Chapter : 28. TION

Exercise : 28A

Question: 1

Solution:

(i) x^{-3}

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$ting w.r.t x,

$\frac{d}{dx}x^{-3} = -3x^{-3-1}$

$= -3x^{-4}$

(ii) $\sqrt[3]{x} = x^{\frac{1}{3}}$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$ting w.r.t x,

$\frac{d}{dx}x^{\frac{1}{3}} = \frac{1}{3}x^{\frac{1}{3}-1}$

$= \frac{1}{3}x^{-\frac{2}{3}}$

Question: 2

(i) $\frac{1}{x} = x^{-1}$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$ting w.r.t x,

$\frac{d}{dx}x^{-1} = -1x^{-1-1}$

$= -x^{-2}$

(ii) $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$ting w.r.t x,

$\frac{d}{dx}x^{\frac{-1}{2}} = \frac{-1}{2}x^{-\frac{1}{2}-1}$

$= \frac{-1}{2}x^{-\frac{3}{2}}$

(iii) $\frac{1}{\sqrt[3]{x}} = x^{\frac{-1}{3}}$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$ting w.r.t x,

$\frac{d}{dx}x^{\frac{-1}{3}} = \frac{-1}{3}x^{\frac{-1}{3}-1}$

$= -\frac{1}{3}x^{-\frac{4}{3}}$

Question: 3

(i) $3x^{-5}$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$

ting with respect to x,

$\frac{d}{dx}3x^{-5} = 3(-5)x^{-5-1}$

$= -15x^{-6}$

(ii) $1/5x = \frac{1}{5}x^{-1}$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$

ting with respect to x,

$\frac{1}{5}\frac{d}{dx}x^{-1} = \frac{-1}{5}x^{-1-1}$

$= -\frac{1}{5}x^{-2}$

(iii) $6.\sqrt[3]{x^2} = 6x^{\frac{2}{3}}$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$

ting with respect to x,

$\frac{d}{dx}6x^{\frac{2}{3}} = 6 \times \frac{2}{3}x^{\frac{2}{3}-1}$

$= 4x^{-\frac{1}{3}}$

Question: 4

(i) $6x^5 + 4x^3 - 3x^2 + 2x - 7$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$

ting with respect to x,

$\frac{d}{dx}(6x^5 + 4x^3 - 3x^2 + 2x - 7) = 30x^{5-1} + 12x^{3-1} - 6x^{2-1} + 2x^{1-1} + 0$

$= 30x^4 + 12x^2 - 6x^1 + 2x$

(ii) $5x^{-3/2} + \frac{4}{\sqrt{x}} + \sqrt{x} - \frac{7}{x}$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$

ting with respect to x,

$\frac{d}{dx}(5x^{-3/2} + \frac{4}{\sqrt{x}} + \sqrt{x} - \frac{7}{x})$

$= 5 \times -\frac{3}{2}x^{-\frac{3}{2}-1} + 4 \times -\frac{1}{2}x^{-\frac{1}{2}-1} + \frac{1}{2}x^{\frac{1}{2}-1} - 7 \times -1x^{-1-1}$

$= -\frac{15}{2}x^{-\frac{5}{2}} - 2x^{-\frac{3}{2}} + \frac{1}{2}x^{\frac{-1}{2}} + 7x^{-2}$

(iii) $ax^3 + bx^2 + cx + d$, where a, b, c, d are constants

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$

ting with respect to x,

$\frac{d}{dx}(ax^3 + bx^2 + cx + d) = 3ax^{3-1} + 2bx^{2-1} + cx^{1-1} + d\times 0$

$= 3ax^2 + 2bx + c$

Question: 5

(i) $4x^3 + 3.2^x + 6.\sqrt[6]{x^{-4}} + 5\cot x$

$= 4x^3 + 3.2^x + 6x^{-\frac{1}{2}} + 5\cot x$

Formulae:

$\frac{d}{dx}x^n = nx^{n-1}$

$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

$\frac{d}{dx}a^x = \log_n(a)\times a^x$

ting with respect to x,

$\frac{d}{dx}(4x^3 + 3.2^x + 6x^{-\frac{1}{2}} + 5\cot x)$

$= 4.3x^{3-1} + 3.\log_n(2).2^x + 6\times -\frac{1}{2}x^{-\frac{1}{2}-1} + 5 \times - \operatorname{cosec}^2 x$

$= 12x^2 + 3.\log_n(2).2^x - 3x^{-\frac{3}{2}} - 5\operatorname{cosec}^2 x$

(ii) $\frac{x}{3} - \frac{3}{x} + \sqrt{x} - \frac{1}{\sqrt{x}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6$

$= \frac{x}{3} - 3x^{-1} + x^{\frac{1}{2}} - x^{-\frac{1}{2}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6$

$\frac{d}{dx}x^n = nx^{n-1}$

$\frac{d}{dx}a^x = \log_n(a)\times a^x$

ting with respect to x,

$$\frac{d}{dx}\left(\frac{x}{3} - 3x^{-1} + x^{\frac{1}{2}} - x^{-\frac{1}{2}} + x^2 - 2^x + 6x^{-\frac{2}{3}} - \frac{2}{3}x^6\right)$$

$$= \frac{1}{3} - (-1) \times 3x^{-1-1} + \frac{1}{2}x^{\frac{1}{2}-1} - \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} + 2x^{2-1} - \log(2).2^x + 6\left(-\frac{2}{3}\right)x^{-\frac{2}{3}-1} - \frac{2}{3} \times 6x^{6-1}$$

$$= \frac{1}{3} + 3x^{-2} + \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} + 2x^1 - \log(2).2^x - 4x^{-\frac{5}{3}} - 4x^5$$

Question: 6

Formulae: -

$\frac{d}{dx}\cot x = -\ \operatorname{cosec}^2 x$

$\frac{d}{dx}\cos x = -\ \sin x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\operatorname{cosec} x = -\ \operatorname{cosec} x \cot x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}k = 0$,k is constant

(i) $4\cot x - \frac{1}{2}\cos x + \frac{2}{\cos x} - \frac{3}{\sin x} + \frac{6\cot x}{\operatorname{cosec} x} + 9$

$= 4\cot x - \frac{1}{2}\cos x + 2\sec x - 3\operatorname{cosec} x + 6\cos x + 9$

ting with respect to x,

$$\frac{d}{dx}\left(4\cot x - \frac{1}{2}\cos x + 2\sec x - 3\operatorname{cosec} x + 6\cos x + 9\right)$$

$= 4(-\operatorname{cosec}^2 x) - \frac{1}{2}(-\sin x) + 2\sec x \times \tan x - 3(-\operatorname{cosec} x \times \cot x) + 6(-\sin x) + 0$

$= -4\operatorname{cosec}^2 x + \frac{1}{2}\sin x + 2\sec x \tan x + 3\operatorname{cosec} x \cot x - 6\sin x$

(ii) -5 tan x + 4 tan x cos x – 3 cot x sec x + 2sec x – 13

= -5 tan x + 4 sinx – 3 cosecx+ 2sec x – 13

ting with respect to x,

$\frac{d}{dx}$(-5 tan x + 4 sinx – 3 cosecx+ 2sec x – 13)

= -5 $\sec^2 x$ + 4cosx -3(- cosecx cotx) + 2 secx tanx – 0

= -5 $\sec^2 x$ + 4cosx + 3 cosecx cotx + 2 secx tanx

Question: 7

Formula:

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g)\frac{d}{dx}g$$

Chain rule -

$$\frac{d}{dx}(uv) = u\frac{d}{dx}v + v\frac{d}{dx}u$$

Where u and v are the functions of x.

(i) $(2x + 3)(3x - 5)$

Applying, Chain rule

Here, $u = 2x + 3$

$V = 3x - 5$

$\frac{d}{dx}(2x + 3)(3x - 5) = (2x + 3)\frac{d}{dx}(3x - 5) + (3x - 5)\frac{d}{dx}(2x + 3)$

$= (2x + 3)(3x^{1-1} + 0) + (3x - 5)(2x^{1-1} + 0)$

$= 6x + 9 + 6x - 10$

$= 12x - 1$

(ii) $x(1 + x)^3$

Applying, Chain rule

Here, $u = x$

$V = (1 + x)^3$

$\frac{d}{dx}x(1 + x)^3 = x\frac{d}{dx}(1 + x)3 + (1 + x)3\frac{d}{dx}(x)$

$= x \times 3 \times (1 + x)^2 + (1 + x)^3(1)$

$= (1 + x)^2(3x + x + 1)$

$= (1 + x)^2(4x + 1)$

(iii) $\left(\sqrt{x} + \frac{1}{x}\right)\left(x - \frac{1}{\sqrt{x}}\right) = (x^{1/2} + x^{-1})(x - x^{-1/2})$

Applying, Chain rule

Here, $u = (x^{1/2} + x^{-1})$

$V = (x - x^{-1/2})$

$\frac{d}{dx}(x^{1/2} + x^{-1})(x - x^{-1/2})$

$= (x^{1/2} + x^{-1})\frac{d}{dx}(x - x^{-1/2}) + (x - x^{-1/2})\frac{d}{dx}(x^{1/2} + x^{-1})$

$= (x^{1/2} + x^{-1})(1 + \frac{1}{2}x^{-3/2}) + (x - x^{-1/2})(\frac{1}{2}x^{-1/2} - x^{-2})$

$= x^{1/2} + x^{-1} + \frac{1}{2}x^{-1} + \frac{1}{2}x^{-5/2} + \frac{1}{2}x^{1/2} - x^{-1} - \frac{1}{2}x^{-1} + x^{-5/2}$

$= \frac{3}{2}x^{1/2} + \frac{3}{2}x^{-5/2}$

(iv) $\left(x - \frac{1}{x}\right)^2$

tion of composite function can be done by

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g)\frac{d}{dx}g$$

Here, $f(g) = g^2$, $g(x) = x - \frac{1}{x}$

$\frac{d}{dx}\left(x - \frac{1}{x}\right)^2 = 2g \times (1 + \frac{1}{x^2})$

$= 2(x - \frac{1}{x})\ (1 + \frac{1}{x^2})$

$= 2(x + \frac{1}{x} - \frac{1}{x} + \frac{1}{x^3})$

$= 2(x + \frac{1}{x^3})$

(v) $\left(x^2 - \frac{1}{x^2}\right)^3$

tion of composite function can be done by

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g)\frac{d}{dx}g$$

Here, $f(g) = g^3$, $g(x) = x^2 - \frac{1}{x^2}$

$\frac{d}{dx}\left(x^2 - \frac{1}{x^2}\right)^3 = 3g^2 \times (2x - \frac{2}{x^3})$

$= 3\left(x^2 - \frac{1}{x^2}\right)^2 (2x - \frac{2}{x^3})$

$= 3(2x^3 - \frac{2}{x} - \frac{2}{x} + \frac{2}{x^5})$

$= 3(2x^3 - \frac{4}{x} + \frac{2}{x^5})$

(vi) $(2x^2 + 5x - 1)(x - 3)$

Applying, Chain rule

Here, $u = (2x^2 + 5x - 1)$

$V = (x - 3)$

$\frac{d}{dx}(2x^2 + 5x - 1)(x - 3)$

$= (2x^2 + 5x - 1)\frac{d}{dx}(x - 3) + (x - 3)\frac{d}{dx}(2x^2 + 5x - 1)$

$= (2x^2 + 5x - 1) \times 1 + (x - 3)(4x + 5)$

$= 2x^2 + 5x - 1 + 4x^2 - 7x - 15$

$= 6x^2 - 2x - 16$

Question: 8

Formula:

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{d}{dx}u - u\frac{d}{dx}v}{u^2}$$

(i) $\frac{3x^2+4x-5}{x}$

Applying, quotient rule

$$\frac{d}{dx}\frac{3x^2 + 4x - 5}{x} = \frac{x\frac{d}{dx}(3x^2 + 4x - 5) - (3x^2 + 4x - 5)\frac{d}{dx}x}{x^2}$$

$= \frac{x(6x+4)-(3x^2+4x-5)1}{x^2}$

$= \frac{6x^2+4x-(3x^2+4x-5)}{x^2}$

$= \frac{3x^2+5}{x^2}$

(ii) $\frac{(x^3+1)(x-2)}{x^2}$

Applying, quotient rule

$$\frac{d}{dx}\frac{(x^3+1)(x-2)}{x^2} = \frac{x^2\frac{d}{dx}(x^3+1)(x-2) - (x^3+1)(x-2)\frac{d}{dx}x^2}{x^4}$$

$$= \frac{x^2\{(x^3+1)\frac{d}{dx}(x-2)+(x-2)\frac{d}{dx}(x^3+1)\}-(x^3+1)(x-2)2x}{x^4}$$

$$= \frac{x^2\{(x^3+1)+(x-2)3x^2\}-(x^3+1)(x-2)2x}{x^4}$$

$$= \frac{x^2\{x^3+1+3x^3-6x^2\}-2(x^4+x)(x-2)}{x^4}$$

$$= \frac{4x^5-6x^4+x^2-2(x^5-2x^4+x^2-2x)}{x^4}$$

$$= \frac{2x^5-2x^4-x^2+4x}{x^4}$$

(iii) $\frac{x-4}{2\sqrt{x}}$

Applying, quotient rule

$$\frac{d}{dx}\frac{x-4}{2\sqrt{x}} = \frac{2\sqrt{x}\frac{d}{dx}(x-4) - (x-4)\frac{d}{dx}2\sqrt{x}}{4x}$$

$$= \frac{2\sqrt{x}-(x-4)2\frac{1}{2}x^{-\frac{1}{2}}}{4x}$$

$$= \frac{2\sqrt{x}-(x-4)x^{-\frac{1}{2}}}{4x}$$

$$= \frac{2\sqrt{x}-x^{\frac{1}{2}}+4x^{-\frac{1}{2}}}{4x}$$

$$= \frac{\sqrt{x}+4x^{-\frac{1}{2}}}{4x}$$

(iv) $\frac{(1+x)\sqrt{x}}{\sqrt[3]{x}}$

Applying, quotient rule

$$\frac{d}{dx}\frac{(1+x)\sqrt{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}\frac{d}{dx}(1+x)\sqrt{x} - (1+x)\sqrt{x}\frac{d}{dx}\sqrt[3]{x}}{x^{\frac{2}{3}}}$$

$$= \frac{\sqrt[3]{x}\left\{(1+x)\frac{d}{dx}\sqrt{x}+\sqrt{x}\frac{d}{dx}(1+x)\right\}-(1+x)\sqrt{x}\cdot\frac{1}{3}x^{\frac{-2}{3}}}{x^{\frac{2}{3}}}$$

$$= \frac{\sqrt[3]{x}\left\{(1+x)\cdot\frac{1}{2}x^{-\frac{1}{2}}+\sqrt{x}\right\}-(1+x)\cdot\frac{1}{3}x^{\frac{-1}{6}}}{x^{\frac{2}{3}}}$$

$$= \frac{\sqrt[3]{x}\left\{\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{\frac{1}{2}}+\sqrt{x}\right\}-\frac{1}{3}\left(x^{\frac{-1}{6}}+x^{\frac{5}{6}}\right)}{x^{\frac{2}{3}}}$$

$$= \frac{\sqrt[3]{x}\left\{\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{\frac{1}{2}}+\sqrt{x}\right\}-\frac{1}{3}\left(x^{\frac{-1}{6}}+x^{\frac{5}{6}}\right)}{x^{\frac{2}{3}}}$$

$$= \frac{\frac{1}{2}x^{-\frac{1}{6}} + \frac{1}{2}x^{\frac{5}{6}} + \sqrt{x} - \frac{1}{3}\left(x^{\frac{-1}{6}} + x^{\frac{5}{6}}\right)}{x^{\frac{2}{3}}}$$

$$= \frac{\frac{1}{6}x^{-\frac{1}{6}} + \frac{1}{6}x^{\frac{5}{6}} + \sqrt{x}}{x^{\frac{2}{3}}}$$

