

## Exercise Solutions

### Question 1:

#### Solution:

$$\text{Pressure} = P = \rho gh$$

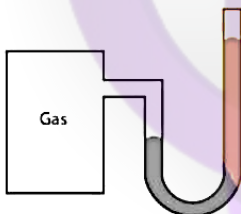
Here  $\rho = 1000 \text{ kg/m}^3$ ,  $g = 10 \text{ m s}^{-2}$  and  $h = 4 \text{ m}$

$$\Rightarrow P = 40000 \text{ N/m}^2$$

Yes, it necessary to state that the tap is closed. If tap is open the level of water will decrease and hence the pressure will decrease, the pressure at the tap is atmospheric.

### Question 2:

#### Solution:



(a) Pressure at the bottom of the tubes should be same when considered for both limbs.

From figure,

$$P_a + \rho_{Hg} h_1 g = P_g + \rho_{Hg} h_2 g$$

$$\Rightarrow P_g = P_a + \rho_{Hg} g(h_1 - h_2)$$

(b) Pressure of mercury at the bottom of tube

$$P = P_a + \rho_{Hg} h_1 g$$

**Question 3:**

**Solution:**

Pressure = Force/Area

$$F = mg = 45 \times 9.8 = 441 \text{ N}$$

and  $A = 900 \text{ cm}^2$  or  $0.09 \text{ m}^2$

$$\text{Therefore, Pressure} = 441 / 0.09 = 4900 \text{ N} \dots(1)$$

Also, pressure can be represented as,  $P = \rho g \Delta h$

Where,  $\Delta h$  is the height difference and  $\rho$  = density

Density of water =  $\rho = 1000 \text{ kg/m}^3$

$$\Rightarrow P = 1000 \times 9.8 \times \Delta h \dots(2)$$

from (1) and (2)

$$4900 = 1000 \times 9.8 \times \Delta h$$

$$\text{or } \Delta h = 50 \text{ cm}$$

**Question 4:**

**Solution:**

(a) Pressure at bottom of the container = pressure due to water + atmospheric pressure.

$$P = \rho g h + P_{\text{atm}}$$

$$= (1000 \times 10 \times 0.2) + 1 \times 10^5$$

$$= 1.02 \times 10^5 \text{ N/m}^2$$

Now, Force exerted on the bottom of the container =  $F = P \times \text{Area}$

$$= (1.02 \times 10^5) \times 0.002$$

$$F = 204 \text{ N}$$

(b) Vertical forces ( $F'$ ) includes the weight of the water and the force due to atmospheric pressure.

$$F' = mg + [P_{\text{atm}} \times \text{area}]$$

$$= (0.5 \times 9.8) + (0.002 \times 1 \times 10^5)$$

$$F' = 205 \text{ N}$$

Therefore, total upward force =  $F' - F = 1 \text{ N}$

**Question 5:****Solution:**

If the glass is covered by a jar and the air inside the jar is completely pumped out.

$$\text{Pressure at the bottom of the glass} = P = \rho gh = 1000 \times 10 \times 0.2 = 2000 \text{ N/m}^2$$

$$\text{The downward force is } F = P \times \text{area} = 2000 \times 0.002 = 4 \text{ N}$$

(b) Vertical force is equal to the weight of the water =  $mg = 0.5 \times 10 = 5 \text{ N}$

The horizontal forces again cancel out. So the total upward force =  $1 \text{ N}$

If glass of different shape is used provided the volume, height and area remain same, no change in answer will occur.

**Question 6:****Solution:**

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Standard atmospheric pressure is always pressure exerted by 76 cm Hg column.

$$P_{\text{atm}} | \text{Hg} = \rho_{\text{Hg}} gh = 13.6 \times 10^3 \times g \times 0.76 \dots(1)$$

$$\text{For water, } P_{\text{atm}} | \text{water} = \rho_{\text{water}} gh = 1000 \times g \times 0.76 \dots(2)$$

$$\text{Atmospheric pressure: } P_{\text{atm}} | \text{Hg} = P_{\text{atm}} | \text{water}$$

equating both the equations, we get

$h = 1033.6$  cm, is the required height of the water.

