Chapter : 10. BINOMIAL THEOREM

Exercise : 10A

Question: 1

Solution:

To find: Expansion of $(1 - 2x)^5$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(1 - 2x)^5$

$\Rightarrow [{}^5C_0(1)^5] + [{}^5C_1(1)^{5-1}(-2x)^1] + [{}^5C_2(1)^{5-2}(-2x)^2] + [{}^5C_3(1)^{5-3}(-2x)^3] + [{}^5C_4(1)^{5-4}(-2x)^4] + [{}^5C_5(-2x)^5]$

$$\Rightarrow \left[\frac{5!}{0!(5-0)!}(1)^5\right] - \left[\frac{5!}{1!(5-1)!}(1)^4(2x)\right] + \left[\frac{5!}{2!(5-2)!}(1)^3(4x^2)\right]$$

$$-\left[\frac{5!}{3!(5-3)!}(1)^2(8x^3)\right] + \left[\frac{5!}{4!(5-4)!}(1)^1(16x^4)\right] - \left[\frac{5!}{5!(5-5)!}(32x^5)\right]$$

$\Rightarrow 1 - 5(2x) + 10(4x^2) - 10(8x^3) + 5(16x^4) - 1(32x^5)$

$\Rightarrow 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

On rearranging

Ans) $-32x^5 + 80x^4 - 80x^3 + 40x^2 - 10x + 1$

Question: 2

Solution:

To find: Expansion of $(2x - 3)^6$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(2x - 3)^6$

$\Rightarrow [{}^6C_0(2x)^6]+[{}^6C_1(2x)^{6-1}(-3)^1]+[{}^6C_2(2x)^{6-2}(-3)^2]+[{}^6C_3(2x)^{6-3}(-3)^3]+ [{}^6C_4(2x)^{6-4}(-3)^4] + [{}^6C_5(2x)^{6-5}(-3)^5] + [{}^6C_6(-3)^6]$

$$\Rightarrow \left[\frac{6!}{0!(6-0)!}(2x)^6\right] - \left[\frac{6!}{1!(6-1)!}(2x)^5(3)\right] + \left[\frac{6!}{2!(6-2)!}(2x)^4(9)\right]$$

$$-\left[\frac{6!}{3!(6-3)!}(2x)^3(27)\right] + \left[\frac{6!}{4!(6-4)!}(2x)^2(81)\right]$$

$$-\left[\frac{6!}{5!(6-5)!}(2x)^1(243)\right] + \left[\frac{6!}{6!(6-6)!}(729)\right]$$

$\Rightarrow [(1)(64x^6)] - [(6)(32x^5)(3)] + [15(16x^4)(9)] - [20(8x^3)(27)] + [15(4x^2)(81)] - [(6)(2x)(243)] + [(1)(729)]$

$\Rightarrow 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

Ans) $64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

Question: 3

Solution:

To find: Expansion of $(3x + 2y)^5$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(3x + 2y)^5$

$\Rightarrow [{}^5C_0(3x)^{5-0}] + [{}^5C_1(3x)^{5-1}(2y)^1] + [{}^5C_2(3x)^{5-2}(2y)^2] + [{}^5C_3(3x)^{5-3}(2y)^3] + [{}^5C_4(3x)^{5-4}(2y)^4] + [{}^5C_5(2y)^5]$

$\Rightarrow \left[\frac{5!}{0!(5-0)!}(243x^5)\right] + \left[\frac{5!}{1!(5-1)!}(81x^4)(2y)\right] + \left[\frac{5!}{2!(5-2)!}(27x^3)(4y^2)\right] + \left[\frac{5!}{3!(5-3)!}(9x^2)(8y^3)\right] + \left[\frac{5!}{4!(5-4)!}(3x)(16y^4)\right] + \left[\frac{5!}{5!(5-5)!}(32y^5)\right]$

$\Rightarrow [1(243x^5)] + [5(81x^4)(2y)] + [10(27x^3)(4y^2)] + [10(9x^2)(8y^3)] + [5(3x)(16y^4)] + [1(32y^5)]$

$\Rightarrow 243x^5 + 810x^4y + 1080x^3y^2 + 720x^2y^3 + 240xy^4 + 32y^5$

Ans) $243x^5 + 810x^4y + 1080x^3y^2 + 720x^2y^3 + 240xy^4 + 32y^5$

Question: 4

Solution:

To find: Expansion of $(2x - 3y)^4$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(2x - 3y)^4$

$\Rightarrow [{}^4C_0(2x)^{4-0}] + [{}^4C_1(2x)^{4-1}(-3y)^1] + [{}^4C_2(2x)^{4-2}(-3y)^2] + [{}^4C_3(2x)^{4-3}(-3y)^3] + [{}^4C_4(-3y)^4]$

$\left[\frac{4!}{0!(4-0)!}(2x)^4\right] - \left[\frac{4!}{1!(4-1)!}(2x)^3(3y)\right] + \left[\frac{4!}{2!(4-2)!}(2x)^2(9y^2)\right] - \left[\frac{4!}{3!(4-3)!}(2x)^1(27y^3)\right] + \left[\frac{4!}{4!(4-4)!}(81y^4)\right]$

$\Rightarrow [1(16x^4)] - [4(8x^3)(3y)] + [6(4x^2)(9y^2)] - [4(2x)(27y^3)] + [1(81y^4)]$

$\Rightarrow 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$

Ans) $16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$

Question: 5

Solution:

To find: Expansion of

$$\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$

$$\Rightarrow \left[{}^6C_0\left(\frac{2x}{3}\right)^{6-0}\right] + \left[{}^6C_1\left(\frac{2x}{3}\right)^{6-1}\left(-\frac{3}{2x}\right)^1\right] + \left[{}^6C_2\left(\frac{2x}{3}\right)^{6-2}\left(-\frac{3}{2x}\right)^2\right] + \left[{}^6C_3\left(\frac{2x}{3}\right)^{6-3}\left(-\frac{3}{2x}\right)^3\right] + \left[{}^6C_4\left(\frac{2x}{3}\right)^{6-4}\left(-\frac{3}{2x}\right)^4\right]$$

$$+ \left[{}^6C_5\left(\frac{2x}{3}\right)^{6-5}\left(-\frac{3}{2x}\right)^5\right] + \left[{}^6C_6\left(-\frac{3}{2x}\right)^6\right]$$

$$\Rightarrow \left[\frac{6!}{0!(6-0)!}\left(\frac{2x}{3}\right)^6\right] - \left[\frac{6!}{1!(6-1)!}\left(\frac{2x}{3}\right)^5\left(\frac{3}{2x}\right)\right] + \left[\frac{6!}{2!(6-2)!}\left(\frac{2x}{3}\right)^4\left(\frac{9}{4x^2}\right)\right] - \left[\frac{6!}{3!(6-3)!}\left(\frac{2x}{3}\right)^3\left(\frac{27}{8x^3}\right)\right] + \left[\frac{6!}{4!(6-4)!}\left(\frac{2x}{3}\right)^2\left(\frac{81}{16x^4}\right)\right] - \left[\frac{6!}{5!(6-5)!}\left(\frac{2x}{3}\right)^1\left(\frac{243}{32x^5}\right)\right]$$

$$+ \left[\frac{6!}{6!(6-6)!}\left(\frac{729}{64x^6}\right)\right]$$

$$\Rightarrow \left[1\left(\frac{64x^6}{729}\right)\right] - \left[6\left(\frac{32x^5}{243}\right)\left(\frac{3}{2x}\right)\right] + \left[15\left(\frac{16x^4}{81}\right)\left(\frac{9}{4x^2}\right)\right] - \left[20\left(\frac{8x^3}{27}\right)\left(\frac{27}{8x^3}\right)\right] + \left[15\left(\frac{4x^2}{9}\right)\left(\frac{81}{16x^4}\right)\right] - \left[6\left(\frac{2x}{3}\right)\left(\frac{243}{32x^5}\right)\right] + \left[1\left(\frac{729}{64x^6}\right)\right]$$

$$\Rightarrow \frac{64}{729}x^6 - \frac{32}{27}x^4 + \frac{20}{3}x^2 - 20 + \frac{135}{4}\frac{1}{x^2} - \frac{243}{8}\frac{1}{x^4} + \frac{729}{64}\frac{1}{x^6}$$

Ans) $\frac{64}{729}x^6 - \frac{32}{27}x^4 + \frac{20}{3}x^2 - 20 + \frac{135}{4}\frac{1}{x^2} - \frac{243}{8}\frac{1}{x^4} + \frac{729}{64}\frac{1}{x^6}$

Question: 6

Solution:

To find: Expansion of $\left(x^2 - \frac{3x}{7}\right)^7$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $\left(x^2 - \frac{3x}{7}\right)^7$

$\Rightarrow \left[{}^7C_0(x^2)^{7-0}\right] + \left[{}^7C_1(x^2)^{7-1}\left(-\frac{3x}{7}\right)^1\right] + \left[{}^7C_2(x^2)^{7-2}\left(-\frac{3x}{7}\right)^2\right] +$

$\left[{}^7C_3(x^2)^{7-3}\left(-\frac{3x}{7}\right)^3\right] + \left[{}^7C_4(x^2)^{7-4}\left(-\frac{3x}{7}\right)^4\right] + \left[{}^7C_5(x^2)^{7-5}\left(-\frac{3x}{7}\right)^5\right] +$

$\left[{}^7C_6(x^2)^{7-6}\left(-\frac{3x}{7}\right)^6\right] + \left[{}^7C_7\left(-\frac{3x}{7}\right)^7\right]$

$\Rightarrow \left[\frac{7!}{0!(7-0)!}(x^2)^7\right] - \left[\frac{7!}{1!(7-1)!}(x^2)^6\left(\frac{3x}{7}\right)\right] + \left[\frac{7!}{2!(7-2)!}(x^2)^5\left(\frac{9x^2}{49}\right)\right] -$

$\left[\frac{7!}{3!(7-3)!}(x^2)^4\left(\frac{27x^3}{343}\right)\right] + \left[\frac{7!}{4!(7-4)!}(x^2)^3\left(\frac{81x^4}{2401}\right)\right] - \left[\frac{7!}{5!(7-5)!}\right.$

$\left.(x^2)^2\left(\frac{243x^5}{16807}\right)\right] + \left[\frac{7!}{6!(7-6)!}(x^2)^1\left(\frac{729x^6}{117649}\right)\right] - \left[\frac{7!}{7!(7-7)!}\left(\frac{2187x^7}{823543}\right)\right]$

$\Rightarrow [1(x^{14})] - \left[7(x^{12})\left(\frac{3x}{7}\right)\right] + \left[21(x^{10})\left(\frac{9x^2}{49}\right)\right] - \left[35(x^8)\left(\frac{27x^3}{343}\right)\right] +$

$\left[35(x^6)\left(\frac{81x^4}{2401}\right)\right] - \left[21(x^4)\left(\frac{243x^5}{16807}\right)\right] + \left[7(x^2)\left(\frac{729x^6}{117649}\right)\right] -$

$\left[1\left(\frac{2187x^7}{823543}\right)\right]$

$\Rightarrow x^{14} - 3x^{13} + \left(\frac{27}{7}\right)x^{12} - \left(\frac{135}{49}\right)x^{11} + \left(\frac{405}{343}\right)x^{10} -$ Ans)

$\left(\frac{729}{2401}\right)x^9 + \left(\frac{729}{16807}\right)x^8 - \left(\frac{2187}{823543}\right)x^7$

$x^{14} - 3x^{13} + \left(\frac{27}{7}\right)x^{12} - \left(\frac{135}{49}\right)x^{11} + \left(\frac{405}{343}\right)x^{10} - \left(\frac{729}{2401}\right)x^9 + \left(\frac{729}{16807}\right)x^8 -$

$\left(\frac{2187}{823543}\right)x^7$

Question: 7

Solution:

To find: Expansion of $\left(x - \frac{1}{y}\right)^5$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $\left(x - \frac{1}{y}\right)^5$

$\Rightarrow {}^5C_0(x)^{5-0} + {}^5C_1(x)^{5-1}\left(-\frac{1}{y}\right)^1 + {}^5C_2(x)^{5-2}\left(-\frac{1}{y}\right)^2 + {}^5C_3(x)^{5-3}\left(-\frac{1}{y}\right)^3 + {}^5C_4(x)^{5-4}\left(-\frac{1}{y}\right)^4 + {}^5C_5\left(-\frac{1}{y}\right)^5$

$\Rightarrow \left[\frac{5!}{0!(5-0)!}(x^5)\right] - \left[\frac{5!}{1!(5-1)!}(x^4)\left(\frac{1}{y}\right)^1\right] + \left[\frac{5!}{2!(5-2)!}(x^3)\left(\frac{1}{y^2}\right)\right]$

$-\left[\frac{5!}{3!(5-3)!}(x^2)\left(\frac{1}{y^3}\right)\right] + \left[\frac{5!}{4!(5-4)!}(x)\left(\frac{1}{y^4}\right)\right] - \left[\frac{5!}{5!(5-5)!}\left(\frac{1}{y^5}\right)\right]$

$\Rightarrow [1(x^5)] - \left[5\left(\frac{x^4}{y}\right)\right] + \left[10\left(\frac{x^3}{y^2}\right)\right] - \left[10\left(\frac{x^2}{y^3}\right)\right] + \left[5\left(\frac{x}{y^4}\right)\right] - [1(y^5)]$

$\Rightarrow x^5 - 5\frac{x^4}{y} + 10\frac{x^3}{y^2} - 10\frac{x^2}{y^3} + 5\frac{x}{y^4} - y^5$

Ans) $x^5 - 5\frac{x^4}{y} + 10\frac{x^3}{y^2} - 10\frac{x^2}{y^3} + 5\frac{x}{y^4} - y^5$

Question: 8

Solution:

To find: Expansion of $(\sqrt{x}+\sqrt{y})^8$

Formula used: (i) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(\sqrt{x}+\sqrt{y})^8$

We can write $\sqrt{x}$ as $x^{\frac{1}{2}}$ and $\sqrt{y}$ as $y^{\frac{1}{2}}$

Now, we have to solve for $\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)^8$

$$\Rightarrow \left[{}^8C_0\left(x^{\frac{1}{2}}\right)^{8-0}\right] + \left[{}^8C_1\left(x^{\frac{1}{2}}\right)^{8-1}\left(y^{\frac{1}{2}}\right)^1\right] + \left[{}^8C_2\left(x^{\frac{1}{2}}\right)^{8-2}\left(y^{\frac{1}{2}}\right)^2\right] +$$
$$\left[{}^8C_3\left(x^{\frac{1}{2}}\right)^{8-3}\left(y^{\frac{1}{2}}\right)^3\right] + \left[{}^8C_4\left(x^{\frac{1}{2}}\right)^{8-4}\left(y^{\frac{1}{2}}\right)^4\right] + \left[{}^8C_5\left(x^{\frac{1}{2}}\right)^{8-5}\left(y^{\frac{1}{2}}\right)^5\right] +$$
$$\left[{}^8C_6\left(x^{\frac{1}{2}}\right)^{8-6}\left(y^{\frac{1}{2}}\right)^6\right] + \left[{}^8C_7\left(x^{\frac{1}{2}}\right)^{8-7}\left(y^{\frac{1}{2}}\right)^7\right] + \left[{}^8C_8\left(y^{\frac{1}{2}}\right)^8\right]$$

$$\Rightarrow \left[\frac{8!}{0!(8-0)!}\left(x^{\frac{8}{2}}\right)\right] + \left[\frac{8!}{1!(8-1)!}\left(x^{\frac{7}{2}}\right)\left(y^{\frac{1}{2}}\right)\right] + \left[\frac{8!}{2!(8-2)!}\left(x^{\frac{6}{2}}\right)\left(y^{\frac{2}{2}}\right)\right] +$$
$$\left[\frac{8!}{3!(8-3)!}\left(x^{\frac{5}{2}}\right)\left(y^{\frac{3}{2}}\right)\right] + \left[\frac{8!}{4!(8-4)!}\left(x^{\frac{4}{2}}\right)\left(y^{\frac{4}{2}}\right)\right] + \left[\frac{8!}{5!(8-5)!}\left(x^{\frac{3}{2}}\right)\left(y^{\frac{5}{2}}\right)\right] +$$
$$\left[\frac{8!}{6!(8-6)!}\left(x^{\frac{2}{2}}\right)\left(y^{\frac{6}{2}}\right)\right] + \left[\frac{8!}{7!(8-7)!}\left(x^{\frac{1}{2}}\right)\left(y^{\frac{7}{2}}\right)\right] + \left[\frac{8!}{8!(8-8)!}\left(y^{\frac{8}{2}}\right)\right]$$

$$\Rightarrow [1(x^4)] + \left[8\left(x^{\frac{7}{2}}\right)\left(y^{\frac{1}{2}}\right)\right] + [28(x^3)(y)] + \left[56\left(x^{\frac{5}{2}}\right)\left(y^{\frac{3}{2}}\right)\right]$$

$$+ [70(x^2)(y^2)] + \left[56\left(x^{\frac{3}{2}}\right)\left(y^{\frac{5}{2}}\right)\right] + [28(x^1)(y^3)] + \left[8\left(x^{\frac{1}{2}}\right)\left(y^{\frac{7}{2}}\right)\right] + [1(y^4)]$$

