Chapter : 1. RELATION

CLASS24

Exercise : 1A

Question: 1

Solution:

dom (R) = {-1, 1, -2, 2} and range (R) = {1, 4}

Question: 2

Solution:

range (R) = {8 27}

Question: 3

Solution:

(i) R = {(2, 8),(3, 27),(5, 125), (7, 343)}

(ii) dom (R) = {2, 3, 5, 7}

(iii) range (R) = {8, 27, 125, 343}

Question: 4

Solution:

{3, 2, 1}

Question: 5

Solution:

dom (R) = {3, 6, 9} and range (R) = {3, 2, 1}

Question: 6

Solution:

dom (R) = {-2, -1, 0, 1, 2} and range (R) = {3, 2, 1, 0}

Question: 7

Let $(R) = \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$

Question: 8

Solution:

dom (R) = {1, 2, 3} and range (R) = {6, 7, 8}

Question: 9

Solution:

Let R = {(A, B) : A ⊂ B)}, i.e., A is a proper subset of B, be a relation defined on S.

Now,

Any set is a subset of itself, but not a proper subset.

⇒ (A,A) ∉ R ∀ A ∈ S

⇒ R is not reflexive.

Let (A,B) ∈ R ∀ A, B ∈ S

⇒ A is a proper subset of B

⇒ all elements of A are in B, but B contains at least one element that is not in A.

⇒ B cannot be a proper subset of A

⇒ (B,A) ∉ R

For e.g. , if B = {1,2,5} then A = {1,5} is a proper subset of B . we observe that B is not a proper subset of A.

⇒ R is not symmetric

Let (A,B) ∈ R and (B,C) ∈ R ∀ A, B,C ∈ S

⇒ A is a proper subset of B and B is a proper subset of C

⇒ A is a proper subset of C

⇒ (A,C) ∈ R

For e.g. , if B = {1,2,5} then A = {1,5} is a proper subset of B .

And if C = {1,2,5,7} then B = {1,2,5} is a proper subset of C.

We observe that A = {1,5} is a proper subset of C also.

⇒ R is transitive.

Thus, R is transitive but not reflexive and not symmetric.

Question: 10

Solution:

In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that, A be the set of all points in a plane and O be the origin. Then, R = {(P, Q) : P, Q ∈ A and OP = OQ)}

Now,

R is Reflexive if (P,P) ∈ R ∀ P ∈ A

∀ P ∈ A , we have

OP=OP

⇒ (P,P) ∈ R

Thus, R is reflexive.

R is Symmetric if (P,Q) ∈ R ⇒ (Q,P) ∈ R ∀ P, Q ∈ A

Let P, Q ∈ A such that,

(P,Q) ∈ R

⇒ OP = OQ

⇒ OQ = OP

⇒ (Q,P) ∈ R

Thus, R is symmetric.

R is Transitive if (P,Q) ∈ R and (Q,S) ∈ R ⇒ (P,S) ∈ R ∀ P, Q, S ∈ A

Let (P,Q) ∈ R and (Q,S) ∈ R ∀ P, Q, S ∈ A

⇒ OP = OQ and OQ = OS

⇒ OP = OS

⇒ (P,S) ∈ R

Thus, R is transitive.

Since R is reflexive, symmetric and transitive it is an equivalence relation on A.

Question: 11

Solution:

Let R = {(a, b) : a ≤ b} be a relation defined on S.

Now,

We observe that any element x ∈ S is less than or equal to itself.

⇒ (x,x) ∈ R ∀ x ∈ S

⇒ R is reflexive.

Let (x,y) ∈ R ∀ x, y ∈ S

⇒ x is less than or equal to y

But y cannot be less than or equal to x if x is less than or equal to y.

⇒ (y,x) ∉ R

For e.g. , we observe that (2,5) ∈ R i.e. 2 < 5 but 5 is not less than or equal to 2 ⇒ (5,2) ∉ R

⇒ R is not symmetric

Let (x,y) ∈ R and (y,z) ∈ R ∀ x, y ,z ∈ S

⇒ x ≤ y and y ≤ z

⇒ x ≤ z

⇒ (x,z) ∈ R

For e.g. , we observe that

(4,5) ∈ R ⇒ 4 ≤ 5 and (5,6) ∈ R ⇒ 5 ≤ 6

And we know that 4 ≤ 6 ∴ (4,6) ∈ R

⇒ R is transitive.

Thus, R is reflexive and transitive but not symmetric.

Question: 12

Solution:

Given that,

A = {1, 2, 3, 4, 5, 6} and R = {(a, b) : a, b ∈ A and b = a + 1}.

∴ R = {(1,2),(2,3),(3,4),(4,5),(5,6)}

Now,

R is Reflexive if $(a,a) \in R \ \forall \ a \in A$

Since, $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \notin R$

Thus, R is not reflexive .

R is Symmetric if $(a,b) \in R \Rightarrow (b,a) \in R \ \forall \ a,b \in A$

We observe that $(1,2) \in R$ but $(2,1) \notin R$.

