional.

$\therefore \frac{BC}{BD} = \frac{AB}{BC}$

$\Rightarrow BC^2 = BD.AB \quad ...(1)$

In Δ ACB and Δ ADC

$\angle ACB = \angle ADC = 90^0\ (Given)$

$\angle CAB = \angle DAC\ (Common)$

By AA similarity-criterion Δ ACB ~ ΔADC

When two triangles are similar, then the ratios of their corresponding sides a
proportional.

$\therefore \frac{AC}{AD} = \frac{AB}{AC}$

$\Rightarrow AC^2 = AD. AB$ ....(2)

Dividing (2) by (1), we get

$\frac{BC^2}{AC^2} = \frac{BD}{AD}$

17.

Sol:

(i)

In right-angled triangle AEC, applying Pythagoras theorem, we have:

$AC^2 = AE^2 + EC^2$

$\Rightarrow b^2 = h^2 + \left(x + \frac{a}{2}\right)^2 = h^2 + x^2 + \frac{a^2}{4} + ax ... (i)$

In right – angled triangle AED, we have:

$AD^2 = AE^2 + ED^2$

$\Rightarrow p^2 = h^2 + x^2 ... (ii)$

Therefore,

from (i) and (ii),

$b^2 = p^2 + ax + \frac{a^2}{x}$

(ii)

In right-angled triangle AEB, applying Pythagoras, we have:

$AB^2 = AE^2 + EB^2$

$\Rightarrow c^2 = h^2 + \left(\frac{a}{2} - x\right)^2 \; (\because BD = \frac{a}{2} \text{ and } BE = BD - x)$

$\Rightarrow c^2 = h^2 + x^2 - \frac{a^2}{4}(\because h^2 + x^2 = p^2)$

$\Rightarrow c^2 = p^2 - ax + \frac{a^2}{x}$

(iii)

Adding (i) and (ii), we get:

$\Rightarrow b^2 + c^2 = p^2 + ax + \frac{a^2}{4} + p^2 - ax + \frac{a^2}{4}$

$= 2p^2 + ax - ax + \frac{a^2 + a^2}{4}$

$= 2p^2 + \frac{a^2}{2}$

(iv)

Subtracting (ii) from (i), we get:

$b^2 - c^2 = p^2 + ax + \frac{a^2}{4} - (p^2 - ax + \frac{a^2}{4})$

$= p^2 - p^2 + ax + ax + \frac{a^2}{4} - \frac{a^2}{4}$

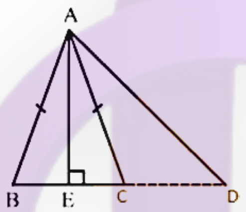
= 2ax

18.

Sol:

Draw AE⊥BC, meeting BC at D.

Applying Pythagoras theorem in right-angled triangle AED, we get:



Since, ABC is an isosceles triangle and AE is the altitude and we know that the altitude is also the median of the isosceles triangle.

So, BE = CE

And DE+CE=DE+BE=BD

$AD^2 = AE^2 + DE^2$

$\Rightarrow AE^2 = AD^2 - DE^2$ …(i)

In ΔACE,

$AC^2 = AE^2 + EC^2$

$\Rightarrow AE^2 = AC^2 - EC^2 \ldots (ii)$

Using (i) and (ii),

$\Rightarrow AD^2 - DE^2 = AC^2 - EC^2$

$\Rightarrow AD^2 - AC^2 = DE^2 - EC^2$

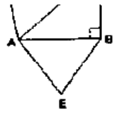
= (DE + CE) (DE – CE)

= (DE + BE) CD

= BD.CD

19. A

Sol:



We have, ABC as an isosceles triangle, right angled at B.

Now, AB = BC

Applying Pythagoras theorem in right-angled triangle ABC, we get:

$AC^2 = AB^2 + BC^2 = 2AB^2\ (\because AB = AC) \ldots (i)$

$\because \Delta ACD \sim \Delta ABE$

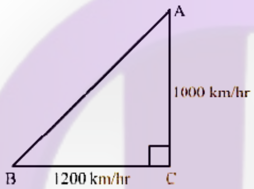
We know that ratio of areas of 2 similar triangles is equal to squares of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta ABE)}{ar(\Delta ACD)} = \frac{AB^2}{AC^2} = \frac{AB^2}{2AB^2}\ [from\ (i)]$

$= \frac{1}{2} = 1 : 2$

20.

