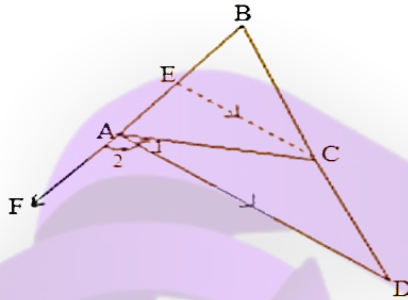


Chapter 6. Triangle

Question-1

The bisector of the exterior angle $\angle A$ of $\triangle ABC$ intersects side BC produced at D . Prove that $\frac{AB}{AC} = \frac{BD}{DC}$.



Solution:

Given: ABC is a triangle; AD is the exterior bisector of $\angle A$ and meets BC produced at D ; BA is produced to F .

To prove: $\frac{AB}{AC} = \frac{BD}{DC}$

Construction: Draw $CE \parallel DA$ to meet AB at E .

Proof: In $\triangle ABC$, $CE \parallel AD$ cut by AC .

$\angle CAD = \angle ACE$ (Alternate angles)

Similarly $CE \parallel AD$ cut by AB

$\angle FAD = \angle AEC$ (corresponding angles)

Since $\angle FAD = \angle CAD$ (given)

$\therefore \angle ACE = \angle AEC$

$\therefore AC = AE$ (by isosceles \triangle theorem)

Now in $\triangle BAD$, $CE \parallel DA$

$\frac{AE}{AB} = \frac{DC}{BD}$ (BPT)

$AB \quad BD$

But $AC = AE$ (proved above)

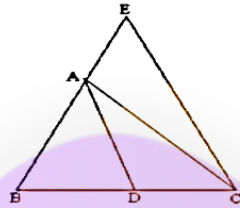
$\therefore \frac{AC}{AB} = \frac{DC}{BD}$

$AB \quad BD$ or

$$\frac{AB}{AC} = \frac{BD}{DC} \text{ (proved).}$$

Question-2

State and prove the converse of angle bisector theorem.



Solution:

Given: ABC is a Δ ; AD divides BC in the ratio of the sides containing the angles $\angle A$ to meet BC at D.

$$\text{i.e. } \frac{AB}{AC} = \frac{BD}{DC}$$

To prove: AD bisects $\angle A$.

Construction: Draw $CE \parallel DA$ to meet BA produced at E.

Proof: In ΔABC , $CE \parallel DA$ cut by AE.

$$\therefore \angle BAD = \angle AEC \text{ (corresponding angle) } \text{---(i)}$$

Similarly $CE \parallel DA$ cut by AC

$$\therefore \angle DAC = \angle ACE \text{ (alternate angles) } \text{---(ii)}$$

In DBEC; $CE \parallel AD$

$$\therefore \frac{AB}{AE} = \frac{BD}{DC} \text{ (BPT)}$$

$$\frac{AB}{AE} = \frac{BD}{DC}$$

But $\frac{AB}{AC} = \frac{BD}{DC}$ (given)

$$\frac{AB}{AE} = \frac{BD}{DC}$$

$$\therefore \frac{AB}{AE} = \frac{AB}{AC}$$

$$\frac{AB}{AE} = \frac{AB}{AC}$$

$$\therefore AE = AC$$

$$\Rightarrow \angle AEC = \angle ACE \text{ (isosceles property) } \text{---(iii)}$$

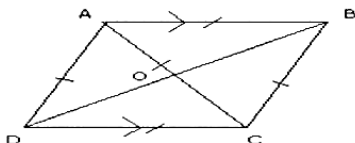
According to equation (i), (ii) and (iii) $\angle BAD = \angle DAC$

\Rightarrow AD bisects $\angle A$.

Question-3

If a parallelogram has all its sides equal and one of its diagonal is equal to a side, show that its diagonals are in the ratio $\sqrt{3} : 1$.

Solution:



Given: ABCD is a parallelogram, where AC and BD are the diagonals, meeting at O. $AB = BC = AC$.

To Prove: $BD : AC :: \sqrt{3} : 1$

Proof: In $\triangle ABC$, $AB = BC = CA$ (given).
 $= a$ (say)

Hence ABC is an equilateral triangle. (Definition of equilateral triangle)

AC and BD are the diagonals of parallelogram ABCD,

$\Rightarrow AC = BD$ (Diagonals of a parallelogram bisect each other)

or $AO = OC$.

i.e BO is the median of the equilateral ABC.

Hence $BO = \frac{\sqrt{3}}{2} a$

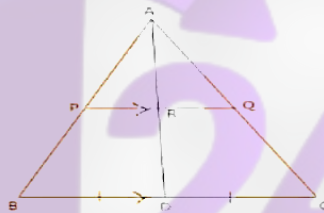
$\therefore BD = \sqrt{3} a$

$\Rightarrow BD : AC :: \sqrt{3} a : a$

$\Rightarrow BD : AC :: \sqrt{3} : 1$.

Question-4

In $\triangle ABC$, P, Q are points on AB and AC respectively and $PQ \parallel BC$. Prove that the median AD bisects PQ.



Solution:

Given: ABC is a triangle, $PQ \parallel BC$; AD is the median which cuts PQ at R.

To prove: AD bisects PQ at R.

Proof: In $\triangle ABD$; $PR \parallel BD$

$$\frac{AP}{PB} = \frac{AR}{RD} \quad (\text{BPT})$$

In $\triangle ACD$, $RQ \parallel DC$

$$\therefore \frac{AR}{RD} = \frac{AQ}{QC} \quad (\text{BPT})$$

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

In $\triangle APR$ and $\triangle ABD$,

$\angle APR = \angle ABD$ (corresponding angles.)