Ans) $(x^4) + 8\left(x^{7/2}\right)\left(y^{1/2}\right) + 28(x^3)(y) + 56\left(x^{5/2}\right)\left(y^{3/2}\right) + 70(x^2)(y^2) + 56\left(x^{3/2}\right)\left(y^{5/2}\right) + 28(x)^1(y)^3 + 8\left(x^{1/2}\right)\left(y^{7/2}\right) + (y)^4$

Question: 9

Solution:

To find: Expansion of

Formula used: (i) $^nC_r = \frac{\left(\sqrt[3]{x}\ n!\sqrt[3]{y}\right)^6}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(\sqrt[3]{x}-\sqrt[3]{y})^6$

We can write $\sqrt[3]{x}$ as $x^{\frac{1}{3}}$ and $\sqrt[3]{y}$ as $y^{\frac{1}{3}}$

Now, we have to solve for $\left(x^{\frac{1}{3}}-y^{\frac{1}{3}}\right)^6$

$$\Rightarrow \left[{}^6C_0\left(x^{\frac{1}{3}}\right)^{6-0}\right] + \left[{}^6C_1\left(x^{\frac{1}{3}}\right)^{6-1}\left(-y^{\frac{1}{3}}\right)^1\right] + \left[{}^6C_2\left(x^{\frac{1}{3}}\right)^{6-2}\left(-y^{\frac{1}{3}}\right)^2\right] + \left[{}^6C_3\left(x^{\frac{1}{3}}\right)^{6-3}\left(-y^{\frac{1}{3}}\right)^3\right] + \left[{}^6C_4\left(x^{\frac{1}{3}}\right)^{6-4}\left(-y^{\frac{1}{3}}\right)^4\right] + \left[{}^6C_5\left(x^{\frac{1}{3}}\right)^{6-5}\left(-y^{\frac{1}{3}}\right)^5\right] + \left[{}^6C_6\left(-y^{\frac{1}{3}}\right)^6\right]$$

$$\Rightarrow \left[{}^6C_0\left(x^{\frac{6}{3}}\right)\right] - \left[{}^6C_1\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right] + \left[{}^6C_2\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] - \left[{}^6C_3\left(x^{\frac{3}{3}}\right)\left(y^{\frac{3}{3}}\right)\right] + \left[{}^6C_4\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{3}}\right)\right] - \left[{}^6C_5\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right] + \left[{}^6C_6\left(y^{\frac{6}{3}}\right)\right]$$

$$\Rightarrow \left[\frac{6!}{0!(6-0)!}(x^2)\right] - \left[\frac{6!}{1!(6-1)!}\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right] + \left[\frac{6!}{2!(6-2)!}\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right]$$

$$- \left[\frac{6!}{3!(6-3)!}(x)(y)\right] + \left[\frac{6!}{4!(6-4)!}\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{3}}\right)\right] - \left[\frac{6!}{5!(6-5)!}\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right]$$

$$+ \left[\frac{6!}{6!(6-6)!}(y^2)\right]$$

$$\Rightarrow [1(x^2)] - \left[6\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right] + \left[15\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] - [20(x)(y)] + \left[15\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{3}}\right)\right] - \left[6\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right] + [1(y^2)]$$

$$\Rightarrow x^2 - 6x^{\frac{5}{3}}y^{\frac{1}{3}} + 15x^{\frac{4}{3}}y^{\frac{2}{3}} - 20xy + 15x^{\frac{2}{3}}y^{\frac{4}{3}} - 6x^{\frac{1}{3}}y^{\frac{5}{3}} + y^2$$

Ans) $x^2 - 6x^{5/3}y^{1/3} + 15x^{4/3}y^{2/3} - 20xy + 15x^{2/3}y^{4/3} - 6x^{1/3}y^{5/3} + y^2$

Question: 10

Solution:

To find: Expansion of $(1 + 2x - 3x^2)^4$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(1 + 2x - 3x^2)^4$

Let $(1+2x) = a$ and $(-3x^2) = b$ ... (i)

Now the equation becomes $(a + b)^4$

$$\Rightarrow [{}^4C_0(a)^{4-0}] + [{}^4C_1(a)^{4-1}(b)^1] + [{}^4C_2(a)^{4-2}(b)^2] + [{}^4C_3(a)^{4-3}(b)^3] + [{}^4C_4(b)^4]$$

$$\Rightarrow [{}^4C_0(a)^4] + [{}^4C_1(a)^3(b)^1] + [{}^4C_2(a)^2(b)^2] + [{}^4C_3(a)(b)^3] + [{}^4C_4(b)^4]$$

(Substituting value of b from eqn. i)

$$\Rightarrow \left[\frac{4!}{0!(4-0)!}(a)^4\right] + \left[\frac{4!}{1!(4-1)!}(a)^3(-3x^2)^1\right] + \left[\frac{4!}{2!(4-2)!}(a)^2(-3x^2)^2\right]$$

$$+ \left[\frac{4!}{3!(4-3)!}(a)(-3x^2)^3\right] + \left[\frac{4!}{4!(4-4)!}(-3x^2)^4\right]$$

(Substituting value of b from eqn. i)

$\Rightarrow [1(1+2x)^4] - [4(1+2x)^3(3x^2)] + [6(1+2x)^2(9x^4)] - [4(1+2x)(27x^6)^3] + [1(81x^8)^4]$ ... (ii)

We need the value of a^4, a^3 and a^2, where a = (1+2x)

For $(1+2x)^4$, Applying Binomial theorem

$(1+2x)^4 \Rightarrow {}^4C_0(1)^{4-0} + {}^4C_1(1)^{4-1}(2x)^1 + {}^4C_2(1)^{4-2}(2x)^2 + {}^4C_3(1)^{4-3}(2x)^3 + {}^4C_4(2x)^4$

$$\Rightarrow \frac{4!}{0!(4-0)!}(1)^4 + \frac{4!}{1!(4-1)!}(1)^3(2x)^1 + \frac{4!}{2!(4-2)!}(1)^2(2x)^2$$

$$+ \frac{4!}{3!(4-3)!}(1)(2x)^3 + \frac{4!}{4!(4-4)!}(2x)^4$$

$\Rightarrow [1] + [4(1)(2x)] + [6(1)(4x^2)] + [4(1)(8x^3)] + [1(16x^4)]$

$\Rightarrow 1 + 8x + 24x^2 + 32x^3 + 16x^4$

We have $(1+2x)^4 = 1 + 8x + 24x^2 + 32x^3 + 16x^4$ ... (iii)

For $(a+b)^3$, we have formula $a^3+b^3+3a^2b+3ab^2$

For, $(1+2x)^3$, substituting a = 1 and b = 2x in the above formula

$\Rightarrow 1^3 + (2x)^3 + 3(1)^2(2x) + 3(1)(2x)^2$

$\Rightarrow 1 + 8x^3 + 6x + 12x^2$

$\Rightarrow 8x^3 + 12x^2 + 6x + 1$ ... (iv)

For $(a+b)^2$, we have formula $a^2+2ab+b^2$

For, $(1+2x)^2$, substituting a = 1 and b = 2x in the above formula

$\Rightarrow (1)^2 + 2(1)(2x) + (2x)^2$

$\Rightarrow 1 + 4x + 4x^2$

$\Rightarrow 4x^2 + 4x + 1$ ... (v)

Putting the value obtained from eqn. (iii),(iv) and (v) in eqn. (ii)

$\Rightarrow 1(1 + 8x + 24x^2 + 32x^3 + 16x^4) - 4(8x^3 + 12x^2 + 6x + 1)(3x^2)$

$+ 6(4x^2 + 4x + 1)(9x^4) - 4(1+2x)(27x^6)^3 + 1(81x^8)$

$\Rightarrow 1(1 + 8x + 24x^2 + 32x^3 + 16x^4) - 4(24x^5 + 36x^4 + 18x^3 + 3x^2)$

$+ 6(36x^6 + 36x^5 + 9x^4) - 4(27x^6 + 54x^7) + 1(81x^8)$

$\Rightarrow 1 + 8x + 24x^2 + 32x^3 + 16x^4 - 96x^5 - 144x^4 - 72x^3 - 12x^2 + 216x^6 + 216x^5 + 54x^4 - 108x^6 - 216x^7 + 81x^8$

On rearranging

Ans) $81x^8 - 216x^7 + 108x^6 + 120x^5 - 74x^4 - 40x^3 + 12x^2 + 8x + 1$

Question: 11

Solution:

To find: Expansion of

$$\left(1+\frac{x}{2}-\frac{2}{x}\right)^4, x \neq 0$$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4$, $x \neq 0$

Let $\left(1+\frac{x}{2}\right)$ = a and $\left(-\frac{2}{x}\right)$= b ... (i)

Now the equation becomes $(a + b)^4$

$\Rightarrow [{}^4C_0(a)^{4-0}] + [{}^4C_1(a)^{4-1}(b)^1] + [{}^4C_2(a)^{4-2}(b)^2] + [{}^4C_3(a)^{4-3}(b)^3] + [{}^4C_4(b)^4]$

$\Rightarrow [{}^4C_0(a)^4] + [{}^4C_1(a)^3(b)^1] + [{}^4C_2(a)^2(b)^2] + [{}^4C_3(a)(b)^3] + [{}^4C_4(b)^4]$

(Substituting value of b from eqn. i)

$$\Rightarrow \left[\frac{4!}{0!(4-0)!}(a)^4\right] + \left[\frac{4!}{1!(4-1)!}(a)^3\left(-\frac{2}{x}\right)^1\right] + \left[\frac{4!}{2!(4-2)!}(a)^2\left(-\frac{2}{x}\right)^2\right] + \left[\frac{4!}{3!(4-3)!}(a)^1\left(-\frac{2}{x}\right)^3\right] + \left[\frac{4!}{4!(4-4)!}\left(-\frac{2}{x}\right)^4\right]$$

(Substituting value of a from eqn. i)

$$\Rightarrow \left[1\left(1+\frac{x}{2}\right)^4\right] - \left[4\left(1+\frac{x}{2}\right)^3\left(\frac{2}{x}\right)\right] + \left[6\left(1+\frac{x}{2}\right)^2\left(\frac{4}{x^2}\right)\right] - \left[4\left(1+\frac{x}{2}\right)^1\left(\frac{8}{x^3}\right)\right] + \left[1\left(\frac{16}{x^4}\right)\right] \ldots\text{(ii)}$$

We need the value of a^4,a^3 and a^2, where a = $\left(1+\frac{x}{2}\right)$

For $\left(1+\frac{x}{2}\right)^4$, Applying Binomial theorem

$$\left(1+\frac{x}{2}\right)^4 = [{}^4C_0(1)^{4-0}] + \left[{}^4C_1(1)^{4-1}\left(\frac{x}{2}\right)^1\right] + \left[{}^4C_2(1)^{4-2}\left(\frac{x}{2}\right)^2\right] + \left[{}^4C_3(1)^{4-3}\left(\frac{x}{2}\right)^3\right] + \left[{}^4C_4\left(\frac{x}{2}\right)^4\right]$$

$$\Rightarrow \left[\frac{4!}{0!(4-0)!}(1)^4\right] + \left[\frac{4!}{1!(4-1)!}(1)^3\left(\frac{x}{2}\right)^1\right] + \left[\frac{4!}{2!(4-2)!}(1)^2\left(\frac{x}{2}\right)^2\right] + \left[\frac{4!}{3!(4-3)!}(1)\left(\frac{x}{2}\right)^3\right] + \left[\frac{4!}{4!(4-4)!}\left(\frac{x}{2}\right)^4\right]$$

$$\Rightarrow [1] + \left[4(1)\left(\frac{x}{2}\right)\right] + \left[6(1)\left(\frac{x^2}{4}\right)\right] + \left[4(1)\left(\frac{x^3}{8}\right)\right] + \left[1\left(\frac{x^4}{16}\right)\right]$$

$$\Rightarrow 1 + 2x + \frac{3}{2}x^2 + \frac{x^3}{2} + \frac{x^4}{16}$$

On rearranging the above eqn.

$$\Rightarrow \frac{1}{16}x^4 + \frac{1}{2}x^3 + \frac{3}{2}x^2 + 2x + 1 \ldots \text{(iii)}$$

We have, $\left(1+\frac{x}{2}\right)^4 = \frac{1}{16}x^4 + \frac{1}{2}x^3 + \frac{3}{2}x^2 + 2x + 1$

For, $(a+b)^3$, we have formula $a^3+b^3+3a^2b+3ab^2$

For, $\left(1+\frac{x}{2}\right)^3$, substituting a = 1 and b = $\frac{x}{2}$ in the above formula

$$\Rightarrow 1^3+\left(\frac{x}{2}\right)^3+3(1)^2\left(\frac{x}{2}\right)+3(1)\left(\frac{x}{2}\right)^2$$

$$\Rightarrow 1+\left(\frac{x^3}{8}\right)+\left(\frac{3x}{2}\right)+\left(\frac{3x^2}{4}\right)$$

$$\Rightarrow \left(\frac{x^3}{8}\right)+\left(\frac{3x^2}{4}\right)+\left(\frac{3x}{2}\right)+1 \ldots (iv)$$

For, $(a+b)^2$, we have formula $a^2+2ab+b^2$

For, $\left(1+\frac{x}{2}\right)^2$, substituting a = 1 and b = $\frac{x}{2}$ in the above formula

$$\Rightarrow (1)^2+2(1)\left(\frac{x}{2}\right)+\left(\frac{x}{2}\right)^2$$

$$\Rightarrow 1+x+\left(\frac{x^2}{4}\right)$$

$$\Rightarrow \frac{x^2}{4}+x+1 \ldots (v)$$

Putting the value obtained from eqn. (iii),(iv) and (v) in eqn. (ii)

$$\Rightarrow \left[1\left(\frac{1}{16}x^4+\frac{1}{2}x^3+\frac{3}{2}x^2+2x+1\right)\right]-\left[4\left(\frac{x^3}{8}+\frac{3x^2}{4}+\frac{3x}{2}+1\right)\left(\frac{2}{x}\right)\right]$$

$$\left[6\left(\frac{x^2}{4}+x+1\right)\left(\frac{4}{x^2}\right)\right]-\left[4\left(1+\frac{x}{2}\right)\left(\frac{8}{x^3}\right)\right]+\left[1\left(\frac{16}{x^4}\right)\right]$$

$$\Rightarrow \frac{1}{16}x^4+\frac{1}{2}x^3+\frac{3}{2}x^2+2x+1-x^2-6x-12-\frac{8}{x}+6+\frac{24}{x}+\frac{24}{x^2}$$

$$-\frac{32}{x^3}-\frac{16}{x^2}+\frac{16}{x^4}$$

On rearranging

Ans) $\frac{1}{16}x^4+\frac{1}{2}x^3+\frac{1}{2}x^2-4x-5+\frac{16}{x}+\frac{8}{x^2}-\frac{32}{x^3}+\frac{16}{x^4}$

