Light Waves

Exercise Solutions

Question 1:
Solution:
We know, $c = v\lambda$

Where c = speed of light = 3×10^8 m/s^2
Minimum wavelength = λ_{min} = 400 nm
Associated frequency = v_{max} = $[3 \times 10^8]/[400 \times 10^{-9}]$ = 7.5×10^{14} Hz

Max wavelength = λ_{max} = 700 nm

and v_{min} = $[3 \times 10^8]/[700 \times 10^{-9}]$ = 4.3×10^{14} Hz

Range of the frequency = 4.3×10^{14} Hz to 7.5×10^{14} Hz

Question 2:

Solution:

(a) frequency = $v_{Na} = c/\lambda_{Na}$ = $[3 \times 10^8]/[589 \times 10^9]$ = 5.09×10^{14} Hz

(b) Wavelength of sodium light in water = $\lambda_{Na'} = \lambda_{Na}/\mu$ = 589/1.33 = 443 nm (approx)

(c)Frequency of light does not change, i.e. 5.09×10^{14} Hz

(d) speed of light = $c' = c/\mu$ = $[3 \times 10^8]/1.33$ = 2.25×10^8 m/s

Question 3:

Solution:
The speed of light in quartz = $c' = c/\mu$ = $[3 \times 10^8]/1.472$ = 2.04×10^8 m/s

Here, wavelength = 400nm

The speed of light in quartz = $[3x10^8]/1.452 = 2.07 \times 10^8$ m/s

Here, wavelength = 760nm

Question 4:

Solution:
Speed of light in a medium of refractive index "μ "

Here c' = 2.04×10^8 m/s

We know, c = speed of light = 3×10^8 m/s^2

Now, c' = c/μ

or μ = c/c' = $[3x10^8]/[2.4x10^8]$ = 1.25

Question 5:

Solution:
Distance between the slits = d = 1 cm
Distance between the slits and the screen= D = 1 m

Wavelength of the light = λ = $5x10^{-7}$ m

(a)
$w = D\lambda/d$

$w = [1x5x10^{-7}]/0.01 = 0.05$ mm

(b) Here w = 1 mm

So, $d = D\lambda/w = [1x5x10^{7}]/0.001 = 0.5$ mm

Question 6:

Solution:

The width of a fringe = w = 1mm = 0.001 m
Distance between the screen and the slit = D = 2.5 m
Separation between the slits = d = 10mm = 0.01 m

We know, $w = D\lambda/d$

$\Rightarrow \lambda = dw/D = [0.001 \times 0.01]/2.5 = 400$ nm

Question 7:

Solution:
Distance between the screen and the slit = D = 1 m
Separation between the slits = d = 1mm = 0.001 m
wavelength of light = $5x10^{-7}$ m

(a)
We know, $w = D\lambda/2d$

$=[1x5x10^{-7}]/[2x0.001] = 0.25$ mm

(b) width of one fringe = w = 0.5 mm

Number of such fringes present in 1 cm region = 10/0.5 = 20

Question 8:

Solution:

Distance between the screen and the slit = D = 200 m
Separation between the slits = d = 0.8 mm = 0.0008 m
wavelength of light = $589x10^{-9}$ m

We know, $w = D\lambda/d$

$= [200 \times 589x10^{-9}]/0.0008 = 1.47$ mm (approx)

Question 9:

Solution:

Separation between the slits = d = 2.0×10^{-3} m
wavelength of light = $500x10^{-9}$ m

We know, $d \sin\theta = \lambda$

For small angle, $\sin\theta = \theta$

Now, $\theta = [500 x10^{-9}]/[2x10^{-3}]$ rad = 0.0014 degree (approx.)

CLASS24

Question 10:

Solution:

Separation between the slits = d = 0.25 mm = 0.25×10^{-3} m
Distance between the screen and the slit = D = 150 cm = 1.5 m
wavelength of light = 500×10^{-9} m

We know, $w = D\lambda/d$

For first time occur, wavelength is 480 x10^-9 m

$w_1 = D\lambda/d = [1.5 \times 480 \times 10^{-9}]/[0.2 \times 10^{-3}]$

For wavelength = 600×10^{-9} m

$w_2 = [1.5 \times 600 \times 10^{-9}]/[0.2 \times 10^{-3}]$

The separation between these two bright fringes is = $w_2 - w_1$ = 0.72 mm

Question 11:

Solution:

The distance of nth fringe from the centre = $w = nD\lambda/d$

Let m^{th} violet fringe overlaps with the nth red fringe, then

$nD\lambda_v/d = nD\lambda_r/d$

=> $m/n = \lambda_r/\lambda_v = 700/400 = 7/4$

Question 12:

Solution:
Let Δx be the thickness of the plate.

