

Some Mechanical Properties of Matter**Exercise Solutions****Question 1:****Solution:**

(a) Stress = Force/Area = mg/A

$$= (10 \times 10) / (4 \times 10^{-6})$$

$$= 2.5 \times 10^{-7} \text{ N/m}^2$$

(b) Strain = Stress/ Y

$$= [2.5 \times 10^{-7}] / [2 \times 10^{11}]$$

$$= 1.25 \times 10^{-4} \text{ m}$$

(c) Let elongation be l

$$l = L \times \text{strain} = 3.75 \times 10^{-4} \text{ m}$$

Question 2:**Solution:**

Area of cross-section of the cylinder = $A = \pi r^2$

$$= \pi \times 0.02^2$$

$$= 4\pi \times 10^{-4} \text{ m}^2$$

(a) stress = Force/Area

$$= [10 \times 10] / [4\pi \times 10^{-4}]$$

$$= 7.96 \times 10^5 \text{ N/m}^2$$

(b) Strain = Stress/Y

$$= [7.96 \times 10^5] / [2 \times 10^{11}]$$

$$= 3.98 \times 10^{-6}$$

(c) Let compression be l

$$l = 2 \times (3.98 \times 10^{-6}) = 8 \times 10^{-6} \text{ m (approx)}$$

Question 3:**Solution:**

The elastic limit of steel is $8 \times 10^8 \text{ N m}^{-2}$ and its Young modulus $2 \times 10^{11} \text{ N m}^{-2}$.

$$\text{Strain} = \text{Stress}/Y \text{ and } \text{Stress} = l/L$$

$$\Rightarrow \text{Stress}/Y = l/L$$

$$\Rightarrow l = (\text{stress } L)/Y$$

$$= [8 \times 10^8 \times 0.5] / [2 \times 10^{11}]$$

$$= 2 \times 10^{-3} \text{ m}$$

Question 4:

Solution:

(a) Let the stress in steel wire be σ_s and in copper σ_c

Now, $\sigma_s = F/A$ and $\sigma_c = F/A$

$$\Rightarrow \sigma_s/\sigma_c = 1$$

(b) Strain, $\epsilon = \sigma/Y$

Strain in the steel wire = $\epsilon_s = [F/A]/[2 \times 10^{11}]$

and strain in the copper wire = $\epsilon_c = [F/A]/[1.3 \times 10^{11}]$

Hence, $\epsilon_c/\epsilon_s = 20/13$

Question 5:**Solution:**

$$[\text{Strain in the steel wire}]/[\text{strain in the copper wire}] = [2 \times 10^{11}]/[1.3 \times 10^{11}]$$

$$= 20/13$$

$$= 1.54 \text{ (approx)}$$

Question 6:

Solution:

(a) Stress on the lower wire = $\sigma = \frac{[(\text{total weight of lower wire})g]}{[\text{Area of cross section of upper wire}]}$

$$= \frac{[m+m_1]g}{A_1}$$

Let say, $A_1 =$ Area of cross section of upper wire

$m =$ weight of the load and $m_1 =$ mass of the block = 10 kg

For maximum load, we take in the maximum stress that the wire can withstand.

$$\Rightarrow 8 \times 10^8 = \frac{[(m+10) \times 10]}{[3 \times 10^{-7}]}$$

$$\Rightarrow m = 14 \text{ kg}$$

The upper wire has a total weight of $m+m_1+m_2$

Hence, stress on the lower wire = $\frac{[(m+m_1+m_2)g]}{A_2}$

For maximum load,

$$\Rightarrow 8 \times 10^8 = \frac{[(m+10+20) \times 10]}{[3 \times 10^{-7}]}$$

Given, $m_2 = 20 \text{ kg}$

$$\Rightarrow m = 18 \text{ kg}$$

(b) For $m_2 = 36 \text{ kg}$.

$$\Rightarrow 8 \times 10^8 = [(m+10+36) \times 10] / [6 \times 10^{-7}]$$

$$\Rightarrow m = 2 \text{ kg}$$

The maximum load that can be put is 2 kg, Upper wire will break first load is increased.

