Chapter : 13. METHOD OF INTEGRATION

Exercise : 13A

Question: 1

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 2x + 9 = t ⇒ 2 dx = dt

$$\int t^5 (\frac{dt}{2}) = \frac{1}{2}\int t^5 dt = \frac{1}{2}\frac{t^6}{6} + c = \frac{t^6}{12} + c$$

$$= \frac{(2x+9)^6}{12} + c$$

Question: 2

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 7 – 3x = t ⇒ -3 dx = dt

$$\int t^4 (\frac{dt}{-3}) = \frac{1}{-3}\int t^4 dt = \frac{1}{-3}\frac{t^5}{5} + c = -\frac{t^5}{15} + c$$

$$= -\frac{(7-3x)^5}{15} + c$$

Question: 3

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 3x - 5 = t ⇒ 3 dx = dt

$$\int t^{0.5} (\frac{dt}{3}) = \frac{1}{3}\int t^{0.5} dt = \frac{1}{3} \times \frac{t^{1.5}}{1.5} + c = \frac{2}{1} \times \frac{t^{1.5}}{9} + c$$

$$= \frac{2(3x-5)^5}{9} + c$$

Question: 4

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 4x + 3 = t ⇒ 4 dx = dt

$$\int t^{-0.5}(\frac{dt}{4}) = \frac{1}{4}\int t^{-0.5}dt = \frac{1}{4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{4} \times \frac{t^{0.5}}{1} + c$$

$$= \frac{\sqrt{4x+3}}{2} + c$$

Question: 5

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 3 – 4x = t ⇒ -4 dx = dt

$$\int t^{-0.5}(\frac{dt}{-4}) = \frac{1}{-4}\int t^{-0.5}dt = \frac{1}{-4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{-4} \times \frac{t^{0.5}}{1} + c$$

$$= -\frac{\sqrt{3-4x}}{2} + c$$

Question: 6

Solution:

Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 2x – 3 = t ⇒ 2 dx = dt

$$\int t^{-\frac{3}{2}}(\frac{dt}{2}) = \frac{1}{2}\int t^{-\frac{3}{2}}dt = \frac{1}{2} \times \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + c = \frac{-2}{2} \times \frac{t^{-0.5}}{1} + c$$

$$= -\frac{1}{\sqrt{2x-3}} + c$$

Question: 7

Solution:

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put 2x – 1 = t ⇒ 2 dx = dt

$$\int e^t(\frac{dt}{2}) = \frac{1}{2}\int e^t dt = \frac{1}{2} \times e^t + c = \frac{e^{2x-1}}{2} + c$$

$$= \frac{e^{(2x-1)}}{2} + c$$

Question: 8

Solution:

Formula =

$$\int e^x dx = e^x + c$$

Therefore ,

Put 1 – 3x = t ⇒ -3 dx = dt

$$\int e^t (\frac{dt}{-3}) = \frac{1}{-3}\int e^t dt = \frac{1}{-3} \times e^t + c = \frac{e^{1-3x}}{-3} + c$$

$$= -\frac{e^{(1-3x)}}{3} + c$$

Question: 9

Solution:

Formula = $\int a^x dx = \frac{a^x}{\log a} + c$

Therefore ,

Put 2 – 3x = t ⇒ -3 dx = dt

$$\int 3^t (\frac{dt}{-3}) = \frac{1}{-3}\int 3^t dt = \frac{1}{-3} \times (\frac{3^t}{\log 3}) + c = \frac{3^t}{-3\log 3} + c$$

$$= -\frac{3^{(2-3x)}}{3\log 3} + c$$

Question: 10

Solution:

Formula = $\int \sin x\, dx = -\cos x + c$

Therefore ,

Put 3x = t ⇒ 3 dx = dt

$$\int \sin t\, (\frac{dt}{3}) = \frac{1}{3}\int \sin t\, dt = \frac{1}{3} \times (-\cos t) + c = \frac{-\cos 3x}{3} + c$$

$$= -\frac{\cos 3x}{3} + c$$

Question: 11

Solution:

Formula = $\int \cos x\, dx = \sin x + c$

Therefore ,

Put 5 + 6x = t ⇒ 6 dx = dt

$$\int \cos t\, (\frac{dt}{6}) = \frac{1}{6}\int \cos t\, dt = \frac{1}{6} \times (\sin t) + c = \frac{\sin 5 + 6x}{6} + c$$

$$= \frac{\sin(5 + 6x)}{6} + c$$

Question: 12

Solution:

Formula

$1 + \cos 2x \int \cos 2\cos^2 x = \sin x + c$

Therefore ,

$$\int \sin x \sqrt{1+\cos 2x}\, dx = \int \sin x \sqrt{2} \cos x + c$$

$$\int \sqrt{2} \sin x \cos x \;\; dx$$

Put sin x =t ⇒ cos x dx = dt

$$\int \sqrt{2} \sin x \cos x \;\; dx = \int \sqrt{2} t\, dt = \sqrt{2}\, \frac{t^2}{2} + c$$

$$= \frac{(\sin x)^2}{\sqrt{2}} + c$$

Question: 13

Solution:

Formula $\int cosec^2 x\, dx = -\cot x + c$

Therefore ,

Put 2x + 5 =t ⇒ 2 dx = dt

$$\int cosec^2 t\, \frac{dt}{2} = -\frac{1}{2} \cot t + c = -\frac{1}{2} \cot(2x+5) + c$$

$$= -\frac{1}{2} \cot(2x+5) + c$$

Question: 14

Solution:

Formula $\int \sin x\, dx = -\cos x + c$

Therefore ,

Put sin x =t ⇒ cos x dx = dt

$$\int t\, dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin x)^2}{2} + c$$

Question: 15

Solution:

Formula $\int \sin x\, dx = -\cos x + c$

Therefore ,

Put sin x =t ⇒ cos x dx = dt

$$\int t^3\, dt = \frac{t^4}{4} + c$$

$$= \frac{(\sin x)^4}{4} + c$$

Question: 16

Evaluate the foll

Solution:

Formula $\int \sin x\, dx = -\cos x + c$

Therefore ,

Put cos x =t ⇒ -sin x dx = dt

$$\int t^{0.5}(-1)dt = -\frac{t^{1.5}}{1.5} + c$$

$$= -\frac{2(\cos x)^{\frac{3}{2}}}{3} + c$$

Question: 17

Solution:

Formula $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c \quad \frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

Therefore ,

Put $\sin^{-1} x$ =t ⇒ $\frac{1}{\sqrt{1-x^2}}$ dx = dt

$$\int t^1 dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin^{-1} x)^2}{2} + c$$

