Chapter : 9. MEAN, MEDIAN, MODE OF GROUPED DATA CUMULATIVE FREQUENCY GRAPH AND OGIVE

Exercise : 9A

Question: 1

Solution:

Mean of the observation is given by –

$$\text{Mean} = \frac{\text{Sum of the given observations}}{\text{Total number of observations}}$$

So, adding the given observations, we get

Sum of the given observations = x + (x + 2) + (x + 4) + (x + 6) + (x + 8)

= x + x + 2 + x + 4 + x + 6 + x + 8

= 5x + 20

Total number of observations = 5

Mean = 11 (Given)

$$\text{Mean} = \frac{5x + 20}{5}$$

$$\Rightarrow 11 = \frac{5x + 20}{5}$$

⇒ 55 = 5x + 20

⇒ 5x = 55 – 20 = 35

⇒ x = 7

Thus, x = 7

Question: 2

Solution:

Mean of the observation is given by –

$$\text{Mean} = \frac{\text{Sum of the given observations}}{\text{Total number of observations}}$$

Total number of observations = 25

Mean of 25 observation = 27 (Given)

We get

$$27 = \frac{\text{Sum of 25 observations}}{25}$$

⇒ Sum of 25 observations = 27 × 25 = 675

If each observation is decreased by 7, the Sum gets affected.

New Sum = 675 – (25 × 7) = 675 – 175 = 500

$$\text{New mean} = \frac{500}{25} = 20$$

Thus, new mean = 20

Question: 3

Solution:

For equal class intervals, we will solve by finding mid points of these classes using direct method.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
1 - 3	2	12	24
3 - 5	4	22	88
5 - 7	6	27	162
7 - 9	8	19	152
TOTAL		80	426

We have got

$\Sigma f_i = 20$ & $\Sigma f_ix_i = 426$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_ix_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{426}{80}$$

$$\Rightarrow \bar{x} = 5.325$$

Thus, mean is 5.325

Question: 4

Solution:

For equal class intervals, we will solve by finding mid points of these classes using direct method.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
0 - 10	5	7	35
10 - 20	15	5	75
20 - 30	25	6	150
30 - 40	35	12	420
40 - 50	45	8	360
50 - 60	55	2	110
TOTAL		40	1150

We have got

$\Sigma f_i = 40$ & $\Sigma f_ix_i = 1150$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{1150}{40}$$

$$\Rightarrow \bar{x} = 28.75$$

Thus, mean is 28.75

Question: 5

Solution:

For equal class intervals, we will solve by finding mid points of these classes using direct method.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
25 - 35	30	6	180
35 - 45	40	10	400
45 - 55	50	8	400
55 - 65	60	12	720
65 - 75	70	4	280
TOTAL		40	1980

We have got

$\Sigma f_i = 40$ & $\Sigma f_ix_i = 1980$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_ix_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{1980}{40}$$

$$\Rightarrow \bar{x} = 49.5$$

Thus, mean is 49.5

Question: 6

Solution:

For equal class intervals, we will solve by finding mid points of these classes using direct method.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
0 - 100	50	6	300
100 - 200	150	9	1350
200 - 300	250	15	3750
300 - 400	350	12	4200
400 - 500	450	8	3600
TOTAL		50	13200

We have got

$\Sigma f_i = 50$ & $\Sigma f_ix_i = 13200$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_ix_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{13200}{50}$$

$$\Rightarrow \bar{x} = 264$$

Thus, mean is 264

Question: 7

Solution:

For equal class intervals, we will solve by finding mid points of these classes using direct method.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
84 - 90	87	8	696
90 - 96	93	10	930
96 - 102	99	16	1584
102 - 108	105	23	2415
108 - 114	111	12	1332
114 - 120	117	11	1287
TOTAL		80	8244

We have got

$\Sigma f_i = 80$ & $\Sigma f_ix_i = 8244$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_ix_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{8244}{80}$$

$$\Rightarrow \bar{x} = 103.05$$

Thus, mean is 103.05

Here, the method being used is direct method as it is easy to calculate the mid - points of the class intervals and the rest calculations were simple and easy.

Question: 8

Solution:

For equal class intervals, we will solve by finding mid points of these classes using direct method.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
0 - 10	5	3	15
10 - 20	15	4	60
20 - 30	25	p	25p
30 - 40	35	3	105
40 - 50	45	2	90
TOTAL		12 + p	270 + 25p

We have got

$\Sigma f_i = 12 + p$ & $\Sigma f_ix_i = 270 + 25p$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_ix_i}{\sum_i f_i}$$

$$\Rightarrow 24 = \frac{270 + 25p}{12 + p}$$

$\Rightarrow 288 + 24p = 270 + 25p$

$\Rightarrow 25p - 24p = 288 - 270$

$\Rightarrow p = 18$

Thus, p is 18

Question: 9

Solution:

For equal class intervals, we will solve by finding mid points of these classes using direct method.

DAILY POCKET ALLOWANCE (Rs.)	MID - POINT(x_i)	NUMBER OF CHILDREN (f_i)	f_ix_i
11 - 13	12	7	84
13 - 15	14	6	84
15 - 17	16	9	144
17 - 19	18	13	234
19 - 21	20	f	20f
21 - 23	22	5	110
23 - 25	24	4	96
TOTAL		44 + f	752 + 20f

We have got

$\Sigma f_i = 44 + f$ and $\Sigma f_i x_i = 752 + 20f$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$\Rightarrow 18 = \frac{752 + 20f}{44 + f}$ ($\because$ given: mean of pocket allowance is 18)

$\Rightarrow 792 + 18f = 752 + 20f$

$\Rightarrow 20f - 18f = 792 - 752$

$\Rightarrow 2f = 40$

$\Rightarrow f = 20$

Thus, f is 20.

Question: 10

Solution:

For equal class intervals, we will solve by finding mid points of these classes using ....

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
0 - 20	10	7	70
20 - 40	30	p	30p
40 - 60	50	10	500
60 - 80	70	9	630
80 - 100	90	13	1170
TOTAL		39 + p	2370 + 30p

We have got

$\Sigma f_i = 39 + p$ and $\Sigma f_i x_i = 2370 + 30p$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$\Rightarrow 54 = \frac{2370 + 30p}{39 + p}$ ($\because$ given: mean of pocket allowance is 54)

$\Rightarrow 2106 + 54p = 2370 + 30p$

$\Rightarrow 54p - 30p = 2370 - 2106$

$\Rightarrow 24p = 264$

$\Rightarrow p = 11$

Thus, p is 11.

Question: 11

Solution:

For equal class intervals, we will solve by finding mid points of these classes using direct method.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
0 - 10	5	7	35
10 - 20	15	10	150
20 - 30	25	x	25x
30 - 40	35	13	455
40 - 50	45	y	45y
50 - 60	55	10	550
60 - 70	65	14	910
70 - 80	75	9	675
TOTAL		63 + x + y	2775 + 25x + 45y

We have got

$\Sigma f_i = 63 + x + y$ and $\Sigma f_ix_i = 2775 + 25x + 45y$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_ix_i}{\sum_i f_i}$$

$\Rightarrow 42 = \frac{2775 + 25x + 45y}{63 + x + y}$ ($\because$ given: mean of pocket allowance is 42)

$\Rightarrow 2646 + 42x + 42y = 2775 + 25x + 45y$

$\Rightarrow 42x - 25x + 42y - 45y = 2775 - 2646$

$\Rightarrow 17x - 3y = 129$ ...(i)

As given in the question, frequency(Σf_i) = 100

And as calculated by us, frequency (Σf_i) = 63 + x + y

Equalizing them, we get

$63 + x + y = 100$

$\Rightarrow x + y = 37$ ...(ii)

We will now solve equations (i) and (ii), multiply eq.(ii) by 3 and then add it to eq.(i), we get

$(17x - 3y) + [3(x + y)] = 129 + 111$

$\Rightarrow 17x - 3y + 3x + 3y = 240$

$\Rightarrow 20x = 240$

$\Rightarrow x = 12$

Substitute $x = 12$ in equation (ii),

$12 + y = 37$

$\Rightarrow y = 37 - 12$

$\Rightarrow y = 25$

Thus, $x = 12$ and $y = 25$.

Question: 12

Solution:

For equal class intervals, we will solve by finding mid points of these classes using direct method.

EXPENDITURE (Rs.)	MID - POINT(x_i)	NUMBER OF FAMILIES(f_i)	f_ix_i
140 - 160	150	5	750
160 - 180	170	25	4250
180 - 200	190	f_1	$190f_1$
200 - 220	210	f_2	$210f_2$
220 - 240	230	5	1150
TOTAL		$35 + f_1 + f_2$	$6150 + 190f_1 + 210f_2$

We have got

$\Sigma f_i = 35 + f_1 + f_2$ and $\Sigma f_ix_i = 6150 + 190f_1 + 210f_2$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$\Rightarrow 188 = \frac{6150 + 190f_1 + 210f_2}{35 + f_1 + f_2}$ ($\because$ given: mean of pocket allowance is 188)

$\Rightarrow 6580 + 188f_1 + 188f_2 = 6150 + 190f_1 + 210f_2$

$\Rightarrow 190f_1 - 188f_1 + 210f_2 - 188f_2 = 6580 - 6150$

$\Rightarrow 2f_1 + 22f_2 = 430$ ...(i)

As given in the question, frequency(Σf_i) = 100

And as calculated by us, frequency (Σf_i) = $35 + f_1 + f_2$

Comparing them, we get

$35 + f_1 + f_2 = 100$

$\Rightarrow f_1 + f_2 = 65$ ...(ii)

We will now solve equations (i) and (ii), multiply eq.(ii) by 2 and then subtracting it from eq.(i), we get

$(2f_1 + 22f_2) - [2(f_1 + f_2)] = 430 - 130$

$\Rightarrow 2f_1 + 22f_2 - 2f_1 - 2f_2 = 300$

$\Rightarrow 20\ f_2 = 300$

$\Rightarrow f_2 = 15$

Substitute $f_2 = 15$ in equation (ii),

$f_1 + 15 = 65$

$\Rightarrow f_1 = 65 - 15$

$\Rightarrow f_1 = 50$

Thus, $f_1 = 50$ and $f_2 = 15$.

Question: 13

Solution:

For equal class intervals, we will solve by finding mid points of these classes using direct method.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
0 - 20	10	7	70
20 - 40	30	f_1	$30f_1$
40 - 60	50	12	600
60 - 80	70	f_2	$70f_2$
80 - 100	90	8	720
100 - 120	110	5	550
TOTAL		$32 + f_1 + f_2$	$1940 + 30f_1 + 70f_2$

We have got

$\Sigma f_i = 32 + f_1 + f_2$ and $\Sigma f_ix_i = 1940 + 30f_1 + 70f_2$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_ix_i}{\sum_i f_i}$$

$\Rightarrow 57.6 = \frac{1940 + 30f_1 + 70f_2}{32 + f_1 + f_2}$ ($\because$ given: mean of pocket allowance is 57.6)

$\Rightarrow 1843.2 + 57.6f_1 + 57.6f_2 = 1940 + 30f_1 + 70f_2$

$\Rightarrow 57.6f_1 - 30f_1 + 57.6f_2 - 70f_2 = 1940 - 1843.2$

$\Rightarrow 27.6f_1 - 12.4f_2 = 96.8$

$\Rightarrow 69f_1 - 31f_2 = 242$ ...(i)

As given in the question, frequency(Σf_i) = 50

And as calculated by us, frequency (Σf_i) = $32 + f_1 + f_2$

Comparing them, we get

$32 + f_1 + f_2 = 50$

$\Rightarrow f_1 + f_2 = 18$ ...(ii)

We will now solve equations (i) and (ii), multiply eq.(ii) by 31 and then adding to eq.(i), we get

$(69f_1 - 31f_2) + [31(f_1 + f_2)] = 242 + 558$

$\Rightarrow 69f_1 - 31f_2 + 31f_1 + 31f_2 = 800$

$\Rightarrow 100f_1 = 800$

$\Rightarrow f_1 = 8$

Substitute $f_1 = 8$ in equation (ii),

$8 + f_2 = 18$

$\Rightarrow f_2 = 18 - 8$

$\Rightarrow f_2 = 10$

Thus, $f_1 = 8$ and $f_2 = 10$.

Question: 14

Solution:

We will find the mean heartbeats per minute by direct method.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
65 - 68	66.5	2	133
68 - 71	69.5	4	278
71 - 74	72.5	3	217.5
74 - 77	75.5	8	604
77 - 80	78.5	7	549.5
80 - 83	81.5	4	326
83 - 86	84.5	2	169
TOTAL		30	2277

We have got

$\Sigma f_i = 30$ & $\Sigma f_i x_i = 2277$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{2277}{30}$$

$$\Rightarrow \bar{x} = 75.9$$

Thus, mean is 75.9 heartbeats per minute.