**Question 7:**

**Solution:**

$$\text{Pressure} = P = \rho gh = 1000 \times 10 \times 500 = 5 \times 10^6 \text{ N/m}^2$$

$$\text{and force} = F = \text{Pressure} \times \text{Area} = (5 \times 10^6) \times 2 = 10^7 \text{ N}$$

**Question 8:**

**Solution:**

Dimensions of rectangular tank :  $3 \text{ m} \times 2 \text{ m} \times 1 \text{ m}$ .

$$(a) \text{ Pressure at the bottom of the tank} = P = \rho gh$$

$$= 1000 \times 10 \times 1 = 10^4 \text{ N/m}^2$$

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Area of the bottom of the tank =  $A = lb = 3 \times 2 = 6\text{m}^2$

Now, force =  $F = \text{Pressure} \times \text{Area} = 10^4 \times 6 = 60000\text{ N}$

(b)  $P = \rho gh = \rho gx$

Area of strip =  $A = 2\text{ dx}$

So, the force on the strip =  $F = PA = \rho gx \times 2\text{ dx}$

=  $(20000\text{xdx})\text{ N}$

(c) Let us first find the perpendicular distance( $d$ ) of the strip from the bottom edge.

$d = h - x = (1 - x)\text{ m}$

Now, Torque = Force  $\times$  perpendicular distance =  $Fd$

=  $(20000x(1-x)\text{dx})\text{ N-m}$

(d) Total force

$$F = 20000 \int_0^1 x dx = 20000 \left[ \frac{x^2}{2} \right]_0^1 = 10000\text{N}$$

(e) Total Torque

$$\tau = \int_0^1 20000x(1-x) dx = \frac{10000}{3}\text{ N-m}$$

**Question 9:**

**Solution:** The density of copper and gold are 8.9 and 19.3 respectively.

$$V = x/8.9 + (36-x)/19.3 \dots(1)$$

## CLASS24

When ornament placed in water, it displaces water equal to its weight. The density of water =  $\rho_{\text{water}} = 1 \text{ g/cm}^3$

Weight of the water displaced =  $w = mg = v \rho_{\text{water}} g = vg \dots (2)$

ornament weighs 34 g in water.(Given)

The buoyant force =  $F_b = mg = (36-34) = 2g \dots (3)$

Buoyant force is equal to weight of the water displaced.

$vg = 2g$

$\Rightarrow [x/8.9 + (36-x)/19.3] g = 2g$

or  $x = 23.14/10.4 = 2.22 \text{ grams (approx)}$

The weight of copper in the ornament is 2.22 grams.

### Question 10:

#### Solution:

Total volume of the Ornament( $y$ )= Volume of gold + volume of cavity

Total volume ( $y$ )=  $36/19.3 + v$

Using, Principle of Flotation,

The volume of the water displaced is also  $y \text{ cm}^3$

The mass of the water displaced =  $y \text{ grams}$

The weight of the water displaced =  $yg$

$yg = 2g$

or  $y = 2$

Therefore,  $36/19.3 + v = 2$

or  $v = 0.13 \text{ cm}^3$

**Question 11:****Solution:**

Volume of metal immersed,  $= V = m/\rho = 0.16/8000 = 0.00002 \text{ m}^3$

Weight of the water displaced  $= F_b = V \times \rho_{\text{water}} \times g$

$= 0.00002 \times 1000 \times 10$

$= 0.2$

Find the normal force exerted by the bottom of the glass on the metal piece:  
Vertical forces consist of the normal reaction(N) from the surface (upwards), the weight of the metal(downwards) and the buoyant force(upwards) to maintain the balance of vertical forces.

$N + F_b = mg$

$\Rightarrow N = (0.16 \times 10) - 0.2 = 1.4 \text{ N}$

The normal reaction is  $1/4 \text{ N}$  in total.