Thus, R is not symmetric .

R is Transitive if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R \ \forall \ a,b,c \in A$

We observe that $(1,2) \in R$ and $(2,3) \in R$ but $(1,3) \notin R$

Thus, R is not transitive.

Exercise : 1B

Question: 1

Solution:

Relation: Let A and B be two sets. Then a relation R from set A to set B is a subset of A x B. Thus, R is a relation to A to B $\Leftrightarrow R \subseteq A \times B$.

If R is a relation from a non-void set B and if $(a,b) \in R$, then we write a R b which is read as 'a is related to b by the relation R'. if $(a,b) \notin R$, then we write a R b, and we say that a is not related to b by the relation R.

Domain: Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R.

Thus, domain of $R = \{a : (a,b) \in R\}$. The domain of $R \subseteq A$.

Range: let R be a relation from a set A to a set B. then the set of all second component or coordinates of the ordered pairs belonging to R is called the range of R.

Example 1: $R = \{(-1, 1), (1, 1), (-2, 4), (2, 4)\}$.

dom (R) = $\{-1, 1, -2, 2\}$ and range (R) = $\{1, 4\}$

Example 2: $R = \{(a, b): a, b \in N \text{ and } a + 3b = 12\}$

dom (R) = $\{3, 6, 9\}$ and range (R) = $\{3, 2, 1\}$

Question: 2

Solution:

Let $R = \{(\Delta_1, \Delta_2) : \Delta_1 \sim \Delta_2\}$ be a relation defined on A.

Now,

R is Reflexive if $(\Delta, \Delta) \in R \ \forall \ \Delta \in A$

We observe that for each $\Delta \in A$ we have,

$\Delta \sim \Delta$ since, every triangle is similar to itself.

$\Rightarrow (\Delta, \Delta) \in R \ \forall \ \Delta \in A$

$\Rightarrow$ R is reflexive.

R is Symmetric if $(\Delta_1, \Delta_2) \in R \Rightarrow (\Delta_2, \Delta_1) \in R \ \forall \ \Delta_1, \Delta_2 \in A$

Let $(\Delta_1, \Delta_2) \in R \ \forall \ \Delta_1, \Delta_2 \in A$

$\Rightarrow \Delta_1 \sim \Delta_2$

$\Rightarrow \Delta_2 \sim \Delta_1$

$\Rightarrow (\Delta_2, \Delta_1) \in R$

$\Rightarrow$ R is symmetric

R is Transitive if $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R \Rightarrow (\Delta_1, \Delta_3) \in R \ \forall \ \Delta_1, \Delta_2, \Delta_3 \in A$

Let $(\Delta_1, \Delta_2) \in R$ and $((\Delta_2, \Delta_3) \in R \ \forall \ \Delta_1, \Delta_2, \Delta_3 \in A$

$\Rightarrow \Delta_1 \sim \Delta_2$ and $\Delta_2 \sim \Delta_3$

$\Rightarrow \Delta_1 \sim \Delta_3$

$\Rightarrow (\Delta_1, \Delta_3) \in R$

$\Rightarrow$ R is transitive.

Since R is reflexive, symmetric and transitive, it is an equivalence relation on A.

Question: 3

Solution:

In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that, $\forall$ a, b $\in$ Z, R = {(a, b) : (a + b) is even }.

Now,

R is Reflexive if (a,a) $\in$ R $\forall$ a $\in$ Z

For any a $\in$ A, we have

a+a = 2a, which is even.

$\Rightarrow$ (a,a) $\in$ R

Thus, R is reflexive.

R is Symmetric if (a,b) $\in$ R $\Rightarrow$ (b,a) $\in$ R $\forall$ a,b $\in$ Z

(a,b) $\in$ R

$\Rightarrow$ a+b is even.

$\Rightarrow$ b+a is even.

$\Rightarrow$ (b,a) $\in$ R

Thus, R is symmetric .

R is Transitive if (a,b) $\in$ R and (b,c) $\in$ R $\Rightarrow$ (a,c) $\in$ R $\forall$ a,b,c $\in$ Z

Let (a,b) $\in$ R and (b,c) $\in$ R $\forall$ a, b,c $\in$ Z

$\Rightarrow$ a+b = 2P and b+c = 2Q

Adding both, we get

a+c+2b = 2(P+Q)

$\Rightarrow$ a+c = 2(P+Q)-2b

$\Rightarrow$ a+c is an even number

$\Rightarrow$ (a, c) $\in$ R

Thus, R is transitive on Z.

Since R is reflexive, symmetric and transitive it is an equivalence relation on Z.

Question: 4

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Solution:

In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that, ∀ a, b ∈Z, aRb if and only if a – b is divisible by 5.