Question: 15

Solution:

We will find the mean marks per student using Assumed - mean method, where A = Assumed mean.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	DEVIATION(d_i) $d_i = x_i - 25$	$f_i d_i$
0 - 10	5	12	- 20	- 240
10 - 20	15	18	- 10	- 180
20 - 30	25 = A	27	0	0
30 - 40	35	20	10	200
40 - 50	45	17	20	340
50 - 60	55	6	30	180
TOTAL		100		300

We have got

$A = 25$, $\Sigma f_i = 100$ & $\Sigma f_i d_i = 300$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_i d_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = 25 + \frac{300}{100}$$

$$\Rightarrow \bar{x} = 28$$

Thus, mean is 28.

Question: 16

Solution:

We will the frequency distribution using Assumed - mean method, where A = Assumed mean.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	DEVIATION(d_i) $d_i = x_i - 150$	$f_i d_i$
100 - 120	110	10	- 40	- 400
120 - 140	130	20	- 20	- 400
140 - 160	150 = A	30	0	0
160 - 180	170	15	20	300
180 - 200	190	5	40	200
TOTAL		80		- 300

We have got

A = 150, $\Sigma f_i = 80$ & $\Sigma f_i d_i = -300$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_i d_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = 150 - \frac{300}{80}$$

$$\Rightarrow \bar{x} = 146.25$$

Thus, mean is 146.25.

Question: 17

Solution:

We will the data using Assumed - mean method, where A = Assumed mean.

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	DEVIATION(d_i) $d_i = x_i - 50$	$f_i d_i$
0 - 20	10	20	- 40	- 800
20 - 40	30	35	- 20	- 700
40 - 60	50 = A	52	0	0
60 - 80	70	44	20	880
80 - 100	90	38	40	1520
100 - 120	110	31	60	1860
TOTAL		220		2760

We have got

$A = 50, \Sigma f_i = 220$ & $\Sigma f_i d_i = 2760$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_i d_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = 50 + \frac{2760}{220}$$

$$\Rightarrow \bar{x} = 62.55$$

Thus, mean is 62.55.

Question: 18

The following tab

Solution:

We will solve this using direct method.

LITERACY RATE(%)	MID - POINT(x_i)	NUMBER OF CITIES(f_i)	f_ix_i
45 - 55	50	4	200
55 - 65	60	11	660
65 - 75	70	12	840
75 - 85	80	9	720
85 - 95	90	4	360
TOTAL		40	2780

We have got

$\Sigma f_i = 40$ and $\Sigma f_ix_i = 2780$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_ix_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{2780}{40}$$

$$\Rightarrow \bar{x} = 69.5$$

Thus, mean is 69.5%.

Question: 19

Solution:

We will the frequency distribution using step - deviation method, where A =Assumed mean and h = length of class interval.

Here, let A = 25 and h = 10

CLASS	MID - POINT(x_i)	DEVIATION(d_i) $d_i = x_i - 25$	FREQUENCY(f_i)	$u_i = d_i/h$	f_iu_i
0 - 10	5	- 20	7	- 2	- 14
10 - 20	15	- 10	10	- 1	- 10
20 - 30	25 = A	0	15	0	0
30 - 40	35	10	8	1	8
40 - 50	45	20	10	2	20
TOTAL			50		4

We have got

$A = 25, h = 10, \Sigma f_i = 50$ & $\Sigma f_i u_i = 4$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_i u_i}{\sum_i f_i} \times h$$

$$\Rightarrow \bar{x} = 25 + \frac{4}{50} \times 10$$

$$\Rightarrow \bar{x} = 25.8$$

Thus, mean is 25.8

Question: 20

Solution:

We will the data using step - deviation method, where A = Assumed mean and h = length of class interval.

Here, let A = 40 and h = 10

CLASS	MID - POINT(x_i)	DEVIATION(d_i) $d_i = x_i - 40$	FREQUENCY(f_i)	$u_i = d_i/h$	f_iu_i
5 - 15	10	- 30	6	- 3	- 18
15 - 25	20	- 20	10	- 2	- 20
25 - 35	30	- 10	16	- 1	- 16
35 - 45	40 = A	0	15	0	0
45 - 55	50	10	24	1	24
55 - 65	60	20	8	2	16
65 - 75	70	30	7	3	21
TOTAL			86		7

We have got

$A = 40$, $h = 10$, $\Sigma f_i = 86$ & $\Sigma f_i u_i = 7$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_i u_i}{\sum_i f_i} \times h$$

$$\Rightarrow \bar{x} = 40 + \frac{7}{86} \times 10$$

$$\Rightarrow \bar{x} = 40.81$$

Thus, mean is 40.81

Question: 21

Solution:

We will find the mean weight of packet using step - deviation method, where A = Assumed mean and h = length of class interval.

Here, let A = 202.5 and h = 1

WEIGHT(g)	MID - POINT(x_i)	DEVIATION(d_i) $d_i = x_i - 202.5$	NUMBER OF PACKETS(f_i)	$u_i = d_i/h$	f_iu_i
200 - 201	200.5	- 2	13	- 2	- 26
201 - 202	201.5	- 1	27	- 1	- 27
202 - 203	202.5 = A	0	18	0	0
203 - 204	203.5	1	10	1	10
204 - 205	204.5	2	1	2	2
205 - 206	205.5	3	1	3	3
TOTAL			70		- 38

We have got

A = 202.5, h = 1, $\Sigma f_i = 70$ & $\Sigma f_iu_i = -38$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_iu_i}{\sum_i f_i} \times h$$

$$\Rightarrow \bar{x} = 202.5 - \frac{38}{70} \times 1$$

$$\Rightarrow \bar{x} = 201.96$$

Thus, mean is 201.96 g.

Question: 22

Solution:

We will the frequency distribution using step - deviation method, where A =Assumed mean and h = length of class interval.

Here, let A = 45 and h = 10

CLASS	MID - POINT(x_i)	DEVIATION(d_i) $d_i = x_i - 45$	FREQUENCY(f_i)	$u_i = d_i/h$	f_iu_i
20 - 30	25	- 20	25	- 2	- 50
30 - 40	35	- 10	40	- 1	- 40
40 - 50	45 = A	0	42	0	0
50 - 60	55	10	33	1	33
60 - 70	65	20	10	2	20
TOTAL			150		- 37

We have got

A = 45, h = 10, $\Sigma f_i = 150$ & $\Sigma f_i u_i = -37$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_i u_i}{\sum_i f_i} \times h$$

$$\Rightarrow \bar{x} = 45 - \frac{37}{150} \times 10$$

$$\Rightarrow \bar{x} = 42.53$$

Thus, mean is 42.53.

Question: 23

Solution:

We will find the mean marks using step - deviation method, where A = Assumed mean and h = length of class interval.

Here, let A = 37.5 and h = 15

MARKS OBTAINED	MID - POINT(x_i)	DEVIATION(d_i) $d_i = x_i - 37.5$	NUMBER OF STUDENTS(f_i)	$u_i = d_i/h$	f_iu_i
0 - 15	7.5	- 30	2	- 2	- 4
15 - 30	22.5	- 15	4	- 1	- 4
30 - 45	37.5 = A	0	5	0	0
45 - 60	52.5	15	20	1	20
60 - 75	67.5	30	9	2	18
75 - 90	82.5	45	10	3	30
TOTAL			50		60

We have got

$A = 37.5$, $h = 15$, $\Sigma f_i = 50$ & $\Sigma f_i u_i = 60$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_i u_i}{\sum_i f_i} \times h$$

$$\Rightarrow \bar{x} = 37.5 + \frac{60}{50} \times 15$$

$$\Rightarrow \bar{x} = 55.5$$

Thus, mean marks are 55.5.

Question: 24

Solution:

We will the frequency distribution using step - deviation method, where A =Assumed mean and h = length of class interval.

Here, let A = 33 and h = 6

AGE(years)	MID - POINT(x_i)	DEVIATION(d_i) $d_i = x_i - 33$	NUMBER OF WORKERS(f_i)	$u_i = d_i/h$	f_iu_i
18 - 24	21	- 12	6	- 2	- 12
24 - 30	27	- 6	8	- 1	- 8
30 - 36	33 = A	0	12	0	0
36 - 42	39	6	8	1	8
42 - 48	45	12	4	2	8
48 - 54	51	18	2	3	6
TOTAL			40		2

We have got

$A = 33$, $h = 6$, $\Sigma f_i = 40$ & $\Sigma f_i u_i = 2$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_i u_i}{\sum_i f_i} \times h$$

$$\Rightarrow \bar{x} = 33 + \frac{2}{40} \times 6$$

$$\Rightarrow \bar{x} = 33.3$$

Thus, mean age is 33.3 years.

Question: 25

Solution:

We will the frequency distribution using step - deviation method, where A =Assumed mean and h = length of class interval.

Here, let A = 550 and h = 20

CLASS	MID - POINT(x_i)	DEVIATION(d_i) $d_i = x_i - 550$	FREQUENCY(f_i)	$u_i = d_i/h$	f_iu_i
500 - 520	510	- 40	14	- 2	- 28
520 - 540	530	- 20	9	- 1	- 9
540 - 560	550 = A	0	5	0	0
560 - 580	570	20	4	1	4
580 - 600	590	40	3	2	6
600 - 620	610	60	5	3	15
TOTAL			40		- 12

We have got

$A = 550$, $h = 20$, $\Sigma f_i = 40$ & $\Sigma f_i u_i = -12$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_i u_i}{\sum_i f_i} \times h$$

$$\Rightarrow \bar{x} = 550 - \frac{12}{40} \times 20$$

$$\Rightarrow \bar{x} = 544$$

Thus, mean is 544.

Question: 26

Solution:

We will find the mean age using step - deviation method, where A = Assumed mean and h = length of class interval.

Here, let A = 42 and h = 5

Since, the class intervals are inclusive type, we'll first convert it into exclusive type by extending the class interval from both the ends.

AGE(years)	MID - POINT(x_i)	DEVIATION(d_i) $d_i = x_i - 550$	NUMBER OF PERSONS(f_i)	$u_i = d_i/h$	f_iu_i
24.5 - 29.5	27	- 15	4	- 3	- 12
29.5 - 34.5	32	- 10	14	- 2	- 28
34.5 - 39.5	37	- 5	22	- 1	- 22
39.5 - 44.5	42 = A	0	16	0	0
44.5 - 49.5	47	5	6	1	6
49.5 - 54.5	52	10	5	2	10
54.5 - 59.5	57	15	3	3	9
TOTAL			70		- 37

We have got

A = 42, h = 5, $\Sigma f_i = 70$ & $\Sigma f_iu_i = -37$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_iu_i}{\sum_i f_i} \times h$$

$$\Rightarrow \bar{x} = 42 - \frac{37}{70} \times 5$$

$$\Rightarrow \bar{x} = 39.36$$

Thus, mean age is 544 years.

Question: 27

Solution:

We will find the average age using step - deviation method, where A = Assumed mean and h =

length of class interval.

Here, let A = 29.5 and h = 10

Since, the class intervals are inclusive type, we'll first convert it into exclusive type by extending the class interval from both the ends.

AGE(years)	MID - POINT(x_i)	DEVIATION(d_i) $d_i = x_i - 29.5$	NUMBER OF CASES(f_i)	$u_i = d_i/h$	f_iu_i
4.5 - 14.5	9.5	- 20	6	- 2	- 12
14.5 - 24.5	19.5	- 10	11	- 1	- 11
24.5 - 34.5	29.5 = A	0	21	0	0
34.5 - 44.5	39.5	10	23	1	23
44.5 - 54.5	49.5	20	14	2	28
54.5 - 64.5	59.5	30	5	3	15
TOTAL			80		43

We have got

A = 29.5, h = 10, $\Sigma f_i = 80$ & $\Sigma f_iu_i = 43$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_iu_i}{\sum_i f_i} \times h$$

$$\Rightarrow \bar{x} = 29.5 + \frac{43}{80} \times 10$$

$$\Rightarrow \bar{x} = 34.88$$

Thus, mean age is 34.88 years.

Question: 28

Solution:

We will find the mean weight using step - deviation method, where A = Assumed mean and h = length of class interval.

Here, let A = 92 and h = 5

Since, the class intervals are inclusive type, we'll first convert it into exclusive type by extending the class interval from both the ends.