$\angle ARP = \angle ADB$ (corresponding angles.)

$\therefore \triangle APR$ is similar to $\triangle ABD$ (AA similarity)

$\therefore \frac{AP}{AB} = \frac{AR}{AD} = \frac{PR}{BD}$ (corresponding sides of similar triangles are proportional)---

-(i)

$$\frac{AP}{AB} = \frac{AR}{AD} = \frac{PR}{BD}$$

Similarly ΔARQ is similar to ΔADC

$$\frac{AQ}{AC} = \frac{AR}{AD} = \frac{RQ}{DC} \text{ ---(ii)}$$

According to equation (i) and (ii),

$$\frac{AR}{AD} = \frac{PR}{BD} = \frac{RQ}{DC}$$

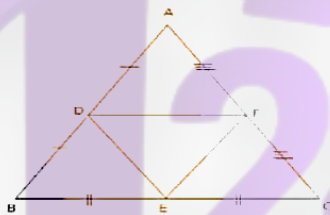
but $BD = DC$ (given)

$$\therefore PR = RQ$$

or AD bisects PQ at R (proved).

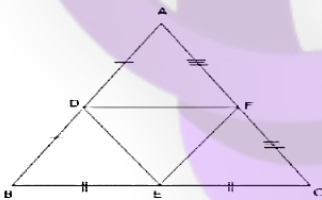
Question-5

Prove that the line joining the midpoints of the sides of the triangle form four triangles, each of which is similar to the original triangle.



Solution:

Given: In ΔABC , D, E, F are the midpoints of AB, BC and AC respectively.



To prove: $\Delta ABC \sim \Delta DEF$

$$\Delta ABC \sim \Delta ADF$$

$$\Delta ABC \sim \Delta BDE$$

$$\Delta ABC \sim \Delta EFC$$

Proof: In ΔABC , D and F are mid points of AB and AC respectively.

$\therefore DF \parallel BC$ (midpoint theorem)

In ΔABC and ΔADF

$\angle A$ is common; $\angle ADF = \angle ABC$ (corresponding angles)

$$\Delta ABC \sim \Delta ADF \text{ (AA similarity) ---(1)}$$

Similarly we can prove $\Delta ABC \sim \Delta BDE$ (AA similarity)---(2)

$$\Delta ABC \sim \Delta EFC \text{ (AA similarity)---(3)}$$

In ΔABC and ΔDEF ;

since D, E, F are the midpoints of AB, BC and AC respectively,

$DF = (1/2) \times BC$; $DE = (1/2) \times AC$; $EF = (1/2) \times AB$; (midpoint theorem)

$$\therefore \frac{AB}{EF} = \frac{BC}{DF} = \frac{CA}{DE} = 2$$

$$\frac{AB}{EF} = \frac{BC}{DF} = \frac{CA}{DE} = 2$$

$\therefore \Delta ABC \sim \Delta EFD$ (SSS similarity) -----(4)

From (1), (2), (3) and (4)

$$\Delta ABC \sim \Delta DEF$$

$$\Delta ABC \sim \Delta ADF$$

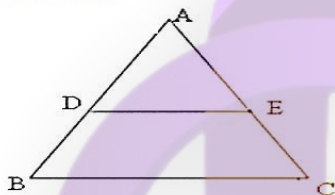
$$\Delta ABC \sim \Delta BDE$$

$$\Delta ABC \sim \Delta EFC.$$

Question-6

In triangle ABC, $DE \parallel BC$ and $AD : DB = 2 : 3$. Determine the ratio of the area triangle ADE to the area triangle ABC.

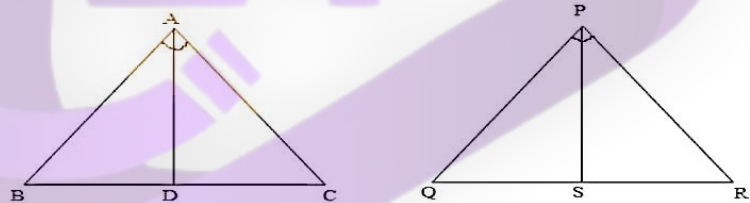
Solution:



$$\therefore \frac{\text{Area } \Delta ADE}{\text{Area } \Delta ABC} = \frac{2^2}{5^2} = \frac{4}{25}$$

Question-7

One angle of a triangle is equal to one angle of another triangle and the bisectors of these equal angles divide the opposite sides in the same ratio. Prove that the triangles are similar.



Solution:

Given: ΔABC and ΔPQR

$$\angle A = \angle P$$

AD and PS bisect $\angle A$ and $\angle P$ respectively.

$$\frac{BD}{DC} = \frac{QS}{SR}$$

$$\frac{BD}{DC} = \frac{QS}{SR}$$

To prove: $\Delta ABC \sim \Delta PQR$

Proof: In ΔABC and ΔPQR

AD bisects $\angle A$

$$\therefore \frac{AB}{AC} = \frac{BD}{DC} \text{ (Angle bisector theorem) } \text{----(1)}$$

$$\frac{AB}{AC} = \frac{BD}{DC}$$

Similarly in ΔPQR ,

$$\frac{PQ}{PR} = \frac{QS}{SR} \text{ (Angle bisector theorem) } \text{----(2)}$$

$$\frac{PQ}{PR} = \frac{QS}{SR}$$

But $\underline{BD} = \underline{QS}$ (given)

$\underline{DC} = \underline{SR}$

\therefore According to equation (1) and (2)

$\underline{AB} = \underline{PQ} \text{ } \underline{AB} = \underline{AC}$

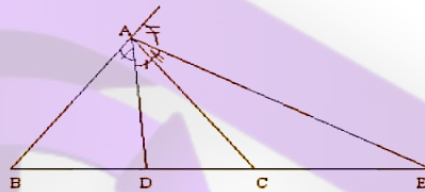
$\underline{AC} = \underline{PR} \text{ } \underline{PQ} = \underline{PR}$

$\angle A = \angle P$ (given)

$\therefore \triangle ABC \sim \triangle PQR$ (SAS similarity).

