Chapter : 24. HYPERBOLA

Exercise : 24

Question: 1

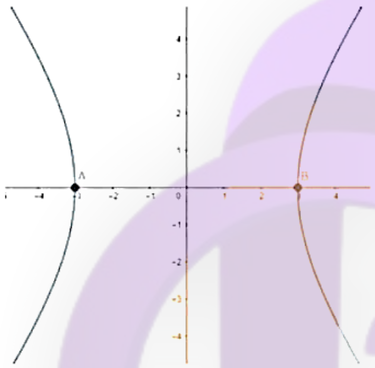
Solution:

Given Equation:

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

a = 3 and b = 4



(i) Length of Transverse axis = 2a = 6 units.

Length of Conjugate axis = 2b = 8 units.

(ii) Coordinates of the vertices = ($\pm$a, 0) = ($\pm$3, 0)

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{16}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$

(iii) Coordinates of the foci = ($\pm$ae, 0) = ($\pm$5, 0)

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{32}{3} = 10.67$ units.

Question: 2

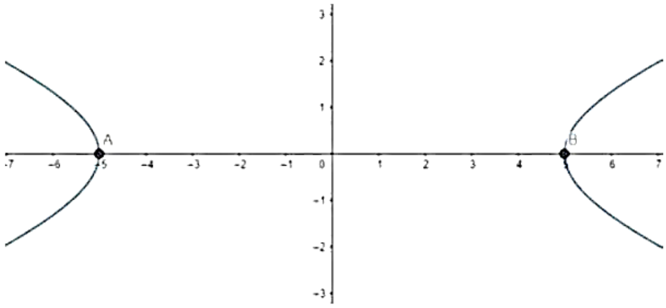
Solution:

Given Equation:

$$\frac{x^2}{25} - \frac{y^2}{4} = 1$$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

a = 5 and b = 2



(i) Length of Transverse axis = 2a = 10 units.

Length of Conjugate axis = 2b = 4 units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm 5, 0)$

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{4}{25}} = \sqrt{\frac{29}{25}} = \frac{\sqrt{29}}{5}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm\sqrt{29}, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{8}{5} = 1.6$ units.

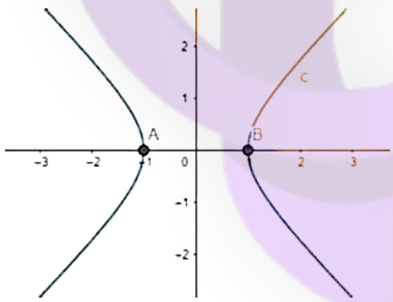
Question: 3

Solution:

Given Equation: $x^2 - y^2 = 1$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

a = 1 and b = 1



(i) Length of Transverse axis = 2a = 2 units.

Length of Conjugate axis = 2b = 2 units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm 1, 0)$

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{1}{1}} = \sqrt{\frac{2}{1}} = \sqrt{2}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm\sqrt{2}, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = 2$ units.

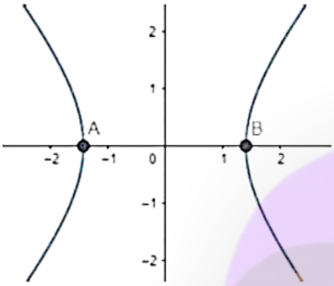
Question: 4

Solution:

Given Equation: $3x^2 - 2y^2 = 6 \Rightarrow \frac{x^2}{2} - \frac{y^2}{3} = 1$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

a = $\sqrt{2}$ and b = $\sqrt{3}$



(i) Length of Transverse axis = 2a = $2\sqrt{2}$ units.

Length of Conjugate axis = 2b = $2\sqrt{3}$ units.

(ii) Coordinates of the vertices = ($\pm$a, 0) = ($\pm\sqrt{2}$, 0)

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{3}{2}} = \sqrt{\frac{5}{2}}$

(iii) Coordinates of the foci = ($\pm$ae, 0) = ($\pm\sqrt{5}$, 0)

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$ units.