WEIGHT(G)	MID - POINT(x_i)	DEVIATION(d_i) $d_i = x_i - 92$	NUMBER OF EGGS(f_i)	$u_i = d_i/h$	f_iu_i
74.5 - 79.5	77	- 15	4	- 3	- 12
79.5 - 84.5	82	- 10	9	- 2	- 18
84.5 - 89.5	87	- 5	13	- 1	- 13
89.5 - 94.5	92 = A	0	17	0	0
94.5 - 99.5	97	5	12	1	12
99.5 - 104.5	102	10	3	2	6
104.5 - 109.5	107	15	2	3	6
TOTAL			60		- 19

We have got

A = 92, h = 5, $\Sigma f_i = 60$ & $\Sigma f_iu_i = -19$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_i u_i}{\sum_i f_i} \times h$$

$$\Rightarrow \bar{x} = 92 - \frac{19}{60} \times 5$$

$$\Rightarrow \bar{x} = 90.42$$

Thus, mean weight is 90 g.

Question: 29

Solution:

We will find the mean marks using step - deviation method, where A = Assumed mean and h = length of class interval.

Here, let A = 17.5 and h = 5

Since, the class intervals are less - than type, we'll first convert it into exclusive type.

WEIGHT(G)	MID - POINT(x_i)	DEVIATION(d_i) $d_i = x_i - 17.5$	NUMBER OF EGGS(f_i)	$u_i = d_i/h$	f_iu_i
0 - 5	2.5	- 15	3	- 3	- 9
5 - 10	7.5	- 10	7	- 2	- 14
10 - 15	12.5	- 5	15	- 1	- 15
15 - 20	17.5 = A	0	24	0	0
20 - 25	22.5	5	16	1	16
25 - 30	27.5	10	8	2	16
30 - 35	32.5	15	5	3	15
35 - 40	37.5	20	2	4	8
TOTAL			80		17

We have got

$A = 17.5, h = 5, \Sigma f_i = 80$ & $\Sigma f_i u_i = 17$

$\because$ mean is given by

$$\bar{x} = A + \frac{\sum_i f_i u_i}{\sum_i f_i} \times h$$

$$\Rightarrow \bar{x} = 17.5 + \frac{17}{80} \times 5$$

$$\Rightarrow \bar{x} = 18.5625$$

Thus, mean marks correct to 2 decimal places are 18.56.

CLASS24

Question: 1

Solution:

To find median,

Assume $\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

AGE(years)	NUMBER OF PATIENTS(f_i)	C_f
0 - 15	5	5
15 - 30	20	5 + 20 = 25
30 - 45	40	25 + 40 = 65
45 - 60	50	65 + 50 = 115
60 - 75	25	115 + 25 = 140
TOTAL	140	

So, N = 140

$\Rightarrow N/2 = 140/2 = 70$

The cumulative frequency just greater than (N/2 =) 70 is 115, so the corresponding median class is 45 - 60 and accordingly we get C_f = 65(cumulative frequency before the median class).

Now, since median class is 45 - 60.

$\therefore$ l = 45, h = 15, f = 50, N/2 = 70 and C_f = 65

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 45 + \left(\frac{70-65}{50}\right) \times 15$$

$= 45 + 1.5$

$= 46.5$

Thus, median age is 46.5 years.

Question: 2

Solution:

To find median,

Assume $\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

MARKS	NUMBER OF STUDENTS(f_i)	C_f
0 - 7	3	3
7 - 14	4	3 + 4 = 7
14 - 21	7	7 + 7 = 14
21 - 28	11	14 + 11 = 25
28 - 35	0	25 + 0 = 25
35 - 42	16	25 + 16 = 41
42 - 49	9	41 + 9 = 50
TOTAL	50	

So, N = 50

⇒ N/2 = 50/2 = 25

The cumulative frequency just greater than (N/2 =) 25 is 41, so the corresponding median class is 35 - 42 and accordingly we get C_f = 25(cumulative frequency before the median class).

Now, since median class is 35 - 42.

∴ l = 35, h = 7, f = 16, N/2 = 25 and C_f = 25

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 35 + \left(\frac{25-25}{16}\right) \times 7$$

= 35 + 0

= 35

Thus, median marks are 35.

Question: 3

Solution:

To find median,

Assume $\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

DAILY WAGES(Rs.)	NUMBER OF WORKERS(f_i)	C_f
0 - 100	40	40
100 - 200	32	40 + 32 = 72
200 - 300	48	72 + 48 = 120
300 - 400	22	120 + 22 = 142
400 - 500	8	142 + 8 = 150
TOTAL	150	

So, N = 150

$\Rightarrow N/2 = 150/2 = 75$

The cumulative frequency just greater than (N/2 =) 75 is 120, so the corresponding median class is 200 - 300 and accordingly we get C_f = 72(cumulative frequency before the median class).

Now, since median class is 200 - 300.

$\therefore$ l = 200, h = 100, f = 48, N/2 = 75 and C_f = 72

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 200 + \left(\frac{75-72}{48}\right) \times 100$$

= 200 + 6.25

= 206.25

Thus, median wage is Rs. 206.25.

Question: 4

Solution:

To find median, Assume

Σf_i = N = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

CLASS	FREQUENCY(f_i)	C_f
5 - 10	5	5
10 - 15	6	5 + 6 = 11
15 - 20	15	11 + 15 = 26
20 - 25	10	26 + 10 = 36
25 - 30	5	36 + 5 = 41
30 - 35	4	41 + 4 = 45
35 - 40	2	45 + 2 = 47
40 - 45	2	47 + 2 = 49
TOTAL	49	

So, N = 49

⇒ N/2 = 49/2 = 24.5

The cumulative frequency just greater than (N/2 =) 24.5 is 25, so the corresponding median class is 15 - 20 and accordingly we get C_f = 11(cumulative frequency before the median class).

Now, since median class is 15 - 20.

∴ l = 15, h = 5, f = 15, N/2 = 24.5 and C_f = 11

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 15 + \left(\frac{24.5-11}{15}\right) \times 5$$

= 15 + 4.5

= 19.5

Thus, median is 19.5.

Question: 5

Solution:

To find median, Assume

Σf_i = N = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

CONSUMPTION (unit)	NUMBER OF CONSUMERS(f_i)	C_f
65 - 85	4	4
85 - 105	5	4 + 5 = 9
105 - 125	13	9 + 13 = 22
125 - 145	20	22 + 20 = 42
145 - 165	14	42 + 14 = 56
165 - 185	7	56 + 7 = 63
185 - 205	4	63 + 4 = 67
TOTAL	67	

So, N = 67

$\Rightarrow$ N/2 = 67/2 = 33.5

The cumulative frequency just greater than (N/2 =) 33.5 is 42, so the corresponding median class is 125 - 145 and accordingly we get C_f = 22(cumulative frequency before the median class).

Now, since median class is 125 - 145.

$\therefore$ l = 125, h = 20, f = 20, N/2 = 33.5 and C_f = 22

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 125 + \left(\frac{33.5-22}{20}\right) \times 20$$

= 125 + 11.5

= 136.5

Thus, median is 136.5.

Question: 6

Solution:

To find median, Assume

Σf_i = N = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

HEIGHT(cm)	NUMBER OF BOYS(f_i)	C_f
135 - 140	6	6
140 - 145	10	6 + 10 = 16
145 - 150	18	16 + 18 = 34
150 - 155	22	34 + 22 = 56
155 - 160	20	56 + 20 = 76
160 - 165	15	76 + 15 = 91
165 - 170	6	91 + 6 = 97
170 - 175	3	97 + 3 = 100
TOTAL	100	

So, N = 100

⇒ N/2 = 100/2 = 50

The cumulative frequency just greater than (N/2 =) 50 is 56, so the corresponding median class is 150 - 155 and accordingly we get $C_f = 34$ (cumulative frequency before the median class).

Now, since median class is 150 - 155.

∴ l = 150, h = 5, f = 22, N/2 = 50 and $C_f = 34$

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 150 + \left(\frac{50-34}{22}\right) \times 5$$

= 150 + 3.636

= 153.64

Thus, median is 153.64 cm.

Question: 7

Solution:

Median(given) = 24, Assume

Σf_i = N = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table, where x is the unknown frequency.

CLASS	FREQUENCY(f_i)	C_f
0 - 10	5	5
10 - 20	25	5 + 25 = 30
20 - 30	x	30 + x
30 - 40	18	30 + x + 18 = 48 + x
40 - 50	7	48 + x + 7 = 55 + x
TOTAL	55 + x	

Median = 24 (as already mentioned in the question)

24 lies between 20 - 30 ⇒ Median class = 20 - 30

∴ l = 20, h = 10, f = x, N/2 = (55 + x)/2 and C_f = 30

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow 24 = 20 + \left(\frac{\frac{55+x}{2} - 30}{x}\right) \times 10$$

$\Rightarrow 24 = 20 + \left(\frac{55 + x - 60}{2x}\right) \times 10$

$\Rightarrow 24 - 20 = (10x - 50)/2x$

$\Rightarrow (4)(2x) = 10x - 50$

$\Rightarrow 8x = 10x - 50$

$\Rightarrow 10x - 8x = 50$

$\Rightarrow 2x = 50$

$\Rightarrow x = 25$

Thus, the unknown frequency is 25.

Question: 8

Solution:

Given: Median = 16 & N = 70

Assume

Σf_i = N = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table, where x is the unknown frequency.

CLASS	FREQUENCY(f_i)	C_f
0 - 5	12	12
5 - 10	a	12 + a
10 - 15	12	12 + a + 12 = 24 + a
15 - 20	15	24 + a + 15 = 39 + a
20 - 25	b	39 + a + b
25 - 30	6	39 + a + b + 6 = 45 + a + b
30 - 35	6	45 + a + b + 6 = 51 + a + b
35 - 40	4	51 + a + b + 4 = 55 + a + b
TOTAL	55 + a + b	

Median = 16 (as already mentioned in the question)

16 lies between 15 - 20 ⇒ Median class = 15 - 20

∴ l = 15, h = 5, f = 15, N/2 = (55 + a + b)/2 and C_f = 24 + a

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow 16 = 15 + \left(\frac{\frac{55+a+b}{2} - (24+a)}{15}\right) \times 5$$

$$\Rightarrow 16 = 15 + \left(\frac{55 + a + b - 48 - 2a}{30}\right) \times 5$$

⇒ 16 – 15 = (7 – a + b)/6

⇒ 6 = 7 – a + b

$\Rightarrow a - b = 1$ ...(i)

And given that N = 70

$\Rightarrow 55 + a + b = 70$

$\Rightarrow a + b = 15$ ...(ii)

Solving equations (i) & (ii), we get

$(a - b) + (a + b) = 1 + 15$

$\Rightarrow 2a = 16$

$\Rightarrow a = 8$

Substituting a = 8 in eq.(i),

$8 - b = 1$

$\Rightarrow b = 7$

Thus, the unknown frequencies are a = 8 and b = 7.

Question: 9

Solution:

Given: Median = 5000 & N = 60

Assume

$\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table, where x is the unknown frequency.

RUNS SCORED	NUMBER OF BATSMEN(f_i)	C_f
2500 - 3500	5	5
3500 - 4500	x	5 + x
4500 - 5500	y	5 + x + y
5500 - 6500	12	5 + x + y + 12 = 17 + x + y
6500 - 7500	6	17 + x + y + 6 = 23 + x + y
7500 - 8500	2	23 + x + y + 2 = 25 + x + y
TOTAL	25 + x + y	

Given,Median = 5000 (as already mentioned in the question)

Sum of frequencies, N = x + y + 25 = 60 [Total No of players]

5000 lies between 4500 - 5500 ⇒ Median class = 4500 - 5500

∴ l = 4500, h = 1000, f = y, N/2 = 60/2=30 and C_f = 5 + x

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow 5000 = 4500 + \left(\frac{\frac{60}{2} - (5 + x)}{y}\right) \times 1000$$

$$\Rightarrow 5000 = 4500 + \left(\frac{30 - 5 - x}{y}\right) \times 1000$$

⇒ 5000 – 4500 = (25000 – 1000x)/y

⇒ 500y = 25000 – 1000x

⇒ 2x + y = 50 ...(i)

And given that N = 60

⇒ 25 + x + y = 60

⇒ x + y = 35 ...(ii)

Solving equations (i) & (ii), we get

(2x + y) – (x + y) = 50 – 35

⇒ x = 15

Substituting x = 15 in eq.(ii),

15 + y = 35

⇒ y = 20

Thus, the unknown frequencies are x = 15 and y = 20.

Question: 10

Solution:

Given: Median = 32.5 & N = 40

Assume

Σf_i = N = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table, where x is the unknown frequency.