Question-8

The bisector of interior angle A of a triangle ABC meets BC in D and the bisector of exterior angle A meets BC produced in E . Prove that $\frac{BD}{BE} = \frac{CD}{CE}$



Solution:

Given: $\triangle ABC$, AD bisects interior $\angle A$ and AE bisects exterior $\angle A$ meeting BC at D and BC produced at E .

To prove: $\frac{BD}{BE} = \frac{CD}{CE}$

Proof: In $\triangle ABC$, AD bisects interior $\angle A$

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$ (Angle Bisector theorem).....(1)

Similarly in $\triangle ABC$, AE bisects exterior $\angle A$

$\therefore \frac{AB}{AC} = \frac{BE}{CE}$ (2)

From equation (1) and (2),

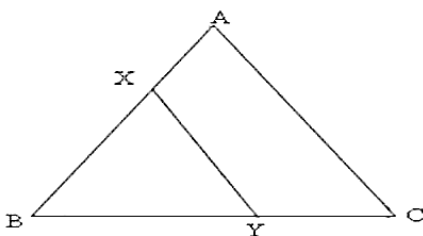
$$\frac{AB}{AC} = \frac{BD}{DC} = \frac{BE}{CE} \Rightarrow \frac{BD}{BE} = \frac{CD}{CE}$$

Hence Proved.

Question-9

In a triangle ABC , $XY \parallel AC$ divides the triangle into two parts equal in areas.

Determine $\frac{AX}{AB}$.



Solution:

Given: ABC is a triangle with XY || AC divides the triangle into two parts equal in areas.

To find: $\frac{AX}{AB}$

Proof:

ar ΔBXY = ar trap. XYCA (Given) \therefore ar ΔBXY = $\frac{1}{2}$ ar ΔABC

In ΔBXY and ΔBAC ,

$\angle BXY = \angle BAC$ (Corresponding angles)

$\angle BYX = \angle BCA$ (Corresponding angles)

$\Delta BXY \sim \Delta BAC$ (AA similarity)

$$\therefore \frac{\text{Area } \Delta BXY}{\text{Area } \Delta BAC} = \frac{BX^2}{AB^2} \quad (\text{Areas of similar triangle})$$

$$\therefore \frac{1}{2} = \frac{BX^2}{AB^2}$$

$$\therefore \frac{1}{\sqrt{2}} = \frac{BX}{AB}$$

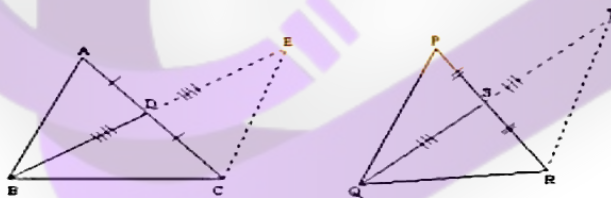
$$\therefore AB - BX = \sqrt{2} BX - BX$$

$$\therefore AX = (\sqrt{2} - 1)BX$$

$$\frac{AX}{AB} = \frac{(\sqrt{2} - 1)BX}{\sqrt{2}BX} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

Question-10

If two sides and a median bisecting the third side of a Δ are respectively proportional to the corresponding sides and the median of another triangle, then prove that the two triangles are similar.



Solution:

Given: ΔABC and ΔPQR where BD and QS are the medians and $\underline{AB} = \underline{BC} = \underline{BD}$

PQ QR QS

To prove: $\Delta ABC \sim \Delta PQR$

Construction: Produce BD and QS to E and T respectively such that $BD = DE$ and $QS = ST$. CE and TR are joined.

Proof: In ΔADB and ΔCDE ,

$AD = DC$ (given)

$\angle ADB = \angle CDE$ (Vertically opposite angles)

$BD = DE$.

$\therefore \Delta ADB \cong \Delta CDE$ (SAS \cong axiom)

Hence $AB = CE$ and $\angle ABD = \angle DEC$.

Similarly $\triangle PQS \cong \triangle RST$,

hence $PQ = TR$ and $\angle PQS = \angle STR$.

Consider $\triangle EBC$ and $\triangle TQR$,

$$\underline{BD} = \underline{2BD} = \underline{BE} \text{ (from given and construction)} \text{-----(1)}$$

$$\underline{QS} = \underline{2QS} = \underline{QT}$$

$AB = CE$ and $PQ = RT$ (proved),

$$\underline{AB} = \underline{CE} \text{----(2)}$$

$$\underline{PQ} = \underline{RT}$$

$$\underline{AB} = \underline{BC} = \underline{BD} \text{ (Given)} \text{-----(3)}$$

$$\underline{PQ} = \underline{QR} = \underline{QS}$$

From (1),(2) and (3),

$$\underline{BE} = \underline{CE} = \underline{BC}$$

$$\underline{QT} = \underline{RT} = \underline{QR}$$

$\therefore \triangle EBC \sim \triangle TQR$ (SSS similarity axiom).