Now,

R is Reflexive if (a,a) ∈ R ∀ a ∈ Z

aRa ⇒ (a-a) is divisible by 5.

a-a = 0 = 0 × 5 [since 0 is multiple of 5 it is divisible by 5]

⇒ a-a is divisible by 5

⇒ (a,a) ∈ R

Thus, R is reflexive on Z.

R is Symmetric if (a,b) ∈ R ⇒ (b,a) ∈ R ∀ a,b ∈ Z

(a,b) ∈ R ⇒ (a-b) is divisible by 5

⇒ (a-b) = 5z for some z ∈ Z

⇒ -(b-a) = 5z

⇒ b-a = 5(-z) [∵ z ∈ Z ⇒ -z ∈ Z]

⇒ (b-a) is divisible by 5

⇒ (b,a) ∈ R

Thus, R is symmetric on Z.

R is Transitive if (a,b) ∈ R and (b,c) ∈ R ⇒ (a,c) ∈ R ∀ a,b,c ∈ Z

(a,b) ∈ R ⇒ (a-b) is divisible by 5

⇒ a-b = $5z_1$ for some z_1∈ Z

(b,c) ∈ R ⇒ (b-c) is divisible by 5

⇒ b-c = $5z_2$ for some z_2∈ Z

Now,

a-b = $5z_1$ and b-c = $5z_2$

⇒ (a-b) + (b-c) = $5z_1 + 5z_2$

⇒ a-c = $5(z_1 + z_2) = 5z_3$ where $z_1 + z_2 = z_3$

⇒ a-c = $5z_3$ [∵ z_1,z_2 ∈ Z ⇒ z_3∈ Z]

⇒ (a-c) is divisible by 5.

⇒ (a, c) ∈ R

Thus, R is transitive on Z.

Since R is reflexive, symmetric and transitive it is an equivalence relation on Z.

Question: 5

Solution:

In order to show R is an equivalence relation we need to show R is Reflexive, Symmetric and

Transitive.

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Given that, ∀ a, b ∈A, R = {(a, b) : |a – b| is even}.

Now,

R is Reflexive if (a,a) ∈ R ∀ a ∈ A

For any a ∈ A, we have

|a-a| = 0, which is even.

⇒ (a,a) ∈ R

Thus, R is reflexive.

R is Symmetric if (a,b) ∈ R ⇒ (b,a) ∈ R ∀ a,b ∈ A

(a,b) ∈ R

⇒ |a-b| is even.

⇒ |b-a| is even.

⇒ (b,a) ∈ R

Thus, R is symmetric .

R is Transitive if (a,b) ∈ R and (b,c) ∈ R ⇒ (a,c) ∈ R ∀ a,b,c ∈ A

Let (a,b) ∈ R and (b,c) ∈ R ∀ a, b,c ∈ A

⇒ |a – b| is even and |b – c| is even

⇒ (a and b both are even or both odd) and (b and c both are even or both odd)

Now two cases arise:

Case 1 : when b is even

Let (a,b) ∈ R and (b,c) ∈ R

⇒ |a – b| is even and |b – c| is even

⇒ a is even and c is even [∵ b is even]

⇒ |a – c| is even [∵ difference of any two even natural numbers is even]

⇒ (a, c) ∈ R

Case 2 : when b is odd

Let (a,b) ∈ R and (b,c) ∈ R

⇒ |a – b| is even and |b – c| is even

⇒ a is odd and c is odd [∵ b is odd]

⇒ |a – c| is even [∵ difference of any two odd natural numbers is even]

⇒ (a, c) ∈ R

Thus, R is transitive on Z.

Since R is reflexive, symmetric and transitive it is an equivalence relation on Z.

Question: 6

Solution:

In order to show R is an equivalence relation we need to show R is Reflexive, Symmetric and Transitive.

Given that, R be the relation in N ×N defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in

N ×N.

<u>R is Reflexive if (a, b) R (a, b) for (a, b) in N ×N</u>

Let (a,b) R (a,b)

⇒ a+b = b+a

which is true since addition is commutative on N.

⇒ R is reflexive.

<u>R is Symmetric if (a,b) R (c,d) ⇒ (c,d) R (a,b) for (a, b), (c, d) in N ×N</u>

Let (a,b) R (c,d)

⇒ a+d = b+c

⇒ b+c = a+d

⇒ c+b = d+a [since addition is commutative on N]

⇒ (c,d) R (a,b)

⇒ R is symmetric.

<u>R is Transitive if (a,b) R (c,d) and (c,d) R (e,f) ⇒ (a,b) R (e,f) for (a, b), (c, d),(e,f) in N ×N</u>

Let (a,b) R (c,d) and (c,d) R (e,f)

⇒ a+d = b+c and c+f = d+e

⇒ (a+d) – (d+e) = (b+c) – (c+f)

⇒ a-e= b-f

⇒ a+f = b+e

⇒ (a,b) R (e,f)

⇒ R is transitive.

Hence, R is an equivalence relation.

Question: 7

Solution:

In order to show R is an equivalence relation we need to show R is Reflexive, Symmetric and Transitive.

