Simple Harmonic Motion

Exercise Solutions

Question 1:

Solution: Amplitude, A = 10cm, r =10cm
Time period, T = 6 sec

W = 2π/T = 2π/6 = π/3 s^{-1}

At t = 0, x = 5 cm

5 = 10 sin(Wx_0 + ϕ) = 10 sin ϕ

sinϕ = 1/2

or ϕ = π/6

Equation of displacement, x = (10) sin(π/3)

At t = 4 sec

x = 10 sin[π/3 x 4 + π/6] = 10 sin[(8π+π)/6]

= 10 sin(3π/2)

= -10

Acceleration = a = $-W^2$ x = $-(\pi/3)^2$ x (-10) = 0.11 cm/s

Question 2:

Solution:
$x = 2$ cm, $v = 1$ m/s or 100 cm/s, and $a = 10\ m/s^2$ or $1000\ cm/s^2$

$a = -\omega^2 x$

$1000 = -2\omega^2$

or $\omega^2 = 500$

or $\omega = 10\sqrt{5}$

$T = 2\pi/\omega$

$= 2\pi/10\sqrt{5}$

= 0.28 sec

Now, $v^2 = \omega^2 (A^2 - x^2)$

$10000 = 5x\ 100(A^2 - 4)$

or A = 4.9 cm or -4.9 cm.

Question 3:

Solution:

Given, K.E. = P.E.
and r = 10 cm

$(1/2)m\omega^2(r^2-y^2) = (1/2)m\omega^2 y^2$

$r^2 - y^2 = y^2$

=> $y = r/\sqrt{2} = 10/\sqrt{2} = 5\sqrt{2}$ cm form the mean position.

Question 4:

Solution:

$V_{max} = r\omega = 10$ cm/s

$\omega^2 = 100/r^2$ ...(1)

And $A_{max} = \omega^2 r = 50$ cm/s

$\omega^2 = 50/y = 50/r$ ...... (2)

From (1) and (2)

$100/r^2 = 50/r$

=> r = 2 cm

Therefore, $\omega = \sqrt{100/r^2} = 5\ s^2$

Find the positions where the speed is 8m/s

$v^2 = \omega^2 (r^2 - y^2)$

=> $64 = 25(4 - y^2)$

or y = 1.2 cm and -1.2 cm from mean position.

Question 5:

Solution:

Given equation, $x = (2.0\ cm) \sin [(100\ s^{-1})\ t + \pi/6]$ (1)

(a) we know, $x = A \sin(\omega t + \phi)$ ...(2)

Comparing (1) and (2) we have

$A = 2$ cm, $\omega = 100$ and $\phi = \pi/6$

Time period = $T = 2\pi/\omega = 2\pi/10 = \pi/50$ s

For spring constant: $\omega^2 = K/m$

or $K = \omega^2 /m = 10/1000 \times 100^2 = 100$ N/m

(b) At t = 0

(1)=> $x = 2 \sin(\pi/6)$

or x = 1 m

Also $v = A\omega \cos(\omega t + \phi)$

On substituting values, we get

$a = 100$ m/s^2

Question 6:

Solution:
Given equation is: $x = 5 \sin (20 t + \pi/3)$ ...(1)

(a) velocity of the particle will be zero at extreme position.
At x = 5

(1)=> 5 = 5 sin (20 t + π/3)

sin (20 t + π/3) = 1 = sin(π/2)

=> 20 t + π/3 = π/2

Or t = π/120 sec
So at π/120 sec it first comes to rest.

(b) a = $\omega^2 x = \omega^2$ [5 sin (20 t + π/3)]

For a = 0, 5 sin (20 t + π/3) = 0

=> sin (20 t + π/3) = sin π

=> 20t = π – π/3 = 2 π/3

Or t = π/30 sec

(c) v = A ω cos(ωt+ π/3)

= (20) 5 cos(20t+ π/3)

Here, cos(20t+ π/3) = -1 = cos π [v is maximum]

=> 20t = π – π/3 = 2 π/3
Or t = π/30 sec

Question 7:

Solution:
$x = 2.0 \cos(50\pi t + \tan^{-1}(0.75))$

$x = 2.0 \cos(50\pi t + 0.643)$

Now, $V = dx/dt$

$= -100 \sin(50\pi t + 0.643)$

As the particle comes to rest for the first time

$50\pi t + 0.643 = \pi$

Or $t = 1.6 \times 10^{-2}$ sec

(b) Acceleration of the particle = $a = dv/dt$

$\Rightarrow a = -100\pi \times 50\pi \cos(50\pi t + 0.643)$

For maximum acceleration, $\cos(50\pi t + 0.643) = -1 = \cos\pi$

Or $t = 1.6 \times 10^{-2}$ sec

(c) particle comes to rest second time after

$(50\pi t + 0.643) = 2\pi$

Or $t = 3.6 \times 10^{-2}$ sec

Question 8:

Solution:
Using Equation : $A = x \sin(\omega t + \phi)$
For the first half journey: $x = A/2$

$1/2 = \sin(\omega t + \phi)$

For $\phi = 0$, $\omega t = \pi/6$

$t = \pi/6\omega$

For second journey: $x = A/2$ to A

$x = A \sin(\omega t + \phi)$

$\sin(\omega t) = \sin(\pi/2)$

or $\omega t = \pi/2$

or $t = \pi/2\omega$

Time taken from $A/2$ to A is

$(\pi/2)\omega - (\pi/6)\omega = (\pi/3)\omega$

Now, $\omega = (2\pi/T)$

Time taken $= \pi/3(2\pi/T) = T/6$

Question 9:

Solution:
$T = 2\pi \sqrt{(m/k)}$

$T = 2\pi \sqrt{(m/0.1)}$

$\sqrt{0.1} = \pi \sqrt{m}$

or $m = 0.01$ kg or 10 g

Question 10:

Solution:
Force due to Gravity = Spring Force.
mg = KA

or K = mg/A

Time Period of the Spring-mass system = $T = 2\pi \sqrt{(m/k)}$

$= 2\pi \sqrt{(mA/mg)}$

$= 2\pi \sqrt{(A/g)}$

Time period for Simple Pendulum = $T = 2\pi \sqrt{(l/g)}$

Time period is equal as frequency is equal.