CLASS	NUMBER OF BATSMEN(f_i)	C_f
0 - 10	f_1	f_1
10 - 20	5	$5 + f_1$
20 - 30	9	$5 + f_1 + 9 = 14 + f_1$
30 - 40	12	$14 + f_1 + 12 = 26 + f_1$
40 - 50	f_2	$26 + f_1 + f_2$
50 - 60	3	$26 + f_1 + f_2 + 3 = 29 + f_1 + f_2$
60 - 70	2	$29 + f_1 + f_2 + 2 = 31 + f_1 + f_2$
TOTAL	$31 + f_1 + f_2$	

Median = 32.5 (as already mentioned in the question)

32.5 lies between 30 - 40 $\Rightarrow$ Median class = 30 - 40

$\therefore$ l = 30, h = 10, f = 12, $N/2 = (31 + f_1 + f_2)/2 = 40/2$ and $C_f = 14 + f_1$

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow 32.5 = 30 + \left(\frac{\frac{40}{2} - (14 + f_1)}{12}\right) \times 10$$

$$\Rightarrow 32.5 = 30 + \left(\frac{20 - 14 - f_1}{12}\right) \times 10$$

$\Rightarrow 32.5 - 30 = (60 - 10f_1)/12$

$\Rightarrow (2.5)(12) = 60 - 10f_1$

$\Rightarrow 30 = 60 - 10f_1$

$\Rightarrow f_1 = 3$ ...(i)

And given that N = 40

$\Rightarrow 31 + f_1 + f_2 = 40$

$\Rightarrow f_1 + f_2 = 9$ ...(ii)

Substituting $f_1 = 3$ in eq.(ii),

$3 + f_2 = 9$

$\Rightarrow f_2 = 6$

Thus, the unknown frequencies are $f_1 = 3$ and $f_2 = 6$.

Question: 11

Solution:

To find median, Assume

Σf_i = N = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table and convert it into exclusive - type by adjusting from both ends of a class.

AGE(years)	FREQUENCY(f_i)	C_f
18.5 - 25.5	35	35
25.5 - 32.5	96	35 + 96 = 131
32.5 - 39.5	68	131 + 68 = 199
39.5 - 46.5	102	199 + 102 = 301
46.5 - 53.5	35	301 + 35 = 336
53.5 - 60.5	4	336 + 4 = 340
TOTAL	340	

So, N = 340

$\Rightarrow N/2 = 340/2 = 170$

The cumulative frequency just greater than (N/2 =) 170 is 199, so the corresponding median class is 32.5 - 39.5 and accordingly we get $C_f = 131$(cumulative frequency before the median class).

Now, since median class is 32.5 - 39.5.

$\therefore l = 32.5, h = 7, f = 68, N/2 = 170$ and $C_f = 131$

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 32.5 + \left(\frac{170-131}{68}\right) \times 7$$

$= 32.5 + 4.014$

$= 36.51$

Thus, median is 36.51 years.

Question: 12

Solution:

To find median, Assume

$\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table and convert it into exclusive - type by adjusting from both ends of a class.

WAGES PER DAY(Rs.)	NUMBER OF WOMEN WOKERS(f_i)	C_f
60.5 - 70.5	5	5
70.5 - 80.5	15	5 + 15 = 20
80.5 - 90.5	20	20 + 20 = 40
90.5 - 100.5	30	40 + 30 = 70
100.5 - 110.5	20	70 + 20 = 90
110.5 - 120.5	8	90 + 8 = 98
TOTAL	98	

So, N = 98

⇒ N/2 = 98/2 = 49

The cumulative frequency just greater than (N/2 =)49 is 70, so the corresponding median class is 90.5 - 100.5 and accordingly we get C_f = 40(cumulative frequency before the median class).

Now, since median class is 90.5 - 100.5.

∴ l = 90.5, h = 10, f = 30, N/2 = 49 and C_f = 40

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 90.5 + \left(\frac{49-40}{30}\right) \times 10$$

= 90.5 + 3

= 93.5

Thus, median is Rs. 93.5.

Question: 13

Solution:

To find median, Assume

$\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table and convert it into exclusive - type by adjusting from both ends of a class.

CLASS	FREQUENCY(f_i)	C_f
0.5 - 5.5	7	7
5.5 - 10.5	10	7 + 10 = 17
10.5 - 15.5	16	17 + 16 = 33
15.5 - 20.5	32	33 + 32 = 65
20.5 - 25.5	24	65 + 24 = 89
25.5 - 30.5	16	89 + 16 = 105
30.5 - 35.5	11	105 + 11 = 116
35.5 - 40.5	5	116 + 5 = 121
40.5 - 45.5	2	121 + 2 = 123
TOTAL	123	

So, N = 123

$\Rightarrow N/2 = 123/2 = 61.5$

The cumulative frequency just greater than (N/2 =)61.5 is 65, so the corresponding median class is 15.5 - 20.5 and accordingly we get C_f = 33(cumulative frequency before the median class).

Now, since median class is 15.5 - 20.5.

$\therefore$ l = 15.5, h = 5, f = 32, N/2 = 61.5 and C_f = 33

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 15.5 + \left(\frac{61.5-33}{32}\right) \times 5$$

= 15.5 + 4.45

= 19.95

Thus, median is 19.95.

Question: 14

Solution:

To find median, Assume

Σf_i = N = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table and convert it into exclusive - type.

MARKS	C_f	NUMBER OF STUDENTS(f_i)
0 - 10	12	12
10 - 20	32	32 - 12 = 20
20 - 30	57	57 - 32 = 25
30 - 40	80	80 - 57 = 23
40 - 50	92	92 - 80 = 12
50 - 60	116	116 - 92 = 24
60 - 70	164	164 - 116 = 48
70 - 80	200	200 - 164 = 36
TOTAL		200

So, N = 200

$\Rightarrow N/2 = 200/2 = 100$

The cumulative frequency just greater than (N/2 =)100 is 116, so the corresponding median class is 50 - 60 and accordingly we get $C_f = 92$(cumulative frequency before the median class).

Now, since median class is 50 - 60.

$\therefore l = 50, h = 10, f = 24, N/2 = 100$ and $C_f = 92$

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 50 + \left(\frac{100-92}{24}\right) \times 10$$

$= 50 + 3.33$

$= 53.33$

Thus, median is 53.33.

Exercise : 9C

Question: 1

Solution:

Here, the maximum class frequency is 45.

The class corresponding to this frequency is the modal class. ⇒ modal class = 30 - 40

∴ lower limit of the modal class (l) = 30

Modal class size (h) = 10

Frequency of the modal class (f_1) = 45

Frequency of class preceding the modal class (f_0) = 35

Frequency of class succeeding the modal (f_2) = 25

Mode is given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \text{Mode} = 30 + \left(\frac{45-35}{2(45)-35-25}\right) \times 10$$

$$\Rightarrow \text{Mode} = 30 + \left(\frac{10}{30}\right) \times 10$$

⇒ Mode = 30 + 3.33 = 33.33

Hence, the mode is 33.33

Question: 2

Solution:

Here, the maximum class frequency is 28.

The class corresponding to this frequency is the modal class. ⇒ modal class = 40 - 60

∴ lower limit of the modal class (l) = 40

Modal class size (h) = 20

Frequency of the modal class (f_1) = 28

Frequency of class preceding the modal class (f_0) = 16

Frequency of class succeeding the modal (f_2) = 20

Mode is given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \text{Mode} = 40 + \left(\frac{28-16}{2(28)-16-20}\right) \times 20$$

$$\Rightarrow \text{Mode} = 40 + \left(\frac{12}{20}\right) \times 20$$

⇒ Mode = 40 + 12 = 52

Hence, the mode is 52.

Question: 3

Solution:

Here, the maximum class frequency is 20.

The class corresponding to this frequency is the modal class. ⇒ modal class = 160 - 165

∴ lower limit of the modal class (l) = 160

Modal class size (h) = 5

Frequency of the modal class (f_1) = 20

Frequency of class preceding the modal class (f_0) = 8

Frequency of class succeeding the modal (f_2) = 12

Mode is given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \text{Mode} = 160 + \left(\frac{20-8}{2(20)-8-12}\right) \times 5$$

$$\Rightarrow \text{Mode} = 160 + \left(\frac{12}{20}\right) \times 5$$

⇒ Mode = 160 + 3 = 163

Hence, the mode is 163 cm.

Mode represents frequency, hence 163 cm is the height of maximum number of students.

To find the mean, we will solve by using direct method.

HEIGHT (cm)	MID - POINT(x_i)	NUMBER OF STUDENTS(f_i)	f_ix_i
150 - 155	152.5	15	2287.5
155 - 160	157.5	8	1260
160 - 165	162.5	20	3250
165 - 170	167.5	12	2010
170 - 175	172.5	5	862.5
TOTAL		60	9670

We have got

$\Sigma f_i = 60$ & $\Sigma f_i x_i = 9670$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{9670}{60}$$

$$\Rightarrow \bar{x} = 161.17$$

Thus, mean is 161.17

Mean represents average, thus 161.7 cm is the average height of all the students.

Question: 4

Solution:

Here, the maximum class frequency is 25.

The class corresponding to this frequency is the modal class. ⇒ modal class = 26 - 30

$\therefore$ lower limit of the modal class (l) = 26

Modal class size (h) = 4

Frequency of the modal class (f_1) = 25

Frequency of class preceding the modal class (f_0) = 20

Frequency of class succeeding the modal (f_2) = 22

Mode is given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \text{Mode} = 26 + \left(\frac{25-20}{2(25)-20-22}\right) \times 4$$

$$\Rightarrow \text{Mode} = 26 + \left(\frac{5}{8}\right) \times 4$$

⇒ Mode = 26 + 2.5 = 28.5

Hence, the mode is 28.5.

Question: 5

Solution:

Expenditure done by maximum number of manual workers is estimated by finding mode.

So here, the maximum class frequency is 40.

The class corresponding to this frequency is the modal class. ⇒ modal class = 1500 - 2000

$\therefore$ lower limit of the modal class (l) = 1500

Modal class size (h) = 500

Frequency of the modal class (f_1) = 40

Frequency of class preceding the modal class (f_0) = 24

Frequency of class succeeding the modal (f_2) = 31

Mode is given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \text{Mode} = 1500 + \left(\frac{40-24}{2(40)-24-31}\right) \times 500$$

$$\Rightarrow \text{Mode} = 1500 + \left(\frac{16}{25}\right) \times 500$$

$\Rightarrow$ Mode = 1500 + 320 = 1820

Hence, the mode is Rs.1820.

Question: 6

Solution:

Here, the maximum class frequency is 150.

The class corresponding to this frequency is the modal class. $\Rightarrow$ modal class = 5000 - 10000

$\therefore$ lower limit of the modal class (l) = 5000

Modal class size (h) = 5000

Frequency of the modal class (f_1) = 150

Frequency of class preceding the modal class (f_0) = 90

Frequency of class succeeding the modal (f_2) = 100

Mode is given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \text{Mode} = 5000 + \left(\frac{150-90}{2(150)-90-100}\right) \times 5000$$

$$\Rightarrow \text{Mode} = 5000 + \left(\frac{60}{110}\right) \times 5000$$

$\Rightarrow$ Mode = 5000 + 2727.27 = 7727.27

Hence, the mode is Rs.7727.27.

Question: 7

Solution:

Here, the maximum class frequency is 24.

The class corresponding to this frequency is the modal class. $\Rightarrow$ modal class = 15 - 20

$\therefore$ lower limit of the modal class (l) = 15

Modal class size (h) = 5

Frequency of the modal class (f_1) = 24

Frequency of class preceding the modal class (f_0) = 18

Frequency of class succeeding the modal (f_2) = 17

Mode is given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$\Rightarrow \text{Mode} = 15 + \left(\frac{24-18}{2(24)-18-17}\right) \times 5$

$\Rightarrow \text{Mode} = 15 + \left(\frac{6}{13}\right) \times 5$

$\Rightarrow$ Mode = 15 + 2.30 = 17.30

Hence, the mode is 17.30 years.

Question: 8

Solution:

Here, the maximum class frequency is 32.

The class corresponding to this frequency is the modal class. $\Rightarrow$ modal class = 85 - 95

$\therefore$ lower limit of the modal class (l) = 85

Modal class size (h) = 10

Frequency of the modal class (f_1) = 32

Frequency of class preceding the modal class (f_0) = 30

Frequency of class succeeding the modal (f_2) = 6

Mode is given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$\Rightarrow \text{Mode} = 85 + \left(\frac{32-30}{2(32)-30-6}\right) \times 10$

$\Rightarrow \text{Mode} = 85 + \left(\frac{2}{28}\right) \times 10$

$\Rightarrow$ Mode = 85 + 0.71 = 85.71

Hence, the mode is 85.71.

Question: 9

Solution:

Since, the given data is in inclusive series, it needs to get converted in exclusive series.

CLASS INTERVAL	0.5 - 5.5	5.5 - 10.5	10.5 - 15.5	15.5 - 20.5	20.5 - 25.5	25.5 - 30.5	30.5 - 35.5	35.5 - 40.5	40.5 - 45.5	45.5 - 50.5
FREQUENCY	3	8	13	18	28	20	13	8	6	4

Here, the maximum class frequency is 28.