**Question 12:****Solution:**

(a) Buoyant force should be equal to the weight of the boat.

$$v_w g = mg$$

$$v \times 1000 = 50$$

$$\text{or } v = 1/20 \text{ m}^3$$

(b) Let  $V_b$  be the volume of boat filled with water before water starts coming in from the sides. here  $V_b = 1 \text{ m}^3$

$$\text{Therefore, buoyant force} = F_b = V_b \rho_w g = 1000 \text{ gN}$$

$$\text{The volume of water} = V_w = m_w / \rho_w = 950 / 1000 = 0.95 \text{ m}^3$$

$$[\text{Here } m_w + 50 = 1000 \text{ or } m_w = 950 \text{ kg}]$$

$$\text{Now, fraction of water filled in the boat} = 0.95 = 19/20$$

**Question 13:****Solution:**

Let the edge of the cube be "x cm".

$$\text{Volume of the cube} = V = x^3 / 10^6 \text{ m}^3 \dots (1)$$

$$\text{And buoyant force} = F_b = m_{mg} + V \rho_i g = 0.5 + 900 v \dots (2)$$

Also, buoyant force is equal to the weight of the water displaced

$$\Rightarrow V \times 1000 \text{ g} = 0.5 + 900$$

$$\Rightarrow 100V = 0.5$$

Using (1),

$$x^3/10^6 = 0.5/100$$

or  $x = 17 \text{ cm}$  (approx)

**Question 14****Solution:**

Density of water =  $\rho_w = 1000 \text{ kg/m}^3$

Density of ice =  $\rho_i = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Density of K.oil =  $\rho_o = 1000 \text{ kg/m}^3$

The weight of the cube of ice =  $w_i = \text{volume} \times \text{density} \times g = (a+b)900g \dots(1)$

Where  $a = \text{volume of cube immersed in water}$  and  $b = \text{volume immersed in K.oil}$

The total buoyant force = sum of forces exerted by the liquids.

$$\Rightarrow F_b = a \rho_w + b \rho_o = a(1000) + b(800) \dots (2)$$

From (1) and (2)

$$a(1000) + b(800) = (a+b)900$$

$$\Rightarrow a/b = 1$$

The ratio of the volumes immersed is 1:1.

**Question 15:**

Solution:

Density of iron =  $\rho_i = 8000 \text{ kg/m}^3$  and density of water =  $1000 \text{ kg m}^{-3}$ .

Let "x cm" be the edge of the iron sheet.

$$\text{Surface area} = 6x^2 \text{ cm}^2$$

$$\text{Volume of the iron sheets} = v = 6x^2 \times 0.1 = (0.6/10^6) \text{ m}^3$$

$$\text{Therefore, mass of the iron box} = m_b = \rho_i \times v = 4.8 \times 10^{-3} x^2 \text{ kg} \dots(1)$$

$$\text{volume of the cube} = V' = x^3 \text{ cm}^3 = (x^3/10^6) \text{ m}^3$$

$$\text{buoyant force} = F_b = V' \rho_w g$$

$$= (x^3/10^6) \times 1000 \times g$$

$$= (x^3/10^3) \text{ gN} \dots\dots (2)$$

Here, buoyant force is equal to the weight of the box.

From (1) and (2)

$$4.8 \times 10^{-3} x^2 = (x^3/10^3)g$$

$$\Rightarrow x = 4.8 \text{ cm}$$

**Question 16:****Solution :**

Total mass of the system = mass of wood ( $m_w$ ) + mass of lead( $m_{pb}$ )

$$= 200 + m_{pb} \dots(1)$$

$$\text{Density of wood} = \rho_w = 0.8 \text{ g/cm}^3$$

$$\text{Volume of the wooden block} = v_w = m_w/\rho_w = 250 \text{ cm}^3$$

$$\text{Density of lead} = \rho_{pb} = 11.3 \text{ g/cm}^3$$

$$\text{Volume of lead piece} = v_{pb} = m_{pb}/\rho_{pb} = (m_{pb}/11.3) \text{ cm}^3$$