$2\pi \sqrt{(A/g)} = 2\pi \sqrt{(l/g)}$

or A = l (proved)

Question 11:

Solution:
x = r = 0.1 m

m = 0.5 kg and T = 0.314 s

Total force exerted on the block = weight of block + spring force

$T = 2\pi \sqrt{(m/k)}$

=> $0.314 = 2\pi \sqrt{(0.5/k)}$

Or k = 200 N/m

Force exerted by the spring on the block = F = k x 201.1 x 0.1 = 20 N

Therefore, max force = F + weight = 20 + 5 = 25 N

Question 12:

Solution:
Time period of simple harmonic motion = $T = 2\pi \sqrt{(m/k)}$

We are given that, m = 2 kg, T = 4s

=> $4 = 2\pi \sqrt{(2/k)}$

Or k = 5 N/m

We know that, force F is proportional to displacement,

F = mg = kx

=> x = mg/k = 4 m

Thus, P.E. = $(1/2)\ kx^2 = 40$ J

Question 13:

Solution:

Time period of oscillation = T = 1/v = (1/5) s

P.E. = $(1/2)kx^2$

= $(1/2)k(0.25)^2$
= 5 J

=>k = 160 N/m

Also, T = $2\pi \sqrt{(m/k)}$

Or m = $(T/2\pi)^2 k$ = 0.16 kg

Question 14:

Solution:

(a) From free body diagram:

$R + m\omega^2 x - mg = 0$

Resultant force is $m\omega^2 x = mg - R$

(b) Normal force = $R = mg - m\omega^2 x$

Or $\omega = \sqrt{k/(M+m)}$

Then, R = mg – mkx/(M+m)

m ω^2 x is max for R to be smallest.
X is maximum. The particle should be at the highest point.

(c) Two blocks may oscillate together, R ≥ 0. At limiting R = 0, hen

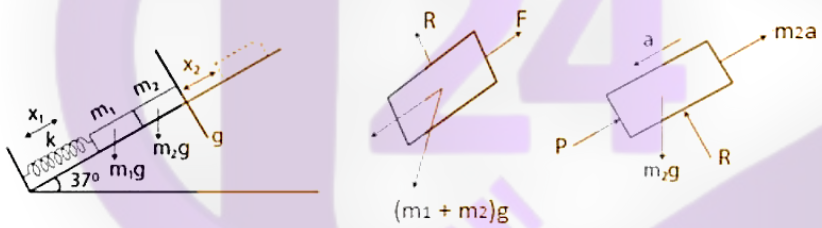
R = mg - m ω^2x = 0

x = mg/m ω = mg(M+m)/mk

Maximum amplitude is, x = g(M+m)/k

Question 15:

Solution:



(a) At equilibrium condition, kx = (m_1+m_2)g sin θ

Or $x = x_1 = [(m_1+m_2)\, g \sin\theta]/k$

(b) When the system is released, it will start to make SHM

$\omega = \sqrt{[k/(m_1+m_2)]}$

When the blocks lose contact, P=0,

$$m_2 g sin\theta = m_2 x_2 \omega^2 = m_2 x_2 \frac{k}{m_1 + m_2}$$

$$x_2 = \frac{(m_1 + m_2) g sin\theta}{k}$$

blocks will lose contact each other when the springs attain its natural length.

(c) Let v be the common speed attained by both the blocks

$(1/2)\,(m_1+m_2)v^2 = (1/2)\,(x_1+x_2)^2 k - (m_1+m_2) g \sin\theta\,(x_1+x_2)$

Where Total compression = x_1+x_2

$v = [\sqrt{(k/(m_1+m_2))}\,]\, g \sin\theta$

Question 16:

Solution:

k = 100 N/m and M = 1 kg and F = 10N

(a) In equilibrium position,

compression, $\delta = F/k = 10/100 = 0.1$ m or 10 cm

(b) The blow imparts speed of 2 m/s to the block towards left.

$PE + KE = (1/2)\, k\delta^2 + (1/2)\, Mv^2$

$= (1/2)\,(100)(0.1)^2 + (1/2)(10)(2)^2$

$= 2.5$ J

(c) Time period = $T = 2\pi\,\sqrt{M/k}$

$= 2\pi\,\sqrt{1/100}$

$= \pi/5$ sec

(d) Let x be the amplitude, which lies between the mean position and the extreme position. So, in the extreme position, compression of the spring is $(x + \delta)$

Since, in the SHM, the total energy remains constant.

$(1/2)k(x+\delta)^2 = (1/2)k\,\delta^2 + (1/2)Mv^2 + Fx$

$= 2.5 + 10x$

So, $50(x + 0.1)^2 = 2.5 + 10x$

Or x = 0.2 m or 20 cm

(e) Potential energy at the left extreme

PE = $(1/2)k(x + \delta)^2 = (1/2) \times 100 \times (0.2 + 0.1)^2 = 4.5$ J

(f) Potential energy at the right extreme

P.E. = $(1/2)k(x + \delta)^2 - F(2x)$

= 4.5 – 10(0.4)

= 0.5 J

The different values in (b), (e), (f) do not violate law of conservation of energy as the work is done by the external force 10N.

