Chapter : 8. TRIGONOMETRIC IDENTITIES

Exercise : 8A

Question: 1 A

Solution:

Consider the left – hand side:

L.H.S. = $(1 - \cos^2\theta) \times \text{cosec}^2\theta$

= $(\sin^2\theta) \times \text{cosec}^2\theta$ ($\because \sin^2\theta + \cos^2\theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 1 B

Solution:

Consider the left – hand side:

L.H.S. = $(1 + \cot^2\theta) \times \sin^2\theta$

= $(\text{cosec}^2\theta) \times \sin^2\theta$ ($\because 1 + \cot^2\theta = \text{cosec}^2\theta$)

= 1

= R.H.S.

Hence, proved.

Question: 2 A

Solution:

Consider the left – hand side:

L.H.S. = $(\sec^2\theta - 1) \times \cot^2\theta$

= $(\tan^2\theta) \times \cot^2\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$)

= 1

= R.H.S.

Hence, proved.

Question: 2 B

Solution:

Consider the left – hand side:

L.H.S. = $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

= $(\tan^2\theta) \times \cot^2\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \text{cosec}^2\theta$)

= 1

= R.H.S.

Hence, proved.

Question: 2 C

Solution:

Consider the left – hand side:

L.H.S. = $(1 - \cos^2 \theta) \sec^2 \theta$

= $(\sin^2\theta) \times (1/\cos^2\theta)$ ($\because \sin^2 \theta + \cos^2 \theta = 1$

= $\tan^2 \theta$

= R.H.S.

Hence, proved.

Question: 3 A

Solution:

Consider the left – hand side:

L.H.S. = $\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}$

= $(\sin^2\theta) + (1/\sec^2 \theta)$ ($\because 1 + \tan^2 \theta = \sec^2 \theta$)

= $(\sin^2\theta) + (\cos^2\theta)$ ($\because \sin^2 \theta + \cos^2 \theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 3 B

Solution:

Consider the left – hand side:

L.H.S. = $\frac{1}{1 + \tan^2 \theta} + \frac{1}{1 + \cot^2 \theta}$

= $(1/\sec^2\theta) + (1/\text{cosec}^2 \theta)$ ($\because 1 + \tan^2 \theta = \sec^2 \theta$ and $1 + \tan^2 \theta = \sec^2 \theta$)

= $(\cos^2 \theta) + (\sin^2\theta)$ ($\because \sin^2 \theta + \cos^2 \theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 4 A

Solution:

Consider the left – hand side:

L.H.S. = $(1 + \cos \theta)(1 - \cos \theta)(1 + \cot^2 \theta)$

= $(1 - \cos^2 \theta) \times \text{cosec}^2 \theta$ ($\because 1 + \cot^2 \theta = \text{cosec}^2 \theta$)

= $(\sin^2 \theta) \times \text{cosec}^2 \theta$ ($\because \sin^2\theta + \cos^2 \theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 4 B

Solution:

To prove: (cosec θ)(1 + cos θ)(cosec θ – cot θ) = 1**Proof:**Consider the left – hand side:

(cosec θ) (1 + cos θ)(cosec θ – cot θ)⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ) = (cosec θ + cosec θ cos θ)(cosec θ – cot θ)since cosec θ = 1/sin θ⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ)= $\left(cosec\ \theta + \frac{\cos\theta}{\sin\theta}\right)(cosec\theta - \cot\theta)$ Also $\cot\theta = \frac{cos\theta}{sin\theta}$ So,⇒ (cosec θ) (1 + cos θ) (cosec θ – cot θ) =(cosec θ + cot θ)(cosec θ – cot θ)

Use the formula $(a + b)(a - b) = a^2 - b^2$

⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ) =$(cosec^2\theta - cot^2\theta)$

Since $cosec^2\theta - cot^2\theta = 1$⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ) =1

= R.H.S.

Hence, proved.

Question: 5 A

Solution:

Consider the left – hand side:

L.H.S. = $\cot^2\theta - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta - 1}{\sin^2\theta}$

$= (-\sin^2\theta) \times \sin^2\theta\ (\because \sin^2\theta + \cos^2\theta = 1)$

= – 1

= R.H.S.

Hence, proved.

Question: 5 B

Solution:

Consider the left – hand side:

L.H.S. = $\tan^2\theta - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta - 1}{\cos^2\theta}$

$= (-\cos^2\theta) \times \cos^2\theta\ (\because \sin^2\theta + \cos^2\theta = 1)$

= – 1

= R.H.S.

Hence, proved.

Question: 5 C

Solution:

Consider the left – hand side:

L.H.S. = $\cos^2\theta + \frac{1}{1+\cot^2\theta}$

$= \cos^2\theta + \frac{1}{\mathrm{cosec}^2\theta}$ $(\because 1 + \cot^2\theta = \mathrm{cosec}^2\theta)$

$= \cos^2\theta + \sin^2\theta$

$= (-\cos^2\theta) \times \cos^2\theta$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= -1$

= R.H.S.

Hence, proved.

Question: 6

Solution:

Consider the left – hand side:

L.H.S. $= \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$= \frac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta}$

$= 2\sec^2\theta$

= R.H.S.

Hence, proved.

Question: 7 A

Solution:

Consider the left – hand side:

L.H.S. = $\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)$

$= \left(\frac{1}{\cos\theta}\right) \times (1-\sin\theta) \times \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$

$= \left(\frac{1}{\cos\theta}\right) \times (1-\sin\theta) \times \left(\frac{1+\sin\theta}{\cos\theta}\right)$

$= \frac{1-\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\cos^2\theta}$

$= 1$

= R.H.S.

Hence, proved.

Question: 7 B

Solution:

To prove: sin θ (1 + tan θ) + cos θ (1+ cot θ) = (sec θ + cosec θ)**Proof:**Consider the left – hand side:

L.H.S. = sin θ (1 + tan θ) + cos θ (1+ cot θ)

$= \sin\theta\,(1 + \frac{\sin\theta}{\cos\theta}) + \cos\theta\,(1 + \frac{\cos\theta}{\sin\theta})$

$= \sin\theta\left(\frac{\cos\theta + \sin\theta}{\cos\theta}\right) + (\cos\theta) \times \left(\frac{\sin\theta + \cos\theta}{\sin\theta}\right)$

$= (\cos\theta + \sin\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= (\cos\theta + \sin\theta)\left(\frac{(\cos^2\theta + \sin^2\theta)}{\cos\theta\sin\theta}\right)$We know $\cos^2\theta + \sin^2\theta = 1$

$= \left(\frac{\cos\theta + \sin\theta}{\cos\theta\sin\theta}\right)$

$= \left(\frac{1}{\sin\theta} + \frac{1}{\cos\theta}\right)$

= cosec θ + sec θ

= R.H.S.

Hence, proved.

Question: 8 A

Solution:

Consider the left – hand side:

$\text{L.H.S.} = 1 + \frac{\cot^2\theta}{1+\text{cosec}\,\theta}$

$= 1 + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{1+\frac{1}{\sin\theta}}$

$= 1 + \frac{\cos^2\theta}{1+\sin\theta} \times \frac{\sin\theta}{\sin^2\theta}$

$= 1 + \frac{\cos^2\theta}{(1+\sin\theta)\sin\theta}$

$= \frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\sin\theta + \sin^2\theta}$

$= \frac{\sin\theta + 1}{\sin\theta(1+\sin\theta)}$

= 1/sin θ

= cosec θ

= R.H.S.

Hence, proved.

Question: 8 B

Solution:

Consider the left – hand side:

$$\text{L.H.S.} = 1 + \frac{\tan^2\theta}{1+\sec\theta}$$

$$= 1 + \frac{\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{1}{\cos\theta}}$$

$$= 1 + \frac{\sin^2\theta}{1+\cos\theta} \times \frac{\cos\theta}{\cos^2\theta}$$

$$= 1 + \frac{\sin^2\theta}{(1+\cos\theta)\cos\theta}$$

$$= \frac{\cos\theta + \cos^2\theta + \sin^2\theta}{\cos\theta + \cos^2\theta}$$

$$= \frac{\cos\theta+1}{\cos\theta(1+\cos\theta)}$$

$= 1/\cos\theta$

$= \sec\theta$

= R.H.S.

Hence, proved.