Question: 5

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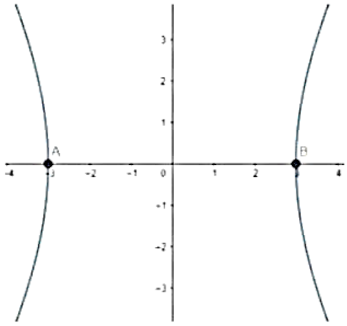
Solution:

Given Equation: $25x^2 - 9y^2 = 225 \Rightarrow$

$$\frac{x^2}{9} - \frac{y^2}{25} = 1$$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

a = 3 and b = 5



(i) Length of Transverse axis = 2a = 6 units.

Length of Conjugate axis = 2b = 10 units.

(ii) Coordinates of the vertices = (±a, 0) = (±3, 0)

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{25}{9}} = \sqrt{\frac{34}{9}} = \frac{\sqrt{34}}{3}$

(iii) Coordinates of the foci = (±ae, 0) = (± $\sqrt{34}$, 0)

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{50}{3} = 16.67$ units.

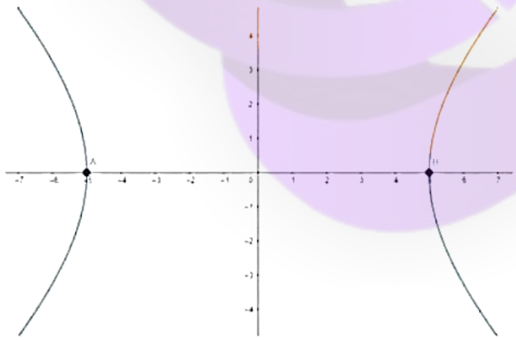
Question: 6

Solution:

Given Equation: $24x^2 - 25y^2 = 600 \Rightarrow \frac{x^2}{25} - \frac{y^2}{24} = 1$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

a = 5 and b = $\sqrt{24} = 2\sqrt{6}$



(i) Length of Transverse axis = 2a = 10 units.

Length of Conjugate axis = 2b = $4\sqrt{6}$ units.

(ii) Coordinates of the vertices = (±a, 0) = (±5, 0)

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{24}{25}} = \sqrt{\frac{49}{25}} = \frac{7}{5}$

(iii) Coordinates of the foci = (±ae, 0) = (±7, 0)

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{48}{5} = 9.6$ units.

Question: 7

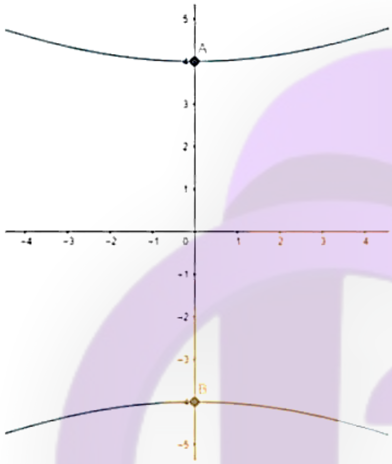
Solution:

Given Equation:

$$\frac{y^2}{16} - \frac{x^2}{49} = 1$$

Comparing with the equation of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ we get,

a = 4 and b = 7



(i) Length of Transverse axis = 2a = 8 units.

Length of Conjugate axis = 2b = 14 units.

(ii) Coordinates of the vertices = $(0, \pm a) = (0, \pm 4)$

(iv) Here, eccentricity, e = $\sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{49}{16}} = \sqrt{\frac{65}{16}} = \frac{\sqrt{65}}{4}$

(iii) Coordinates of the foci = $(0, \pm ae) = (0, \pm\sqrt{65}\,)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{98}{4} = 24.5$ units.

Question: 8

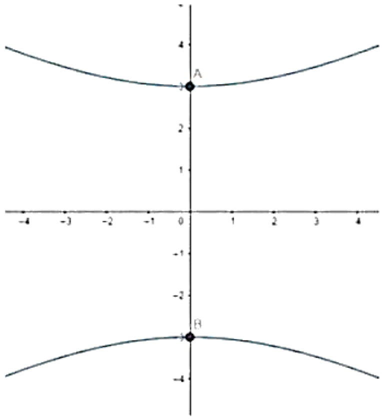
Solution:

Given Equation:

$$\frac{y^2}{9} - \frac{x^2}{27} = 1$$

Comparing with the equation of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ we get,

a = 3 and b = $\sqrt{27} = 3\sqrt{3}$



(i) Length of Transverse axis = 2a = 6 units.