Question 17:

Solution:

(a) Equivalent spring constant, $k = k_1+k_2$ (parallel)

$T = 2\pi \sqrt{m/k} = 2\pi \sqrt{[m/(k_1+k_2)]}$

b) When we displace the block m towards left through displacement x,

The resulting force = $F = F_1 + F_2 = (k_1+k_2)$

Now, acceleration = $a = F/m = (k_1+k_2)x/m$

The time period can be calculates as:

$$Time\ period\ T = 2\pi\sqrt{\frac{displacement}{acceleration}}$$

$$= 2\pi\sqrt{\frac{x}{\frac{m(k_1 + k_2)}{m}}}$$

$$= 2\pi\sqrt{\frac{x}{k}}$$

c) In series connection equivalent spring constant be k, So

$1/k = 1/k_1 + 1/k_2 = (k_1+k_2)/ (k_1 k_2)$

Or $k = (k_1 k_2)/ k_1+k_2)$

And time period:

$$T = 2\pi\sqrt{\frac{m}{k}} = \frac{2\pi\sqrt{m(k_1 + k_2)}}{k_1 k_2}$$

Question 18:
Solution:

(a) F = kx

and acceleration = F/m

Time period = $T = 2\pi \sqrt{m/k}$

Amplitude =maximum displacement = F/k

(b) The energy stored in the spring when the block passes through the equilibrium position

= $(1/2)\, kx^2$

= $(1/2)\, k\, (F/k)^2$

= $F^2/2k$

c) At the mean position, potential energy is zero, K.E. = $F^2/2k$

Question 19:

Solution:

Consider that particle is pushed slightly against the spring C through displacement x.

Total resultant force on the particle kx due to spring and $kx/\sqrt{2}$ due to spring A and B, is

$$kx + \sqrt{\left(\frac{kx}{\sqrt{2}}\right)^2 + \left(\frac{kx}{\sqrt{2}}\right)^2} = kx + kx = 2kx$$

Now, acceleration = 2kx/m and Time period = $T = 2\pi \sqrt{(m/2k)}$

Question 20:
Solution:

Total resulting force acting on the mass m

$$F = kx + \sqrt{\left(\frac{kx}{2}\right)^2 + \left(\frac{kx}{2}\right)^2 + 2\left(\frac{kx}{2}\right)\left(\frac{kx}{2}\right)\cos 120^\circ}$$

$$= kx + \frac{kx}{2} = \frac{3kx}{2}$$

Acceleration = a = F/m = 3kx/2m

and a/x = 3k/2

so, ω = √(3k/2m)

Therefore, Time period = T = 2π /ω = 2π x √(2m/3k)

Question 21:

Solution:
Since k_2 and k_3 are in series, let k4 be equivalent spring constant.
So,
$1/k_4 = 1/k_3 + 1/k_2 = (k_3+k_2)/ (k_3 k_2)$

Or $k_4 = k_3k_2 /[k_3+k_2]$

k_4 and k_1 are parallel. k be equivalent spring constant k.

$k = k_4 + k_1 = k_3k_2 /[k_3+k_2] + k_1$

$= [k_3k_2 + k_1k_2 + k_3k_1]/[k_3+k_2]$

Now,
$T = 2\pi \sqrt{(m/k)}$

$$= 2\pi \sqrt{\frac{m(k_3 + k_2)}{k_3k_2 + k_1k_2 + k_3k_1}}$$

and

$$Frequency = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k_3k_2 + k_1k_2 + k_3k_1}{m(k_3 + k_2)}}$$

$$Amplitude\ x = \frac{F}{k} = \frac{F(k_3 + k_2)}{k_3k_2 + k_1k_2 + k_3k_1}$$

Question 22:

Solution:

Since k_1 , k_2 and k_3 are in series, and k be equivalent spring constant.
So,
$1/k = 1/k_1 + 1/k_2 + 1/k_3$

So, $k = (k_1 k_2 k_3)/(k_3 k_2 + k_1 k_2 + k_3 k_1)$

And , $T = 2\pi/\omega = 2\pi\sqrt{(m/k)}$

Substituting the value of k, we have

$$T = 2\pi\sqrt{M\left(\frac{1}{k_1} + \frac{1}{k_3} + \frac{1}{k_2}\right)}$$

Also,

$$P.E_1 = \frac{1}{2}k_1 x_1{}^2 = \frac{1}{2}k_1\left(\frac{Mg}{k_1}\right)^2 = \frac{M^2 g^2}{2k_1}$$

And,

$$P.E_2 = \frac{M^2 g^2}{2k_2}, P.E_3 = \frac{M^2 g^2}{2k_3}$$

[For force = F = Mg,
at spring constant k_1, $x_1 = Mg/K_1$
Similarly, $x_2 = Mg/K_2$ and $x_3 = Mg/K_3$]

Question 23:

Solution:

When mass m is hanging, let l be the extension in the spring.

so, $T_1 = kl = mg$

When a force F is applied, let x be the further extension. This implies
$T_2 = k(l + x)$

Therefore, $T = T_2 - T_1 = k(l + x) - kl = kx$

Acceleration = kl/m

and Time period = $t = 2\pi \sqrt{(m/k)}$

Question 24:

Solution:
The Initial extension of the spring in the mean position = $\delta = mg/k$

If r is the radius of the pull, then $v = r\omega$

At t any instant, total energy for the system executing SHM is constant

$(1/2)\ [mv^2 + I\omega^2 + k\{(x+\delta)^2 - \delta^2\} - mgx] = \text{constant}$

or $(1/2)\ [mv^2 + I\omega^2 + kx^2 - kx\delta - mgx] = \text{constant}$

Put the value of v and δ as rω

$(1/2)\ [mv^2 + I(v/r)^2 + kx^2] = \text{constant}$

Differentiating above w.r.t. “t” , we get

$$mv\frac{dv}{dt} + \frac{1}{r^2}v\frac{dv}{dt} + kx\frac{dx}{dt} = 0$$

$$=> \ a\left(m + \frac{1}{r^2}\right) = kx \ [as\ v = \frac{dx}{dt}\ , a = \frac{dv}{dt}]$$

$$\frac{a}{x} = \frac{k}{m + \frac{1}{r^2}} = \omega^2$$

and,

$$T = 2\pi\sqrt{\frac{m + \frac{1}{r^2}}{k}}$$

Question 25:

Solution:

Here, total energy of the system should be constant.