Given that, ∀ a, b ∈ S, R = {(a, b) : a = ± b }

Now,

<u>R is Reflexive if (a,a) ∈ R ∀ a ∈ S</u>

For any a ∈ S, we have

a = ±a

⇒ (a,a) ∈ R

Thus, R is reflexive.

<u>R is Symmetric if (a,b) ∈ R ⇒ (b,a) ∈ R ∀ a,b ∈ S</u>

(a,b) ∈ R

⇒ a = ± b

⇒ b = ± a

⇒ (b,a) ∈ R

Thus, R is symmetric .

R is Transitive if (a,b) ∈ R and (b,c) ∈ R ⇒ (a,c) ∈ R ∀ a,b,c ∈ S

Let (a,b) ∈ R and (b,c) ∈ R ∀ a, b,c ∈ S

⇒ a = ± b and b = ± c

⇒ a = ± c

⇒ (a, c) ∈ R

Thus, R is transitive.

Hence, R is an equivalence relation.

Question: 8

Solution:

Given that, ∀ A, B ∈ S, R = {(A, B) : d(A, B) < 2 units}.

Now,

R is Reflexive if (A,A) ∈ R ∀ A ∈ S

For any A ∈ S, we have

d(A,A) = 0, which is less than 2 units

⇒ (A,A) ∈ R

Thus, R is reflexive.

R is Symmetric if (A, B) ∈ R ⇒ (B,A) ∈ R ∀ A,B ∈ S

(A, B) ∈ R

⇒ d(A, B) < 2 units

⇒ d(B, A) < 2 units

⇒ (B,A) ∈ R

Thus, R is symmetric.

R is Transitive if (A, B) ∈ R and (B,C) ∈ R ⇒ (A,C) ∈ R ∀ A,B,C ∈ S

Consider points A(0,0),B(1.5,0) and C(3.2,0).

d(A,B)=1.5 units < 2 units and d(B,C)=1.7 units < 2 units

d(A,C)= 3.2 ≮ 2

⇒ (A, B) ∈ R and (B,C) ∈ R ⇒ (A,C) ∉ R

Thus, R is not transitive.

Thus, R is reflexive, symmetric but not transitive.

Question: 9

Solution:

Given that, ∀ a, b ∈ S, R = {(a, b) : $a^2 + b^2 = 1$ }

Now,

R is Reflexive if (a,a) ∈ R ∀ a ∈ S

For any a ∈ S, we have

$a^2+a^2 = 2a^2 \neq 1$

$\Rightarrow (a,a) \notin R$

Thus, R is not reflexive.

R is Symmetric if $(a,b) \in R \Rightarrow (b,a) \in R \ \forall \ a,b \in S$

$(a,b) \in R$

$\Rightarrow a^2 + b^2 = 1$

$\Rightarrow b^2 + a^2 = 1$

$\Rightarrow (b,a) \in R$

Thus, R is symmetric .

R is Transitive if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R \ \forall \ a,b,c \in S$

Let $(a,b) \in R$ and $(b,c) \in R \ \forall \ a, b,c \in S$

$\Rightarrow a^2 + b^2 = 1$ and $b^2 + c^2 = 1$

Adding both, we get

$a^2 + c^2 + 2b^2 = 2$

$\Rightarrow a^2 + c^2 = 2 - 2b^2 \neq 1$

$\Rightarrow (a, c) \notin R$

Thus, R is not transitive.

Thus, R is symmetric but neither reflexive nor transitive.

Question: 10

Solution:

We have, $R = \{(a, b) : a = b^2\}$ relation defined on N.

Now,

We observe that, any element $a \in N$ cannot be equal to its square except 1.

$\Rightarrow (a,a) \notin R \ \forall \ a \in N$

For e.g. $(2,2) \notin R \because 2 \neq 2^2$

$\Rightarrow$ R is not reflexive.

Let $(a,b) \in R \ \forall \ a, b \in N$

$\Rightarrow a = b^2$

But b cannot be equal to square of a if a is equal to square of b.

$\Rightarrow (b,a) \notin R$

For e.g., we observe that $(4,2) \in R$ i.e $4 = 2^2$ but $2 \neq 4^2 \Rightarrow (2,4) \notin R$

$\Rightarrow$ R is not symmetric

Let $(a,b) \in R$ and $(b,c) \in R \ \forall \ a, b,c \in N$

$\Rightarrow a = b^2$ and $b = c^2$

$\Rightarrow a \neq c^2$

$\Rightarrow (a,c) \notin R$

For e.g., we observe that

$(16,4) \in R \Rightarrow 16 = 4^2$ and $(4,2) \in R \Rightarrow 4 = 2^2$

But $16 \neq 2^2$

$\Rightarrow (16,2) \notin R$

$\Rightarrow$ R is not transitive.

Thus, R is neither reflexive nor symmetric nor transitive.

Question: 11

Solution:

We have, R = {(a, b) : a > b} relation defined on N.

