Exercise Solutions

Solution:

Given: A wave pulse passing on a string with a speed of 40 cm s^{-1} in the negative x-direction has its maximum at x = 0 at t = 0.

We know, V = x/t

or x = vt

=> x = 0.4 x 5 m =2 m

or x = 200 cm along the negative x-axis.

Question 2:

Solution:

(a) dimensions of A, a and T

[A] = [L]
[a] = [L] and
[T] = [T]

(b) Find the wave speed:
Wave speed = v = λ/T = a/T

(c)(c)

The whole structure depends upon the exponent.

$$\text{Let } Y = -\left(\frac{x}{a}+\frac{t}{T}\right)^2 = -\left(\frac{1}{T^2}\right)\left(\frac{xT}{a}+t\right)^2$$

$$\text{or } Y = f\left(t+\frac{x}{V}\right)$$

Now,

Case 1: If y = f(t-x/v), then wave is travelling in positive direction.

Case 2: If y = f(t + x/v), x/v), then wave is travelling in negative direction.

(d) wave speed = v = a/T

The maximum pulse at t = T is (a/T) x T = a -> negative x-axis
and maximum pulse at t = 2T is (a/T) x 2T = 2a -> along negative x-axis

So, the wave travels in negative x-axis direction.

Question 3:

Solution:

Using relation: x = vt

At t = 1 s the pulse will be at 10 cm.
At t = 2 s the pulse will be, x = 2(10) cm = 20 cm
At t = 3 s the pulse will be, x = 3(10)cm = 30 cm

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Question 4:

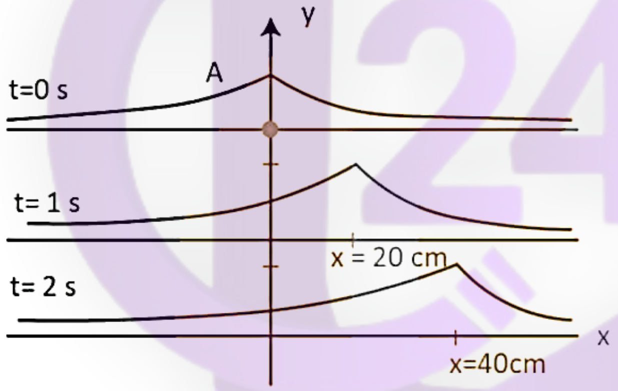
Solution:

$y = a^3/(x^2+a^2)$
For maximum, $dy/dx = 0$
=> $x = vt$
again $dx/dt = v$

At $t = 0$ s, $x = 0$ cm

At $t = 1$ s, $x = 20$ cm

At $t = 2$ s, $x = 40$ cm



Question 5:

Solution: AT $x = 0$, $f(t) = a \sin(t/T)$

=> wavelength = $\lambda = vT$
So, general equation of wave,
$Y = A \sin((t/T) - (x/vT))$

Question 6:

Solution:

(a) dimensions of A and a

$[A]=[L]$ and $[a]=[L]$

(b) wave velocity is v (given). So, the time period will be,

$T = \lambda/v$

Here $\lambda = a$
=> $T = a/v$

Therefore, $Y = \sin(x/\lambda - t/T)$

$= A \sin (x/a - vt/a)$

$= A \sin [(x-vt)/a]$

Question 7:

Solution:
The general equation:

$Y = A \sin(x/\lambda - t/T)$

Here $\lambda = a$ also $T = a/v$

$Y = A \sin(x/a - vt/a)$

$\Rightarrow A \sin(x/a) = A \sin(x/a + vt_0/a)$

To sustain equality, the equation must be,

$Y = A \sin(x/a - vt/a + vt_0/a)$

Question 8:

Solution:
The equation of a wave travelling on a string:

$y = (0.10 \text{ mm}) \sin[(31.4 \text{ m}^{-1}) x + (314 \text{ s}^{-1}) t]$.

We know, The structure of the equation is $y = A \sin(kx + \omega t)$

(a) Negative x -direction.