Sol:



Let A be the first aeroplane flied due north at a speed of 1000 km/hr and B be the second aeroplane flied due west at a speed of 1200 km/hr

Distance covered by plane A in $1\frac{1}{2}\ hours = 1000 \times \frac{3}{2} = 1500\ km$

Distance covered by plane B in $1\frac{1}{2}\ hours = 1200 \times \frac{3}{2} = 1800\ km$

Now, In right triangle ABC

By using Pythagoras theorem, we have

$AB^2 = BC^2 + CA^2$

$= (1800)^2 + (1500)^2$

$= 3240000 + 2250000$

$= 5490000$

$\therefore AB^2 = 5490000$

$\Rightarrow$ AB = $300\sqrt{61}\,m$

Hence, the distance between two planes after $1\frac{1}{2}$ $hours\ is\ 300\sqrt{61}m$

CLASS24

21.

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Sol:

(a) In right triangle ALD

Using Pythagoras theorem, we have

$AC^2 = AL^2 + LC^2$

$= AD^2 - DL^2 + (DL + DC)^2$ [Using (1)]

$= AD^2 - DL^2 + (DL + \frac{BC}{2})^2$ [$\because$ AD is a median]

$= AD^2 - DL^2 + DL^2 + (\frac{BC}{2})^2 + BC.DL$

$\therefore AC^2 = AD^2 + BC.DL + (\frac{BC}{2})^2$ …. (2)

(b) In right triangle ALD

Using Pythagoras theorem, we have

$AL^2 = AD^2 - DL^2$ ... (3)

Again, In right triangle ABL

Using Pythagoras theorem, we have

$AB^2 = AL^2 + LB^2$

$= AD^2 - DL^2 + LB^2$ [$Using$ (3)]

$= AD^2 - DL^2 + (BD - DL)^2$

$= AD^2\ DL^2 + (\frac{1}{2}BC - DL)^2$

$= AD^2 - DL^2 + (\frac{BC}{2})^2 - BC.DL + DL^2$

$\therefore AB^2 = AD^2 - BC.DL + (\frac{BC}{2})^2$ ….. (4)

(c) Adding (2) and (4), we get,

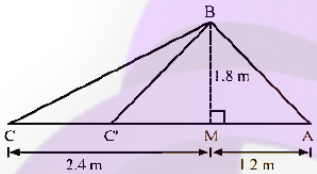
$= AC^2 + AB^2 = AD^2 + BC.DL + (\frac{BC}{2})^2 + AD^2 - BC.DL + (\frac{BC}{2})^2$

$= 2\ AD^2 + \frac{BC^2}{4} + \frac{BC^2}{4}$

$= 2\ AD^2 + \frac{1}{2} BC^2$

22.

Sol:



Naman pulls in the string at the rate of 5 cm per second.

Hence, after 12 seconds the length of the string he will pulled is given by

12 × 5 = 60 cm or 0.6 m

Now, in ΔBMC

By using Pythagoras theorem, we have

$BC^2 = CM^2 + MB^2$

$= (2.4)^2 + (1.8)^2$

= 9

∴ BC = 3 m

Now, BC' = BC – 0.6

= 3 – 0.6

= 2.4 m

Now, In ΔBC'M

By using Pythagoras theorem, we have

$C'M^2 = BC'^2 - MB^2$

$= (2.4)^2 - (1.8)^2$
$= 2.52$
$\therefore$ C'M = 1.6 m
The horizontal distance of the fly from him after 12 seconds is given by
C'A = C'M + MA
= 1.6 + 1.2
= 2.8 m

Exercise – 4E

1.

Sol:
The two triangles are similar if and only if
1. The corresponding sides are in proportion.
2. The corresponding angles are equal.

2.

Sol:
If a line is draw parallel to one side of a triangle intersect the other two sides, then it divides the other two sides in the same ratio.

3.