Question: 9

Solution:

Consider the left – hand side:

$$\text{L.H.S.} = \frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta}$$

$$= \frac{\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\times\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta} \times \sin^2\theta$$

$$= 1 \times \frac{\sin\theta}{\cos\theta}$$

$= \tan\theta$

= R.H.S.

Hence, proved.

Question: 10

Solution:

Consider the left – hand side:

$$\text{L.H.S.} = \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\sin^2\theta}{1+\sin^2\theta} + \frac{\cos^2\theta}{1+\cos^2\theta}$$

$$= \frac{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}{(1+\sin^2\theta)(1+\cos^2\theta)}$$

$$= \frac{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}$$

= 1

= R.H.S.

Hence, proved.

Question: 11

Solution:

Consider the left – hand side:

L.H.S. = $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

Adding both the fractions, we get

$= \frac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{\sin\theta(1+\cos\theta)}$

As $\sin^2\theta + \cos^2\theta = 1$, we have

$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

= $2/\sin\theta$

= 2cosec θ

= R.H.S.

Hence, proved.

Question: 12

Prove :

Solution:

Consider L.H.S. = $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{(\sin\theta-\cos\theta)\cos\theta} + \frac{\cos^2\theta}{(\cos\theta-\sin\theta)\sin\theta}$

$= \frac{1}{(\sin\theta-\cos\theta)}\left(\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right)$

$= \frac{1}{(\sin\theta-\cos\theta)}\left(\frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta}\right)$

$= \frac{\sin^3\theta-\cos^3\theta}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$

$= \frac{\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$

$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta.\cos\theta} + \frac{\sin\theta.\cos\theta}{\sin\theta.\cos\theta}$

$= \frac{1}{\sin\theta.\cos\theta} + 1$

= sec θ cosec θ + 1

= R.H.S.

Hence, proved.

Question: 13

Solution:

Consider the left – hand side:

L.H.S. $= \frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$

$= \frac{\cos^2\theta}{1\frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$

$= \frac{\cos^3\theta}{\cos\theta - \sin\theta} - \frac{\sin^3\theta}{\cos\theta - \sin\theta}$

$= \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$

$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{\cos\theta - \sin\theta}$

$= \cos^2\theta + \sin^2\theta + \sin\theta\cos\theta$

$= 1 + \cos\theta\sin\theta$

= R.H.S.

Hence, proved.

Question: 14

Solution:

$\frac{\cos\theta}{1-\tan\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta} = \frac{\cos\theta}{1\frac{\sin\theta}{\cos\theta}} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$

$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta}$

$= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)}$

$= \cos\theta + \sin\theta$

= R.H.S.

Hence, proved.

Question: 15

Solution:

Consider L.H.S. $= (1 + \tan^2\theta)(1 + \cot^2\theta)$

$= (\sec^2\theta)(\operatorname{cosec}^2\theta)$

$= \frac{1}{\cos^2\theta} \times \frac{1}{\sin^2\theta}$

$= \frac{1}{1-\sin^2\theta} \times \frac{1}{\sin^2\theta}$

$= \frac{1}{\sin^2\theta - \sin^4\theta}$

= R.H.S.

Hence, proved.

Question: 16

Solution:

Consider L.H.S. $= \frac{\tan\theta}{(1+\tan^2\theta)^2} + \frac{\cot\theta}{(1+\cot^2\theta)^2}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)^2} + \frac{\frac{\cos\theta}{\sin\theta}}{\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right)^2}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}\right)^2} + \frac{\frac{\cos\theta}{\sin\theta}}{\left(\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}\right)^2}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{1}{\cos^2\theta}\right)^2} + \frac{\frac{\cos\theta}{\sin\theta}}{\left(\frac{1}{\sin^2\theta}\right)^2}$

$= \left(\frac{\sin\theta}{\cos\theta} \times \cos^4\theta\right) + \left(\frac{\cos\theta}{\sin\theta} \times \sin^4\theta\right)$

$= \sin\theta\,(\cos^3\theta) + \cos\theta\,(\sin^3\theta)$

$= \sin\theta\cos\theta\,(\cos^2\theta + \sin^2\theta)$

$= \sin\theta\cos\theta$

= R.H.S.

Hence, proved.

Question: 17 A

Solution:

Consider L.H.S. $= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

[Using $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$]

$= (\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= [\{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta\} - \sin^2\theta\cos^2\theta]$

$(\because (a^2 + b^2) = (a + b)^2 - 2ab)$

$= [1 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta]$

$= 1 - 3\sin^2\theta\cos^2\theta$

= R.H.S.

Hence, proved.

Question: 17 B

Prove each of the

Solution:

Consider L.H.S. = $\sin^2 \theta + \cos^4 \theta$

$= (\sin^2 \theta) + (\cos^2 \theta)^2$

$= (\sin^2 \theta) + (1 - \sin^2 \theta)^2$

$= (\sin^2 \theta) + 1 + \sin^4 \theta - 2\sin^2 \theta$

$= 1 - \sin^2 \theta + \sin^4 \theta$

$= \cos^2 \theta + \sin^4 \theta$

= R.H.S.

Hence, proved.

Question: 17 C

Solution:

Consider L.H.S. = $\operatorname{cosec}^4 \theta - \operatorname{cosec}^2 \theta$

$= (\operatorname{cosec}^2 \theta)^2 - (\operatorname{cosec}^2 \theta)$

$= (1 + \cot^2 \theta)^2 - (\operatorname{cosec}^2 \theta)$

$= 1 + \cot^4 \theta + 2\cot^2 \theta - (\operatorname{cosec}^2 \theta)$

$= 1 + \cot^4 \theta + \cot^2 \theta - (\operatorname{cosec}^2 \theta - \cot^2 \theta)$

$= 1 + \cot^4 \theta + \cot^2 \theta - 1$

$= \cot^4 \theta + \cot^2 \theta$

= R.H.S.

Hence, proved.

Question: 18 A

Solution:

Consider L.H.S. = $\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$

$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}$

$= \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta + \cos^2\theta}$

$= \cos^2 \theta - \sin^2 \theta$

= R.H.S.

Hence, proved.

Question: 18 B

Solution:

Consider L.H.S. = $\frac{1-\tan^2\theta}{\cot^2\theta-1}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}-1}$

$= \frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}}$

$= \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$

$= \sin^2\theta / \cos^2\theta$

$= \tan^2\theta$

= R.H.S.

Hence, proved.

Question: 19 A

Solution:

Consider L.H.S. = $\frac{\tan\theta}{\sec\theta-1} + \frac{\tan\theta}{\sec\theta+1}$

$= \frac{\tan\theta(\sec\theta+1)+\tan\theta(\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}$

$= \frac{\tan\theta\sec\theta+\tan\theta+\tan\theta\sec\theta-\tan\theta}{\sec^2\theta-1}$

$= \frac{2\tan\theta\sec\theta}{\tan^2\theta}$

$= \frac{2\sec\theta}{\tan\theta}$

$= [2(1/\cos\theta)]/[\sin\theta/\cos\theta]$

$= [2/\sin\theta]$

$= 2\operatorname{cosec}\theta$

= R.H.S.

Hence, proved.

Question: 19 B

Solution:

Consider L.H.S. = $\frac{\cot\theta}{\operatorname{cosec}\theta+1} + \frac{\operatorname{cosec}\theta+1}{\cot\theta}$

$= \frac{\cot^2\theta+(\operatorname{cosec}\theta+1)^2}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{\cot^2\theta+\operatorname{cosec}^2\theta+1+2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{\operatorname{cosec}^2\theta+\operatorname{cosec}^2\theta+2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{2\operatorname{cosec}^2\theta+2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{2\operatorname{cosec}\theta(\operatorname{cosec}\theta+1)}{(\operatorname{cosec}\theta+1)}$

= 2 cosec θ/cot θ

= 2 (1/sin θ)/(cos θ/sin θ)

= 2/ cos θ

= 2 sec θ

= R.H.S.

Hence, proved.