Therefore, the volume of water displaced would be equal to the total volume.

$$V = v_w + v_{pb} = 250 + (m_{pb}/11.3) \text{ cm}^3$$

$$F_b = v \rho_w g = (250 + m_{pb}/11.3) \times 1 \times g \dots(2)$$

Now, (1) = (2), we get

$$200 + m_{pb} = (250 + m_{pb}/11.3) \times 1 \times g$$

$$\Rightarrow m_{pb} = 54.8 \text{ g}$$

**Question 17:****Solution:**

volume immersed is only due to the wooden block.

$$\text{The buoyant force} = F_b = 250\text{g}$$

Now,

Total mass =  $F_b$

$$\Rightarrow 200 + m_{pb} = 250 \text{ g}$$

$$\text{or } m_{pb} = 50 \text{ g}$$

The mass of the lead piece is 50 g.

**Question 18:****Solution:**

Volume immersed in mercury =  $v_m = (12^3/5) \text{ cm}^3$

Buoyant force on the block is equal to the weight of the mercury displaced =  $\rho_m \times v_m$

$$= 13.6 \times (12^3/5) \text{ g} \dots (1)$$

Volume of block in water =  $v_w = 12^3 h \text{ cm}^3$

Volume of block in mercury =  $v'_m = 12^2(12-h) \text{ cm}^3$

Now, buoyant force =  $F_b = [v_w \rho_w + v'_m \rho_m]g$

$$= 12^2 h + 12^2 (12-h)13.6 \text{ g} \dots (2)$$

The buoyant force is equal to the weight of the cube.

$$(1)=(2)$$

$$\Rightarrow 13.6 \times (12^3/5) \text{ g} = 12^2 h + 12^2 (12-h)13.6 \text{ g}$$

Solving above equation, we have

$$h = 10.4 \text{ cm}$$

Therefore, water needs to be poured to a height of 10.4 cm.

**Question 19:****Solution:**

A hollow spherical body of inner and outer radii 6 cm and 8 cm respectively floats half-submerged in water.

Find density of the material of the sphere:

Here, Mass of the water displaced = Mass of the material,

$$\rho_w \times V_i = v_b \times \rho$$

$$\Rightarrow 1000 \times \left(\frac{2}{3}\right) \pi 8^3 = \left(\frac{4}{3}\right) \pi (8^3 - 6^3) \rho$$

Solving above equation, we have

$$\rho = 865 \text{ kg/m}^3$$

**Question 20:****Solution:**

A solid sphere of radius 5 cm floats in water. If a maximum load of 0.1 kg can be put on it without wetting the load

$$\text{Volume of sphere} = v = \frac{4\pi r^3}{3} = \left(\frac{5\pi}{3}\right) \times 10^{-4} \text{ m}^3$$

$$\text{Weight of water displaced} = \rho_w v g = \left(\frac{5\pi}{3}\right) \text{ N}$$

Now, Weight of sphere + Load = weight of displaced water

$$\rho_s v g + 0.1 \times 10 = \rho_w v g$$

$$\text{or } \rho_s = [\rho_w v g - 1]/v g$$

$$\text{Therefore, specific gravity} = \rho_s/\rho_w = 1 - 1/[5\pi/3] = 0.81$$

Which is the specific gravity of sphere.