$\Rightarrow \angle DBC = \angle SQR$ and $\angle DEC = \angle STR$ ----(4) (corresponding angles of similar triangles are proportional)

But $\angle ABD = \angle DEC$ and $\angle PQS = \angle STR$ (proved)----(5)

$\therefore \angle ABD = \angle PQS$ (from (4) and (5)) ----(6)

From (5) and (6),

$$\angle ABC = \angle PQR \text{----I}$$

In $\triangle ABC$ and $\triangle PQR$,

$$\underline{AB} = \underline{BC} \text{ (given)}$$

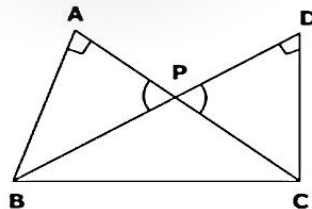
$$\underline{PQ} = \underline{QR}$$

And $\angle ABC = \angle PQR$ (from I)

$\therefore \triangle ABC \sim \triangle PQR$ (SAS Similarity).

Question-11

Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same sides of BC . If AC and DB intersect at P , prove that $AP \times PC = BP \times PD$.



Solution:

Given: Two right triangles ABC and BDC on the same hypotenuse AC and BD intersect at P.

To prove: $AP \times PC = BP \times PD$

Proof: In $\triangle ABP$ and $\triangle DCP$

$$\angle A = \angle D (= 90^\circ) \text{ (given)}$$

$$\angle APB = \angle DPC \text{ (vertically opposite angles)}$$

$$\therefore \triangle ABP \sim \triangle DCP \text{ (AA similarity axiom)}$$

$$\therefore \frac{AB}{DC} = \frac{BP}{CP} = \frac{AP}{DP} \text{ (corresponding sides of similar } \triangle \text{ s are proportional)}$$

$$\dots\dots\dots(1)$$

$$\frac{DC}{CP} = \frac{BP}{DP}$$

$$\text{From (1) } \frac{BP}{CP} = \frac{AP}{DP}$$

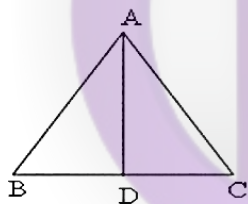
$$\frac{BP}{CP} = \frac{AP}{DP}$$

By cross multiplication,

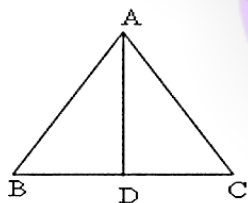
$$BP \times DP = AP \times PC \text{ (proved).}$$

Question-12

If ABC is an equilateral triangle of side 2a prove that the altitude AD = $a\sqrt{3}$ and $3AB^2 = 4AD^2$.



Solution:



Given: $\triangle ABC$ is an equilateral triangle of side 2a. AD is the altitude of triangle.

To Prove: $AD = a\sqrt{3}$ and $3AB^2 = 4AD^2$

Proof:

In rt. $\triangle ADC$,

$$AD^2 = AC^2 - DC^2$$

$$= (2a)^2 - a^2$$

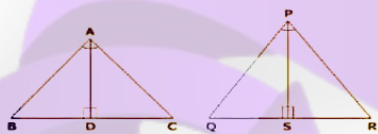
$$= 4a^2 - a^2$$

$$= 3a^2$$

$$\begin{aligned}
 \therefore AD &= a\sqrt{3} \\
 3AB^2 &= 3(2a)^2 \\
 &= 3(2a)^2 \\
 &= 3 \cdot 4a^2 \\
 &= 4(a\sqrt{3})^2 \\
 &= 4AD^2 \\
 \therefore 3AB^2 &= 4AD^2.
 \end{aligned}$$

Question-13

Two isosceles Δ s have equal vertical angles and their areas are in the ratio 9 : 16. Find the ratio of their corresponding heights (altitudes).



Solution:

Given: ΔABC and ΔPQR are isosceles and $\angle A = \angle P$. AD , PS are the altitudes and $\frac{\Delta ABC}{\Delta PQR} = \frac{9}{16}$.

To find: $\frac{AD}{PS}$

Proof: In ΔABC , $\angle B = \angle C$ (isosceles Δ property)

Similarly in ΔPQR , $\angle Q = \angle R$.

$\angle A = \angle P$ (given)

$$\therefore \angle B = \angle C = \frac{180^\circ - \angle A}{2}$$

Since $\angle A = \angle P$

$$\angle B = \angle C = \angle Q = \angle R$$

$$\therefore \Delta ABC \sim \Delta PQR \text{ (AA)}$$

If 2 triangles are similar then the ratio of areas will be equal to the square of the corresponding sides,

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AD^2}{PS^2} \dots\dots\dots(1)$$

In ΔABD , ΔPQS

$$\angle D = \angle S (= 90^\circ)$$

$$\angle B = \angle Q \text{ (given)}$$

$$\therefore \Delta ABD \sim \Delta PQS \text{ (AA)}$$

$$\therefore \frac{AB}{PQ} = \frac{AD}{PS} \dots\dots\dots(2)$$

$$\frac{AB}{PQ} = \frac{AD}{PS}$$

$$\text{According to equation (1)} \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PS^2} = \frac{AB^2}{PQ^2}$$

$$\therefore \frac{9}{16} = \frac{AD^2}{PS^2}$$

$$\Rightarrow \frac{AD}{PS} = \frac{3}{4}$$

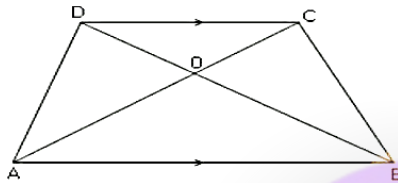
$$\frac{AD}{PS} = \frac{3}{4}$$

\therefore The ratio of their corresponding heights is 3 : 4.