Question: 20 A

Solution:

Consider L.H.S. = $\frac{\sec\theta-1}{\sec\theta+1}$

Multiply and divide by (sec θ + 1):

$= \frac{\sec\theta-1}{\sec\theta+1} \times \frac{\sec\theta+1}{\sec\theta+1}$

$= \frac{\sec^2\theta-1}{(\sec\theta+1)^2}$

$= \frac{\tan^2\theta}{(1+\sec\theta)^2}$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\left(\frac{1+\cos\theta}{\cos\theta}\right)^2}$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{(1+\cos\theta)^2}{\cos^2\theta}}$

$= \frac{\sin^2\theta}{(1+\cos\theta)^2}$

= R.H.S.

Hence, proved.

Question: 20 B

Solution:

Consider L.H.S. = $\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}$

Multiply and divide by (sec θ + tan θ):

$= \frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta} \times \frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}$

$= \frac{\sec^2\theta-\tan^2\theta}{(\sec\theta+\tan\theta)^2}$

$= \frac{1}{\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)^2}$

$= \frac{1}{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \frac{\cos^2\theta}{(1+\sin\theta)^2}$

= R.H.S.

Hence, proved.

Question: 21 A

Solution:

Consider L.H.S. = $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

Multiply and divide by (1 + sin θ):

$= \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

= (1 + sin θ)/cos θ

= (1/cos θ) + (sin θ/cos θ)

= sec θ + tan θ

= R.H.S.

Hence, proved.

Question: 21 B

Solution:

Consider L.H.S. = $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

Multiply and divide by (1 – cos θ):

$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

= (1 – cos θ)/sin θ

= (1/sin θ) – (cos θ/sin θ)

= cosec θ – cot θ

= R.H.S.

Hence, proved.

Question: 21 C

Solution:

Consider L.H.S. = $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

Multiply and divide by (1 + cos θ) in first part and (1 – cos θ) in the second part:

$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

$= [(1 + \cos\theta)/\sin\theta] + [(1 - \cos\theta)/\sin\theta]$

$= [(1/\sin\theta) + (\cos\theta/\sin\theta)] + [(1/\sin\theta) - (\cos\theta/\sin\theta)]$

$= [\text{cosec}\,\theta + \cot\theta] + [\text{cosec}\,\theta - \cot\theta]$

$= 2\,\text{cosec}\,\theta$

= R.H.S.

Hence, proved.

Question: 22

Solution:

Consider L.H.S. $= \frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta} + \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

Using identities $(a^3 + b^3) = (a + b)(a^2 + b^2 - ab)$ and $(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$

$\therefore$ L.H.S. $= \frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\cos\theta\sin\theta)}{(\cos\theta+\sin\theta)} + \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)}{(\cos\theta-\sin\theta)}$

$= (\cos^2\theta + \sin^2\theta - \cos\theta\sin\theta) + (\cos^2\theta + \sin^2\theta + \cos\theta\sin\theta)$

$= (1 - \cos\theta\sin\theta) + (1 + \cos\theta\sin\theta)$

$= 2$

= R.H.S.

Hence, proved.

Question: 23

Solution:

Consider L.H.S. $= \frac{\sin\theta}{(\cot\theta+\text{cosec}\,\theta)} - \frac{\sin\theta}{(\cot\theta+\text{cosec}\,\theta)}$

$= \frac{\sin\theta}{\left(\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}\right)} - \frac{\sin\theta}{\left(\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)}$

$$= \frac{\sin^2\theta}{1+\cos\theta} - \frac{\sin^2\theta}{\cos\theta-1}$$
$$= \frac{\sin^2\theta}{1+\cos\theta} + \frac{\sin^2\theta}{1-\cos\theta}$$
$$= \sin^2\theta\left(\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta}\right)$$
$$= \sin^2\theta\left(\frac{1-\cos\theta+1+\cos\theta}{1-\cos^2\theta}\right)$$
$$= \sin^2\theta \times \frac{2}{\sin^2\theta} \quad [As, \sin^2\theta + \cos^2\theta = 1]$$
$$= 2$$

= R.H.S.

Hence, proved.

Question: 24 A

Solution:

Consider L.H.S. = $\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}+\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$

$=\frac{(\sin\theta-\cos\theta)^2+(\sin\theta+\cos\theta)^2}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}$

$=\frac{\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

$=\frac{1-2\sin\theta\cos\theta+1+2\sin\theta\cos\theta}{\sin^2\theta-(1-\sin^2\theta)}$

$=\frac{2}{2\sin^2\theta-1}$

= R.H.S.

Hence, proved.

Question: 24 B

Solution:

Consider L.H.S. = $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

$=\frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

$=\frac{1+2\sin\theta\cos\theta+1-2\sin\theta\cos\theta}{1-\cos^2\theta-coss^2\theta}$

$=\frac{2}{1-2\cos^2\theta}$

= R.H.S.

Hence, proved.

Question: 25

Solution:

Consider L.H.S. = $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$

$=\frac{(1-\sin^2\theta)+\cos\theta}{\sin\theta(1+\cos\theta)}$

$=\frac{(\cos^2\theta)+\cos\theta}{\sin\theta(1+\cos\theta)}$

$=\frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

= $\cos\theta/\sin\theta$

= $\cot\theta$

= R.H.S.

Hence, proved.

Question: 26 A

Prove each of the

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Solution:

Consider L.H.S. = $\frac{\text{cosec}\,\theta+\cot\theta}{\text{cosec}\,\theta-\cot\theta}$

Multiply and divide by (cosec θ + cot θ):

$= \frac{\text{cosec}\,\theta+\cot\theta}{\text{cosec}\,\theta-\cot\theta} \times \frac{\text{cosec}\,\theta+\cot\theta}{\text{cosec}\,\theta+\cot\theta}$

$= \frac{(\text{cosec}\,\theta+\cot\theta)^2}{\text{cosec}^2\,\theta-\cot^2\theta}$

$= (\text{cosec}\,\theta + \cot\theta)^2$

Thus, proved.

Also, consider $(\text{cosec}\,\theta + \cot\theta)^2 = \text{cosec}^2\,\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta$

$= 1 + \cot^2\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta$ ($\because 1 + \cot^2\theta = \text{cosec}^2\,\theta$)

$= (1 + 2\cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta)$

= R.H.S.

Hence, proved.

Question: 26 B

Solution:

Consider L.H.S. = $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}$

Multiply and divide by (sec θ + tan θ):

$= \frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} \times \frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}$

$= \frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}$

$= (\sec\theta + \tan\theta)^2$

Thus, proved.

Also, consider $(\sec\theta + \tan\theta)^2 = \sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$

$= 1 + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$)

$= (1 + 2\tan^2\theta + 2\sec\theta\tan\theta)$

= R.H.S.

Hence, proved.

Question: 27 A

Solution:

Consider L.H.S. = $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$

Multiply and divide by ((1 + cos θ) + sin θ):

$= \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta} \times \frac{1+\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta}$

$= \frac{(1+\cos\theta+\sin\theta)^2}{(1+\cos\theta)^2-\sin^2\theta}$

$= \frac{1+\cos^2\theta+\sin^2\theta+2\cos\theta+2\sin\theta+2\cos\theta\sin\theta}{1+\cos^2\theta+2\cos\theta-(1-\cos^2\theta)}$

$= \frac{1+1+2\cos\theta+2\sin\theta(1+\cos\theta)}{2\cos^2\theta+2\cos\theta}$

$= \frac{2(1+\cos\theta)+2\sin\theta(1+\cos\theta)}{2\cos\theta(1+\cos\theta)}$

$= \frac{2(1+\cos\theta)(1+\sin\theta)}{2\cos\theta(1+\cos\theta)}$

$= \frac{1+\sin\theta}{\cos\theta}$

= R.H.S.

Thus, proved.

Question: 27 B

Solution:

Consider L.H.S. $= \frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta}$

Multiply and divide by $(\cos\theta + 1) + \sin\theta)$:

$= \frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta} \times \frac{\cos\theta+1+\sin\theta}{\cos\theta+1+\sin\theta}$

$= \frac{(1+\sin\theta)^2-\cos^2\theta}{(\sin\theta+\cos\theta)^2-1}$

$= \frac{1+\sin^2\theta+2\sin\theta-(1-\sin^2\theta)}{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}$

$= \frac{2\sin\theta(1+\sin\theta)}{2\sin\theta\cos\theta}$

$= \frac{1+\sin\theta}{\cos\theta}$

= R.H.S.