Length of Conjugate axis = 2b = $6\sqrt{3}$ units.

(ii) Coordinates of the vertices = $(0, \pm a) = (0, \pm 3)$

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{27}{9}} = \sqrt{\frac{36}{9}} = \frac{6}{3} = 2$

(iii) Coordinates of the foci = $(0, \pm ae) = (0, \pm 6)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{54}{3} = 18$ units.

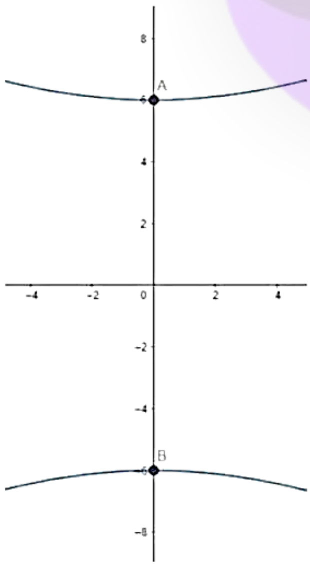
Question: 9

Solution:

Given Equation: $3y^2 - x^2 = 108 \Rightarrow \frac{y^2}{36} - \frac{x^2}{108} = 1$

Comparing with the equation of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ we get,

a = 6 and b = $\sqrt{108} = 6\sqrt{3}$



(i) Length of Transverse axis = 2a = 12 units.

Length of Conjugate axis = 2b = $12\sqrt{3}$ units.

(ii) Coordinates of the vertices = (0, ±a) = (0, ±6)

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{108}{36}} = \sqrt{1+3} = 2$

(iii) Coordinates of the foci = (0, ±ae) = (0, ±12)

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{216}{6} = 36$ units.

Question: 10

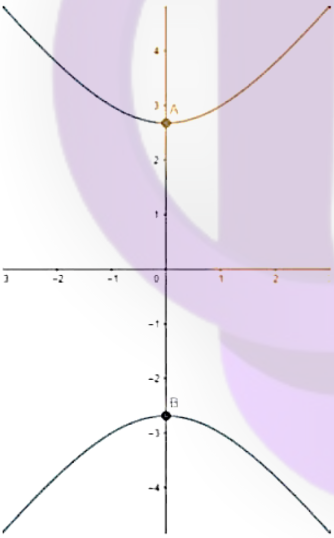
Solution:

Given Equation: $5y^2 - 9x^2 = 36 \Rightarrow$

$$\frac{y^2}{36/5} - \frac{x^2}{4} = 1$$

Comparing with the equation of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ we get,

a = $\sqrt{\frac{36}{5}} = \frac{6}{\sqrt{5}}$ and b = 2



(i) Length of Transverse axis = 2a = $\frac{12}{\sqrt{5}}$ units.

Length of Conjugate axis = 2b = 4 units.

(ii) Coordinates of the vertices = (0, ±a) = (0, ± $\frac{6}{\sqrt{5}}$)

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{4}{36/5}} = \sqrt{1+\frac{20}{36}} = \frac{\sqrt{56}}{6} = \frac{\sqrt{14}}{3}$

(iii) Coordinates of the foci = $(0, \pm ae) = (0, \pm \frac{6}{\sqrt{5}} . \frac{\sqrt{14}}{3}) = (0, \frac{2\sqrt{14}}{\sqrt{5}})$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{8}{6/\sqrt{5}} = \frac{8\sqrt{5}}{6} = \frac{4\sqrt{5}}{3}$ units.

Question: 11

Solution:

Given: Vertices at (±6, 0) and foci at (±8, 0)

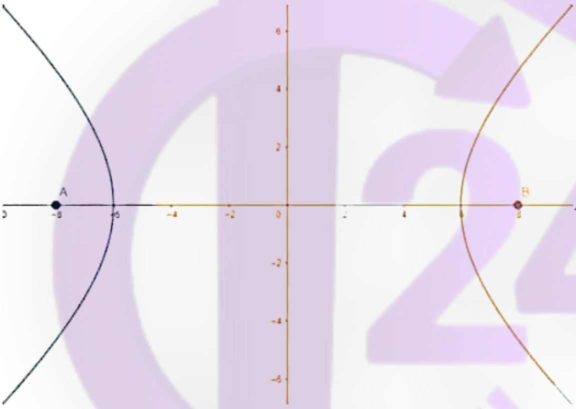
Need to find: The equation of the hyperbola.