Question 7:**Solution:**

We know, stress = F/A and strain = l/L

Young's modulus = $Y = FL/AI$

$$= [100 \times 2] / [2 \times 10^{-4} \times 0.01]$$

$$= 1 \times 10^8 \text{ N/m}^2$$

Question 8:**Solution:**

We know, stress = T/A and strain = l/L

Young's modulus = $Y = TL/AI$

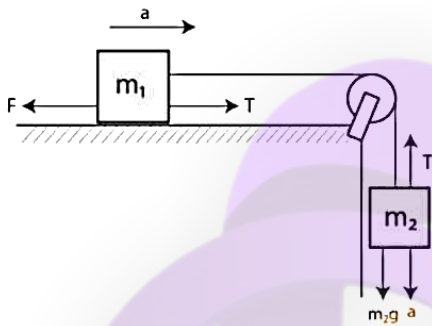
$$\Rightarrow T = YAI/L$$

$$= [1.9 \times 10^{11} \times 4 \times 10^{-4} \times 0.001] / [2]$$

$$\Rightarrow T = 3.8 \times 10^4 \text{ N}$$

Question 9:

Solution:



Force = $F = m_2 (g/2)$ (given)

From free body diagram:

$$T - F = m_1 a \quad [\text{for mass } m_1]$$

$$\Rightarrow T - m_2 g / 2 = m_1 a \quad \dots(1)$$

$$m_2 g - T = m_2 a \quad \dots(2) \quad [\text{for mass } m_2]$$

Using (1) and (2)

$$\Rightarrow T - \frac{m_2 g}{2} = m_1 \left(g - \frac{T}{m_2} \right)$$

$$\Rightarrow T = \frac{m_2 g(2m_1 + m_2)}{2(m_1 + m_2)}$$

We know, stress = T/A
and Young's modulus = $Y = \text{stress/strain}$

Therefore, strain = T/AY

$$= [m_2 g(2m_1 + m_2)] / [2AY(m_1 + m_2)]$$

Question 10:**Solution:**

At the point of release, the tension = $T = mg + mv^2/r$

Force which brings elongation is centrifugal force = $F = mv^2/r \dots(1)$

From work energy theorem,
 $(1/2)mv^2 = mgr(1 - \cos\theta)$

$$\text{or } v^2 = 2gr(1 - \cos\theta)$$

$$(1) \Rightarrow F = (m/r) 2gr(1 - \cos\theta) = 2mg(1 - \cos\theta)$$

Again, Young's modulus = $Y = FL/AI$

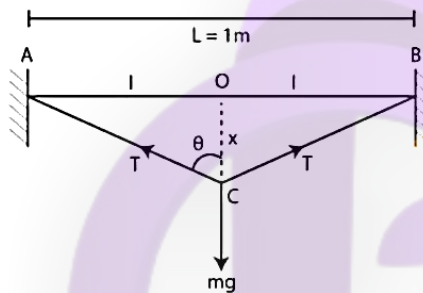
$$\Rightarrow Y = [2mg(1 - \cos\theta)L]/AI$$

$$\cos\theta = 1 - \frac{2 \times 10^{11} \times \pi \times 0.25 \times 10^{-6} \times 0.002}{2 \times 20 \times 10 \times 4} = 0.803$$

$$\Rightarrow \theta = \cos^{-1}(0.803) \text{ or } 36.4^\circ \text{ (approx)}$$

Question 11:

Solution:



From the figure,

$$\cos \theta = \frac{x}{\sqrt{x^2 + l^2}}$$

$$= \frac{x}{l} \left(1 - \frac{x^2}{2l^2} + \dots \right)$$

[Using binomial theorem]

$$\text{If } x \ll l, \frac{x^2}{l^2} = 0$$

$$\Rightarrow \cos \theta = \frac{x}{l}$$

Again, change in length,

CLASS24

$$\Delta L = (AC + CB) - AB = 2(l^2 + x^2)^{1/2} - 2l$$

$$\Delta L = 2l \left(1 + \frac{x^2}{l^2} \right)^{1/2} - 2l$$
$$= \frac{x^2}{l}$$

Now, Using Young's modulus,

$$T = (YA \Delta l)/L$$

$$\Rightarrow T = 8 \times 10^5 \times (x^2/l)$$

Forces are balance, $2T \cos \theta = mg$

$$\Rightarrow 2 \times 8 \times 10^5 \times (x^2/l) \times (x/l) = 2.16 \times 10$$

$$\Rightarrow x = 1.5 \text{ cm}$$

Question 12:

Solution:

The longitudinal strain = $T/A Y$

Where T = tension, A = cross-sectional area and Y = Young's modulus

$$\Rightarrow \text{longitudinal strain} = 20 / [0.01 \times 10^{-4} \times 1.1 \times 10^{11}] = 1.82 \times 10^{-4}$$

The area of circular section be of the diameter D , $A = \pi D^2/4$

$$\Rightarrow dA = 2\pi D dD/4$$

$$\Rightarrow dA = 2 \times 0.01 \times 5.82 \times 10^{-5}$$

$$= 1.164 \times 10^{-6} \text{ cm}^2$$

Question 13:**Solution:**

Let v be initial volume and v' be the final volume.