### Question 21:

#### Solution:

$$\text{The weight of iron block} = w_i = m_i g - v_i \rho_a g = m_i [1 - (1/\rho_i) \rho_a] g$$

$$\text{and weight of wooden block} = w_w = m_w g - v_w \rho_a g = m_w [1 - (1/\rho_w) \rho_a] g$$

Now, ratio of  $w_i$  and  $w_w$ :

$$\frac{W_i}{W_w} = \frac{m_i \left[ 1 - \frac{1}{\rho_i} \rho_a \right] g}{m_w \left[ 1 - \frac{1}{\rho_w} \rho_a \right] g}$$

$$\Rightarrow \frac{W_i}{W_w} = \frac{1 - \frac{1.293}{7800}}{1 - \frac{1.293}{800}} = 1.0015$$

### Question 22:

#### Solution:

$$\text{Buoyant force} = F_b = v \rho_w g = (\pi r^2 x) \rho_w g$$

Where  $v$  = volume of the immersed object

$$\text{and } ma = (\pi r^2 x) \rho_w g$$

$$\text{or } a = (\pi r^2 x \rho_w g) / 2000$$

Now, time period =  $T = 2\pi \sqrt{(\text{displacement}/\text{acceleration})}$

$$\Rightarrow T = 0.5 \text{ sec (approx)}$$

**Question 23 :**

**Solution:**

(a) In the equilibrium condition, the weight of the cylinder, is supported by the spring and the buoyant force.

$$kx + v \rho_w g = mg$$

where  $v$  is volume =  $\pi r^2(h/2)$

$$\Rightarrow 500x + (\pi(0.05)^2 \times 0.1 \times 10 \times 1000) = \pi((0.05)^2 \times 0.2 \times 8000 \times 10)$$

Solving above equation, we get

$$x = 23.5 \text{ cm}$$

(b) Driving force =  $F = kx + v \rho_w g$

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$$\Rightarrow ma = kx + \pi r^2 \times \rho_w g$$

$$\Rightarrow a = 2\pi \sqrt{(\text{displacement}/\text{acceleration})/m}$$

$$\text{Time period, } T = 2\pi \sqrt{(\text{displacement}/\text{acceleration})}$$

$$= 2\pi \sqrt{0.5x/[500+(\pi(0.05)^2 \times 1000 \times 10)x]}$$

$$= 0.935 \text{ sec}$$

### Question 24:

#### Solution:

(a) The weight of the block is balanced by the spring and the buoyant force.

$$\Rightarrow mg = kx + v \rho_w g$$

$$\Rightarrow 0.5 \times 10 = 50x + (0.5/800) \times 1000 \times 10$$

Solving for  $x$ ,

$$x = 2.5 \text{ cm}$$

$$(b) a = kx/m$$

$$w^2 x = kx/m$$

$$\text{Time period} = T = 2\pi \sqrt{m/k} = (\pi/5) \text{ sec}$$

### Question 25:

**Solution:**

The weight of the remaining ice will be balanced by the buoyant force provided from the melted water.

$$mg = v \rho_w g$$

$$\text{The mass of the ice} = m = x^3 \times \rho_i$$

$$\Rightarrow x^3 \times 0.9 = x^2 \times h \times 1$$

$$\Rightarrow h = 0.9 x$$

$$\text{Volume of water formed} = 4^3 - x^3 = \pi r^2 h - x^2 h$$

$$x = 2.26 \text{ cm}$$

**Question 26:****Solution:**

Balance all the vertical forces on the tube.

$$P_a A + \rho_0 a l = P_a A + \rho h A g$$

Where  $P_a$  is atmospheric pressure.

$$\Rightarrow \rho_0 l = \rho h$$

$$\text{or } h = \rho_0 l / \rho$$

**Question 27:**

**Solution:**

The sum of the volume flow from Alaknanda and Bhagirathi is equal to the volume flow in Ganga.

