

Dispersion and Spectra

Exercise Solutions

Question 1:

Solution:

Refractive index of flint glass = $\mu_f = 1.620$

Refracting angle of flint prism = $A_f = 6.0^\circ$

Refractive index of crown glass = $\mu_c = 1.518$

Now,

For zero net deviation of mean ray:

$$(\mu_c - 1)A_c = (\mu_f - 1)A_f$$

$$\Rightarrow A_c = [(\mu_f - 1)/(\mu_c - 1)]A_f$$

$$= [1.620 - 1]/[1.518 - 1] \times 6^\circ$$

$$= 7.2^\circ$$

Question 2:

Solution:

(a) Dispersive power = $\omega = [\mu_v - \mu_r]/[\mu_y - 1]$

Where μ_r = Refractive index of red light = 1.56

μ_y = Refractive index of yellow light = 1.60 and

μ_v = Refractive index of violet light = 1.68

$$\Rightarrow \omega = 0.2$$

(b) Angular dispersion = $\delta = [\mu_v - \mu_r]A$

Here, Refracting angle of prism = $A = 6.0^\circ$

$$\Rightarrow \delta = (1.68 - 1.56)60^\circ = 7.2^\circ$$

Question 3:**Solution:**

The focal length of a lens is given by

$$1/f = (\mu - 1)[1/R_1 - R_2]$$

$$(\mu - 1) = \frac{1}{f} \frac{1}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$$\text{Let } k = 1/\left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\text{Then, } \mu - 1 = k/f$$

$$\text{So, } \mu_r - 1 = k/100 \dots(1)$$

$$\mu_v - 1 = k/98 \dots\dots(2) \text{ and}$$

$$\mu_y - 1 = k/96 \dots\dots(3)$$

Where, μ_r = Refractive index for the red color

μ_v = Refractive index for the violet color

μ_y = Refractive index for the yellow color

Now,

$$\text{Dispersive power} = \omega = [\mu_v - \mu_r]/[\mu_y - 1]$$

$$= [(\mu_v - \mu_r) - (\mu_r - 1)]/[\mu_y - 1]$$

$$= [k/96 - k/100]/[k/98]$$

$$\Rightarrow \omega = 0.0408$$

Question 4:**Solution:**

$$\mu_v - \mu_r = 0.014 \text{ [given]}$$

$$\mu_y = [\text{Real depth}]/[\text{Apparent depth}] = 2/1.30 = 1.515$$

$$\text{Dispersive power} = \omega = [\mu_v - \mu_r]/[\mu_y - 1]$$

$$= 0.014/[1.515 - 1]$$

$$= 0.027$$

Question 5:**Solution:**

$$\text{Dispersive power} = \omega = [\mu_v - \mu_r]/[\mu_y - 1]$$

$$\text{Here, } \mu_v = 1.65, \mu_r = 1.61, \omega = 0.07 \text{ and } \delta_y = 4^\circ$$

$$\Rightarrow 0.07 = [1.65 - 1.61]/[\mu_y - 1]$$

$$\Rightarrow \mu_y - 1 = 0.04/0.07 = 4/7$$

$$\text{Again, } \delta = (\mu - 1)A$$

$$\Rightarrow A = \delta_y/[\mu_y - 1] = 4/(4/7) = 7^\circ$$

Question 6:

Solution:

Minimum Deviations by Red = $\delta_r = 38.4^\circ$

Minimum Deviations by Yellow = $\delta_y = 38.7^\circ$

Minimum Deviations by Violet = $\delta_v = 39.2^\circ$

$$\text{Dispersive power} = \omega = [\mu_v - \mu_r] / [\mu_y - 1]$$

We know, $\delta = (\mu - 1)A$

$$\omega = [\mu_v - \mu_r] / [\mu_y - 1] = [(\mu_v - 1) - (\mu_r - 1)] / [\mu_y - 1]$$

$$= [(\delta_v/A) - (\delta_r/A)] / (\delta_v/A)$$

$$= [\delta_v - \delta_r] / \delta_v$$

$$= [39.2 - 38.4] / 38.7$$

$$= 0.0204$$

Question 7:

Solution: Two prisms of identical geometrical shape are combined.

Let A = Angle of the prisms.

$$\delta = (\mu_{v1} - 1)A - (\mu_{v2} - 1)A$$

Where, Refractive index of violet light through first prism = $\mu_{v1} = 1.52$

Refractive index of violet light from second prism = $\mu_{v2} = 1.62$

Deviation of violet light through prism = $\delta = 1^\circ$

$$\text{Or, } \delta = (\mu_{v1} - \mu_{v2})A$$

$$\Rightarrow A = \delta / (\mu_{v1} - \mu_{v2})$$

On substituting the values, we get

$$A = 10^\circ$$

Question 8:

, and (b) there is no net deviation in the yellow ray.