(v) $\frac{ax^2+bx+c}{\sqrt{x}}$

Applying, quotient rule

$$\frac{d}{dx}\frac{ax^2+bx+c}{\sqrt{x}} = \frac{\sqrt{x}\frac{d}{dx}(ax^2+bx+c) - (ax^2+bx+c)\frac{d}{dx}\sqrt{x}}{x}$$

$$= \frac{\sqrt{x}(2ax+b) - \frac{1}{2}(ax^2+bx+c)x^{-\frac{1}{2}}}{x}$$

$$= \frac{\frac{3}{2}ax^{\frac{3}{2}} + \frac{1}{2}bx^{\frac{1}{2}} - \frac{1}{2}cx^{-\frac{1}{2}}}{x}$$

(vi) $\frac{a+b\cos x}{\sin x}$

Applying, quotient rule

$$\frac{d}{dx}\frac{a+b\cos x}{\sin x} = \frac{\sin x\frac{d}{dx}(a+b\cos x) - (a+b\cos x)\frac{d}{dx}\sin x}{\sin^2 x}$$

$$= \frac{\sin x(-b\sin x) - (a+b\cos x)\cos x}{\sin^2 x}$$

$$= \frac{-b\sin^2 x - a\cos x - b\cos^2 x}{\sin^2 x}$$

$$= \frac{-b(1) - a\cos x}{\sin^2 x}$$

Question: 9

Formulae:

$\frac{d}{dx}x^n = nx^{n-1}$

$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\sin x = \cos x$

(i) If $y = 6x^5 - 4x^4 - 2x^2 + 5x - 9$, find $\frac{dy}{dx}$ at $x = -1$.

ting with respect to x,

$\frac{d}{dx}(6x^5 - 4x^4 - 2x^2 + 5x - 9)$

$= 30x^4 - 16x^3 - 4x + 5$

substituting $x = -1$

$\left(\frac{dy}{dx}\right)_{x=-1} = 30(-1)^4 - 16(-1)^3 - 4(-1) + 5$

= 30+16+4+5

= 55

(ii) If $y = (\sin x + \tan x)$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{3}$.

ting with respect to x,

$\frac{d}{dx}(\sin x + \tan x) = \cos x + \sec^2 x$

Substituting $x = \frac{\pi}{3}$

$\left(\frac{dy}{dx}\right)_{x = \pi/3} = \cos\frac{\pi}{3} + \sec^2\frac{\pi}{3}$

$= \frac{1}{2} + 4$

$= \frac{5}{2}$

(iii) If $y = \frac{(2-3\cos x)}{\sin x}$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$.

ting with respect to x,

$\frac{d}{dx}(2\text{cosec } x - 3\cot x) = 2(-\text{cosec}x \cot x) - 3(-\text{cosec}^2 x)$

Substituting $x = \frac{\pi}{4}$

$\left(\frac{dy}{dx}\right)_{x = \pi/4} = 2(-\text{cosec}\frac{\pi}{4}\cot\frac{\pi}{4}) - 3(-\text{cosec}^2\frac{\pi}{4})$

$= -2\times\sqrt{2} + 3\times 2$

$= 6 - 2\times\sqrt{2}$

Question: 10

If To show: $2x.\frac{dy}{dx} + y = 2\sqrt{x}$ ting

with respect to x $\frac{dy}{dx} =$

$\frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}}$

Now,

LHS = $2x.\frac{dy}{dx} + y$

LHS = $2x \times (\frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}}) + \sqrt{x} + \frac{1}{\sqrt{x}}$

LHS = $\sqrt{x} - \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{1}{\sqrt{x}}$

LHS = $2\sqrt{x}$

∴ LHS = RHS

Question: 11

If To prove: $(2xy)\left(\frac{dy}{dx}\right) = \left(\frac{x}{a} - \frac{a}{x}\right)$.

ting y with respect to x

$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}\right) = \frac{1}{2\sqrt{ax}} - \frac{\sqrt{a}}{2x^{\frac{3}{2}}}$

Now,

LHS = $(2xy)\left(\frac{dy}{dx}\right)$

LHS = $2x\left(\sqrt{\frac{x}{a}}+\sqrt{\frac{a}{x}}\right)\left(\frac{1}{2\sqrt{ax}}-\frac{\sqrt{a}}{2x^{\frac{3}{2}}}\right)$

LHS = $(\sqrt{\frac{x}{a}}+\sqrt{\frac{a}{x}})(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}})$

LHS = $\left(\frac{x}{a}-\frac{a}{x}\right)$

∴ LHS = RHS

Question: 12

If $y=\sqrt{\frac{1+\cos 2x}{1-\cos 2x}}$

Formula:

Using double angle formula:

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$\therefore 1 + \cos 2x = 2\cos^2 x$

$1 - \cos 2x = 2\sin^2 x$

$\therefore y=\sqrt{\frac{2\cos^2 x}{2sinn^2 x}}$

$=\sqrt{\cot^2 x}$

$= \cot x$

ting y with respect to x

$\frac{dy}{dx}=\frac{d}{dx}(\cot x)$

$= - \text{cosec}^2\, x$

Question: 13

Formula:

Using Half angle formula,

$\cos x = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}$

$\therefore y = \cos x$

ting y with respect to x

$\frac{dy}{dx}=\frac{d}{dx}\cos x$

$= -\sin x$

Exercise : 28B

Question: 1
Solution:

Let f(x) = ax + b

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad ...(i)$$

f(x) = ax + b

f(x + h) = a(x + h) + b

= ax + ah + b

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{ax + ah + b - (ax + b)}{h}$$

$$= \lim_{h\to 0} \frac{ax + ah + b - ax - b}{h}$$

$$= \lim_{h\to 0} \frac{ah}{h}$$

$$= \lim_{h\to 0} a$$

f'(x) = a

Hence, f'(x) = a

Question: 2

Solution:

Let $f(x) = ax^2 + \frac{b}{x}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad ...(i)$$

$$f(x) = ax^2 + \frac{b}{x}$$

$$f(x+h) = a(x+h)^2 + \frac{b}{(x+h)}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{\left[a(x+h)^2 + \frac{b}{(x+h)}\right] - \left[ax^2 + \frac{b}{x}\right]}{h}$$

$$= \lim_{h\to 0} \frac{a(x+h)^2 + \frac{b}{(x+h)} - ax^2 - \frac{b}{x}}{h}$$

$$= \lim_{h\to 0} \frac{a[(x+h)^2 - x^2] + b\left[\frac{1}{x+h} - \frac{1}{x}\right]}{h}$$

$$= \lim_{h\to 0} \frac{a[x^2 + h^2 + 2xh - x^2] + h\left[\frac{x-(x+h)}{x(x+h)}\right]}{h}$$

$= \lim_{h\to 0} \frac{a[h^2 + 2xh] + b\left[\frac{x - x - h}{x(x+h)}\right]}{h}$

$= \lim_{h\to 0} \frac{a[h^2 + 2xh] + b\left[\frac{-h}{x(x+h)}\right]}{h}$

$= \lim_{h\to 0} \left[\frac{ah(h+2x)}{h} + \frac{b(-h)}{hx(x+h)}\right]$

Taking 'h' common from both the numerator and denominator, we get

$= \lim_{h\to 0} \left[a(h+2x) - \frac{b}{x(x+h)}\right]$

Putting h = 0, we get

$= a[(0) + 2x] - \frac{b}{x(x+0)}$

$= 2ax - \frac{b}{x^2}$

Hence, $f'(x) = 2ax - \frac{b}{x^2}$

Question: 3

Solution:

Let $f(x) = 3x^2 + 2x - 5$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ...(i)

$f(x) = 3x^2 + 2x - 5$

$f(x + h) = 3(x + h)^2 + 2(x + h) - 5$

$= 3(x^2 + h^2 + 2xh) + 2x + 2h - 5$

$[\because (a + b)^2 = a^2 + b^2 + 2ab]$

$= 3x^2 + 3h^2 + 6xh + 2x + 2h - 5$

Putting values in (i), we get

$f'(x) = \lim_{h\to 0} \frac{3x^2 + 3h^2 + 6xh + 2x + 2h - 5 - (3x^2 + 2x - 5)}{h}$

$= \lim_{h\to 0} \frac{3x^2 + 3h^2 + 6xh + 2x + 2h - 5 - 3x^2 - 2x + 5}{h}$

$= \lim_{h\to 0} \frac{3h^2 + 6xh + 2h}{h}$

$= \lim_{h\to 0} 3h + 6x + 2$

Putting h = 0, we get

$f'(x) = 3(0) + 6x + 2$

$= 6x + 2$

Hence, f'(x) = 6x + 2

Question: 4

Solution:

Let $f(x) = x^3 - 2x^2 + x + 3$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$f(x) = x^3 - 2x^2 + x + 3$

$f(x + h) = (x + h)^3 - 2(x + h)^2 + (x + h) + 3$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - 2(x+h)^2 + (x+h) + 3 - [x^3 - 2x^2 + x + 3]}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^3 - 2(x+h)^2 + (x+h) + 3 - x^3 + 2x^2 - x - 3}{h}$$

$$= \lim_{h\to 0} \frac{[(x+h)^3 - x^3] - 2[(x+h)^2 - x^2] + [x + h - x]}{h}$$

Using the identities:

$(a + b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b$

$(a + b)^2 = a^2 + b^2 + 2ab$

$$= \lim_{h\to 0} \frac{[x^3 + h^3 + 3xh^2 + 3x^2h - x^3] - 2[x^2 + h^2 + 2xh - x^2] + h}{h}$$

$$= \lim_{h\to 0} \frac{[h^3 + 3xh^2 + 3x^2h] - 2[h^2 + 2xh] + h}{h}$$

$$= \lim_{h\to 0} \frac{h[h^2 + 3xh + 3x^2] - 2h[h + 2x] + h}{h}$$

$$= \lim_{h\to 0} h^2 + 3xh + 3x^2 - 2h - 4x + 1$$

Putting h = 0, we get

$f'(x) = (0)^2 + 2x(0) + 3x^2 - 2(0) - 4x + 1$

$= 3x^2 - 4x + 1$

Hence, $f'(x) = 3x^2 - 4x + 1$

Question: 5

Solution:

Let $f(x) = x^8$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$f(x) = x^8$

$f(x + h) = (x + h)^8$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^8 - x^8}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^8 - x^8}{(x+h) - x}$$

[Add and subtract x in denominator]

$$= \lim_{z \to x} \frac{z^8 - x^8}{z - x} \text{ where } z = x + h \text{ and } z \to x \text{ as } h \to 0$$

$$= 8x^{8-1}\left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$= 8x^7$

Hence, $f'(x) = 8x^7$

Question: 6

Solution:

Let $f(x) = \frac{1}{x^3}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \ldots(i)$$

$$f(x) = \frac{1}{x^3}$$

$$f(x+h) = \frac{1}{(x+h)^3}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^{-3} - x^{-3}}{(x+h) - x}$$

[Add and subtract x in denominator]

$$= \lim_{z \to x} \frac{z^{-3} - x^{-3}}{z - x} \text{ where } z = x + h \text{ and } z \to x \text{ as } h \to 0$$

$$= (-3)x^{-3-1}\left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$= -3x^{-4}$

$$= -\frac{3}{x^4}$$

Hence, $f'(x) = -\frac{3}{x^4}$

Question: 7

Solution:

Let $f(x) = \frac{1}{x^5}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$f(x) = \frac{1}{x^5}$

$f(x+h) = \frac{1}{(x+h)^5}$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{(x+h)^5} - \frac{1}{x^5}}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^{-5} - x^{-5}}{(x+h) - x}$$

[Add and subtract x in denominator]

$$= \lim_{z \to x} \frac{z^{-5} - x^{-5}}{z - x} \text{ where } z = x + h \text{ and } z \to x \text{ as } h \to 0$$

$= (-5)x^{-5-1}\left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$

$= -5x^{-6}$

$= -\frac{5}{x^6}$

Hence, $f'(x) = -\frac{5}{x^6}$

Question: 8

Solution:

Let $f(x) = \sqrt{ax + b}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$f(x) = \sqrt{ax + b}$

$f(x+h) = \sqrt{a(x+h) + b}$

$= \sqrt{ax + ah + b}$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{ax + ah + b} - \sqrt{ax + b}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of

$\sqrt{ax+ah+b}-\sqrt{ax+b}$

$$=\lim_{h\to 0}\frac{\sqrt{ax+ah+b}-\sqrt{ax+b}}{h}\times\frac{\sqrt{ax+ah+b}+\sqrt{ax+b}}{\sqrt{ax+ah+b}+\sqrt{ax+b}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$=\lim_{h\to 0}\frac{\left(\sqrt{ax+ah+b}\right)^2-\left(\sqrt{ax+b}\right)^2}{h\left(\sqrt{ax+ah+b}+\sqrt{ax+b}\right)}$$

$$=\lim_{h\to 0}\frac{ax+ah+b-ax-b}{h\left(\sqrt{ax+ah+b}+\sqrt{ax+b}\right)}$$

$$=\lim_{h\to 0}\frac{ah}{h\left(\sqrt{ax+ah+b}+\sqrt{ax+b}\right)}$$

$$=\lim_{h\to 0}\frac{a}{\sqrt{ax+ah+b}+\sqrt{ax+b}}$$

Putting **h** = 0, we get

$$=\frac{a}{\sqrt{ax+a(0)+b}+\sqrt{ax+b}}$$

$$=\frac{a}{\sqrt{ax+b}+\sqrt{ax+b}}$$

$$=\frac{a}{2\sqrt{ax+b}}$$

Hence, $f'(x)=\frac{a}{2\sqrt{ax+b}}$

Question: 9

Solution:

Let $f(x) = \sqrt{5x-4}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ...(i)

$f(x) = \sqrt{5x-4}$

$f(x+h) = \sqrt{5(x+h)-4}$

$= \sqrt{5x+5h-4}$

Putting values in (i), we get

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{5x+5h-4}-\sqrt{5x-4}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of $\sqrt{5x+5h-4}-\sqrt{5x-4}$

$$=\lim_{h\to 0}\frac{\sqrt{5x+5h-4}-\sqrt{5x-4}}{h}\times\frac{\sqrt{5x+5h-4}+\sqrt{5x-4}}{\sqrt{5x+5h-4}+\sqrt{5x-4}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h \to 0} \frac{\left(\sqrt{5x + 5h - 4}\right)^2 - \left(\sqrt{5x - 4}\right)^2}{h\left(\sqrt{5x + 5h - 4} + \sqrt{5x - 4}\right)}$$

$$= \lim_{h \to 0} \frac{5x + 5h - 4 - 5x + 4}{h\left(\sqrt{5x + 5h - 4} + \sqrt{5x - 4}\right)}$$

$$= \lim_{h \to 0} \frac{5h}{h\left(\sqrt{5x + 5h - 4} + \sqrt{5x - 4}\right)}$$

$$= \lim_{h \to 0} \frac{5}{\sqrt{5x + 5h - 4} + \sqrt{5x - 4}}$$

Putting **h** = 0, we get

$$= \frac{5}{\sqrt{5x + 5(0) - 4} + \sqrt{5x - 4}}$$

$$= \frac{5}{\sqrt{5x - 4} + \sqrt{5x - 4}}$$

$$= \frac{5}{2\sqrt{5x - 4}}$$

Hence, $f'(x) = \frac{5}{2\sqrt{5x-4}}$

Question: 10

Solution:

Let $f(x) = \frac{1}{\sqrt{x+2}}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ...(i)

$$f(x) = \frac{1}{\sqrt{x + 2}}$$

$$f(x + h) = \frac{1}{\sqrt{x + h + 2}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x + h + 2}} - \frac{1}{\sqrt{x + 2}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sqrt{x + 2} - \sqrt{x + h + 2}}{\left(\sqrt{x + h + 2}\right)\left(\sqrt{x + 2}\right)}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of $\sqrt{x + 2} - \sqrt{x + h + 2}$

$$= \lim_{h \to 0} \frac{\sqrt{x + 2} - \sqrt{x + h + 2}}{h\left(\sqrt{x + h + 2}\right)\left(\sqrt{x + 2}\right)} \times \frac{\sqrt{x + 2} + \sqrt{x + h + 2}}{\sqrt{x + 2} + \sqrt{x + h + 2}}$$

Using the formula:

(a + b)(a – b) = (a² – b²)

$$= \lim_{h\to 0} \frac{\left(\sqrt{x+2}\right)^2 - \left(\sqrt{x+h+2}\right)^2}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2}+\sqrt{x+h+2})}$$

$$= \lim_{h\to 0} \frac{x+2-x-h-2}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2}+\sqrt{x+h+2})}$$

$$= \lim_{h\to 0} \frac{-h}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2}+\sqrt{x+h+2})}$$

$$= \lim_{h\to 0} \frac{-1}{(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2}+\sqrt{x+h+2})}$$

Putting h = 0, we get

$$= \frac{-1}{(\sqrt{x+0+2})(\sqrt{x+2})(\sqrt{x+2}+\sqrt{x+0+2})}$$

$$= \frac{-1}{\left(\sqrt{x+2}\right)^2(2\sqrt{x+2})}$$

$$= \frac{-1}{2\left(\sqrt{x+2}\right)^3}$$

Hence, $f'(x) = \frac{-1}{2\left(\sqrt{x+2}\right)^3}$

Question: 11

Solution:

Let $f(x) = \frac{1}{\sqrt{2x+3}}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$$f(x) = \frac{1}{\sqrt{2x+3}}$$

$$f(x+h) = \frac{1}{\sqrt{2x+2h+3}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{2x+2h+3}} - \frac{1}{\sqrt{2x+3}}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sqrt{2x+3}-\sqrt{2x+2h+3}}{(\sqrt{2x+2h+3})(\sqrt{2x+3})}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of $\sqrt{2x+3} - \sqrt{2x+2h+3}$

$$= \lim_{h\to 0} \frac{\sqrt{2x+3} - \sqrt{2x+2h+3}}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})} \times \frac{\sqrt{2x+3} + \sqrt{2x+2h+3}}{\sqrt{2x+3} + \sqrt{2x+2h+3}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h\to 0} \frac{\left(\sqrt{2x+3}\right)^2 - \left(\sqrt{2x+2h+3}\right)^2}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

$$= \lim_{h\to 0} \frac{2x+3-2x-2h-3}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

$$= \lim_{h\to 0} \frac{-2h}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

$$= \lim_{h\to 0} \frac{-2}{(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

Putting h = 0, we get

$$= \frac{-2}{(\sqrt{2x+0+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+0+3})}$$

$$= \frac{-2}{\left(\sqrt{2x+3}\right)^2 (2\sqrt{2x+3})}$$

$$= \frac{-2}{2\left(\sqrt{2x+3}\right)^3}$$

$$= \frac{-1}{\left(\sqrt{2x+3}\right)^3}$$

Hence, $f'(x) = \frac{-1}{\left(\sqrt{2x+3}\right)^3}$

Question: 12

Solution:

Let $f(x) = \frac{1}{\sqrt{6x-5}}$

We need to find the derivative of f(x) i.e. f′(x)

We know that,

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$$f(x) = \frac{1}{\sqrt{6x-5}}$$

$$f(x+h) = \frac{1}{\sqrt{6x+6h-5}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{6x+6h-5}} - \frac{1}{\sqrt{6x-5}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sqrt{6x-5} - \sqrt{6x+6h-5}}{(\sqrt{6x+6h-5})(\sqrt{6x-5})}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of $\sqrt{6x-5} - \sqrt{6x+6h-5}$

$$= \lim_{h \to 0} \frac{\sqrt{6x-5} - \sqrt{6x+6h-5}}{h(\sqrt{6x+6h-5})(\sqrt{6x-5})} \times \frac{\sqrt{6x-5} + \sqrt{6x+6h-5}}{\sqrt{6x-5} + \sqrt{6x+6h-5}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h \to 0} \frac{\left(\sqrt{6x-5}\right)^2 - \left(\sqrt{6x+6h-5}\right)^2}{h(\sqrt{6x+6h-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6h-5})}$$

$$= \lim_{h \to 0} \frac{6x - 5 - 6x - 6h + 5}{h(\sqrt{6x+6h-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6h-5})}$$

$$= \lim_{h \to 0} \frac{-6h}{h(\sqrt{6x+6h-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6h-5})}$$

$$= \lim_{h \to 0} \frac{-6}{(\sqrt{6x+6h-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6h-5})}$$

Putting **h** = 0, we get

$$= \frac{-6}{(\sqrt{6x+6(0)-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6(0)-5})}$$

$$= \frac{-6}{\left(\sqrt{6x-5}\right)^2(2\sqrt{6x-5})}$$

$$= \frac{-6}{2\left(\sqrt{6x-5}\right)^3}$$

$$= \frac{-3}{\left(\sqrt{6x-5}\right)^3}$$

Hence, $f'(x) = \frac{-3}{\left(\sqrt{6x-5}\right)^3}$

Question: 13

Solution:

Let $f(x) = \frac{1}{\sqrt{2-3x}}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ...(i)

$$f(x) = \frac{1}{\sqrt{2-3x}}$$

$$f(x+h) = \frac{1}{\sqrt{2-3(x+h)}} = \frac{1}{\sqrt{2-3x-3h}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{2-3x-3h}} - \frac{1}{\sqrt{2-3x}}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sqrt{2-3x} - \sqrt{2-3x-3h}}{\sqrt{2-3x-3h}(\sqrt{2-3x})}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of $\sqrt{2-3x} - \sqrt{2-3x-3h}$

$$= \lim_{h\to 0} \frac{\sqrt{2-3x} - \sqrt{2-3x-3h}}{h\sqrt{2-3x-3h}(\sqrt{2-3x})} \times \frac{\sqrt{2-3x} + \sqrt{2-3x-3h}}{\sqrt{2-3x} + \sqrt{2-3x-3h}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h\to 0} \frac{\left(\sqrt{2-3x}\right)^2 - \left(\sqrt{2-3x-3h}\right)^2}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h\to 0} \frac{2-3x-2+3x+3h}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h\to 0} \frac{3h}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h\to 0} \frac{3}{(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

Putting h = 0, we get

$$= \frac{3}{(\sqrt{2-3x-3(0)})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3(0)})}$$

$$= \frac{3}{\left(\sqrt{2-3x}\right)^2 (2\sqrt{2-3x})}$$

$$= \frac{3}{2\left(\sqrt{2-3x}\right)^3}$$

Hence, $f'(x) = \frac{3}{2\left(\sqrt{2-3x}\right)^3}$

Question: 14

Solution:

Let $f(x) = \frac{2x+3}{3x+2}$

We need to find the derivative of f(x) i.e. f′(x)

We know that,

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$$f(x) = \frac{2x+3}{3x+2}$$

$$f(x+h) = \frac{2(x+h)+3}{3(x+h)+2} = \frac{2x+2h+3}{3x+3h+2}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0}\frac{\frac{2x+2h+3}{3x+3h+2} - \frac{2x+3}{3x+2}}{h}$$

$$= \lim_{h\to 0}\frac{\frac{(2x+2h+3)(3x+2)-(2x+3)(3x+3h+2)}{(3x+3h+2)(3x+2)}}{h}$$

$$= \lim_{h\to 0}\frac{6x^2+4x+6xh+4h+9x+6-[6x^2+6xh+4x+9x+9h+6]}{h((3x+3h+2)(3x+2))}$$

$$= \lim_{h\to 0}\frac{6x^2+4x+6xh+4h+9x+6-6x^2-6xh-4x-9x-9h-6}{h((3x+3h+2)(3x+2))}$$

$$= \lim_{h\to 0}\frac{-5h}{h((3x+3h+2)(3x+2))}$$

$$= \lim_{h\to 0}\frac{-5}{((3x+3h+2)(3x+2))}$$

Putting h = 0, we get

$$= \frac{-5}{((3x+3(0)+2)(3x+2))}$$

$$= \frac{-5}{(3x+2)(3x+2)}$$

$$= \frac{-5}{(3x+2)^2}$$

Hence, $f'(x) = \frac{-5}{(3x+2)^2}$

Question: 15

Solution:

Let $f(x) = \frac{5-x}{5+x}$

We need to find the derivative of f(x) i.e. f′(x)

We know that,

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ...(i)

$$f(x) = \frac{5-x}{5+x}$$

$$f(x+h) = \frac{5-(x+h)}{5+(x+h)} = \frac{5-x-h}{5+x+h}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0}\frac{\frac{5-x-h}{5+x+h} - \frac{5-x}{5+x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(5-x-h)(5+x)-(5-x)(5+x+h)}{(5+x+h)(5+x)}}{h}$$

$$= \lim_{h\to 0} \frac{25+5x-5x-x^2-5h-xh-[25+5x+5h-5x-x^2-xh]}{h(5+x+h)(5+x)}$$

$$= \lim_{h\to 0} \frac{25-x^2-5h-xh-25-5h+x^2+xh}{h(5+x+h)(5+x)}$$

$$= \lim_{h\to 0} \frac{-10h}{h(5+x+h)(5+x)}$$

$$= \lim_{h\to 0} \frac{-10}{(5+x+h)(5+x)}$$

Putting h = 0, we get

$$= \frac{-10}{(5+x+0)(5+x)}$$

$$= \frac{-10}{(5+x)(5+x)}$$

$$= \frac{-10}{(5+x)^2}$$

Hence, $f'(x) = \frac{-10}{(5+x)^2}$

Question: 16

Solution:

Let $f(x) = \frac{x^2+1}{x}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$$f(x) = \frac{x^2+1}{x}$$

$$f(x+h) = \frac{(x+h)^2+1}{x+h} = \frac{x^2+h^2+2xh+1}{x+h}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{\frac{x^2+h^2+2xh+1}{x+h} - \frac{x^2+1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x^2+h^2+2xh+1)(x)-(x^2+1)(x+h)}{(x+h)(x)}}{h}$$

$$= \lim_{h\to 0} \frac{x^3+xh^2+2x^2h+x-[x^3+x^2h+x+h]}{h(x+h)(x)}$$

$$= \lim_{h\to 0} \frac{x^3+xh^2+2x^2h+x-x^3-x^2h-x-h}{h(x+h)(x)}$$

$$= \lim_{h \to 0} \frac{xh^2 + x^2h - h}{h(x+h)(x)}$$

$$= \lim_{h \to 0} \frac{xh + x^2 - 1}{(x+h)(x)}$$

Putting **h** = 0, we get

$$= \frac{x(0) + x^2 - 1}{(x+0)(x)}$$

$$= \frac{x^2 - 1}{(x)^2}$$

Hence, $f'(x) = \frac{x^2-1}{x^2}$

Question: 17

Solution:

Let $f(x) = \sqrt{\cos 3x}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$f(x) = \sqrt{\cos 3x}$

$f(x+h) = \sqrt{\cos 3(x+h)}$

$= \sqrt{\cos(3x + 3h)}$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of $\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}$

$$= \lim_{h \to 0} \frac{\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}}{h} \times \frac{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h \to 0} \frac{\left(\sqrt{\cos(3x+3h)}\right)^2 - \left(\sqrt{\cos 3x}\right)^2}{h\left(\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}\right)}$$

$$= \lim_{h \to 0} \frac{\cos(3x+3h) - \cos 3x}{h\left(\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}\right)}$$

Using the formula:

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$= \lim_{h \to 0} \frac{-2\sin\frac{3x+3h+3x}{2}\sin\frac{3x+3h-3x}{2}}{h(\sqrt{\cos(3x+3h)}+\sqrt{\cos 3x})}$$

$$= \lim_{h \to 0} \frac{-2\sin\frac{6x+3h}{2}\sin\frac{3h}{2}}{h\sqrt{\cos(3x+3h)}+\sqrt{\cos 3x}}$$

$$= -2\lim_{h \to 0} \frac{\sin\frac{3h}{2}}{\frac{3h}{2}} \times \frac{3}{2}\lim_{h \to 0} \sin(\frac{6x+3h}{2}) \times \lim_{h \to 0} \frac{1}{\sqrt{\cos(3x+3h)}+\sqrt{\cos 3x}}$$

[Here, we multiply and divide by $\frac{3}{2}$]

$$= -2 \times \frac{3}{2}\lim_{h \to 0} \frac{\sin\frac{3h}{2}}{\frac{3h}{2}} \times \lim_{h \to 0} \sin(\frac{6x+3h}{2}) \times \lim_{h \to 0} \frac{1}{\sqrt{\cos(3x+3h)}+\sqrt{\cos 3x}}$$

$$= -3 \times (1) \times \lim_{h \to 0} \sin(\frac{6x+3h}{2}) \times \lim_{h \to 0} \frac{1}{\sqrt{\cos(3x+3h)}+\sqrt{\cos 3x}}$$

$$\left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

Putting h = 0, we get

$$= -3 \times \sin[\frac{6x+3(0)}{2}] \times \frac{1}{\sqrt{\cos(3x+3(0))}+\sqrt{\cos 3x}}$$

$$= -3\sin 3x \times \frac{1}{2\sqrt{\cos 3x}}$$

$$= -\frac{3\sin 3x}{2(\cos 3x)^{\frac{1}{2}}}$$

Hence, $f'(x) = -\frac{3\sin 3x}{2(\cos 3x)^{\frac{1}{2}}}$

Question: 18

Solution:

Let $f(x) = \sqrt{\sec x}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$f(x) = \sqrt{\sec x}$

$f(x+h) = \sqrt{\sec(x+h)}$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{\sec(x+h)}-\sqrt{\sec x}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of $\sqrt{\sec(x+h)}-\sqrt{\sec x}$

$$= \lim_{h\to 0} \frac{\sqrt{\sec(x+h)} - \sqrt{\sec x}}{h} \times \frac{\sqrt{\sec(x+h)} + \sqrt{\sec x}}{\sqrt{\sec(x+h)} + \sqrt{\sec x}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h\to 0} \frac{\left(\sqrt{\sec(x+h)}\right)^2 - \left(\sqrt{\sec x}\right)^2}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h\to 0} \frac{\sec(x+h) - \sec(x)}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h\to 0} \frac{\frac{\cos x - \cos(x+h)}{\cos(x+h)\cos x}}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h\to 0} \frac{\cos x - \cos(x+h)}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

Using the formula:

$$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$$

$$= \lim_{h\to 0} \frac{2\sin\frac{x+(x+h)}{2}\sin\frac{(x+h)-x}{2}}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h\to 0} \frac{2\sin\frac{2x+h}{2}\sin\frac{h}{2}}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= 2\lim_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2}\lim_{h\to 0}\sin\left(\frac{2x+h}{2}\right) \times \lim_{h\to 0} \frac{1}{(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

[Here, we multiply and divide by $\frac{1}{2}$]

$$= 2 \times \frac{1}{2}\lim_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} \times \lim_{h\to 0}\sin\left(x+\frac{h}{2}\right) \times \lim_{h\to 0} \frac{1}{(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= (1) \times \lim_{h\to 0}\sin\left(x+\frac{h}{2}\right) \times \lim_{h\to 0} \frac{1}{(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$\left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

Putting h = 0, we get

$$= \sin[x + \frac{0}{2}] \times \frac{1}{\cos(x+0)\cos x\left(\sqrt{\sec(x+0)} + \sqrt{\sec x}\right)}$$

$$= \sin x \times \frac{1}{\cos x \cos x\left(\sqrt{\sec x} + \sqrt{\sec x}\right)}$$

$$= \frac{\sin x}{\cos^2 x(2\sqrt{\sec x})}$$

$$= \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \times \frac{1}{2\sqrt{\sec x}}$$

$$= \tan x \times \sec x \times \frac{1}{2\sqrt{\sec x}} \left[\because \frac{\sin x}{\cos x} = \tan x\right] \& \left[\frac{1}{\cos x} = \sec x\right]$$

$$= \frac{1}{2}\tan x\sqrt{\sec x}$$

Hence, $f'(x) = \frac{1}{2}\tan x\sqrt{\sec x}$

Question: 19

Solution:

Let $f(x) = \tan^2 x$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$f(x) = \tan^2 x$

$f(x + h) = \tan^2(x + h)$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{\tan^2(x+h) - \tan^2 x}{h}$$

$$= \lim_{h\to 0} \frac{[\tan(x+h) - \tan x][\tan(x+h) + \tan x]}{h}$$

Using:

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \lim_{h\to 0} \frac{\left[\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}\right]\left[\frac{\sin(x+h)}{\cos(x+h)} + \frac{\sin x}{\cos x}\right]}{h}$$

$$= \lim_{h\to 0} \frac{\left[\frac{\sin(x+h)\cos x - \sin x\cos(x+h)}{\cos(x+h)\cos x}\right]\left[\frac{\sin(x+h)\cos x + \sin x\cos(x+h)}{\cos(x+h)\cos x}\right]}{h}$$

$$= \lim_{h\to 0} \frac{\{\sin[(x+h) - x]\}\{\sin[(x+h) + x]\}}{h[\cos^2(x+h)\cos^2 x]}$$

[∵ sin A cos B – sin B cos A = sin(A – B)

& sin A cos B + sin B cos A = sin(A + B)]

$$= \lim_{h\to 0}\frac{[\sin h][\sin(2x+h)]}{h[\cos^2(x+h)\cos^2 x]}$$

$$= \frac{1}{\cos^2 x}\lim_{h\to 0}\frac{\sin h}{h} \times \lim_{h\to 0}\sin(2x+h) \times \lim_{h\to 0}\frac{1}{\cos^2(x+h)}$$

$$= \frac{1}{\cos^2 x} \times (1) \times \lim_{h\to 0}\sin(2x+h) \times \lim_{h\to 0}\frac{1}{\cos^2(x+h)}$$

$$\left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

Putting h = 0, we get

$$= \frac{1}{\cos^2 x} \times \sin(2x+0) \times \frac{1}{\cos^2(x+0)}$$

$$= \frac{1}{\cos^2 x} \times \sin 2x \times \frac{1}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} \times 2\sin x\cos x \times \sec^2 x$$

[∵ sin2x = 2sinxcosx]

$$= 2\frac{\sin x}{\cos x} \times \sec^2 x \left[\because \frac{1}{\cos x} = \sec x\right]$$

$= 2\tan x \sec^2 x$

$$\left[\because \frac{\sin x}{\cos x} = \tan x\right]$$

Hence, $f'(x) = 2\tan x \sec^2 x$

Question: 20

Solution:

Let f(x) = sin (2x + 3)

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ...(i)

f(x) = sin (2x + 3)

f(x + h) = sin [2(x + h) + 3]

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0}\frac{\sin[2(x+h)+3] - \sin(2x+3)}{h}$$

Using the formula:

$$\sin A - \sin B = 2\sin\frac{A-B}{2}\cos\frac{A+B}{2}$$

$$= \lim_{h\to 0}\frac{2\sin\frac{2(x+h)+3-(2x+3)}{2}\cos\frac{2(x+h)+3+2x+3}{2}}{h}$$

$$= \lim_{h\to 0}\frac{2\sin\frac{2x+2h+3-2x-3}{2}\cos\frac{2x+2h+6+2x}{2}}{h}$$

$$= \lim_{h\to 0}\frac{2\sin\frac{2h}{2}\cos\frac{4x+2h+6}{2}}{h}$$

$$= \lim_{h\to 0}\frac{2\sin(h)\cos(2x+h+3)}{h}$$

$$= 2\lim_{h\to 0}\frac{\sin h}{h}\times\lim_{h\to 0}\cos(2x+h+3)$$

$$= 2(1)\times\lim_{h\to 0}\cos(2x+h+3)$$

$$\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

Putting h = 0, we get

$= 2\cos(2x + 0 + 3)$

$= 2\cos(2x + 3)$

Hence, $f'(x) = 2\cos(2x + 3)$

Question: 21

Solution:

Let $f(x) = \tan(3x + 1)$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad ...(i)$$

$f(x) = \tan(3x + 1)$

$f(x + h) = \tan[3(x + h) + 1]$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0}\frac{\tan[3(x+h)+1]-\tan[3x+1]}{h}$$

Using the formula:

$$\tan A - \tan B = \frac{\sin(A-B)}{\cos A\cos B}$$

$$= \lim_{h\to 0}\frac{\frac{\sin[3(x+h)+1-(3x+1)]}{\cos[3(x+h)+1]\cos[3x+1]}}{h}$$

$$= \lim_{h\to 0}\frac{\frac{\sin[3x+3h+1-3x-1]}{\cos[3(x+h)+1]\cos[3x+1]}}{h}$$

$$= \lim_{h\to 0}\frac{\sin 3h}{h[\cos[3(x+h)+1]\cos[3x+1]]}$$

$$= \lim_{h\to 0}\frac{\sin 3h}{h}\times\lim_{h\to 0}\frac{1}{\cos[3(x+h)+1]\cos[3x+1]}$$

$$= \lim_{h\to 0}\frac{\sin 3h}{3h}\times 3\times\lim_{h\to 0}\frac{1}{\cos[3(x+h)+1]\cos[3x+1]}$$

$$= 3(1)\times\lim_{h\to 0}\frac{1}{\cos[3(x+h)+1]\cos[3x+1]}$$

$\left[\because \lim_{x\to 0}\frac{\sin 3x}{3x} = 1\right]$

Putting h = 0, we get

$= 3 \times \frac{1}{\cos[3(x+0)+1]\cos[3x+1]}$

$= \frac{3}{\cos[3x+1]\cos[3x+1]}$

$= \frac{3}{\cos^2(3x+1)}$

$= 3\sec^2(3x+1)\left[\because \frac{1}{\cos x} = \sec x\right]$

Hence, $f'(x) = 3\sec^2(3x+1)$

Exercise : 28C

Question: 1

Solution:

To find: tion of $x^2 \sin x$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

(iii) $\frac{d\sin x}{dx} = \cos x$

Let us take $u = x^2$ and $v = \sin x$

$u' = \frac{du}{dx} = \frac{d(x^2)}{dx} = 2x$

$v' = \frac{dv}{dx} = \frac{d(\sin x)}{dx} = \cos x$

Putting the above obtained values in the formula:-

$(uv)' = u'v + uv'$

$(x^2 \sin x)' = 2x \times \sin x + x^2 \times \cos x$

$= 2x\sin x + x^2\cos x$

Ans) $2x\sin x + x^2\cos x$

Question: 2

tion of $e^x \cos x$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii) $\frac{de^x}{dx} = e^x$

(iii) $\frac{d\cos x}{dx} = -\sin x$

Let us take $u = e^x$ and $v = \cos x$

$u' = \frac{du}{dx} = \frac{de^x}{dx} = e^x$

$$v' = \frac{dv}{dx} = \frac{dcosx}{dx} = -sinx$$

Putting the above obtained values in the formula:-

(uv)′ = u′v + uv′

$(e^x cosx)' = e^x \times cosx + e^x \times -sinx$

$= e^x cosx - e^x sinx$

$= e^x (cosx - sinx)$

Ans) $e^x (cosx - sinx)$

Question: 3

tion of e^x cot x

Formula used: (i) (uv)′ = u′v + uv′ (Leibnitz or product rule)

(ii) $\frac{de^x}{dx} = e^x$

(iii) $\frac{dcotx}{dx} = -cosec^2x$

Let us take $u = e^x$ and v = cotx

$$u' = \frac{du}{dx} = \frac{de^x}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{dcotx}{dx} = -cosec^2x$$

Putting the above obtained values in the formula:-

(uv)′ = u′v + uv′

$(e^x cotx)' = e^x \times cotx + e^x \times -cosec^2x$

$= e^x cotx - e^x cosec^2x$

$= e^x (cotx - cosec^2x)$

Ans) $e^x (cotx - cosec^2x)$

Question: 4

tion of x^n cot x

Formula used: (i) (uv)′ = u′v + uv′ (Leibnitz or product rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

(iii) $\frac{dcotx}{dx} = -cosec^2x$

Let us take $u = x^n$ and v = cotx

$$u' = \frac{du}{dx} = \frac{dx^n}{dx} = nx^{n-1}$$

$$v' = \frac{dv}{dx} = \frac{dcotx}{dx} = -cosec^2x$$

Putting the above obtained values in the formula :-

(uv)′ = u′v + uv′

$(x^n cotx)' = nx^{n-1} \times cotx + x^n \times -cosec^2x$

$= nx^{n-1}cotx - x^n cosec^2x$

$= x^n (nx^{-1}\cot x - \operatorname{cosec}^2 x)$

Ans) $x^n (nx^{-1}\cot x - \operatorname{cosec}^2 x)$

Question: 5

tion of $x^3 \sec x$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

(iii) $\frac{d\sec x}{dx} = \sec x \tan x$

Let us take $u = x^3$ and $v = \sec x$

$$u' = \frac{du}{dx} = \frac{dx^3}{dx} = 3x^2$$

$$v' = \frac{dv}{dx} = \frac{d\sec x}{dx} = \sec x \tan x$$

Putting the above obtained values in the formula :-

$(uv)' = u'v + uv'$

$(x^3 \sec x)' = 3x^2 \times \sec x + x^3 \times \sec x \tan x$

$= 3x^2 \sec x + x^3 \sec x \tan x$

$= x^2 \sec x(3 + x \tan x)$

Ans) $x^2 \sec x(3 + x \tan x)$

Question: 6

tion of $(x^2 + 3x + 1) \sin x$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

(iii) $\frac{d\sin x}{dx} = \cos x$

Let us take $u = x^2 + 3x + 1$ and $v = \sin x$

$$u' = \frac{du}{dx} = \frac{d(x^2 + 3x + 1)}{dx} = 2x + 3$$

$$v' = \frac{dv}{dx} = \frac{d\sin x}{dx} = \cos x$$

Putting the above obtained values in the formula :-

$(uv)' = u'v + uv'$

$[(x^2 + 3x + 1) \sin x]' = (2x + 3) \times \sin x + (x^2 + 3x + 1) \times \cos x$

$= \sin x (2x + 3) + \cos x (x^2 + 3x + 1)$

Ans) $(2x + 3) \sin x + (x^2 + 3x + 1) \cos x$

Question: 7

tion of $x^4 \tan x$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

(iii) $\frac{d\tan x}{dx} = \sec^2 x$

Let us take $u = x^4$ and $v = \tan x$

$$u' = \frac{du}{dx} = \frac{dx^4}{dx} = 4x^3$$

$$v' = \frac{dv}{dx} = \frac{d\tan x}{dx} = \sec^2 x$$

Putting the above obtained values in the formula:-

$(uv)' = u'v + uv'$

$(x^4 \tan x)' = 4x^3 \times \tan x + x^4 \times \sec^2 x$

$= 4x^3 \tan x + x^4 \sec^2 x$

$= x^3 (4\tan x + x\sec^2 x)$

Ans) $x^3 (4\tan x + x\sec^2 x)$

Question: 8

tion of $(3x - 5)(4x^2 - 3 + e^x)$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

(iii) $\frac{de^x}{dx} = e^x$

Let us take $u = (3x - 5)$ and $v = (4x^2 - 3 + e^x)$

$$u' = \frac{du}{dx} = \frac{d(3x - 5)}{dx} = 3$$

$$v' = \frac{dv}{dx} = \frac{d(4x^2 - 3 + e^x)}{dx} = (8x + e^x)$$

Putting the above obtained values in the formula :-

$(uv)' = u'v + uv'$

$[(3x - 5)(4x^2 - 3 + e^x)]' = 3 \times (4x^2 - 3 + e^x) + (3x - 5) \times (8x + e^x)$

$= 12x^2 - 9 + 3e^x + 24x^2 + 3xe^x - 40x - 5e^x$

$= 36x^2 + x(3e^x - 40) - 9 - 2e^x$

Ans) $36x^2 + x(3e^x - 40) - 9 - 2e^x$

Question: 9

tion of $(x^2 - 4x + 5)(x^3 - 2)$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take $u = (x^2 - 4x + 5)$ and $v = (x^3 - 2)$

$$u' = \frac{du}{dx} = \frac{d(x^2 - 4x + 5)}{dx} = 2x - 4$$

$$v' = \frac{dv}{dx} = \frac{d(x^3 - 2)}{dx} = 3x^2$$

Putting the above obtained values in the formula:-

$(uv)' = u'v + uv'$

$[(x^2 - 4x + 5)(x^3 - 2)]' = (2x - 4)\times(x^3 - 2) + (x^2 - 4x + 5)\times(3x^2)$

$= 2x^4 - 4x - 4x^3 + 8 + 3x^4 - 12x^3 + 15x^2$

$= 5x^4 - 16x^3 + 15x^2 - 4x + 8$

Ans) $5x^4 - 16x^3 + 15x^2 - 4x + 8$

Question: 10

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take $u = (x^2 + 2x - 3)$ and $v = (x^2 + 7x + 5)$

$$u' = \frac{du}{dx} = \frac{d(x^2 + 2x - 3)}{dx} = 2x + 2$$

$$v' = \frac{dv}{dx} = \frac{d(x^2 + 7x + 5)}{dx} = 2x + 7$$

Putting the above obtained values in the formula :-

$(uv)' = u'v + uv'$

$[(x^2 + 2x - 3)(x^2 + 7x + 5)]'$

$= (2x + 2) \times (x^2 + 7x + 5) + (x^2 + 2x - 3) \times (2x + 7)$

$= 2x^3 + 14x^2 + 10x + 2x^2 + 14x + 10 + 2x^3 + 7x^2 + 4x^2 + 14x - 6x - 21$

$= 4x^3 + 27x^2 + 32x - 11$

Ans) $4x^3 + 27x^2 + 32x - 11$

Question: 11

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii) $\frac{d\tan x}{dx} = \sec^2 x$

(iii) $\frac{d\sec x}{dx} = \sec x \tan x$

(iv) $\frac{d\cot x}{dx} = -\text{cosec}^2 x$

(v) $\frac{d\,\text{cosec}x}{dx} = -\text{cosec}x \cot x$

Let us take $u = (\tan x + \sec x)$ and $v = (\cot x + \text{cosec } x)$

$$u' = \frac{du}{dx} = \frac{d(\tan x + \sec x)}{dx} = \sec^2 x + \sec x \tan x = \sec x(\sec x + \tan x)$$

$$v' = \frac{dv}{dx} = \frac{d(\cot x + \text{cosec}x)}{dx}$$

$= -\text{cosec}^2 x + (-\text{cosec}x \cot x) = -\text{cosec}x(\text{cosec}x + \cot x)$

Putting the above obtained values in the formula:-

$(uv)' = u'v + uv'$

$[(\tan x + \sec x)(\cot x + \text{cosec } x)]'$

= [secx (secx + tanx)] × [(cot x + cosec x)] + [(tan x + sec x)] × [-cosecx (cosec

= (secx +tanx) [secx(cotx + cosecx) - cosecx(cosecx + cotx)]

= (secx + tanx) (secx – cosecx) (cotx + cosecx)

Ans) (secx + tanx) (secx – cosecx) (cotx + cosecx)

Question: 12

tion of (x^3 cos x – 2^x tan x)

Formula used: (i) (uv)′ = u′v + uv′ (Leibnitz or product rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

(iii) $\frac{dcosx}{dx}$ = -sinx

(iv) $\frac{da^x}{dx} = a^x loga$

(v) $\frac{dtanx}{dx} = sec^2 x$

Here we have two function (x^3 cos x) and (2^x tan x)