Let, the equation of the parabola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Vertices of the parabola is at (±6, 0)

That means a = 6

The foci are given at (±8, 0)



A and B are the foci.

That means, ae = 8, where e is the eccentricity.

$\Rightarrow 6e = 8$ [As a = 6]

$\Rightarrow e = \frac{8}{6} = \frac{4}{3}$

We know that,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Therefore,

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \frac{4}{3}$$

$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{16}{9}$ [Squaring both sides]

$$\Rightarrow \frac{b^2}{a^2} = \frac{16}{9} - 1 = \frac{7}{9}$$

$\Rightarrow b^2 = a^2 \frac{7}{9}$

$\Rightarrow b^2 = 36 \times \frac{7}{9} = 4 \times 7 = 28$ [As a = 6]

So, the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{36} - \frac{y^2}{28} = 1$ [Answer]

Question: 12

Solution:

Given: Vertices at (0, ±5) and foci at (0, ±8)

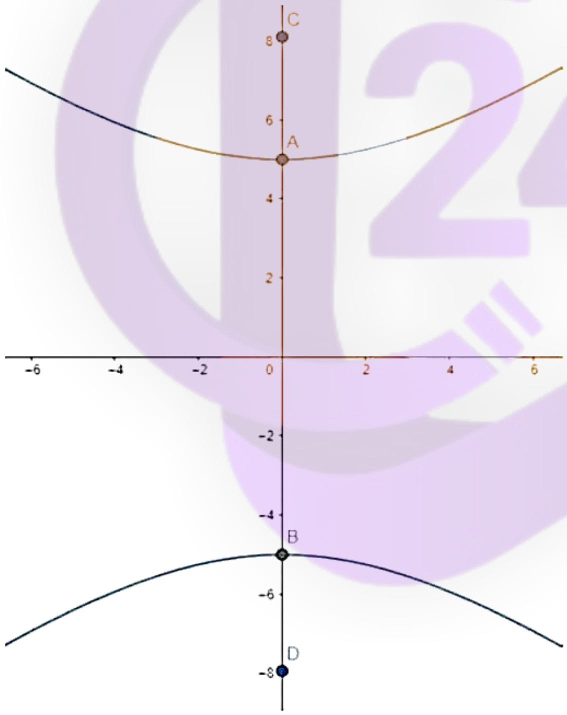
Need to find: The equation of the hyperbola.

Let, the equation of the parabola be: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Vertices of the parabola are at (0, ±5)

That means a = 5

The foci are given at (0, ±8)



A and B are the vertices. C and D are the foci.

That means, ae = 8, where e is the eccentricity.

$\Rightarrow 5e = 8$ [As a = 5]

$\Rightarrow e = \frac{8}{5}$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \frac{8}{5}$$

$$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{64}{25} \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{64}{25} - 1 = \frac{39}{25}$$

$$\Rightarrow b^2 = a^2 \frac{39}{25}$$

$$\Rightarrow b^2 = 25 \times \frac{39}{25} = 39 \text{ [As a = 5]}$$

So, the equation of the hyperbola is,

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \frac{y^2}{25} - \frac{x^2}{39} = 1 \text{ [Answer]}$$

Question: 13

Solution:

Given: Foci are $(\pm\sqrt{29}, 0)$, the transverse axis is of the length 10

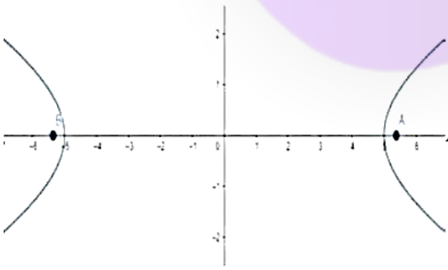
Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The transverse axis is of the length 10, i.e., 2a = 10

Therefore, a = 5

The foci are given at $(\pm\sqrt{29}, 0)$



A and B are the foci.