Question: 12

Solution:

To find: Expansion of $(3x^2 - 2ax + 3a^2)^3$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(3x^2 - 2ax + 3a^2)^3$

Let, $(3x^2 - 2ax) = p$ ... (i)

The equation becomes $(p + 3a^2)^3$

$$\Rightarrow [{}^3C_0(p)^{3-0}]+[{}^3C_1(p)^{3-1}(3a^2)^1]+[{}^3C_2(p)^{3-2}(3a^2)^2]+[{}^3C_3(3a^2)^3]$$

$$\Rightarrow [{}^3C_0(p)^3]+[{}^3C_1(p)^2(3a^2)]+[{}^3C_2(p)(9a^4)]+[{}^3C_3(27a^6)]$$

Substituting the value of p from eqn. (i)

$$\Rightarrow \left[\frac{3!}{0!(3-0)!}(3x^2-2ax)^3\right]+\left[\frac{3!}{1!(3-1)!}(3x^2-2ax)^2(3a^2)\right]$$

$$+\left[\frac{3!}{2!(3-2)!}(3x^2-2ax)(9a^4)\right]+\left[\frac{3!}{3!(3-3)!}(27a^6)\right]$$

$\Rightarrow [1(3x^2-2ax)^3]+[3(3x^2-2ax)^2(3a^2)]+[3(3x^2-2ax)(9a^4)]+[1(27a^6)^3]$ ... (ii)

We need the value of p^3 and p^2, where $p = 3x^2 - 2ax$

For, $(a+b)^3$, we have formula $a^3+b^3+3a^2b+3ab^2$

For, $(3x^2 - 2ax)^3$, substituting $a = 3x^2$ and $b = -2ax$ in the above formula

$\Rightarrow [(3x^2)^3]+[(-2ax)^3]+[3(3x^2)^2(-2ax)]+[3(3x^2)(-2ax)^2]$

$\Rightarrow 27x^6 - 8a^3x^3 - 54ax^5 + 36a^2x^4$ ... (iii)

For, $(a+b)^2$, we have formula $a^2+2ab+b^2$

For, $(3x^2 - 2ax)^3$, substituting $a = 3x^2$ and $b = -2ax$ in the above formula

$\Rightarrow [(3x^2)^2]+[2(3x^2)(-2ax)]+[(-2ax)^2]$

$\Rightarrow 9x^4 - 12x^3a + 4a^2x^2$ ... (iv)

Putting the value obtained from eqn. (iii) and (iv) in eqn. (ii)

$\Rightarrow [1(27x^6 - 8a^3x^3 - 54ax^5 + 36a^2x^4)] +$
$[3(9x^4 - 12x^3a + 4a^2x^2)(3a^2)] + [3(3x^2 - 2ax)(9a^4)] + [1(27a^6)]$

$\Rightarrow 27x^6 - 8a^3x^3 - 54ax^5 + 36a^2x^4 + 81a^2x^4 - 108x^3a^3 + 36a^4x^2 + 81a^4x^2 - 54a^5x + 27a^6$

On rearranging

Ans) $27x^6 - 54ax^5 + 117a^2x^4 - 116x^3a^3 + 117a^4x^2 - 54a^5x + 27a^6$

Question: 13

Solution:

To find: Value of $(\sqrt{2}+1)^6+(\sqrt{2}-1)^6$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(a+1)^6 = [{}^6C_0a^6]+[{}^6C_1a^{6-1}1]+[{}^6C_2a^{6-2}1^2]+[{}^6C_3a^{6-3}1^3]+[{}^6C_4a^{6-4}1^4]+[{}^6C_5a^{6-5}1^5]+[{}^6C_61^6]$

$\Rightarrow {}^6C_0a^6 + {}^6C_1a^5 + {}^6C_2a^4 + {}^6C_3a^3 + {}^6C_4a^2 + {}^6C_5a + {}^6C_6$ ... (i)

$(a-1)^6 = [{}^6C_0a^6]+[{}^6C_1a^{6-1}(-1)^1]+[{}^6C_2a^{6-2}(-1)^2]+[{}^6C_3a^{6-3}(-1)^3]+[{}^6C_4a^{6-4}(-1)^4]+[{}^6C_5a^{6-5}(-1)^5]+[{}^6C_6(-1)^6]$

$\Rightarrow {}^6C_0a^6 - {}^6C_1a^5 + {}^6C_2a^4 - {}^6C_3a^3 + {}^6C_4a^2 - {}^6C_5a + {}^6C_6$ ... (ii)

Adding eqn. (i) and (ii)

$(a+1)^6 + (a-1)^6 = [{}^6C_0a^6 + {}^6C_1a^5 + {}^6C_2a^4 + {}^6C_3a^3 + {}^6C_4a^2 + {}^6C_5a + {}^6C_6] + [{}^6C_0a^6 - {}^6C_1a^5 + {}^6C_2a^4 - {}^6C_3a^3 + {}^6C_4a^2 - {}^6C_5a + {}^6C_6]$

$\Rightarrow 2[{}^6C_0a^6 + {}^6C_2a^4 + {}^6C_4a^2 + {}^6C_6]$

$\Rightarrow 2\left[\left(\frac{6!}{0!(6-0)!}a^6\right) + \left(\frac{6!}{2!(6-2)!}a^4\right) + \left(\frac{6!}{4!(6-4)!}a^2\right) + \left(\frac{6!}{6!(6-6)!}\right)\right]$

$\Rightarrow 2[(1)a^6 + (15)a^4 + (15)a^2 + (1)]$

$\Rightarrow 2[a^6 + 15a^4 + 15a^2 + 1] = (a+1)^6 + (a-1)^6$

Putting the value of $a = \sqrt{2}$ in the above equation

$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 = 2[(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1]$

$\Rightarrow 2[8 + 15(4) + 15(2) + 1]$

$\Rightarrow 2[8 + 60 + 30 + 1]$

$\Rightarrow 2[99]$

$\Rightarrow 198$

Ans) 198

Question: 14

Solution:

To find: Value of $(\sqrt{3}+1)^5 - (\sqrt{3}-1)^5$

Formula used: (I) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(a+1)^5 = {}^5C_0a^5 + {}^5C_1a^{5-1}1 + {}^5C_2a^{5-2}1^2 + {}^5C_3a^{5-3}1^3 + {}^5C_4a^{5-4}1^4 + {}^5C_51^5$

$\Rightarrow {}^5C_0a^5 + {}^5C_1a^4 + {}^5C_2a^3 + {}^5C_3a^2 + {}^5C_4a + {}^5C_5 \ldots$ (i)

$(a-1)^5 = [{}^5C_0a^5] + [{}^5C_1a^{5-1}(-1)^1] + [{}^5C_2a^{5-2}(-1)^2] + [{}^5C_3a^{5-3}(-1)^3] + [{}^5C_4a^{5-4}(-1)^4] + [{}^5C_5(-1)^5]$

$\Rightarrow {}^5C_0a^5 - {}^5C_1a^4 + {}^5C_2a^3 - {}^5C_3a^2 + {}^5C_4a - {}^5C_5 \ldots$ (ii)

Substracting (ii) from (i)

$(a+1)^5 - (a-1)^5 = [{}^5C_0a^5 + {}^5C_1a^4 + {}^5C_2a^3 + {}^5C_3a^2 + {}^5C_4a + {}^5C_5] - [{}^5C_0a^5 - {}^5C_1a^4 + {}^5C_2a^3 - {}^5C_3a^2 + {}^5C_4a - {}^5C_5]$

$\Rightarrow 2[{}^5C_1a^4 + {}^5C_3a^2 + {}^5C_5]$

$\Rightarrow 2\left[\left(\frac{5!}{1!(5-1)!}a^4\right) + \left(\frac{5!}{3!(5-3)!}a^2\right) + \left(\frac{5!}{5!(5-5)!}\right)\right]$

$\Rightarrow 2[(5)a^4 + (10)a^2 + (1)]$

$\Rightarrow 2[5a^4 + 10a^2 + 1] = (a+1)^5 - (a-1)^5$

Putting the value of $a = \sqrt{3}$ in the above equation

$(\sqrt{3}+1)^5 - (\sqrt{3}-1)^5 = 2[5(\sqrt{3})^4 + 10(\sqrt{3})^2 + 1]$

$\Rightarrow 2[(5)(9) + (10)(3) + 1]$

$\Rightarrow 2[45+30+1]$

$\Rightarrow 152$

Ans) 152

Question: 15

Solution:

To find: Value of $(2+\sqrt{3})^7+(2-\sqrt{3})^7$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(a+b)^7 = [{}^7C_0a^7] + [{}^7C_1a^{7-1}b] + [{}^7C_2a^{7-2}b^2] + [{}^7C_3a^{7-3}b^3] + [{}^7C_4a^{7-4}b^4] + [{}^7C_5a^{7-5}b^5] + [{}^7C_6a^{7-6}b^6] + [{}^7C_7b^7]$

$\Rightarrow {}^7C_0a^7 + {}^7C_1a^6b + {}^7C_2a^5b^2 + {}^7C_3a^4b^3 + {}^7C_4a^3b^4 + {}^7C_5a^2b^5 + {}^7C_6a^1b^6 + {}^7C_7b^7$ ... (i)

$(a-b)^7 = [{}^7C_0a^7] + [{}^7C_1a^{7-1}(-b)] + [{}^7C_2a^{7-2}(-b)^2] + [{}^7C_3a^{7-3}(-b)^3] + [{}^7C_4a^{7-4}(-b)^4] + [{}^7C_5a^{7-5}(-b)^5] + [{}^7C_6a^{7-6}(-b)^6] + [{}^7C_7(-b)^7]$

$\Rightarrow {}^7C_0a^7 - {}^7C_1a^6b + {}^7C_2a^5b^2 - {}^7C_3a^4b^3 + {}^7C_4a^3b^4 - {}^7C_5a^2b^5 + {}^7C_6a^1b^6 - {}^7C_7b^7$ ... (ii)

Adding eqn. (i) and (ii)

$(a+b)^7 + (a-b)^7 = [{}^7C_0a^7 + {}^7C_1a^6b + {}^7C_2a^5b^2 + {}^7C_3a^4b^3 + {}^7C_4a^3b^4 + {}^7C_5a^2b^5 + {}^7C_6a^1b^6 + {}^7C_7b^7] + [{}^7C_0a^7 - {}^7C_1a^6b + {}^7C_2a^5b^2 - {}^7C_3a^4b^3 + {}^7C_4a^3b^4 - {}^7C_5a^2b^5 + {}^7C_6a^1b^6 - {}^7C_7b^7]$

$\Rightarrow 2[{}^7C_0a^7 + {}^7C_2a^5b^2 + {}^7C_4a^3b^4 + {}^7C_6a^1b^6]$

$\Rightarrow 2\left[\left[\frac{7!}{0!(7-0)!}a^7\right] + \left[\frac{7!}{2!(7-2)!}a^5b^2\right] + \left[\frac{7!}{4!(7-4)!}a^3b^4\right] + \left[\frac{7!}{6!(7-6)!}a^1b^6\right]\right]$

$\Rightarrow 2[(1)a^7 + (21)a^5b^2 + (35)a^3b^4 + (7)ab^6]$

$\Rightarrow 2[a^7 + 21a^5b^2 + 35a^3b^4 + 7ab^6] = (a+b)^7 + (a-b)^7$

Putting the value of $a = 2$ and $b = \sqrt{3}$ in the above equation

$(2+\sqrt{3})^7+(2-\sqrt{3})^7$

$= 2\left[\{2^7\} + \{21(2)^5(\sqrt{3})^2\} + \{35(2)^3(\sqrt{3})^4\} + \{7(2)(\sqrt{3})^6\}\right]$

$= 2[128 + 21(32)(3) + 35(8)(9) + 7(2)(27)]$

$= 2[128 + 2016 + 2520 + 378]$

$= 10084$

Ans) 10084

Question: 16

Solution:

To find: Value of $(\sqrt{3}+\sqrt{2})^6-(\sqrt{3}-\sqrt{2})^6$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(a+b)^6 = {}^6C_0a^6 + {}^6C_1a^{6-1}b + {}^6C_2a^{6-2}b^2 + {}^6C_3a^{6-3}b^3 + {}^6C_4a^{6-4}b^4 + {}^6C_5a^{6-5}b^5 + {}^6C_6b^6$

$\Rightarrow {}^6C_0a^6 + {}^6C_1a^5b + {}^6C_2a^4b^2 + {}^6C_3a^3b^3 + {}^6C_4a^2b^4 + {}^6C_5ab^5 + {}^6C_6b^6$ ... (i)

$(a-b)^6 = [{}^6C_0a^6] + [{}^6C_1a^{6-1}(-b)] + [{}^6C_2a^{6-2}(-b)^2] + [{}^6C_3a^{6-3}(-b)^3] + [{}^6C_4a^{6-4}(-b)^4] + [{}^6C_5a^{6-5}(-b)^5] + [{}^6C_6(-b)^6]$

$\Rightarrow {}^6C_0a^6 - {}^6C_1a^5b + {}^6C_2a^4b^2 - {}^6C_3a^3b^3 + {}^6C_4a^2b^4 - {}^6C_5ab^5 + {}^6C_6b^6$ ... (ii)

Substracting (ii) from (i)

$(a+b)^6 - (a-b)^6 = [{}^6C_0a^6 + {}^6C_1a^5b + {}^6C_2a^4b^2 + {}^6C_3a^3b^3 + {}^6C_4a^2b^4 + {}^6C_5ab^5 + {}^6C_6b^6] - [{}^6C_0a^6 - {}^6C_1a^5b + {}^6C_2a^4b^2 - {}^6C_3a^3b^3 + {}^6C_4a^2b^4 - {}^6C_5ab^5 + {}^6C_6b^6]$

$= 2[{}^6C_1a^5b + {}^6C_3a^3b^3 + {}^6C_5ab^5]$

$= 2\left[\left\{\frac{6!}{1!(6-1)!}a^5a\right\} + \left\{\frac{6!}{3!(6-3)!}a^3b^3\right\} + \left\{\frac{6!}{5!(6-5)!}ab^5\right\}\right]$

$= 2[(6)a^5b + (20)a^3b^3 + (6)ab^5]$

$\Rightarrow (a+b)^6 - (a-b)^6 = 2[(6)a^5b + (20)a^3b^3 + (6)ab^5]$

Putting the value of $a = \sqrt{3}$ and $b = \sqrt{2}$ in the above equation

$\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6$

$\Rightarrow 2\left[(6)(\sqrt{3})^5(\sqrt{2}) + (20)(\sqrt{3})^3(\sqrt{2})^3 + (6)(\sqrt{3})(\sqrt{2})^5\right]$

$\Rightarrow 2[54(\sqrt{6}) + 120(\sqrt{6}) + 24(\sqrt{6})]$

$\Rightarrow 396\sqrt{6}$

Ans) $396\sqrt{6}$

Question: 17

Prove that

Solution:

To prove: $\sum_{r=0}^{n} {}^nC_r \cdot 3^r = 4^n$

Formula used: $\sum_{r=0}^{n} {}^nC_r \cdot a^{n-r} b^r = (a+b)^n$

Proof: In the above formula if we put a = 1 and b = 3, then we will get $\sum_{r=0}^{n} {}^nC_r \cdot 1^{n-r} 3^r = (1+3)^n$

Therefore,

$$\sum_{r=0}^{n} {}^nC_r \cdot 3^r = (4)^n$$

Hence Proved.