$(1/2)k(2x)^2 + (1/2)mv^2 + (1/2)mv^2 = \text{constant}$

[The center of mass of the system should not change during the motion. So if the block m on the left side moves towards right side by a distance x, the block on the right side moves towards left side by a distance x. So total compression of the spring is 2x.]

$2kx^2 + mv^2 = \text{constant}$

derivation of above equation,

(2x)(2k) dx/dt = 2mv dv/dt = 0

or ma + 2kx = 0

or a/x = -2k/m

and ω = √(2k/m)

and T = 2 π √(m/2k)

Question 26:

Solution:

Driving force = mg sinθ and acceleration = F/m = g sinθ

For small θ, sin θ = 0

Therefore, a = gθ = gx/L

or a is directly proportion to x

So, Motion is simple harmonic, therefore time period:

$$T = 2\pi\sqrt{\frac{x}{gx/L}} = 2\pi\sqrt{\frac{L}{g}}$$

Question 27:

Solution:
Here k = 100 N/m
amplitude = 0.1 m and total mass(M) = 1+3 = 4kg

$T = 2\pi\sqrt{(M/k)} = 2\pi/5$ sec

And, Frequency = $v = 5/2\pi$ Hz

At the mean position, let v is the velocity of 1 kg block.

K.E. = $(1/2)\, mv^2 = (1/2)kx^2$ = P.E.

or v = 1 m/s

Now, let V be the velocity of the 4 kg weight agin,

v = 4V

or V = (1/4) m/s

Now the two blocks have velocity 1/4 m/s at its mean position.

K.E. = $(1/2)MV^2 = 1/8$ J

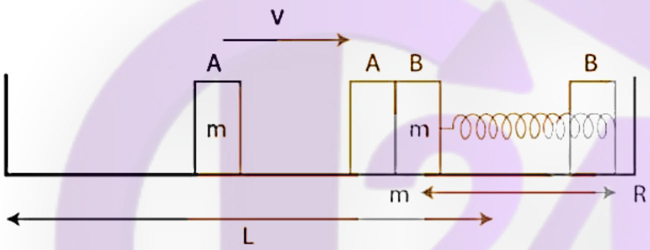
When the blocks are going to the extreme position, there will be only potential energy,
P.E. = $(1/2)kx^2 = 1/8$

(1/2)(100)x^2 = 1/8

or x = 5 cm

Question 28:

Solution:



As per given statement, collision is elastic and surface is frictionless. When the block A moves with the velocity v and collides with the block B, it transfers all energy to the block B. Block A will move a distance x against a spring, again the block B will return to the original point and completes half of the oscillation.

So, the time period of block B is T i.e.

$$T = \frac{2\pi\sqrt{(m/k)}}{2} = \pi\sqrt{\frac{m}{k}}$$

Block B collides with block A and comes to rest. Resultantly, block A moves a further distance.
Let us consider "L" be the distance moved by block to return to its original position.

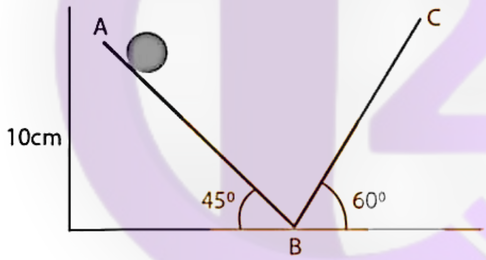
Therefore, time taken by the block to move = L/v + L/v = 2L/v

Time period of the periodic motion is:

$$2\left(\frac{L}{v}\right) + \pi\sqrt{\frac{m}{k}}$$

Question 29:

Solution:



Let the time taken to travel AB and BC be t_1 and t_2 respectively. And, s_1 is distance travelled along AB.

Acceleration For part AB = a_1 = g sin 45° and s_1 = (0.1)/sin45° = 2 m

Let v be the velocity at point B:
Therefore, $v^2 - u^2 = 2 a_1 s_1$

Or $v^2 = 2 g \sin 45^o \times (0.1)/\sin 45^o = 2$

Or $v = \sqrt{2}$ m/s

Now,
$t_1 = (v-u)/a_1 = 0.2$ sec
[on substituting the values]

For Part BC:
$A_2 = g \sin 60^o$ and $v = 0$ and $u = \sqrt{2}$ m/s

Also, time taken t_2 is:

$$t_2 = \frac{0-\sqrt{2}}{-g\left(\frac{\sqrt{3}}{2}\right)} = \frac{2\sqrt{2}}{\sqrt{3}g} = 0.15\ sec$$

Total time period = $t_1 + t_2 = 2(0.2 + 0.15) = 0.7$ sec

Question 30:

Solution:

Let x_1, x_2 are the amplitude of oscillation of m and M respectively.

(a) $mx_1 = Mx_2$
[law of conservation of momentum]

or $(m/M)x_1 = x_2$

And, $(1/2)\ kx_0^2 = (1/2)k(x_1 + x_2)^2$

Therefore, $x_o = x_1 + x_2$

Putting value of x_2

$x_o = x_1 + (m/M)x_1$

=> $x_1 = Mx_0/(M+m)$

and $x_2 = x_0 - x_1 = mx_0/(M+m)$

(b)
Let v_1, v_2 are the velocities of m and M be respectively.