That means, $ae = \pm\sqrt{29}$, where e is the eccentricity.

$$\Rightarrow 5e = \sqrt{29} \text{ [As a = 5]}$$

$$\Rightarrow e = \frac{\sqrt{29}}{5}$$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \frac{\sqrt{29}}{5}$$

$$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{29}{25} \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{29}{25} - 1 = \frac{4}{25}$$

$$\Rightarrow b^2 = a^2 \frac{4}{25}$$

$$\Rightarrow b^2 = 25 \times \frac{4}{25} = 4 \text{ [As a = 5]}$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{25} - \frac{y^2}{4} = 1 \text{ [Answer]}$$

Question: 14

Solution:

Given: Foci are (±5, 0), the conjugate axis is of the length 8

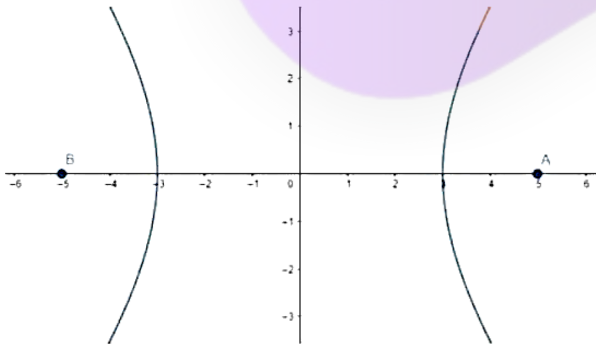
Need to find: The equation of the hyperbola and eccentricity.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The conjugate axis is of the length 8, i.e., 2b = 8

Therefore, b = 4

The foci are given at (±5, 0)



A and B are the foci.

That means, ae = 5, where e is the eccentricity.

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$\Rightarrow a\sqrt{1+\frac{b^2}{a^2}} = 5$

$\Rightarrow \sqrt{1+\frac{b^2}{a^2}} = \frac{5}{a}$

$\Rightarrow 1+\frac{b^2}{a^2} = \frac{25}{a^2}$ [Squaring both sides]

$\Rightarrow \frac{b^2}{a^2} = \frac{25}{a^2} - 1 = \frac{25-a^2}{a^2}$

$\Rightarrow b^2 = 25 - a^2$

$\Rightarrow a^2 = 25 - b^2 = 25 - 16 = 9$

So, the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

Eccentricity, $e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1-\frac{16}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$ [Answer]

Question: 15

Solution:

Given: Foci are $(\pm 3\sqrt{5}, 0)$ the length of the latus rectum is 8 units

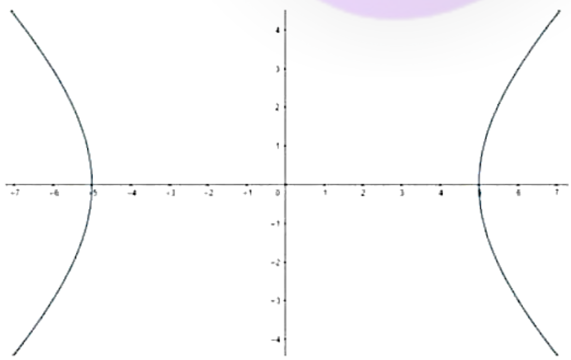
Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The length of the latus rectum is 8 units.

Therefore, $\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a$ ---- (1)

The foci are given at $(\pm 3\sqrt{5}, 0)$



That means, ae $= 3\sqrt{5}$, where e is the eccentricity.

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

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Therefore,

$\Rightarrow a\sqrt{1 + \frac{b^2}{a^2}} = 3\sqrt{5}$

$\Rightarrow a\frac{\sqrt{a^2 + b^2}}{a} = 3\sqrt{5}$

$\Rightarrow a^2 + b^2 = 45$ [Squaring both sides]

$\Rightarrow a^2 + 4a = 45$ [From (1)]

$\Rightarrow a^2 + 4a - 45 = 0$

$\Rightarrow a^2 + 9a - 5a - 45 = 0$

$\Rightarrow (a + 9)(a - 5) = 0$

So, either a = 5 or, a = -9

That means, either b = $2\sqrt{5}$ or, b = $\sqrt{-36}$

The value of b = $\sqrt{-36}$ is not a valid one. So, the b value and its corresponding a value is not acceptable.