Now,

We observe that, any element $a \in N$ cannot be greater than itself.

$\Rightarrow (a,a) \notin R\ \forall\ a \in N$

$\Rightarrow$ R is not reflexive.

Let $(a,b) \in R\ \forall\ a, b \in N$

$\Rightarrow$ a is greater than b

But b cannot be greater than a if a is greater than b.

$\Rightarrow (b,a) \notin R$

For e.g., we observe that $(5,2) \in R$ i.e $5 > 2$ but $2 \ngtr 5 \Rightarrow (2,5) \notin R$

$\Rightarrow$ R is not symmetric

Let $(a,b) \in R$ and $(b,c) \in R\ \forall\ a, b,c \in N$

$\Rightarrow a > b$ and $b > c$

$\Rightarrow a > c$

$\Rightarrow (a,c) \in R$

For e.g., we observe that

$(5,4) \in R \Rightarrow 5 > 4$ and $(4,3) \in R \Rightarrow 4 > 3$

And we know that $5 > 3\ \therefore (5,3) \in R$

$\Rightarrow$ R is transitive.

Thus, R is transitive but not reflexive not symmetric.

Question: 12

Solution:

Given that, A = {1, 2, 3} and R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3)}.

Now,

R is reflexive $\because (1,1),(2,2),(3,3) \in R$

R is not symmetric $\because (1,2),(2,3) \in R$ but $(2,1),(3,2) \notin R$

R is not transitive $\because (1,2) \in R$ and $(2,3) \in R \Rightarrow (1,3) \notin R$

Thus, R is reflexive but neither symmetric nor transitive.

Question: 13

Solution:

Given that, A = {1, 2, 3} and R = {1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (3, 2)}

Now,

R is reflexive ∵ (1,1),(2,2),(3,3),(4,4) ∈ R

R is not symmetric ∵ (1,2),(1,3),(3,2) ∈ R but (2,1),(3,1),(2,3) ∉ R

R is transitive ∵ (1,3) ∈ R and (3,2) ∈ R ⇒ (1,2) ∈ R

Thus, R is reflexive and transitive but not symmetric.

Exercise : OBJECTIVE QUESTIONS

Question: 1

Solution:

Given set A = {1, 2, 3}

And R = {(1, 1), (2, 2), (3, 3), (1, 3), (3, 2), (1, 2)}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , (1,1) ∈ R , (2,2) ∈ R , (3,3) ∈ R

Therefore , R is reflexive ……. (1)

Check for symmetric

Since (1,3) ∈ R but (3,1) ∉ R

Therefore , R is not symmetric ……. (2)

Check for transitive

Here , (1,3) ∈ R and (3,2) ∈ R and (1,2) ∈ R

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (B)

Question: 2

Solution:

Given set A = {a, b, c}

And R = {(a, a), (a, b), (b, a)}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , (b,b) ∉ R and (c,c) ∉ R

Therefore , R is not reflexive ……. (1)

Check for symmetric

Since , (a,b) ∈ R and (b,a) ∈ R

Therefore , R is symmetric ……. (2)

Check for transitive

Here , (a,b) ∈ R and (b,a) ∈ R and (a,a) ∈ R

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (C)

Question: 3

Solution:

Given set A = {1, 2, 3}

And R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , (1,1) ∈ R , (2,2) ∈ R , (3,3) ∈ R

Therefore , R is reflexive ……. (1)

Check for symmetric

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Since , (1,2) ∈ R and (2,1) ∈ R

(2,3) ∈ R and (3,2) ∈ R

Therefore , R is symmetric ……. (2)

Check for transitive

Here , (1,2) ∈ R and (2,3) ∈ R but (1,3) ∉ R

Therefore , R is not transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

Question: 4

Solution:

According to the question ,

Given set S = {x, y, z}

And R = {(x, y), (y, z), (x, z) , (y, x), (z, y), (z, x)}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , (x,x) ∉ R , (y,y) ∉ R , (z,z) ∉ R

Therefore , R is not reflexive ……. (1)

Check for symmetric

Since , (x,y) ∈ R and (y,x) ∈ R

(z,y) ∈ R and (y,z) ∈ R

(x,z) ∈ R and (z,x) ∈ R

Therefore , R is symmetric ……. (2)

Check for transitive

Here , (x,y) ∈ R and (y,x) ∈ R but (x,x) ∉ R

Therefore , R is not transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (B)

Question: 5

Mark the tick aga

Solution:

According to the question ,

Given set S = {x, y, z}

And R = {(x, x), (y, y), (z, z)}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , (x,x) ∈ R , (y,y) ∈ R , (z,z) ∈ R

Therefore , R is reflexive ……. (1)

Check for symmetric

Since , (x,x) ∈ R and (x,x) ∈ R

(y,y) ∈ R and (y,y) ∈ R

(z,z) ∈ R and (z,z) ∈ R

Therefore , R is symmetric ……. (2)