Question: 18

Solution:

Formula $\int \sin t\, dx = -\cos t + c \quad \frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$

Therefore ,

Put $\tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$$\int \sin 2t\, dt = \frac{-\cos 2t}{2} + c$$

$$= -\frac{\cos(2\tan^{-1} x)}{2} + c$$

Question: 19

Solution:

Formula $\int \cos t\, dx = \sin t + c \quad \frac{d(\log x)}{dx} = \frac{1}{x}$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int \cos t\, dt = \sin t + c$$

$$= \sin(\log x) + c$$

Question: 20

Evaluate the foll

Solution:

Formula $\int cosec^2 x\, dx = -\cot x + c$ $\frac{d(\log x)}{dx} = \frac{1}{x}$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int cosec^2 t \frac{dt}{1} = -\cot t + c = -\cot(\log x) + c$$

$= -\cot(\log x) + c$

Question: 21

Solution:

Formula $\frac{d(\log x)}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \log x$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int \frac{dt}{t} = \log t + c = \log(\log x) + c$$

$= \log(\log x) + c$

Question: 22

Solution:

Formula $\frac{d(\log x)}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \log x$

$$\int \frac{(x+1)(x+\log x)^2}{x} dx = \int \frac{x+1}{x} \times \frac{(x+\log x)^2}{1} dx$$

$$= \int (1+\frac{1}{x}) \times \frac{(x+\log x)^2}{1} dx$$

Therefore ,

Put $x + \log x = t \Rightarrow (1+\frac{1}{x})dx = dt$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$= \frac{(x+\log x)^3}{3} + c$$

Question: 23

Solution:

Formula $\frac{d(\log x)}{dx} = \frac{1}{x}$ $\int \frac{1}{x} dx = \log x$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int t^2 dt = \frac{t^3}{3} + c = \frac{(\log x)^3}{3} + c$$

$= \frac{(\log x)^3}{3} + c$

CLASS24

Question: 24

Solution:

Formula $\int \cos t \, dx = \sin t + c \; \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

Therefore ,

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\int \cos t \; 2dt = 2 \sin t + c$$

$= 2 \sin(\sqrt{x}) + c$

Question: 25

Solution:

Formula = $\int e^x dx = e^x + c \; \frac{d(\tan x)}{dx} = sec^2 x$

Therefore ,

Put tan x = t $\Rightarrow sec^2 x$ dx = dt

$$\int e^t dt = e^t + c$$

$= e^{\tan x} + c$

Question: 26

Solution:

Formula = $\int e^x dx = e^x + c \; \frac{d(cos^2 x)}{dx} = 2 \cos x \,(-\sin x) = -\sin 2x$

Therefore ,

Put $cos^2 x$ = t $\Rightarrow -\sin 2x$ dx = dt

$$\int -e^t dt = -e^t + c$$

$= -e^{cos^2 x} + c$

Question: 27

Solution:

Formula = $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put ax+b = t ⇒ adx = dt

$$\int \sin t \cos t \frac{dt}{a} = \frac{1}{a} \int \sin t \cos t \, dt$$

Put sin t = z �cos t dt = dz

$$\frac{1}{a}\int zdz = \frac{1}{a}\times\frac{z^2}{2}+c$$

$$= \frac{(\sin ax + b)^2}{2a} + c$$

Question: 28

Solution:

Formula = $\int \cos x\, dx = \sin x + c$

$\cos 3x = 3\cos x - 4cos^3 x$

Therefore ,

$$\int (\frac{3\cos x}{4} - \frac{\cos 3x}{4})dx = \frac{3\sin x}{4} - \frac{\sin 3x}{4\times 3} + c$$

$$= \frac{3\sin x}{4} - \frac{\sin 3x}{12} + c$$

Question: 29

Solution:

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put $-\frac{1}{x} = \mathrm{t} \Rightarrow \frac{1}{x^2}dx = dt$

$$\int e^t(dt) = \int e^t dt = e^t + c = e^{-\frac{1}{x}} + c$$

$$= e^{-\frac{1}{x}} + c$$

Question: 30

Solution:

Formula = $\int \cos x\, dx = \sin x + c$

Therefore ,

Put $-\frac{1}{x} = \mathrm{t} \Rightarrow \frac{1}{x^2}dx = dt$

$$\int \cos t\,(dt) = \int \cos t\, dt = \sin t + c = \sin(-\frac{1}{x}) + c$$

$$= -\sin\frac{1}{x} + c$$

Question: 31

Solution:

Formula = $\int e^x dx = e^x + c$

Therefore ,

$$\int \frac{e^x}{1+e^{2x}}dx$$

Put $e^x = t \Rightarrow e^x dx = dt$

$$\int \frac{1}{1+t^2}(dt) = \int \frac{1}{1+t^2} dt = \tan^{-1} t + c$$

$$= \tan^{-1}(e^x) + c$$

Question: 32

Solution:

Formula = $\int e^x dx = e^x + c$

Therefore ,

Put $e^{2x} - 2 = t \Rightarrow 2e^{2x} dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{2}\right) = \frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t + c$$

$$= \frac{1}{2}\log(e^{2x} - 2) + c$$

Question: 33

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sin x) = t $\Rightarrow \frac{\cos x}{\sin x} dx = dt$ �$\cot x \, dx = dt$

$$\int t \, dt = \frac{t^2}{2} + c$$

$$= \frac{(\log \sin x)^2}{2} + c$$

Question: 34

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sin x) = t $\Rightarrow \frac{\cos x}{\sin x} dx = dt$ �$\cot x \, dx = dt$

$$\int \frac{1}{t} dt = \log t + c$$

$$= \log(\log \sin x) + c$$

Question: 35

Solution:

Formula = $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put $x^2 + 1 = t \Rightarrow 2x \, dx = dt$

$$\int \sin t \ dt = -\cos t + c$$

$$= -\cos(x^2 + 1) + c$$

Question: 36

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sec x + tan x)= t

$$\frac{1}{\sec x + \tan x} \times (\sec x \tan x + sec^2 x)\ dx = dt$$

$$\frac{1}{\sec x + \tan x} \times \sec x\ (\sec x + \tan x)\ dx = dt$$

Sec x dx = dt

$$\int t\,dt = \frac{t^2}{2} + c$$

$$= \frac{(\log(\sec x + \tan x))^2}{2} + c$$

Question: 37

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\tan\sqrt{x}$ = t

$$sec^2\sqrt{x} \times \left(\frac{1}{2\sqrt{x}}\right) dx = dt$$

$$\int t\,dt = \frac{t^2}{2} + c$$

$$= \frac{(\tan\sqrt{x})^2}{2} + c$$

Question: 38

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\tan^{-1} x^2$ = t $\Rightarrow \frac{1}{1+(x^2)^2} \times 2x \times dx = dt$ � $\frac{2x}{1+x^4} dx = dt$

$$\int t\left(\frac{dt}{2}\right) = \frac{1}{2}\int t dt = \frac{t^2}{4} + c$$

$$= \frac{(\tan^{-1} x^2)^2}{4} + c$$

Question: 39

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\sin^{-1} x^2 = t \Rightarrow \frac{1}{\sqrt{1-(x^2)^2}} \times 2x \times dx = dt$ � $\frac{2x}{\sqrt{1-x^4}} dx = dt$

$$\int t\left(\frac{dt}{2}\right) = \frac{1}{2}\int t dt = \frac{t^2}{4} + c$$

$$= \frac{(\sin^{-1} x^2)^2}{4} + c$$

Question: 40

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\sin^{-1} x^1 = t \Rightarrow \frac{1}{\sqrt{1-(x^2)^1}} \times dx = dt$ � $\frac{1}{\sqrt{1-x^2}} dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{1}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log \sin^{-1} x + c$$

Question: 41

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put 2 + log x = t $\Rightarrow \frac{1}{x} \times dx = dt$

$$\int \sqrt{t}\left(\frac{dt}{1}\right) = \int \sqrt{t} dt = \frac{2t^{1.5}}{3} + c$$

$$= \frac{2(2 + \log x)^{\frac{3}{2}}}{3} + c$$

Question: 42

Solution:

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put 1 + tan x = t $\Rightarrow sec^2 x \times dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(1 + \tan x) + c$$

Question: 43

Solution:

Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

Put 1 + cos x = t $\Rightarrow -\sin x \times dx = dt$

$$\int \left(\frac{-dt}{t}\right) = -\int \frac{1}{t} dt = -\log t + c$$

$= -\log(1 + \cos x) + c$

Question: 44

Solution:

Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

$$\int \left(\frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}}\right) dx = \int \left(\frac{\cos x + \sin x}{\cos x - \sin x}\right) dx$$

Put cos x - sin x = t ⇒ (- cos x - sin x) dx = dt

$$\int \left(\frac{-dt}{t}\right) = -\int \frac{1}{t} dt = -\log t + c$$

$= -\log(\cos x - \sin x) + c$

Question: 45

Solution:

(i)

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put x + log (sec x) = t $\Rightarrow 1 + \frac{1}{\sec x} \times \sec x \tan x \, dx = dt$

$(1 + \tan x)dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$= \log(x + \log(\sec x)) + c$

(ii)

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $x + cos^2 x$ = t $\Rightarrow 1 + 2\cos x \times (-\sin x)dx = dt$

$(1 - \sin 2x)dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$= \log(x + cos^2 x) + c$

Question: 46

Solution:

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $a^2 + b^2 sin^2 x = t$ �$b^2 \times 2 \sin x \times \cos x\, dx = dt$

$(b^2 \sin 2x)dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{b^2}\right) = \frac{1}{b^2}\int \frac{1}{t} dt = \frac{1}{b^2}\log t + c$$

$$= \frac{1}{b^2}\log|a^2 + b^2 sin^2 x| + c$$

Question: 47

Solution:

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $a^2 cos^2 x + b^2 sin^2 x = t$

$(a^2 \times 2\cos x \times (-\sin x) + b^2 \times 2 \sin x \times \cos x)dx = dt$

$(b^2 - a^2) \sin 2x\ dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{b^2 - a^2}\right) = \frac{1}{b^2 - a^2}\int \frac{1}{t} dt = \frac{1}{b^2 - a^2}\log t + c$$

$$= \frac{1}{b^2 - a^2}\log|a^2 cos^2 x + b^2 sin^2 x| + c$$

Question: 48

Solution:

Formula = $\int \cos x\, dx = \sin x + c$

Therefore ,

Put 3cos x + 2sin x = t ⇒ (2cos x - 3sin x) dx = dt

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$= \log(3\cos x + 2 \sin x) + c$

Question: 49

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $2x^2 + 3 = t$ ⇒ (4x) dx = dt

$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$

$= \log(2x^2 + 3) + c$

Question: 50

Solution:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $x^2+2x+3= t \Rightarrow (2x+2)\ dx = dt$ �2(x+1)dx=dt

$\int \frac{1}{t}\left(\frac{dt}{2}\right) = \frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t + c$

$= \frac{1}{2}\log(x^2 + 2x + 3) + c$

Question: 51

Solution:

To find: Value of $\int \frac{4x - 5}{(2x^2 - 5x + 1)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{4x - 5}{(2x^2 - 5x + 1)} dx$ ... (i)

Let $2x^2 - 5x + 1 = t$

$\Rightarrow \frac{d(2x^2 - 5x + 1)}{dx} = \frac{dt}{dx}$

$\Rightarrow 4x - 5 = \frac{dt}{dx}$

$\Rightarrow (4x - 5)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$ [$2x^2 - 5x + 1 = t$]

$I = \log|t| + c$

$I = \log|2x^2 - 5x + 1| + c$

Ans) $\log|2x^2 - 5x + 1| + c$

Question: 52

Solution:

To find: Value of $\int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$ ... (i)

Let $3x^3 - 2x^2 + 5x + 1 = t$

$$\Rightarrow \frac{d(3x^3 - 2x^2 + 5x + 1)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow 9x^2 - 4x + 5 = \frac{dt}{dx}$$

$\Rightarrow (9x^2 - 4x + 5)dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t} \; [\, 3x^3 - 2x^2 + 5x + 1 = t \,]$$

$I = \log|t| + c$

$I = \log|3x^3 - 2x^2 + 5x + 1| + c$

Ans) $\log|3x^3 - 2x^2 + 5x + 1| + c$

Question: 53

Solution:

To find: Value of $\int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$ ... (i)

Let $\log(\tan x) = t$

$$\Rightarrow \frac{d(\log(\tan x))}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(\log(\tan x))}{d\tan x}\frac{d\tan x}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{\tan x}\sec^2 x = \frac{dt}{dx}$$

$$\Rightarrow \sec x \operatorname{cosec} x = \frac{dt}{dx}$$

$\Rightarrow (\sec x \operatorname{cosec} x)dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t} \; [\, \log(\tan x) = t \,]$$