Hence, the acceptable value of a is 5 and b is $2\sqrt{5}$

So, the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{25} - \frac{y^2}{20} = 1$ [Answer]

Question: 16

Solution:

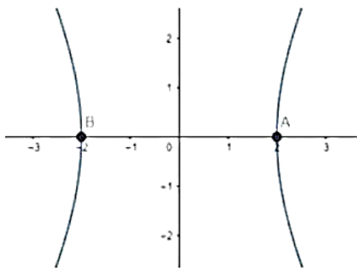
Given: Vertices are (±2, 0) and the eccentricity is 2

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Vertices are (±2, 0), that means, a = 2

And also given, the eccentricity, e = 2



A and B are the foci.

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = 2$

$\Rightarrow 1 + \frac{b^2}{a^2} = 4$ [Squaring both sides]

$\Rightarrow \frac{b^2}{a^2} = 3$

$\Rightarrow b^2 = 3a^2 = 3 \times 4 = 12$ [As a = 2]

So, the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{4} - \frac{y^2}{12} = 1$ [Answer]

Question: 17

Solution:

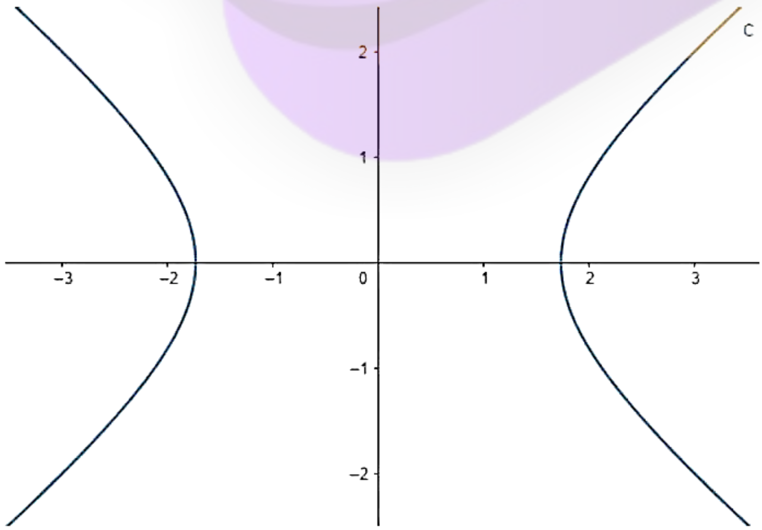
Given: Foci are $(\pm\sqrt{5}, 0)$, and the eccentricity is $\sqrt{\frac{5}{3}}$

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The eccentricity, e = $\sqrt{\frac{5}{3}}$

And also given, foci are $(\pm\sqrt{5}, 0)$



That means, $ae = \sqrt{5}$

$\Rightarrow a = \frac{\sqrt{5}}{e}$

$\Rightarrow a = \frac{\sqrt{5}}{\sqrt{\frac{5}{3}}}$ [As $e = \sqrt{\frac{5}{3}}$]

$\Rightarrow a = \sqrt{3}$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{5}{3}}$

$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{5}{3}$ [Squaring both sides]

$\Rightarrow \frac{b^2}{a^2} = \frac{5}{3} - 1 = \frac{2}{3}$

$\Rightarrow b^2 = \frac{2}{3}a^2 = \frac{2}{3} \times 3 = 2$ [As $a = \sqrt{3}$]

So, the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{3} - \frac{y^2}{2} = 1$ [Answer]

Question: 18

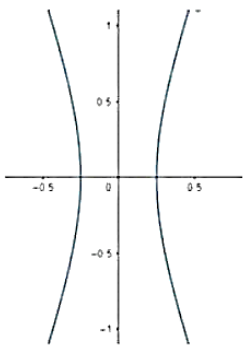
Solution:

Given: The length of latus rectum is 4, and the eccentricity is 3

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The length of the latus rectum is 4 units.