Thus, proved.

Question: 28

Solution:

Consider L.H.S. $= \frac{\sin\theta}{\sec\theta+\tan\theta-1} + \frac{\cos\theta}{\operatorname{cosec}\theta+\cot\theta-1}$

$= \frac{\sin\theta}{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}-1} + \frac{\cos\theta}{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}-1}$

$= \frac{\sin\theta\cos\theta}{1+\sin\theta-\cos\theta} + \frac{\cos\theta\sin\theta}{1+\cos\theta-\sin\theta}$

$= \sin\theta\cos\theta \times \left(\frac{1}{1+\sin\theta-\cos\theta} + \frac{1}{1+\cos\theta-\sin\theta}\right)$

$= \sin\theta\cos\theta \times \left(\frac{1+\sin\theta-\cos\theta+1+\cos\theta-\sin\theta}{(1+\sin\theta-\cos\theta)(1+\cos\theta-\sin\theta)}\right)$

$= \sin\theta\cos\theta \times \frac{2}{1-(\sin\theta-\cos\theta)^2}$

$= \sin\theta\cos\theta \times \frac{2}{1-(\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta)}$

$= \sin\theta\cos\theta \times \frac{2}{1-1+2\sin\theta\cos\theta}$

= sin θ cos θ/ sin θ cos θ

= 1

= R.H.S.

Question: 29

Solution:

Consider L.H.S. = $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

$= \frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

$= \frac{2}{\sin^2\theta-\cos^2\theta}$

Thus, prove.

Also, consider $\frac{2}{\sin^2\theta-\cos^2\theta}=\frac{2}{\sin^2\theta-(1-\sin^2\theta)}$

$= \frac{2}{(2\sin^2\theta-1)}$

= R.H.S.

Hence, proved.

Question: 30

Solution:

Consider L.H.S. = $\frac{\cos\theta\,\mathrm{cosec}\,\theta-\sin\theta\sec\theta}{\cos\theta+\sin\theta}$

$= \frac{\cos\theta\left(\frac{1}{\sin\theta}\right)-\sin\theta\left(\frac{1}{\cos\theta}\right)}{\cos\theta+\sin\theta}$

$= \frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta(\cos\theta+\sin\theta)}$

$= \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta\sin\theta(\cos\theta+\sin\theta)}$

$= \frac{\cos\theta-\sin\theta}{\cos\theta\sin\theta}$

= (1/sin θ) – (1/cos θ)

= cosec θ – sec θ

= R.H.S.

Hence, proved.

Question: 31

Solution:

Consider L.H.S. = (1 + tan θ + cot θ)(sin θ – cos θ)

= sin θ – cos θ + tan θ sin θ – tan θ cos θ + cot θ sin θ – cot θ cos θ

= sin θ – cos θ + tan θ sin θ – sin θ + cos θ – cot θ cos θ

= tan θ sin θ – cot θ cos θ

$= \frac{\sin\theta}{\cos\theta} \times \sin\theta - \frac{\cos\theta}{\sin\theta} \times \cos\theta$

$= \frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}$

$\left[\text{since, } \sin\theta = \frac{1}{\mathrm{cosec}\theta} \text{ and } \cos\theta = \frac{1}{\sec\theta}\right]$ $= \text{R.H.S.}$

$= \frac{\sec\theta}{\mathrm{cosec}^2\theta} - \frac{\mathrm{cosec}\theta}{\sec^2\theta}$

Hence, proved.

Question: 32

Solution:

Consider L.H.S. $= \frac{\cot^2\theta(\sec\theta - 1)}{(1+\sin\theta)} + \frac{\sec^2\theta(\sin\theta - 1)}{(1+\sec\theta)}$

$= \frac{\left(\frac{\cos^2\theta}{\sin^2\theta}\right)\left(\frac{1}{\cos\theta} - 1\right)}{(1+\sin\theta)} + \frac{\left(\frac{1}{\cos^2\theta}\right)(\sin\theta - 1)}{\left(1+\frac{1}{\cos\theta}\right)}$

$= \frac{\cos\theta(1-\cos\theta)}{(1+\sin\theta)\sin^2\theta} + \frac{(\sin\theta-1)\cos\theta}{\cos^2\theta(1+\cos\theta)}$ $\frac{(\sin\theta-1)\cos\theta}{(1-\sin^2\theta)(1+\cos\theta)}$

$= \frac{\cos\theta(1-\cos\theta)}{(1+\sin\theta)(1-\cos^2\theta)} + \frac{(\sin\theta-1)\cos\theta}{(1-\sin\theta)(1+\sin\theta)(1+\cos\theta)}$ $(1+\sin\theta)(1-\cos\theta)(1+\cos\theta)$

$= \frac{\cos\theta}{(1+\sin\theta)(1+\cos\theta)}$

$= \frac{\cos\theta}{(1+\sin\theta)(1+\cos\theta)} -$

$= 0$

$= \text{R.H.S.}$

Hence, proved.

Question: 33

Solution:

Consider L.H.S. $= \left\{\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\mathrm{cosec}^2\theta - \sin^2\theta}\right\}(\sin^2\theta\cos^2\theta)$

$= \left\{\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right\}(\sin^2\theta\cos^2\theta)$

$= \left\{\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right\}(\sin^2\theta\cos^2\theta)$

$= \left\{\frac{\cos^2\theta(1-\sin^4\theta)+\sin^2\theta(1-\cos^4\theta)}{(1-\cos^4\theta)(1-\sin^4\theta)}\right\}(\sin^2\theta\cos^2\theta)$

$= \frac{\cos^2\theta+\sin^2\theta-\cos^2\theta\sin^4\theta-\cos^4\theta\sin^2\theta}{(1-\cos^2\theta)(1+\cos^2\theta)(1-\sin^2\theta)(1+\sin^2\theta)}(\sin^2\theta\cos^2\theta)$

$= \frac{1-\cos^2\theta\sin^2\theta(\cos^2\theta+\sin^2\theta)}{\sin^2\theta\cos^2\theta(1+\cos^2\theta)(1+\sin^2\theta)}(\sin^2\theta\cos^2\theta)$

$= \frac{1-\sin^2\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}$

$= \frac{1-\sin^2\theta\cos^2\theta}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta}$

$= \frac{1-\sin^2\theta\cos^2\theta}{1+1+\sin^2\theta\cos^2\theta}$

$= \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

= R.H.S.

Question: 34

Solution:

Consider the left – hand side $= \frac{\sin A-\sin B}{\cos A+\cos B} + \frac{\cos A-\cos B}{\sin A+\sin B}$

$= \frac{(\sin A-\sin B)(\sin A+\sin B)+(\cos A-\cos B)(\cos A+\cos B)}{(\cos A+\cos B)(\sin A+\sin B)}$

$= \frac{\sin^2 A-\sin^2 B+\cos^2 A-\cos^2 B}{(\cos A+\cos B)(\sin A+\sin B)}$

$= \frac{\sin^2 A+\cos^2 A-(\sin^2 B+\cos^2 B)}{(\cos A+\cos B)(\sin A+\sin B)}$

$= \frac{1-1}{(\cos A+\cos B)(\sin A+\sin B)}$

= 0

= R.H.S.

Hence, proved.

Question: 35

Solution:

Consider the L.H.S. $= \frac{\tan A+\tan B}{\cot A+\cot B}$

$= \frac{\tan A+\tan B}{\frac{1}{\tan A}+\frac{1}{\tan B}}$

$= \frac{\tan A+\tan B}{\frac{\tan A+\tan B}{\tan A\tan B}}$

$= \frac{(\tan A+\tan B)(\tan A\tan B)}{(\tan A+\tan B)}$

= tan A tan B

= R.H.S.

Hence, proved.

Question: 36 A

Solution:

If the given equation is an identity, then it is true for every value of θ.

So, let θ = 60°

So, for θ = 60°, consider the L.H.S. = $\cos^2 60° + \cos 60°$

= $(1/2)^2 + (1/2)$

= (1/4) + (1/2)

= 3/4 ≠ 1

Therefore, L.H.S. ≠ R.H.S.

Thus, the given equation is not an identity.

Question: 36 B

Solution:

If the given equation is an identity, then it is true for every value of θ.