Question-14

The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Solution:



Given: ABCD is a trapezium with $AB \parallel CD$ and the diagonals AC and BD intersect at 'O'.

To prove: $\frac{OA}{OC} = \frac{OB}{OD}$

Proof:

In the figure consider the triangle OAB and OCD

$\angle DOC = \angle AOB$ (Vertically opposite angles are equal)

since $AB \parallel DC$,

$\angle DCO = \angle OAB$ (Alternate angles are equal)

\therefore By AA corollary of similar triangles.

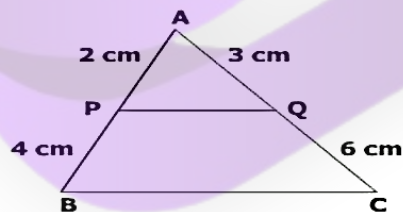
$\therefore \triangle OAB \sim \triangle OCB$ When the two triangle are similar, the side are proportionally.

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

Hence proved.

Question-15

P and Q are the points on the sides AB and AC respectively of a $\triangle ABC$. If $AP = 2$ cm, $PB = 4$ cm, $AQ = 3$ cms, $QC = 6$ cm, prove that $BC = 3PQ$.



Solution:

Given: $\triangle ABC$, PQ are points on AB and AC such that $AP = 2$ cm, $BP = 4$ cm, $AQ = 3$ cm, $QC = 6$ cm

To prove: $BC = 3PQ$

Proof: In $\triangle ABC$, $\frac{AP}{PB} = \frac{2}{4} = \frac{1}{2}$, $\frac{AQ}{QC} = \frac{3}{6} = \frac{1}{2}$

As $\frac{AP}{PB} = \frac{AQ}{QC}$

$PB = QC$

According to converse of BPT, $PQ \parallel BC$

In ΔAPQ and ΔABC

$\therefore \angle APQ = \angle ABC$ (Corresponding angles)

$\angle A$ is Common

$\therefore \Delta APQ \sim \Delta ABC$ (AAS similarity)

$\therefore \frac{AP}{AB} = \frac{AQ}{AC}$ (corresponding sides of similar Δ s are proportional)

But $\frac{AP}{AB} = \frac{PQ}{BC}$

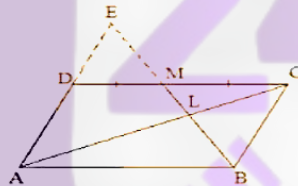
$\therefore \frac{PQ}{BC} = \frac{1}{3}$

$BC = 6$

$\therefore 3PQ = BC$ (Proved).

Question-16

Through the midpoint of M of the side CD of a parallelogram $ABCD$, the line BM is drawn intersecting AC in L and AD produced in E . Prove that $EL = 2BL$.



Solution:

Given: $ABCD$ is a parallelogram, M is the midpoint of CD . BM intersects AC at L and AD produced at E .

To prove: $EL = 2BL$

Proof: In ΔBMC and ΔEDM

$\angle DME = \angle BMC$ (Vertically opposite angles)

$DM = MC$ (given)

$\angle DEM = \angle MBC$ (alternate angles)

$\therefore \Delta BMC \cong \Delta EDM$ (ASA congruence)

$\therefore DE = BC$ (c.p.c.t)

But $BC = AD$ (opposite sides of parallelogram $ABCD$)

$\therefore AD = DE \Rightarrow AE = 2AD = 2BC$

In $\triangle AEL$ and $\triangle CBL$

$\angle ALE = \angle BLC$ (Vertically opposite angles)

$\angle AEL = \angle LBC$ (alternate angles)

$\therefore \triangle AEL \sim \triangle CBL$ (AA similarity axiom)

$$\Rightarrow \frac{AE}{BC} = \frac{AL}{LC} = \frac{EL}{BL}$$

$$\Rightarrow \frac{EL}{BL} = \frac{AE}{BC}$$

$$\Rightarrow \frac{EL}{BL} = \frac{AD-DE}{BC}$$

$$\Rightarrow \frac{EL}{BL} = \frac{BC+BC}{BC}$$

$$\Rightarrow \frac{EL}{BL} = \frac{2BC}{BC}$$

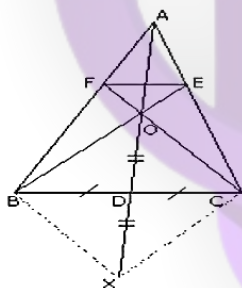
$$\Rightarrow \frac{EL}{BL} = 2$$

$\therefore EL = 2 BL$

Question-17

The side BC of a triangle ABC is bisected at D; O is any point in AD. BO, CO produced meet AC, AB in E, F respectively, and AD is produced to X so that D is the mid point of OX. Prove that $AO : AX = AF : AB$ and show that EF is parallel to BC.

Solution:



Given : The side BC of a triangle ABC is bisected at D; O is any point in AD. BO, CO produced meet AC, AB in E, F respectively, and AD is produced to X so that D is the mid point of OX.

To Prove : $AO : AX = AF : AB$ and show that EF is parallel to BC.