$I = \log|t| + c$

$I = \log|\log(\tan x)| + c$

Ans) $\log|\log(\tan x)| + c$

Question: 54

Solution:

To find: Value of

$$\int \frac{(1 + \cos x)}{(x + \sin x)^3} dx$$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{(1+\cos x)}{(x+\sin x)^3}dx$ ... (i)

Let $x + \sin x = t$

$\Rightarrow \frac{d(x+\sin x)}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(x)}{dx} + \frac{d(\sin x)}{dx} = \frac{dt}{dx}$

$\Rightarrow (1+\cos x) = \frac{dt}{dx}$

$\Rightarrow (1 + \cos x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t^3}$ [$x + \sin x = t$]

$\Rightarrow I = -\frac{1}{2t^2} + c$

$I = -\frac{1}{2(x+\sin x)^2} + c$

Ans) $-\frac{1}{2(x+\sin x)^2} + c$

Question: 55

Solution:

To find: Value of $\int \frac{\sin x}{(1+\cos x)^2}dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1+\cos x)^2}dx$ ... (i)

Let $1 + \cos x = t$

$\Rightarrow \frac{d(1+\cos x)}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(1)}{dx} + \frac{d(\cos x)}{dx} = \frac{dt}{dx}$

$\Rightarrow (0 - \sin x) = \frac{dt}{dx}$

$\Rightarrow (-\sin x)dx = dt$

Putting this value in equation (i)

$I = \int -\frac{dt}{t^2}$ [$1 + \cos x = t$]

$\Rightarrow I = \frac{1}{t} + c$

$I = \frac{1}{1 + \cos x} + c$

Ans) $\frac{1}{1 + \cos x} + c$

Question: 56

Solution:

To find: Value of $\int \frac{(2x + 3)}{\sqrt{x^2 + 3x - 2}} dx$

Formula used: $\int x^n dx = \frac{1}{n + 1} x^{n + 1} + c$

We have, $I = \int \frac{\sin x}{(1 + \cos x)^2} dx$ ... (i)

Let $x^2 + 3x - 2 = t$

$\Rightarrow (2x + 3) = \frac{dt}{dx}$

$\Rightarrow (2x + 3)\ dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{\sqrt{t}}$ [$x^2 + 3x - 2 = t$]

$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$

$I = 2t^{\frac{1}{2}} + c$

$I = 2\sqrt{x^2 + 3x - 2} + c$

Ans) $2\sqrt{x^2 + 3x - 2} + c$

Question: 57

Solution:

To find: Value of $\int \frac{(2x - 1)}{\sqrt{x^2 - x - 1}} dx$

Formula used: $\int x^n dx = \frac{1}{n + 1} x^{n + 1} + c$

We have, $I = \int \frac{\sin x}{(1 + \cos x)^2} dx$ ... (i)

Let $x^2 - x - 1 = t$

$\Rightarrow \frac{d(x^2 - x - 1)}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{d(x^2)}{dx} - \frac{d(x)}{dx} - \frac{d(1)}{dx} = \frac{dt}{dx}$

$\Rightarrow (2x - 1) = \frac{dt}{dx}$

$\Rightarrow (2x - 1)\ dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^{\frac{1}{2}}} \; [\, x^2 - x - 1 = t \,]$$

$$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\Rightarrow I = \frac{2\sqrt{t}}{1} + c$$

$$I = \frac{2\sqrt{x^2 - x - 1}}{1} + c$$

Ans) $2\sqrt{x^2 - x - 1} + c$

Question: 58

Solution:

To find: Value of $\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$ ... (i)

$$I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}}$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{\left(\sqrt{x+a}\right)^2 - \left(\sqrt{x+b}\right)^2} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{(x+a) - (x+b)} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{x + a - x - b} dx$$

$$I = \frac{1}{a-b}\left[\int \sqrt{x+a}\, dx - \int \sqrt{x+b}\, dx\right]$$

$$I = \frac{1}{a-b}\left[\int (x+a)^{\frac{1}{2}} dx - \int (x+b)^{\frac{1}{2}} dx\right]$$

$$I = \frac{1}{a-b}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$I = \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$

Question: 59

Solution:

To find: Value of $\int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}}$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}}$ ... (i)

$$I = \int \frac{dx}{\sqrt{1-3x}-\sqrt{5-3x}} \times \frac{\sqrt{1-3x}+\sqrt{5-3x}}{\sqrt{1-3x}+\sqrt{5-3x}}$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{\left(\sqrt{1-3x}\right)^2-\left(\sqrt{5-3x}\right)^2}dx$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{(1-3x)-(5-3x)}dx$$

$$I = \int \frac{\sqrt{1-3x}+\sqrt{5-3x}}{1-3x-5+3x}dx$$

$$I = -\frac{1}{4}\left[\int \sqrt{1-3x}\,dx + \int \sqrt{5-3x}\,dx\right]$$

$$I = -\frac{1}{4}\left[\int (1-3x)^{\frac{1}{2}}dx + \int (5-3x)^{\frac{1}{2}}dx\right]$$

$$I = -\frac{1}{4}\left[\frac{(1-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)} + \frac{(5-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)}\right]$$

$$I = -\frac{2}{-9\times 4}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$$

$$I = \frac{1}{18}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{1}{18}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$

Question: 60

Solution:

To find: Value of $\int \frac{x^2}{(1+x^6)}dx$

Formula used: $\int \frac{1}{1+x^2}dx = \tan^{-1}x$

We have, $I = \int \frac{x^2}{(1+x^6)}dx$ ... (i)

$$I = \int \frac{x^2}{1+(x^3)^2}dx$$

Let $x^3 = t$

$$\Rightarrow \frac{d(x^3)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (3x^2) = \frac{dt}{dx}$$

$$\Rightarrow (x^2)dx = \frac{dt}{3}$$

Putting this value in equation (i)

$I = \frac{1}{3}\int \frac{dt}{1+t^2}$ [1 + cosx = t]

$\Rightarrow I = \frac{1}{3}\tan^{-1}(t) + c$

$I = \frac{1}{3}\tan^{-1}(x^3) + c$

Ans) $\frac{1}{3}\tan^{-1}(x^3) + c$

Question: 61

Solution:

To find: Value of $\int \frac{x^3}{(1+x^8)}dx$

Formula used: $\int \frac{1}{1+x^2}dx = \tan^{-1}x$

We have, $I = \int \frac{x^3}{(1+x^8)}dx$ ... (i)