Question: 18

Using binominal t

Solution:

(i) $(101)^4$

To find: Value of $(101)^4$

Formula used: (i) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

101 = (100+1)

Now $(101)^4 = (100+1)^4$

$(100+1)^4 = [{}^4C_0(100)^{4-0}] + [{}^4C_1(100)^{4-1}(1)^1] + [{}^4C_2(100)^{4-2}(1)^2] + [{}^4C_3(100)^{4-3}(1)^3] + [{}^4C_4(1)^4]$

$\Rightarrow [{}^4C_0(100)^4] + [{}^4C_1(100)^3(1)^1] + [{}^4C_2(100)^2(1)^2] + [{}^4C_3(100)^1(1)^3] + [{}^4C_4(1)^4]$

$\Rightarrow \left[\frac{4!}{0!(4-0)!}(100000000)\right] + \left[\frac{4!}{1!(4-1)!}(1000000)\right] + \left[\frac{4!}{2!(4-2)!}(10000)\right] + \left[\frac{4!}{3!(4-3)!}(100)\right] + \left[\frac{4!}{4!(4-4)!}(1)\right]$

$\Rightarrow [(1)(100000000)] + [(4)(1000000)] + [(6)(10000)] + [(4)(100)] + [(1)(1)]$

= 104060401

Ans) 104060401

(ii) $(98)^4$

To find: Value of $(98)^4$

Formula used: (I) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

98 = (100-2)

Now $(98)^4 = (100-2)^4$

$(100-2)^4 = [{}^4C_0(100)^{4-0}] + [{}^4C_1(100)^{4-1}(-2)^1] + [{}^4C_2(100)^{4-2}(-2)^2] + [{}^4C_3(100)^{4-3}(-2)^3] + [{}^4C_4(-2)^4]$

$\Rightarrow [{}^4C_0(100)^4] - [{}^4C_1(100)^3(2)] + [{}^4C_2(100)^2(4)] - [{}^4C_3(100)^1(8)] + [{}^4C_4(16)]$

$\Rightarrow \left[\frac{4!}{0!(4-0)!}(100000000)\right] - \left[\frac{4!}{1!(4-1)!}(1000000)(2)\right] + \left[\frac{4!}{2!(4-2)!}(10000)(4)\right] - \left[\frac{4!}{3!(4-3)!}(100)(8)\right] + \left[\frac{4!}{4!(4-4)!}(16)\right]$

$\Rightarrow [(1)(100000000)] - [(4)(1000000)(2)] + [(6)(10000)(4)] - [(4)(100)(8)] + [(1)(16)]$

= 92236816

Ans) 92236816

(iii) $(1.2)^4$

To find: Value of $(1.2)^4$

Formula used: (i) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

1.2 = (1 + 0.2)

Now $(1.2)^4 = (1 + 0.2)^4$

$(1+0.2)^4 = [{}^4C_0(1)^{4-0}] + [{}^4C_1(1)^{4-1}(0.2)^1] + [{}^4C_2(1)^{4-2}(0.2)^2] + [{}^4C_3(1)^{4-3}(0.2)^3] + [{}^4C_4(0.2)^4]$

$\Rightarrow [{}^4C_0(1)^4] + [{}^4C_1(1)^3(0.2)^1] + [{}^4C_2(1)^2(0.2)^2] + [{}^4C_3(1)^1(0.2)^3] + [{}^4C_4(0.2)^4]$

$\Rightarrow \left[\frac{4!}{0!(4-0)!}(1)\right] + \left[\frac{4!}{1!(4-1)!}(1)(0.2)\right] + \left[\frac{4!}{2!(4-2)!}(1)(0.04)\right] + \left[\frac{4!}{3!(4-3)!}(1)(0.008)\right] + \left[\frac{4!}{4!(4-4)!}(0.0016)\right]$

$\Rightarrow [(1)(1)] + [(4)(1)(0.2)] + [(6)(1)(0.04)] + [(4)(1)(0.008)] + [(1)(0.0016)]$

$= 2.0736$

Ans) 2.0736

Question: 19

Solution:

To prove: $(2^{3n} - 7n - 1)$ is divisible by 49, where n N

Formula used: $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(2^{3n} - 7n - 1) = (2^3)^n - 7n - 1$

$\Rightarrow 8^n - 7n - 1$

$\Rightarrow (1+7)^n - 7n - 1$

$\Rightarrow {}^nC_01^n + {}^nC_11^{n-1}7 + {}^nC_21^{n-2}7^2 + \ldots\ldots + {}^nC_{n-1}7^{n-1} + {}^nC_n7^n - 7n - 1$

$\Rightarrow {}^nC_0 + {}^nC_17 + {}^nC_27^2 + \ldots\ldots + {}^nC_{n-1}7^{n-1} + {}^nC_n7^n - 7n - 1$

$\Rightarrow 1 + 7n + 7^2[{}^nC_2 + {}^nC_37 + \ldots + {}^nC_{n-1}\,7^{n-3} + {}^nC_n\,7^{n-2}] - 7n - 1$

$\Rightarrow 7^2[{}^nC_2 + {}^nC_37 + \ldots + {}^nC_{n-1}\,7^{n-3} + {}^nC_n\,7^{n-2}]$

$\Rightarrow 49[{}^nC_2 + {}^nC_37 + \ldots + {}^nC_{n-1}\,7^{n-3} + {}^nC_n\,7^{n-2}]$

$\Rightarrow$ 49K, where $K = ({}^nC_2 + {}^nC_37 + \ldots + {}^nC_{n-1}\,7^{n-3} + {}^nC_n7^{n-2})$

Now, $(2^{3n} - 7n - 1) = 49K$

Therefore $(2^{3n} - 7n - 1)$ is divisible by 49

Question: 20

Solution:

To prove: $(2+\sqrt{x})^4 + (2-\sqrt{x})^4 = 2(16+24x+x^2)$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(a+b)^4 = {}^4C_0a^4 + {}^4C_1a^{4-1}b + {}^4C_2a^{4-2}b^2 + {}^4C_3a^{4-3}b^3 + {}^4C_4b^4$

$\Rightarrow {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3a^1b^3 + {}^4C_4b^4$ ... (i)

$(a-b)^4 = {}^4C_0a^4 + {}^4C_1a^{4-1}(-b) + {}^4C_2a^{4-2}(-b)^2 + {}^4C_3a^{4-3}(-b)^3 + {}^4C_4(-b)^4$

$\Rightarrow {}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4$ ... (ii)

Adding (i) and (ii)

$(a+b)^4 + (a-b)^7 = [{}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3a^1b^3 + {}^4C_4b^4] + [{}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4]$

$\Rightarrow 2[{}^4C_0a^4 + {}^4C_2a^2b^2 + {}^4C_4b^4]$

$\Rightarrow 2\left[\left(\frac{4!}{0!(4-0)!}a^4\right) + \left(\frac{4!}{2!(4-2)!}a^2b^2\right) + \left(\frac{4!}{4!(4-4)!}b^4\right)\right]$

$\Rightarrow 2[(1)a^4 + (6)a^2b^2 + (1)b^4]$

$\Rightarrow 2[a^4 + 6a^2b^2 + b^4]$

Therefore, $(a+b)^4 + (a-b)^7 = 2[a^4 + 6a^2b^2 + b^4]$

Now, putting a = 2 and b = $(\sqrt{x})$ in the above equation.

$(2+\sqrt{x})^4 + (2-\sqrt{x})^4 = 2[(2)^4 + 6(2)^2(\sqrt{x})^2 + (\sqrt{x})^4]$

$= 2(16+24x+x^2)$

Hence proved.

Question: 21

Solution:

To find: 7^{th} term in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x}\right)^8$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

For 7^{th} term, r+1=7

$\Rightarrow r = 6$

In, $\left(\frac{4x}{5} + \frac{5}{2x}\right)^8$

7^{th} term = T_{6+1}

$\Rightarrow {}^8C_6\left(\frac{4x}{5}\right)^{8-6}\left(\frac{5}{2x}\right)^6$

$\Rightarrow \frac{8!}{6!(8-6)!}\left(\frac{4x}{5}\right)^2\left(\frac{5}{2x}\right)^6$

$\Rightarrow (28)\left(\frac{16x^2}{25}\right)\left(\frac{15625}{64x^6}\right)$

$\Rightarrow \frac{4375}{x^4}$

Ans) $\frac{4375}{x^4}$

Question: 22

Solution:

To find: 9^{th} term in the expansion of $\left(\frac{a}{b}-\frac{b}{2a^2}\right)^{12}$

Formula used: (i) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

For 9^{th} term, r+1=9

$\Rightarrow r = 8$

In, $\left(\frac{a}{b}-\frac{b}{2a^2}\right)^{12}$

9^{th} term = T_{8+1}

$\Rightarrow {}^{12}C_8\left(\frac{a}{b}\right)^{12-8}\left(\frac{-b}{2a^2}\right)^8$

$\Rightarrow \frac{12!}{8!(12-8)!}\left(\frac{a}{b}\right)^4\left(\frac{-b}{2a^2}\right)^8$

$\Rightarrow 495\left(\frac{a^4}{b^4}\right)\left(\frac{b^8}{256a^{16}}\right)$

$\Rightarrow \left(\frac{495b^4}{256a^{12}}\right)$

Ans) $\left(\frac{495\,b^4}{256\,a^{12}}\right)$

Question: 23

Solution:

To find: 16^{th} term in the expansion of $(\sqrt{x}-\sqrt{y})^{17}$

Formula used: (i) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

For 16^{th} term, r+1=16

$\Rightarrow r = 15$

In, $(\sqrt{x}-\sqrt{y})^{17}$

16^{th} term = T_{15+1}

$\Rightarrow {}^{17}C_{15}\,(\sqrt{x})^{17-15}(-\sqrt{y})^{15}$

$\Rightarrow \frac{17!}{15!(17-15)!}(\sqrt{x})^2(-\sqrt{y})^{15}$

$\Rightarrow 136(x)(-y)^{\frac{15}{2}}$

$\Rightarrow -136xy^{\frac{15}{2}}$Ans) $-136xy^{\frac{15}{2}}$

Question: 24

Solution:

To find: 13^{th} term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$

Formula used: (i) ${}^{n}C_{r} = \frac{n!}{(n-r)!(r)!}$

(ii) $T_{r+1} = {}^{n}C_{r}\, a^{n-r}\, b^{r}$

For 13^{th} term, r+1=13

$\Rightarrow r = 12$

In, $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$

13^{th} term = T_{12+1}

$\Rightarrow {}^{18}C_{12}\,(9x)^{18-12}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$

$\Rightarrow \frac{18!}{12!(18-12)!}(9x)^{6}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$

$\Rightarrow 18564(531441x^{6})\left(\frac{1}{531441x^{6}}\right)$

$\Rightarrow 18564$

Question: 25

Solution:

<u>To find</u> : coefficients of x^7 and x^8

<u>Formula :</u> $t_{r+1} = \binom{n}{r} a^{n-r} b^{r}$

Here, a=2, $b = \frac{x}{3}$

We have, $t_{r+1} = \binom{n}{r} a^{n-r} b^{r}$

$\therefore t_{r+1} = \binom{n}{r}(2)^{n-r}\left(\frac{x}{3}\right)^{r}$

$= \binom{n}{r}\frac{2^{n-r}}{3^{r}}x^{r}$

To get a coefficient of x^7, we must have,

$x^7 = x^r$

• r = 7

Therefore, the coefficient of $x^7 = \binom{n}{7}\frac{2^{n-7}}{3^{7}}$

And to get the coefficient of x^8 we must have,

$x^8 = x^r$

• r = 8

Therefore, the coefficient of $x^8 = \binom{n}{8}\frac{2^{n-8}}{3^{8}}$

<u>Conclusion</u> :

• coefficient of $x^7 = \binom{n}{7}\frac{2^{n-7}}{3^{7}}$

- coefficient of $x^8 = \binom{n}{8}\frac{2^{n-8}}{3^8}$

Question: 26

Solution:

To Find: the ratio of the coefficient of x^{15} to the term independent of x

Formula : $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

Here, $a=x^2$, $b = \frac{2}{x}$ and $n=15$

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{15}{r} (x^2)^{15-r} \left(\frac{2}{x}\right)^r$

$= \binom{15}{r} (x)^{30-2r} (2)^r (x)^{-r}$

$= \binom{15}{r} (x)^{30-2r-r} (2)^r$

$= \binom{15}{r} (2)^r (x)^{30-3r}$

To get coefficient of x^{15} we must have,

$(x)^{30-3r} = x^{15}$

- $30 - 3r = 15$
- $3r = 15$
- $r = 5$

Therefore, coefficient of $x^{15} = \binom{15}{5} (2)^5$

Now, to get coefficient of term independent of xthat is coefficient of x^0 we must have,

$(x)^{30-3r} = x^0$

- $30 - 3r = 0$
- $3r = 30$
- $r = 10$

Therefore, coefficient of $x^0 = \binom{15}{10} (2)^{10}$

But $\binom{15}{10} = \binom{15}{5}$ ....$[\because \binom{n}{r} = \binom{n}{n-r}]$

Therefore, the coefficient of $x^0 = \binom{15}{5} (2)^{10}$

Therefore,

$$\frac{\text{coefficient of } x^{15}}{\text{coefficient of } x^0} = \frac{\binom{15}{5} (2)^5}{\binom{15}{5} (2)^{10}}$$

$$= \frac{1}{(2)^5}$$

$= \frac{1}{32}$

Hence, coefficient of x^{15} : coefficient of x^0 = 1:32

<u>Conclusion</u> : The ratio of coefficient of x^{15} to coefficient of x^0 = 1:32

Question: 27

Solution:

<u>To Prove</u> : coefficient of x^{10} in $(1\text{-}x^2)^{10}$: coefficient of x^0 in $\left(x - \frac{2}{x}\right)^{10}$ = 1:32

For $(1\text{-}x^2)^{10}$,

Here, a=1, b=$-x^2$ and n=15

We have formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{10}{r} (1)^{10-r}(-x^2)^r$

$= - \binom{10}{r} (1) (x)^{2r}$

To get coefficient of x^{10} we must have,

$(x)^{2r} = x^{10}$

• 2r = 10

• r = 5

Therefore, coefficient of $x^{10} = -\binom{10}{5}$

For $\left(x - \frac{2}{x}\right)^{10}$,

Here, a=x, $b = \frac{-2}{x}$ and n=10

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{10}{r} (x)^{10-r}\left(\frac{-2}{x}\right)^r$

$= \binom{10}{r} (x)^{10-r} (-2)^r (x)^{-r}$

$= \binom{10}{r} (x)^{10-r-r} (-2)^r$

$= \binom{10}{r} (-2)^r (x)^{10-2r}$

Now, to get coefficient of term independent of xthat is coefficient of x^0 we must have,

$(x)^{10\text{-}2r} = x^0$

• 10 - 2r = 0

• 2r = 10

- r = 5

Therefore, coefficient of $x^0 = -\binom{10}{5}(2)^5$

Therefore,

$$\frac{\text{coefficient of } x^{10} \text{ in } (1-x^2)^{10}}{\text{coefficient of } x^0 \text{ in } \left(x-\frac{2}{x}\right)^{10}} = \frac{-\binom{15}{5}}{-\binom{15}{5}(2)^5}$$

$$= \frac{1}{(2)^5}$$

$$= \frac{1}{32}$$

Hence,

coefficient of x^{10} in $(1\text{-}x^2)^{10}$: coefficient of x^0 in $\left(x-\frac{2}{x}\right)^{10}$ = 1:32

Question: 28

Solution:

To Find : term independent of x, i.e. coefficient of x^0

Formula : $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Therefore, the expansion of $\left(x-\frac{2}{x}\right)^{10}$ is given by,

$$\left(x-\frac{2}{x}\right)^{10} = \sum_{r=0}^{10}\binom{10}{r}(x)^{10-r}\left(\frac{-2}{x}\right)^r$$

$$= \binom{10}{0}(x)^{10}\left(\frac{-2}{x}\right)^0 + \binom{10}{1}(x)^9\left(\frac{-2}{x}\right)^1 + \binom{10}{2}(x)^8\left(\frac{-2}{x}\right)^2 + \cdots\ldots\ldots + \binom{10}{10}(x)^0\left(\frac{-2}{x}\right)^{10}$$

$$= x^{10} + \binom{10}{1}(x)^9(-2)\frac{1}{x} + \binom{10}{2}(x)^8(-2)^2\frac{1}{x^2} + \cdots + \binom{10}{10}(x)^0(-2)^{10}\frac{1}{x^{10}}$$

$$= x^{10} - (2)\binom{10}{1}(x)^8 + (2)^2\binom{10}{2}(x)^6 + \cdots\ldots\ldots + (2)^{10}\binom{10}{10}\frac{1}{x^{10}}$$

Now,

$$(91 + x + 2x^3)\left(x-\frac{2}{x}\right)^{10}$$

$$= (91 + x + 2x^3)\left(x^{10} - (2)\binom{10}{1}(x)^8 + (2)^2\binom{10}{2}(x)^6 + \cdots\ldots\ldots + (2)^{10}\binom{10}{10}\frac{1}{x^{10}}\right)$$

Multiplying the second bracket by 91 , x and $2x^3$

$$= \left\{91x^{10} - 91(2)\binom{10}{1}(x)^8 + 91(2)^2\binom{10}{2}(x)^6 + \cdots + 91(2)^{10}\binom{10}{10}\frac{1}{x^{10}}\right\}$$
$$+ \left\{x.x^{10} - x.(2)\binom{10}{1}(x)^8 + x.(2)^2\binom{10}{2}(x)^6 + \cdots \ldots \ldots + x.(2)^{10}\binom{10}{10}\frac{1}{x^{10}}\right\}$$
$$+ \left\{2x^3.x^{10} - 2x^3.(2)\binom{10}{1}(x)^8 + 2x^3.(2)^2\binom{10}{2}(x)^6 + \cdots \ldots \ldots + 2x^3.(2)^{10}\binom{10}{10}\frac{1}{x^{10}}\right\}$$

In the first bracket, there will be a 6^{th} term of x^0 having coefficient $91(-2)^5\binom{10}{5}$

While in the second and third bracket, the constant term is absent.