(b) $k = 31.4 \text{ m}^{-1}$

$\Rightarrow 2\pi/\lambda = 3.14$

or $\lambda = 20$ cm

Again, $\omega = 314\ s^{-1}$

=> $2\pi f = 314$

=> $f = 50\ sec^{-1}$

Therefore, wave speed = $v = \lambda f = 20 \times 50 = 1000$ cm/s

(c) Max displacement = 0.10 mm
Max. velocity = $a\omega = (0.1) \times 10^{-1} \times 314 = 3.14$ cm/sec

Question 9:

Solution:
Here, $\lambda = 2$ cm, $V = 2.0$ m/s and $A = 0.20$ cm

(a) Equation of wave along the x-axis

$y = A \sin(kx - wt)$

$k = 2\pi/\lambda = \pi\ cm^{-1}$

and $T = \lambda/v = 2/2000 = 10^{-3}$ sec

This implies, $\omega = 2\pi/T = 2\pi\ 10^{3}$ sec

So, the wave equation is,

$y = (0.2) \sin\pi\ x - (2\pi\ 10^{3})t$

(b) At x = 2 cm and t = 0

y = (0.2)sin(π/2) = 0

Therefore, particle velocity, v = rω cos(πx) = 0.2 x 2000π x cos 2π = 400π

= 400 x 3.14

= 4π m/s

Question 10:

Solution:
(a) T = 2x0.01 sec = 20 min
λ = 2x2 = 4 cm

(b) v = dy/dt = d/dt [sin2π(x/4 - t/0.02)]

= - cos 2π (x/4 - t/0.02) x 1/0.02

=> v = -50 cos 2π (x/4 - t/0.02)

At x = 1 and t = 0.01 sec, v = -50 cos 2(1/4-1/2) = 0

(c)
At x = 3 cm, t = 0.01 sec
v = -50 cos 2π(3/4 - 1/2) = 0

At x = 5 cm, t = 0.01 sec, v = 0

At x = 7 cm and t = 0.011 sec, v =0

At x = 1 cm, and t = 0.011 sec

v = -50 cos 2π[1/4 - (0.011/0.02)]

= -9.7 cm/sec

Question 11:

Solution:
frequency of vibration = f = 1/T = 50 Hz

Any two neighbouring mean positions always remain at half of the wave length, λ = 4 cm

Now, wave speed = v = λf = 2 m/s

Question 12:

Solution:

V = 200 m/s (given)

(a) amplitude = A = 1 mm

(b) the wavelength = λ= 4 cm

(c) the wave number = n = $2\pi/\lambda$ = 1.57 cm^{-1}

(d) the frequency of the wave = f = 1/T = $(26/\lambda)/20$ = 5 Hz
Where T = λ/v

Question 13:

Solution:

(a) v = λ/T

or λ = vt = 20 cm

(b) Phase shift difference = $(2\pi/\lambda)x$ = $2\pi/20 \times 10 = \pi$ rad

$y_1 = a \sin(\omega t - kx)$

=> $1.5 = a \sin(\omega t - kx)$

the displacement of a particle at x = 10 cm

$y_2 = a \sin(\omega t - kx + \pi)$

=> $-a \sin(\omega t - kx) = -1.5$ mm

Therefore, displacement of a particle is -1.5 mm

Question 14:

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Solution:

Mass = 5g, Length = 64cm and Force = 8 N (given)

So, density= ρ = (5/64) g/cm

Now, $F = \rho v^2$

or $v^2 = 8 \times (64\ cm/5g)$

or v = 32 m/s

Question 15:

Solution:

(a) velocity of wave = $v^2 = T/m = (16\times10^5)/0.4$

or v = 2000 cm/s

Time taken to reach other end = 20/2000 = 0.01 sec

Time taken to see the pulse again in the original position = 0.01 x 2 = 0.02 s

(b) At t = 0.01 s, there will be a though at the right end as it is reflected.

Question 16:

Solution:

(a) The distance travelled by the wave = 20+20 = 40 cm

time = t = x/v = 40/20 = 2 sec

(b) The string regains its original shape after completing a periodic distance i.e. (30+30) cm = 60cm.

time period = 60/20 = 3 sec

(c) frequency = n = (1/3) sec^{-1}

n = (1/2l)√(T/m)

m = mass per unit length = 0.5 g/cm

=> (1/3) = 1/(2x30) x √(T/0.5)

=> T = 2 x 10^{-3} N

Question 17:

Solution: Let v_1 and v_2 are the velocities of wires.
ρ_1 and ρ_2 are the respective densities.