Here v_1 of m with respect to M. So, the total energy must be constant.

$$\frac{1}{2}Mv^2 + \frac{1}{2}m(v_1 - v_2)^2 + \frac{1}{2}k(x_1 + x_2)^2 = constant$$
………(1)

Again, from law of conservation of momentum,

$Mx_2 = mx_1$

=> $x_1 = (M/m)\ x_2$

and $Mv_2 = m(v_1 - v_2)$

=> $v_1 - v_2 = (M/m)v_2$

(1)=>

$$\frac{1}{2}Mv_2^2 + \frac{1}{2}m\left(\frac{M}{m}\right)^2 v_2^2 + \frac{1}{2}kx_2^2\left(1+\frac{M}{m}\right)^2 = constant$$

or

$$Mv_2^2 + k\left(1+\frac{M}{m}\right)x_2^2 = constant$$

On differentiating w.r.t. t, we get

$$M\,2v_2\frac{dv_2}{dt} + k\frac{M+m}{m}\,2x_2\frac{dx_2}{dt} = 0$$

$$Ma_2 + k\frac{M+m}{m}\,x_2 = 0$$

$$\frac{a_2}{x_2} = -k\frac{M+m}{mM} = \omega^2$$

$$\omega = \sqrt{\frac{k(m+M)}{Mm}}$$

So, the time period is

$$T = 2\pi\sqrt{\frac{Mm}{k(m+M)}}$$

Question 31:

Solution:
Let x be the displacement of the uniform plate towards left.

At the displaced position, $R_1 + R_2 = mg$

Moment about g is,
$R_1(l/2 - x) = R_2 (l/2 + x) = (mg – R_1) (l/2 + x)$

or $R_1 = [mg(2x+l)]/2l$

Now, $F_1 = \mu R_1 = [\mu mg(2x+l)]/2l$

Similarly, $F_2 = \mu R_2 = [\mu mg(2x+l)]/2l$

Since $F_1 > F_2$

$F_1 - F_2 = ma = [2\mu mg/l]\ x$

or $a/x = 2\mu g/l = \omega^2$

=> $\omega = \sqrt{(\mu g/l)}$

Therefore, time period, $T = 2\pi \sqrt{(l/\mu g)}$

Question 32:

Solution:
Given $g = \pi^2$ m s^{-2} or 10 m/s and T = 2 s

We know, $T = 2\pi \sqrt{(l/g)}$

or l = 1 cm

Question 33:
Find the length of the pendulum if $g = \pi^2$ m s^{-2}.

Solution:
On comparing given equation with equation of SHM

$\omega = \pi\ s^{-1}$

and T = 2 sec

[As, $2\pi/T = \pi$]

Again, we know $T = 2\pi\sqrt{(l/g)}$

On comparing equations for T

=> l = 1 m, which is the length of pendulum.

Question 34:
Solution :

Let time period of given clock = T_1 =2.04 sec

We know, time period of normal clock = T = 2.00 sec

Therefore, Change in time lag = T_1 - T = 2.04 - 2.00 = 0.04 sec per oscillation

Number of oscillations in a day= N = 24×60×60×0 = 43200

So, total time lag = change in time lag × N

$= (0.04) \times (43200)$

= 1728 sec or 28.8 minutes

Question 35:

Solution:
When clock is at g^1 it loses 24 seconds in 24 hours
So, $T = 2\pi\sqrt{(l/g)}$

$\Rightarrow l = (T^2/4\pi^2)\, g$ ......(1)

This length does not change in the new place T_1^2

$l = (T_1^2/4\pi^2)\, g_1$...... (1)

From equations (1) and (2)

$g_1 = (T_2/T_1^2)\, g$

$= (9.8x4)/(2.0005)^2$

$= 9.746\ m/s^2$

Question 36:

Solution:

(a) Here, $l = 5m$ and $g = 9.8\ m/s^2$

We know, Time period $= T = 2\pi\sqrt{(l/g)}$

$= 2(3.14)\sqrt{(5/9.8)} = 4.485$

Frequency = $f = 1/T = 1/2\pi \times (1.4) = (0.7)/\pi$ Hz

(b) Frequency on moon:

$f_{moon} = 1/2\pi \sqrt{(l/g)} = 1/2\pi \sqrt{(l.67/5)}$

$= 1/(2\pi\sqrt{3})$ HZ

Question 37:

Solution:
Tension in the string is maximum at mean position

$T_{max} = mg + mv^2/l$

where m is mass, v = velocity and θ = angular amplitude

Tension is minimum at extreme position = $T_{min} = mg \cos\theta$

Given: $T_{max} = 2\ T_{min}$

$mg + mv^2/l = 2mg \cos \theta$

or $\cos \theta = l/2 + v^2/2gl$

From the law of conservation energy: $v^2 = 2gl(1 - \cos \theta)$

Substituting the values, we get

$\cos \theta = (1/2) + [2gl(1-\cos\theta)]/2gl$

or $\theta = \cos^{-1}(3/4)$

Question 38:

Solution:
Let "m" be the mass of the block and "R" Radius of the concave surface.