$I = \int \frac{x^3}{1+(x^4)^2}dx$

Let $x^4 = t$

$\Rightarrow \frac{d(x^4)}{dx} = \frac{dt}{dx}$

$\Rightarrow (4x^3) = \frac{dt}{dx}$

$\Rightarrow (x^3)dx = \frac{dt}{4}$

Putting this value in equation (i)

$I = \frac{1}{4}\int \frac{dt}{1+t^2}$ [1 + cosx = t]

$\Rightarrow I = \frac{1}{4}\tan^{-1}(t) + c$

$I = \frac{1}{4}\tan^{-1}(x^4) + c$

Ans) $\frac{1}{4}\tan^{-1}(x^4) + c$

Question: 62

Solution:

To find: Value of $\int \frac{x}{(1+x^4)}dx$

Formula used: $\int \frac{1}{1+x^2}dx = \tan^{-1}x$

We have, $I = \int \frac{x}{(1+x^4)}dx$ ... (i)

$I = \int \frac{x}{1+(x^2)^2} dx$

Let $x^2 = t$

$\Rightarrow \frac{d(x^2)}{dx} = \frac{dt}{dx}$

$\Rightarrow (2x) = \frac{dt}{dx}$

$\Rightarrow (x)dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \frac{1}{2}\int \frac{dt}{1+t^2}$ [$1 + \cos x = t$]

$\Rightarrow I = \frac{1}{2}\tan^{-1}(t) + c$

$I = \frac{1}{2}\tan^{-1}(x^2) + c$

Ans) $\frac{1}{2}\tan^{-1}(x^2) + c$

Question: 63

Solution:

To find: Value of $\int \frac{x^5}{\sqrt{1+x^3}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{x^5}{\sqrt{1+x^3}} dx$ ... (i)

Let $1 + x^3 = t$

$\Rightarrow x^3 = t - 1$

$\Rightarrow \frac{d(x^3)}{dx} = \frac{d(t-1)}{dx}$

$\Rightarrow (3x^2) = \frac{dt}{dx}$

$\Rightarrow x^2 \, dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \frac{x^3 x^2}{\sqrt{1 + x^3}} dx$

$I = \int \frac{(t-1)}{t^{\frac{1}{2}}} \frac{dt}{3}$ [$1 + x^3 = t$]

$\Rightarrow I = \frac{1}{3}\int \frac{t}{t^{\frac{1}{2}}} dt - \frac{1}{3}\int \frac{1}{t^{\frac{1}{2}}} dt$

$\Rightarrow I = \frac{1}{3}\left[\int t^{\frac{1}{2}}dt - \int t^{-\frac{1}{2}}dt\right]$

$\Rightarrow I = \frac{1}{3}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right]$

$\Rightarrow I = \frac{2}{3}\left[\frac{(1+x^3)^{\frac{3}{2}}}{3} - \frac{(1+x^3)^{\frac{1}{2}}}{1}\right]$

$\Rightarrow I = \frac{2(1+x^3)^{\frac{3}{2}}}{9} - \frac{2(1+x^3)^{\frac{1}{2}}}{3} + c$

Ans) $\frac{2(1+x^3)^{\frac{3}{2}}}{9} - \frac{2(1+x^3)^{\frac{1}{2}}}{3} + c$

Question: 64

Solution:

To find: Value of $\int \frac{x}{\sqrt{1+x}}dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{x}{\sqrt{1+x}}dx$ ... (i)

Let $1 + x = t$

$\Rightarrow x = t - 1$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int \frac{t-1}{\sqrt{t}}dx$ [$1 + x = t$]

$\Rightarrow I = \int \sqrt{t}dt - \int \frac{1}{\sqrt{t}}dt$

$\Rightarrow I = \left[\int t^{\frac{1}{2}}dt - \int t^{-\frac{1}{2}}dt\right]$

$\Rightarrow I = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right] + c$

$\Rightarrow I = 2\left[\frac{(1+x)^{\frac{3}{2}}}{3} - \frac{(1+x)^{\frac{1}{2}}}{1}\right] + c$

$\Rightarrow I = \frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$

Ans) $\frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$

Question: 65

Evaluate the foll

CLASS24

Solution:

To find: Value of $\int \frac{1}{x\sqrt{x^4-1}}dx$

Formula used: $\int \frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1}x + c$

We have, $I = \int \frac{1}{x\sqrt{x^4-1}}dx$ ... (i)

Multiplying numerator and denominator with x

$$I = \int \frac{x}{x^2\sqrt{(x^2)^2-1}}dx$$

Let $x^2 = t$

$$\Rightarrow 2x = \frac{dt}{dx}$$

$$\Rightarrow xdx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2-1}}\ [x^2 = t]$$

$$\Rightarrow I = \frac{1}{2}\sec^{-1}t + c$$

$$\Rightarrow I = \frac{1}{2}\sec^{-1}(x^2) + c$$

Ans) $\frac{1}{2}\sec^{-1}(x^2) + c$

Question: 66

Solution:

To find: Value of $\int x\sqrt{x-1}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{x-1}\,dx$ ... (i)

Let $x - 1 = t$

$x = t + 1$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$$I = \int (t+1)\sqrt{t}\,dt\ [x = t + 1]$$

$$\Rightarrow I = \int t\sqrt{t}dx + \int \sqrt{t}\,dx$$

$$\Rightarrow I = \int t^{\frac{3}{2}}dx + \int t^{\frac{1}{2}}dx$$

$$\Rightarrow I = \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$\Rightarrow I = \frac{2}{5}(x - 1)^{\frac{5}{2}} + \frac{2}{3}(x - 1)^{\frac{3}{2}} + c$

Ans) $\frac{2}{5}(x - 1)^{\frac{5}{2}} + \frac{2}{3}(x - 1)^{\frac{3}{2}} + c$

Question: 67

Solution:

To find: Value of $\int (1 - x)\sqrt{1 + x}\, dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int (1 - x)\sqrt{1 + x}\, dx$ ... (i)

Let $1 + x = t$

x = t - 1

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int \{1 - (t - 1)\}\sqrt{t}\, dt\ [\ x = t - 1\ ]$

$\Rightarrow I = \int \{1 - t + 1\}\sqrt{t}\, dt$

$\Rightarrow I = \int \{2 - t\}\sqrt{t}\, dt$

$\Rightarrow I = \int 2\sqrt{t}\, dt - \int t\sqrt{t}\, dt$

$\Rightarrow I = 2\int t^{\frac{1}{2}} dx - \int t^{\frac{3}{2}} dx$

$\Rightarrow I = 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + c$

$\Rightarrow I = \frac{4}{3}(1 + x)^{\frac{3}{2}} - \frac{2}{5}(1 + x)^{\frac{5}{2}} + c$