Therefore, $T_1 = T_2$

=> $\rho_1 v_1^2 = \rho_2 v_2^2$

or $\rho_1/\rho_2 = v_2^2/v_1^2$

Given that, $V_1 = 2V_2$

$\rho_1/\rho_2 = v_2^2/4v_2^2 = 1/4$

or $\rho_1/\rho_2 = 0.25$

Question 18:

Solution: A transverse wave described by
$y = (0.02 \text{ m}) \sin(1.0 \text{ m}^{-1}) x + (30 \text{ s}^{-1})t]$

Speed = $v = \omega/k$

here, $\omega = 30 \text{ sec}^{-1}$ and $k = 1 \text{ m}^{-1}$

=> $v = 30$ m/s

But $T = \rho v^2$

$= 1.2 \times 90 \times 10^{-4}$ N

or $T = 0.108$ N

Question 19:

Solution:

Amplitude = A = 1 cm, tension = T = 90N, frequency = f = 200/2 = 100 Hz and mass=m = 0.1 kg/m

(a)

$v = \sqrt{(T/\rho)}$

$= \sqrt{(90/0.1)} = 30$ m/s

Again, $v = \lambda f = 30$ cm

(b) $y = 10 \cos 2\pi[x/30 - t/0.01]$

At t = 0 and x = 0, it has maxima, consists a phase of $\pi/2$

=> $y = (1) \sin[2\pi x/30 - 2\pi t/0.01 + \pi/2]$

the required equation is, $y = (1) \cos[2\pi x/30 - 2\pi t/0.01]$

(c) The velocity of the particle,

$y' = -(1) (2\pi/0.01) \sin[2\pi x/30 - 2\pi t/0.01]$

At x = 50 cm and t = 10 s

$y' = -5.4$ m/s

Now the acceleration is :

y'' = -(1) (2π/0.01)^2 cos[2πx/30 - 2πt/0.01]

At x = 50 cm and t = 10 s

y'' = 2 km/s^2

Question 20:

Solution:
Here l = 40 cm , spring constant = k = 160 N/m and mass = 10 g

Mass per unit length = m = 10/40 = (1/4) g/cm
Now, deflection = x = 1 cm = 0.01 m

=> T = kx = 1.6 N = 16 x 10^4 dyne

Also, v = √(T/m) = 8 x 10^2 cm/s = 800 cm/s

Therefore, time taken by the pulse to reach the spring:

t = 40/800 = 0.05 sec

Question 21:

Solution:

Force due to gravity on AB:
$T_{AB} = (3.2 + 3.2)9.8\ N = 62.72\ N$

Force due to gravity on CD:

$T_{AB} = 3.2 \times 9.8 = 31.36\ N$

The velocities are:

$v_{AB} = \sqrt{(T_{AB}/\rho_{AB})} = 79\ m/s$

and
$v_{CD} = \sqrt{(T_{CD}/\rho_{CD})} = 63\ m/s$

Question 22:

Solution:

Mass density = ρ = (0.0045/2.25) kg/m

force on the string = T = 20 N

Now, speed of the wave = v = $\sqrt{(T/\rho)}$ = 100 m/s

Therefore, time taken = t = 2.25/100 = 0.02 sec

Question 23:

Solution:

T = ma + mg = (4x2 + 4x10) = 48 N

And, speed = v = $\sqrt{(T/\rho)}$ = 50 m/s

Question 24:

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Solution:

Tension = T = mg

Speed = $v_1 = \sqrt{(mg/\rho)}$

At motion:

Tension = T = $m\sqrt{(a^2+g^2)}$ and

Speed = $v_2 = \sqrt{[(m\sqrt{(a^2 + g^2)})/\rho]}$

Now,

$$\frac{v_2}{v_1} = \frac{\sqrt{\frac{m\sqrt{a^2+g^2}}{\rho}}}{\sqrt{\frac{mg}{\rho}}}$$

$$\left(\frac{v_2}{v_1}\right)^4 = \frac{a^2+g^2}{g^2}$$

$$g^2 A = a^2 + g^2 \,;\; \left(\frac{v_2}{v_1}\right)^4 = A$$

or $(A - 1)g^2 = a^2$

$a^2 = 0.140 \times 100$

or $a = 3.74\ m/s^2$

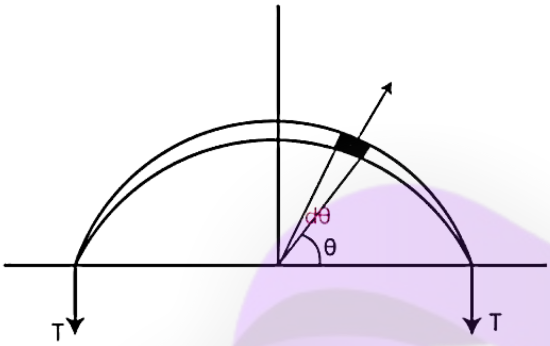
Question 25:

Solution:

R = Radius of the loop, m = mass per unit length of the string

ω = angular velocity, V = linear velocity of the string

The force for a small potion in the ring:
$dF = (mRd\theta)\omega^2 R$



$dF = 2\ (mRd\theta)\omega^2 R \sin\theta$

$$F = \int_0^{\frac{\pi}{2}} 2(m\ Rd\theta)\omega^2 R \sin\theta$$

or $F = 2mR^2\ \omega^2$

But whole of this process was for half of the ring:

$2T = 2mR^2\ \omega^2$

Or $T = mR^2\ \omega^2$

Now, velocity, $v = \sqrt{(T/m)} = R\omega$
Which is the speed of the disturbance.

Question 26:

Solution:

(a) Downward weight for the element = (mx)g = Tension in the string of upper part

velocity of transverse vibration = v = √(T/m) = √(mglm) = √gx

(b) For small displacement dx, dt = dx/√gx

Total time:

$$\int_0^T dt = \frac{1}{\sqrt{g}}\int_0^L \frac{dx}{\sqrt{x}}$$

= √(4L/g)

(c) Suppose, it will meet the pulse after y distance.
To get the in between time, we integrate,

$$\int_0^t dt = \frac{1}{\sqrt{g}}\int_0^y \frac{dx}{\sqrt{x}}$$

$$t = \sqrt{4\frac{y}{g}}$$

Therefore, distance travelled by the particle in this time is (L-y)

We know the relation, $S = ut + (1/2)gt^2$

=> L – y(1/2)g x $\sqrt{(4y/g)}^{\,2}$ at u = 0
=> L – y = 2y
=> y = L/3, which shows that, the particle meet at distance L/3 from the lower end.

Question 27:

Solution:

Suppose v and v' are wave speeds in string A and B respectively.
$T = 4.8$ and $m = 1.2 \times 10^{-2}$ and $T' = 7.5$

Now,
$v = \sqrt{(T/m)} = 20$ m/s and
$v' = \sqrt{(T'/m)} = 20$ m/s $= 25$ m/s

t = 0 in string A,

$t_1 = 0+20$ ms $= 20 \times 10^{-3} = 0.02$ sec

In 0.02 sec A has travelled 20 x 0.02 = 0.4 mt

relative speed between A and B = 25-20 = 5 m/s

Time taken for B for overtake A = s/v = 0.4/5 = 0.08 sec

Question 28:

Solution:

Average power of the source $= P = 2\pi^2 mva^2f^2$...(1)

$v = \sqrt{(T/m)} = 100$ m/s and $m = 0.01$ kg/m
$r = 0.5 \times 10^{-3}$ and $f = 100$
(Given)

On substituting the values, (1)=> P = 49 mW

Question 29:

Solution:

Here A = 1 mm = 10^{-3} mA, m = 6 g/m = 6 x 10^{-3} kg/m

F = 200 Hz and T = 60 N

(a) Average power of the source = P = $2\pi^2 mvA^2f^2$ = 0.47 W

(b) Length of string = 2m

And t = 2/100 = 0.02 sec

So, Energy = $2\pi^2 mvtA^2f^2$ = 9.46 mJ

Question 30:

Solution: Given, m = 0.01 kg/m, T = 49N, r = 0.5 x 10^{-3} m and f = 440 Hz

(a) Let the wavelength of the wave be λ.