Sol:
If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

4. **Sol:**
The line segment connecting the midpoints of two sides of a triangle is parallel to the third side and is equal to one half of the third side.

5.

Sol:
If the corresponding angles of two triangles are equal, then their corresponding sides are proportional and hence the triangles are similar.

6. **Sol:**
If two angles are correspondingly equal to the two angles of another triangle, then the two triangles are similar.

7.

Sol:

If the corresponding sides of two triangles are proportional then their corresponding angles are equal, and hence the two triangles are similar.

8.

Sol:

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional then the two triangles are similar.

9.

Sol:

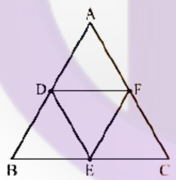
The square of the hypotenuse is equal to the sum of the squares of the other two sides. Here, the hypotenuse is the longest side and it's always opposite the right angle.

10.

Sol:

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

11. **Sol:**



By using mid theorem i.e., the segment joining two sides of a triangle at the midpoints of those sides is parallel to the third side and is half the length of the third side.

∴ DF || BC

And $DF = \frac{1}{2}BC$

⟹ DF = BE

Since, the opposite sides of the quadrilateral are parallel and equal.

Hence, BDFE is a parallelogram

Similarly, DFCE is a parallelogram.

Now, in ΔABC and ΔEFD

$\angle ABC = \angle EFD$ $(Opposite\ angles\ of\ a\ parallelogram)$

$\angle BCA = \angle EDF$ $(Opposite\ angles\ of\ a\ parallelogram)$

By AA similarity criterion, ΔABC ~ ΔEFD

If two triangles are similar, then the ratio of their areas is equal to the square corresponding sides.

$\therefore \frac{area(\Delta DEF)}{area(\Delta ABC)} = \left(\frac{DF}{BC}\right)^2 = \left(\frac{DF}{2\,DF}\right)^2 = \frac{1}{4}$

Hence, the ratio of the areas of ΔDEF and ΔABC is 1 : 4.

12. **Sol:**

Now, In ΔABC and ΔPQR

$\angle A = \angle P = 70^0 \quad (Given)$

$\frac{AB}{PQ} = \frac{AC}{PR} \quad \left[\because \frac{3}{4.5} = \frac{6}{9} \Rightarrow \frac{1}{1.5} = \frac{1}{1.5}\right]$

By SAS similarity criterion, ΔABC~ ΔPQR

13.

Sol:

When two triangles are similar, then the ratios of the lengths of their corresponding sides are equal.

Here, ΔABC ~ΔDEF

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{AB}{2\,AB} = \frac{6}{EF}$

⟹ EF = 12 cm

14. **Sol:**

In ΔADE and ΔABC

$\angle ADE = \angle ABC \quad (Corresponding\ angles\ in\ DE \parallel BC)$

$\angle AED = \angle ACB \quad (Corresponding\ angles\ in\ DE \parallel BC$

By AA similarity criterion, ΔADE ~ ΔABC

If two triangles are similar, then the ratio of their corresponding sides are proportional

$\therefore \frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{AD}{AD+DB} = \frac{AE}{AE+EC}$

$\Rightarrow \frac{x}{x+3x+4} = \frac{x+3}{x+3+3x+19}$

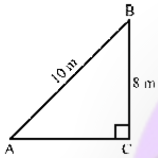
$\Rightarrow \frac{x}{4x+4} = \frac{x+3}{x+3+3x+19}$

$\Rightarrow \frac{x}{2x+2} = \frac{x+3}{2x+11}$

$\Rightarrow 2x^2 + 11x = 2x^2 + 2x + 6x + 6$
$\Rightarrow 3x = 6$
$\Rightarrow x = 2$
Hence, the value of x is 2.

15.

Sol:



Let AB be A ladder and B is the window at 8 m above the ground C.
Now, In right triangle ABC
By using Pythagoras theorem, we have
$AB^2 = BC^2 + CA^2$
$\Rightarrow 10^2 = 8^2 + CA^2$
$\Rightarrow CA^2 = 100 - 64$
$\Rightarrow CA^2 = 36$
$\Rightarrow CA = 6m$
Hence, the distance of the foot of the ladder from the base of the wall is 6 m.