We have two te them separately

Let us assume g(x) = (x^3 cos x)

And h(x) = (2^x tan x)

Therefore, f(x) = g(x) – h(x)

⇒ f’(x) = g’(x) – h’(x) ... (i)

Applying product rule on g(x)

Let us take u = x^3 and v = cos x

$u' = \frac{du}{dx} = \frac{d(x^3)}{dx} = 3x^2$

$v' = \frac{dv}{dx} = \frac{d(cosx)}{dx} = -sinx$

Putting the above obtained values in the formula:-

(uv)′ = u′v + uv′

$[x^3 cosx]' = 3x^2 \times cosx + x^3 \times -sinx$

= $3x^2cosx - x^3sinx$

= x^2 (3cosx – x sinx)

g’(x) = x^2 (3cosx – x sinx)

Applying product rule on h(x)

Let us take u = 2^x and v = tan x

$u' = \frac{du}{dx} = \frac{d(2^x)}{dx} = 2^x log2$

$v' = \frac{dv}{dx} = \frac{d(tan\, x)}{dx} = sec^2 x$

Putting the above obtained values in the formula:-

(uv)′ = u′v + uv′

$[2^x \tan x]' = 2^x \log 2 \times \tan x + 2^x \times \sec^2 x$

$= 2^x (\log 2 \tan x + \sec^2 x)$

$h'(x) = 2^x (\log 2 \tan x + \sec^2 x)$

Putting the above obtained values in eqn. (i)

$f'(x) = x^2 (3\cos x - x \sin x) - 2^x (\log 2 \tan x + \sec^2 x)$

Ans) $x^2 (3\cos x - x \sin x) - 2^x (\log 2 \tan x + \sec^2 x)$

Exercise : 28D

Question: 1

Solution:

To find: Differentiation of $\frac{2^x}{x}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{da^x}{dx} = a^x \log a$

Let us take $u = 2^x$ and $v = x$

$u' = \frac{du}{dx} = \frac{d(2^x)}{dx} = 2^x \log 2$

$v' = \frac{dv}{dx} = \frac{d(x)}{dx} = 1$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left(\frac{2^x}{x}\right)' = \frac{2^x \log 2 \times x - 2^x \times 1}{(x)^2}$

$= \frac{2^x(x\log 2 - 1)}{x^2}$

Ans) $= \frac{2^x(x\log 2 - 1)}{x^2}$

Question: 2

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{d\log x}{dx} = \frac{1}{x}$

Let us take $u = \log x$ and $v = x$

$u' = \frac{du}{dx} = \frac{d(\log x)}{dx} = \frac{1}{x}$

$v' = \frac{dv}{dx} = \frac{d(x)}{dx} = 1$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left(\frac{\log x}{x}\right)' = \frac{\frac{1}{x} \times x - \log x \times 1}{(x)^2}$

$= \frac{1 - \log x}{x^2}$

Ans) $= \frac{1 - \log x}{x^2}$

Question: 3

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{de^x}{dx} = e^x$

Let us take $u = e^x$ and $v = (1+x)$

$u' = \frac{du}{dx} = \frac{d(e^x)}{dx} = e^x$

$v' = \frac{dv}{dx} = \frac{d(1+x)}{dx} = 1$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left(\frac{e^x}{(1+x)}\right)' = \frac{e^x \times (1+x) - e^x \times 1}{(1+x)^2}$

$= \frac{xe^x}{(1+x)^2}$

Ans) $= \frac{xe^x}{(1+x)^2}$

Question: 4

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{de^x}{dx} = e^x$

(iii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take $u = e^x$ and $v = (1+x^2)$

$u' = \frac{du}{dx} = \frac{d(e^x)}{dx} = e^x$

$v' = \frac{dv}{dx} = \frac{d(1+x^2)}{dx} = 2x$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left(\frac{e^x}{(1+x^2)}\right)' = \frac{e^x \times (1+x^2) - e^x \times 2x}{(1+x^2)^2}$

$= \frac{e^x(x^2-2x+1)}{(1+x^2)^2}$

$= \frac{e^x(x-1)^2}{(1+x^2)^2}$

Ans) = $\frac{e^x(x-1)^2}{(1+x^2)^2}$

Question: 5

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take u = $(2x^2 - 4)$ and v = $(3x^2 + 7)$

$$u' = \frac{du}{dx} = \frac{d(2x^2-4)}{dx} = 4x$$

$$v' = \frac{dv}{dx} = \frac{d(3x^2+7)}{dx} = 6x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{(2x^2-4)}{(3x^2+7)}\right]' = \frac{4x \times (3x^2+7) - (2x^2-4) \times 6x}{(3x^2+7)^2}$$

$$= \frac{12x^3 + 28x - 12x^3 + 24x}{(3x^2+7)^2}$$

$$= \frac{52x}{(3x^2+7)^2}$$

Ans) = $\frac{52x}{(3x^2+7)^2}$

Question: 6

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take u = $(x^2 + 3x - 1)$ and v = $(x + 2)$

$$u' = \frac{du}{dx} = \frac{d(x^2+3x-1)}{dx} = 2x + 3$$

$$v' = \frac{dv}{dx} = \frac{d(x+2)}{dx} = 1$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{x^2+3x-1}{x+2}\right)' = \frac{(2x+3) \times (x+2) - (x^2+3x-1) \times 1}{(x+2)^2}$$

$$= \frac{2x^2+7x+6-x^2-3x+1}{(x+2)^2}$$

$$= \frac{x^2+4x+7}{(x+2)^2}$$

Ans) = $\frac{x^2+4x+7}{(x+2)^2}$

Question: 7

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{dx^n}{dx} = nx^{n-1}$ CLASS24

Let us take $u = (x^2 - 1)$ and $v = (x^2 + 7x + 1)$

$$u' = \frac{du}{dx} = \frac{d(x^2-1)}{dx} = 2x$$

$$v' = \frac{dv}{dx} = \frac{d(x^2+7x+1)}{dx} = 2x + 7$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$$\left[\frac{(x^2-1)}{(x^2+7x+1)}\right]' = \frac{2x \times (x^2+7x+1) - (x^2-1) \times (2x+7)}{(x^2+7x+1)^2}$$

$$= \frac{2x^3 + 14x^2 + 2x - 2x^3 - 7x^2 + 2x + 7}{(x^2+7x+1)^2}$$

$$= \frac{7x^2 + 4x + 7}{(x^2+7x+1)^2}$$

Ans) $= \frac{7x^2 + 4x + 7}{(x^2+7x+1)^2}$

Question: 8

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take $u = (5x^2 + 6x + 7)$ and $v = (2x^2 + 3x + 4)$

$$u' = \frac{du}{dx} = \frac{d(5x^2 + 6x + 7)}{dx} = 10x + 6$$

$$v' = \frac{dv}{dx} = \frac{d(2x^2 + 3x + 4)}{dx} = 4x + 3$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$$\left(\frac{5x^2 + 6x + 7}{2x^2 + 3x + 4}\right)' = \frac{(10x+6)\times(2x^2+3x+4) - (5x^2+6x+7)\times(4x+3)}{(2x^2+3x+4)^2}$$

$$= \frac{20x^3+30x^2+40x+12x^2+18x+24 - 20x^3-15x^2-24x^2-18x-28x-21}{(2x^2+3x+4)^2}$$

$$= \frac{3x^2+12x+3}{(2x^2+3x+4)^2}$$

$$= \frac{3(x^2+4x+1)}{(2x^2+3x+4)^2}$$

Ans) $= \frac{3(x^2+4x+1)}{(2x^2+3x+4)^2}$

Question: 9

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{dx^n}{dx} = nx^{n-1}$ CLASS24

Let us take u = (x) and v = $(a^2 + x^2)$

$u' = \frac{du}{dx} = \frac{d(x)}{dx} = 1$

$v' = \frac{dv}{dx} = \frac{d(a^2+x^2)}{dx} = 2x$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left[\frac{x}{(a^2+x^2)}\right]' = \frac{1 \times (a^2+x^2) - (x) \times (2x)}{(a^2+x^2)^2}$

$= \frac{a^2+x^2 - 2x^2}{(a^2+x^2)^2}$

$= \frac{a^2 - x^2}{(a^2+x^2)^2}$

Ans) $= \frac{a^2 - x^2}{(a^2+x^2)^2}$

Question: 10

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{dx^n}{dx} = nx^{n-1}$

(iii) $\frac{d\sin x}{dx} = \cos x$

Let us take u = (x^4) and v = (sinx)

$u' = \frac{du}{dx} = \frac{d(x^4)}{dx} = 4x^3$

$v' = \frac{dv}{dx} = \frac{d(\sin x)}{dx} = \cos x$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left[\frac{x^4}{\sin x}\right]' = \frac{4x^3 \times (\sin x) - (x^4) \times (\cos x)}{(\sin x)^2}$

$= \frac{x^3[4(\sin x) - x(\cos x)]}{(\sin x)^2}$

Ans) $= \frac{x^3[4(\sin x) - x(\cos x)]}{(\sin x)^2}$

Question: 11

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{dx^n}{dx} = nx^{n-1}$

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Let us take $u = (\sqrt{a}+\sqrt{x})$ and $v = (\sqrt{a}-\sqrt{x})$

$$u' = \frac{du}{dx} = \frac{d(\sqrt{a}+\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

$$v' = \frac{dv}{dx} = \frac{d(\sqrt{a}-\sqrt{x})}{dx} = -\frac{1}{2\sqrt{x}}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}-\sqrt{x}}\right]' = \frac{\frac{1}{2\sqrt{x}} \times (\sqrt{a}-\sqrt{x}) - (\sqrt{a}+\sqrt{x}) \times -\frac{1}{2\sqrt{x}}}{(\sqrt{a}-\sqrt{x})^2}$$

$$= \frac{\frac{\sqrt{a}}{2\sqrt{x}} - \frac{1}{2} + \frac{\sqrt{a}}{2\sqrt{x}} + \frac{1}{2}}{(\sqrt{a}-\sqrt{x})^2}$$

$$= \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a}-\sqrt{x})^2}$$

$$\text{Ans)} = \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a}-\sqrt{x})^2}$$

Question: 12

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{d\cos x}{dx} = -\sin x$

(iii) $\frac{d\log x}{dx} = \frac{1}{x}$

Let us take $u = (\cos x)$ and $v = (\log x)$

$$u' = \frac{du}{dx} = \frac{d(\cos x)}{dx} = -\sin x$$

$$v' = \frac{dv}{dx} = \frac{d(\log x)}{dx} = \frac{1}{x}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\cos x}{\log x}\right]' = \frac{-\sin x \times (\log x) - (\cos x) \times \left(\frac{1}{x}\right)}{(\log x)^2}$$

$$= \frac{-x\sin x(\log x) - (\cos x)}{x(\log x)^2}$$

$$\text{Ans)} = \frac{-x\sin x(\log x) - (\cos x)}{x(\log x)^2}$$

Question: 13

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{d\cot x}{dx} = -\mathrm{cosec}^2 x$

(iii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take u = (2cotx) and v = $(\sqrt{x})$

$$u' = \frac{du}{dx} = \frac{d(2\cot x)}{dx} = -2\mathrm{cosec}^2 x$$

$$v' = \frac{dv}{dx} = \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{2\cot x}{\sqrt{x}}\right]' = \frac{-2\mathrm{cosec}^2 x \times (\sqrt{x}) - (2\cot x) \times \left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2}$$

$$= \frac{-2x\mathrm{cosec}^2 x - (\cot x)}{\sqrt{x}(\sqrt{x})^2}$$

$$\text{Ans)} = \frac{-2x\mathrm{cosec}^2 x - \cot x}{x^{3/2}}$$

Question: 14

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where v ≠0 (Quotient rule)(ii) $\frac{d\cos x}{dx} = -\sin x$

(iii) $\frac{d\sin x}{dx} = \cos x$

Let us take u = (sinx) and v = (1 + cosx)

$$u' = \frac{du}{dx} = \frac{d(\sin x)}{dx} = \cos x$$

$$v' = \frac{dv}{dx} = \frac{d(1 + \cos x)}{dx} = -\sin x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\sin x}{(1 + \cos x)}\right]' = \frac{\cos x \times (1 + \cos x) - (\sin x) \times (-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$$

$$= \frac{\cos x + 1}{(1 + \cos x)^2}$$

$$= \frac{1}{(1 + \cos x)}$$

$$\text{Ans)} = \frac{1}{1 + \cos x}$$

Question: 15

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)(ii) $\frac{d\sin x}{dx}$ = cos

Let us take u = (1 + sinx) and v = (1 - sinx)

$u' = \frac{du}{dx} = \frac{d(1+\sin x)}{dx} = \cos x$

$v' = \frac{dv}{dx} = \frac{d(1-\sin x)}{dx} = -\cos x$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)

$\left[\frac{1+\sin x}{1-\sin x}\right]' = \frac{\cos x \times (1-\sin x) - (1+\sin x) \times (-\cos x)}{(1-\sin x)^2}$

$= \frac{\cos x - \cos x\sin x + \cos x + \cos x\sin x}{(1-\sin x)^2}$

$= \frac{2\cos x}{(1-\sin x)^2}$

Ans) = $\frac{2\cos x}{(1-\sin x)^2}$

Question: 16

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)(ii) $\frac{d\cos x}{dx} = -\sin x$

Let us take u = (1 - cosx) and v = (1 + cosx)

$u' = \frac{du}{dx} = \frac{d(1-\cos x)}{dx} = \sin x$

$v' = \frac{dv}{dx} = \frac{d(1+\cos x)}{dx} = -\sin x$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)

$\left[\frac{1-\cos x}{1+\cos x}\right]' = \frac{\sin x \times (1+\cos x) - (1-\cos x) \times (-\sin x)}{(1+\cos x)^2}$

$= \frac{\sin x + \sin x\cos x + \sin x - \sin x\cos x}{(1+\cos x)^2}$

$= \frac{2\sin x}{(1+\cos x)^2}$

Ans) = $\frac{2\sin x}{(1+\cos x)^2}$

Question: 17

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)(ii) $\frac{d\sin x}{dx} = \cos x$

(iii) $\frac{d\cos x}{dx} = -\sin x$

Let us take u = (sinx - cosx) and v = (sinx + cosx)

$u' = \frac{du}{dx} = \frac{d(\sin x - \cos x)}{dx} = (\cos x + \sin x)$

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$v' = \frac{dv}{dx} = \frac{d(\sin x + \cos x)}{dx} = (\cos x - \sin x)$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left[\frac{\sin x - \cos x}{\sin x + \cos x}\right]' = \frac{(\cos x + \sin x) \times (\sin x + \cos x) - (\sin x - \cos x) \times (\cos x - \sin x)}{(\sin x + \cos x)^2}$

$= \frac{\sin^2 x + \cos^2 x + 2\sin x\cos x - (\sin x - \cos x) \times -(\sin x - \cos x)}{(\sin x + \cos x)^2}$

$= \frac{\sin^2 x + \cos^2 x + 2\sin x\cos x + \sin^2 x + \cos^2 x - 2\sin x\cos x}{(\sin x + \cos x)^2}$

$= \frac{2(\sin^2 x + \cos^2 x)}{\sin^2 x + \cos^2 x + 2\sin x\cos x}$

$= \frac{2}{1 + \sin 2x}$

Ans) $= \frac{2}{1 + \sin 2x}$

Question: 18

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{d\sec x}{dx} = \sec x \tan x$

(iii) $\frac{d\tan x}{dx} = \sec^2 x$

Let us take u = (secx - tanx) and v = (secx + tanx)

$u' = \frac{du}{dx} = \frac{d(\sec x - \tan x)}{dx} = (\sec x \tan x - \sec^2 x)$