Therefore, the coefficient of term independent of x, i.e. constant term in the above expansion

$$= 91(-2)^5\binom{10}{5}$$

$$= -91.(2)^5\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}$$

$=-91(2)^5\ (252)$

Conclusion : coefficient of term independent of x $= -91(2)^5\ (252)$

Question: 29

Solution:

To Find : coefficient of x

Formula : $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Therefore, expansion of $(1\text{-}x)^{16}$ is given by,

$$(1-x)^{16} = \sum_{r=0}^{16}\binom{16}{r}(1)^{16-r}(-x)^r$$

$$= \binom{16}{0}(1)^{16}(-x)^0 + \binom{16}{1}(1)^{15}(-x)^1 + \binom{16}{2}(1)^{14}(-x)^2 + \cdots \ldots \ldots + \binom{16}{16}(1)^0(-x)^{16}$$

$$= 1 - \binom{16}{1}x + \binom{16}{2}x^2 + \cdots \ldots \ldots + \binom{16}{16}x^{16}$$

Now,

$$(1-3x+7x^2)(1-x)^{16} = (1-3x+7x^2)\left(1 - \binom{16}{1}x + \binom{16}{2}x^2 + \cdots \ldots \ldots + \binom{16}{16}x^{16}\right)$$

Multiplying the second bracket by 1 , (-3x) and $7x^2$

$$= \left(1 - \binom{16}{1} x + \binom{16}{2} x^2 + \cdots \ldots\ldots + \binom{16}{16} x^{16}\right)$$
$$+ \left(-3x + 3x\binom{16}{1} x - 3x\binom{16}{2} x^2 + \cdots \ldots\ldots - 3x\binom{16}{16} x^{16}\right)$$
$$+ \left(7x^2 - 7x^2\binom{16}{1} x + 7x^2\binom{16}{2} x^2 + \cdots \ldots\ldots + 7x^2\binom{16}{16} x^{16}\right)$$

In the above equation terms containing x are

$-\binom{16}{1}$ x and -3x

Therefore, the coefficient of x in the above expansion

$$= -\binom{16}{1} - 3$$

=-16-3

=-19

<u>Conclusion</u> : coefficient of x =-19

Question: 30

Solution:

(i) Here, a=x, b=3 and n=8

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{8}{r} (x)^{8-r}(3)^r$$

$$= \binom{8}{r} (3)^r (x)^{8-r}$$

To get coefficient of x^5 we must have,

$(x)^{8-r} = x^5$

- 8 - r = 5
- r = 3

Therefore, coefficient of $x^5 = \binom{8}{3}(3)^3$

$$= \frac{8 \times 7 \times 6}{3 \times 2 \times 1} \cdot (27)$$

= 1512

(ii) Here, $a=3x^2$, $b = \frac{-1}{3x}$ and n=9

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{9}{r} (3x^2)^{9-r}\left(\frac{-1}{3x}\right)^r$$

$$= \binom{9}{r} (3)^{9-r} (x^2)^{9-r} \left(\frac{-1}{3}\right)^r (x)^{-r}$$

$= \binom{9}{r} (3)^{9-r} (x)^{18-2r} \left(\frac{-1}{3}\right)^r (x)^{-r}$

$= \binom{9}{r} (3)^{9-r} (x)^{18-2r-r} \left(\frac{-1}{3}\right)^r$

$= \binom{9}{r} (3)^{9-r} \left(\frac{-1}{3}\right)^r (x)^{18-3r}$

To get coefficient of x^6 we must have,

$(x)^{18-3r} = x^6$

- $18 - 3r = 6$
- $3r = 12$
- $r = 4$

Therefore, coefficient of $x^6 = \binom{9}{4} (3)^{9-4} \left(\frac{-1}{3}\right)^4$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} . (3)^5 \left(\frac{1}{3}\right)^4$

$= 126 \times 3$

$= 378$

(iii) Here, $a=3x^2$, $b = \frac{-a}{3x^3}$ and $n=10$

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{10}{r} (3x^2)^{10-r} \left(\frac{-a}{3x^3}\right)^r$

$= \binom{10}{r} (3)^{10-r} (x^2)^{10-r} \left(\frac{-a}{3}\right)^r (x)^{-3r}$

$= \binom{10}{r} (3)^{10-r} (x)^{20-2r} \left(\frac{-a}{3}\right)^r (x)^{-3r}$

$= \binom{10}{r} (3)^{10-r} (x)^{20-2r-3r} \left(\frac{-a}{3}\right)^r$

$= \binom{10}{r} (3)^{10-r} \left(\frac{-a}{3}\right)^r (x)^{20-5r}$

To get coefficient of x^{-15} we must have,

$(x)^{20-5r} = x^{-15}$

- $20 - 5r = -15$
- $5r = 35$
- $r = 7$

Therefore, coefficient of $x^{-15} = \binom{10}{7} (3)^{10-7} \left(\frac{-a}{3}\right)^7$

But $\binom{10}{7} = \binom{10}{3}$ .... $\left[\because \binom{n}{r} = \binom{n}{n-r}\right]$

Therefore, qthe coefficient of $x^{-15} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} . (3)^3 \left(\frac{-a}{3}\right)^7$

$= 120 \, . \, (-a)^7 \left(\frac{1}{3}\right)^4$

$= (-a)^7 \frac{120}{3^4}$

$= (-a)^7 \frac{40}{27}$

(iv) Here, a=a, b=-2b and n=12

We have formula,

$t_{r+1} = \binom{n}{r} \, a^{n-r} \, b^r$

$= \binom{12}{r} \, (a)^{12-r}(-2b)^r$

$= \binom{12}{r} \, (-2)^r \, (a)^{12-r} \, (b)^r$

To get coefficient of a^7b^5 we must have,

$(a)^{12-r} \, (b)^r = a^7b^5$

• r = 5

Therefore, coefficient of $a^7b^5 = \binom{12}{5} \, (-2)^5$

$= \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} . (-32)$

= 792. (-32)

= -25344

Question: 31

Solution:

For $\left(3x - \frac{1}{2x}\right)^8$,

a=3x, $b = \frac{-1}{2x}$ and n=8

We have a formula,

$t_{r+1} = \binom{n}{r} \, a^{n-r} \, b^r$

$= \binom{8}{r} \, (3x)^{8-r} \left(\frac{-1}{2x}\right)^r$

$= \binom{8}{r} \, (3)^{8-r} \, (x)^{8-r} \, \left(\frac{-1}{2}\right)^r (x)^{-r}$

$= \binom{8}{r} \, (3)^{8-r} \, (x)^{8-r-r} \, \left(\frac{-1}{2}\right)^r$

$= \binom{8}{r} \, (3)^{8-r} \left(\frac{-1}{2}\right)^r \, (x)^{8-2r}$

To get coefficient of x^3 we must have,

$(x)^{8-2r} = (x)^3$

• 8 - 2r = 3

- $2r = 5$
- $r = 2.5$

As $\binom{8}{r} = \binom{8}{2.5}$ is not possible

Therefore, the term containing x^3 does not exist in the expansion of $\left(3x - \frac{1}{2x}\right)^8$

Question: 32

Solution:

For $\left(2x^2 - \frac{1}{x}\right)^{20}$,

$a=2x^2$, $b = \frac{-1}{x}$ and $n=20$

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{20}{r} (3x^2)^{20-r} \left(\frac{-1}{x}\right)^r$$

$$= \binom{20}{r} (3)^{20-r} (x^2)^{20-r} (-1)^r (x)^{-r}$$

$$= \binom{20}{r} (3)^{20-r} (x)^{40-2r} (-1)^r (x)^{-r}$$

$$= \binom{20}{r} (3)^{20-r} (x)^{40-2r-r} (-1)^r$$

$$= \binom{20}{r} (3)^{20-r} (-1)^r (x)^{40-3r}$$

To get coefficient of x^9 we must have,

$(x)^{40-3r} = (x)^9$

- $40 - 3r = 9$
- $3r = 31$
- $r = 10.3333$

As $\binom{20}{r} = \binom{20}{10.3333}$ is not possible

Therefore, the term containing x^9 does not exist in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$

Question: 33

Solution:

For $\left(x^2 + \frac{1}{x}\right)^{12}$,

$a=x^2$, $b = \frac{1}{x}$ and $n=12$

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$= \binom{12}{r} (x^2)^{12-r} \left(\frac{1}{x}\right)^r$

$= \binom{12}{r} (x)^{24-2r} (x)^{-r}$

$= \binom{12}{r} (x)^{24-2r-r}$

$= \binom{12}{r} (x)^{24-3r}$

To get coefficient of x^{-1} we must have,

$(x)^{24-3r} = (x)^{-1}$

- $24 - 3r = -1$
- $3r = 25$
- $r = 8.3333$

As $\binom{20}{r} = \binom{20}{8.3333}$ is not possible

Therefore, the term containing x^{-1} does not exist in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$

Question: 34

Solution:

To Find : General term, i.e. t_{r+1}

For $(x^2 - y)^6$

$a=x^2$, $b=-y$ and $n=6$

General term t_{r+1} is given by,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{6}{r} (x^2)^{6-r}(-y)^r$

Conclusion : General term $= \binom{6}{r} (x^2)^{6-r}(-y)^r$

Question: 35

Solution:

To Find : 5^{th} term from the end

Formulae :

- $t_{r+1} = \binom{n}{r} a^{n-r} b^r$
- $\binom{n}{r} = \binom{n}{n-r}$

For $\left(x - \frac{1}{x}\right)^{12}$,

$a=x$, $b = \frac{-1}{x}$ and $n=12$

As n=12 ,therefore there will be total (12+1)=13 terms in the expansion

Therefore,

5^{th} term from the end = $(13-5+1)^{th}$ i.e. 9^{th} term from the starting.

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

For t_9, r=8

$\therefore t_9 = t_{8+1}$

$= \binom{12}{8} (x)^{12-8} \left(\frac{-1}{x}\right)^8$

$= \binom{12}{4} (x)^4 (x)^{-8}$ ....$[\because \binom{n}{r} = \binom{n}{n-r}]$

$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} (x)^{4-8}$

$= 495\ (x)^{-4}$

Therefore, a 5^{th} term from the end $= 495\ (x)^{-4}$

Conclusion : 5^{th} term from the end $= 495\ (x)^{-4}$

Question: 36

Solution:

To Find : 4^{th} term from the end

Formulae :

- $t_{r+1} = \binom{n}{r} a^{n-r} b^r$
- $\binom{n}{r} = \binom{n}{n-r}$

For $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$,

$a = \frac{4x}{5}$, $b = \frac{-5}{2x}$ and n=9

As n=9 ,therefore there will be total (9+1)=10 terms in the expansion

Therefore,

4^{th} term from the end = $(10-4+1)^{th}$, i.e. 7^{th} term from the starting.

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

For t_7, r=6

$\therefore t_7 = t_{6+1}$

$= \binom{10}{6} \left(\frac{4x}{5}\right)^{10-6} \left(\frac{-5}{2x}\right)^6$

$= \binom{10}{4} \left(\frac{4x}{5}\right)^4 \left(\frac{-5}{2x}\right)^6$ ....$[\because \binom{n}{r} = \binom{n}{n-r}]$

$= \binom{10}{4} \frac{(4)^4}{(5)^4} (x)^4 \frac{(-5)^6}{(2)^6} (x)^{-6}$

$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} (100)\ (x)^{-2}$

$= 21000\ (x)^{-2}$

Therefore, a 4^{th} term from the end $= 21000\ (x)^{-2}$

Conclusion : 4^{th} term from the end $= 21000\ (x)^{-2}$

Question: 37

Solution:

To Find :

I. 4^{th} term from the beginning

II. 4^{th} term from the end

Formulae :

- $t_{r+1} = \binom{n}{r} a^{n-r} b^{r}$
- $\binom{n}{r} = \binom{n}{n-r}$

For $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^{n}$,

$a = \sqrt[3]{2}$, $b = \frac{1}{\sqrt[3]{3}}$ and n=9

As n=n ,therefore there will be total (n+1) terms in the expansion

Therefore,

I. For the 4^{th} term from the starting.

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^{r}$

For t_4, r=3

$\therefore\ t_4 = t_{3+1}$

$= \binom{n}{3} (\sqrt[3]{2})^{n-3} \left(\frac{1}{\sqrt[3]{3}}\right)^{3}$

$= \binom{n}{3} (2)^{\frac{n-3}{3}} \frac{1}{3}$

$= \binom{n}{3} . \frac{(2)^{\frac{n-3}{3}}}{3}$

$= \frac{n!}{(n-3)! \times 3!} . \frac{(2)^{\frac{n-3}{3}}}{3}$

Therefore, a 4^{th} term from the starting $= \frac{n!}{(n-3)! \times 3!} . \frac{(2)^{\frac{n-3}{3}}}{3}$

Now,

II. For the 4^{th} term from the end

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

For $t_{(n-2)}$, r = (n-2)-1 = (n-3)

$\therefore\ t_{(n-2)} = t_{(n-3)+1}$

$= \binom{n}{n-3} (\sqrt[3]{2})^{n-(n-3)} \left(\frac{1}{\sqrt[3]{3}}\right)^{(n-3)}$

$= \binom{n}{3} (\sqrt[3]{2})^3 (3)^{\frac{-(n-3)}{3}}$ ....$\left[\because \binom{n}{r} = \binom{n}{n-r}\right]$

$= \binom{n}{4} (2) (3)^{\frac{3-n}{3}}$

$= \frac{n!}{(n-4)! \times 4!} (2) (3)^{\frac{3-n}{3}}$

Therefore, a 4^{th} term from the end $= \frac{n!}{(n-4)! \times 4!} (2) (3)^{\frac{3-n}{3}}$

<u>Conclusion</u> :

I. 4^{th} term from the beginning $= \frac{n!}{(n-3)! \times 3!} \cdot \frac{(2)^{\frac{n-3}{3}}}{3}$

II. 4^{th} term from the end $= \frac{n!}{(n-4)! \times 4!} (2) (3)^{\frac{3-n}{3}}$

Question: 38

Solution:

(i) For $(3 + x)^6$,

a=3, b=x and n=6

As n is even, $\left(\frac{n+2}{2}\right)^{th}$ is the middle term

Therefore, the middle term $= \left(\frac{6+2}{2}\right)^{th} = \left(\frac{8}{2}\right)^{th} = (4)^{th}$

General term t_{r+1} is given by,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