Therefore, $\frac{2b^2}{a} = 4 \Rightarrow b^2 = 2a$ ---- (1)

And also given, the eccentricity, e = 3

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = 3$

$\Rightarrow 1 + \frac{b^2}{a^2} = 9$ [Squaring both sides]

$\Rightarrow \frac{b^2}{a^2} = 8$

$\Rightarrow b^2 = 8a^2$

$\Rightarrow 2a = 8a^2$ [From (1)]

$\Rightarrow a = \frac{1}{4}$

Therefore,

$b^2 = 2a = 2 \times \frac{1}{4} = \frac{1}{2}$

So, the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{1/16} - \frac{y^2}{1/2} = 1 \Rightarrow 16x^2 - 2y^2 = 1$ [Answer]

Question: 19

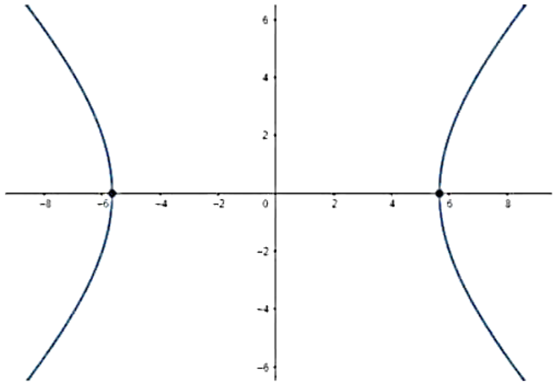
Solution:

Given: Eccentricity is $\sqrt{2}$, and the distance between foci is 16

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Distance between the foci is 16, i.e., 2ae = 16



And also given, the eccentricity, e = $\sqrt{2}$

Therefore,

$$2a\sqrt{2} = 16$$

$$a = \frac{16}{2\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2} \text{ ---- (1)}$$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{2}$$

$$\Rightarrow 1 + \frac{b^2}{a^2} = 2 \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{b^2}{a^2} = 1$$

$$\Rightarrow b^2 = a^2 = 32 \text{ [From (1)]}$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1 \Rightarrow x^2 - y^2 = 32 \text{ [Answer]}$$

Question: 20

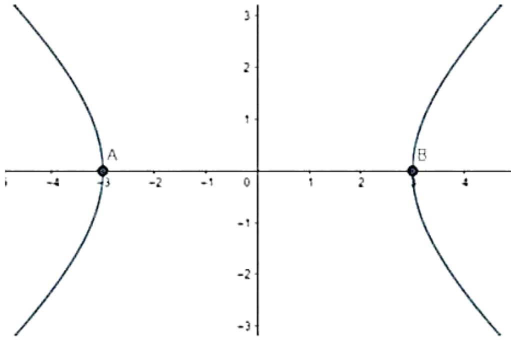
Solution:

Given: Vertices are (0, ±3) and the eccentricity is $\frac{4}{3}$

Need to find: The equation of the hyperbola and coordinates of foci.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Vertices are (±3, 0), that means, a = 3



A and B are vertices.

And also given, the eccentricity, $e = \frac{4}{3}$

We know that,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Therefore,

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \frac{4}{3}$$

$$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{16}{9} \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{16}{9} - 1 = \frac{7}{9}$$

$$\Rightarrow b^2 = \frac{7}{9}a^2 = \frac{7}{9} \times 9 = 7 \text{ [As a = 3]}$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{9} - \frac{y^2}{7} = 1$$

Coordinates of the foci = $(\pm ae, 0) = (\pm 4, 0)$ [Answer]

Question: 21

Solution:

Given: Foci are $(0, \pm 13)$, the conjugate axis is of the length 24

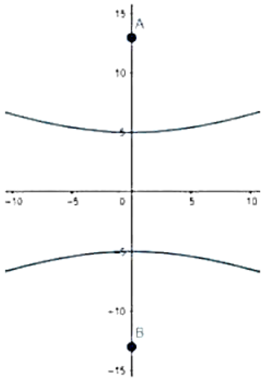
Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

The conjugate axis is of the length 24, i.e., $2b = 24$

Therefore, $b = 12$

The foci are given at $(0, \pm 13)$



A and B are the foci.

That means, ae = 13, where e is the eccentricity.