Construction: Join BX and CX.

Proof: In quadrilateral BOCX, $BD = DC$ and $DO = DX$ (given)

\therefore BOCX is a parallelogram (When the diagonals of a quadrilateral bisect each other, then the quad. is a parallelogram)

$\therefore BX \parallel CO$ (Definition of a parallelogram)

or $BX \parallel FO$.

In $\triangle ABX$, $BX \parallel FO$ (proved).

$\therefore AO : AX = AF : AB$ (using B.P.T) -----(i)

Similarly, $AO : AX = AE : AC$ -----(ii)

From (i) and (ii),

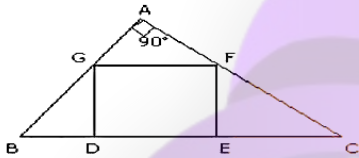
$AF : AB = AE : AC$

By corollary to B.P.T, EF is parallel to BC.

Question-18

ABC is a triangle in which $\angle BAC = 90^\circ$ and DEFG is a square, prove that $DE^2 = BD \times EC$.

Solution:



Given: ABC is a triangle in which $\angle BAC = 90^\circ$ and DEFG is a square.

To prove: $DE^2 = BD \times EC$.

Proof: In $\triangle AGF$ and $\triangle DBG$,

$\angle AGF = \angle GBD$ (corresponding angles)

$\angle GAF = \angle BDG$ (each = 90°)

$\therefore \triangle AGF \sim \triangle DBG$. -----(i)

Similarly, $\triangle AFG \sim \triangle ECF$ (AA Similarity) -----(ii)

From (i) and (ii), $\triangle DBG \sim \triangle ECF$.

$$\frac{BD}{EF} = \frac{BG}{FC} = \frac{DG}{EC}$$

$$\frac{BD}{EF} = \frac{DG}{EC}$$

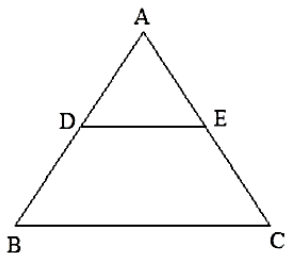
$$EF \times DG = BD \times EC. \text{ -----(iii)}$$

Also DEFG is a square $\Rightarrow DE = EF = FG = DG$ -----(iv)

From (iii) and (iv), $DE^2 = BD \times EC$.

Question-19

In fig. $DE \parallel BC$. If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, find the value x .



Solution:

Given: ABC is a triangle, $DE \parallel BC$, $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$.

To find: x

In $\triangle ABC$, we have

$DE \parallel BC$

Therefore [By Thale's theorem]

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$AD \times EC = AE \times DB$$

$$x(x - 1) = (x - 2)(x + 2)$$

$$x^2 - x = x^2 - 4$$

$$x = 4$$

Question-20

In a $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. If $AD = 4$ cm, $AE = 8$ cm, $DB = x - 4$ and $EC = 3x - 19$, find x .

Solution:

Given: In $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. $AD = 4$ cm, $AE = 8$ cm, $DB = x - 4$ and $EC = 3x - 19$.

To find: x .

In $\triangle ABC$, we have $DE \parallel BC$

Therefore $\frac{AD}{DB} = \frac{AE}{EC}$ [By Thale's theorem]

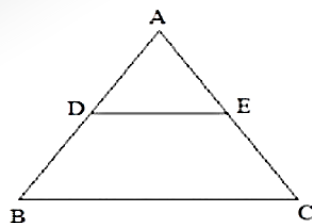
$$\frac{4}{x - 4} = \frac{8}{3x - 19}$$

$$4(3x - 19) = 8(x - 4)$$

$$12x - 76 = 8x - 32$$

$$4x = 44$$

$$x = 11$$



Question-21

In a ΔABC , AD is the bisector of $\angle A$, meeting side BC at D. If AC = 4.2 cm, DC = 6 cm, BC = 10 cm, find AB.

Solution:

Given: In a ΔABC , AD is the bisector of $\angle A$, meeting side BC at D. AC = 4.2 cm, DC = 6 cm and BC = 10 cm.

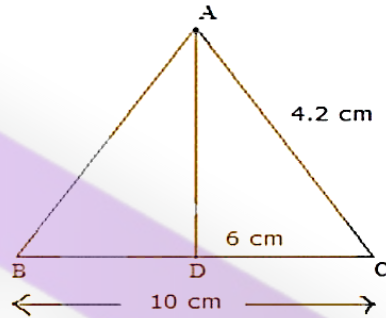
To find: AB.

In ΔABC ,

$$\frac{AB}{AC} = \frac{BD}{DC} \text{ [By internal bisector theorem]}$$

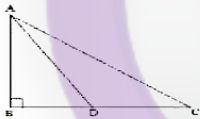
$$AB = 4 \times 4.2 / 6 = 2.8 \text{ cm}$$

$$\therefore AB = 2.8 \text{ cm}$$



Question-22

In ΔABC , $\angle B = 90^\circ$ and D is the mid-point of BC. Prove that $AC^2 = AD^2 + 3CD^2$.



Solution:

Given: In ΔABC , $\angle B = 90^\circ$ and D is the mid-point of BC.