Let the depth of the rivers be "d"

$$v_A \times 12 \times d + v_B \times 8 \times d = v_G \times 16 \times d$$

$$\Rightarrow v_G = 23 \text{ km/h}$$

**Question 28:****Solution:**

(a) The total volume is equal to area times velocity.

$$v_A \times a_v = v$$

Where  $v_A$  is the velocity in tube.

$$\Rightarrow 4 \times 10^{-2} \times v_A = 1$$

$$\Rightarrow v_A = 25 \text{ cm/s}$$

(b) Let the velocity in tube B,  $v_B$

For steady flow:  $v_A \times a_A = v_B \times a_B$

$$\Rightarrow 25 \times 4 \times 10^{-2} = v_B \times 2 \times 10^{-2}$$

$$\Rightarrow V_B = 50 \text{ cm/s}$$

(c) From Bernoulli equation,

$$P_A + (1/2)\rho v_A^2 = P_B + (1/2)\rho v_B^2$$

$$\Rightarrow P_A - P_B = (1/2)\rho [v_B^2 - v_A^2]$$

$$= (1/2) \times 1000 \times (0.5^2 - 0.25^2)$$

$$= 94 \text{ N/m}^2$$

**Question 29:**

**Solution:**

$V_A = 25 \text{ cm/s}$  and  $V_B = 50 \text{ cm/s}$  [from previous problem solution]

As changing the orientation doesn't change the volume of water flowing from the tubes.

(c) From Bernoulli's equation

$$P_A + \frac{1}{2}\rho v_A^2 + \rho gh_A = P_B + \frac{1}{2}\rho v_B^2 + \rho gh_B$$

$$\Rightarrow P_A - P_B = \frac{1}{2} \times 1000 \times [0.5^2 - 0.25^2] - 1000 \times 10 \times \frac{15}{16 \times 10^2}$$

$$= 0$$

Therefore, pressure difference is  $0 \text{ N/m}^2$

**Question 30:**

**Solution:**

(a) The total volume is equal to area times velocity.

$$V = a_B \times v_B$$

Where,  $v = 1 \text{ cm}^3$ ,  $a_A =$  area of cross-section of tube A =  $4 \text{ mm}^2$  and  $a_B =$  area of cross-section of tube B =  $2 \text{ mm}^2$

$$\Rightarrow v_B = 50 \text{ cm/s}$$

(b) let  $v_b$  be the velocity in tube

$$v_A \times a_A = v_B \times a_B$$

$$\Rightarrow v_A = 25 \text{ cm/s}$$

(c) From Bernoulli equation

$$P_A + \frac{1}{2} \rho v_A^2 + \rho g h_A = P_B + \frac{1}{2} \rho v_B^2 + \rho g h_B$$

$$\begin{aligned} \Rightarrow P_A - P_B &= \frac{1}{2} \times 1000 \times [0.5^2 - 0.25^2] + 1000 \times 10 \times \frac{15}{16 \times 10^2} \\ &= 187.5 \text{ N/m}^2 \end{aligned}$$

$$\Rightarrow P_A - P_B = 188 \text{ N/m}^2 \text{ (approx)}$$

**Question 31:**

**Solution:**

(a) From equation of continuity,

$$v_A \times a_A = v_B \times a_B$$

$$0.1 \times 1 = 0.5 \times v_B$$

$$\Rightarrow v_B = 20 \text{ cm/s}$$

(b) From Bernoulli's equation

$$P_A + \frac{1}{2} \rho v_A^2 + \rho g h_A = P_B + \frac{1}{2} \rho v_B^2 + \rho g h_B$$

$$\Rightarrow P_B - P_A = \rho g [h_B - h_A] - \frac{1}{2} \rho [v_B^2 - v_A^2]$$

$$= 1000 \times 10 \times 0.05 - \frac{1}{2} \times 1000 \times [0.2^2 - 0.1^2]$$

$$= 485 \text{ N/m}^2$$

**Question 32:****Solution:**

Equation of continuity:  $v_A \times a_A = v_B \times a_B$

$$\Rightarrow v_B = 2 v_A$$

Where  $v_A$  and  $v_B$  speed of water at A and speed of water at B respectively.