Therefore, for 4^{th}, r=3

Therefore, the middle term is

$t_4 = t_{3+1}$

$= \binom{6}{3} (3)^{6-3} (x)^3$

$= \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \cdot (3)^3 (x)^3$

= (20). (27) x^3

= 540 x^3

(ii) For $\left(\frac{x}{3} + 3y\right)^8$,

$a = \frac{x}{3}$, b=3y and n=8

As n is even, $\left(\frac{n+2}{2}\right)^{th}$ is the middle term

Therefore, the middle term $= \left(\frac{8+2}{2}\right)^{th} = \left(\frac{10}{2}\right)^{th} = (5)^{th}$

General term t_{r+1} is given by,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Therefore, for 5^{th}, r=4

Therefore, the middle term is

$$t_5 = t_{4+1}$$

$$= \binom{8}{4} \left(\frac{x}{3}\right)^{8-4} (3y)^4$$

$$= \binom{8}{4} \left(\frac{x}{3}\right)^4 (3)^4 (y)^4$$

$$= \binom{8}{4} \frac{(x)^4}{(3)^4} (3)^4 (y)^4$$

$$= \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} . (x)^4 (y)^4$$

$= (70). x^4 y^4$

(iii) For $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$,

$a = \frac{x}{a}$, $b = \frac{-a}{x}$ and n=10

As n is even, $\left(\frac{n+2}{2}\right)^{th}$ is the middle term

Therefore, the middle term $= \left(\frac{10+2}{2}\right)^{th} = \left(\frac{12}{2}\right)^{th} = (6)^{th}$

General term t_{r+1} is given by,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Therefore, for 6^{th}, r=5

Therefore, the middle term is

$$t_6 = t_{5+1}$$

$$= \binom{10}{5} \left(\frac{x}{a}\right)^{10-5} \left(\frac{-a}{x}\right)^5$$

$$= \binom{10}{5} \left(\frac{x}{a}\right)^5 (-a)^5 \left(\frac{1}{x}\right)^5$$

$$= \binom{10}{5} \frac{(x)^5}{(a)^5} (-a)^5 \left(\frac{1}{x}\right)^5$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} . (-1)$$

= -252

(iv) For $\left(x^2 - \frac{2}{x}\right)^{10}$,

$a = x^2$, $b = \frac{-2}{x}$ and $n = 10$

As n is even, $\left(\frac{n+2}{2}\right)^{th}$ is the middle term

Therefore, the middle term $= \left(\frac{10+2}{2}\right)^{th} = \left(\frac{12}{2}\right)^{th} = (6)^{th}$

General term t_{r+1} is given by,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

Therefore, for the 6^{th} middle term, r=5

Therefore, the middle term is

$t_6 = t_{5+1}$

$= \binom{10}{5} (x^2)^{10-5} \left(\frac{-2}{x}\right)^5$

$= \binom{10}{5} (x^2)^5 (-2)^5 \left(\frac{1}{x}\right)^5$

$= \binom{10}{5} \frac{(x)^{10}}{(x)^5} (-32)$

$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} . (-32)\ (x)^5$

$= -252\ (32)\ x^5$

$= -8064\ x^5$

Question: 39 A

Solution:

For $(x^2 + a^2)^5$,

$a = x^2$, $b = a^2$ and $n = 5$

As n is odd, there are two middle terms i.e.

I. $\left(\frac{n+1}{2}\right)^{th}$ and II. $\left(\frac{n+3}{2}\right)^{th}$

General term t_{r+1} is given by,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

I. The first, middle term is $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{5+1}{2}\right)^{th} = \left(\frac{6}{2}\right)^{th} = (3)^{rd}$

Therefore, for the 3^{rd} middle term, r=2

Therefore, the first middle term is

$t_3 = t_{2+1}$

$= \binom{5}{2} (x^2)^{5-2} (a^2)^2$

$= \binom{5}{2} (x^2)^3 (a)^4$

$= \binom{5}{2} (x)^6 (a)^4$

$= \frac{5 \times 4}{2 \times 1}. (x)^6 (a)^4$

$= 10. a^4. x^6$

II. The second middle term is $\left(\frac{n+3}{2}\right)^{th} = \left(\frac{5+3}{2}\right)^{th} = \left(\frac{8}{2}\right)^{th} = (4)^{th}$

Therefore, for the 4^{th} middle term, r=3

Therefore, the second middle term is

$t_4 = t_{3+1}$

$= \binom{5}{3} (x^2)^{5-3} (a^2)^3$

$= \binom{5}{3} (x^2)^2 (a)^6$

$= \binom{5}{2} (x)^4 (a)^6$ ……..$\left[\because \binom{n}{r} = \binom{n}{n-r}\right]$

$= \frac{5 \times 4}{2 \times 1}. (x)^4 (a)^6$

$= 10. a^6. x^4$

Question: 39 B

Solution:

For $\left(x^4 - \frac{1}{x^3}\right)^{11}$,

a= x^4, $b = \frac{-1}{x^3}$ and n=11

As n is odd, there are two middle terms i.e.

II. $\left(\frac{n+1}{2}\right)^{th}$ and II. $\left(\frac{n+3}{2}\right)^{th}$

General term t_{r+1} is given by,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

I. The first middle term is $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{11+1}{2}\right)^{th} = \left(\frac{12}{2}\right)^{th} = (6)^{th}$

Therefore, for the 6^{th} middle term, r=5

Therefore, the first middle term is

$t_6 = t_{5+1}$

$= \binom{11}{5} (x^4)^{11-5} \left(\frac{-1}{x^3}\right)^5$

$= \binom{11}{5} (x^4)^6 (-1)^5 \left(\frac{1}{x^3}\right)^5$

$= \binom{11}{5} (x)^{24} (-1) \frac{1}{x^{15}}$

$= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} . (x)^9 (-1)$

$= - 462. x^9$

II. The second middle term is $\left(\frac{n+3}{2}\right)^{th} = \left(\frac{11+3}{2}\right)^{th} = \left(\frac{14}{2}\right)^{th} = (7)^{th}$

Therefore, for the 7^{th} middle term, r=6

Therefore, the second middle term is

$t_7 = t_{6+1}$

$= \binom{11}{6} (x^4)^{11-6} \left(\frac{-1}{x^3}\right)^6$

$= \binom{11}{5} (x^4)^5 (-1)^6 \left(\frac{1}{x^3}\right)^6$ ....$[\because \binom{n}{r} = \binom{n}{n-r}]$

$= \binom{11}{5} (x)^{20} (1) \frac{1}{x^{18}}$

$= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} . (x)^2$

$= 462. x^2$

Question: 39 C

Solution:

For $\left(\frac{p}{x} + \frac{x}{p}\right)^9$,

$a = \frac{p}{x}, b = \frac{x}{p}$ and n=9

As n is odd, there are two middle terms i.e.

I. $\left(\frac{n+1}{2}\right)^{th}$ and II. $\left(\frac{n+3}{2}\right)^{th}$

General term t_{r+1} is given by,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

I. The first middle term is $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{9+1}{2}\right)^{th} = \left(\frac{10}{2}\right)^{th} = (5)^{th}$

Therefore, for 5^{th} middle term, r=4

Therefore, the first middle term is

$t_5 = t_{4+1}$

$= \binom{9}{4} \left(\frac{p}{x}\right)^{9-4} \left(\frac{x}{p}\right)^4$

$= \binom{9}{4} \left(\frac{p}{x}\right)^5 (x)^4 \left(\frac{1}{p}\right)^4$

$= \binom{9}{4} \left(\frac{p}{x}\right)$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}.(p).(x)^{-1}$

$= 126p.x^{-1}$

II. The second middle term is $\left(\frac{n+3}{2}\right)^{th} = \left(\frac{9+3}{2}\right)^{th} = \left(\frac{12}{2}\right)^{th} = (6)^{th}$

Therefore, for the 6^{th} middle term, r=5

Therefore, the second middle term is

$t_6 = t_{5+1}$

$= \binom{9}{5}\left(\frac{p}{x}\right)^{9-5}\left(\frac{x}{p}\right)^{5}$

$= \binom{9}{4}\left(\frac{p}{x}\right)^{4}(x)^5\left(\frac{1}{p}\right)^{5}$ ....$[\because \binom{n}{r} = \binom{n}{n-r}]$

$= \binom{9}{4}\left(\frac{x}{p}\right)$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}.\left(\frac{1}{p}\right).(x)$

$= 126\left(\frac{1}{p}\right).(x)$

Question: 39 D

Solution:

For $\left(3x - \frac{x^3}{6}\right)^9$,

$a=3x, b = \frac{-x^3}{6}$ and n=9

As n is odd, there are two middle terms i.e.

I. $\left(\frac{n+1}{2}\right)^{th}$ and II. $\left(\frac{n+3}{2}\right)^{th}$

General term t_{r+1} is given by,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

I. The first middle term is $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{9+1}{2}\right)^{th} = \left(\frac{10}{2}\right)^{th} = (5)^{th}$

Therefore, for 5^{th} middle term, r=4

Therefore, the first middle term is

$t_5 = t_{4+1}$

$= \binom{9}{4}(3x)^{9-4}\left(\frac{-x^3}{6}\right)^{4}$

$= \binom{9}{4}(3x)^5(x^3)^4\left(\frac{1}{6}\right)^{4}$

$= \binom{9}{4}(3)^5(x)^5(x)^{12}\left(\frac{1}{6}\right)^{4}$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \cdot \frac{243}{1296} (x)^{17}$

$= \frac{189}{8} (x)^{17}$

II. The second middle term is $\left(\frac{n+3}{2}\right)^{th} = \left(\frac{9+3}{2}\right)^{th} = \left(\frac{12}{2}\right)^{th} = (6)^{th}$

Therefore, for the 6^{th} middle term, r=5

Therefore, the second middle term is

$t_6 = t_{5+1}$

$= \binom{9}{5} (3x)^{9-5} \left(\frac{-x^3}{6}\right)^5$

$= \binom{9}{4} (3x)^4 (-x^3)^5 \left(\frac{1}{6}\right)^5$ ………$\left[\because \binom{n}{r} = \binom{n}{n-r}\right]$

$= \binom{9}{4} (3)^4 (x)^4 (-x)^{15} \left(\frac{1}{6}\right)^5$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \cdot \frac{81}{7776} (-x)^{19}$

$= -\frac{21}{16} (x)^{19}$

Question: 40 A

Solution:

To Find : term independent of x, i.e. x^0

For $\left(2x + \frac{1}{3x^2}\right)^9$

a=2x, $b = \frac{1}{3x^2}$ and n=9

We have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{9}{r} (2x)^{9-r} \left(\frac{1}{3x^2}\right)^r$

$= \binom{9}{r} (x)^{9-r} (2)^{9-r} \left(\frac{1}{3}\right)^r \left(\frac{1}{x^2}\right)^r$

$= \binom{9}{r} (x)^{9-r} \frac{(2)^{9-r}}{(3)^r} (x)^{-2r}$

$= \binom{9}{r} \frac{(2)^{9-r}}{(3)^r} (x)^{9-r-2r}$

$= \binom{9}{r} \frac{(2)^{9-r}}{(3)^r} (x)^{9-3r}$

Now, to get coefficient of term independent of xthat is coefficient of x^0 we must have,

$(x)^{9-3r} = x^0$

• 9 - 3r = 0

• $3r = 9$

• $r = 3$

Therefore, coefficient of x^0= $\binom{9}{3}\frac{(2)^{9-3}}{(3)^3}$

$$= \frac{9 \times 8 \times 7}{3 \times 2 \times 1}\frac{(2)^6}{(3)^3}$$

$$= \frac{1792}{3}$$

<u>Conclusion</u> : coefficient of x^0= $\frac{1792}{3}$

Question: 40 B

Solution:

<u>To Find</u> : term independent of x, i.e. x^0

For $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^6$

$a = \frac{3x^2}{2}$, $b = -\frac{1}{3x}$ and n=6

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{6}{r}\left(\frac{3x^2}{2}\right)^{6-r}\left(-\frac{1}{3x}\right)^r$$

$$= \binom{6}{r}\left(\frac{3}{2}\right)^{6-r}(x^2)^{6-r}\left(\frac{-1}{3}\right)^r\left(\frac{1}{x}\right)^r$$

$$= \binom{6}{r}\left(\frac{3}{2}\right)^{6-r}\left(\frac{-1}{3}\right)^r (x)^{12-2r}(x)^{-r}$$

$$= \binom{6}{r}\left(\frac{3}{2}\right)^{6-r}\left(\frac{-1}{3}\right)^r (x)^{12-2r-r}$$

$$= \binom{6}{r}\left(\frac{3}{2}\right)^{6-r}\left(\frac{-1}{3}\right)^r (x)^{12-3r}$$

Now, to get coefficient of term independent of xthat is coefficient of x^0 we must have,

$(x)^{12-3r} = x^0$

• $12 - 3r = 0$

• $3r = 12$

• $r = 4$

Therefore, coefficient of x^0= $\binom{6}{4}\left(\frac{3}{2}\right)^{6-4}\left(\frac{-1}{3}\right)^4$

$$= \binom{6}{2}\left(\frac{3}{2}\right)^2\frac{1}{81} \text{ ....}\left[\because \binom{n}{r} = \binom{n}{n-r}\right]$$

$$= \frac{6 \times 5}{2 \times 1}.\frac{9}{4}.\frac{1}{81}$$

$$= \frac{15}{36}$$

Conclusion : coefficient of $x^0 = \frac{15}{36}$

Question: 40 C

Solution:

To Find : term independent of x, i.e. x^0

For $\left(x - \frac{1}{x^2}\right)^{3n}$

a=x, $b = -\frac{1}{x^2}$ and N=3n

We have a formula,

$$t_{r+1} = \binom{N}{r} a^{N-r} b^r$$

$$= \binom{3n}{r} (x)^{3n-r} \left(-\frac{1}{x^2}\right)^r$$

$$= \binom{3n}{r} (x)^{3n-r}(-1)^r \left(\frac{1}{x^2}\right)^r$$

$$= \binom{3n}{r} (x)^{3n-r}(-1)^r (x)^{-2r}$$

$$= \binom{3n}{r} (-1)^r (x)^{3n-r-2r}$$

$$= \binom{3n}{r} (-1)^r (x)^{3n-3r}$$

Now, to get coefficient of term independent of xthat is coefficient of x^0 we must have,

$(x)^{3n-3r} = x^0$

- $3n - 3r = 0$
- $3r = 3n$
- $r = n$

Therefore, coefficient of $x^0 = \binom{3n}{n} (-1)^n$

Conclusion : coefficient of $x^0 = \binom{3n}{n} (-1)^n$

Question: 40 D

Solution:

To Find : term independent of x, i.e. x^0

For $\left(3x - \frac{2}{x^2}\right)^{15}$

a=3x, $b = \frac{-2}{x^2}$ and n=15

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{15}{r} (3x)^{15-r} \left(\frac{-2}{x^2}\right)^r$$

$= \binom{15}{r} (3)^{15-r} (x)^{15-r} (-2)^r \left(\frac{1}{x^2}\right)^r$

$= \binom{15}{r} (3)^{15-r} (x)^{15-r} (-2)^r (x)^{-2r}$

$= \binom{15}{r} (3)^{15-r} (-2)^r (x)^{15-r-2r}$

$= \binom{15}{r} (3)^{15-r} (-2)^r (x)^{15-3r}$

Now, to get coefficient of term independent of xthat is coefficient of x^0 we must have,

$(x)^{15-3r} = x^0$

- 15 - 3r = 0
- 3r = 15
- r = 5

Therefore, coefficient of $x^0 = \binom{15}{5} (3)^{15-5} (-2)^5$

$= \frac{15 \times 14 \times 13 \times 12 \times 11}{5 \times 4 \times 3 \times 2 \times 1} . (3)^{10} . (-32)$

$= -3003 . (3)^{10} . (32)$

Conclusion : coefficient of $x^0 = -3003 . (3)^{10} . (32)$

Question: 41

Solution:

To Find : coefficient of x^5

For $(1+x)^3$

a=1, b=x and n=3

We have a formula,

$$(1 + x)^3 = \sum_{r=0}^{3} \binom{3}{r} (1)^{3-r} x^r$$

$= \binom{3}{0} (1)^3 x^0 + \binom{3}{1} (1)^2 x^1 + \binom{3}{2} (1)^1 x^2 + \binom{3}{3} (1)^0 x^3$

$= 1 + 3x + 3x^2 + x^3$

For $(1-x)^6$

a=1, b=-x and n=6

We have formula,

$$(1 - x)^6 = \sum_{r=0}^{6} \binom{6}{r} (1)^{6-r} (-x)^r$$

$$= \binom{6}{0} (1)^6 (-x)^0 + \binom{6}{1} (1)^5 (-x)^1 + \binom{6}{2} (1)^4 (-x)^2 + \binom{6}{3} (1)^3 (-x)^3 + \binom{6}{4} (1)^2 (-x)^4 + \binom{6}{5} (1)^1 (-x)^5 + \binom{6}{6} (1)^0 (-x)^6$$

We have a formula ,

$\binom{n}{r} = \frac{n!}{(n-r)! \times r!}$

By using this formula, we get,×

$(1-x)^6 = 1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6$

$\therefore (1+x)^3(1-x)^6$

$= (1 + 3x + 3x^2 + x^3)(1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6)$

Coefficients of x^5 are

$x^0.x^5 = 1\times(-6) = -6$

$x^1.x^4 = 3\times15 = 45$

$x^2.x^3 = 3\times(-20) = -60$

$x^3.x^2 = 1\times15 = 15$

Therefore, Coefficients of $x^5 = -6+45-60+15 = -6$

Conclusion : Coefficients of $x^5 = -6$

Question: 42

Solution:

To Find : numerically greatest term

For $(2+3x)^9$,

$a=2$, $b=3x$ and $n=9$

We have relation,

$t_{r+1} \geq t_r$ or $\frac{t_{r+1}}{t_r} \geq 1$

we have a formula,

$t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$= \binom{9}{r} 2^{9-r} (3x)^r$

$= \frac{9!}{(9-r)! \times r!} 2^{9-r} (3)^r (x)^r$

$\therefore t_r = \binom{n}{r-1} a^{n-r+1} b^{r-1}$

$= \binom{9}{r-1} 2^{9-r+1} (3x)^{r-1}$

$= \frac{9!}{(9-r+1)! \times (r-1)!} 2^{10-r} (3)^{r-1} (x)^{r-1}$

$= \frac{9!}{(10-r)! \times (r-1)!} 2^{10-r} (3)^{r-1} (x)^{r-1}$

$\therefore \frac{t_{r+1}}{t_r} \geq 1$

$$\therefore \frac{\frac{9!}{(9-r)! \times r!} 2^{9-r} (3)^r (x)^r}{\frac{9!}{(10-r)! \times (r-1)!} 2^{10-r} (3)^{r-1} (x)^{r-1}} \geq 1$$

$$\therefore \frac{9!}{(9-r)! \times r!} 2^{9-r} (3)^r (x)^r \geq \frac{9!}{(10-r)! \times (r-1)!} 2^{10-r} (3)^{r-1} (x)^{r-1}$$

$$\therefore \frac{9!}{(9-r)! \times r(r-1)!} 2^{9-r} (3)(3)^{r-1} (x)(x)^{r-1}$$
$$\geq \frac{9!}{(10-r)(9-r)! \times (r-1)!} (2) 2^{9-r} (3)^{r-1} (x)^{r-1}$$

$$\therefore \frac{1}{r} (3)(x) \geq \frac{1}{(10-r)} (2)$$

At $x = 3/2$

$$\therefore \frac{1}{r} (3) \frac{3}{2} \geq \frac{1}{(10-r)} (2)$$

$$\therefore \frac{9}{4} \geq \frac{r}{(10-r)}$$

$$\therefore 9(10-r) \geq 4r$$

$$\therefore 90 - 9r \geq 4r$$

- $90 \geq 13r$
- $r \leq 6.923$

therefore, r=6 and hence the 7^{th} term is numerically greater.

By using formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$t_7 = \binom{9}{7} 2^{9-7} (3x)^7$$

$$= \binom{9}{2} 2^2 (3)^7 (x)^7$$

Conclusion : the 7^{th} term is numerically greater with value $\binom{9}{2} 2^2 (3)^7 (x)^7$

Question: 43

Solution:

For $(1 + x)^{2n}$

a=1, b=x and N=2n

We have, $t_{r+1} = \binom{N}{r} a^{N-r} b^r$

For the 2^{nd} term, r=1

$$\therefore t_2 = t_{1+1}$$

$$= \binom{2n}{1} (1)^{2n-1} (x)^1$$

$$= (2n)\, x \cdots\cdots\cdots \left[\because \binom{n}{1} = n\right]$$

Therefore, the coefficient of 2^{nd} term = (2n)

For the 3^{rd} term, r=2

$\therefore t_3 = t_{2+1}$

$= \binom{2n}{2} (1)^{2n-2} (x)^2$

$= \frac{(2n)!}{(2n-2)! \times 2!} x^2$

$= \frac{(2n)(2n-1)(2n-2)!}{(2n-2)! \times 2} x^2$ ....(n! = n. (n-1)!)

$= (n)(2n-1)\, x^2$

Therefore, the coefficient of 3^{rd} term = (n)(2n-1)

For the 4^{th} term, r=3

$\therefore t_4 = t_{3+1}$

$= \binom{2n}{3} (1)^{2n-3} (x)^3$

$= \frac{(2n)!}{(2n-3)! \times 3!} x^3$

$= \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)! \times 6} x^3$ ....(n! = n. (n-1)!)

$= \frac{(n)(2n-1).2(n-1)}{3} x^3$

$= \frac{2(n)(2n-1).(n-1)}{3} x^3$

Therefore, the coefficient of 3^{rd} term $= \frac{2(n)(2n-1).(n-1)}{3}$

As the coefficients of 2^{nd}, 3^{rd} and 4^{th} terms are in A.P.

Therefore,

2×coefficient of 3^{rd} term = coefficient of 2^{nd} term + coefficient of the 4^{th} term

$\therefore 2 \times (n)(2n-1) = (2n) + \frac{2(n)(2n-1).(n-1)}{3}$

Dividing throughout by (2n),

$\therefore 2n - 1 = 1 + \frac{(2n-1)(n-1)}{3}$

$\therefore 2n - 1 = \frac{3 + (2n-1)(n-1)}{3}$

- 3 (2n-1) = 3 + (2n-1)(n-1)
- 6n – 3 = 3 + ($2n^2$ - 2n – n + 1)
- 6n – 3 = 3 + $2n^2$ - 3n + 1
- 3 + $2n^2$ - 3n + 1 - 6n + 3 = 0
- $2n^2$ - 9n + 7 = 0

Conclusion : If the coefficients of 2^{nd}, 3^{rd} and 4^{th} terms of $(1 + x)^{2n}$ are in A.P. then $2n^2$ - 9n + 7 = 0

Question: 44

Solution:

Given : 3^{rd} term from the end =45

To Find : 6^{th} term

For $(y^{1/2} + x^{1/3})^n$,

a= $y^{1/2}$, b= $x^{1/3}$

We have, $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

As n=n, therefore there will be total (n+1) terms in the expansion.

3^{rd} term from the end = $(n+1-3+1)^{th}$ i.e. $(n-1)^{th}$ term from the starting

For $(n-1)^{th}$ term, r = (n-1-1) = (n-2)

$t_{(n-1)} = t_{(n-2)+1}$

$= \binom{n}{n-2} \left(y^{\frac{1}{2}}\right)^{n-(n-2)} \left(x^{\frac{1}{3}}\right)^{(n-2)}$

$= \binom{n}{2} \left(y^{\frac{1}{2}}\right)^{2} (x)^{\frac{n-2}{3}}$ ....∵ $\binom{n}{n-r} = \binom{n}{r}$

$= \frac{n(n-1)}{2} (y) (x)^{\frac{n-2}{3}}$

Therefore 3^{rd} term from the end $= \frac{n(n-1)}{2} (y) (x)^{\frac{n-2}{3}}$

Therefore coefficient 3^{rd} term from the end $= \frac{n(n-1)}{2}$

$\therefore 45 = \frac{n(n-1)}{2}$

- 90 = n (n-1)
- 10 (9) = n (n-1)

Comparing both sides, n=10

For 6^{th} term, r=5

$t_6 = t_{5+1}$

$= \binom{10}{5} \left(y^{\frac{1}{2}}\right)^{10-5} \left(x^{\frac{1}{3}}\right)^{5}$

$= \binom{10}{5} (y)^{\frac{5}{2}} (x)^{\frac{5}{3}}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} (y)^{\frac{5}{2}} (x)^{\frac{5}{3}}$

$= 252 (y)^{\frac{5}{2}} (x)^{\frac{5}{3}}$

Conclusion : 6^{th} term $= 252 (y)^{\frac{5}{2}} (x)^{\frac{5}{3}}$

Question: 45

Solution:

Given : $t_{17} = t_{18}$

To Find : value of a

For $(2 + a)^{50}$

A=2, b=a and n=50

We have, $t_{r+1} = \binom{n}{r} A^{n-r} b^{r}$

For the 17^{th} term, r=16

$\therefore t_{17} = t_{16+1}$

$= \binom{50}{16} (2)^{50-16} (a)^{16}$

$= \binom{50}{16} (2)^{34} (a)^{16}$

For the 18^{th} term, r=17

$\therefore t_{18} = t_{17+1}$

$= \binom{50}{17} (2)^{50-17} (a)^{17}$

$= \binom{50}{17} (2)^{33} (a)^{17}$

As 17^{th} and 18^{th} terms are equal

$\therefore t_{18} = t_{17}$

$\therefore \binom{50}{17} (2)^{33} (a)^{17} = \binom{50}{16} (2)^{34} (a)^{16}$

$\therefore \binom{50}{17} (2)^{33} (a)^{17} = \binom{50}{16} (2)^{34} (a)^{16}$

$\therefore \frac{50!}{(50-17)! \times (17)!} (2)^{33} (a)^{17} = \frac{50!}{(50-16)! \times (16)!} (2)^{34} (a)^{16}$

.......$\left[\because \binom{n}{r} = \frac{n!}{(n-r)! \times (r)!}\right]$

$\therefore \frac{(a)^{17}}{(a)^{16}} = \frac{50!}{(50-16)! \times (16)!} \cdot \frac{(50-17)! \times (17)!}{50!} \cdot \frac{(2)^{34}}{(2)^{33}}$

$\therefore a = \frac{(50-17) \times (50-16)! \times 17 \times (16)!}{(50-16)! \times (16)!} .(2)$

.......$[\because n! = n(n-1)!]$

$\therefore a = (50-17) \times 17 .(2)$

• a = 1122

Conclusion : value of a = 1122

Question: 46

Solution:

To Find : Coefficients of x^4

For $(1+x)^n$

a=1, b=x

We have a formula,

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} (1)^{n-r} x^r$$

$$= \binom{n}{0} (1)^n x^0 + \binom{n}{1} (1)^{n-1} x^1 + \binom{n}{2} (1)^{n-2} x^2 + \cdots + \binom{n}{n} (1)^{n-n} x^n$$

$$= \binom{n}{0} x^0 + \binom{n}{1} x + \binom{n}{2} x^2 + \cdots + \binom{n}{n} x^n$$

For $(1-x)^n$

a=1, b=-x and n=n

We have formula,

$$(1-x)^n = \sum_{r=0}^{n} \binom{n}{r} (1)^{n-r} (-x)^r$$

$$= \binom{n}{0} (1)^n (-x)^0 + \binom{n}{1} (1)^{n-1} (-x)^1 + \binom{n}{2} (1)^{n-2} (-x)^2 + \cdots + \binom{n}{n} (1)^{n-n} (-x)^n$$

$$= \binom{n}{0} (-x)^0 - \binom{n}{1} (x)^1 + \binom{n}{2} (x)^2 + \cdots + \binom{n}{n} (-x)^n$$

$$\therefore (1+x)^3(1-x)^6$$

$$= \left\{\binom{n}{0} x^0 + \binom{n}{1} x + \binom{n}{2} x^2 + \cdots + \binom{n}{n} x^n\right\}\left\{\binom{n}{0} (-x)^0 - \binom{n}{1} (x)^1 + \binom{n}{2} (x)^2 + \cdots + \binom{n}{n} (-x)^n\right\}$$

Coefficients of x^4 are

$x^0.x^4 = \binom{n}{0} \times \binom{n}{4} = C_0C_4$

$x^1.x^3 = \binom{n}{1} \times (-1)\binom{n}{3} = -\binom{n}{1}\binom{n}{3} = - C_1C_3$

$x^2.x^2 = \binom{n}{2} \times \binom{n}{2} = C_2C_2$

$x^3.x^1 = \binom{n}{3} \times (-1)\binom{n}{1} = -\binom{n}{3}\binom{n}{1} = - C_3C_1$

$x^4.x^0 = \binom{n}{4} \times \binom{n}{0} = C_4C_0$

Therefore, Coefficient of x^4

$= C_4C_0 - C_1C_3 + C_2C_2 - C_3C_1 + C_4C_0$

Let us assume, n=4, it becomes

${}^4C_4\, {}^4C_0 - {}^4C_1\, {}^4C_3 + {}^4C_2\, {}^4C_2 - {}^4C_3\, {}^4C_1 + {}^4C_4\, {}^4C_0$

We know that ,

$$\binom{n}{r} = \frac{n!}{(n-r)! \times r!}$$

By using above formula, we get,

${}^4C_4\, {}^4C_0 - {}^4C_1\, {}^4C_3 + {}^4C_2\, {}^4C_2 - {}^4C_3\, {}^4C_1 + {}^4C_4\, {}^4C_0$

$= (1)(1) - (4)(4) + (6)(6) - (4)(4) + (1)(1)$

$= 1 - 16 + 36 - 16 + 1$

$= 6$

$= {}^4C_2$

Therefore, in general,

$C_4C_0 - C_1C_3 + C_2C_2 - C_3C_1 + C_4C_0 = C_2$

Therefore, Coefficient of $x^4 = C_2$

Conclusion :

- Coefficient of $x^4 = C_2$
- $C_4C_0 - C_1C_3 + C_2C_2 - C_3C_1 + C_4C_0 = C_2$

Question: 47

Solution:

To Prove : coefficient of x^n in $(1+x)^{2n} = 2 \times$ coefficient of x^n in $(1+x)^{2n-1}$

For $(1+x)^{2n}$,

a=1, b=x and m=2n

We have a formula,

$t_{r+1} = \binom{m}{r} a^{m-r} b^r$

$= \binom{2n}{r} (1)^{2n-r} (x)^r$

$= \binom{2n}{r} (x)^r$

To get the coefficient of x^n, we must have,

$x^n = x^r$

- $r = n$

Therefore, the coefficient of $x^n = \binom{2n}{n}$

$= \frac{(2n)!}{n! \times (2n-n)!}$ ....$\left(\because \binom{n}{r} = \frac{n!}{r! \times (n-r)!}\right)$

$= \frac{(2n)!}{n! \times n!}$

$= \frac{2n \times (2n-1)!}{n! \times n(n-1)!}$ ....$(\because n! = n(n-1)!)$ $\frac{2 \times (2n-1)!}{n! \times (n-1)!}$

.........cancelling n

Therefore, the coefficient of x^n in $(1+x)^{2n} = \frac{2 \times (2n-1)!}{n! \times (n-1)!}$ ....eq(1)

Now for $(1+x)^{2n-1}$,

a=1, b=x and m=2n-1

We have formula,

$t_{r+1} = \binom{m}{r} a^{m-r} b^r$

$= \binom{2n-1}{r} (1)^{2n-1-r} (x)^r$

$= \binom{2n-1}{r} (x)^r$

To get the coefficient of x^n, we must have,

$x^n = x^r$

- $r = n$

Therefore, the coefficient of x^n in $(1+x)^{2n-1} = \binom{2n-1}{n}$

$$= \frac{(2n-1)!}{n! \times (2n-1-n)!}$$

$$= \frac{1}{2} \times \frac{2 \times (2n-1)!}{n! \times (n-1)!}$$

.....multiplying and dividing by 2

Therefore,

coefficient of x^n in $(1+x)^{2n-1}$ = [illegible]× coefficient of x^n in $(1+x)^{2n}$ or

coefficient of x^n in $(1+x)^{2n}$ = 2 × coefficient of x^n in $(1+x)^{2n-1}$

Hence proved.

