Gravitation

CLASS24

Exercise Solutions

Question 1:

Solution:

The mass of each ball = m = 10 kg

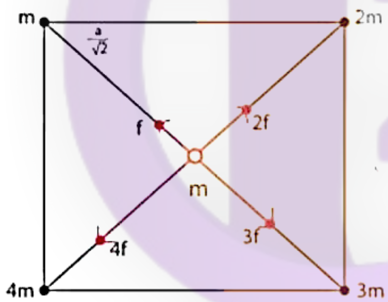
Distance of separation = r = 10 cm or 0.010 cm

Force = $GMm/r^2 = [6.67 \times 10^{-11} \times 10^2]/(0.010)^2$

$= 6.67 \times 10^{-7}$

Question 2:

Solution:



The gravitational force at the center = vector sum of all the forces acting on it.

The distance between the center particle with others, say $r = a/\sqrt{2}$

Force acting between particles of mass m and center particle = $F_m = GMm/r^2 = 2Gm^2/a^2$

Force acting between particles of mass m and center particle = $F_2m = GM(2m)/r^2 = (4Gm^2)/r^2 = 2F_m$

Similarly, we can calculate of mass 3m and 4m along with the center particle:
F_{3m} =3 F_m and F_{4m} = 4 F_m

The net force:

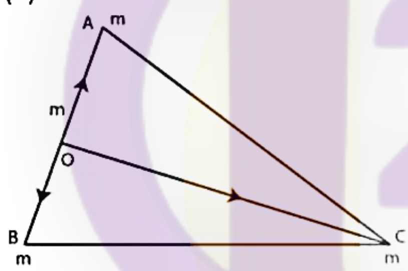
$F_{net} = 2 F_m \cos\theta = 4 F_m (1/\sqrt{2}) = 2\sqrt{2} F_m$

=> $F_{net} = 2\sqrt{2} F_m = 2\sqrt{2} \times 2Gm^2/a^2 = [4\sqrt{2} Gm^2]/a^2$

Question 3:

Solution:

(a)



If "m" is the mid point of a side, then

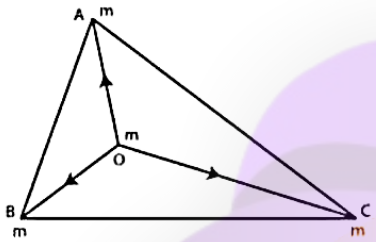
$F_{OA} = 4Gm^2/a^2$ in OA direction
$F_{OB} = 4Gm^2/a^2$ in OB direction

=> $F_{OC} = 4Gm^2/3a^2$ in OC direction

[As, Equal and opposite cancel each other]

So, net gravitational force on m is $4Gm^2/a^2$

(b)



If point "O" is the centroid, then

$F_{OA} = 3Gm^2/a^2$ and $F_{OB} = 3Gm^2/a^2$

So, resultant force is

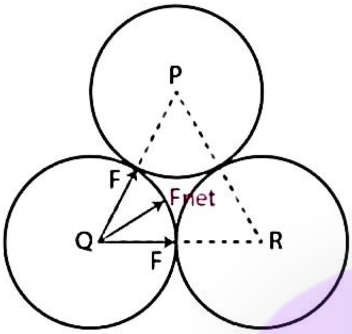
$$= \sqrt{2(\frac{3Gm^2}{a^2})^2 - 2(\frac{3Gm^2}{a^2})^2 \times \frac{1}{2}} = \frac{3Gm^2}{a^2}$$

Since $F_{OC} = 3Gm^2/a^2$
[Equal and opposite to F cancel each other]

=> Net gravitational force become zero.

Question 4:

Solution:
Distance between the centers of two spheres = r= 2a



Force on one sphere due to another = $F = GM^2/4a^2$

Net force = $F_{net} = 2F \cos\theta = 2F \cos 30^o$

$= 2 \times \sqrt{3}/2 \times GM^2/4a^2$

=> $F_{net} = \sqrt{3}GM^2/4a^2$

Question 5:

Solution:
Let A, B, C and D are four particles of mass , moving in a circle of radius R.