$\mu \Delta x - \Delta x = \lambda/2$

=> $\Delta x = \lambda/[2(\mu-1)$

Question 13:

Solution:

(a) The optical path length in vacuum is "t" and that introduce due to the plate is "μt" .

change in optical path length = $\mu t - t$

(b) To have a dark fringe at the centre the pattern should shift by one half of a fringe.

$\mu t - t = \lambda/2$

=> $t = \lambda/[2(\mu-1]$

Question 14:
Solution:

Optical path difference = $\mu t - t$

If there was no film present, there is no path difference at the center. But due to the presence of the film, we have a path difference of λ.

For path difference λ we have 1 fringe shift.
For path difference $\mu t - t$ we have $(\mu t - t)/\lambda$ fringe shift.

$\Rightarrow \mu t - t = n\lambda$

$\Rightarrow n = [\mu t - t]/\lambda = [(1.45-1)0.02 \times 10^{-3}]/[620 \times 10^{-9}] = 14.5$ fringes (approx)

Question 15:

Solution:

The number of fringe shifted = $n = (\mu-1)t/\lambda$
Corresponding shift = Number of fringes shifted x fringe width

$= (\mu-1)t/\lambda \times \lambda D/d$

$= (\mu-1)tD/d$

When the distance between the screen and the slits is doubled
Fringe width = $\lambda(2D)/d$

Form above equations,

$(\mu-1)tD/d = \lambda(2D)/d$

$\Rightarrow \lambda = [(1.6-1) \times (1.964 \times 10^{-6})]/2 = 589.2$ nm

Question 16:

Solution:

(a) The fringe width = w = $D\lambda/d$

=> w = $[1 \times 590 \times 10^{-9}]/[0.12 \times 10^{-2}]$

= 4.9×10^{-4} m

(b) The optical path difference:

$\Delta x = \mu_m t - \mu_p t$

μ_m = refractive index of mica and μ_p = refractive index of polystyrene

$\Delta x = (1.58 - 1.55)5 \times 10^{-4}$

= 1.5×10^{-5} m

We know, $\Delta x = n\lambda$

Number of fringe shifts = n = $\Delta x/\lambda$

= $[1.5 \times 10^{-5}]/[590 \times 10^{-9}]$

= 25.42

There are 25 fringes and 0.42th of a fringe.

Therefore,

maximum on one side = 0.42w = 0.021 cm

another side = 0.58w = 0.028 cm
[using value of w]

Question 17:

Solution:

The change of path difference due to the two slabs = $(\mu_1 - \mu_2)t$

For having a minimum at P_0, the path difference should change by $\lambda/2$

=> $\lambda/2 = (\mu_1 - \mu_2)t$

=> $t = \lambda/[2(\mu_1 - \mu_2)]$

Question 18:

Solution:
I = original intensity of light and I' = intensity after passing from the paper.

$I' = (4/9)I$

=> $I'/I = 4/9$

Again, we know

$I'/I = a'^2/a^2$

Where a be the initial amplitude and that from the paper be a'.

=>
$I'/I = a'^2/a^2 = 4/9$

$a'/a = 2/3 = k$ (constant)

Therefore, $a' = 2k$ and $a = 3k$

For maximum amplitude = $a' + a = 5k$

For minimum amplitude = $a' - a = k$

Ratio of maximum intensity to minimum:

$I_{max}/I_{min} = (5k)^2/k^2 = 25$

(b) μ (refractive index) = 1.45
Wavelength of light(λ) = 600×10^{-9} m
$t = 0.02 \times 10^{-2}$
optical path difference = $(\mu-1)t = n\lambda$

using values, we get
=> n = 15

Question 19:

Solution:
Distance between the slits = d = 0.28 mm = 0.28×10^{-3} m
Distance between the slits and the screen= D = 48 cm = 0.48 m

Wavelength of the light = λ = 700×10^{-9} m

Refractive index of water= $\mu = 1.33$

We know, fringe width = $w = D\lambda/d$

As light is passing through water, its wavelength changes. So, the new wavelength is

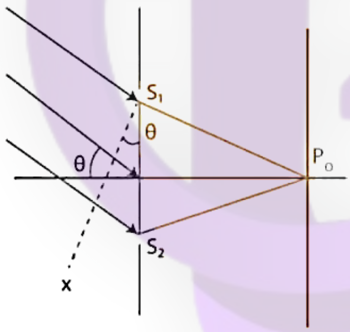
$\lambda' = \lambda/\mu = [700\times10^{-9}]/1.33 = 5.26\times10^{-7}$ m

Hence,
$w = D\lambda'/d = [0.48\times5.26\times10^{-7}]/[0.28 \times 10^{-3}]$

= 0.9 mm

Question 20:

Solution:
From figure, the wavefronts are making an angle with the normal to the slit passing through S_1 and S_2.