Ans) $\frac{4}{3}(1 + x)^{\frac{3}{2}} - \frac{2}{5}(1 + x)^{\frac{5}{2}} + c$

Question: 68

Solution:

To find: Value of $\int x\sqrt{x^2 - 1}\, dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{x^2 - 1}\, dx$ ... (i)

Let $x^2 - 1 = t$

$\Rightarrow 2x = \frac{dt}{dx}$

CLASS24

$\Rightarrow xdx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int \frac{1}{2}\sqrt{t}\,dt\ [\ x = x^2 - 1\ ]$

$\Rightarrow I = \frac{1}{2}\int t^{\frac{1}{2}}\,dx$

$\Rightarrow I = \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow I = \frac{1}{3}\ t^{\frac{3}{2}} + c$

$\Rightarrow I = \frac{1}{3}\ (x^2 - 1)^{\frac{3}{2}} + c$

Ans) $\frac{1}{3}\ (x^2 - 1)^{\frac{3}{2}} + c$

Question: 69

Solution:

To find: Value of $\int x\sqrt{3x - 2}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{3x - 2}\,dx$ ... (i)

Let $3x - 2 = t$

$\Rightarrow 3x = t + 2$

$\Rightarrow x = \frac{t + 2}{3}$

$\Rightarrow 3 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \left(\frac{t + 2}{3}\right)\sqrt{t}\,\frac{dt}{3}\ [\ t = 3x - 2\ ]$

$\Rightarrow I = \frac{1}{9}\left[\int t^{\frac{3}{2}}\,dx\ + 2\int t^{\frac{1}{2}}\,dx\right]$

$\Rightarrow I = \frac{1}{9}\left[\frac{t^{\frac{5}{2}}}{\frac{5}{2}} + 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$

$\Rightarrow I = \frac{1}{9}\left[\frac{2}{5}(3x - 2)^{\frac{5}{2}} + \frac{4}{3}(3x - 2)^{\frac{3}{2}}\right] + c$

$\Rightarrow I = \frac{2}{45}(3x - 2)^{\frac{5}{2}} + \frac{4}{27}(3x - 2)^{\frac{3}{2}} + c$

$\Rightarrow I = \frac{2}{45}(3x - 2)^{\frac{5}{2}} + \frac{4}{27}(3x - 2)^{\frac{3}{2}} + c$

Ans) $\frac{2}{45}(3x - 2)^{\frac{5}{2}} + \frac{4}{27}(3x - 2)^{\frac{3}{2}} + c$

Question: 70

Solution:

To find: Value of $\int \frac{dx}{x\cos^2(1+\log x)}$

Formula used: $\int \sec^2 x \, dx = \tan x + c$

We have, $I = \int \frac{dx}{x\cos^2(1+\log x)}$ ... (i)

Let $1 + \log x = t$

$\Rightarrow \frac{1}{x} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{x}dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{\cos^2(t)}$ [$t = 1 + \log x$]

$\Rightarrow I = \int \sec^2 t \, dt$

$\Rightarrow I = \tan(t) + c$

$\Rightarrow I = \tan(1 + \log x) + c$

Ans) $\tan(1 + \log x) + c$

Question: 71

Solution:

To find: Value of $\int x^2 \sin x^3 \, dx$

Formula used: $\int \sin x \, dx = -\cos x + c$

We have, $I = \int x^2 \sin x^3 \, dx$ ... (i)

Let $x^3 = t$

$\Rightarrow 3x^2 = \frac{dt}{dx}$

$\Rightarrow x^2 dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \sin t \frac{dt}{3}$ [$t = x^3$]

$\Rightarrow I = \frac{1}{3}\left[\int \sin t \, dt\right]$

CLASS24

$\Rightarrow I = \frac{1}{3}(-\cos t) + c$

$\Rightarrow I = \frac{1}{3}(-\cos x^3) + c$

Ans) $\frac{-\cos x^3}{3} + c$

Question: 72

Solution:

To find: Value of $\int (2x+4)\sqrt{x^2+4x+3}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int (2x+4)\sqrt{x^2+4x+3}\,dx$ ... (i)

Let $x^2 + 4x + 3 = t$

$\Rightarrow (2x+4) = \frac{dt}{dx}$

$\Rightarrow (2x+4)dx = dt$

Putting this value in equation (i)

$I = \int \sqrt{t}\,dt\ [\ t = (2x+4)\ ]$

$\Rightarrow I = \int t^{\frac{1}{2}}\,dx$

$\Rightarrow I = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow I = \frac{2}{3}\left[(t)^{\frac{3}{2}}\right] + c$

$\Rightarrow I = \frac{2}{3}\left[(x^2+4x+3)^{\frac{3}{2}}\right] + c$

Ans) $\frac{2}{3}\left[(x^2+4x+3)^{\frac{3}{2}}\right] + c$

Question: 73

Solution:

To find: Value of $\int \frac{\sin x}{(\sin x - \cos x)}\,dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{\sin x}{(\sin x - \cos x)}\,dx$ ... (i)

$\Rightarrow I = \frac{1}{2}\int \frac{2\sin x}{(\sin x - \cos x)}\,dx$

CLASS24

$\Rightarrow I = \frac{1}{2}\int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x - \cos x)}\,dx$

$\Rightarrow I = \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}\,dx + \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)}\,dx$

Let $\sin x - \cos x = t$

$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$

$\Rightarrow (\cos x + \sin x)dx = dt$

Putting this value in equation (i)

$I = \frac{1}{2}\int \frac{dt}{t} + \frac{1}{2}\int dx$

$\Rightarrow I = \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + c$

$\Rightarrow I = \frac{x}{2} + \frac{1}{2}\log|\sin x - \cos x| + c$

Ans) $\frac{x}{2} + \frac{1}{2}\log|\sin x - \cos x| + c$

Question: 74

Solution:

To find: Value of $\int \frac{dx}{(1 - \tan x)}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \tan x)}$ ... (i)

$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\sin x}{\cos x}\right)}$

$\Rightarrow I = \int \frac{dx}{\left(\frac{\cos x - \sin x}{\cos x}\right)}$

$\Rightarrow I = \frac{1}{2}\int \frac{2\cos x dx}{(\cos x - \sin x)}$

$I = \frac{1}{2}\int \frac{(\cos x + \sin x) + (\cos x - \sin x)dx}{(\cos x - \sin x)}$