$v' = \frac{dv}{dx} = \frac{d(\sec x + \tan x)}{dx} = (\sec x \tan x + \sec^2 x)$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left[\frac{\sec x - \tan x}{\sec x + \tan x}\right]' = \frac{(\sec x\tan x - \sec^2 x)(\sec x + \tan x) - (\sec x - \tan x)(\sec x\tan x + \sec^2 x)}{(\sec x + \tan x)^2}$

$= \frac{(\sec x\tan x - \sec^2 x)(\sec x + \tan x) - (\sec x - \tan x)(\sec x)(\tan x + \sec x)}{(\sec x + \tan x)^2}$

$= \frac{(\sec x + \tan x)[(\sec x\tan x - \sec^2 x) - (\sec x - \tan x)(\sec x)]}{(\sec x + \tan x)^2}$

$= \frac{(\sec x + \tan x)[(\sec x\tan x - \sec^2 x) - (\sec^2 x - \sec x\tan x)]}{(\sec x + \tan x)^2}$

$= \frac{(\sec x + \tan x)[2\sec x\tan x - 2\sec^2 x]}{(\sec x + \tan x)^2}$

$= \frac{2secx[tanx-secx]}{(secx + tanx)}$

Ans) $= \frac{2secx[tanx-secx]}{(secx + tanx)}$

Question: 19

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)(ii) $\frac{dsinx}{dx} = cosx$

(iii) $\frac{dlogx}{dx} = \frac{1}{x}$

(iv) $\frac{de^x}{dx} = e^x$

Let us take u = $(e^x + sinx)$ and v = $(1 + logx)$

$u' = \frac{du}{dx} = \frac{d(e^x + sinx)}{dx} = (e^x + cosx)$

$v' = \frac{dv}{dx} = \frac{d(1 + logx)}{dx} = \frac{1}{x}$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)

$\left[\frac{e^x + sinx}{1 + logx}\right]' = \frac{(e^x + cosx)\times(1 + logx) - (e^x + sinx)\times\left(\frac{1}{x}\right)}{(1 + logx)^2}$

$= \frac{x(e^x + cosx)(1 + logx) - (e^x + sinx)}{x(1 + logx)^2}$

Ans) $= \frac{x(e^x + cosx)(1 + logx) - (e^x + sinx)}{x(1 + logx)^2}$

Question: 20

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)(ii) $\frac{dsinx}{dx} = cosx$

(iii) $\frac{dsecx}{dx} = secx\ tanx$

(iv) $\frac{de^x}{dx} = e^x$

(v) (uv)′ = u′v + uv′ (Leibnitz or product rule)

Let us take u = $(e^x sinx)$ and v = (secx)

$u' = \frac{du}{dx} = \frac{d(e^x sinx)}{dx}$

Applying Product rule

(gh)′ = g′h + gh′

Taking g = e^x and h = sinx

= e^xsinx + e^xcosx

u′ = e^xsinx + e^xcosx

$v' = \frac{dv}{dx} = \frac{d(\sec x)}{dx} = \sec x \tan x$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left[\frac{e^x \sin x}{\sec x}\right]' = \frac{(e^x \sin x + e^x \cos x) \times (\sec x) - (e^x \sin x) \times (\sec x \tan x)}{(\sec x)^2}$

$= \frac{(e^x \sin x + e^x \cos x) - (e^x \sin x) \times (\tan x)}{(\sec x)}$

$= \cos x\, [(e^x \sin x + e^x \cos x) - (e^x \sin x) \times (\tan x)]$

$= [(e^x \sin x \cos x + e^x \cos^2 x) - (e^x \sin x \cos x) \times (\tan x)]$

$= \left[(e^x \sin x \cos x + e^x \cos^2 x) - \left(e^x \sin^2 x\right)\right]$

$= (e^x \sin x \cos x + e^x \cos^2 x - e^x \sin^2 x$

$= (e^x \sin x \cos x + e^x \cos^2 x - e^x \sin^2 x$

$= e^x \sin x \cos x + e^x \cos 2x$

$= e^x (\sin x \cos x + \cos 2x)$

Ans) $= e^x (\sin x \cos x + \cos 2x)$

Question: 21

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{d\cot x}{dx} = -\mathrm{cosec}^2 x$

(iii) $\frac{dx^n}{dx} = nx^{n-1}$

(iv) $\frac{da^x}{dx} = a^x \log a$

(v) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

Let us take $u = (2^x \cot x)$ and $v = (\sqrt{x})$

$u' = \frac{du}{dx} = \frac{d(2^x \cot x)}{dx}$

Applying Product rule

$(gh)' = g'h + gh'$

Taking $g = 2^x$ and $h = \cot x$

$= (2^x \log 2) \cot x + 2^x (-\mathrm{cosec}^2 x)$

$u' = (2^x \log 2) \cot x - 2^x (\mathrm{cosec}^2 x)$

$u' = 2^x [\log 2 \cot x - \mathrm{cosec}^2 x]$

$v' = \frac{dv}{dx} = \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

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$$\left[\frac{2^x \cot x}{\sqrt{x}}\right]' = \frac{\{2^x[\log 2 \cot x - \operatorname{cosec}^2 x] \times \sqrt{x}\} - \left\{(2^x \cot x) \times \left(\frac{1}{2\sqrt{x}}\right)\right\}}{(\sqrt{x})^2}$$

$$= \frac{\{2^x[\log 2 \cot x - \operatorname{cosec}^2 x] \times \sqrt{x}\} - \left\{(2^x \cot x) \times \left(\frac{1}{2\sqrt{x}}\right)\right\}}{x}$$

$$= \frac{\{2^x[\log 2 \cot x - \operatorname{cosec}^2 x] \times \sqrt{x}\} - \left\{\left(2^{x-1} \cot x\right) \times \left(\frac{1}{\sqrt{x}}\right)\right\}}{x}$$

$$= \frac{\{x2^x[\log 2 \cot x - \operatorname{cosec}^2 x]\} - \left\{\left(2^{x-1} \cot x\right)\right\}}{x\sqrt{x}}$$

$$= \frac{\{2^x[x\log 2 \cot x - x\operatorname{cosec}^2 x]\} - \left\{\left(2^{x-1} \cot x\right)\right\}}{x^{\frac{3}{2}}}$$

$$\text{Ans)} = \frac{\{2^x[x\log 2 \cot x - x\operatorname{cosec}^2 x]\} - \left\{\left(2^{x-1} \cot x\right)\right\}}{x^{\frac{3}{2}}}$$

Question: 22

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)(ii) $\frac{de^x}{dx} = e^x$

(iii) $\frac{dx^n}{dx} = nx^{n-1}$

(iv) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

Let us take $u = e^x(x-1)$ and $v = (x+1)$

$$u' = \frac{du}{dx} = \frac{d[e^x(x-1)]}{dx}$$

Applying Product rule

$(gh)' = g'h + gh'$

Taking $g = e^x$ and $h = x - 1$

$[e^x(x-1)]' = e^x(x-1) + e^x(1)$

$= e^x(x-1) + e^x$

$u' = e^x x$

$$v' = \frac{dv}{dx} = \frac{d(x+1)}{dx} = 1$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$$\left[\frac{e^x(x-1)}{(x+1)}\right]' = \frac{(e^x x)(x+1) - [e^x(x-1)](1)}{(x+1)^2}$$

$$= \frac{e^x x^2 + e^x x - e^x x + e^x}{(x+1)^2}$$

$$= \frac{e^x x^2 + e^x}{(x+1)^2}$$

$$= \frac{e^x(x^2+1)}{(x+1)^2}$$

Ans) $= \frac{e^x(x^2+1)}{(x+1)^2}$

Question: 23

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\sec x}{dx} = \sec x \tan x$

(iii) $\frac{d\tan x}{dx} = \sec^2 x$ (iii) $\frac{dx^n}{dx} = nx^{n-1}$

(iv) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

Let us take $u = (x \tan x)$ and $v = (\sec x + \tan x)$

$u' = \frac{du}{dx} = \frac{d[x \tan x]}{dx}$

Applying Product rule for finding u'

$(gh)' = g'h + gh'$

Taking $g = x$ and $h = \tan x$

$[x \tan x]' = (1)(\tan x) + x(\sec^2 x)$

$= \tan x + x\sec^2 x$

$u' = \tan x + x\sec^2 x$

$v' = \frac{dv}{dx} = \frac{d(\sec x + \tan x)}{dx} = \sec x \tan x + \sec^2 x$

$v' = \sec x(\tan x + \sec x)$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$$\left[\frac{x \tan x}{(\sec x+\tan x)}\right]' = \frac{(\tan x + x\sec^2 x)(\sec x + \tan x) - [x \tan x][\sec x(\tan x + \sec x)]}{(\sec x+\tan x)^2}$$

$$= \frac{(\sec x + \tan x)[(\tan x + x\sec^2 x)-(x \tan x)(\sec x)]}{(\sec x+\tan x)^2}$$

$$= \frac{[\tan x + x\sec^2 x - x \tan x\sec x]}{(\sec x+\tan x)}$$

$$= \frac{\tan x + x\sec x(\sec x - \tan x)}{(\sec x+\tan x)}$$

Ans) $= \frac{\tan x + x\sec x(\sec x - \tan x)}{(\sec x+\tan x)}$

Question: 24

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take u = (ax^2+bx+c) and v = (px^2+qx+r)

$$u' = \frac{du}{dx} = \frac{d[ax^2+bx+c]}{dx} = 2ax + b$$

$$v' = \frac{dv}{dx} = \frac{d(px^2+qx+r)}{dx} = 2px + q$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{ax^2+bx+c}{px^2+qx+r}\right]' = \frac{(2ax + b)(px^2+qx+r) - [ax^2+bx+c][2px + q]}{(px^2+qx+r)^2}$$

$$= \frac{2apx^3+2aqx^2+2axr+bpx^2+bqx+br-[2apx^3+qax^2+2bpx^2+bqx+2pcx+cq]}{(px^2+qx+r)^2}$$

$$= \frac{(aq-bp)x^2+2(ra-pc)x+br-cp}{(px^2+qx+r)^2}$$

$$\text{Ans)} = \frac{(aq-bp)x^2+2(ra-pc)x+br-cp}{(px^2+qx+r)^2}$$

Question: 25

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\sin x}{dx} = \cos x$

(iii) $\frac{d\cos x}{dx} = -\sin x$ (iv) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

Let us take u = (sinx-xcosx) and v = (xsinx + cosx)

$$u' = \frac{du}{dx} = \frac{d[\sin x - x\cos x]}{dx}$$

Applying Product rule for finding the term xcosx in u'

(gh)' = g'h + gh'

Taking g = xand h = cosx

[x cosx]' = (1) (cosx) + x (-sinx)

[x cosx]' = cosx – x sinx

Applying the above obtained value for finding u'

u' = cosx – (cosx – x sinx)

u' = x sinx

$$v' = \frac{dv}{dx} = \frac{d(x\sin x + \cos x)}{dx}$$

Applying Product rule for finding the term xsinx in v'

(gh)' = g'h + gh'

Taking g = xand h = sinx

$[x \sin x]' = (1)(\sin x) + x(\cos x)$

$[x \sin x]' = \sin x + x \cos x$

Applying the above obtained value for finding v'

$v' = \sin x + x \cos x - \sin x$

$v' = x \cos x$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{(\sin x - x\cos x)}{(x\sin x + \cos x)}\right]' = \frac{(x \sin x)(x \sin x + \cos x) - (\sin x - x \cos x)(x \cos x)}{(x\sin x + \cos x)^2}$$

$$= \frac{(x^2 \sin^2 x + x\sin x\cos x) - (x\sin x\cos x - x^2 \cos^2 x)}{(x\sin x + \cos x)^2}$$

$$= \frac{x^2 \sin^2 x + x\sin x\cos x - x\sin x\cos x + x^2 \cos^2 x)}{(x\sin x + \cos x)^2}$$

$$= \frac{x^2(\sin^2 x + \cos^2 x)}{(x\sin x + \cos x)^2}$$

$$= \frac{x^2}{(x\sin x + \cos x)^2}$$

$$\text{Ans)} = \frac{x^2}{(x\sin x + \cos x)^2}$$

Question: 26

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\sin x}{dx} = \cos x$

(iii) $\frac{d\cos x}{dx} = -\sin x$

We can write cotx as $\frac{\cos x}{\sin x}$

Let us take u = cosx and v = sinx

$u' = (\cos x)' = -\sin x$

$v' = (\sin x)' = \cos x$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{\cos x}{\sin x}\right)' = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$= \frac{-1}{\sin^2 x}$

$= -\text{cosec}^2 x$

Ans).$-\text{cosec}^2 x$

(ii) To find: Differentiation of secx

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\cos x}{dx} = -\sin x$

We can write secx as $\frac{1}{\cos x}$

Let us take u = 1 and v = cosx

u' = (1)' = 0

v' = (cosx)' = -sinx

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left(\frac{1}{\cos x}\right)' = \frac{(0)(\cos x) - (1)(-\sin x)}{(\cos x)^2}$

$= \frac{\sin x}{\cos^2 x} = \sec x \tan x$

Ans).$-\text{cosec}^2 x$

Exercise : 28E

Question: 1

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\sin nu) = \cos(nu)\frac{d}{dx}(nu)$

Let us take y = sin 4x.

So, by using the above formula, we have

$\frac{d}{dx}(\sin 4x) = \cos(4x) \times \frac{d}{dx}(4x) = 4\cos 4x.$

tion of y = sin 4x is 4cos4x

Question: 2

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\cos nu) = -\sin(nu)\frac{d}{dx}(nu)$.

Let us take y = cos5x.

So, by using the above formula, we have

$\frac{d}{dx}(\cos 5x) = -\sin(5x) \times \frac{d}{dx}(5x) = -5\sin 5x$.

tion of y = cos 5x is - 5sin5x

Question: 3

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\tan nu) = \sec^2(nu).\frac{d}{dx}(nu)$.