$$\text{Change in volume} = \Delta V = -(v - v')$$

$$\text{and pressure, } P = -B(\Delta v/v)$$

$$= B(1 - v'/v)$$

$$\text{Therefore, } dP = B (dv'/v)$$

$$\text{We are given that, } dv'/v' = (0.01/100)$$

$$\Rightarrow dP = B \times dv'/v' \times v'/v$$

$$[\text{Assuming, } v'/v = 1]$$

$$\Rightarrow dP = B (dv'/v')$$

$$= 2.1 \times 10^9 \times (0.01/100)$$

$$= 2.1 \times 10^5 \text{ N/m}^2$$

Question 14:**Solution:**

The bulk strain at depth h :

CLASS24

$$\Delta v/v = P/B = \rho gh/B$$

Now, the density at depth is $\rho' = \rho (v/v')$

V' is compressed volume at depth h and $v' = v - \Delta v$

$$\Rightarrow \rho' = \rho \times [v/(v-\Delta v)]$$

$$= \rho B/(B-\rho gh)$$

$$= [1030 \times 2 \times 10^9] / [2 \times 10^9 - (1030 \times 10 \times 400)]$$

$$= 1032 \text{ kg/m}^3$$

Therefore, change in density = $1032 - 1030 = 2 \text{ kg/m}^3$

Question 15:

Solution:

$$\text{Shearing strain} = \Delta x/d = \Delta x/0.0005 = 200 \Delta x$$

$$\text{Shearing Stress} = F/A = 10/(4 \times 10^{-4}) = 25000 \text{ N/m}^2$$

$$\text{Rigidity modulus} = (\text{shearing stress})/(\text{shearing strain})$$

$$\Rightarrow 8.4 \times 10^{10} = 25000/200\Delta x$$

$$\Rightarrow \Delta x = 1.5 \times 10^{-9} \text{ m}$$

Question 16:

Solution:

Length of thread = $L = 5 \text{ cm}$ or 0.05 m

Surface tension of water = $S = 0.076 \text{ N/m}$

Therefore, force due to surface tension: $F = SL = 0.076 \times 0.05$

$$= 3.8 \times 10^{-3} \text{ N}$$

Question 17:**Solution:**

(a) excess pressure inside a drop of mercury of radius 2 mm :

$$P = 2S/r = [2 \times 0.465] / 0.002 = 465 \text{ N/m}^2$$

(b) excess pressure inside a soap bubble of radius 4 mm

$$P = 4S/r = [4 \times 0.03] / 0.004 = 30 \text{ N/m}^2$$

(c) excess pressure inside an air bubble of radius 4 mm formed inside a tank of water.

$$P = 2S/r = [2 \times 0.076] / 0.004 = 38 \text{ N/m}^2$$

Question 18:

Solution:

(a) Force exerted by air above:

$$F = PA = 10^5 \times 10^{-6} = 0.1 \text{ N}$$

(b) Force exerted by the mercury below the surface area:

Let P' be the required pressure,

$$\Rightarrow P' = P_0 + 2T/r$$

$$\text{Now, } F = P'A = (P_0 + 2T/r)r$$

$$= (0.1 + (2 \times 0.465)/(4 \times 10^{-3})) \times 10^{-6}$$

$$= 0.10023 \text{ N}$$

(c) Force exerted by the mercury surface in contact with it:

$$F = PA = (2T/r) A$$

$$= (2 \times 0.465)/(4 \times 10^{-3})$$

$$= 0.00023 \text{ N}$$

Question 19:

Solution:

Surface tension of water = $T = 7.5 \times 10^{-2} \text{ N/m}$

Considering $\cos \theta = 1$

Radius of capillary A = $r_A = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$

Height of water level in capillary A:

$$h_A = [2T \cos\theta]/[r_A \rho g]$$

$$= [2 \times 7.5 \times 10^{-2}] / [0.5 \times 10^{-3} \times 1000 \times 10]$$

$$= 3 \times 10^{-2} \text{ m or } 3 \text{ cm}$$

Radius of capillary B = $r_B = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$

Height of water level in capillary B:

$$h_B = [2T \cos\theta]/[r_B \rho g]$$

$$= [2 \times 7.5 \times 10^{-2}] / [1 \times 10^{-3} \times 1000 \times 10]$$

$$= 15 \times 10^{-3} \text{ m or } 1.5 \text{ cm}$$

Radius of capillary C = $r_C = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