Speed of the transverse wave = v = √(T/m) = √(49/0.01) = 70 m/s

Also, v = f/λ

or λ = f/v = 70/440 = 16 cm

(b) y = A sin(ωt - kx)

Therefore, v = dy/dt = Aω cos(ωt - kx)

Now,

v_{max} = (dy/dt) = Aw

= (0.50) x 10^{-3} x 2π x 440

= 1.381 m/s

and

$a = d^2y/dt^2$

=> $a = -A\omega^2 \sin(\omega t - kx)$

$a_{max} = -A\omega^2$

$= 0.50 \times 10^{-3} \times 4\pi^2 \times 440^2$

$= 3.8 \text{ km/s}^2$

(c) Average rate = $p = 2\pi^2 vA^2f^2$

$= 2 \times 10 \times 0.01 \times 70 \times (0.50\times10^{-3})^2 \times 440^2$

= 0.67 W

Question 31:

Solution: Consider equation of waves:

$y' = r \sin \omega t$ and $y'' = r \sin(\omega t + \pi/2)$

Now, $y = y' + y''$

$$y = r[2(\sin\frac{2\omega t + \frac{\pi}{2}}{2})(\cos - \frac{\pi}{\omega})]$$

Or $y = \sqrt{2}\, r \sin(\omega t + \pi/4)$

The amplitude is $4\sqrt{2}$ mm

Question 32:
Solution:

Distance travelled by any classical wave = s = vt
At t = 4 => s = 4 x 10^{-3} x 50 x 10 = 2 mm
At t = 6 => s = 6 x 10^{-3} x 50 x 10 = 3 mm
At t = 8 => s = 8 x 10^{-3} x 50 x 10 = 4 mm
At t = 12 => s = 12 x 10^{-3} x 50 x 10 = 6 mm

Question 33:

Solution:

(a) Wave speed = wave length × wave frequency

v = 100 x 0.02 = 2 m/s

In 0.015 sec, the path travelled by wave,

x = 0.015 x 2 = 0.03 m

The phase difference:

$\phi = (2\pi x)/\lambda = (2\pi)/0.02 \times 0.03 = 3\pi$

(b) Again, for path difference, x = 0.04 m

$\phi = 4\pi$

(c) Two waves have same amplitude if their frequency and wavelength are same.

Now, consider two wave equations, $y' = r \sin \omega t$ and $y'' = r \sin (\omega t + \phi)$

$\Rightarrow y = y' + y''$

$= 2r \sin (\omega t + \phi/2) \cos (\phi/2)$

Therefore, resultant amplitude, $A = 2r\cos(\phi/2)$

For $A = 0$, $\phi = 3\pi$

For $A = 4$, $\phi = 4\pi$

Question 34:

Solution:

Fundamental frequency = $f = v/2L = 30$ Hz

Question 35:

Solution:

Fundamental frequency = $f = v/2L = 1/2L \times \sqrt{(T/m)}$ = 1 g/m

Question 36:

Solution:

Fundamental frequency = f = v/2L = 1/2L x $\sqrt{(T/m)}$ = 62.5 Hz

frequency of 4th harmonic = F_4 = 4 x 62.5 = 250 Hz

Now, v = $F_4 \lambda_4$

or λ_4 = 250/v = 40 cm

Question 37:

Solution:

Fundamental frequency = f = (1/2L)$\sqrt{(T/m)}$

or f = $\sqrt{150\,T}$

Also, f = 261.63 Hz (given)

=> 261.63 = $\sqrt{150\,T}$

=> T = 1480 N (approx.)