Force between A and B = $F_{AB} = GM^2/(\sqrt{2}R)^2 = GM^2/2R^2$

Force between A and D = $F_{AD} = GM^2/(\sqrt{2}R)^2 = GM^2/2R^2 = F_{AB}$

Net force in downward direction = $F_D = 2F_{AB} \cos 45^o = \sqrt{2}\, F_{AE}$

Force between A and C = $F_{AC} = GM^2/(2R)^2 = GM^2/4R^2$

Now,
Net force on particle A = $F_{net} = F_D + F_{AC}$

$$F_{net} = \left(\frac{2\sqrt{2}+1}{4}\right)\frac{GM^2}{R^2}$$

For moving along the circle, F_net = mv²/R

$$\frac{Mv^2}{R} = \left(\frac{2\sqrt{2}+1}{4}\right)\frac{GM^2}{R^2}$$

$$v = \sqrt{\left(\frac{2\sqrt{2}+1}{4}\right)\frac{GM}{R}}$$

Question 6:

Solution:

Mass of the moon= M = 7.4× 10^{22} kg
Radius =R = 1740 km and Distance of the point from surface= R' = 1000 km

Total distance from the center = r = 1740+1000 = 2740 km

Now,
Find Acceleration due to gravity:
g = GM/r^2

$$g = \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{(2740 \times 10^3)^2} = 0.65 m/s^2$$

Question 7:

Solution:
let m_1 and m_2 masses of bodies, where m_1 = 10 kg and m_2 = 20 kg

Initial separation, say r_1 = 10 m and Final separation, say r_2 = 0.5 m
Let v_1 be the initial velocity and v_2 be the final velocity, where $v_1 = v_2 = 0$ m/s

Let us consider v_1', v_2' are the final velocities.

Now,
$m_1 v_1' + m_2 v_2' = 0$

=> $v_1' = -(20/10)v_2' = -2 v_2'$

[From momentum conservation]

Again, from using the conservation of energy:
$PE_{initial} + KE_{initial} = PE_{final} + KE_{final}$

$$\Rightarrow \frac{Gm_1m_2}{r_1^2} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{Gm_1m_2}{r_2^2} + \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

Substituting the values, we get

$$\Rightarrow v_2'^2 = \frac{2 \times 6.67 \times 10^{-11}}{3} = 44.47 \times 10^{-11} = 2.10 \times 10^{-5} m/s$$

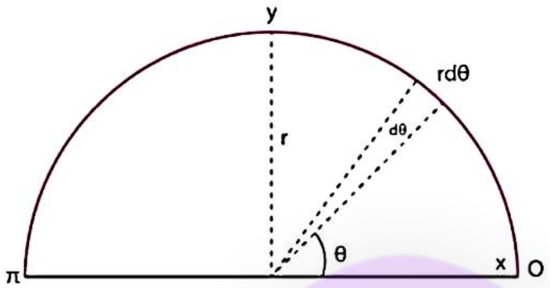
and,

$$v_1' = 2v_2' = 4.20 \times 10^{-5} m/s$$

Question 8:

Solution:
Let us take a small element on the wire. The arc length of the element is $r\,d\theta$.
=> Mass of the element = $dM = (M/L)\, r\, d\theta$
Also, $r = L/\pi$

=> $dM = Md\theta/\pi$



The force on particle due to element = dF = GmdM/r^2= (GMmπdθ)/L^2
Therefore,

$$F = \int dF = \int_0^{\pi} \frac{GMm\pi}{L^2} \cos\theta d\theta$$

$$F = \frac{2\pi GM m}{L^2}$$

Question 9:

Solution:
A small section of rod is at "x" distance mass of the element = dm = (M/L).dx

$dE_1 = [G(dm)]/(d^2+x^2) = dE_2$

So resultant $dE = 2\ dE_1 \sin\theta$

$$2 \times \frac{Gdm}{d^2 + x^2} \times \frac{d}{\sqrt{d^2 + x^2}} = \frac{2(GM)dx}{L(d^2 + x^2)(\sqrt{d^2 + x^2})}$$

Now, the total gravitational force:

$$E = \int_0^{\frac{L}{2}} \frac{2Gmddx}{L(d^2 + x^2)^{3/2}}$$

Solving above equation, we get

$E = 2GM/[d\sqrt{(L^2+4d^2)}]$

Question 10:

Solution:

The gravitational force on m due to shell of M_2 is zero. Ma is at distance $(R_1 + R_2)/2$

The gravitational force:

$$F = \frac{GM_1m}{r^2} = \frac{GM_1m}{[(R_1 + R_2)/2]^2}$$

$$F = \frac{4GM_1m}{(R_1 + R_2)^2}$$

Question 11:

Solution:

Let us assume that tunnel doesn't change the gravitational field distribution of earth.
Mass of the sphere:

$$\frac{M'}{\frac{4}{3\pi x^3}} = \frac{M_e}{\frac{4}{3\pi R^3}}$$

or

$$M' = \frac{x^3}{R^3} M_e$$

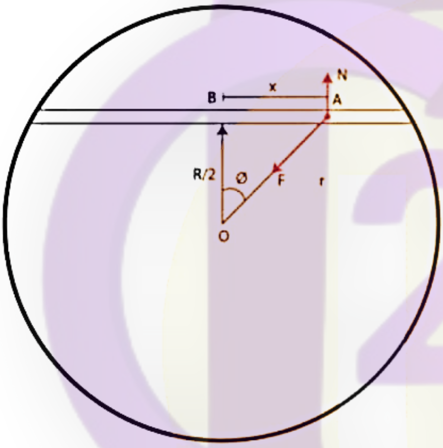
Where M_e is the mass of the earth .

The gravitational force on the particle at distance x,

$F = GMM'/x^2 = GM_e/R^3$

Question 12:

Solution:
Let M_E be the mass of the earth.



From figure,

$N = F \cos \phi$

here, $\cos \phi = R/2r$ and $F = GM_E\, mr/R^3$

thus, $N = GM_E\, mr/R^3 \times R/2r$

or $N = GMm/2R^2$

Question 13:

Solution:

(a) distance of the particle from the center of solid sphere:

l = x – r

Gravitational force on the object:
F = Gmm'/r^3

Here, the mass of the sphere "m" and m' is the place at distance x from O.

=> F = Gmm'(x-r)/r^3

(b) 2r < x < 2R, then F is due to only sphere

F = Gmm'/(x-r)^2

(c) If x > 2R, the gravitational force is due to both shell and sphere,

Force due to shell: F = GMm'/(x-R)^2

Force due to sphere: F = GMm'/(x-r)^2

So, resultant force = GMm'/(x-R)^2 + GMm'/(x-r)^2

Question 14:
Solution:

At point P_1
Gravitational force due to sphere = M = $GM/(3a+a)^2 = GM/16a^2$

At point P_2,Gravitational force due to sphere and shell

= $GM/(a+4a+a)^2 + GM/(4a+a)^2$

= $(61/900)\ GM/a^2$

Question 15:

Solution:

we know, the field inside the shell is zero. Let the gravitational field at A due to the first part be E and the gravitational field at B due to the second part be E'.

Therefore, $E + E' = 0$

or $E = -E'$

Hence, the fields are equal is magnitude and opposite in direction

Question 16:

Solution:
Let the mass of 0.10 kg be at a distance x from 2 kg mass and at the distance of (2-x) from the 4 kg mass.

Force between 0.1 kg mass and 4 kg mass = Force between 0.1 kg mass and 2 kg mass

$(2x0.1)/x^2 = -(4x0.1)/(2-x)^2$

$x = 2/2.414$

or x = 0.83 m from the 2 kg mass.

Now,

The gravitational potential energy is given by

$$V = \sum_{i \neq j} \frac{Gm_i m_j}{r_{ij}} = \frac{G0.1 \times 2}{0.83} + \frac{G0.1 \times 4}{1.17} + \frac{G2 \times 4}{2}$$

$$= 0.24GJ$$

$$= -3.06 \times 10^{-10}$$

Question 17:

Solution:

work done = W = U_f - U_i

Where U_f = Final potential energy and U_i = Initial potential energy

Here, U_f = $-3Gm^2/2a$ and U_i = $-3Gm^2/a$

Now, W = $3Gm^2/2a$

Question 18:

Solution:

U_f = Final potential energy = 0

[As the particle is to be taken away, we assume the final point to be approximately at infinite distance]

and U_i = Initial potential energy = $(-GM_s m)/r$

Here, m = Mass of the particle = 100 g or 0.1 kg

M_s = Mass of sphere = 10 kg

And r = radius of sphere = 10 cm or 0.1 m

On putting values, we get

$U_i = -6.67 \times 10^{-10}$

Now, work done = $W = -(U_f - U_i)$

=> $W = 6.67 \times 10^{-10}$ J

Question 19:

(a) Find the magnitude of the gravitational force acting on a particle of mass 2 kg placed at the origin.
(b) Find the potential at the points (12m, 0) and (0, 5m) if the potential at the origin is taken to be zero.
(c) Find the change in gravitational Potential energy if a particle of mass 2 kg is taken from the origin to the point (12m, 5m).
(d) Find the change in potential energy If the particle is taken from (12m, 0) to (0, 5m).

Solution:

(a) force on the particle

F = mE = 2[5i+12j] = 10i + 24j N

[given mass of the particle = m = 2 kg]

Magnitude of F = 26 N

(b) Potential at (12, 0):

V = -E.r = -12i [5i+12j] = -60 J/kg

Potential at (0, 5):

V = -E.r = -5i [5i+12j] = -60 J/kg

(c) potential energy at (12,5) m:

V = [5i+12j] [2i + 5j] = -120 J/kg

And potential energy at the origin is zero.

Therefore, the change in potential energy is -240 J.

(d) Change in potential energy = 0
[from part (b), potential energy of the particle would be same at both the points.]

Question 20:
Solution:

(a) V= 20N Kg^{-1} (x+y)

Dimension of V = $[MLT^{-2}]/M \times L = L^2T^{-2}$

Dimension of j/kg = $[ML^2T^{-2}]/M = L^2T^{-2}$

Hence dimensions are correct.

(b)

$$E = -\frac{\partial V}{\partial x} - \frac{\partial V}{\partial y}$$

Or E = -20i – 20j N/kg

E is independent of the coordinate.

(c) Force = mE
= 0.5 x [-20(i+j)]
= -10i – 10j

Magnitude of Force = 10√2 N

Question 21:

Solution:

Electric field = E = 2i + 3j

Angle made by E with the x-axis:

$$\cos\theta = \frac{E.i}{|E||i|} = \frac{(2i + 3i).i}{\sqrt{13}} = \frac{2}{\sqrt{13}}$$

$$\sec\theta = \frac{\sqrt{13}}{2}$$

we know,

$$\tan^2\theta = \sec^2\theta - 1$$

$$= \left(\frac{\sqrt{13}}{2}\right)^2 - 1$$

or

$$\tan\theta = \frac{3}{2}$$

Equation of line is $y = -(2/3)x + 5/3$ made angle with x-axis is

$\tan\phi = -2/3$

Now,

$\tan(\phi - \theta) = [\tan\phi - \tan\theta]/[1+\tan\phi \tan\theta]$

= infinity

Now, the angle between the electric filed and the line = $\phi - \theta = 90^o$
Since product of both the slope is -1, the direction of field and the displacement are perpendicular, is done by the particle on the line.

Question 22:

Solution:
Let h be the height

Therefore, $(1/2)\ Gm/R^2 = GM/(R+h)^2$

or $2R^2 = (R+h)^2$

or $h = (\sqrt{2} - 1)R$

Question 23:

Solution:
Height of Mount Everest = h = 8848 m or 8.848 km

Acceleration due to gravity at a height h, say g'

$g' = g(1 - 2h/R)$

$= 9.8(1 - 640/(6400 \times 10^3)$

$= 9.799\ m/s^2$

Question 24:

Solution:
Let g' be the acceleration due to gravity

$g' = g\ (1 - 2h/R)$

$= 98(1-0.64/6400)$

$= 9.799\ m/s^2$

Question 25:

Solution:
Let g' be the acceleration due to gravity at equation and that of pole g

Angular velocity of earth = $\omega = 2\pi/T$

$= 2\pi/(24 \times 3600)$ rad/s

Now, acceleration due to gravity at equator:

$g' = g - \omega^2 R$

$= 9.8 - [2\pi/(24 \times 3600)]^2 \times 64000$

$= 9.767\ m/s^2$

And the weight at equator = mg' = 1 x 9.767 N = 0.997 kg

Question 26:

Solution:

Acceleration due to gravity at equator = g' = g- ω^2 R

Acceleration due to gravity at a height above south pole = g'' = g(1-2h/R)

Now g' = g''

=> g- ω^2 R = g(1-2h/R)

=> h = ω^2 R^2/2g

Or h = [4 π x 6400000^2]/[$(4x3600)^2$x2x9.8]

= 10 km (approx)

Question 27:

Solution:
for apparent g at equator be zero.

g' = g - ω^2 R = 0

or g = ω^2 R

=> ω = √(g/R) = √(9.8/6400000)

= 1.237 x 10^{-3} rad/s

Now, T = 2 π / ω

$= [2 \times 3.14]/[1.237 \times 10^{-3} \times 3600]$

= 1.4 h (approx.)

Question 28:

Solution:

a) the speed of the ship is equal to earth's rotation when the ship is stationary point.

speed $= \omega R$

(b) tension in the string at the equator

$T_0 = mg' = mg - m\omega^2 R$

$mg - T_0 = m\omega^2 R$

Difference between T_0 and the earth's attraction on the bob.

(c) angular speed of the ship is v/R about its center.

Total angular speed $= \omega' = \omega - v/R$

And $T = mg - m\omega'^2 R$ [tension given]

$\Rightarrow T = mg - m(\omega - V/R)^2 R$

$\Rightarrow T = mg - [m\omega^2 + mv^2/R^2 - 2m\omega v/R)R]$

$\Rightarrow T = mg - m\omega^2 R - mv^2/R + 2m\omega v$

From part (b),

$T_0 = mg' = mg - m\omega^2 R$

$\Rightarrow T = T_0 - mv^2/R + 2m\omega v$

$\Rightarrow T = T_0 + 2m\omega v$

Neglect, mv^2/R, As small quantity.

Question 29:

Solution:

From Kepler's third law, the time period of an orbit is proportional to the cube of the radius of the orbit.

$T^2 \; \alpha \; R^3$

$T_m^2/T^2 = R_{MS}^3/R_{SE}^3$

$R_{MS}/R_{SE} = (3.534)^{1/3} = 1.52$

Question 30:

Solution:

For an orbit, the time period:

$T^2 = 4\pi^2 a^3/GM$

[here $a = 3.84 \times 10^5$ km and $T = 27.3 \times 24 \times 3600$ sec]

Or $M = 6.02 \times 10^{24}$ kg

Question 31:

Solution:

For an orbit, the time period:

$T^2 = 4\pi^2 a^3/GM$

Or $M = 4\pi^2 a^3/G T^2$ ....(1)
Where M = mass of mars.

Here, Radius of mars=a = 9.4×10^3 km or 9.4×10^6 m and Time = T = 27540 s

Now, (1)=> $M = [4\pi^2 (9.4 \times 10^6)^3]/[6.67 \times 10^{-11} \times 27540^2]$

Or $M = 6.5 \times 10^{23}$ kg

Question 32:

Solution:

(a) Radius of the orbit = a = 2000 + 6400 = 8400 km or 8.4×10^6 m

Therefore, the speed = $v = \sqrt{(GM)}/a$

$v = \sqrt{[(6.67 \times 10^{-11} \times 6 \times 10^{24})/(8.4 \times 10^6)]}$ = 6.9 km/s (approx.)

(b) $KE = (1/2)mv^2$
Here m = 1000 kg (mass of satellite)

$KE = (1/2) \times 1000 \times 6900^2 = 2.38 \times 10^{10}$

(c) Potential energy at infinity is zero. Hence, the potential energy at a radius, a

PE = -GMm/a

= $[-6.67 \times 10^{-11} \times 6 \times 10^{24} \times 1000]/[8.4 \times 10^{6}]$

= -4.76×10^{10}

(d) Time period

$T^2 = 4\pi^2 a^3/GM$

= $[4\pi^2(8.4 \times 10^6)^3]/[6.67 \times 10^{-11} \times 6 \times 10^{24}]$

= 2.12 hours

Question 33:

Solution:
(a) Time period of revolution of satellite: T = 24x3600 = 86400 sec

Let "a" be the radius of the orbit.