So, let θ = 30°

So, for θ = 30°, consider the L.H.S. = $\sin^2 30° + \sin 30°$

= $(1/2)^2 + (1/2)$

= (1/4) + (1/2)

= 3/4 ≠ 2

Therefore, L.H.S. ≠ R.H.S.

Thus, the given equation is not an identity.

Question: 36 C

Solution:

If the given equation is an identity, then it is true for every value of θ.

So, let θ = 30°

So, for θ = 30°, consider the L.H.S. = $\tan^2 30° + \sin 30°$

= $(1/\sqrt{3})^2 + (1/2)$

= (1/3) + (1/2)

= 5/6

Consider the R.H.S. = $\cos^2 30° = (\sqrt{3}/2)^2$

= 3/4

Therefore, L.H.S. ≠ R.H.S.

Thus, the given equation is not an identity.

Question: 37

Prove that: (sin

Solution:

Consider R.H.S. = $(2\cos^3 \theta - \cos \theta) \tan \theta$

= $\cos \theta(2\cos^2 \theta - 1)\left(\frac{\sin \theta}{\cos \theta}\right)$

= $(2\cos^2 \theta - 1)\sin \theta$

Consider L.H.S. = $(\sin \theta - 2 \sin^3 \theta)$

= $\sin \theta(1 - 2 \sin^2 \theta)$

= $\sin \theta[1 - 2(1 - \cos^2 \theta)]$

= $\sin \theta [1 - 2 + 2\cos^2 \theta]$

$= \sin\theta\,(2\cos^2\theta - 1)$

Therefore, L.H.S. = R.H.S.

Hence, proved.

Exercise : 8B

Question: 1

Solution:

Given: $a\cos\theta + b\sin\theta = m$ ....(1)

$a\sin\theta - b\cos\theta = n$ ....(2)

Square equation (1) and (2) on both sides:

$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta = m^2$ ....(3)

$a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\sin\theta = n^2$ ....(4)

Add equation (3) and (4):

$[a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta] + [a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\sin\theta] = m^2 + n^2$

$\Rightarrow a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta) = m^2 + n^2$

$\Rightarrow a^2 + b^2 = m^2 + n^2$

Hence, proved.

Question: 2

Solution:

Given: $a\sec\theta + b\tan\theta = x$........ (1)

$a\tan\theta + b\sec\theta = y$........ (2)

Square equation (1) and (2) on both sides:

$a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta = x^2$..........(3)

$a^2\tan^2\theta + b^2\sec^2\theta + 2ab\sec\theta\tan\theta = y^2$...........(4)

Subtract equation (4) from (3):

$[a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta] - [a^2\tan^2\theta + b^2\sec^2\theta + 2ab\sec\theta\tan\theta] = x^2 - y^2$

$\Rightarrow a^2(\sec^2\theta - \tan^2\theta) + b^2(\tan^2\theta - \sec^2\theta) = x^2 - y^2$

$\Rightarrow a^2 - b^2 = x^2 - y^2$ ($\because \sec^2\theta = 1 + \tan^2\theta$)

Hence, proved.

Question: 3

If Given: $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$........ (1)

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$........ (2)

Square equation (1) and (2) on both sides:

$\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{xy}{ab}\cos\theta\sin\theta = 1$ .... (3)

$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{xy}{ab}\cos\theta\sin\theta = 1$ .... (4)

Add equation (3) and (4):

$\frac{x^2}{a^2}(\sin^2\theta + \cos^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 1+1$

$\Rightarrow \frac{x^2}{a^2}(1) + \frac{y^2}{b^2}(1) = 2$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

Hence, proved.

Question: 4

Solution:

Given: $(\sec\theta + \tan\theta) = m$.................(1)

$(\sec\theta - \tan\theta) = n$.................(2)

Multiply equation (1) and (2):

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = mn$

$(\sec^2\theta - \tan^2\theta) = mn$

$1 = mn$ ($\because 1 + \tan^2\theta = \sec^2\theta$)

Therefore, mn = 1.

Hence, proved.

Question: 5

Solution:

Given: $(\text{cosec}\,\theta + \cot\theta) = m$ ....(1)

$(\text{cosec}\,\theta - \cot\theta) = n$ .... (2)

Multiply equation (1) and (2):

$(\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = mn$

$(\text{cosec}^2\theta - \cot^2\theta) = mn$

$1 = mn$ ($\because 1 + \cot^2\theta = \text{cosec}^2\theta$)

Therefore, mn = 1.

Hence, proved.

Question: 6

Solution:

Given: $x = a\cos^3\theta$

$y = b\sin^3\theta$

Consider L.H.S. $= \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3}$

$= \left(\frac{a\cos^3\theta}{a}\right)^{2/3} + \left(\frac{b\sin^3\theta}{b}\right)^{2/3}$

$= (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3}$

$= (\cos^2\theta + (\sin^2\theta)$

= 1 = R.H.S.

Hence, proved.

Question: 7

Solution:

Given: $\tan\theta + \sin\theta = m$ ....(1)

$\tan\theta - \sin\theta = n$ ....(2)

Square equation (1) and (2) on both sides:

$\tan^2\theta + \sin^2\theta + 2\sin\theta\tan\theta = m^2$ ....(3)

$\tan^2\theta + \sin^2\theta - 2\sin\theta\tan\theta = n^2$ ....(4)

Subtract equation (4) from (3):

$[\tan^2\theta + \sin^2\theta + 2\sin\theta\tan\theta] - [\tan^2\theta + \sin^2\theta - 2\sin\theta\tan\theta] = m^2 - n^2$

$\Rightarrow 4\sin\theta\tan\theta = m^2 - n^2$

Square both sides:

$\Rightarrow 16\sin^2\theta\tan^2\theta = (m^2 - n^2)^2$

Therefore, $(m^2 - n^2)^2 = 16\sin^2\theta\tan^2\theta$

Also, $16mn = 16 \times (\tan\theta + \sin\theta) \times (\tan\theta - \sin\theta)$

$= 16(\tan^2\theta - \sin^2\theta)$

$= 16[(\sin^2\theta/\cos^2\theta) - \sin^2\theta]$

$= 16[\sin^2\theta\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right)]$

$= 16\sin^2\theta(\sin^2\theta/\cos^2\theta)$

$= 16\sin^2\theta\tan^2\theta$

Therefore, $(m^2 - n^2)^2 = 16mn$

Hence, proved.

Question: 8

Solution:

Given: $(\cot\theta + \tan\theta) = m$

$(\sec\theta - \cos\theta) = n$

Since, $m = \cot\theta + \tan\theta$

$= (1/\tan\theta) + \tan\theta$

$= \frac{1+\tan^2\theta}{\tan\theta}$

$= \sec^2\theta/\tan\theta$

$= 1/(\sin\theta\ \cos\theta)$

Also, $n = \sec\theta - \cos\theta$

$= (1/\cos\theta) - \cos\theta$

$= (1 - \cos^2\theta)/\cos\theta$

= $\sin^2 \theta/\cos \theta$

Now, consider the left – hand side:

$$(m^2n)^{2/3} - (mn^2)^{2/3} = \left[\left(\frac{1}{\sin\theta\cos\theta}\right)^2 \times \frac{\sin^2\theta}{\cos\theta}\right]^{2/3} - \left[\left(\frac{1}{\sin\theta\cos\theta}\right) \times \left(\frac{\sin^2\theta}{\cos\theta}\right)^2\right]^{2/3}$$

$$= \left[\frac{\sin^2\theta}{\sin^2\theta\cos^3\theta}\right]^{2/3} - \left[\frac{\sin^4\theta}{\sin\theta\cos^3\theta}\right]^{2/3}$$

$$= \left[\frac{1}{\cos^3\theta}\right]^{2/3} - \left[\frac{\sin^3\theta}{\cos^3\theta}\right]^{2/3}$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

= $(1 - \sin^2 \theta)\cos^2 \theta$

= $\cos^2 \theta/ \cos^2 \theta$

= 1

Question: 9

Solution:

Given: $(\operatorname{cosec} \theta - \sin \theta) = a^3$

$(\sec \theta - \cos \theta) = b^3$

Since, $a^3 = (\operatorname{cosec} \theta - \sin \theta)$

= $(1/\sin \theta) - \sin \theta$

$$= \frac{1-\sin^2\theta}{\sin\theta}$$

= $\cos^2 \theta/\sin \theta$

Therefore, $a^2 = (a^3)^{2/3} = (\cos^2 \theta/\sin \theta)^{2/3}$

Also, $b^3 = \sec \theta - \cos \theta$

= $(1/\cos \theta) - \cos \theta$

= $(1 - \cos^2 \theta)/\cos \theta$

= $\sin^2 \theta/\cos \theta$

Therefore, $b^2 = (b^3)^{2/3} = (\sin^2 \theta/\cos \theta)^{2/3}$

Now, consider the left – hand side:

$$a^2 b^2 (a^2 + b^2) = \left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{2/3} \times \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}} \times \left(\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{\frac{2}{3}} + \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}}\right)$$

$$= \left(\frac{\cos^2\theta\sin^2\theta}{\cos\theta\sin\theta}\right)^{2/3} \times \left(\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{\frac{2}{3}} + \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}}\right)$$

= $[\cos^3 \theta]^{2/3} + [\sin^3 \theta]^{2/3}$

= $\sin^2 \theta + \cos^2 \theta$

= 1 = R.H.S.