The class corresponding to this frequency is the modal class. $\Rightarrow$ modal class = 15.5 - 20.5

$\therefore$ lower limit of the modal class (l) = 15.5

Modal class size (h) = 5

Frequency of the modal class (f_1) = 28

Frequency of class preceding the modal class (f_0) = 18

Frequency of class succeeding the modal (f_2) = 20

Mode is given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \text{Mode} = 15.5 + \left(\frac{28-18}{2(28)-18-20}\right) \times 5$$

$$\Rightarrow \text{Mode} = 15.5 + \left(\frac{10}{18}\right) \times 5$$

$\Rightarrow$ Mode = 15.5 + 2.78 = 23.28

Hence, the mode is 23.28.

Question: 10

Solution:

To find frequencies, we have Sum of frequencies that is, 181.

Using Sum of frequencies = 181,

x + 15 + 18 + 30 + 50 + 48 + x = 181

$\Rightarrow$ 2x + 161 = 181

$\Rightarrow$ 2x = 181 – 161 = 20

$\Rightarrow$ x = 10

Thus we have,

Age (in years)	5 - 7	7 - 9	9 - 11	11 - 13	13 - 15	15 - 17	17 - 19
Number of students	10	15	18	30	50	48	10

Here, the maximum class frequency is 50.

The class corresponding to this frequency is the modal class. $\Rightarrow$ modal class = 13 - 15

$\therefore$ lower limit of the modal class (l) = 13

Modal class size (h) = 2

Frequency of the modal class (f_1) = 50

Frequency of class preceding the modal class (f_0) = 30

Frequency of class succeeding the modal (f_2) = 48

Mode is given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \text{Mode} = 13 + \left(\frac{50-30}{2(50)-30-48}\right) \times 2$$

$$\Rightarrow \text{Mode} = 13 + \left(\frac{20}{22}\right) \times 2$$

⇒ Mode = 13 + 1.82 = 14.82

Hence, the mode is 14.82.

Exercise : 9D

Question: 1

Solution:

To find mean, we will solve by direct method:

CLASS INTERVAL	MID - POINT(x_i)	NUMBER OF BATSMEN(f_i)	f_ix_i
0 - 10	5	4	20
10 - 20	15	4	60
20 - 30	25	7	175
30 - 40	35	10	350
40 - 50	45	12	540
50 - 60	55	8	440
60 - 70	65	5	325
TOTAL		50	1910

We have got

$\Sigma f_i = 50$ & $\Sigma f_ix_i = 1910$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_ix_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{1910}{50}$$

$$\Rightarrow \bar{x} = 38.2$$

To find median,

Assume $\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

CLASS INTERVAL	NUMBER OF WORKERS(f_i)	C_f
0 - 10	4	4
10 - 20	4	4 + 4 = 8
20 - 30	7	8 + 7 = 15
30 - 40	10	15 + 10 = 25
40 - 50	12	25 + 12 = 37
50 - 60	8	37 + 8 = 45
60 - 70	5	45 + 5 = 50
TOTAL	50	

So, N = 50

$\Rightarrow N/2 = 50/2 = 25$

The cumulative frequency just greater than (N/2 =) 25 is 37, so the corresponding median class is 40 - 50 and accordingly we get C_f = 25(cumulative frequency before the median class).

Now, since median class is 40 - 50.

$\therefore l = 40, h = 10, f = 16, N/2 = 25$ and $C_f = 25$

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 40 + \left(\frac{25 - 25}{16}\right)10$$

= 40 + 0

= 40

And we know that,

Mode = 3(Median) – 2(Mean)

= 3(40) – 2(38.2)

= 120 – 76.4

= 43.6

Hence, mean is 38.2, median is 40 and mode is 43.6.

Question: 2

Solution:

To find mean, we will solve by direct method:

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
0 - 20	10	6	60
20 - 40	30	8	240
40 - 60	50	10	500
60 - 80	70	12	840
80 - 100	90	6	540
100 - 120	110	5	550
120 - 140	130	3	390
TOTAL		50	3120

We have got

$\Sigma f_i = 50$ & $\Sigma f_i x_i = 3120$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{3120}{50}$$

$$\Rightarrow \bar{x} = 62.4$$

To find median,

Assume $\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

CLASS	FREQUENCY(f_i)	C_f
0 - 20	6	6
20 - 40	8	6 + 8 = 14
40 - 60	10	14 + 10 = 24
60 - 80	12	24 + 12 = 36
80 - 100	6	36 + 6 = 42
100 - 120	5	42 + 5 = 47
120 - 140	3	47 + 3 = 50
TOTAL	50	

So, N = 50

⇒ N/2 = 50/2 = 25

The cumulative frequency just greater than (N/2 =) 25 is 36, so the corresponding median class is 60 - 80 and accordingly we get C_f = 24(cumulative frequency before the median class).

Now, since median class is 60 - 80.

∴ l = 60, h = 20, f = 12, N/2 = 25 and C_f = 24

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 60 + \left(\frac{25-24}{12}\right) \times 20$$

= 60 + 1.67

= 61.67

And we know that,

Mode = 3(Median) – 2(Mean)

= 3(61.67) – 2(62.4)

= 185.01 – 124.8

= 60.21

Hence, mean is 62.4, median is 61.67 and mode is 60.21.

Question: 3

Solution:

To find mean, we will solve by direct method:

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
0 - 50	25	2	50
50 - 100	75	3	225
100 - 150	125	5	625
150 - 200	175	6	1050
200 - 250	225	5	1125
250 - 300	275	3	825
300 - 350	325	1	325
TOTAL		25	4225

We have got

$\Sigma f_i = 25$ & $\Sigma f_i x_i = 4171$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{4225}{25}$$

$$\Rightarrow \bar{x} = 169$$

To find median,

Assume $\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

CLASS	FREQUENCY(f_i)	C_f
0 - 50	2	2
50 - 100	3	2 + 3 = 5
100 - 150	5	5 + 5 = 10
150 - 200	6	10 + 6 = 16
200 - 250	5	16 + 5 = 21
250 - 300	3	21 + 3 = 24
300 - 350	1	24 + 1 = 25
TOTAL	25	

So, N = 25

$\Rightarrow N/2 = 25/2 = 12.5$

The cumulative frequency just greater than (N/2 =) 12.5 is 16, so the corresponding median class is 150 - 200 and accordingly we get $C_f = 10$ (cumulative frequency before the median class).

Now, since median class is 150 - 200.

$\therefore$ l = 150, h = 50, f = 6, N/2 = 12.5 and $C_f = 10$

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 150 + \left(\frac{12.5-10}{6}\right) \times 50$$

$= 150 + 20.83$

$= 170.83$

And we know that,

Mode = 3(Median) – 2(Mean)

$= 3(170.83) – 2(169)$

$= 512.49 – 338$

$= 174.49$

Hence, mean is 169, median is 170.83 and mode is 174.49.

Question: 4

Solution:

To find mean, we will solve by direct method:

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
25 - 35	30	7	210
35 - 45	40	31	1240
45 - 55	50	33	1650
55 - 65	60	17	1020
65 - 75	70	11	770
75 - 85	80	1	80
TOTAL		100	4970

We have got

$\Sigma f_i = 100$ & $\Sigma f_ix_i = 4970$

∵ mean is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{4970}{100}$$

$$\Rightarrow \bar{x} = 49.7$$

To find median,

Assume $\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

CLASS	FREQUENCY(f_i)	C_f
25 - 35	7	7
35 - 45	31	7 + 31 = 38
45 - 55	33	38 + 33 = 71
55 - 65	17	71 + 17 = 88
65 - 75	11	88 + 11 = 99
75 - 85	1	99 + 1 = 100
TOTAL	100	

So, N = 100

⇒ N/2 = 100/2 = 50

The cumulative frequency just greater than (N/2 =) 50 is 71, so the corresponding median class is 45 - 55 and accordingly we get C_f = 38(cumulative frequency before the median class).

Now, since median class is 45 - 55.

$\therefore$ l = 45, h = 10, f = 33, N/2 = 50 and C_f = 38

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 45 + \left(\frac{50-38}{33}\right) \times 10$$

= 45 + 3.64

= 48.64

And we know that,

Mode = 3(Median) – 2(Mean)

= 3(48.64) – 2(49.7)

= 145.92 – 99.4

= 46.52

Hence, mean is 49.7, median is 48.64 and mode is 46.52.

Question: 5

Solution:

To find mean, we will solve by direct method:

HEIGHT (cm.)	MID - POINT(x_i)	TOTAL NUMBER OF GIRLS(f_i)	f_ix_i
120 - 130	125	2	250
130 - 140	135	8	1080
140 - 150	145	12	1740
150 - 160	155	20	3100
160 - 170	165	8	1320
TOTAL		50	7490

We have got

$\Sigma f_i = 50$ & $\Sigma f_ix_i = 7490$

∵ mean is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$\Rightarrow \bar{x} = \frac{7490}{50}$$

$$\Rightarrow \bar{x} = 149.8$$

To find median,

Assume $\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

HEIGHT (cm.)	TOTAL NUMBER OF GIRLS(f_i)	C_f
120 - 130	2	2
130 - 140	8	2 + 8 = 10
140 - 150	12	10 + 12 = 22
150 - 160	20	22 + 20 = 42
160 - 170	8	42 + 8 = 50
TOTAL	50	

So, N = 50

⇒ N/2 = 50/2 = 25

The cumulative frequency just greater than (N/2 =) 25 is 42, so the corresponding median class is 150 - 160 and accordingly we get C_f = 22(cumulative frequency before the median class).

Now, since median class is 150 - 160.

∴ l = 150, h = 10, f = 20, N/2 = 25 and C_f = 22

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 150 + \left(\frac{25-22}{20}\right) \times 10$$

= 150 + 1.5

= 151.5

And we know that,

Mode = 3(Median) – 2(Mean)

= 3(151.5) – 2(149.8)

= 454.5 – 299.6

= 154.9

Hence, mean is 149.8, median is 151.5 and mode is 154.9.

Question: 6

Solution:

To find mean, we will solve by direct method:

DAILY INCOME (Rs.)	MID - POINT(x_i)	NUMBER OF WORKERS(f_i)	f_ix_i
100 - 120	110	12	1320
120 - 140	130	14	1820
140 - 160	150	8	1200
160 - 180	170	6	1020
180 - 200	190	10	1900
TOTAL		50	7260

We have got

$\Sigma f_i = 50$ & $\Sigma f_ix_i = 7260$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_ix_i}{\sum_i f_i}$$

$\Rightarrow \bar{x} = \frac{7260}{50}$

$\Rightarrow \bar{x} = 145.2$

To find median,

Assume $\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

DAILY INCOME (Rs.)	NUMBER OF WORKERS(f_i)	C_f
100 - 120	12	12
120 - 140	14	12 + 14 = 26
140 - 160	8	26 + 8 = 34
160 - 180	6	34 + 6 = 40
180 - 200	10	40 + 10 = 50
TOTAL	50	

So, N = 50

$\Rightarrow N/2 = 50/2 = 25$

The cumulative frequency just greater than (N/2 =) 25 is 26, so the corresponding median class is 120 - 140 and accordingly we get C_f = 12(cumulative frequency before the median class).

Now, since median class is 120 - 140.

$\therefore$ l = 120, h = 20, f = 14, N/2 = 25 and C_f = 12

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$\Rightarrow \text{Median} = 120 + \left(\frac{25-12}{14}\right) \times 20$

$= 120 + 18.57$

$= 138.57$

And we know that,

Mode = 3(Median) – 2(Mean)

= 3(138.57) – 2(145.2)

= 415.71 – 290.4

= 125.31

Hence, mean is 145.2, median is 138.57 and mode is 125.31.

Question: 7

Solution:

To find mean, we will solve by direct method:

DAILY EXPENDITURE (Rs.)	MID - POINT(x_i)	NUMBER OF HOUSEHOLDS(f_i)	f_ix_i
100 - 150	125	6	750
150 - 200	175	7	1225
200 - 250	225	12	2700
250 - 300	275	3	825
300 - 350	325	2	650
TOTAL		30	6150

We have got

$\Sigma f_i = 30$ & $\Sigma f_ix_i = 6150$

$\because$ mean is given by

$\bar{x} = \frac{\sum_i f_ix_i}{\sum_i f_i}$

$\Rightarrow \bar{x} = \frac{6150}{30}$

$\Rightarrow \bar{x} = 205$

To find median,

Assume $\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table.