To Prove: $AC^2 = AD^2 + 3CD^2$

Proof:

In ΔABD ,

$$AD^2 = AB^2 + BD^2$$

$$AB^2 = AD^2 - BD^2 \dots\dots\dots(i)$$

In ΔABC ,

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = AC^2 - BC^2 \dots\dots\dots(ii)$$

Equating (i) and (ii)

$$AD^2 - BD^2 = AC^2 - BC^2$$

$$AD^2 - BD^2 = AC^2 - (BD + DC)^2$$

$$AD^2 - BD^2 = AC^2 - BD^2 - DC^2 - 2BD \times DC$$

$$AD^2 = AC^2 - DC^2 - 2DC^2 \text{ (DC = BD)}$$

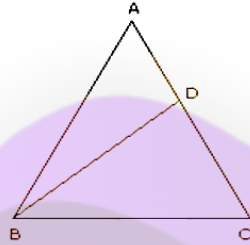
$$AD^2 = AC^2 - 3DC^2$$

Question-23

ABC is a triangle in which $AB = AC$ and D is a point on the side AC such that $BC^2 = AC \times CD$. Prove that $BD = BC$.

Solution:

Given: A ΔABC in which $AB = AC$. D is a point on AC such that $BC^2 = AC \times CD$.



To prove: $BD = BC$

Proof: Since $BC^2 = AC \times CD$

Therefore $BC \times BC = AC \times CD$

$AC/BC = BC/CD$ (i)

Also $\angle ACB = \angle BCD$

Since $\Delta ABC \sim \Delta BDC$ [By SAS Axiom of similar triangles]

$AB/AC = BD/BC$ (ii)

But $AB = AC$ (Given)(iii)

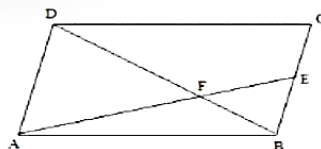
From (i), (ii) and (iii) we get

$BD = BC$.

Question-24

The diagonal BD of a parallelogram ABCD intersects the segment AE at the point F, where E is any point on the side BC. Prove that $DF \times EF = FB \times FA$.

Solution:



Given: The diagonal BD of parallelogram ABCD intersects the segment AE at F, where E is any point on BC.

To prove: $DF \times EF = FB \times FA$

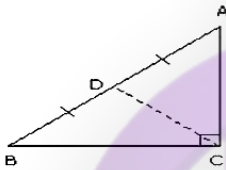
Proof: In triangles AFD and BFE,
 $\angle FAD = \angle FEB$ (Alternate angles)
 $\angle AFD = \angle BFE$ (Vertically opposite angles)
 Therefore $\triangle ADF \sim \triangle BFE$ (AA similarity)
 $\frac{DF}{FA} = \frac{FB}{EF}$
 Hence $DF \times EF = FB \times FA$

Question-25

ABC is a triangle, right-angled at C and $AC = \sqrt{3} BC$. Prove that $\angle ABC = 60^\circ$.

Solution:

Given: $\triangle ABC$ is right angled at C and $AC = \sqrt{3}BC$.



To prove: $\angle ABC = 60^\circ$.

Proof:

Let D be the midpoint of AB. Join CD.
 Now, $AB^2 = BC^2 + AC^2 = BC^2 + (\sqrt{3}BC)^2 = 4BC^2$
 Therefore $AB = 2BC$.
 Now, $BD = \frac{1}{2} AB = \frac{1}{2} (2BC) = BC$.

But, D being the midpoint of hypotenuse AB, it is equidistant from all the three vertices.

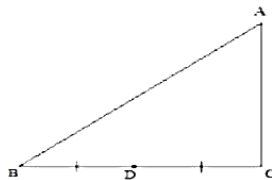
Therefore $CD = BD = DA$ or $CD = \frac{1}{2} AB = BC$.

Thus, $BC = BD = CD$,
 i.e., $\triangle BCD$ is an equilateral triangle.

Hence, $\angle ABC = 60^\circ$.

Question-27

Let ABC be a triangle, right-angled at C. If D is the mid-point of BC, prove that $AB^2 = 4AD^2 - 3AC^2$.



Solution:

Given: ABC be a triangle, right-angled at C and D is the mid-point of BC.

To Prove: $AB^2 = 4AD^2 - 3AC^2$.

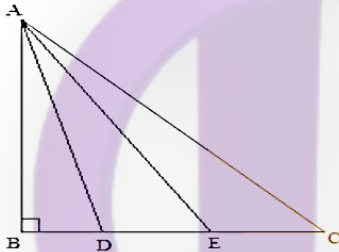
Proof:

From right triangle ACB, we have,

$$\begin{aligned}
 AB^2 &= AC^2 + BC^2 \\
 &= AC^2 + (2CD)^2 = AC^2 + 4CD^2 \quad [\text{Since } BC = 2CD] \\
 &= AC^2 + 4(AD^2 - AC^2) \quad [\text{From right } \triangle ACD] \\
 &= 4AD^2 - 3AC^2.
 \end{aligned}$$

Question-28

In the given figure, points D and E trisect BC and $\angle B = 90^\circ$. Prove that $8AE^2 = 3AC^2 + 5AD^2$.



Solution:

Given: In $\triangle ABC$, points D and E trisect on BC and $\angle B = 90^\circ$.

To Prove: $8AE^2 = 3AC^2 + 5AD^2$.

Proof: Let ABC be the triangle in which $\angle B = 90^\circ$. Let the points D and E trisect BC.