$T^2 = 4\pi^2 a^3/GM$

Or $a^3 = GM T^2/4\pi^2$

= $[6.67 \times 10^{-11} \times 6 \times 10^{24} \times 86400^2]/[4\pi^2]$

= 7.56×10^{22}

Or a = 42300 km (approx.)

(b) A complete revolution takes 24 hours, therefore a quarter of revolution is 24/4 = 6 hours

Question 34:

Solution:

Weight at north pole, $W_p \propto 1/R^2$

Let h is distance of the satellite from earth.

Weight of satellite at equator, $W_e \propto 1/(R+h)^2$

Now, ratio is $W_p/ W_e = (R+h)^2/R^2$

$W_e = [W_pR^2]/[(R+h)^2]$

We are given, $W_p = 10$ N and h = 36000 km [Height of the geostationary satellite,]

Therefore, $W_e = [10\text{x}6400^2]/[(6400+36000)^2] = 0.23$ N(approx.)

Question 35:

Solution:

The time period of revolution: $T^2 = 4\pi^2a^3/GM$

Or $GM = 4\pi^2(R_2)^3/ T^2$

Or $GM/R_1^2 = 4\pi^2(R_2)^3/ T^2 R_1^2$

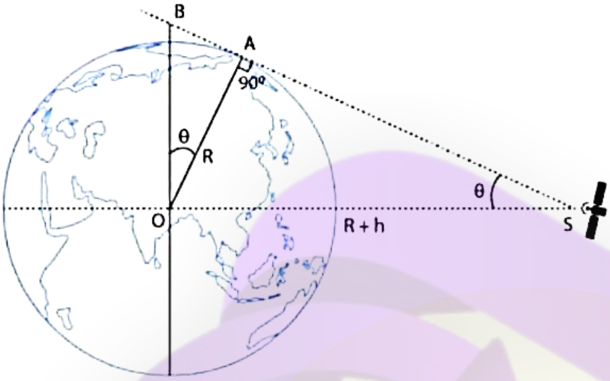
Now, Acceleration due to gravity :

$g = GM/ R_1^2$

or $g = 4\pi^2(R_2)^3/ T^2 R_1^2$

Question 36:

Solution:



form figure, angle BOA = angle OSA

In triangle AOS

sinθ = AO/OS

= R/(R+h)

= 6400/(6400+36000)

= 0.15 (approx)

or θ = $\sin^{-1}(0.15)$

Question 37:

Solution:
Let KE_i be initial KE and PE_i initial potential energy of the system.

$KE_i = (1/2)mv^2$ and $PE_i = -GMm/R$

$KE_f = 0$ [at the maximum height]

And PE at height h is h = 6400 km
And $PE_f = -GMm/(R+h)$

Now, From conservation of energy, $KE_i + PE_i = KE_f + PE_f$

=> $(1/2)mv^2 – GMm/R = -GMm/(R+h)$

=> $(1/2)mv^2 = GMm[(-R+R+h)/R(R+h)] = GMmh/R(R+h)$

Or $v^2 = 2GMh/R(R+h)$

$= [2x6.67x10^{-11}x6x10^{24}x6400x10^3]/[2x(6400x10^3)^2]$

= 7.9 km/s

Question 38:
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Solution:
Let KE_i be initial KE and PE_i initial potential energy of the system.

$KE_i = (1/2)mv_i^2$ and $PE_i = -GMm/R$

Final KE = $KE_f = (1/2)mv_f^2$

Final potential energy = $PE_f = 0$

Using energy conservation, we have
$KE_i + PE_i = KE_f + PE_f$

$$\Rightarrow \frac{1}{2}mv_i^2 - \frac{GMm}{R} = \frac{1}{2}mv_f^2$$

$$\Rightarrow \frac{15^2}{2} - \frac{6.67 \times 10^{-7} \times 6 \times 10^{24}}{6400} = \frac{1}{2}v_f^2$$

$$\Rightarrow v_f = \times\ 10^4 m/s = 10km/_s$$

Question 39:

Solution:
We have, $(1/2)\ mv^2 = GMm/R$

or $R = 2GM/v^2$

$R = [2x6.67x10^{-11}x6x10^{24}]/[(3x10^8)^2]$

R = 9 mm (approx.)