DAILY EXPENDITURE (Rs.)	NUMBER OF HOUSEHOLDS(f_i)	C_f
100 - 150	6	6
150 - 200	7	6 + 7 = 13
200 - 250	12	13 + 12 = 25
250 - 300	3	25 + 3 = 28
300 - 350	2	28 + 2 = 30
TOTAL	30	

So, N = 30

$\Rightarrow N/2 = 30/2 = 15$

The cumulative frequency just greater than (N/2 =) 15 is 25, so the corresponding median class is 200 - 250 and accordingly we get $C_f = 13$(cumulative frequency before the median class).

Now, since median class is 200 - 250.

$\therefore$ l = 200, h = 50, f = 12, N/2 = 15 and $C_f = 13$

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow \text{Median} = 200 + \left(\frac{15-13}{12}\right) \times 50$$

$= 200 + 8.33$

= 208.33

Hence, mean is 205 and median is 208.33

Exercise : 9E

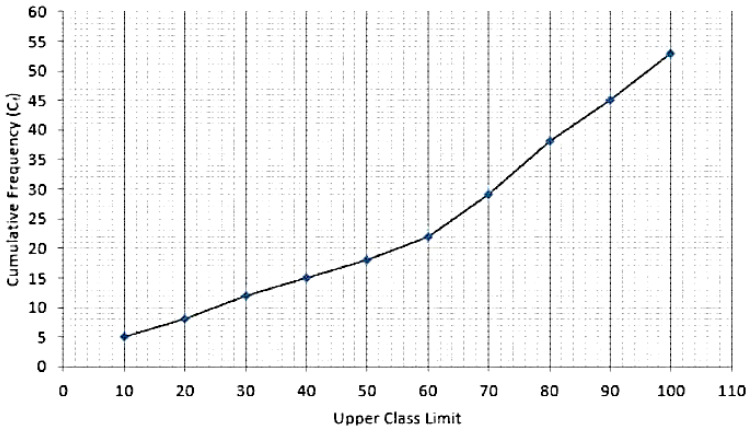
Question: 1

Solution:

The frequency distribution table for 'less than' type is:

MARKS	CUMULATIVE FREQUENCY (C_f)
Less than 10	5
Less than 20	5 + 3 = 8
Less than 30	8 + 4 = 12
Less than 40	12 + 3 = 15
Less than 50	15 + 3 = 18
Less than 60	18 + 4 = 22
Less than 70	22 + 7 = 29
Less than 80	29 + 9 = 38
Less than 90	38 + 7 = 45
Less than 100	45 + 8 = 53

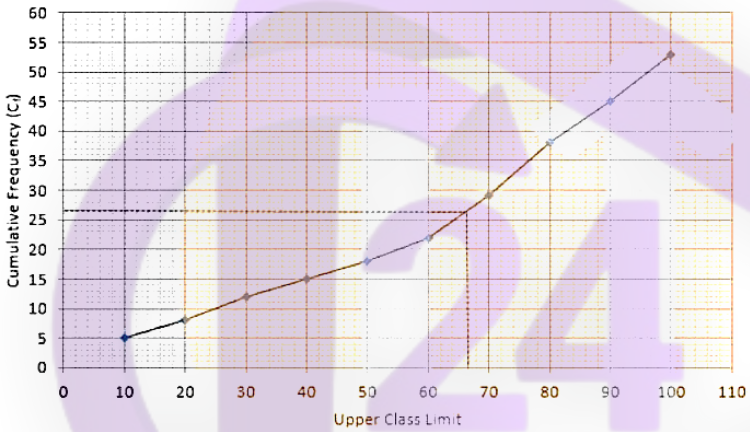
Lets plot a graph of 'less than ogive', taking upper limits of the class intervals on x - axis and cumulative frequencies on y - axis.



As we have N = 53 by the frequency table.

N/2 = 53/2 = 26.5

Mark 26.5 on y - axis and the corresponding point on x - axis would be the median.



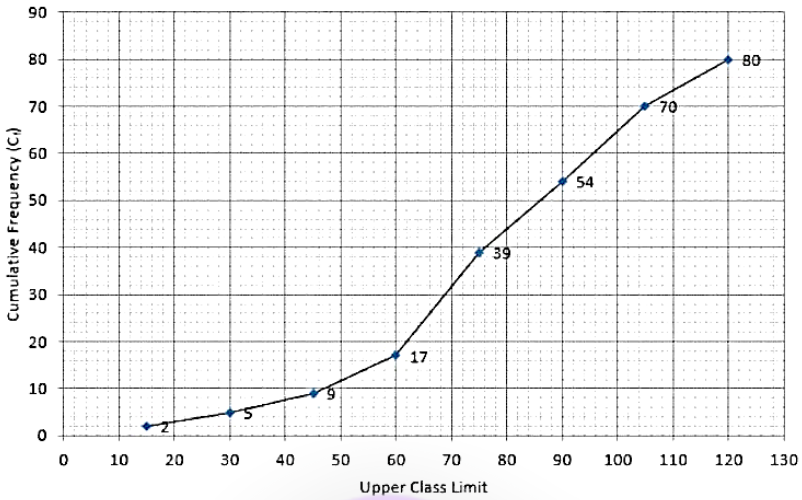
The corresponding point on x - axis is 66.4.

Hence, median is 66.4

Question: 2

Solution:

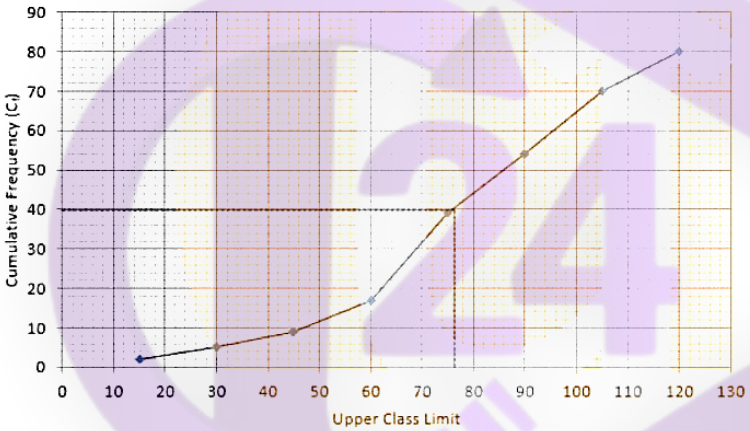
Lets plot a graph of 'less than ogive', taking upper limits of the class intervals on x - axis and cumulative frequencies on y - axis.



As we have N = 80 by the frequency table.

N/2 = 80/2 = 40

Mark 40 on y - axis and the corresponding point on x - axis would be the median.



The corresponding point on x - axis is 76.

Hence, median is 76.

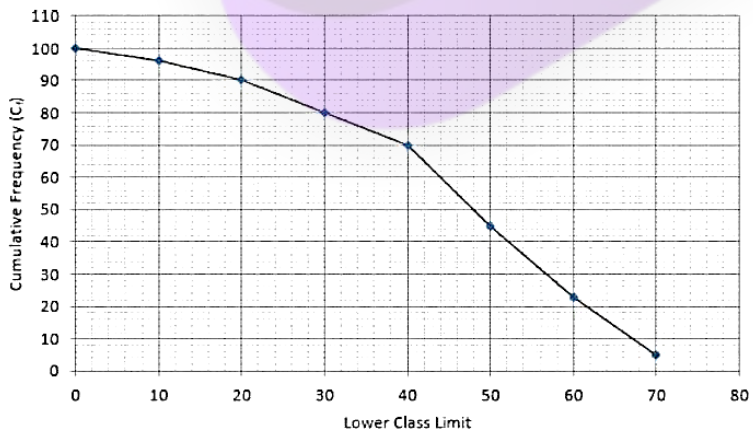
Question: 3

Solution:

The frequency distribution table for 'more than' type is:

MARKS	CUMULATIVE FREQUENCY (C_f)
more than 0	96 + 4 = 100
more than 10	90 + 6 = 96
more than 20	80 + 10 = 90
more than 30	70 + 10 = 80
more than 40	45 + 25 = 70
more than 50	23 + 22 = 45
more than 60	5 + 18 = 23
more than 70	5

Lets plot a graph of 'more than' ogive, taking lower limits of the class intervals on x - axis and cumulative frequencies on y - axis.



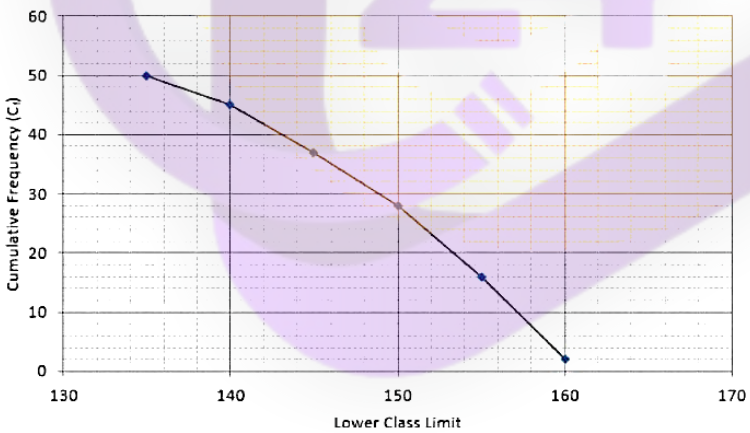
Question: 4

Solution:

The frequency distribution table for 'more than' type is:

HEIGHT(cm)	CUMULATIVE FREQUENCY (C_f)
more than 135	45 + 5 = 50
more than 140	37 + 8 = 45
more than 145	28 + 9 = 37
more than 150	16 + 12 = 28
more than 155	2 + 14 = 16
more than 160	2

Lets plot a graph of 'more than' ogive, taking lower limits of the class intervals on x - axis and cumulative frequencies on y - axis.



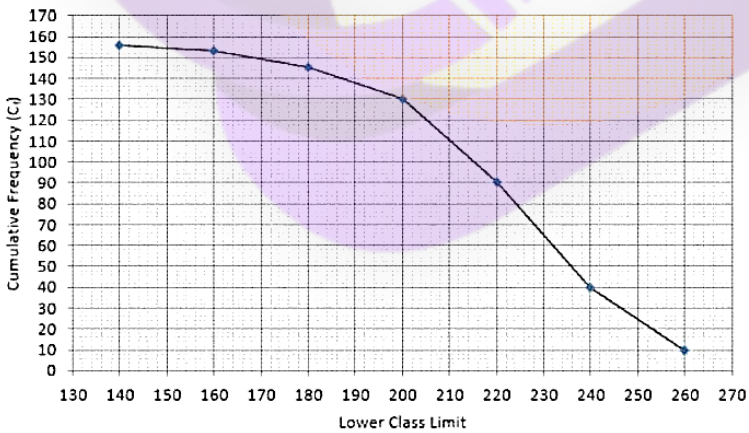
Question: 5

Solution:

The frequency distribution table for 'more than' type is:

HEIGHT(cm)	CUMULATIVE FREQUENCY (C_f)
more than 140	153 + 3 = 156
more than 160	145 + 8 = 153
more than 180	130 + 15 = 145
more than 200	90 + 40 = 130
more than 220	40 + 50 = 90
more than 240	10 + 30 = 40
more than 260	10

Lets plot a graph of 'more than' ogive, taking lower limits of the class intervals on x - axis and cumulative frequencies on y - axis.



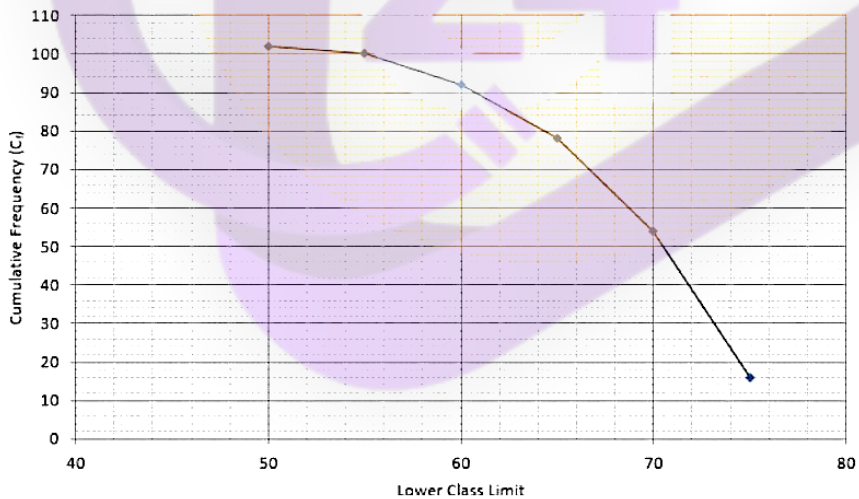
Question: 6

Solution:

The frequency distribution table for 'more than' type is:

HEIGHT(cm)	CUMULATIVE FREQUENCY (C_f)
more than 50	100 + 2 = 102
more than 55	92 + 8 = 100
more than 60	78 + 14 = 92
more than 65	54 + 24 = 78
more than 70	16 + 38 = 54
more than 75	16

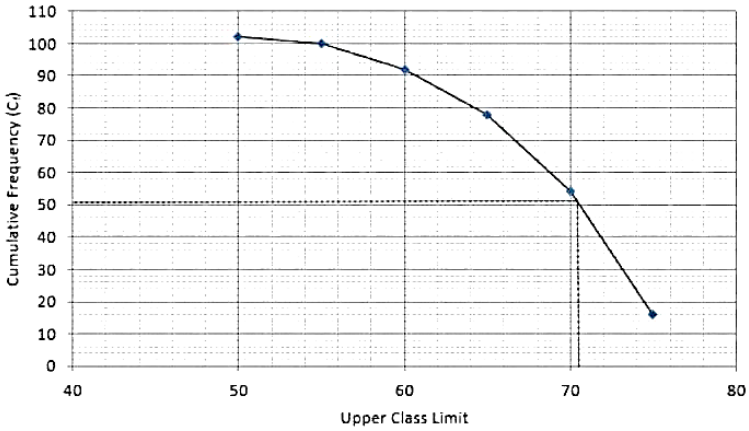
Lets plot a graph of 'more than' ogive, taking lower limits of the class intervals on x - axis and cumulative frequencies on y - axis.