Height of water level in capillary C:

$$h_C = [2T \cos\theta]/[r_C \rho g]$$

$$= [2 \times 7.5 \times 10^{-2}] / [1.5 \times 10^{-3} \times 1000 \times 10]$$

$$= 15 / 1.5 \times 10^{-3} \text{ m or } 1 \text{ cm}$$

Question 20:**Solution:**

We know, The rise or depression in capillary = $h = [2T \cos\theta]/[r\rho g]$

Let the angle of mercury, $\theta = 0^\circ$

depression in mercury = $-2 \text{ cm} = -0.02 \text{ cm}$ (Given)

For mercury = $h_{hg} = -0.02 = [2 T_{hg}]/[r\rho_{hg} g] \dots(1)$

Let h_w be the rise in water.

$h_w = [2T_w]/[r_w w g] \dots\dots (2)$

Dividing (2) by (1), we get

$$h_w/h_{hg} = (0.075/0.465) \times 13.6$$

$$= 2.19$$

Now, height of the water level = $h_w = 2 \times 2.19 = 4.38 \text{ cm}$

Question 21:**Solution:**

Rise of mercury column = $h = [2 T \cos\theta]/[r\rho g]$

Substituting given values,

$$h = [2 \times 0.466 \times \cos 130^\circ]/[0.001 \times 13600 \times 10]$$

$$= -0.005 \text{ m or } -0.5 \text{ cm}$$

[The negative sign is used as mercury goes down in a capillary tube.]

$$\text{Height of mercury in the tube} = h+H = -0.5 + 76 = 75.5 \text{ cm}$$

Question 22:**Solution:**

$$\text{Pressure} = p = 2T/r$$

Where T = surface tension = 0.075 N/m (given)

and r = Radius of the capillary tube = 0.5 mm or 0.0005 m (given)

$$\Rightarrow p = 300 \text{ N/m}^2$$

$$\text{Pressure at a depth of } 5 \text{ cm} = p' = \rho gh = 1000 \times 9 \times 0.05 = 490 \text{ N/m}^2$$

Let P be the pressure of water just below surface, then

$$\text{Difference in pressure} : \Delta P = (P + p') - (P + p)$$

$$= (P + 490) - (P + 300)$$

$$= 190 \text{ N/m}^2$$

Question 23: Find the surface energy of water kept in a cylindrical vessel of radius 6.0 cm . Surface tension of water 0.075 J m^{-2} .

Solution:

$$\text{Area of surface} = A = \pi r^2$$

where $r = 6 \text{ cm} = 0.06 \text{ m}$ (given)

$$\Rightarrow A = 0.0113 \text{ m}^2$$

$$\text{Now, surface energy} = E = SA = 0.075 \times 0.0113 = 8.4 \times 10^4 \text{ J}$$

Question 24:**Solution:**

$$\text{Volume of drop} = v = \frac{4}{3}\pi r^3$$

$$\text{Volume of each drop after division} = v' = v/8 = \frac{1}{6}\pi r^3$$

Now the new radius of each drop be r' :

$$\frac{4}{3}\pi r^3 = \frac{1}{6}\pi r'^3$$

$$\text{or } r' = r/2$$

$$\text{The total area of all droplets} = A' = 8 \times 4\pi r'^2 = 8\pi r^2$$

$$\text{Therefore, increase in area} = 8\pi r^2 - 4\pi r^2 = 4\pi r^2$$

$$\text{Therefore, increase in energy} = E = S \times (\text{increase in area}) = 2.34 \times 10^{-5} \text{ J}$$

$$[S = 0.465 \text{ J m}^{-2} \text{ and } r = 0.002 \text{ cm}]$$

Question 25:**Solution:**

Let h be the height of water raised in the capillary.

$$h = [2T \cos\theta]/r\rho g \dots (1)$$

$$\text{Here } T = \text{Surface tension of water} = 0.076 \text{ N/m}, \theta = 0^\circ, r = 0.001, \rho = 1000 \text{ and } g = 10]$$

$$\Rightarrow h = 15 \text{ cm}$$

(b) length of capillary, $h' = h/2 \dots (A)$, then

$$h' = [2T \cos\theta]/r\rho g \dots(2)$$

Dividing (2) by (1)

$$h'/h = \cos \theta$$

using (A)

or $\theta = 60$ degrees

Question 26:**Solution:**

(a) depression of mercury column in the capillary

$$h = [2T \cos\theta]/r\rho g$$

$$= [2 \times 0.465 \times \cos 135^\circ] / [0.001 \times 13600 \times 9.8]$$

$$= 4.9 \text{ cm}$$

(b) Let ϕ be the angle made by mercury.