Hence, proved.

Question: 10

Solution:

Given: $2 \sin \theta + 3 \cos \theta = 2$

Consider $(2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 4\sin^2 \theta + 9 \cos^2 \theta + 12\sin \theta \cos \theta + 9 \sin^2 \theta + 4 \cos^2 \theta - 12 \sin \theta \cos \theta$

$\Rightarrow (2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13\sin^2 \theta + 13 \cos^2 \theta$

$\Rightarrow (2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13(\sin^2 \theta + \cos^2 \theta)$

$\Rightarrow (2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13$

$\Rightarrow (2)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13$

$\Rightarrow (3 \sin \theta - 2 \cos \theta)^2 = 13 - 4$

$\Rightarrow (3 \sin \theta - 2 \cos \theta)^2 = 9$

$\Rightarrow (3 \sin \theta - 2 \cos \theta) = \pm 3$

Hence, proved.

Question: 11

Solution:

Given: $(\sin \theta + \cos \theta) = \sqrt{2} \cos \theta$ **To show:** $\cot \theta = (\sqrt{2} + 1)$ **Solution:** $(\sin \theta + \cos \theta) = \sqrt{2} \cos \theta$

~~Divide both sides by sin θ,~~ $\Rightarrow \frac{\sin\theta + \cos\theta}{\sin\theta} = \frac{\sqrt{2}\cos\theta}{\sin\theta}$

$\Rightarrow \frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{2}\cos\theta}{\sin\theta}$ ~~Since~~ $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\Rightarrow 1 + \cot \theta = \sqrt{2} \cot \theta$

$\Rightarrow 1 = \sqrt{2} \cot \theta - \cot \theta$

$\Rightarrow (\sqrt{2} - 1)\cot \theta = 1$

$\Rightarrow \cot \theta = \frac{1}{\sqrt{2}-1}$

$\Rightarrow \cot \theta = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$\Rightarrow \cot \theta = \frac{\sqrt{2}+1}{2-1}$

$\Rightarrow \cot \theta = \sqrt{2} + 1$

Question: 12

Solution:

Given: $\cos \theta + \sin \theta = \sqrt{2}\sin \theta$

Consider $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = \sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta$

$\Rightarrow (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2\sin^2 \theta + 2 \cos^2 \theta$

$\Rightarrow (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2(\sin^2 \theta + \cos^2 \theta)$

$\Rightarrow (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$

$\Rightarrow (\sqrt{2}\sin \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$

$\Rightarrow (\sin \theta - \cos \theta)^2 = 2 - 2\sin^2 \theta$

$\Rightarrow (\sin\theta - \cos\theta)^2 = 2(1 - \sin^2\theta)$

$\Rightarrow (\sin\theta - \cos\theta)^2 = 2(\cos^2\theta)$

$\Rightarrow (\sin\theta - \cos\theta) = \pm\sqrt{2}\cos\theta$

Hence, proved.

Question: 13

Solution:

(i) Given: $\sec\theta + \tan\theta = p$ ……(1)

Then, $(\sec\theta + \tan\theta) \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} = p$

$\Rightarrow \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} = p$

$\Rightarrow \frac{1}{\sec\theta - \tan\theta} = p$

$\Rightarrow \sec\theta - \tan\theta = (1/p)$ ……(2)

Adding equation (1) and (2), we get:

$2\sec\theta = p + (1/p)$

$\Rightarrow \sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

Therefore, $\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

(ii) Given: $\sec\theta + \tan\theta = p$ ……(1)

Then, $(\sec\theta + \tan\theta) \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} = p$

$\Rightarrow \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} = p$

$\Rightarrow \frac{1}{\sec\theta - \tan\theta} = p$

$\Rightarrow \sec\theta - \tan\theta = (1/p)$ ……(2)

Subtracting equation (2) from (1), we get:

$2\tan\theta = p - (1/p)$

$\Rightarrow \tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$

(iii) Since $\sin\theta = \tan\theta/\sec\theta$

$= \frac{\frac{1}{2}\left(p - \frac{1}{p}\right)}{\frac{1}{2}\left(p + \frac{1}{p}\right)}$

$= \frac{\left(p - \frac{1}{p}\right)}{\left(p + \frac{1}{p}\right)}$

$= \frac{p^2 - 1}{p^2 + 1}$

Question: 14

Solution:

Given: $\tan A = n \tan B$

Therefore, $\tan B = \frac{\tan A}{n}$ -

Thus, $\cot B = \frac{n}{\tan A}$ Squaring both sides, we get,

$\Rightarrow \cot^2 B = n^2/\tan^2 A$ ......(1)

Also, sin A = m sin B

Therefore, sin B = sin A/m

Thus, cosec B = m/sin A

$\Rightarrow \text{cosec}^2 B = m^2/\sin^2 A$ ......(2)

Now, subtract equation (2) from (1):

$\text{cosec}^2 B - \cot^2 B = \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A}$

$\Rightarrow 1 = \frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A}$

$\Rightarrow 1 = \frac{m^2 - n^2 \cos^2 A}{\sin^2 A}$

$\Rightarrow m^2 - n^2 \cos^2 A = \sin^2 A$

$\Rightarrow m^2 - n^2 \cos^2 A = 1 - \cos^2 A$

$\Rightarrow m^2 - 1 = n^2 \cos^2 A - \cos^2 A$

$\Rightarrow (n^2 - 1)\cos^2 A = m^2 - 1$

$\Rightarrow \cos^2 A = (m^2 - 1)/(n^2 - 1)$

Hence, proved.

Question: 15

Solution:

Given: m = (cos θ – sin θ)

n = (cos θ + sin θ)

Now, $\frac{m}{n} = \frac{(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)}$

Multiply numerator and denominator by cos θ – sin θ :

Therefore, $\frac{m}{n} = \frac{(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)} \times \frac{\cos\theta - \sin\theta}{\cos\theta - \sin\theta}$

$= \frac{(\cos\theta - \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}$

Now, $\frac{n}{m} = \frac{(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)}$

Multiply numerator and denominator by cos θ + sin θ :

Therefore, $\frac{n}{m} = \frac{(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)} \times \frac{\cos\theta + \sin\theta}{\cos\theta + \sin\theta}$

$= \frac{(\cos\theta + \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}$

Now, consider $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \sqrt{\frac{(\cos\theta - \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}} + \sqrt{\frac{(\cos\theta + \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}}$

$= \frac{\cos\theta - \sin\theta}{\sqrt{(\cos^2\theta - \sin^2\theta)}} + \frac{\cos\theta + \sin\theta}{\sqrt{(\cos^2\theta - \sin^2\theta)}}$

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$= \frac{1}{\sqrt{(\cos^2\theta - \sin^2\theta)}}(\cos\theta - \sin\theta + \cos\theta + \sin\theta)$

$= \frac{2\cos\theta}{\sqrt{(\cos^2\theta - \sin^2\theta)}}$

Divide numerator and denominator by cos θ:

$= \frac{2}{\sqrt{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}}$

$= \frac{2}{\sqrt{(1 - \tan^2\theta)}}$

Therefore, $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{2}{\sqrt{(1 - \tan^2\theta)}}$

Hence, proved.