Check for transitive

Here , (x,x) ∈ R and (y,y) ∈ R and (z,z) ∈ R

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

Question: 6

Solution:

According to the question ,

Given set Z = {1, 2, 3 ,4 …..}

And R = {(a, b) : a,b ∈ Z and (a-b) is divisible by 3}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

(a - a) = 0 which is divisible by 3

(a,a) ∈ R where a ∈ Z

Therefore , R is reflexive .... (1)

Check for symmetric

Consider , (a,b) ∈ R

∴ (a - b) which is divisible by 3

- (a - b) which is divisible by 3

(since if 6 is divisible by 3 then -6 will also be divisible by 3)

∴ (b - a) which is divisible by 3 ⇒ (b,a) ∈ R

For any (a,b) ∈ R ; (b,a) ∈ R

Therefore , R is symmetric .... (2)

Check for transitive

Consider , (a,b) ∈ R and (b,c) ∈ R

∴ (a - b) which is divisible by 3

and (b - c) which is divisible by 3

[(a-b)+(b-c)] is divisible by 3] (if 6 is divisible by 3 and 9 is divisible by 3 then 6+9 will also be divisible by 3)

∴ (a - c) which is divisible by 3 ⇒ (a,c) ∈ R

Therefore (a,b) ∈ R and (b,c) ∈ R then (a,c) ∈ R

Therefore , R is transitive .... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

Question: 7

Solution:

According to the question ,

Given set N = {1, 2, 3 ,4 .....}

And R = {(a, b) : a,b ∈ N and a is a factor of b}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

a is a factor of a

(2,2) , (3,3)... (a,a) where a ∈ N

Therefore , R is reflexive .... (1)

Check for symmetric

a R b ⇒ a is factor of b

b R a ⇒ b is factor of a as well

Ex _ (2,6) ∈ R

But (6,2) ∉ R

Therefore , R is not symmetric .... (2)

Check for transitive

a R b ⇒ a is factor of b

b R c ⇒ b is a factor of c

a R c ⇒ b is a factor of c also

Ex _(2,6) , (6,18)

∴ (2,18) ∈ R

Therefore , R is transitive .... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (B)

Question: 8

Solution:

According to the question ,

Given set Z = {1, 2, 3 ,4 .....}

And R = {(a, b) : a,b ∈ Z and a ≥ b}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relatio

Check for reflexive

Consider , (a,a) (b,b)

∴ $a \geq a$ and $b \geq b$ which is always true.

Therefore , R is reflexive ……. (1)

Check for symmetric

$a R b \Rightarrow a \geq b$

$b R a \Rightarrow b \geq a$

Both cannot be true.

Ex _ If a=2 and b=1

∴ $2 \geq 1$ is true but $1 \geq 2$ which is false.

Therefore , R is not symmetric ……. (2)

Check for transitive

$a R b \Rightarrow a \geq b$

$b R c \Rightarrow b \geq c$

∴ $a \geq c$

Ex _a=5 , b=4 and c=2

∴ 5≥4 , 4≥2 and hence 5≥2

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (C)

Question: 9

Solution:

According to the question ,

Given set S = {…….,-2,-1,0,1,2 …..}

And R = {(a, b) : a,b ∈ S and $|a| \leq b$ }

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

∴ $|a| \leq a$ and which is not always true.

Ex_if a=-2

∴ $|-2| \leq -2 \Rightarrow 2 \leq -2$ which is false.

Therefore , R is not reflexive ……. (1)

Check for symmetric

a R b ⇒ $|a| \leq b$

b R a ⇒ $|b| \leq a$

Both cannot be true.

Ex _ If a=-2 and b=-1

∴ $2 \leq -1$ is false and $1 \leq -2$ which is also false.

Therefore , R is not symmetric ……. (2)

Check for transitive

a R b ⇒ $|a| \leq b$

b R c ⇒ $|b| \leq c$

∴ $|a| \leq c$

Ex _a=-5 , b= 7 and c=9

∴ $5 \leq 7$, $7 \leq 9$ and hence $5 \leq 9$

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (C)

Question: 10

Solution:

According to the question ,

Given set S = {…….,-2,-1,0,1,2 …..}

And R = {(a, b) : a,b ∈ S and $|a - b| \leq 1$ }

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

∴ $|a - a| \leq 1$ and which is always true.

Ex_if a=2

∴ $|2-2| \le 1 \Rightarrow 0 \le 1$ which is true.

Therefore , R is reflexive .... (1)

Check for symmetric

a R b ⇒ $|a - b| \le 1$

b R a ⇒ $|b - a| \le 1$

Both can be true.

Ex _ If a=2 and b=1

∴ $|2 - 1| \le 1$ is true and $|1-2| \le 1$ which is also true.

Therefore , R is symmetric .... (2)

Check for transitive

a R b ⇒ $|a - b| \le 1$

b R c ⇒ $|b - c| \le 1$

∴ $|a - c| \le 1$ will not always be true

Ex _a=-5 , b= -6 and c= -7

∴ $|6-5| \le 1$, $|7 - 6| \le 1$ are true But $|7 - 5| \le 1$ is false.