In right triangle M S_1 S_2, at M

$\sin\theta = MS_1/S_1S_2$

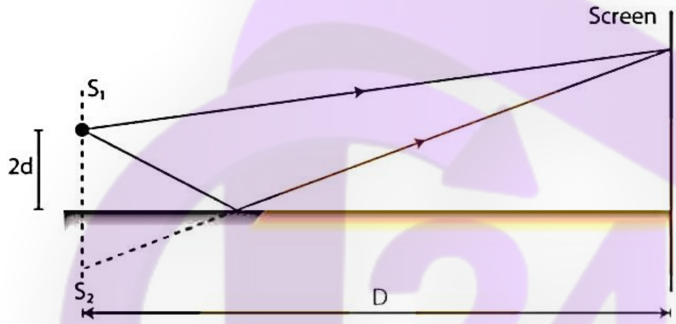
=> $MS_1 = S_1S_2 \sin\theta = d \sin\theta$

If the path difference is $\lambda/2$

Then $d \sin \theta = \lambda/2$

$\Rightarrow \theta = \sin^{-1}(\lambda/2d)$

Question 21:
Solution:



(a) There is a phase difference of π between direct light and reflecting light, the intensity just above the mirror will be zero.

(b) 2d = equivalent slot separation and D is the distance between slit and screen.

For bright fringe = $\Delta x = y(2d)/D = n\lambda$

As there is phase reverse of $\lambda/2$

=> $y(2d)/D + \lambda/2 = n\lambda$

=> $y = \lambda D/4d$

Question 22:

Solution:

separation between the slit = 2d = 2mm = 22×10^{-3} m

Now, fringe width = $w = D\lambda/d$

=> $w = [1 \times 700 \times 10^{-9}]/[2 \times 10^{-3}] = 0.35 \times 10^{-3}$ m = 0.35 mm

Question 23: Consider the situation of the previous problem. If the mirror reflects only 64% of the light energy falling on it, what will be the ratio of the maximum to the minimum intensity in the interference pattern observed on the screen?

Solution:

$I'/I = a'^2/a^2 = 64/100$

$a'/a = 4/5 = k$

Here, k is some constant.

Therefore, $a' = 4k$ and $a = 5k$.

Now,
For maximum amplitude = $a' + a = 9k$

For minimum amplitude = a' - a = k

Ratio of maximum intensity to minimum:

$I_{max}/I_{min} = (9k)^2/k^2 = 81/1$

Question 24:

Solution:

It's clear from the figure that, the apparent distance of the screen from the slits is $D = 2D_1 + D_2$.
Both rays will be shifted by pi and hence will form a normal interference pattern.

So, fringe width = $w = D\lambda/d$

$= [2D_1 + D_2]\ \lambda/d$

Question 25:

Solution:

Distance between the screen and the slit = D = 0.5 m

Separation between the slits = d = 0.5 mm = 0.0005 m

Let Δx be the path difference at a point y above the center on the screen

=> $\Delta x = yd/D$ ...(1)

Also, condition for minima: $\Delta x = (n + 1/2)\lambda$ .....(2)

From (1) and (2)

$yd/D = (n + 1/2)\lambda$

At y = 1 mm = 0.0001 m

Putting all the values, we have

=> $\lambda = [10^{-6}/(n + 1/2)]$ m

For n = 0 => λ = 2000 nm [Out of range]

For n = 1 => λ = 667 nm [in range]

For n = 2 => λ = 400 nm [in range]

For n = 3 => λ = 286 nm [Out of range]

(b) Δx = n λ [for maxima]

Δx = n λ = yd/D

=> λ = yd/Dn

For n = 1 => λ = 1000 nm [in range]

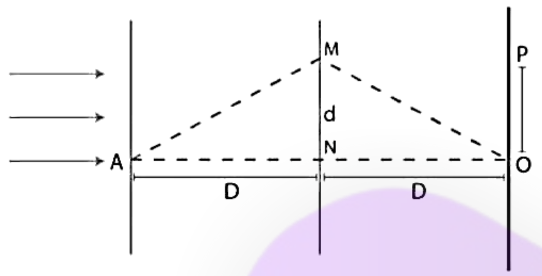
For n = 2 => λ = 500 nm [in range]

For n = 3 => λ = 333 nm [Out of range]

Hence maximum intensity would be for λ = 500 nm

Question 26:

Solution:



Form figure,
Δx = AMO - ANO

Here AM = MO = $\sqrt{[D^2 + d^2]}$ AND AN = NO = D

=> $\Delta x = 2(AM - AN) = 2\{\sqrt{[D^2 + d^2]} - D\}$

For minima at O,

$2\{\sqrt{[D^2 + d^2]} - D\} = (n + (1/2))\lambda$

Solving above equation for d, we get

$d = \sqrt{[D\lambda/2]}$

(b) width of the dark fringe = $w = D\lambda/d$

Now, the location x is given by

$x = D\lambda/[2\sqrt{(D\lambda/2)}]$

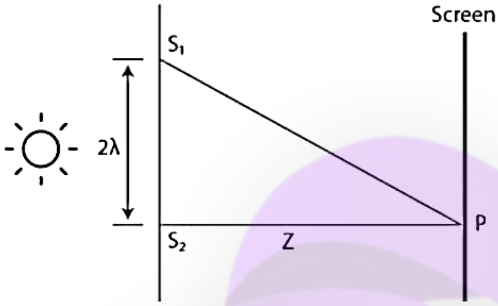
=> $x = d$

(c) As $x = w/2$

=> $w = 2x = 2d$

Question 27:

Solution:



For minimum intensity

$S_1P - S_2P = x = (2n+1)\lambda/2$

Form diagram,

$\sqrt{[Z^2 + (2\lambda)^2]} - Z = (2n+1)\lambda/2$

Taking square both the sides and solving above equation, we have

$Z = [16\lambda^2 - (2n+1)^2\lambda^2]/[4(2n+1)\lambda]$

Now,

For n = -1 => $Z = -15\lambda/4$

For n = 0 => $Z = 15\lambda/4$

For n = 1 => $Z = 7\lambda/12$ and

For n = 2 => $Z = -9\lambda/20$

Therefore, $Z = 7\lambda/12$ is the smallest distance for which there will be minimum intensity.

Question 28:

Solution:

(a) $BP_0 - AP_0 = \lambda/3$

Form diagram,

$BP_0 - AP_0 = \sqrt{D^2+d^2} - D = \lambda/3$

Solving above equation,

=> $d = \sqrt{2D\lambda/3}$

[Hint: Ignore $\lambda^2/9$ from the above expression, as it is small value]

(b) Here $d/2 + d = 3d/2$, be the distance of P_0 from the line in the middle of B and C.

Path difference between waves coming from B and C : $3d/2 \times d/D = \lambda$
[Using value of d from part (a)]

Here $2\pi/3$ is the phase difference of the wave coming from A.

If "a" be the amplitude from each slit, then contribution from B and C is 2a.

Therefore, resultant intensity taking in consideration the phase difference:

$A^2 = (2a)^2 + a^2 + 2a^2 \cos(2\pi/3) = 5a^2 - 2a^2 = 3a^2$

Now, the ratio of total intensity by individual intensity :

$I_{total}/I = A^2/a^2 = 3$

=> $I_{total} = 3I$. (Hence proved)

Question 29:

Solution:

Distance between the screen and the slit = D = 2 m
Separation between the slits = d = 2 mm = 2×10^{-3} m
Wavelength of light = λ = 600×10^{-9} m

from question, $I_m/I = 4a^2/^2$

=> $I = I_m/4$

At y = 0.5 cm = 0.005 m, path difference = Δx = yd/D

=> Δx = [0.005 x 0.002]/2 = 5×10^{-6} m

The phase difference:

$\phi = 2\pi\Delta x/\lambda = 50\pi/3 = 2\pi/3$

Now, the resultant amplitude, A is:

$A^2 = a^2 + a^2 + 2a^2 \cos(2\pi/3) = a^2$

=> A = a

Let the intensity of the resulting wave at point 0.5 cm be I.

=> $I/I\_max = A^2/(2a)^2$

=> I/0.2 = 1/4

=> I = 0.2/4 = 0.05 W/m^2 .

Question 30:

Solution:
Let I_{max} = maximum intensity and
I = intensity at y.

(a) $I_{max}/I = 2/1$

$$\frac{4a^2}{4a^2 cos^2[\phi/2]} = \frac{2}{1}$$

$\cos^2[\phi/2] = 1/2$

=> $\phi = \pi/2$

The path difference:
$\Delta x = \lambda\phi/2\pi = \lambda/4$

=> $y = \Delta x D/d = \lambda D/4d$

(b) when intensity is (1/4) times the maximum:

$I_{max}/I = 4/1$

$$\frac{4a^2}{4a^2 cos^2[\phi/2]} = \frac{4}{1}$$

$\cos^2 [\phi/2] = 1/4$

=> $\phi = 2\pi/3$

The path difference:
$\Delta x = \lambda\phi/2\pi = \lambda/3$

=> $y = \Delta x D/d = \lambda D/3d$