Question: 48

Solution:

Given : $a = \frac{p}{2}$, b=2 and n=8

To find : middle term

Formula :

- The middle term $= \left(\frac{n+2}{2}\right)$
- $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

Here, n is even.

Hence,

$$\left(\frac{n+2}{2}\right) = \left(\frac{8+2}{2}\right) = 5$$

Therefore, 5^{th} tthe erm is the middle term.

For t_5, r=4

We have, $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$$\therefore\ t_5 = \binom{8}{4} \left(\frac{p}{2}\right)^{8-4} 2^4$$

$$\therefore\ t_5 = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} . \left(\frac{p}{2}\right)^4 . (16)$$

$$\therefore\ t_5 = 70. \left(\frac{p^4}{16}\right) . (16)$$

$\therefore\ t_5 = 70\ p^4$

Conclusion : The middle term is $70\ p^4$.

Exercise : 10B

Question: 1

Solution:

To show: the term independent of x in the expansion of $\left(x - \frac{1}{x}\right)^{10}$ is -252.

Formula Used:

General term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\ x^{n-r}\ y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $\left(x - \frac{1}{x}\right)^{10}$, we get

$T_{r+1} = {}^{10}C_r \times x^{10-r} \times \left(\frac{-1}{x}\right)^r$

For finding the term which is independent of x,

10-2r=5

r=5

Thus, the term which would be independent of x is T_6

$T_6 = {}^{10}C_5 \times x^{10-5} \times \left(\frac{-1}{x}\right)^5$

$T_6 = {}^{10}C_5 \times x^{10-5} \times \left(\frac{-1}{x}\right)^5$

$T_6 = -\ {}^{10}C_5$

$T_6 = -\ \frac{10!}{5!(10-5)!}$

$T_6 = -\ \frac{10!}{5! \times 5!}$

$T_6 = -\ \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2}$

$T_6 = -\ \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2}$

$T_6 = 252$

Thus, the term independent of x in the expansion of $\left(x - \frac{1}{x}\right)^{10}$ is -252.

Question: 2

Solution:

To prove: that. If the coefficients of x^2 and x^3 in the expansion of $(3 + px)^9$ are the same then $p = \frac{9}{7}$.

Formula Used:

General term, T_{r+1} of binomial expansion$(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\ x^{n-r}\ y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $(3 + px)^9$, we get

$T_{r+1} = {}^9C_r \times 3^{9-r} \times (px)^r$

For finding the term which has x^2 in it, is given by

r=2

Thus, the coefficients of x^2 are given by,

$T_3 = {}^9C_2 \times 3^{9-2} \times (px)^2$

$T_3 = {}^9C_2 \times 3^7 \times p^2 \times x^2$

For finding the term which has x^2 in it, is given by

r=3

Thus, the coefficients of x^3 are given by,

$T_3 = {}^9C_3 \times 3^{9-3} \times (px)^3$

$T_3 = {}^9C_3 \times 3^6 \times p^3 \times x^3$

As the coefficients of x^2 and x^3 in the expansion of $(3 + px)^9$ are the same.

${}^9C_3 \times 3^6 \times p^3 = {}^9C_2 \times 3^7 \times p^2$

${}^9C_3 \times p = {}^9C_2 \times 3$

$$\frac{9!}{3! \times 6!} \times p = \frac{9!}{2! \times 7!} \times 3$$

$$\frac{9!}{3 \times 2! \times 6!} \times p = \frac{9!}{2! \times 7 \times 6!} \times 3$$

$$p = \frac{9}{7}$$

Thus, the value of p for which coefficients of x^2 and x^3 in the expansion of $(3 + px)^9$ are the same is $\frac{9}{7}$

Question: 3

Solution:

To show: that the coefficient of x^{-3} in the expansion of $\left(x - \frac{1}{x}\right)^{11}$ is -330.

Formula Used:

General term, T_{r+1} of binomial expansion$(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\ x^{n-r}\ y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $\left(x-\frac{1}{x}\right)^{11}$, we get

$T_{r+1}= {}^{11}C_r \times x^{11-r} \times \left(\frac{-1}{x}\right)^r$

For finding the term which has x^{-3} in it , is given by

11-2r=3

2r=14

r=7

Thus, the term which the term which has x^{-3} in it is T_8

$T_8= {}^{11}C_7 \times x^{11-7} \times \left(\frac{-1}{x}\right)^7$

$T_8= -{}^{11}C_7 \times x^{-3}$

$T_8= -\frac{11!}{7!(11-7)!}$

$T_6= -\frac{11\times10\times9\times8\times7!}{7!\times4\times3\times2}$

T_6=-330

Thus, the coefficient of x^{-3} in the expansion of $\left(x-\frac{1}{x}\right)^{11}$ is -330.

Question: 4

Solution:

To show: that the middle term in the expansion of $\left(\frac{2x^2}{3}+\frac{3}{2x^2}\right)^{10}$ is 252.

Formula Used:

General term, T_{r+1} of binomial expansion$(x+y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Total number of terms in the expansion is 11

Thus, the middle term of the expansion is T_6 and is given by,

$T_6 = {}^{10}C_5 \times \left(\frac{2x^2}{3}\right)^5 \times \left(\frac{3}{2x^2}\right)^5$

$T_6= {}^{10}C_5$

$T_6= \frac{10\times9\times8\times7\times6\times5!}{5!\times5\times4\times3\times2}$

T_6=252

Thus, the middle term in the expansion of $\left(\frac{2x^2}{3}+\frac{3}{2x^2}\right)^{10}$ is 252.

Question: 5

Solution:

To show: that the coefficient of x^4 in the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is -330.

Formula Used:

General term, T_{r+1} of binomial expansion$(x+y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$, we get

$T_{r+1} = {}^{10}C_r \times \left(\frac{x}{2}\right)^{10-r} \times \left(\frac{-3}{x^2}\right)^r$

For finding the term which has x^4 in it, is given by

10-3r=4

3r=6

r=2

Thus, the term which has x^4 in it isT_3

$T_3 = {}^{10}C_2 \times \left(\frac{x}{2}\right)^8 \times \left(\frac{-3}{x^2}\right)^2$

$T_3 = \frac{10!\times 9}{2!\times 8!\times 2^8}$

$T_3 = \frac{10\times 9\times 8!\times 9}{2\times 8!\times 2^8}$

$T_3 = \frac{405}{256}$

Thus, the coefficient of x^4 in the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is $\frac{405}{256}$

Question: 6

Solution:

To prove: that there is no term involving x^6 in the expansion of $\left(2x^2-\frac{3}{x}\right)^{11}$

Formula Used:

General term, T_{r+1} of binomial expansion$(x+y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $\left(2x^2-\frac{3}{x}\right)^{11}$, we get

$T_{r+1} = {}^{11}C_r \times (2x^2)^{11-r} \times \left(\frac{-3}{x}\right)^r$

For finding the term which has x^6 in it, is given by

22-2r-r=6

3r=16

$r = \frac{16}{3}$

Since, $r = \frac{16}{3}$ is not possible as r needs to be a whole number

Thus, there is no term involving x^6 in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$.

Question: 7

Solution:

To show: that the coefficient of x^4 in the expansion of $(1 + 2x + x^2)^5$ is 212.

Formula Used:

We have,

$(1 + 2x + x^2)^5 = (1 + x + x + x^2)^5$

$= (1 + x + x(1+x))^5$

$= (1 + x)^5(1 + x)^5$

$= (1 + x)^{10}$

General term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where s

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term,

$T_{r+1} = {}^{10}C_r \times x^{10-r} \times (1)^r$

10-r=4

r=6

Thus, the coefficient of x^4 in the expansion of $(1 + 2x + x^2)^5$ is given by,

${}^{10}C_4 = \frac{10!}{4!6!}$

${}^{10}C_4 = \frac{10\times9\times8\times7\times6!}{24\times6!}$

${}^{10}C_4 = 210$

Thus, the coefficient of x^4 in the expansion of $(1 + 2x + x^2)^5$ is 210

Question: 8

Solution:

To find: the number of terms in the expansion of $\left(\sqrt{2} + 1\right)^5 + \left(\sqrt{2} - 1\right)^5$

Formula Used:

Binomial expansion of $(x + y)^n$ is given by,

$$(x + y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} \times y^r$$

Thus,

$$(\sqrt{2}+1)^5+(\sqrt{2}-1)^5$$
$$=\left((\sqrt{2})^5+(\sqrt{2})^4\binom{5}{1}+\cdots+\binom{5}{5}\right)$$
$$+\left((\sqrt{2})^5-(\sqrt{2})^4\binom{5}{1}+\cdots-\binom{5}{5}\right)$$

So, the no. of terms left would be 6

Thus, the number of terms in the expansion of $(\sqrt{2}+1)^5+(\sqrt{2}-1)^5$ is 6

Question: 9

Solution:

To find: the term independent of x in the expansion of $\left(x-\frac{1}{3x^2}\right)^9$?

Formula Used:

A general term, T_{r+1} of binomial expansion $(x+y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $\left(x-\frac{1}{3x^2}\right)^9$, we get

$T_{r+1} = {}^9C_r \times x^{9-r} \times \left(\frac{-1}{3x^2}\right)^r$

$T_{r+1} = {}^9C_r \times x^{9-r} \times (-1) \times 3x^{-2r}$

$T_{r+1} = {}^9C_r \times (-1) \times 3x^{9-3r}$

For finding the term which is independent of x,

9-3r=0

r=3

Thus, the term which would be independent of x is T_4

Thus, the term independent of x in the expansion of $\left(x-\frac{1}{x}\right)^{10}$ is T_4 i.e 4^{th} term

Question: 10

Solution:

To find: that the middle term in the expansion of $\left(\frac{2x^2}{3}+\frac{3}{2x^2}\right)^{10}$ is 252.

Formula Used:

A general term, T_{r+1} of binomial expansion $(x+y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Total number of terms in the expansion is 11

Thus, the middle term of the expansion is T_6 and is given by,

$T_6 = {}^{10}C_5 \times \left(\frac{2x^2}{3}\right)^5 \times \left(\frac{3}{2x^2}\right)^5$

$T_6 = {}^{10}C_5$

$T_6 = \frac{10\times9\times8\times7\times6\times5!}{5!\times5\times4\times3\times2}$

$T_6 = 252$

Thus, the middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is 252.

Question: 11

Solution:

To find: the coefficient of x^7y^2 in the expansion of $(x + 2y)^9$

Formula Used:

A general term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $(x + 2y)^9$, we get

$T_{r+1} = {}^9C_r \times x^{9-r} \times (2y)^r$

The value of r for which coefficient of x^7y^2 is defined

r=2

Hence, the coefficient of x^7y^2 in the expansion of $(x + 2y)^9$ is given by:

$T_3 = {}^9C_3 \times x^{9-2} \times (2y)^2$

$T_3 = {}^9C_3 \times 4 \times x^7 \times (y)^2$

$T_3 = \frac{9!}{3!\times6!} \times 4 \times x^7 \times (y)^2$

$T_3 = \frac{9\times8\times7\times6!}{6\times6!} \times 4 \times x^7 \times (y)^2$

$T_3 = 336$

Thus, the coefficient of x^7y^2 in the expansion of $(x + 2y)^9$ is 336

Question: 12

Solution:

To find: the value of r with respect to the binomial expansion of $(1 + x)^{34}$ where the coefficients of the $(r - 5)$th and $(2r - 1)$th terms are equal to each other

Formula Used:

The general term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the $(r - 5)$th term, we get

$T_{r-5} = {}^{34}C_{r-6} \times x^{r-6}$

Thus, the coefficient of (r – 5)th term is ${}^{34}C_{r-6}$

Now, finding the (2r – 1)th term, we get

$T_{2r-1} = {}^{34}C_{2r-2} \times (x)^{2r-2}$

Thus, coefficient of (2r – 1)th term is ${}^{34}C_{2r-2}$

As the coefficients are equal, we get

${}^{34}C_{2r-2} = {}^{34}C_{r-6}$

2r-2=r-6

r=-4

Value of r=-4 is not possible

2r-2+r-6=34

3r=42

r=14

Thus, value of r is 14

Question: 13

Solution:

To find: 4th term from the end in the expansion of $\left(\frac{3}{x^2} - \frac{x^3}{6}\right)^7$

Formula Used:

A general term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Total number of terms in the expansion is 8

Thus, the 4th term of the expansion is T_5 and is given by,

$T_5 = {}^7C_5 \times \left(\frac{3}{x^2}\right)^3 \times \left(\frac{-x^3}{6}\right)^4$

$T_5 = \frac{7\times6\times5!}{2\times5!} \times \frac{3\times3\times3}{6\times6\times6\times6} \times x^{-18}$

$T_5 = \frac{7\times6\times5!}{2\times5!} \times \frac{3\times3\times3}{6\times6\times6\times6} \times x^{-18}$

$T_5 = \frac{7}{16} x^{-18}$

Thus, a 4th term from the end in the expansion of $\left(\frac{3}{x^2} - \frac{x^3}{6}\right)^7$ is $T_5 = \frac{7}{16} x^{-18}$

Question: 14

Solution:

To find: the coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$.

Formula Used:

Binomial expansion of $(x + y)^n$ is given by,

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} \times y^r$$

Thus,

$(1 + x)(1 - x)^n$.

$$= (1+x)\left(\binom{n}{0}(-x) + \binom{n}{1}(-x)^1 + \binom{n}{2}(-x)^2 + .. + \binom{n}{n-1}(-x)^{n-1} + \binom{n}{n}(-x)^n\right)$$

Thus, the coefficient of $(x)^n$ is,

${}^nC_n - {}^nC_{n-1}$ (If n is even)

$-{}^nC_n + {}^nC_{n-1}$ (If n is odd)

Thus, the coefficient of $(x)^n$ is, ${}^nC_n - {}^nC_{n-1}$ (If n is even)and $-{}^nC_n + {}^nC_{n-1}$ (If n is odd)

Question: 15

Solution:

To find: the value of n with respect to the binomial expansion of $(a + b)^n$ where the coefficients of the 4^{th} and 13^{th} terms are equal to each other

Formula Used:

A general term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the 4^{th} term, we get

$T_4 = {}^nC_3 \times a^{n-3} \times (b)^3$

Thus, the coefficient of a 4^{th} term is ${}^nC_3$

Now, finding the 13^{th} term, we get

$T_{13} = {}^nC_{12} \times a^{n-12} \times (b)^{12}$

Thus, coefficient of 4^{th} term is ${}^nC_{12}$

As the coefficients are equal, we get

${}^nC_{12} = {}^nC_3$

Also, ${}^nC_r = {}^nC_{n-r}$

${}^nC_{n-12} = {}^nC_3$

n-12=3

n=15

Thus, value of n is 15

Question: 16

Solution:

To find: the positive value of m for which the coefficient of x^2 in the expansion o

Formula Used:

General term, T_{r+1} of binomial expansion $(x + y)^n$ is given by,

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$ where

${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, finding the general term of the expression, $(1 + x)^m$, we get

$T_{r+1} = {}^mC_r \times 1^{m-r} \times (x)^r$

$T_{r+1} = {}^mC_r \times (x)^r$

The coefficient of $(x)^2$ is ${}^mC_2$

${}^mC_2 = 6$

$\frac{m!}{2(m-2)!} = 6$

$$\frac{m(m-1)(m-2)!}{2(m-2)!} = 6$$

$m^2 - m - 6 = 0$

$(m-3)(m+2) = 0$

m=3,-2

Since m cannot be negative. Therefore,

m=3

Thus, positive value of m is 3 for which the coefficient of x2 in the expansion of $(1 + x)^m$ is 6