Also, Driving force F = mgsinθ

Therefore, a = g sinθ

[for small angle, sinθ = θ]

So, a = gθ = 1(approx) ...(1)

and
If x is the displacement of block from mean postion then sinθ = x/R

θ = x/R (approx)

(1)=> a = gx/R.....(2)

From equation of SHM, a = ω^2x..... (3)

From (2) and (3), ω = √g/R

Time period=T = 2π/ω = 2π (√R/g)

Question 39:

Solution: Torque on the ball = F x perpendicular distance

= (mg sinθ) r

Moment of inertia of ball = I = I_cm + mr^2

For spherical surface: I_{cm} = (2/5) mr^2

Therefore, I = (2/5) mr^2 + mr^2

= (7/5) mr^2

Angular acceleration of a ball = α = [Torque on the ball]/I

= 5gsinθ/7r

Now, Angular acceleration about the center of the surface:

At effective distance, R-r : α_o = 5gsinθ/7(R-r)

For small angle, sinθ=θ(approx)

Again, linear acceleration, a = αr = 5gsinθ/7r . r

= 5gsinθ/7

Question 40:

Solution:
Let M be the mass of earth: M = ρ vg = GM/R^2 = (4/3)πR^3 ρ.... (1)

Gravity at depth 'd' :

g' = GM'/r^2

Where r is the distance from center of the earth to that point.

$r = R - 1600 = 6400 - 1600 = 4800$ km

and $M' = (4/3)\pi r^3 \rho$ .... (2)

From (1) and (2)

$M/M' = R^3/r^3$

or $M' = (Mr^3)/R^3$

Therefore, $g' = G/r^2 . (Mr^2/R^3) = GM/R^2 . r/R$

$g' = g.r/R$

$= [9.8 \times 4800 \times 10^3]/[6400 \times 10^3]$

$= 7.35\ m/s^2$

Time period $= T = 2\pi\sqrt{(l/g')}$

$= 2\pi \sqrt{(0.4/7.35)}$

$= 1.47$ s

Question 41:

Solution:
(a) Time period of the oscillation of a particle and center of the earth acts as mean position.

Time period $= T = 2\pi/\omega = 2\pi \sqrt{(R/S)}$

Velocity = $v = \omega \sqrt{(A^2-R^2)}$
Where "A" be the amplitude.

$\sqrt{SR} = \sqrt{(S/R)}\ \sqrt{(A^2-R^2)}$

Solving above, we have

or $A = \sqrt{2}\ R$

Let t_1 time when particle is at distance 'R'

Form equation, $y = A \sin \omega t$

$R = \sqrt{2}\ R \sin \omega t_1$

or $\omega t_1 = 3\pi/4$

Let t_1 time when particle is at distance '-R'

$-R = \sqrt{2}\ R \sin \omega t_2$

or $\omega t_2 = 5\pi/4$

So, $\omega t_2 - \omega t_1 = \pi/2$

or $t_2 - t_1 = \pi/2\omega = \pi/2 \cdot \sqrt{R/g}$

(b)
When body is dropped from a height 'R'. Final velocity of the body on reaching the ground is 'v' is:

$(1/2)\ mv^2 = GMm/(R+R)$
[change in kinetic energy = change in potential energy]

$v^2 = gR$

or $v = \sqrt{gR}$

Therefore, time taken to cover the length of tunnel will be same,[as in part (a)]

$t_2 - t_1 = \pi/2 . \sqrt{R/g}$

(c)
It projected upward and reaches back to the surface with the same velocity i.e. $\sqrt{gR}$

From there it enters tunnel with same velocity as in part (a) and hence time taken to cover the tunnel.

Question 42:

Solution:

(a)
Mass of the earth with reduced radius 'l':

$M' = (4/3)\ \pi l^3 \rho$

where 'l' be the distance from the center of the earth to the particle at distance "x"

Mass of the earth with radius R
$M = (4/3)\pi R^3 \rho$

Now, $M'/M = l^3/R^3$

or $M' = M(l/r)^3$

The gravitational force on the particle will be

$$F = \frac{GM^1M}{l^2} = \frac{GM\left(\frac{l^3}{R^3}\right)m}{l^2} = \frac{GMml}{R^3}$$

...(1)

Also,

$l^2 = x^2 + (R^2/2)^2$

$l = (x^2 + R^2/4)^{(1/2)}$

(1)=>

$$F = \frac{GMm}{R^3}\left(x^2 + \frac{R^2}{4}\right)^{\frac{1}{2}}$$

(b) Let the component of force along tunnel:

$F_x = F \cos \theta = Fx/l$

$= GMm/R^3 \, . \, x$

Let the perpendicular component be F_y

$F_y = F \sin \theta = FR/2l$

$= GMm/2R^2 \, . \, x$

(c) the walls exert the same amount of force which the particle exerts on them

$F_N = F_Y$

$= GMm/2R^2$

(d) Resultant force on particle is the same force that is acting along the tunnel

$F_r = F_x = GMm/R^3 \, . \, x$

(e) For a body to be in S.H.M, a directly proportional to x

$a = F_x/m = GMm/MR^3 . x$

$= GM/R^3 . x$

If GM/R^3 is constant

$a = kx$

=> a α x

Question 43:
Solution:

(a) Time period = $T = 2\pi \sqrt{(l/g)}$

Replace g value by net acceleration.

$l/g = 4/\pi^2$

or $l = 4/\pi^2 . g$ .....(1)

When car accelerates with acceleration a_o, net acceleration on the bob:

$a = \sqrt{(g^2 + a_o^2)}$

$$T_1 = 2\pi\sqrt{\frac{l}{(g^2 + a_0^2)^{\frac{1}{2}}}}$$

$$3.99 = 2\pi\sqrt{\frac{l}{(g^2 + a_0^2)^{\frac{1}{2}}}}$$

$l = (3.99)^2/4\pi^2 [g^2 + a_o^2]^{1/2}$ ……(2)

From (1) and (2)

$4/\pi^2 . g = (3.99)^2/4\pi^2 [g^2 + a_o^2]^{1/2}$

or $a_o = 0.1$ g (approx)

Question 44:

Solution:
If a_0 be the elevator is moving with acceleration.