new height is $h' = h/2$

$$\text{Now, } h' = [2T \cos\phi]/r\rho g$$

From above equations, $h'/h = \cos\phi/\cos(135^\circ)$

$$\Rightarrow \cos\phi = -0.3535$$

or $\phi = 111$ degrees (approx)

Question 27:**Solution:**

The weight of water in unit volume = $W = \rho h dg \times 1$

and given force = $F = 2Tl = 2 \times 0.0075 \times 1 = 0.150 \text{ N}$

Now, $W = \rho h dg = 1000h \times 0.001 \times 10 = 10h \text{ N}$

or $10h = 0.150$

or $h = 0.15/10 = 1.5 \text{ cm}$

Question 28: Consider an ice cube of edge 1.0 cm kept in a gravity-free hail. Find the surface area of the water when the ice melts Neglect the difference in densities of ice and water.

Solution:

Volume of ice cube of edge = $V = 1 \text{ cm}^3 = 10^{-6} \text{ m}^3$

keeping in mind that, volumes do not change when state changes.

As gravity has no effect here, the water after melting will form a sphere,

$$(4/3)\pi r^3 = 10^{-6}$$

$$\text{or } r = [(10^{-6} \times 3)/4\pi]^{1/3} \text{ m}$$

Now, The surface area of the sphere

$$A = 4\pi r^2$$

Using value of r , and solving we get

$$A = (36\pi)^{1/3} \text{ cm}^2$$

Question 29:**Solution:**

The loop will take circular shape after pricking. Radius can be calculated by using relation,

$$l = 2\pi R$$

$$\Rightarrow R = (6.28) / [2 \times 3.14]$$

$$= 1 \text{ cm or } 10^{-2} \text{ m}$$

$2T \sin(d\theta)$ force inward direction is balanced by surface tension force in outward direction.

Therefore, $2T \sin(d\theta) = \text{length of arc} \times \text{surface tension}$

for small angle, $\sin(d\theta) = d\theta$

$$2T d\theta = 2R d\theta \times S$$

Where S is the surface tension.

$$\text{Now, } T = SR$$

$$= 0.030 \times 10^{-2}$$

$$= 3 \times 10^{-4} \text{ N}$$

Question 30:

Solution:

(a) viscous force = $F = 6\pi\eta r v$

$$= 6 \times 0.8 \times 3.14 \times 0.001 \times 0.01$$

$$= 1.5 \times 10^{-4} \text{ N}$$

(b) The hydrostatic force exerted by the glycerin on the sphere:

$$\text{Force} = F_B = \left(\frac{4}{3}\right) \pi r^3 \times \rho g$$

$$= \left(\frac{4}{3}\right) \times 3.14 \times 0.001^3 \times 1260 \times 9.8$$

$$= 5.2 \times 10^{-5} \text{ N (approx)}$$

(c) Terminal velocity with which the sphere will move down without acceleration

$$\text{viscous force} = F = 6\pi\eta r v = 6\pi \times 0.8 \times 0.001 \times v = 0.0048 \pi v$$

The upwards force is due to the buoyant force in glycerin.

$$F_B = 5.2 \times 10^{-5} \text{ N}$$

$$\text{and the weight downwards, } F_m = 50 \times 10^{-6} \times 9.8$$

$$= 4.9 \times 10^{-4} \text{ N}$$

As per situation, there should be no acceleration downwards:

$$F + F_B = F_m$$

$$\Rightarrow 0.0048\pi v + 5.2 \times 10^{-5} = 4.9 \times 10^{-4}$$

$$\Rightarrow v = 0.029 \text{ m/s}$$

$$\Rightarrow v = 2.9 \text{ cm/s}$$

Question 31:**Solution:**

Viscous force is balanced by the weight:

$$6\pi\eta rv = \rho vg$$

$$\Rightarrow v = [1000 \times (4/3) \pi r^3 \times 9.9] / [6\pi\eta r]$$

$$\Rightarrow v = [2000 \times 0.002^2 \times 1.1] / [1.8 \times 10^{-5}] = 4.9 \text{ m/s}$$

Question 32: Water flows at a speed of 6 cm s^{-1} through a tube of radius 1 cm . Coefficient of viscosity of water at room temperature is 0.01 poise . Calculate the Reynolds number. Is it a steady flow?

Solution:

$$\text{Reynolds number} = N = \rho v(2r) / \eta$$

$$= [1000 \times 0.06 \times 2 \times 0.01] / [0.001]$$

$$N = 1200 \text{ (approx)}$$