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$a_A$  = Area of cross-section of tube A and  $a_B$  = Area of cross-section of tube B

Also, given  $h$  = difference in height in the two columns = 2 cm

Pressure difference between A and B:  $P_A - P_B = \rho gh = 1000 \times 10 \times 0.02 = 200 \text{ N/m}^2$

By Bernoulli's equation,

$$P_A + (1/2)\rho[v_B^2 - v_A^2]$$

$$200 = (1/2) \times 1000 \times [4v_A^2 - v_A^2]$$

$$\text{or } v_A^2 = 400/(3 \times 100)$$

$$\text{or } v_A = 36.51 \text{ cm/s}$$

$$\text{Rate of flow} = v_A \times a_A = 36.51 \times 4 = 146 \text{ cm}^3/\text{s (approx)}$$

### Question 33:

#### Solution:

Equation of continuity:  $v_A \times a_A = v_B \times a_B$

Where  $v_A$  and  $v_B$  are velocities of flow at A and B

$$v_A \times 5 = 500$$

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$$\Rightarrow v_A = 100 \text{ cm/s or } 1 \text{ m/s}$$

Similarly,  $v_B = 250 \text{ cm/s or } 2.5 \text{ m/s}$

Now, the pressure difference between the points,

$$P_A - P_B = \rho_{\text{Hg}} gh = 13.6 \times 10 \times h$$

From the Bernoulli's equation:

$$P_A + \frac{1}{2} \rho v_A^2 = P_B + \frac{1}{2} \rho v_B^2$$

$$P_A - P_B = \rho_{\text{Hg}} gh = \frac{1}{2} \rho_w [v_B^2 - v_A^2]$$

$$\Rightarrow 13.6 \times 980 \times h = \frac{1}{2} \times 1 \times [250^2 - 100^2]$$

$$\Rightarrow h = \frac{26250}{13.6 \times 980} = 1.969 \text{ cm}$$

### Question 34:

**Solution:**

(a) The velocity of water exiting at "a"

$$v_A = \sqrt{2gh}$$

$$\Rightarrow v_A = \sqrt{(2 \times 10 \times 9.8)} = 4 \text{ m/s}$$

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$$(b) v_B = \sqrt{2g \cdot h/2} = \sqrt{2 \times 10 \times 0.4} = \sqrt{8} \text{ m/s}$$

$$(c) \text{ volume flow} = V = \text{Area} \times \text{height} = Ah$$

$$\text{Therefore, } dv/dt = A dh/dt \dots(1)$$

$$\text{We know, } V = av dt$$

$$\text{or } V = a (\sqrt{2gh}) t$$

Differentiating above equation, we have

$$dV/dt = A \times \sqrt{2gh} \dots(2)$$

From (1) and (2)

$$a (\sqrt{2gh}) = A dh/dt$$

$$\Rightarrow 2 \times 10^{-6} (\sqrt{2gh}) = 0.4 dh/dt$$

$$\Rightarrow dh/(\sqrt{2gh}) = 5 \times 10^{-6} dt$$

(d) integrating above equation, we have

$$5 \times 10^{-6} \int_0^t dt = \frac{1}{\sqrt{2g}} \int_{0.8}^{0.4} \frac{dh}{\sqrt{h}}$$

$$\Rightarrow t = \frac{1}{\sqrt{20}} \times 2 \times \left[ (0.4)^{\frac{1}{2}} - (0.8)^{\frac{1}{2}} \right] \times \frac{1}{5 \times 10^{-6}}$$

solving for t,

$$t = 6.51 \text{ h}$$

**Question 35:**

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**Solution 35:** Water level is maintained in a cylindrical vessel up to a fixed height  $H$ .

Height of water above the hole =  $H-h$

Velocity with which water =  $v = \sqrt{2g[H-h]}$  and time of flight be  $t$

$$t = \sqrt{2h/g}$$

The horizontal distance travelled =  $x = vt = \sqrt{2g[H-h]} \times \sqrt{2h/g}$

$$\Rightarrow x = \sqrt{4[Hh-h^2]}$$

To maximize this function:  $dx/dh = 0$

$$d/dh \sqrt{4[Hh-h^2]} = 0$$

$$h = H/2$$