Time period = T = $2\pi\sqrt{[l/(g+a_0)]}$

$\pi/3 = 2\pi\sqrt{[l/(g+a_0)]}$

$1/6 = \sqrt{[l/(g+a_0)]}$

$g + a_0 = 36\ l$

[Here g = 32 ft/s^2]

$a_o = 4\ ft/s^2$

Question 45:

Solution:

When car is moving with uniform velocity:

$T = 2\pi \sqrt{(l/g)}$

$l = 4/\pi^2 . g$....... (1)

The net acceleration on the bob, when car accelerates with acceleration a_o

$a = \sqrt{(g^2+a_o^2)}$

$$T_1 = 2\pi\sqrt{\frac{l}{(g^2+a_0^2)^{\frac{1}{2}}}}$$

$$3.99 = 2\pi\sqrt{\frac{l}{(g^2+a_0^2)^{\frac{1}{2}}}}$$

$$\frac{(3.99)^2}{4\pi^2} = \frac{l}{(g^2+a_0^2)^{\frac{1}{2}}}$$

$l = (3.99)^2/4\pi^2 (g^2 + a_o^2)^{1/2}$...... (2)

From (1) and (2)

$4/\pi^2 . g = (3.99)^2/4\pi^2 (g^2 + a_o^2)^{1/2}$

$a_o = 0.1$ g (approx)

Question 46:

Solution:

The tension in the string will be due to force due to gravity and Force due to centripetal force

$$T = \sqrt{m^2 g^2 + \frac{(m^2 v^4)}{r^2}}$$

$$= m\sqrt{g^2 + \frac{v^4}{r^2}}$$

$$= m\left(g^2 + \frac{v^4}{r^2}\right)^{\frac{1}{2}}$$

a = $(g^2 + v^4/r^2)^{1/2}$ is the acceleration on the bob.

(b) Time period of the small oscillation

T = 2π √(l/a)

a = $(g^2 + v^4/r^2)1/2$

and Time period,

$$T = 2\pi\sqrt{\frac{l}{\left(g^2 + \frac{v^4}{r^2}\right)^{\frac{1}{2}}}}$$

Question 47:

Solution:

(a) Length = l = 3cm or 0.03m and g = 9.8

Time period = T = $2\pi \sqrt{(l/g)}$

= 0.3476 sec

(b) Lady is in circular motion on merry-go-round of radius r=2m.

A centripetal force also acts on the earring = $F_c = mv^2/r$ and v = 4m/s

The resultant acceleration on the ear-ring:

$a = \sqrt{(g^2+v^4/r^2)}$

Therefore, Time period = T = $2\pi\sqrt{(l/a)}$

= 0.30 sec

Question 48:

Solution:

(a) Time period for a physical pendulum = T = $2\pi \sqrt{(I/mgl)}$ ...(1)

let l is the distance between centre of mass and point of suspension

$I = I_{cm} + mx^2 = 0.5 - 0.2 = 0.3$ m

Now, $I = m_l^2/12 + m(0.3)^2$

$= 0.08 + 0.09$

$= 0.17$ m

Therefore, $T = 1.51$ sec
(on putting values in (1))

(b) when a ring is suspended through a point on its periphery,

$I = I_{cm} + mr^2$

$= mr^2 + mr^2$

$I = 2\,mr^2$ ...(2)

[here , I_{cm} of circle = mr^2]

Now, If $l = r$,

$T = 2\pi\sqrt{(I/mgr)}$

using (2),

$T = 2\pi\sqrt{(2r/g)}$

(c) when axis pass through corner of the square
$I_z = I_x + I_y + mr^2$

$= ma^2/12 + ma^2/12 + ma^2/12$

$= 2ma^2/3$

Now,

$$T = \frac{2\pi\sqrt{a\sqrt{2}}}{3g}$$

(d) Uniform disc of a mass m and radius r suspended through a point r/2 away from cente.

Here l = r/2

$I = I_{cm} + ml^2$

$= mr^2/2 + m(r/2)^2$

$= (3/4)\ mr^2$

Now,

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{3mr^2}{4mg\frac{r}{2}}} = 2\pi\sqrt{\frac{3r}{2g}}$$

Question 49:

Solution:

Time period for a physical pendulum = T = 2π √(I/mgr) ...(1)

Where, r = distance between center of mass and point of suspension

Given, r = l/2

$I = I_{cm} + ml^2$

$= ml^2/12 + ml^2/4$

$= ml^2/3$

Therefore,

(1)=> $T = 2\pi \sqrt{(2l/3g)}$ ...(2)

Again, time period of simple pendulum = $T = 2\pi \sqrt{(L/g)}$..... (3)

From (1) and (2)

$L = 2l/3$

Question 50:

Solution:

Let x be the distance between centre of mass and point of suspension, choose any.

$I = I_{cm} + mx^2$

$= mr^2 + mx^2$

Now, time period is

$$T = 2\pi\sqrt{\frac{I}{mgx}}$$

$$= 2\pi\sqrt{\frac{\frac{mr^2}{2}+mx^2}{mgx}}$$

$$= 2\pi\sqrt{\frac{r^2}{2gx}+\frac{x}{g}}$$

The condition for minimum of T : dt/dx = 0

$$\frac{dT}{dx} = \frac{d(2\pi\sqrt{\frac{r^2}{2gx}+\frac{x}{g}})}{dx} = 0$$

Consider $\left(-\frac{r^2}{2gx^2}+\frac{1}{g}\right) = 0$

We get,

$$\frac{r^2}{2gx^2} = \frac{1}{g}$$

=> $x = r/\sqrt{2}$

Minimum Time period:

$$T_{min} = 2\pi\sqrt{\frac{r^2}{2gr/\sqrt{2}}+\frac{r/\sqrt{2}}{g}} = 2\pi\sqrt{\frac{r\sqrt{2}}{g}}$$

Question 51:

Solution:
Length of the pendulum = l = 18 + 2 = 20 cm
and, r = 0.02

Time period : T = 2π √(I/mgl)

Where l is the distance between center of mass and point of suspension.