Join AD and AE. Then,

$$AC^2 = AB^2 + BC^2$$

$$3AC^2 = 3AB^2 + 3BC^2 \quad \dots\dots\dots(i)$$

$$AD^2 = AB^2 + BD^2$$

$$5AD^2 = 5AB^2 + 5BD^2 \quad \dots\dots\dots(ii)$$

$$\text{Therefore } 3AC^2 + 5AD^2 = 8AB^2 + 3BC^2 + 5BD^2$$

$$= 8AB^2 + 3 \cdot \left(\frac{3}{2}BE\right)^2 + 5\left(\frac{1}{2}BE\right)^2$$

$$= 8AB^2 + \left(\frac{27}{4} + \frac{5}{4}\right)BE^2$$

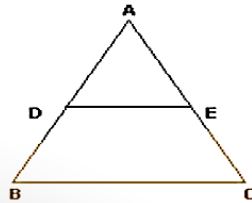
$$= 8AB^2 + 8BE^2$$

$$= 8(AB^2 + BE^2)$$

$$= 8AE^2.$$

Question-29

In fig., ABC is a triangle in which $AB = AC$. D and E are points on the sides AB and AC respectively such that $AD = AE$. Show that the points B, C, E and D are concyclic.



Solution:

Given: In $\triangle ABC$, $AB = AC$. D and E are points on the sides AB and AC respectively such that $AD = AE$.

To Prove: Points B, C, E and D are concyclic.

Proof: In order to prove that the points B, C, E and D are concyclic, it is sufficient to show that $\angle ABC + \angle CED = 180^\circ$ and $\angle ACB + \angle BDE = 180^\circ$.

In $\triangle ABC$, we have

$$AB = AC \text{ and } AD = AE$$

$$AB - AD = AC - AE$$

$$DB = EC$$

Thus, we have

$$AD = AE \text{ and } DB = EC.$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$DE \parallel BC$$

[By the converse of Thale's Theorem]

$$\angle ABC = \angle ADE$$

[Corresponding angles]

$$\angle ABC + \angle BDE = \angle ADE + \angle BDE$$

[adding $\angle BDE$ on both sides]

$$\angle ABC + \angle BDE = 180^\circ$$

$$\angle ACB + \angle BDE = 180^\circ$$

[Since $AB = AC$ Therefore $\angle ABC = \angle ACB$]

Again $DE \parallel BC$

$$\angle ACB = \angle AED$$

$$\angle ACB + \angle CED = \angle AED + \angle CED$$

[Adding $\angle CED$ on both sides]

$$\angle ACB + \angle CED = 180^\circ$$

$$\angle ABC + \angle CED = 180^\circ$$

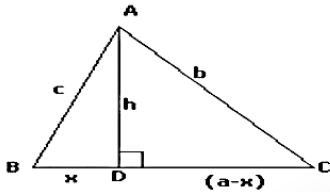
[Since $\angle ABC = \angle ACB$]

Therefore BDEC is a cyclic quadrilateral.

Hence, B, C, E and D are concyclic points.

Question-30

In fig, $\angle B < 90^\circ$ and segment $AD \perp BC$, show that $b^2 = h^2 + a^2 + x^2 - 2ax$



Solution:

Given: In $\triangle ABC$, $\angle B < 90^\circ$ and segment $AD \perp BC$.

To prove: $b^2 = h^2 + a^2 + x^2 - 2ax$

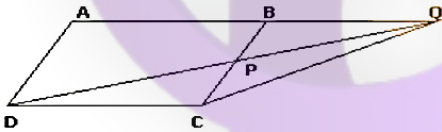
Proof:

$$b^2 = h^2 + (a - x)^2$$

$$b^2 = h^2 + a^2 + x^2 - 2ax.$$

Question-31

In the given figure, ABCD is a parallelogram P is a point on BC, such that $BP : PC = 1 : 2$. DP produced meets AB produced at Q. Given area of triangle CPQ = 20 m^2 , calculate the area of triangle DCP.



Solution:

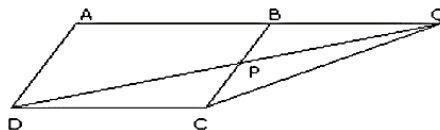
Given: ABCD is a parallelogram P is a point on BC, such that $BP : PC = 1 : 2$.

DP produced meets AB produced at Q.

Area of triangle CPQ = 20 m^2 .

To Find: Area of triangle DCP.

Construction: Join DB.



$\angle BPQ = \angle DPC$ (Vertically opposite angles)

$\angle BQP = \angle PDC$ (alternate angles, $BQ \parallel DC$, DQ meets them)
 $\therefore \Delta BPQ \sim \Delta CPD$ (AA similarity)

$BP/CP = \frac{1}{2}$ (Given)

$$\frac{\text{area } \Delta BPQ}{\text{area } \Delta CPD} = \left(\frac{BP}{CP}\right)^2 = \frac{1}{4}$$

$\therefore \text{Area } \Delta CPD = \text{Area } \Delta BPQ$

$$\frac{\text{area } \Delta BPQ}{\text{area } \Delta CPD} = \left(\frac{BP}{CP}\right) = \frac{1}{2} \text{ and area } \Delta CPQ = 20 \text{ cm}^2 \text{ (Given)}$$

$$\text{Area } \Delta BPQ = 10 \text{ cm}^2$$

$$\text{Area } \Delta CPD = 40 \text{ cm}^2$$

$$\frac{\text{area } \Delta DBC}{\text{area } \Delta DPC} = \frac{3}{2} \text{ (Proportional to bases BC and PC)}$$

$$\frac{\text{area } \Delta DBC}{40} = \frac{3}{2} \therefore \text{Area } \Delta DBC = 40 \times \frac{3}{2} = 60 \text{ cm}^2.$$