Let us take y = tan3x

So, by using the above formula, we have

$\frac{d}{dx}\tan 3x = \sec^2(3x) \times \frac{d}{dx}(3x) = 3\sec^2(3x)$ tion of y = tan3x is $3\sec^2(3x)$

Question: 4

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\cos nu) = -\sin nu\frac{d}{dx}(nu)$ and $\frac{dx^n}{dx} = nx^{n-1}$

Let us take $y = \cos x^3$

So, by using the above formula, we have

$\frac{d}{dx}\cos x^3 = -\sin(x^3) \times \frac{d}{dx}(x^3) = -3x^2\sin(x^3)$ tion of y = $\cos x^3$ is $-3x^2\sin(x^3)$

Question: 5

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\cot^a nu) = a\cot^{a-1}(nu) \times \frac{d}{dx}(\cot nu) \times \frac{d}{dx}(nu)$ and $\frac{dx^n}{dx} = nx^{n-1}$

Let us take $y = \cot^2 x$

So, by using the above formula, we have

$\frac{d}{dx}\cot^2 x = 2\cot(x) \times \frac{d\cot x}{dx} \times \frac{dx}{dx} = -2\cot x\,(\mathrm{cosec}^2 x)$.

tion of $y = \cot^2 x$ is $-2\cot x\,(\mathrm{cosec}^2 x)$

Question: 6

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\tan^a nu) = a\tan^{a-1} nu \times \frac{d(\tan nu)}{dx} \times \frac{d(nu)}{dx}$ and $\frac{d\,x^n}{dx} = nx^{n-1}$

Let us take $y = \tan^3 x$

So, by using the above formula, we have

$\frac{d}{dx}\tan^3 x = 3\tan^2(x) \times \frac{d(\tan x)}{dx} \times \frac{dx}{dx} = 3\tan^2 x \times (\sec^2 x).$

tion of $y = \tan^3 x$ is $3\tan^2 x \times (\sec^2 x)$

Question: 7

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\tan\sqrt{nu}) = \sec^2\sqrt{nu} \times \frac{d}{dx}(\sqrt{nu})\,\frac{d}{dx}(nu).$

Let us take $y = \tan\sqrt{x}$

So, by using the above formula, we have

$\frac{d}{dx}\tan\sqrt{x} = \sec^2(\sqrt{x}) \times \frac{d}{dx}(\sqrt{x})\,\frac{d}{dx}(x). = \frac{1}{2}.\frac{\sec 2(\sqrt{x})}{\sqrt{x}}$

tion of $y = \tan\sqrt{x}$ is $\frac{1}{2}.\frac{\sec 2(\sqrt{x})}{\sqrt{x}}$

Question: 8

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(e^{a^t}) = e^{a^t} \times \frac{d}{dx}(a^t)$ and $\frac{d\,x^n}{dx} = nx^{n-1}$

Let us take $y = e^{x^2}$

So, by using the above formula, we have

$\frac{d}{dx}e^{x^2} = e^{x^2} \times \frac{d}{dx}(x^2) = 2xe^{x^2}$

tion of $y = e^{x^2}$ is $2xe^{x^2}$

Question: 9

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx}$: $\frac{}{dx}$ $\frac{}{dx}$

Formula used: $\frac{d}{dx}(e^a) = e^a \times \frac{da}{dx}$ and $\frac{d\,x^n}{dx} = nx^{n-1}$

Let us take $y = e^{cotx}$

So, by using the above formula, we have

$\frac{d}{dx}e^{cotx} = e^{cotx} \times \frac{dcotx}{dx} = -\ e^{cotx}\ cosec^2x.$

tion of $y = e^{cotx}$ is $-\ e^{cotx}\ cosec^2x$

Question: 10

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\sqrt{sinnu}) = \frac{1}{2\sqrt{sinnu}} \times \frac{d}{dx}(sinnu) \times \frac{d}{dx}(nu)$ and $\frac{d\,x^n}{dx} = nx^{n-1}$

Let us take $y = \sqrt{sinx}$

So, by using the above formula, we have

$\frac{d}{dx}\sqrt{sinx} = \frac{1}{2\sqrt{sinx}} \times \frac{d}{dx}(\sin x)\frac{d}{dx}(x) = \frac{1}{2\sqrt{sinx}}cosx$

tion of $y = \sqrt{sinx}$ is $\frac{1}{2\sqrt{sinx}}$ cosx **Question: 11**

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take $y = (5 + 7x)^6$

So, by using the above formula, we have

$\frac{d}{dx}(5 + 7x)^6 = 6(5 + 7x)^5 \times \frac{d}{dx}(5 + 7x) = 6(5 + 7x)^5 \times 7 = 42(5 + 7x)^5$

tion of $y = (5 + 7x)^6$ is $42(5 + 7x)^5$

Question: 12

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take y= $(3 - 4x)^5$

So, by using the above formula, we have

$\frac{d}{dx}(3 - 4x)^5 = 4(3 - 4x)^5 \times \frac{d}{dx}(3 - 4x) = 4(3 - 4x)^5 \times (-4) = -16(3 - 4x)^5$

tion of $y = (3 - 4x)^5$ is $-16(3 - 4x)^5$

Question: 13

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take y= $(3x^2 - x + 1)^4$

So, by using the above formula, we have

$\frac{d}{dx}(3x^2 - x + 1)^4 = 4(3x^2 - x + 1)^3 \times \frac{d}{dx}(3x^2 - x + 1) = 4(3x^2 - x + 1)^3 \times (3\times 6x - 1) = 4(3x^2 - x + 1)^3(6x - 1)$

tion of $y = (3x^2 - x + 1)^4$ is $4(3x^2 - x + 1)^3(6x - 1)$

Question: 14

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take y= $(ax^2 + bx + c)$

So, by using the above formula, we have

$\frac{d}{dx}(ax^2 + bx + c) = 2ax + b$

tion of $y = (ax^2 + bx + c)$ is $2ax + b$

Question: 15

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take y= $\frac{1}{(x^2 - x + 3)^3} = (x^2 - x + 3)^{-3}$

So, by using the above formula, we have

$\frac{d}{dx}(x^2 - x + 3)^{-3} = -3(x^2 - x + 3)^{-4} \times (2x - 1) = -3\frac{1}{(x^2 - x + 3)^{-4}}(2x - 1)$

tion of $y = (x^2 - x + 3)^{-3}$ is $\frac{-3(2x-1)}{(x^2-x+3)^{-4}}$

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Question: 16

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}\sin^2(ax+b) = 2\sin(ax+b)\frac{d}{dx}\sin(ax+b)\frac{d}{dx}(ax+b)$

Let us take $y = \sin^2(2x+3)$

So, by using above formula, we have

$\frac{d}{dx}\sin^2(2x+3) = 2\sin(2x+3)\frac{d}{dx}\sin(2x+3)\frac{d}{dx}(2x+3) = 4\sin(2x+3)\cos(2x+3).$

tion of $y = \sin^2(2x+3)$ is $4\sin(2x+3)\cos(2x+3)$

Question: 17 te

the**Solution:**

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\cos^a nu) = a\cos^{a-1}nu\,\frac{d}{dx}(\cos nu)\,\frac{d}{dx}(nu)$

Let us take $y = \cos^2(x^3)$

So, by using the above formula, we have

$\frac{d}{dx}\cos^2(x^3) = 2\cos x^3(-\sin(x^3))3x^2 = -6x^2\cos(x^3)\sin x^3$ tion

of $y = \cos^2(x^3)$ is $-6x^2\cos(x^3)\sin x^3$

Question: 18 te

the**Solution:**

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\sqrt{\sin u^a}) = \frac{1}{2\sqrt{\sin u^a}} \times \frac{d}{dx}(\sin u^a) \times \frac{d}{dx}(u^a)$

Let us take $y = \sqrt{\sin x^3}$

So, by using the above formula, we have

$\frac{d}{dx}\sqrt{\sin x^3} = \frac{1}{2\sqrt{\sin x^3}} \times \frac{d}{dx}(\sin x^3) \times \frac{d}{dx}(x^3) = \frac{1}{2\sqrt{\sin x^3}} \times (\cos x^3) \times 3x^2 = \frac{3x^2(\cos x^3)}{2\sqrt{\sin x^3}}$

tion of $y = \sqrt{\sin x^3}$ is $\frac{3x^2(\cos x^3)}{2\sqrt{\sin x^3}}$

Question: 19

te the

Solution:

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx}$ = v $\frac{du}{dx}$ + u $\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\sqrt{usinu}) = \frac{1}{2\sqrt{usinu}} \times \frac{d}{dx}(usinu)$

Let us take y = $\sqrt{x\sin x}$

So, by using the above formula, we have

$\frac{d}{dx}\sqrt{x\sin x} = \frac{1}{2\sqrt{xsinx}} \times \frac{d}{dx}(xsinx) = \frac{1}{2\sqrt{xsinx}} \times (sinx + xcosx) = \frac{(sinx + xcosx)}{2\sqrt{xsinx}}$

tion of y = $\sqrt{xsin\,x}$ is $\frac{(sinx + xcosx)}{2\sqrt{xsinx}}$

Question: 20

Solution:

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\sqrt{cot\sqrt{x}}) = \frac{1}{2\sqrt{cot\sqrt{x}}} \times \frac{d}{dx}(cot\sqrt{x}) \cdot \frac{d}{dx}(\sqrt{x})$

Let us take y = $\sqrt{cot\sqrt{x}}$

So, by using the above formula, we have

$\frac{d}{dx}\sqrt{cot\sqrt{x}} = \frac{1}{2\sqrt{cot\sqrt{x}}} \times \frac{d}{dx}cot\sqrt{x} \times \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{cot\sqrt{x}}} \times (-sec^2\sqrt{x}) \times \frac{1}{2\sqrt{x}} = \frac{-sec^2\sqrt{x}}{4\sqrt{x}\sqrt{cot\sqrt{x}}}$

tion of y = $\sqrt{cot\sqrt{x}}$ is $\frac{-sec^2\sqrt{x}}{4\sqrt{x}\sqrt{cot\sqrt{x}}}$

Question: 21

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Let us take y = cos 3x sin 5x

So, by using the above formula, we have

$\frac{d}{dx}(cos\,3x\,sin\,5x) = sin\,5x\frac{d(cos3x)}{dx} + cos\,3x\frac{d(sin\,5x)}{dx} = sin5\,x\,(-3sin\,3x) + cos\,3\,x(5cos\,5\,x) = 5cos\,(3\,x)\,cos\,(5\,x) - 3\,sin\,(5x)\,3sin\,(3x)$

tion of y = cos 3x sin 5x is 5cos (3 x) cos (5 x) - 3 sin (5x) 3sin (3x)

Question: 22

Solution:

To Find: tion

NOTE : When 2 functions are in the product then we used product rule i.e $\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Let us take y = sin x sin 2x

So, by using the above formula, we have

$\frac{d}{dx}(\sin x \sin 2x) = \sin x \frac{d(\sin 2x)}{dx} + \sin 2x \frac{d(\sin x)}{dx} = \sin x\ (2\cos 2x) + \sin 2x(\sin x) = 2\sin(x)\cos(2x) + \sin 2x(\sin x)$

tion of y = sin x sin 2x is 2sin (x)cos (2x) + sin 2x(sin x)

Question: 23 t

Solution:

Let $y = \cos(\sin\sqrt{ax+b})$, $z = \sin\sqrt{ax+b}$ and $w = \sqrt{ax+b}$

Formula : $\frac{d(\cos x)}{dx} = -\sin x$ and $\frac{d(\sin x)}{dx} = \cos x$

$\frac{d(\sqrt{ax+b})}{dx} = \frac{1}{2} \times (ax+b)^{\frac{1}{2}-1} \times a$

According to the chain rule of tion

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dw} \times \frac{dw}{dx}$$

$$= -\sin(\sin\sqrt{ax+b}) \times \cos\sqrt{ax+b} \times \frac{1}{2} \times (ax+b)^{-\frac{1}{2}} \times a$$

$$= -\frac{a}{2}\sin(\sin\sqrt{ax+b}) \times \cos\sqrt{ax+b} \times (ax+b)^{-\frac{1}{2}}$$

Question: 24

Solution:

Let $y = e^{2x}\sin 3x$, $z = e^{2x}$ and $w = \sin 3x$

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\sin x)}{dx} = \cos x$

According to product rule of tion

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\sin 3x \times (2 \times e^{2x})] + [e^{2x} \times 3\cos 3x]$$

$$= e^{2x} \times [2\sin 3x + 3\cos 3x]$$

Question: 25

Solution:

Let $y = e^{3x}\cos 2x$, $z = e^{3x}$ and $w = \cos 2x$

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to the product rule of tion

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\cos 2x \times (3 \times e^{3x})] + [e^{3x} \times (-2\sin 2x)]$$

$= e^{3x} \times [3\cos 2x - 2\sin 2x]$

Question: 26

Solution:

Let $y = e^{-5x} \cot 4x$, $z = e^{-5x}$ and $w = \cot 4x$

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

According to the product rule of tion

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$= [\cot 4x \times (-5e^{-5x})] + [e^{-5x} \times (-4\operatorname{cosec}^2 4x)]$

$= -e^{-5x} \times [5\cot 4x + 4\operatorname{cosec}^2 4x]$

Question: 27

Solution:

Let $y = \cos(x^3 . e^x)$, $z = x^3 . e^x$, $m = e^x$ and $w = x^3$

Formula : $\frac{d(e^x)}{dx} = e^x$, $\frac{d(x^n)}{dx} = n \times x^{n-1}$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to the product rule of tion

$$dz/_{dx} = w \times \frac{dm}{dx} + m \times \frac{dw}{dx}$$

$= [x^3 \times (e^x)] + [e^x \times (3x^2)]$

$= e^x \times [x^3 + 3x^2]$

According to the chain rule of tion

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$= -\sin(x^3 \times e^x) \times \{ e^x \times [x^3 + 3x^2]\}$

Question: 28

Solution:

Let $y = e^{(x\sin x + \cos x)}$, $z = x\sin x + \cos x$, $m = x$ and $w = \sin x$

Formula : $\frac{d(e^x)}{dx} = e^x$, $\frac{d(\sin x)}{dx} = \cos x$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to the product rule of tion

$$dz/_{dx} = w \times \frac{dm}{dx} + m \times \frac{dw}{dx} + \frac{d(\cos x)}{dx}$$

$= [\sin x \times (1)] + [x \times (\cos x)] - \sin x$

$= x \cos x$

According to the chain rule of tion

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$= e^{(x\sin x + \cos x)} \times (x\cos x)$

Question: 29

Solution:

Let $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$, $u = e^x + e^{-x}$, $v = e^x - e^{-x}$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to the quotient rule of tionIf $y = \frac{u}{v}$

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(e^x - e^{-x}) \times (e^x - e^{-x}) \; - \; (e^x + e^{-x}) \times (e^x + e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x})^2 \; - \; (e^x + e^{-x})^2}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x} \; + \; e^x + e^{-x}) \, (e^x - e^{-x} - \; e^x - e^{-x})}{(e^x - e^{-x})^2}$$

$(a^2 - b^2 = (a - b)(a + b))$

$$= \frac{(2 \, e^x) \, (-2e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{-4}{(e^x - e^{-x})^2}$$

Question: 30

Solution:

Let $y = \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$, $u = e^{2x} + e^{-2x}$, $v = e^{2x} - e^{-2x}$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to the quotient rule of tionIf y =

$\frac{u}{v}$

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(e^{2x} - e^{-2x}) \times (2e^{2x} - 2e^{-2x}) \; - \; (e^{2x} + e^{-2x}) \times (2e^{2x} + 2e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

$$= \frac{2(e^{2x} - e^{-2x})^2 \; - \; 2(e^{2x} + e^{-2x})^2}{(e^{2x} - e^{-2x})^2}$$

$$= \frac{2 \, (e^{2x} - e^{-2x} \; + \; e^{2x} + e^{-2x}) \, (e^{2x} - e^{-2x} - \; e^{2x} - e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

(a² – b² = (a - b)(a + b)

$$= \frac{2(\,2\,e^{2x}\,)\,(-2e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

$$= \frac{-8}{(e^{2x} - e^{-2x})^2}$$

Question: 31

te w.r**Solution:**

Let $y = \sqrt{\frac{1-x^2}{1+x^2}}$, $u = 1 - x^2$, $v = 1 + x^2$, $z = \frac{1-x^2}{1+x^2}$

Formula : $\frac{d(x^2)}{dx} = 2x$

According to the quotient rule of tionIf $z = \frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 + x^2) \times (-2x) - (1 - x^2) \times (2x)}{(1 + x^2)^2}$$

$$= \frac{-2x - 2x^3 - 2x \; + \; 2x^3}{(1 + x^2)^2}$$

$$= \frac{-4x}{(1 + x^2)^2}$$

According to chain rule of tion

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\frac{1}{2} \times \left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{-4x}{(1+x^2)^2}\right]$$

$$= \left[\frac{-2x}{1} \times \left(\frac{1-x^2}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1+x^2)^{2-\frac{1}{2}}}\right]$$

$$= \left[-2x \times (1 - x^2)^{-\frac{1}{2}}\right] \times (1 + x^2)^{-\frac{3}{2}}$$

Question: 32

Solution:

Let $y = \sqrt{\frac{a^2-x^2}{a^2+x^2}}$, $u = a^2 - x^2$, $v = a^2 + x^2$, $z = \frac{a^2-x^2}{a^2+x^2}$

Formula : $\frac{d(x^2)}{dx} = 2x$

According to the quotient rule of tionIf z =

$\frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{\left(a^2 + x^2\right) \times (-2x) \; - \; (a^2 - x^2) \times (2x)}{\left(a^2 + x^2\right)^2}$$

$$= \frac{-2xa^2 - 2x^3 - 2xa^2 \; + \; 2x^3}{(1 + x^2)^2}$$

$$= \frac{-4\,x\,a^2}{(1 + x^2)^2}$$

According to the chain rule of tion

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\frac{1}{2} \times \left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{-4x\,a^2}{(a^2 + x^2)^2}\right]$$

$$= \left[\frac{-2xa^2}{1} \times \left(\frac{a^2 - x^2}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(a^2 + x^2)^{2-\frac{1}{2}}}\right]$$

$$= \left[-2xa^2 \times (a^2 - x^2)^{-\frac{1}{2}}\right] \times (a^2 + x^2)^{-\frac{3}{2}}$$