$I = I_{cm} + ml^2$

$= (2/3)\ mr^2 + ml^2$

= 0.0403 m

and

T = 2π √[(0.0403/(0.20mg)]

= 0.9009 sec

If we consider simple pendulum equation, then time period is

$T_1 = 2\pi \sqrt{l/g}$

$= 2\pi \sqrt{(0.2/9.8}$

= 0.897 sec

Difference in time: $T - T_1$

= 0.9009 - 0.897

= 0.0033 sec

Percentage of Time = 0.3 % more than simple pendulum.

Question 52:

Solution:

(a) Time period = $T = 2\pi \sqrt{(I/mgl)}$ ...(1)

$I = I_{cm} + mr^2$

$= mr^2 + mr^2$

$= 2\ mr^2$

At T = 2 sec

(1)=>
r = 50 cm (approx)

(b) From law of conservation of energy

Δ KE = Δ PE

or $(1/2)\, I\omega^2 - 0 = mgr(1-\cos\theta)$

At $\theta = 2^o$

$\omega^2 = 0.11$ m/s

(c) Acceleration of the particle is due to centripetal force on the circle.

$a = v^2/2r$

here v = 0.11 and r = 0.5

=> a = 0.012 m/s^2

(d) Acceleration of particle when it is at extreme position,

Gicen: $g = \pi^2$ m s^{-1} and T = 2 sec

Here $T = 2\pi/\omega$

Angular acceleration = $\alpha = \omega^2\, \theta$

$= (\pi^2 \times 2\pi)/180$

$= \pi^3/90$

Linear acceleration = $a = \alpha(2r) = 34$ cm/s^2

[here r = 100]

Question 53:

Solution:

Moment of inertia about centre of mass of a uniform disc

$I = mr^2/2$

Now, Time period $T = 2\pi \sqrt{(I/k)}$; where K torsional constant

Therefore, $K = 4\pi^2 I/T^2$

or $K = 2\pi^2 mr^2/T^2$

Question 54:

Solution:

Torque on the rod (τ) is directly proportion to θ

Therefore, $\tau = K\theta$ where k is torsional constant

Now, the work done, W is

$W = \int_0^{\theta_0} k\theta d\theta = K\theta^2/2$

From work energy theorem,

$(1/2)\, I\omega^2 = K\,\theta^2/2$ where $I = mL^2/2$

$= \sqrt{(2K\theta^2)/(mL^2)}$

Tension in the string is due to centripetal force and weight and resultant is,

$T = \sqrt{(F^2 m^2 g^2)}$

$= \sqrt{[(mr\omega^2)^2 + m^2g^2]}$

or $T = \sqrt{[(K^2\theta^4/L^2) + m^2g^2]}$

Question 55:
Solution:

(a) amplitudes of S.H.M

Both S.H.Ms have same time period and direction and hence their equations

$y_1 = A_1 \sin \omega t$ and $y_2 = A_2 \sin \omega t$

Resultant S.H.M equation, $y = y_1 + y_2$

=> $y = 3 \sin \omega t + 4 \sin \omega t = 7 \sin \omega t$

Amplitude of resultant S.H.M is 7cm

(b) S.H.Ms have a phase difference of $\pi/3$ between them

$y_1 = A_1 \sin \omega t$ and $y_2 = A_2 \sin \omega t + \pi/3$

Resultant amplitude = $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\pi/3)$

$A^2 = 9 + 16+2x3x4x(1/2)$

or A = 6.1 cm

(c) At phase difference π/2

$y_1 = A_1 \sin \omega t$ and $y_2 = A_2 \sin \omega t + \pi/2$

Resultant amplitude = $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\pi/2)$

$A^2 = 9 + 16 + 2x3x4x0$

or A = 5 cm

Question 56:

Solution:
S.H.Ms are
$x_1 = A \sin \omega t$ and $x_2 = A \sin(\omega t+\pi/3)$ and $x_3 = A \sin(\omega t+2\pi/3)$

Resultant, $x = x_1 + x_2 + x_3$

Resultant amplitude between first two => $R = x_1 + x_2$

$R = \sqrt{3} A$

Resultant amplitude of all three S.H.Ms will be the resultant of R and x_3

Let R' be the resultant of all three S.H.Ms

$R' = \sqrt{[A_3^2 + R^2 + 2A_3R \cos(\pi/2)]}$

= 2A

Question 57:

Solution:

Given:

$x_1 = 2.0 \sin(100 \pi t)$ and $x_2 = 2.0 \sin(120 \pi t + \pi/3)$, where x is in centimeter and t in second.

Resultant wave = $x = x_1 + x_2$

$x = 2[\sin 100\pi t + \sin (120\pi t + \pi/3)]$

(a) At t = 0.0125 sec

$x = 2[\sin 100\pi(0.0125) + \sin (120\pi(0.0125) + \pi/3)]$

= - 2.41 cm

(b) At t = 0.025 sec

$x = 2[\sin 100\pi(0.025) + \sin (120\pi(0.025) + \pi/3)]$

= 0.27 cm

Question 58:

Solution:

$x = x_0 \sin \omega t$ and $s = s_0 \sin \omega t$ and angle = 45^o

Amplitude of resultant motion = $R^2 = x^2 + s^2$

$= x_0^2 + s_0^2 + 2x_0 s_0 \cos(45^o)$

or $R = [x_0^2 + s_0^2 + \sqrt{2}\, x_0 s_0]^{(1/2)}$