Exercise : 8C

Question: 1

Solution:

Consider $(1 - \sin^2\theta)\sec^2\theta = (\cos^2\theta) \times \sec^2\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

= 1

Question: 2

Solution:

Consider $(1 - \cos^2\theta)\,\mathrm{cosec}^2\theta = (\sin^2\theta) \times \mathrm{cosec}^2\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

= 1

Question: 3

Solution:

Consider $(1 + \tan^2\theta)\cos^2\theta = (\sec^2\theta) \times \cos^2\theta$

$(\because 1 + \tan^2\theta = \sec^2\theta)$

= 1

Question: 4

Solution:

Consider $(1 + \cot^2\theta) \times \sin^2\theta = (\mathrm{cosec}^2\theta) \times \sin^2\theta$

$(\because 1 + \cot^2\theta = \mathrm{cosec}^2\theta)$

= 1

Question: 5

Solution:

Consider $\sin^2\theta + \frac{1}{1+\tan^2\theta}$

$= (\sin^2\theta) + (1/\sec^2\theta)$

$(\because 1 + \tan^2\theta = \sec^2\theta)$

$= (\sin^2\theta) + (\cos^2\theta)$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1$

Question: 6

Solution:

Consider $\cot^2\theta - \frac{1}{\sin^2\theta}$

$= (\cot^2\theta) - (\text{cosec}^2\theta)$

$= -(\text{cosec}^2\theta - \cot^2\theta)$

$(\because 1 + \cot^2\theta = \text{cosec}^2\theta)$

$= -1$

Question: 7

Solution:

Consider $\sin\theta\cos(90° - \theta) + \cos\theta\sin(90° - \theta) = \sin\theta\sin\theta + \cos\theta\cos\theta$

$= \sin^2\theta + \cos^2\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1$

Question: 8

Solution:

Consider $\text{cosec}^2(90° - \theta) - \tan^2\theta = \sec^2\theta - \tan^2\theta$

$(\because 1 + \tan^2\theta = \sec^2\theta)$

$= 1$

Question: 9

Solution:

Consider $\sec^2\theta\,(1 + \sin\theta)(1 - \sin\theta) = \sec^2\theta\,(1 - \sin^2\theta)$

$= \sec^2\theta\cos^2\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1$

Question: 10

Solution:

Consider $\text{cosec}^2 \theta (1 + \cos \theta)(1 - \cos \theta) = \text{cosec}^2 \theta (1 - \cos^2 \theta)$

$= \text{cosec}^2 \theta \sin^2 \theta$

$(\because \sin^2 \theta + \cos^2 \theta = 1)$

$= 1$

Question: 11

Solution:

Consider $\sin^2 \theta \cos^2 \theta (1 + \tan^2 \theta)(1 + \cot^2 \theta)$

$= \sin^2 \theta \cos^2 \theta (\sec^2 \theta)(\text{cosec}^2 \theta)$ $(\because 1 + \cot^2 \theta = \text{cosec}^2 \theta$ and $1 + \tan^2 \theta = \sec^2 \theta)$

$= \sin^2 \theta (\text{cosec}^2 \theta) \cos^2 \theta (\sec^2 \theta)$

$= 1 \times 1$

$= 1$

Question: 12

Solution:

Consider $(1 + \tan^2 \theta)(1 + \sin \theta)(1 - \sin \theta)$

$= (1 + \tan^2 \theta)(1 - \sin^2 \theta)$ $(\because \sin^2 \theta + \cos^2 \theta = 1$ and $1 + \tan^2 \theta = \sec^2 \theta)$

$= (\sec^2 \theta) (\cos^2 \theta)$

$= 1$

Question: 13

Solution:

Consider $3 \cot^2 \theta - 3 \text{cosec}^2 \theta = -3(\text{cosec}^2 \theta - \cot^2 \theta)$

$= -3(1)$ $(\because 1 + \cot^2 \theta = \text{cosec}^2 \theta)$

$= -3$

Question: 14

Solution:

Consider $4 \tan^2 \theta - \frac{4}{\cos^2 \theta} = 4 \tan^2 \theta - 4 \sec^2 \theta$

$= 4(\tan^2 \theta - \sec^2 \theta)$

$= 4(-1)$ $(\because 1 + \tan^2 \theta = \sec^2 \theta)$

$= -4$

Question: 15

Solution:

Consider $\frac{\tan^2 \theta - \sec^2 \theta}{\cot^2 \theta - \text{cosec}^2 \theta} = \frac{-1}{-1}$ $(\because 1 + \tan^2 \theta = \sec^2 \theta$ and $1 + \cot^2 \theta = \text{cosec}^2 \theta)$

$= 1$

Question: 16

Solution:

Give: $\sin\theta = 1/2$

Therefore $\operatorname{cosec}\theta = 1/\sin\theta$

$= 2$

Consider $3\cot^2\theta + 3 = 3(\cot^2\theta + 1)$

$= 3\operatorname{cosec}^2\theta$ ($\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta$)

$= 3(2)^2$

$= 3 \times 4$

$= 12$

Question: 17

Solution:

Give: $\cos\theta = 2/3$

Therefore $\sec\theta = 1/\cos\theta$

$= 3/2$

Consider $4\tan^2\theta + 4 = 4(\tan^2\theta + 1)$

$= 4\sec^2\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$)

$= 4(3/2)^2$

$= 4 \times (9/4)$

$= 9$

Question: 18

Solution:

Given: $\cos\theta = 7/25$

Therefore $\sin\theta = \sqrt{(1 - \cos^2\theta)}$

$= \sqrt{(1 - (49/625))}$

$= \sqrt{[(625 - 49)/625]}$

$= \sqrt{(576/625)}$

$= 24/25$

Thus, $\tan\theta = \sin\theta/\cos\theta = (24/25)/(7/25)$

$= 24/7$

Also, $\cot\theta = 1/\tan\theta = 7/24$

Therefore, $\tan\theta + \cot/\theta = (24/7) + (7/24)$

$= (576 + 49)/(24 \times 7)$

$= 625/168$

Question: 19

Solution:

Given: cos θ = 2/3

Thus, sec θ = 1/cos θ

= 3/2

Now, consider $\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{\frac{3}{2} - 1}{\frac{3}{2} + 1}$

= [(1/2)/(5/2)]

= 1/5

Question: 20

Solution:

Given: 5 tan θ = 4

Therefore, tan θ = 4/5

Now, consider $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$ and divide numerator and denominator by cos θ:

$= \frac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$

$= \frac{1 - \tan\theta}{1 + \tan\theta}$

$= \frac{1 - \frac{4}{5}}{1 + \frac{4}{5}}$

= (1/5)/(9/5)

= 1/9

Question: 21

Solution:

Given: 3 cot θ = 4

Therefore, cot θ = 4/3

Therefore, tan θ = 3/4

Now, consider $\frac{2\cos\theta + \sin\theta}{4\cos\theta - \sin\theta}$ and divide numerator and denominator by cos θ:

$= \frac{2\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{4\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}$

$= \frac{2 + \tan\theta}{4 - \tan\theta}$

$= \frac{2 + \frac{3}{4}}{4 - \frac{3}{4}}$

= (11/4)/(13/4)

= 11/13

Question: 22

If cot θ = 1/√3 w

Solution:

Given: $\cot\theta = 1/\sqrt{3}$

Thus, $\tan\theta = 1/\cot\theta = \sqrt{3}$

Therefore $\sec\theta = \sqrt{(1 + \tan^2\theta)}$

$= \sqrt{(1 + 3)}$

$= \sqrt{(4)}$

$= 2$

Therefore $\sec^2\theta = 4$

Now, $\cos^2\theta = 1/\sec^2\theta = 1/4$

So, consider $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{1-\cos^2\theta}{1+1-\sin^2\theta}$

$= \frac{1-\cos^2\theta}{1+\cos^2\theta}$

$= \frac{1-\frac{1}{4}}{1+\frac{1}{4}}$

$= (3/4)/(5/4)$

$= 3/5$

Question: 23

Solution:

Given: $\tan\theta = 1/\sqrt{5}$

$\therefore \tan^2\theta = 1/5$

Consider $\frac{(\mathrm{cosec}^2\theta-\sec^2\theta)}{(\mathrm{cosec}^2\theta+\sec^2\theta)} = \frac{\left(\frac{1}{\sin^2\theta}-\frac{1}{\cos^2\theta}\right)}{\left(\frac{1}{\sin^2\theta}+\frac{1}{\cos^2\theta}\right)}$

Multiply numerator and denominator by $\sin\theta$:

$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\frac{1}{5}}{1+\frac{1}{5}}$

$= 4/6$

$= 2/3$

Question: 24

Solution:

We are given that: $\cot A = 4/3$

$\Rightarrow \tan(90° - A) = 4/3$

Since $A + B = 90°$, therefore $B = 90° - A$

Therefore, $\tan(90° - A) = \tan B = 4/3$

Question: 25

If $\cos B = 3/5$ an

Solution:

We are given that: cos B = 3/5

⇒ sin (90° – B) = 3/5

Since A + B = 90°, therefore A = 90° – B

Therefore, sin (90° – B) = sin A = 3/5

Question: 26

Solution:

We are given that: √3sin θ = cos θ

∴sin θ /cos θ = 1/√3

⇒ tan θ = 1/√3

⇒ tan θ = tan 30°

On comparing both sides, we get,

θ = 30°

Question: 27

Solution:

Consider tan 10° tan 20° tan 70° tan 80°

= tan 10° tan 20° tan (90° – 20°) tan (90° – 10°)

= tan 10° tan 20° cot 0° cot 10°

= tan 10° cot 10° tan 20° cot 20°

= 1 × 1

= 1

Question: 28

Solution:

Consider tan 1° tan 2°... tan 88° tan 89°

= tan 1° tan 2°... tan 44° tan 45° tan 46° ...tan 88° tan 89°

= tan 1° tan 2°...tan 44° tan 45° tan (90° – 44°) ...tan (90° – 2°) tan (90° – 1°)

= tan 1° tan 2° ... tan 44 ° tan 45° cot 44° ...cot 2° cot 1°

= tan 1° cot 1° tan 2° cot 2°... tan 44 ° cot 44 ° tan 45°

= 1 × 1 × ...× 1

= 1

Question: 29

Solution:

Consider cos 1° cos 2° cos 3° ... cos 180°

= cos 1° cos 2° cos 3° ...× cos 90° × ... × cos 180°

= cos 1° cos 2° cos 3° ... × 0 × ... × cos 180°

$= 0$ ($\because \cos 90° = 0$)

Question: 30

Solution:

Given: $\tan A = 5/12$

Consider $(\sin A + \cos A) \sec A = (\sin A + \cos A)(1/\cos A)$

$= (\sin A/\cos A) + (\cos A/\cos A)$

$= \tan A + 1$

$= (5/12) + 1$

$= 17/12$

Question: 31

Solution:

We are given that: $\sin \theta = \cos (\theta - 45°)$

$\therefore$We can rewrite it as: $\cos (90° - \theta) = \cos (\theta - 45°)$

On comparing both sides, we get,

$90° - \theta = \theta - 45°$

$\Rightarrow \theta + \theta = 90° + 45°$

$\Rightarrow 2\theta = 135°$

$\Rightarrow \theta = 65.5°$

Question: 32

Solution:

Consider $\frac{\sin 50°}{\cos 40°} + \frac{\operatorname{cosec} 40°}{\sec 50°} - 4 \cos 50° \operatorname{cosec} 40°$

$= \frac{\sin 50°}{\cos(90° - 50°)} + \frac{\operatorname{cosec} 40°}{\sec(90° - 40°)} - 4 \cos 50° \operatorname{cosec} (90° - 40°)$

$= \frac{\sin 50°}{\sin 50°} + \frac{\operatorname{cosec} 40°}{\operatorname{cosec} 40°} - 4 \cos 50° \sec 50°$

$= 1 + 1 - 4$

$= -2$

Question: 33

Solution:

Consider $\sin 48° \sec 42° + \cos 48° \operatorname{cosec} 42°$

$= \sin 48° \sec (90° - 48°) + \cos 48° \operatorname{cosec} (90° - 48°)$

$= \sin 48° \operatorname{cosec} 48° + \cos 48° \sec 48°$

$= 1 + 1$

$= 2$

Question: 34

Solution:

Given: $x = a \sin \theta$

$y = b \cos \theta$

Then $b^2x^2 + a^2y^2 = b^2(a \sin \theta)^2 + a^2(b \cos \theta)^2$

$= a^2b^2 \sin^2 \theta + a^2 b^2 \cos^2 \theta$

$= a^2b^2 (\sin^2 \theta + \cos^2 \theta)$

$= (a^2b^2) \times 1$

$= a^2b^2$

Question: 35

Solution:

Given: $5x = \sec \theta$, and $5/x = \tan \theta$

Consider $5(x^2 - (1/x^2)) = \frac{5}{5}\left(5x^2 - \frac{5}{x^2}\right)$

$= \frac{1}{5}\left(25x^2 - \frac{25}{x^2}\right)$

$= \frac{1}{5}\left((5x)^2 - \left(\frac{5}{x}\right)^2\right)$

$= (1/5)\ [\sec^2 \theta - \tan^2 \theta]$

$= (1/5)[1]$

$= 1/5\ (\because \sec^2 x - \tan^2 x = 1)$

Question: 36

Solution:

Given: $2x = \text{cosec}\ \theta$, and $2/x = \cot \theta$

Consider $2(x^2 - (1/x^2)) = \frac{2}{2}\left(2x^2 - \frac{2}{x^2}\right)$

$= \frac{1}{2}\left(4x^2 - \frac{4}{x^2}\right)$

$= \frac{1}{2}\left((2x)^2 - \left(\frac{2}{x}\right)^2\right)$

$= (1/2)(\text{cosec}^2 \theta - \cot^2 \theta)$

$= 1/2\ (\because \text{cosec}^2 x - \cot^2 x = 1)$

Question: 37

If $\sec \theta + \tan \theta$

Solution:

Given: $\sec \theta + \tan \theta = x$ ......(1)

Then, $(\sec \theta + \tan \theta) \times \frac{\sec \theta - \tan \theta}{\sec \theta - \tan \theta} = x$

$\Rightarrow \frac{\sec^2 \theta - \tan^2 \theta}{\sec \theta - \tan \theta} = x$

$\Rightarrow \frac{1}{\sec \theta - \tan \theta} = x$

$\Rightarrow \sec\theta - \tan\theta = (1/x)$ ......(2)

Adding equation (1) and (2), we get:

$2\sec\theta = x + (1/x)$

$= (x^2 + 1)/x$

$\Rightarrow \sec\theta = (x^2 + 1)/2x$

Therefore, $\sec\theta = (x^2 + 1)/2x$

Question: 38

Solution:

Consider $\frac{\cos 38°\operatorname{cosec} 52°}{\tan 18°\tan 35°\tan 60°\tan 72°\tan 55°}$

$= \frac{\cos 38°\operatorname{cosec}(90°-38°)}{\tan 18°\tan(90°-55°)\tan 60°\tan(90°-18°)\tan 55°}$

$= \frac{\cos 38°\sec 38°}{\tan 18°\cot 55°\tan 60°\cot 18°\tan 55°}$

$= \frac{\cos 38°\sec 38°}{\tan 18°\cot 18°\cot 55°\tan 55°\tan 60°}$

$= \frac{1}{1\times 1\times \tan 60°}$

$= \cot 60°$

$= 1/\sqrt{3}$

Question: 39

Solution:

Given: $\sin\theta = x$

Therefore, $\operatorname{cosec}\theta = 1/x$

Using the identity $1 + \cot^2\theta = \operatorname{cosec}^2\theta$, we get

$\cot\theta = \sqrt{(\operatorname{cosec}^2\theta - 1)}$

$= \sqrt{((1/x)^2 - 1)}$

$= \sqrt{\frac{x^2-1}{x^2}}$

$= \frac{\sqrt{x^2-1}}{x}$

Question: 40

If $\sec\theta = x$, wri

Solution:

Given: $\sec\theta = x$

Using the identity $1 + \tan^2\theta = \sec^2\theta$, we get

$\tan\theta = \sqrt{(\sec^2\theta - 1)}$

$= \sqrt{(x^2 - 1)}$