Solution:

Total deviation for yellow ray produced by the prism combination:

$$\delta_v - \delta_r = \delta_{cy} - \delta_{fy} + \delta_{cy} = 2\delta_{cy} - \delta_{fy} = 2(\mu_{cy} - 1)A - (\mu_{fy} - 1)A'$$

Similarly, angular dispersion produced by the combination:

$$\delta_v - \delta_r = 2(\mu_{vc} - 1)A - (\mu_{vf} - 1)A'$$

(a) for net angular dispersion to be zero, $\delta_v - \delta_r = 0$

$$\Rightarrow 2(\mu_{vc} - 1)A - (\mu_{vf} - 1)A' = 0$$

$$\Rightarrow 2(\mu_{vc} - 1)A = (\mu_{vf} - 1)A'$$

$$\Rightarrow A'/A = [2(\mu_{vc} - 1)] / [(\mu_{vf} - 1)]$$

(b) For net deviation in the yellow ray to be zero, $\delta_y = 0$

$$2(\mu_{cy} - 1)A - (\mu_{fy} - 1)A' = 0$$

$$\Rightarrow A'/A = [2(\mu_{cy} - 1)] / [\mu_{fy} - 1]$$

Question 9:

Solution:

A thin prism of crown glass ($\mu_r = 1.515$, $\mu_v = 1.525$) and a thin prism of flint glass ($\mu_r = 1.612$, $\mu_v = 1.632$) are placed in contact with each other.

Since, they are similarly directed, the total deviation produced

$$\delta = \delta_c + \delta_f = (\mu_c - 1)A + (\mu_f - 1)A$$

$$= (\mu_c + \mu_f - 2)A$$

So, angular dispersion of the combination:

$$\delta_v - \delta_r = (\mu_{cv} + \mu_{fv} - 2)A - (\mu_{cr} + \mu_{fr})A$$

$$= (\mu_{cv} + \mu_{fv} - \mu_{cr} - \mu_{fr})A$$

$$= (1.525 + 1.632 - 1.515 - 1.612)A$$

$$= 0.15^\circ$$

Question 10:**Solution:**

Here, For first prism:

$$A_1 = 60, \omega_1 = 0.07, \mu_1 = 1.50$$

For Second Prism

$$A_2 = ?, \omega_2 = 0.08 \text{ and } \mu_2 = 1.60$$

The combination produces no deviation in the mean ray.

$$(a) \delta = (\mu_2 - 1)A_2 - (\mu_1 - 1)A_1 = 0$$

$$\Rightarrow (1.6 - 1)A_2 - (1.5 - 1)6 = 0$$

$$\Rightarrow A_2 = 5^\circ$$

(b) When a beam of white light passes through it,

$$\text{Net angular dispersion} = (\mu_2 - 1)\omega_2 A_2 - (\mu_1 - 1)\omega_1 A_1$$

$$= (1.60 - 1)5^\circ - (1.60 - 1)5^\circ$$

$$= 0.03^\circ$$

(c) If the prisms are similarly directed.

$$\delta = (\mu_2 - 1)A_2 - (\mu_1 - 1)A_1 = 0$$

$$= (1.60 - 1)5^\circ - (1.50 - 1)6^\circ$$

$$\Rightarrow \delta = 6^\circ$$

(d) Similarly, if the prisms are similarly directed,

Net angular dispersion :

$$\mu_v - \mu_r = (\mu_2 - 1)\omega_2 A_2 - (\mu_1 - 1)\omega_1 A_1$$

$$= (1.6 - 1) \times 0.08 \times 5 - (1.50 - 1) \times 0.07 \times 6$$

$$= 0.45^\circ$$

Question 11:

Solution:

Refractive index of a material M1 changes = $(\mu_{v1} - \mu_{r1}) = 0.014$

Refractive index of a material M2 changes = $(\mu_{v2} - \mu_{r2}) = 0.024$

Prism angle of a material M₁ = A₁ = 5.3°

Prism angle of a material M₂ = A₂ = 3.7°

(a) When the prisms are oppositely directed,

Angular dispersion = $\delta = (\mu_{v2} - \mu_{r2})A_2 - (\mu_{v1} - \mu_{r1})A_1$

$$\Rightarrow \delta = (0.024) \times 3.7^\circ - (0.014) \times 5.3^\circ = 0.0146^\circ$$

(b) When they are similarly directed,

Angular dispersion = $\delta = (\mu_{v2} - \mu_{r2})A_2 + (\mu_{v1} - \mu_{r1})A_1$

$$\Rightarrow \delta = (0.024) \times 3.7^\circ + (0.014) \times 5.3^\circ = 0.0163^\circ$$