Question 38:

Solution:

Fundamental frequency of First Harmonic = 256/2 = 128 Hz
Here, Second harmonic= 2 × First Harmonic

When the fundamental wave is produced, $\lambda/2 = 1.5$

=> $\lambda = 3$ m

speed of the wave = f λ = 128 x 3 = 384 m/s

Question 39:
Solution:
Mass of the wire = 12 gm
Length of the wire between two pulleys (L) = 1.5 m

so, Mass per unit length = m = (12/1.5) g/m = 8×10^{-3} kg/m

Tension in the wire = T = 9g = 90 N

Now, Fundamental frequency = f' = (1/2L) $\sqrt{(T/m)}$

Second harmonic = 2(First Harmonic)

=> f = 2 f' = (1/1.5) x $\sqrt{(90/8 \times 10^{-3})}$

= 70.7 Hz

Question 40:

Solution:

Using relation, $L = n\lambda/2$

Here $n = 4$ and $L = 1$ m

=> $\lambda = 0.5$

Also, $v = f\lambda = \sqrt{(T/m)}$

=> $T = f^2\lambda^2 m = 128^2 \times 0.5^2 \times 40 \times 10^{-3}$

= 164 N

Question 41:

Solution:

(a) Two frequencies are $v_1 = 240$ Hz and $v_2 = 320$ Hz

So, Maximum fundamental frequency = $v_2 - v_1$
= 320 − 240 = 80 Hz

(b) Given v = 40 m/s

=> L x 80 = 0.5 x 40

=> L = 0.25 m

Question 42:

Solution:
Let n be the frequency, L is length of the string and λ be the distance between two consecutive nodes.

Therefore, $L = n\lambda$

for next higher frequency, say (n+1) the distance between two consecutive nodes is λ', then

$L = (n+1)\lambda'$

Equating Equations, we get

$n\lambda = (n+1)\lambda'$

or $\lambda' = n(\lambda - \lambda')$

Here $\lambda = 2$ cm and $\lambda' = 1.6$ cm

On putting values,

$n = 4$

$\Rightarrow L = 4 \times 2 = 8$ cm

Question 43 :

Solution :

f = 660 Hz and v = 220 m/s

Wave length = λ = v/f = 1/3 m

(a) Number of loops = n =3

Therefore, L = (nλ)/2 = (3/2) x (1/3) = 1/2 m = 50 cm

(b) resultant wave equation

$$y = 2A\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi ft}{\lambda}$$

$$y = (0.5)\sin[(0.06\pi\ \text{cm}^{-1})x]\cos[(1320\pi\ \text{s}^{-1})t]$$

[On Substituting the values of λ,f and A]

Question 44:

Solution:

We know that, f a 1/l or f = v/l (where v = constant for a medium)

l_1 = 30 cm or 0.3 m (given)

f_1 = 196 Hz and f_2 = 220 Hz (given)

Now, $f_1/f_2 = l_2/l_1$

=> l_2 = 26.7 cm

Again, f_3 = 247 Hz

so, $f_3/f_1 = l_1/l_3 = 0.3/l_3$

=> l_3 = 0.224 m = 22.4 cm
in same way, we have l_4 = 20 cm

Question 45:

Solution:
Fundamental frequency = f_o =200 Hz
nth harmonic = f' = n × fundamental frequency

and frequency of the highest harmonic = 14 kHz = 14000 Hz

Now, f'/f_o = 14000/200

nf_o/f_o = 70

=> n = 70
The highest audible to man is 70th harmonic.

Question 46:

Solution:

(a) gcd of 90,150 and 210 is 30

So, f = 30 Hz

(b) Let f_1, f_2 and f_3 are Three resonant frequencies of the string.

$f_1 = 3f$, $f_2 = 5f$ and $f_3 = 7f$

(c) nth overtone is (n+1)th frequency.

So, 3f is 2nd overtone and 3rd harmonic.
5f is 4th overtone and 6th harmonic.
and 7f is 6th overtone and 7th harmonic.

(d) We know, $f_1 = (3/2)v$

so, 90 = (3/2x80) k

=> k = 48 m/s

Question 47:

Solution: The ratio of mass per unit length of the wires:

$\rho_1/\rho_2 \times r_1^2/r_2^2 = (1/2) \times (9/1) = 9/2$

Fundamental frequency of wire = $(1/2l)\sqrt{(T/\mu)}$

Thus, $f_1/f_2 = 2:3$

Question 48:

Solution:

We know, $f = (1/2L)\sqrt{(T/m)}$

Given, $f_d = 2f_2$

SO, $\sqrt{(T_1/T_2)} = 2$

Now, $T_1 + T_2 = 48 + 12 = 60$ N

By replacing the relations, $T_1 = 48$ N and $T_2 = 12$ N

Taking moment about a point, $T_2 \times 0.4 = 48x + 12(0.2)$
on solving above equation, we have x = 5 cm

Therefore, mass should be placed at a distance 5cm from the left end.