As we have N = 102 by the frequency table.

N/2 = 102/2 = 51

Mark 51 on y - axis and the corresponding point on x - axis would be the median.



The corresponding point on x - axis is 70.5.

Hence, median is 70.5.

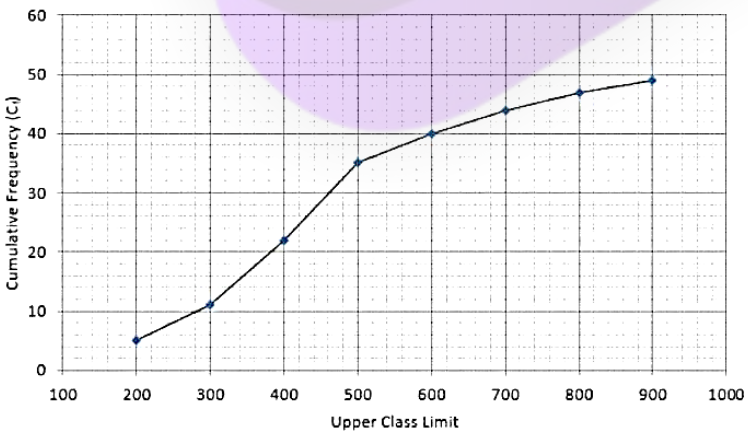
Question: 7

Solution:

The frequency distribution table for 'less than' type is:

MARKS	CUMULATIVE FREQUENCY (C_f)
Less than 200	5
Less than 300	5 + 6 = 11
Less than 400	11 + 11 = 22
Less than 500	22 + 13 = 35
Less than 600	35 + 5 = 40
Less than 700	40 + 4 = 44
Less than 800	44 + 3 = 47
Less than 900	47 + 2 = 49

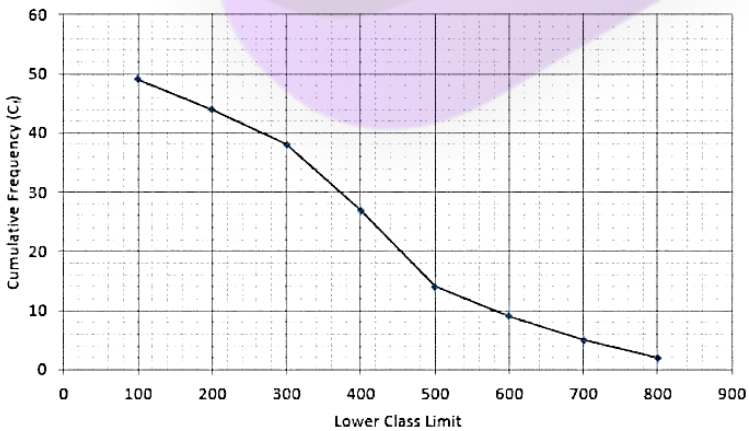
Lets plot the graph of 'less than ogive', taking upper limits of the class intervals on x - axis and cumulative frequencies on y - axis.



The frequency distribution table for 'more than' type is:

MARKS	CUMULATIVE FREQUENCY (C_f)
more than 100	44 + 5 = 49
more than 200	38 + 6 = 44
more than 300	27 + 11 = 38
more than 400	14 + 13 = 27
more than 500	9 + 5 = 14
more than 600	5 + 4 = 9
more than 700	2 + 3 = 5
more than 800	2

Lets plot a graph of 'more than' ogive, taking lower limits of the class intervals on x - axis and cumulative frequencies on y - axis.



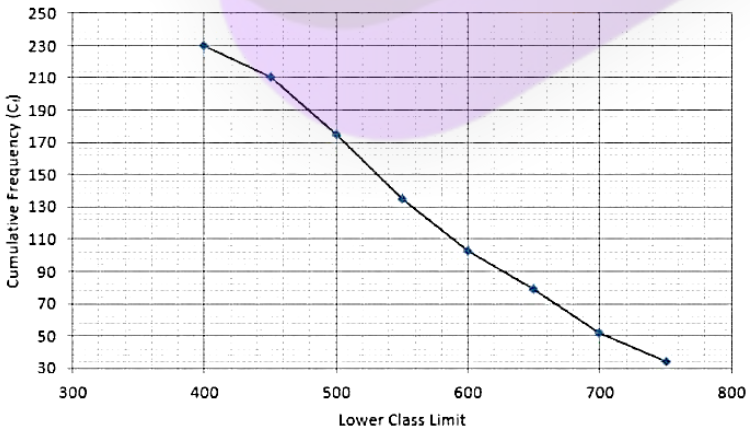
Question: 8

Solution:

The frequency distribution table for 'more than' type is:

HEIGHT(cm)	CUMULATIVE FREQUENCY (C_f)
more than 400	210 + 20 = 230
more than 450	175 + 35 = 210
more than 500	135 + 40 = 175
more than 550	103 + 32 = 135
more than 600	79 + 24 = 103
more than 650	52 + 27 = 79
more than 700	34 + 18 = 52
More than 750	34

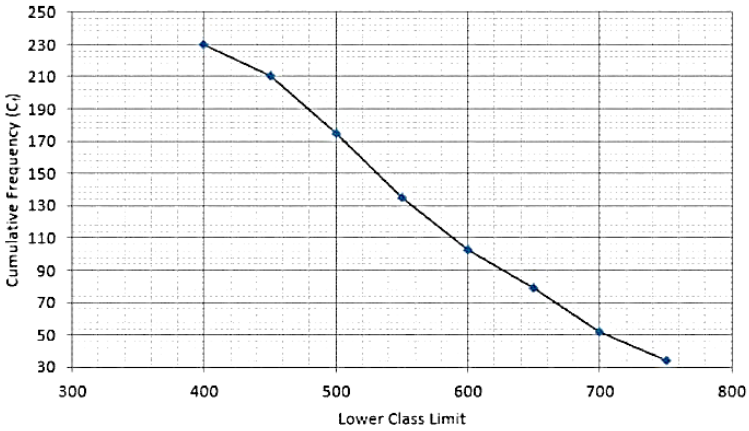
Lets plot a graph of 'more than' ogive, taking lower limits of the class intervals on x - axis and cumulative frequencies on y - axis.



As we have N = 230 by the frequency table.

N/2 = 230/2 = 115

Mark 115 on y - axis and the corresponding point on x - axis would be the median.



The corresponding point on x - axis is 590.

Hence, median is 590.

Question: 9

Solution:

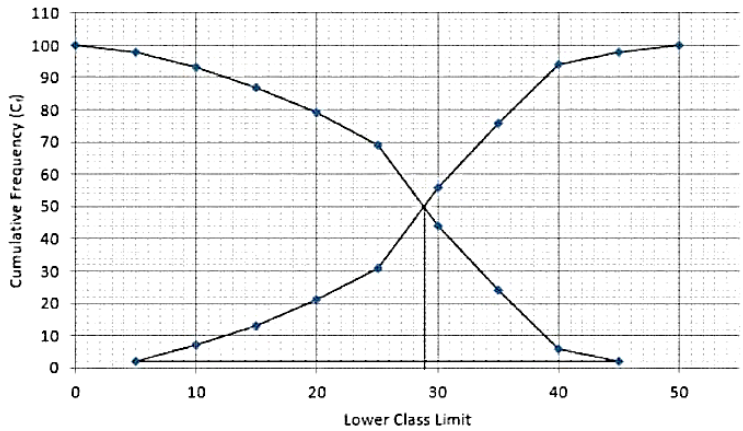
(i) The frequency distribution table for 'less than' type is:

MARKS	CUMULATIVE FREQUENCY (C_f)
Less than 5	2
Less than 10	2 + 5 = 7
Less than 15	7 + 6 = 13
Less than 20	13 + 8 = 21
Less than 25	21 + 10 = 31
Less than 30	31 + 25 = 56
Less than 35	56 + 20 = 76
Less than 40	76 + 18 = 94
Less than 45	94 + 4 = 98
Less than 50	98 + 2 = 100

(ii) The frequency distribution table for 'more than' type is:

MARKS	CUMULATIVE FREQUENCY (C_f)
More than 0	98 + 2 = 100
More than 5	93 + 5 = 98
More than 10	87 + 6 = 93
More than 15	79 + 8 = 87
More than 20	69 + 10 = 79
More than 25	44 + 25 = 69
More than 30	24 + 20 = 44
More than 35	6 + 18 = 24
More than 40	2 + 4 = 6
More than 45	2

Plotting points for 'less - than ogive' and 'more - than ogive' on the graph,



In this type of graph where 'less than ogive' and more than ogive' are plotted in the same graph, median is found on x - axis by the intersection of these two ogives.

Here, median = 29.5

Question: 10

Solution:

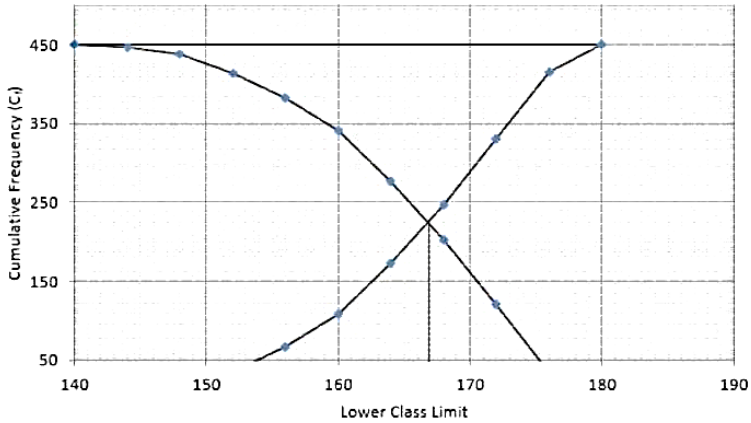
(i) The frequency distribution table for 'less than' type is:

MARKS	CUMULATIVE FREQUENCY (C_f)
Less than 144	3
Less than 148	3 + 9 = 12
Less than 152	12 + 24 = 36
Less than 156	36 + 31 = 67
Less than 160	67 + 42 = 109
Less than 164	109 + 64 = 173
Less than 168	173 + 75 = 248
Less than 172	248 + 82 = 330
Less than 176	330 + 86 = 416
Less than 180	416 + 34 = 450

(ii) The frequency distribution table for 'more than' type is:

MARKS	CUMULATIVE FREQUENCY (C_f)
More than 140	447 + 3 = 450
More than 144	438 + 9 = 447
More than 148	414 + 24 = 438
More than 152	383 + 31 = 414
More than 156	341 + 42 = 383
More than 160	277 + 64 = 341
More than 164	202 + 75 = 277
More than 168	120 + 82 = 202
More than 172	34 + 86 = 120
More than 176	34

Plotting points for 'less - than ogive' and 'more - than ogive' on the graph,



In this type of graph where 'less than ogive' and more than ogive' are plotted in the same graph, median is found on x - axis by the intersection of these two ogives.

Here, median = 166

Exercise : 9F

Question: 1

Solution:

To find median class,

Assume $\Sigma f_i = N$ = Sum of frequencies,

f_i = frequency

and C_f = cumulative frequency

Lets form a table.

AGE(years)	NUMBER OF PATIENTS(f_i)	C_f
0 - 10	4	4
10 - 20	4	4 + 4 = 8
20 - 30	8	8 + 8 = 16
30 - 40	10	16 + 10 = 26
40 - 50	12	26 + 12 = 38
50 - 60	8	38 + 8 = 46
60 - 70	4	46 + 4 = 50
TOTAL	50	

So, N = 50

⇒ N/2 = 50/2 = 25

The cumulative frequency just greater than (N/2 =) 25 is 26, so the corresponding median class is 30 - 40.