We know that, $e = \sqrt{1+\frac{b^2}{a^2}}$

Therefore,

$\Rightarrow a\sqrt{1+\frac{b^2}{a^2}} = 13$

$\Rightarrow a\frac{\sqrt{a^2+b^2}}{a} = 13$

$\Rightarrow a^2 + b^2 = 169$ [Squaring both sides]

$\Rightarrow a^2 = 169 - b^2 = 169 - 144 = 25$ [As b = 12]

So, the equation of the hyperbola is,

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \frac{y^2}{25} - \frac{x^2}{144} = 1$ [Answer]

Question: 22

Solution:

Given: Foci are (0, ±10) and the length of latus rectum is 9 units

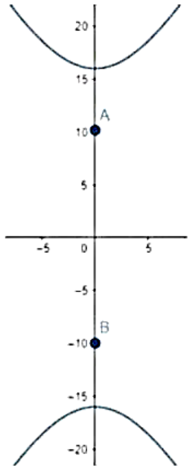
Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

The length of the latus rectum is 9 units.

Therefore, $\frac{2b^2}{a} = 9 \Rightarrow b^2 = \frac{9}{2}a$ ---- (1)

The foci are given at (0, ±10)



That means, ae = 10, where e is the eccentricity.

We know that, $e=\sqrt{1+\frac{b^2}{a^2}}$

Therefore,

$\Rightarrow a\sqrt{1+\frac{b^2}{a^2}}=10$

$\Rightarrow a\frac{\sqrt{a^2+b^2}}{a}=10$

$\Rightarrow a^2+b^2=100$ [Squaring both sides]

$\Rightarrow a^2+\frac{9}{2}a=100$ [From (1)]

$\Rightarrow 2a^2+9a-200=0$

$\Rightarrow 2a^2+25a-16a-200=0$

$\Rightarrow (2a+25)(a-16)=0$

So, either a = 16 or, $a=-\frac{25}{2}$

That means, either $b=\sqrt{\frac{9}{2}\times 16}=6\sqrt{2}$ or, $b=\sqrt{-\frac{9\times 25}{2\times 2}}$

The value of $b=\sqrt{-\frac{9\times 25}{2\times 2}}$ is not a valid one. So, the b value and its corresponding a value is not acceptable.

Hence, the acceptable value of a is 16 and b is $6\sqrt{2}$

So, the equation of the hyperbola is,

$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1 \Rightarrow \frac{y^2}{256}-\frac{x^2}{72}=1$ [Answer]

Question: 23

Find the equation

Solution:

Given: Foci at $(0, \pm\sqrt{14})$ and passing through the point P(3, 4)

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

It passes through the point P(3, 4)

So putting the values of (x, y) we get,

$$\frac{3^2}{a^2} - \frac{4^2}{b^2} = 1 \Rightarrow \frac{9}{a^2} - \frac{16}{b^2} = 1 \text{ ---- (1)}$$

Foci at $(0, \pm\sqrt{14})$

So, $ae = \sqrt{14}$

We know, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$$\Rightarrow a\sqrt{1 + \frac{b^2}{a^2}} = \sqrt{14}$$

$$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{14}$$

$$\Rightarrow a^2 + b^2 = 14 \text{ [Squaring on both sides]}$$

$$\Rightarrow a^2 = 14 - b^2 \text{ ---- (2)}$$

Comparing (1) and (2) we get,

$$\frac{9}{14 - b^2} - \frac{16}{b^2} = 1$$

$$\frac{9}{14 - b^2} = 1 + \frac{16}{b^2} = \frac{b^2 + 16}{b^2}$$

$$9b^2 = 14b^2 - b^4 + 224 - 16b^2$$

$$b^4 + 11b^2 - 224 = 0$$

Solving the equations we get,

$$b_1 = \sqrt{\frac{1}{2}(-11 + 3\sqrt{113})}$$

$$b_2 = -\sqrt{\frac{1}{2}(-11 + 3\sqrt{113})}$$

$$b_3 = (-i)\sqrt{\frac{1}{2}(11 + 3\sqrt{113})}$$

$$b_4 = i\sqrt{\frac{1}{2}(11 + 3\sqrt{113})}$$

With the help of any of these values of b we can't find out the equation of the h

* This is the only process we can apply in this standard.