Question 49:

Solution:

Calculate the mass per unit length of aluminium and steel wire using given values.

We know, $\rho = M/V$

$M/l = \rho A$

Here, $m = M/l$

so, $m = \rho A$

For aluminium:

Put the value into the formula

$m_a = 2.6 \times 3 \times 10^{-2} = 7.8 \times 10^{-2}$ g/cm

For steel:

$m_s = 7.8 \times 10^{-2}$ g/cm

Now, $v = \sqrt{(T/m)}$

Here, T = 40 N and $m = 7.8 \times 10^{-2}$ g/cm

=> v = 71.6 m/s

For minimum frequency, there would be maximum wavelength. And, for maximum wavelength, minimum number of loops are to be produced.

Wavelength = $\lambda = 2d = 2 \times 20 = 40$ cm

The minimum frequency of a tuning fork :
$f = v/\lambda$
Given v = 71.6 m/s and λ = 0.4 m

=> f = 179 Hz

Question 50:

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Solution:
Let L be the length of string.

Velocity of wave = v = $\sqrt{(T/m)}$

(a) wavelength = λ = velocity/frequency

=> $\lambda = \sqrt{(T/m)} \times 1/[(1/2L)\sqrt{(T/m)}] = 2L$

Now, wave number = $k = 2\pi/\lambda$

$= 2\pi/2L = \pi/L$

(b) Equation of the stationary wave:

$y = A \cos(2\pi x/\lambda) \sin(2\pi Vt/\lambda)$

As, $v = V/2L$

=> $y = A \cos(\pi x/L) \sin(2\pi/vt)$

Question 51:

Solution:
(a) Vibrating in first overtone means, n=2

We know, $L = n\lambda/2$
here, $\lambda = L = 2$ m

Again, $f = v/\lambda = 100$ Hz

(b) Suppose, the stationary wave equation:

$y = 2A \cos(2\pi x/\lambda) \sin(2\pi vt/\lambda)$

$= 0.5 \cos(2\pi x/2) \sin(2\pi(200)t/\lambda)$

$= 0.5 \cos[(\pi m^{-1})x] \sin[(200)\pi s^{-1}t]$

Question 52:

Solution:
The stationary wave equation

$y = (0.4 \text{ cm}) \sin[(0.314 \text{ cm}^{-1})x] \cos[(600\pi \text{ s}^{-1})t]$

(a)frequency of vibration:
$\omega = 600\pi$

So, $2\pi f = 600\pi$

$\Rightarrow f = 300$ Hz

(b) positions of the nodes:
$\sin(0.314x) = 0$

$\Rightarrow 0.314x = k\pi = k \times 3.14$

$\Rightarrow x = 10$ K

Nodes are at 0, 10 cm, 20 cm and 30 cm.

(c) length of the string:

Length = $3\lambda/2 = 3 \times 10 = 30$ cm

(d) the wavelength and the speed of two travelling waves that can interface to give this vibration

wave equation = $y = (0.4 \text{ cm}) \sin[0.314 x] \cos(600\pi t)$

=> $\lambda = 20$ cm

So, $v = \omega/k = 6000$ m/s = 60 m/s

Question 53:

Solution: Equation of standing wave
$y = (0.4 \text{ cm}) \sin(0.314 \text{ cm}^{-1})x] \cos[(600\pi \text{ s}^{-1})t]$.

Here, $K = 0.314 = \pi/10$

We know, $\lambda = 2\pi/K = 20$ cm

Now, $L = (n\lambda/2)$

For smallest length, n = 1

=> $L = 20/2 = 10$ cm

Question 54:

Solution:

Strain = $\Delta l/l = 0.125 \times 10^{-2}$ and $f = 1/2L\ \sqrt{(T/m)}$

=> $T = 248.19$ N

Now, stress = Tension/Area = 248.19×10^{6}

Therefore, Young modulus = stress/strain = 19852×10^{8} N/m^{2}