Question: 33

Solution:

Let $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$, $u = 1 + \sin x$, $v = 1 - \sin x$, $z = \frac{1+\sin x}{1-\sin x}$

Formula : $\frac{d(\sin x)}{dx} = \cos x$

According to the quotient rule of tionIf $z = \frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 - \sin x) \times (\cos x) \; - \; (1 + \sin x) \times (-\cos x)}{(1 - \sin x)^2}$$

$$= \frac{\cos x \; - \sin x \cos x + \cos x + \sin x \cos x}{(1 - \sin x)^2}$$

$$= \frac{2 \cos x}{(1 - \sin x)^2}$$

According to the chain rule of tion

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\frac{1}{2} \times \left(\frac{1 + \sin x}{1 - \sin x}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{2 \cos x}{(1 - \sin x)^2}\right]$$

$= \left[\frac{\cos x}{1} \times \left(\frac{1+\sin x}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1-\sin x)^{2-\frac{1}{2}}}\right]$

$= \left[\cos x \times (1+\sin x)^{-\frac{1}{2}}\right] \times (1-\sin x)^{-\frac{3}{2}}$

Question: 34

te w.r**Solution:**

Let $y = \sqrt{\frac{1+e^x}{1-e^x}}$, $u = 1 + e^x$, $v = 1 - e^x$, $z = \frac{1+e^x}{1-e^x}$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to the quotient rule of tionIf $z = \frac{u}{v}$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1-e^x) \times (e^x) - (1+e^x) \times (-e^x)}{(1-e^x)^2}$$

$$= \frac{e^x - e^{2x} + e^x + e^{2x}}{(1-e^x)^2}$$

$$= \frac{2e^x}{(1-e^x)^2}$$

According to chain rule of tion

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\frac{1}{2} \times \left(\frac{1+e^x}{1-e^x}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{2e^x}{(1-e^x)^2}\right]$$

$$= \left[\frac{e^x}{1} \times \left(\frac{1+e^x}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1-e^x)^{2-\frac{1}{2}}}\right]$$

$$= \left[e^x \times (1+e^x)^{-\frac{1}{2}}\right] \times (1-e^x)^{-\frac{3}{2}}$$

Question: 35

Solution:

Let $y = \frac{e^{2x}+x^3}{\operatorname{cosec} 2x}$, $u = e^{2x} + x^3$, $v = \operatorname{cosec} 2x$

Formula: $\frac{d(e^x)}{dx} = e^x$, $\frac{d(x^n)}{dx} = n \times x^{n-1}$ and $\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x$

According to the quotient rule of tion

If $y = \frac{u}{v}$

$$\frac{dy}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(\text{cosec } 2x) \times (2e^{2x} + 3x^2) \; - \; (e^{2x} + x^3) \times (-2 \text{ cosec } 2x \; \cot 2x)}{(\text{cosec } 2x)^2}$$

$$= \frac{2e^{2x} \text{ cosec } 2x \; + 3x^2 \text{ cosec } 2x \; + 2\, e^{2x} \text{ cosec } 2x \; \cot 2x + 2x^3 \text{ cosec } 2x \; \cot 2x}{(\text{cosec } 2x)^2}$$

$$= \frac{2e^{2x} \text{ cosec } 2x\, (\, 1 + \cot 2x\,) \; + \; 3x^2 \text{ cosec } 2x\, (\, 1 + \cot 2x\,)}{(\text{cosec } 2x)^2}$$

$$= \frac{(1 + \cot 2x)(2e^x \text{cosec } 2x \; + \; 3x^2 \text{cosec } 2x)}{(\text{cosec } 2x)^2}$$

$$= \frac{(1 + \cot 2x)(2e^x \; + \; 3x^2)(\text{cosec } 2x)}{(\text{cosec } 2x)^2}$$

$$= \frac{(1 + \cot 2x)(2e^x + 3x^2)}{(\text{cosec } 2x)^1}$$

$$= (\, 1 + \cot 2x\,)(2\, e^x \; + \; 3x^2)(\sin 2x)$$

Question: 36

Solution:

Let $y = \sin(\sqrt{\sin x + \cos x}\,)$, $z = \sqrt{\sin x + \cos x}$

Formula : $\frac{d(\cos x)}{dx} = -\sin x$ and $\frac{d(\sin x)}{dx} = \cos x$

� $\frac{d(\sqrt{\sin x + \cos x})}{dx} = \frac{1}{2} \times (\sin x + \cos x)^{\frac{1}{2} - 1} \times (\cos x - \sin x)$

According to the chain rule of tion

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \cos(\sin \sqrt{\sin x + \cos x}) \times \frac{1}{2} \times (\sin x + \cos x)^{\frac{1}{2} - 1} \times (\cos x - \sin x)$$

$$= \cos(\sin \sqrt{\sin x + \cos x}) \times \frac{1}{2} \times (\sin x + \cos x)^{-\frac{1}{2}} \times (\cos x - \sin x)$$

Question: 37

Solution:

Let $y = e^x \log(\sin 2x)$, $z = e^x$ and $w = \log(\sin 2x)$

Formula : $\frac{d(e^x)}{dx} = e^x$, $\frac{d(\log x)}{dx} = \frac{1}{x}$ and $\frac{d(\sin x)}{dx} = \cos x$

According to the product rule of tion

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\log(\sin 2x) \times (e^x)] + [e^x \times \frac{1}{\sin 2x} \times 2 \cos 2x\,]$$

$$= e^x \times [\log(\sin 2x) + \frac{2 \cos 2x}{\sin 2x}]$$

$$= e^x \times [\log(\sin 2x) + 2 \cot 2x\,]$$

Question: 38

Solution:

Let $y = \cos\left(\frac{1-x^2}{1+x^2}\right)$, $u = 1 - x^2$, $v = 1 + x^2$, $z = \frac{1-x^2}{1+x^2}$

Formula : $\frac{d(x^2)}{dx} = 2x$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to the quotient rule of tionIf $z = \frac{u}{v}$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1+x^2) \times (-2x) - (1-x^2) \times (2x)}{(1+x^2)^2}$$

$$= \frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2}$$

$$= \frac{-4x}{(1+x^2)^2}$$

According to the chain rule of tion

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[-\sin\frac{1-x^2}{1+x^2}\right] \times \left[\frac{-4x}{(1+x^2)^2}\right]$$

$$= \left[\sin\frac{1-x^2}{1+x^2}\right] \times \left[\frac{4x}{(1+x^2)^2}\right]$$

Question: 39

Solution:

Let $y = \sin\left(\frac{1+x^2}{1-x^2}\right)$, $u = 1 + x^2$, $v = 1 - x^2$, $z = \frac{1+x^2}{1-x^2}$

Formula : $\frac{d(x^2)}{dx} = 2x$ and $\frac{d(\sin x)}{dx} = \cos x$

According to the quotient rule of tionIf $z = \frac{u}{v}$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1-x^2) \times (2x) - (1+x^2) \times (-2x)}{(1-x^2)^2}$$

$$= \frac{2x - 2x^3 + 2x + 2x^3}{(1+x^2)^2}$$

$$= \frac{4x}{(1+x^2)^2}$$

According to the chain rule of tion

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$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\cos\frac{1+x^2}{1-x^2}\right] \times \left[\frac{4x}{(1+x^2)^2}\right]$$

Question: 40

Solution:

Let $y = \frac{\sin x + x^2}{\cot 2x}$, $u = \sin x + x^2$, $v = \cot 2x$

Formula: $\frac{d(\sin x)}{dx} = \cos x, \frac{d(x^n)}{dx} = n \times x^{n-1}$ and $\frac{d(\cot x)}{dx} = -\mathrm{cosec}^2 x$

According to the quotient rule of tion

If $y = \frac{u}{v}$

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(\cot 2x) \times (\cos x + 2x) - (\sin x + x^2) \times (-2\,\mathrm{cosec}^2 2x)}{(\cot 2x)^2}$$

$$= \frac{\cot 2x \cos x + 2x \cot 2x + 2\,\mathrm{cosec}^2 2x \sin x + 2x^2 \mathrm{cosec}^2 2x}{(\mathrm{cosec}\, 2x)^2}$$

$$= \frac{\cot 2x (\cos x + 2x) + 2\,\mathrm{cosec}^2 2x (\sin x + x^2)}{(\mathrm{cosec}\, 2x)^2}$$

$$= \frac{2\,\mathrm{cosec}^2 2x (\sin x + x^2)}{(\mathrm{cosec}\, 2x)^2} + \frac{\cot 2x (\cos x + 2x)}{(\mathrm{cosec}\, 2x)^2}$$

$$= \frac{2(\sin x + x^2)}{1} + \frac{\cos 2x\ (\cos x + 2x)}{\sin 2x \frac{1}{\sin^2 2x}}$$

$= 2(\sin x + x^2) + \cos 2x \sin 2x (\cos x + 2x)$

Question: 41

Solution:

Let $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, , $u = \cos x - \sin x$, $v = \cos x + \sin x$

Formula: $\frac{d(\sin x)}{dx} = \cos x$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to the quotient rule of tionIf y =

$\frac{u}{v}$

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(\cos x + \sin x) \times (-\sin x - \cos x) - (\cos x - \sin x) \times (-\sin x + \cos x)}{(\cos x + \sin x)^2}$$

$$= \frac{-(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x + \sin x)^2}$$

$$= -\frac{(\cos x + \sin x)^2}{(\cos x + \sin x)^2} - \frac{(\cos x - \sin x)^2}{(\cos x + \sin x)^2}$$

$$= -\frac{1}{1} - y^2 \; (y = \frac{\cos x - \sin x}{\cos x + \sin x})$$

$$\frac{dy}{dx} + y^2 + 1 = 0$$

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Question: 42

Solution:

Let $y = \frac{\cos x + \sin x}{\cos x - \sin x}$, , $u = \cos x + \sin x$, $v = \cos x - \sin x$

Formula: $\frac{d(\sin x)}{dx} = \cos x$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to the quotient rule of tionIf $y = \frac{u}{v}$

$$\frac{dy}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(\cos x - \sin x) \times (-\sin x + \cos x) - (\cos x + \sin x) \times (-\sin x - \cos x)}{(\cos x - \sin x)^2}$$

$$= \frac{(\cos x - \sin x)^2 + (\cos x + \sin x)^2}{(\cos x - \sin x)^2}$$

$$= \frac{(\cos^2 x + \sin^2 x - 2\cos x \sin x) + (\cos^2 x + \sin^2 x + 2\cos x \sin x)}{(\cos x - \sin x)^2}$$

$$= \frac{2(\cos^2 x + \sin^2 x)}{(\cos x - \sin x)^2}$$

$$= \frac{(1)}{(\cos x - \sin x)^2 / 2} \quad (\cos^2 x + \sin^2 x) = 1$$

$$= \frac{1}{\left(\frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}}\right)^2}$$

$$= \frac{1}{\left(\frac{\cos x \cos 45^\circ}{1} - \frac{\sin x \sin 45^\circ}{1}\right)^2}$$

$= \frac{1}{\cos^2(x + \frac{\pi}{4})}$ [cos a cos b - sin a sin b = cos (a + b)]

$= \sec^2(x + \frac{\pi}{4})$

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Question: 43

Solution:

Let $y = \sqrt{\frac{1-x^1}{1+x^1}}$, $u = 1 - x^1$, $v = 1 + x^1$, $z = \frac{1-x^1}{1+x^1}$

Formula : $\frac{d(x^1)}{dx} = 1$

According to quotient rule of tionIf $z = \frac{u}{v}$

$$dz/dx = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} = \frac{(1+x^1) \times (-1) - (1-x^1) \times (1)}{(1+x^1)^2}$$

$$= \frac{-1 - x^1 - 1 + x}{(1+x^1)^2}$$

$$= \frac{-2}{(1+x)^2}$$

According to the chain rule of tion

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\frac{1}{2} \times \left(\frac{1-x^1}{1+x^1}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{-2}{(1+x^1)^2}\right]$$

$$= \left[\frac{-1}{1} \times \left(\frac{1-x^1}{1+x}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1+x^1)^2}\right]$$

$$= \left[-1 \times \frac{(1-x^1)^{-\frac{1}{2}}}{(1+x^1)^{1-\frac{1}{2}}}\right] \times \left[\frac{1}{(1+x^1)^1}\right] \times \frac{1-x}{1-x}$$

(Muliplying and dividing by 1-x)

$$= \left[-1 \times \frac{(1-x^1)^{1-\frac{1}{2}}}{(1+x^1)^{\frac{1}{2}}}\right] \times \frac{1}{(1-x)(1+x)}$$

$$= \left[-1 \times \frac{(1-x^1)^{\frac{1}{2}}}{(1+x^1)^{\frac{1}{2}}}\right] \times \frac{1}{(1-x)(1+x)} = -\frac{y}{1-x^2}$$

Therefore $(1-x^2)\frac{dy}{dx} = -y$

$$(1-x^2)\frac{dy}{dx} + y = 0$$

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Solution:

$$y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$$

$$y = \sqrt{\frac{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}} = \sqrt{\frac{1-\sin x}{1+\sin x}}$$

u =1-sin x, v = 1 + sin x , $z=\frac{1-\sin x}{1+\sin x}$

Formula : $\frac{d(\sin x)}{dx} = \cos x$

According to quotient rule of tionIf $z = \frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 + \sin x) \times (-\cos x) \;-\; (1 - \sin x) \times (\cos x)}{(1 + \sin x)^2}$$

$$= \frac{-\cos x - \sin x \cos x - \cos x + \sin x \cos x}{(1 + \sin x)^2}$$

$$= \frac{-2\cos x}{(1 + \sin x)^2}$$

According to the chain rule of tion

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\frac{1}{2} \times \left(\frac{1-\sin x}{1+\sin x}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{-2\cos x}{(1+\sin x)^2}\right]$$

$$= \left[-\frac{\cos x}{1} \times \left(\frac{1-\sin x}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1+\sin x)^{2-\frac{1}{2}}}\right]$$

$$= \left[\cos x \times (1 + \sin x)^{-\frac{1}{2}}\right] \times (1 - \sin x)^{-\frac{3}{2}} \times \left(\frac{1+\sin x}{1+\sin x}\right)^{\frac{3}{2}}$$

(Multiplying and dividing by $(1 + \sin x)^{\frac{3}{2}}$)

$$= \left[\cos x \times (1 + \sin x)^{\frac{3}{2}-\frac{1}{2}}\right] \times (1 - \sin x)^{-\frac{3}{2}} \times \left(\frac{1}{1+\sin x}\right)^{\frac{3}{2}}$$

$$= \left[\cos x \times (1 + \sin x)^{\frac{3}{2}-\frac{1}{2}}\right] \times (1 - \sin x)^{-\frac{3}{2}} \times (1 + \sin x)^{-\frac{3}{2}}$$

$$= [\cos x \times (1 + \sin x)^1] \times (1 - \sin^2 x)^{-\frac{3}{2}}$$

$$= [\cos x \times (1 + \sin x)^1] \times (\cos^2 x)^{-\frac{3}{2}}$$

$$= [\cos x \times (1 + \sin x)^1] \times (\cos x)^{-3}$$

$$= [(1 + \sin x)^1] \times (\cos x)^{-3+1}$$

$$= \frac{1+\sin x}{\cos^2 x}$$

$$= \frac{1}{\cos^1 x} \times \frac{1+\sin x}{\cos^1 x}$$

$$= \sec x \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$$

$$= \sec x\,(\sec x + \tan x)$$

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