Therefore , R is not transitive .... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

Question: 11

Solution:

According to the question ,

Given set S = {.......,-2,-1,0,1,2 .....}

And R = {(a, b) : a,b ∈ S and (1 + ab) > 0 }

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

∴ $(1 + a \times a) > 0$ which is always true because a×a will always be positive.

Ex_if a=2

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$\therefore$ (1 + 4) > 0 $\Rightarrow$ (5) > 0 which is true.

Therefore , R is reflexive ……. (1)

Check for symmetric

a R b $\Rightarrow$ (1 + ab) > 0

b R a $\Rightarrow$ (1 + ba) > 0

Both the equation are the same and therefore will always be true.

Ex _ If a=2 and b=1

$\therefore$ (1 + 2×1) > 0 is true and (1+1×2) > which is also true.

Therefore , R is symmetric ……. (2)

Check for transitive

a R b $\Rightarrow$ (1 + ab) > 0

b R c $\Rightarrow$ (1 + bc) > 0

$\therefore$(1 + ac) > 0 will not always be true

Ex _a=-1 , b= 0 and c= 2

$\therefore$ (1 + -1×0) > 0 , (1 + 0×2) > 0 are true

But (1 + -1×2) > 0 is false.

Therefore , R is not transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

Question: 12

Solution:

According to the question ,

Given set S = {…All triangles in plane….}

And R = {(Δ_1 , Δ_2) : Δ_1 , $\Delta_2 \in$ S and $\Delta_1 \equiv \Delta_2$}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (Δ_1, Δ_1)

$\therefore$ We know every triangle is congruent to itself.

(Δ_1, Δ_1) ∈ R all Δ_1 ∈ S

Therefore , R is reflexive ……. (1)

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Check for symmetric

$(\Delta_1, \Delta_2) \in R$ then Δ_1 is congruent to Δ_2

$(\Delta_2, \Delta_1) \in R$ then Δ_2 is congruent to Δ_1

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric ……. (2)

Check for transitive

Let $\Delta_1, \Delta_2, \Delta_3 \in S$ such that $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$

Then $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$

$\Rightarrow \Delta_1$ is congruent to Δ_2, and Δ_2 is congruent to Δ_3

$\Rightarrow \Delta_1$ is congruent to Δ_3

$\therefore (\Delta_1, \Delta_3) \in R$

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

Question: 13

Solution:

According to the question ,

Given set S = {…….,-2,-1,0,1,2 …..}

And $R = \{(a, b) : a,b \in S \text{ and } a^2 + b^2 = 1\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if $(a, a) \in R$ for every $a \in A$

Symmetric

The relation is Symmetric if $(a, b) \in R$, then $(b, a) \in R$

Transitive

Relation is Transitive if $(a, b) \in R$ & $(b, c) \in R$, then $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

$\therefore a^2 + a^2 = 1$ which is not always true

Ex_if a=2

$\therefore 2^2 + 2^2 = 1 \Rightarrow 4 + 4 = 1$ which is false.

Therefore , R is not reflexive ……. (1)

Check for symmetric

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a R b ⇒ $a^2 + b^2 = 1$

b R a ⇒ $b^2 + a^2 = 1$

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric ……. (2)

Check for transitive

a R b ⇒ $a^2 + b^2 = 1$

b R c ⇒ $b^2 + c^2 = 1$

∴ $a^2 + c^2 = 1$ will not always be true

Ex _a=-1 , b= 0 and c= 1

∴ $(-1)^2 + 0^2 = 1$, $0^2 + 1^2 = 1$ are true

But $(-1)^2 + 1^2 = 1$ is false.

Therefore , R is not transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

Question: 14

Solution:

According to the question ,

R = {(a, b) , (c, d) : a + d = b + c }

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a, b) R (a, b)

(a, b) R (a, b) ⇔ a + b = a + b

which is always true .

Therefore , R is reflexive ……. (1)

Check for symmetric

(a, b) R (c, d) ⇔ a + d = b + c

(c, d) R (a, b) ⇔ c + b = d + a

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric ……. (2)

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Check for transitive

(a, b) R (c, d) ⇔ a + d = b + c

(c, d) R (e, f) ⇔ c + f = d + e

On adding these both equations we get , a + f = b + e

Also,

(a, b) R (e, f) ⇔ a + f = b + e

∴ It will always be true

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

Question: 15

Solution:

According to the question ,

O is the origin

R = {(P, Q) : OP = OQ }

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (P , P) ∈ R ⇔ OP = OP

which is always true .