Hence, median class = 30 - 40

Question: 2

Solution:

Here, the maximum class frequency is 27.

The class corresponding to this frequency is the modal class. ⇒ modal class = 40 - 50

∴ lower limit of the modal class (l) = 40

Question: 3

Solution:

Here, the maximum class frequency is 30.

The class corresponding to this frequency is the modal class. ⇒ modal class = 150

∴ lower limit of the modal class (l) = 150

The class mark is found by,

$$\frac{150 + 200}{2} = \frac{350}{2} = 175$$

∴ Class mark is 175.

Question: 4

Solution:

Since we have 25 observations, that is odd number of observations, median is found at $\left(\frac{n+1}{2}\right)^{th}$ position.

So since, n = 25

⇒ Median will be found at $\left(\frac{25+1}{2}\right)^{th}$ position. ⇒ Median = 13^{th} observation

Question: 5

Solution:

Given: mode = 1000 and median = 1250

The empirical relationship between mean, median and mode is,

Mode = 3(Median) – 2(Mean)

⇒ 2(Mean) = 3(Median) – Mode

⇒ Mean = [3(Median) – Mode]/2

⇒ Mean = [3(1250) – 1000]/2

⇒ Mean = [3750 – 1000]/2 = 2750/2 = 1375

∴ mean = 1375

Question: 6

Solution:

Here, the maximum class frequency is 25.

The class corresponding to this frequency is the modal class. ⇒ modal class = 40 - 60

To find median class,

Assume Σf_i = N = Sum of frequencies,

f_i = frequency

and C_f = cumulative frequency

Lets form a table.

MARKS OBTAINED	NUMBER OF STUDENTS(f_i)	C_f
0 - 20	4	4
20 - 40	6	4 + 6 = 10
40 - 60	25	10 + 25 = 35
60 - 80	10	35 + 10 = 45
80 - 100	5	45 + 5 = 50
TOTAL	50	

So, N = 50

⇒ N/2 = 50/2 = 25

The cumulative frequency just greater than (N/2 =) 25 is 35, so the corresponding median class is 40 - 60.

∴ modal class = 40 - 60 and median class = 40 - 60

Question: 7

Solution:

Class mark is given by $\frac{\text{Upper limit + Lower limit}}{2}$

Class mark of class 10 - 25 = $\frac{10+25}{2} = \frac{35}{2} = 17.5$

Class mark of class 35 - 55 = $\frac{35+55}{2} = \frac{90}{2} = 45$

∴ Class mark of class 10 - 25 is 17.5 and 35 - 55 is 45.

Question: 8

Solution:

We have got

A = 25, $\Sigma f_i = 50$ & $\Sigma f_i d_i = 110$

∵ By Assumed - mean method, mean is given by

$\bar{x} = A + \frac{\sum_i f_i d}{\sum_i f_i}$

$\Rightarrow \bar{x} = 25 + \frac{110}{50}$

$\Rightarrow \bar{x} = 27.2$

Thus, mean is 27.2

Question: 9

Solution:

According to the question,

$4 = \frac{X}{36}$ and $3 = \frac{Y}{64}$

$\Rightarrow X = 36 \times 4 = 144$ and $Y = 64 \times 3 = 192$

We have, $X = 144$ and $Y = 192$

Mean of distribution $(X + Y = 144 + 192 =)\ 336$ is,

Mean $= 336/(36 + 64) = 336/100 = 3.36$

Hence, mean = 3.36

Question: 10

Solution:

Given: number of classes = 12,

Class width = 2.5, and

Lowest class boundary = 8.1

Upper class is given by,

Upper class boundary = Lower class boundary + (width × number of classes)

Substituting values,

⇒ Upper class boundary = 8.1 + (2.5 × 12)

⇒ Upper class boundary = 8.1 + 30 = 38.1

Hence, upper class boundary is 38.1

Question: 11

Solution:

Since there are 10 observations, that is, even number of observations, median is found by taking average of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2} + 1\right)^{th}$ observations.

So, median is found at average of $\left(\frac{10}{2}\right)^{th} = (5)^{th}$ and $\left(\frac{10}{2} + 1\right)^{th} = (6)^{th}$ observations.

$(5)^{th}$ observation = x and $(6)^{th}$ observation = x + 2

Taking average,

Median $= (x + x + 2)/2$

$\Rightarrow 63 = (2x + 2)/2$ [$\because$ given is median = 63]

⇒ 126 = 2x + 2

⇒ 2x = 126 – 2

⇒ 2x = 124

⇒ x = 124/2 = 62

∴ x = 62

Question: 12

Solution:

As median is the "middle" number of the sorted list of numbers, and given is median of 19 observations observed to be 30.

⇒ 30 is the middle most value amongst 19 observations.

If two more observations (8 and 32) are added, where 8 is less than 30 and 32 is more than 30. 30 is still the middlemost value as the two values are added on either side of 30.

Hence, median of 21 observations are 30.

Question: 13

Solution:

Arranging the values x/5, x/4, x/2, x and x/3 in ascending order, we get

x/5, x/4, x/3, x/2 and x

Here, median is x/3 as it is the middle value amongst all values.

Given: median = 8

⇒ x/3 = 8

⇒ x = 24

Hence, x = 24

Question: 14

Solution:

Here, the maximum class frequency is 23.

The class corresponding to this frequency is the modal class. ⇒ modal class = 12 - 15

Lets form a table.

CLASS	FREQUENCY(f_i)	C_f
3 - 6	7	7
6 - 9	13	7 + 13 = 20
9 - 12	10	20 + 10 = 30
12 - 15	23	30 + 23 = 53
15 - 18	4	53 + 4 = 57
18 - 21	21	57 + 21 = 78
21 - 24	16	78 + 16 = 94
TOTAL	94	

Since, modal class = 12 - 15, the corresponding cumulative frequency is 53.

Question: 15

Solution:

Here, the maximum class frequency is 18.

The class corresponding to this frequency is the modal class. ⇒ modal class = 40 - 60

∴ lower limit of the modal class (l) = 40

Modal class size (h) = 20

Frequency of the modal class (f_1) = 18

Frequency of class preceding the modal class (f_0) = 6

Frequency of class succeeding the modal (f_2) = 10

Mode is given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$\Rightarrow \text{Mode} = 40 + \left(\frac{18-6}{2(18)-6-10}\right) \times 20$

$\Rightarrow \text{Mode} = 40 + \left(\frac{12}{20}\right) \times 20$

$\Rightarrow$ Mode = 40 + 12 = 52

Hence, the mode is 52.

Question: 16

Solution:

In a 'less than type' cumulative frequency distribution, upper limit of the classes are considered.

It is given by,

AGE(in years)	CUMULATIVE FREQUENCY(C_f)
Less than 20	60
Less than 30	102
Less than 40	157
Less than 50	227
Less than 60	280
Less than 70	300

Question: 17

Solution:

To find p and q, solve by finding cumulative frequency,

CLASS	FREQUENCY (f)	CUMULATIVE FREQUENCY (C_f)
100 - 200	11	11
200 - 300	12	p = 11 + 12 = 23
300 - 400	10	33
400 - 500	q	46 = 33 + q ⇒ q = 13
500 - 600	20	66
600 - 700	14	80

⇒ p = 11 + 12 = 23

And 46 = 33 + q ⇒ q = 46 – 33 = 13

∴ p = 23 and q = 13

Lets form the table again,

CLASS	FREQUENCY (f_i)	CUMULATIVE FREQUENCY (C_f)
100 - 200	11	11
200 - 300	12	23
300 - 400	10	33
400 - 500	13	46
500 - 600	20	66
600 - 700	14	80
TOTAL	80	

For modal class,

Here, the maximum class frequency is 20.

The class corresponding to this frequency is the modal class. ⇒ modal class = 500 - 600

To find median class,

Assume $\Sigma f_i = N$ = Sum of frequencies,

f_i = frequency

and C_f = cumulative frequency

So, N = 80

⇒ N/2 = 80/2 = 40

The cumulative frequency just greater than (N/2 =) 40 is 46, so the corresponding median class is 400 - 500.

∴ modal class = 500 - 600 and median class = 400 - 500

Question: 18

Solution:

In a 'less than type' cumulative frequency distribution, lower limits of the classes are considered.

It is given by,

MONTHLY CONSUMPTION (in units)	CUMULATIVE FREQUENCY(C_f)
More than 65	60 + 4 = 64
More than 85	55 + 5 = 60
More than 105	42 + 13 = 55
More than 125	22 + 20 = 42
More than 145	8 + 14 = 22
More than 165	8

Question: 19

Solution:

The frequency distribution table is:

LIFE TIME (in days)	NUMBER OF BULBS
0 - 50	7
50 - 100	21 - 7 = 14
100 - 150	52 - 21 = 31
150 - 200	79 - 52 = 27
200 - 250	91 - 79 = 12
250 - 300	100 - 91 = 9

Question: 20

Solution:

(a) To convert the given frequency distribution into continuous form, adjust the end - limits of each class.

MARKS OBTAINED (in percent)	NUMBER OF STUDENTS (f_i)
10.5 - 20.5	141
20.5 - 30.5	221
30.5 - 40.5	439
40.5 - 50.5	529
50.5 - 60.5	495
60.5 - 70.5	322
70.5 - 80.5	153

(b) To find median class,

Assume $\Sigma f_i = N$ = Sum of frequencies,

f_i = frequency

and C_f = cumulative frequency

MARKS OBTAINED (in percent)	NUMBER OF STUDENTS (f_i)	C_f
10.5 - 20.5	141	141
20.5 - 30.5	221	141 + 221 = 362
30.5 - 40.5	439	362 + 439 = 801
40.5 - 50.5	529	801 + 529 = 1330
50.5 - 60.5	495	1330 + 495 = 1825
60.5 - 70.5	322	1825 + 322 = 2147
70.5 - 80.5	153	2147 + 153 = 2300
TOTAL	2300	

So, N = 2300

$\Rightarrow N/2 = 2300/2 = 1150$

The cumulative frequency just greater than (N/2 =) 1150 is 1825, so the corresponding median class is 50.5 - 60.5.

$\therefore$ median class = 50.5 - 60.5

The class mark of 50.5 - 60.5 is $\frac{50.5 + 60.5}{2} = \frac{111}{2} = 55.5$

(c) For modal class,

Here, the maximum class frequency is 529.

The class corresponding to this frequency is the modal class. $\Rightarrow$ modal class = 40.5 - 50.5

The cumulative frequency corresponding to the modal class is 1330

Question: 21

Solution:

CLASS	MID - POINT(x_i)	FREQUENCY(f_i)	f_ix_i
0 - 10	5	8	40
10 - 20	15	p	15p
20 - 30	25	12	300
30 - 40	35	13	455
40 - 50	45	10	450
TOTAL		43 + p	1245 + 15p

We have got

$\Sigma f_i = 43 + p$ and $\Sigma f_i x_i = 1245 + 15p$

$\because$ mean is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$\Rightarrow 27 = \frac{1245 + 15p}{43 + p}$ ($\because$ given: mean of pocket allowance is 27)

$\Rightarrow 1161 + 27p = 1245 + 15p$

$\Rightarrow 27p - 15p = 1245 - 1161$

$\Rightarrow 12p = 84$

$\Rightarrow p = 84/12$

$\Rightarrow p = 7$

Thus, p = 7

Question: 22

Solution:

Given: Median = 24

Let the unknown frequency be x.

Assume

$\Sigma f_i = N$ = Sum of frequencies,

h = length of median class,

l = lower boundary of the median class,

f = frequency of median class

and C_f = cumulative frequency

Lets form a table, where x is the unknown frequency.

AGE (in years)	NUMBER OF PERSONS(f_i)	C_f
0 - 10	5	5
10 - 20	25	5 + 25 = 30
20 - 30	x	30 + x
30 - 40	18	30 + x + 18 = 48 + x
40 - 50	7	48 + x + 7 = 55 + x
TOTAL	55 + x	

Median = 24 (as already mentioned in the question)

24 lies between 20 - 30 ⇒ Median class = 20 - 30

∴ l = 20, h = 10, f = x, N/2 = (55 + x)/2 and C_f = 30

Median is given by,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C_f}{f}\right) \times h$$

$$\Rightarrow 24 = 20 + \left(\frac{\frac{55+x}{2} - 30}{x}\right) \times 10$$

$$\Rightarrow 24 = 20 + \left(\frac{55 + x - 60}{2x}\right) \times 10$$

⇒ 24 – 20 = (5x – 25)/x

⇒ 4x = 5x – 25

⇒ 5x – 4x = 25

⇒ x = 25