Therefore , R is reflexive ……. (1)

Check for symmetric

(P , Q) ∈ R ⇔ OP = OQ

(Q , P) ∈ R ⇔ OQ = OP

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric ……. (2)

Check for transitive

(P , Q) ∈ R ⇔ OP = OQ

(Q , R) ∈ R ⇔ OQ = OR

On adding these both equations, we get , OP = OR

Also,

(P , R) ∈ R ⇔ OP = OR

∴ It will always be true

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

Question: 16

Solution:

According to the question ,

Q is set of all rarional numbers

R = {(a, b) : a * b = a + 2b }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a + 2b

And , b * a = b + 2a

Both equations will not always be true .

Therefore , * is not commutative ……. (1)

Check for associative

Consider , (a * b) * c = (a + 2b) * c = a+2b + 2c

And , a * (b * c) = a * (b+2c) = a+2(b+2c) = a+2b+4c

Both the equation are not the same and therefore will not always be true.

Therefore , * is not associative ……. (2)

Now , according to the equations (1) , (2)

Correct option will be (C)

Question: 17

Solution:

According to the question ,

Q = { a,b }

R = {(a, b) : a * b = a + ab }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a + ab

And , b * a = b + ba

Both equations will not always be true .

Therefore , * is not commutative .... (1)

Check for associative

Consider , (a * b) * c = (a + ab) * c = a+ab + (a+ab)c=a+ab+ac+abc

And , a * (b * c) = a * (b+bc) = a+a(b+bc) = a+ab+abc

Both the equation are not the same and therefore will not always be true.

Therefore , * is not associative .... (2)

Now , according to the equations (1) , (2)

Correct option will be (B)

Question: 18

Solution:

According to the question ,

Q = { Positive rationals }

R = {(a, b) : a * b = ab/2 }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = ab/2

And , b * a = ba/2

Both equations are the same and will always true .

Therefore , * is commutative .... (1)

Check for associative

Consider , (a * b) * c = (ab/2) * c = $\frac{\frac{ab}{2}\times c}{2}$ = abc/4

And , a * (b * c) = a * (bc/2) = $\frac{a\times\frac{bc}{2}}{2}$ = abc/4

Both the equation are the same and therefore will always be true.

Therefore , * is associative .... (2)

Now , according to the equations (1) , (2)

Correct option will be (D)

Question: 19

Solution:

According to the question ,

Q = { All integers }

R = {(a, b) : a * b = a – b + ab }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a – b + ab

And , b * a = b – a + ba

Both equations are not the same and will not always be true .

Therefore , * is not commutative ……. (1)

Check for associative

Consider , (a * b) * c = (a – b + ab) * c

= a – b + ab – c +(a – b + ab)c

=a – b +ab – c +ac – bc + abc

And , a * (b * c) = a * (b – c + bc)

= a - (b – c + bc) + a(b – c + bc)

=a – b + c – bc + ab – ac + abc

Both the equation are not the same and therefore will not always be true.

Therefore , * is not associative ……. (2)

Now , according to the equations (1) , (2)

Correct option will be (C)

Question: 20

Solution:

According to the question ,

Q = { All integers }

R = {(a, b) : a * b = a + b - ab }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a + b - ab

And , b * a = b + a - ba

Both equations are the same and will always be true .

Therefore , * is commutative ……. (1)

Check for associative

Consider , (a * b) * c = (a + b - ab) * c

= a + b - ab + c -(a + b - ab)c

=a + b – ab + c – ac – bc + abc

And , a * (b * c) = a * (b + c - bc)

= a + (b + c - bc) - a(b + c - bc)

=a + b + c – bc - ab – ac + abc

Both the equation are the same and therefore will always be true.

Therefore , * is associative ……. (2)

Now , according to the equations (1) , (2)

Correct option will be (D)

Question: 21

Solution:

According to the question ,

Q = { All integers }

R = {(a, b) : a * b = a^b }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a^b

And , b * a = b^a

Both equations are not the same and will not always be true .

Therefore , * is not commutative ……. (1)

Check for associative

Consider , (a * b) * c = (a^b) * c = $(a^b)^c$

And , a * (b * c) = a * (b^c)= $a^{(b^c)}$

Ex a=2 b=3 c=4

(a * b) * c = (2^3) * c = $(8)^4$

a * (b * c) = 2 * (3^4)= $2^{(81)}$

Both the equation are not the same and therefore will not always be true.

Therefore , * is not associative ……. (2)

Now , according to the equations (1) , (2)

Correct option will be (C)

Question: 22

Solution:

According to the question ,

R = {(a, b) : a * b = a + b + ab }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a + b + ab

And , b * a = b + a + ba

Both equations are same and will always be true .

Therefore , * is commutative .... (1)

Check for associative

Consider , (a * b) * c = (a + b + ab) * c

= a + b + ab + c + (a + b + ab)c

=a + b + c + ab + ac + bc + abc

And , a * (b * c) = a * (b + c + bc)

= a + b + c + bc + a(b + c + bc)

=a +b + c + ab + bc + ac + abc

Both the equation are same and therefore will always be true.

Therefore , * is associative .... (2)

Now , according to the equations (1) , (2)

Correct option will be (D)