$I = \frac{1}{2}\int \frac{(\cos x + \sin x)}{(\cos x - \sin x)}dx + \frac{1}{2}\int \frac{(\cos x - \sin x)}{(\cos x - \sin x)}dx$

Let $(\cos x - \sin x) = t$

$\Rightarrow (-\sin x - \cos x) = \frac{dt}{dx}$

$\Rightarrow (\sin x + \cos x)dx = -dt$

Putting this value in equation (i)

$I = -\frac{1}{2}\int \frac{dt}{(t)}\, dx + \frac{1}{2}\int dx$

$\Rightarrow I = -\frac{1}{2}\log|\cos x - \sin x| + \frac{1}{2}x + c$

$\Rightarrow I = \frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + c$

Ans) $\frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + c$

Question: 75

Solution:

To find: Value of $\int \frac{dx}{(1 - \cot x)}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \cot x)}$ ... (i)

$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\cos x}{\sin x}\right)}$

$\Rightarrow I = \int \frac{dx}{\left(\frac{\sin x - \cos x}{\sin x}\right)}$

$\Rightarrow I = \frac{1}{2}\int \frac{2\sin x\, dx}{(\sin x - \cos x)}$

$I = \frac{1}{2}\int \frac{(\sin x + \cos x) + (\sin x - \cos x)dx}{(\sin x - \cos x)}$

$I = \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx + \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)}dx$

Let $(\sin x - \cos x) = t$

$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$

$\Rightarrow (\cos x + \sin x)dx = dt$

Putting this value in equation (i)

$I = \frac{1}{2}\int \frac{dt}{(t)}\, dx + \frac{1}{2}\int dx$

$\Rightarrow I = \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + c$

Ans) $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$

Question: 76

Solution:

To find: Value of $\int \frac{\cos 2x}{(\sin x + \cos x)^2}dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{\cos 2x}{(\sin x + \cos x)^2}dx$ ... (i)

$$\Rightarrow I = \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2}dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2}dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)}dx$$

Let $(\cos x + \sin x) = t$

$$\Rightarrow (-\sin x + \cos x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x - \sin x)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$$\Rightarrow I = \log|t| + c$$

$$\Rightarrow I = \log|\cos x + \sin x| + c$$

Ans) $\log|\cos x + \sin x| + c$

Question: 77

Solution:

To find: Value of $\int \frac{(\cos x - \sin x)}{(1 + \sin 2x)}dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{(\cos x - \sin x)}{(1 + \sin 2x)}dx$ ... (i)

$$\Rightarrow I = \int \frac{\cos x - \sin x}{\cos^2 x + \sin^2 x + 2\sin x\cos x}dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\cos x + \sin x)^2}dx$$

Let $(\sin x + \cos x) = t$

$$\Rightarrow (\cos x - \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x - \sin x)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^2}$$

$$\Rightarrow I = -\frac{1}{t} + c$$

$\Rightarrow I = -\frac{1}{\sin x + \cos x} + c$

Ans) $\frac{-1}{\sin x + \cos x} + c$

Question: 78

Solution:

To find: Value of $\int \frac{(x+1)(x+\log x)^2}{x} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(x+1)(x+\log x)^2}{x} dx$ ... (i)

Let $(x + \log x) = t$

$\Rightarrow \left(1 + \frac{1}{x}\right) = \frac{dt}{dx}$

$\Rightarrow \left(\frac{x+1}{x}\right) = \frac{dt}{dx}$

Putting this value in equation (i)

$I = \int t^2 dt$

$\Rightarrow I = \frac{t^3}{3} + c$

$\Rightarrow I = \frac{(x+\log x)^3}{3} + c$

Ans) $\frac{(x+\log x)^3}{3} + c$

Question: 79

Solution:

To find: Value of $\int x\sin^3 x^2 \cos x^2 dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int x\sin^3 x^2 \cos x^2 dx$ ... (i)

Let $(\sin x^2) = t$

$\Rightarrow (\sin x^2 . 2x) = \frac{dt}{dx}$

$\Rightarrow (\sin x^2 . x)dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int t^3 \frac{dt}{2}$

$I = \frac{1}{2}\int t^3 dt$

$\Rightarrow I = \frac{1}{2}\frac{t^4}{4} + c$

$\Rightarrow I = \frac{t^4}{8} + c$

$\Rightarrow I = \frac{\sin^4 x^2}{8} + c$

Ans) $\frac{\sin^4 x^2}{8} + c$

Question: 80

Solution:

To find: Value of $\int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$

Formula used: $\int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + c$

We have, $I = \int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$ ... (i)

Let $(\tan x) = t$

$\Rightarrow (\sec^2 x) = \frac{dt}{dx}$

$\Rightarrow (\sec^2 x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{\sqrt{1 - t^2}}$

$\Rightarrow I = \sin^{-1}(t) + c$

$\Rightarrow I = \sin^{-1}(\tan x) + c$

Ans) $\sin^{-1}(\tan x) + c$

Question: 81

Solution:

To find: Value of $\int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5)\, dx$

Formula used: $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

We have, $I = \int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5)\, dx$ ... (i)

Let $(2e^{-x} + 5) = t$

$\Rightarrow (2e^{-x}(-1)) = \frac{dt}{dx}$

$\Rightarrow (e^{-x})dx = \frac{dt}{-2}$

Putting this value in equation (i)

$$I = \int \operatorname{cosec}^2(t)\,\frac{dt}{-2}$$

$$I = \frac{1}{-2}\int \operatorname{cosec}^2(t)\,dt$$

$$\Rightarrow I = \frac{1}{-2}(-\cot t) + c$$

$$\Rightarrow I = \frac{1}{2}\cot(2e^{-x} + 5) + c$$

Ans) $\frac{1}{2}\cot(2e^{-x} + 5) + c$

Question: 82

Solution:

To find: Value of $\int 2x\sec^3(x^2+3)\tan(x^2+3)dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int 2x\sec^2(x^2+3)\sec(x^2+3)\tan(x^2+3)dx$ ... (i)

Let $\sec(x^2+3) = t$

$$\Rightarrow \sec(x^2+3) = \frac{dt}{dx}$$

$$\Rightarrow \sec(x^2+3)\tan(x^2+3).2x = \frac{dt}{dx}$$

$$\Rightarrow \sec(x^2+3)\tan(x^2+3).2x = \frac{dt}{dx}$$

Putting this value in equation (i)

$$I = \int t^2\,dt$$

$$\Rightarrow I = \frac{t^3}{3} + c$$

$$\Rightarrow I = \frac{\sec^3(x^2+3)}{3} + c$$

Ans) $\frac{\sec^3(x^2+3)}{3} + c$

Question: 83

Solution:

To find: Value of $\int \frac{\sin 2x}{(a+b\cos x)^2}dx$

Formula used: (i) $\int \frac{1}{x}dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, ... (i)

$$I = \int \frac{\sin 2x}{(a+b\cos x)^2}dx$$

$I = \int \frac{2\sin x\cos x}{(a + b\cos x)^2} dx$

Let $(a + b\cos x) = t$

$\Rightarrow (\cos x) = \frac{t - a}{b}$

$\Rightarrow (\sin x)dx = \frac{dt}{-b}$

Putting this value in equation (i)

$I = \frac{2}{-b^2} \int \frac{t - a}{t^2} dt$

$I = \frac{2}{-b^2}\left[\int \frac{t}{t^2} dt - \int \frac{a}{t^2} dt\right]$

$I = \frac{2}{-b^2}\left[\int \frac{1}{t} dt - a\int \frac{1}{t^2} dt\right]$

$I = \frac{2}{-b^2}\left[\log|t| - a\left(-\frac{1}{t}\right) + c\right]$

$I = -\frac{2}{b^2}\left[\log|a + b\cos x| + \left(\frac{a}{a + b\cos x}\right)\right] + c$

Ans) $-\frac{2}{b^2}\left[\log|a + b\cos x| + \left(\frac{a}{a + b\cos x}\right)\right] + c$

Question: 84

Solution:

To find: Value of $\int \frac{dx}{(3 - 5x)}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(3 - 5x)}$ ... (i)

Let $(3 - 5x) = t$

$\Rightarrow (-5) = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{-5}$

Putting this value in equation (i)

$I = \int \frac{1}{t} \frac{dt}{-5}$

$I = \frac{1}{-5} \int \frac{dt}{t}$

$\Rightarrow I = \frac{1}{-5} \log|t| + c$

$\Rightarrow I = -\frac{1}{5} \log|3 - 5x| + c$

Ans) $-\frac{1}{5}\log|3 - 5x| + c$

Question: 85

Solution:

To find: Value of

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \sqrt{1+x}\, dx$ ... (i)

Let $(1 + x) = t$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int \sqrt{t}\, dt$

$I = \int t^{\frac{1}{2}}\, dt$

$\Rightarrow I = \frac{2}{3}(1+x)^{\frac{3}{2}} + c$

Ans) $\frac{2}{3}(1+x)^{\frac{3}{2}} + c$

Question: 86

Solution:

To find: Value of $\int x^2 e^{x^3} \cos(e^{x^3})dx$

Formula used: $\int \cos x\, dx = \sin x + c$

We have, $I = \int x^2 e^{x^3} \cos(e^{x^3})dx$ ... (i)

Let $e^{x^3} = t$

$\Rightarrow e^{x^3}.\, 3x^2 = \frac{dt}{dx}$

$\Rightarrow e^{x^3}.\, x^2.\, dx = \frac{dt}{3}$

Putting this value in equation (i)

$I = \int \cos(t)\, \frac{dt}{3}$

$I = \frac{\sin(t)}{3} + c$

$I = \frac{\sin\left(e^{x^3}\right)}{3} + c$

Ans) $\frac{\sin\left(e^{x^3}\right)}{3} + c$

Question: 87

Evaluate the foll

Solution:

To find: Value of $\int \frac{e^{m\tan^{-1}x}\,dx}{(1+x^2)}$

Formula used: $\int e^t dx = e^t + c$

We have, $I = \int \frac{e^{m\tan^{-1}x}\,dx}{(1+x^2)}$ ... (i)

Let $(m\tan^{-1}x) = t$

$\Rightarrow m\left(\frac{1}{1+x^2}\right) = \frac{dt}{dx}$

$\Rightarrow \left(\frac{1}{1+x^2}\right)dx = \frac{dt}{m}$

Putting this value in equation (i)

$I = \int e^t \frac{dt}{m}$

$\Rightarrow I = \frac{e^t}{m} + c$

$\Rightarrow I = \frac{e^{m\tan^{-1}x}}{m} + c$

Ans) $\frac{e^{m\tan^{-1}x}}{m} + c$

Question: 88

Solution:

To find: Value of $\int \frac{(x+1)e^x\,dx}{\cos^2(xe^x)}$

Formula used: $\int \sec^2 x\,dx = \tan x + c$

We have, $I = \int \frac{(x+1)e^x\,dx}{\cos^2(xe^x)}$ ... (i)

Let $(xe^x) = t$

$\Rightarrow xe^x + e^x.1 = \frac{dt}{dx}$

$\Rightarrow e^x(x+1) = \frac{dt}{dx}$

Putting this value in equation (i)

$I = \int \frac{dt}{\cos^2(t)}$

$\Rightarrow I = \int \sec^2(t)\,dt$

$\Rightarrow I = \tan(t) + c$

$\Rightarrow I = \tan(xe^x) + c$

Ans) $\tan(xe^x) + c$

Question: 89

Solution:

To find: Value of $\int \frac{e^{\sqrt{x}}\cos(e^{\sqrt{x}})dx}{\sqrt{x}}$

Formula used: $\int \cos x \, dx = \sin x + c$

We have, $I = \int \frac{e^{\sqrt{x}}\cos(e^{\sqrt{x}})dx}{\sqrt{x}}$ ... (i)

Let $(e^{\sqrt{x}}) = t$

$\Rightarrow e^{\sqrt{x}} \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$

Putting this value in equation (i)

$I = \int \cos(t) \, 2dt$

$I = 2\sin(e^{\sqrt{x}}) + c$

Ans) $2\sin(e^{\sqrt{x}}) + c$

Question: 90

Solution:

To find: Value of $\int \sqrt{e^x - 1}\, dx$

Formula used: $\int \frac{1}{x^2+1} dx = \tan^{-1} x + c$

We have, $I = \int \sqrt{e^x - 1}\, dx$ ... (i)

Let $(e^x - 1) = t^2$

$\Rightarrow e^x - 1 = t^2$

$\Rightarrow e^x = t^2 + 1$

$\Rightarrow e^x = \frac{2tdt}{dx}$

$\Rightarrow dx = \frac{2tdt}{e^x}$

$\Rightarrow dx = \frac{2t}{t^2+1} dt$

Putting this value in equation (i)

$I = \int \sqrt{t^2} \frac{2t}{t^2+1} dt$

$\Rightarrow I = \int \frac{2t^2}{t^2+1} dt$

$\Rightarrow I = 2\int \frac{t^2+1-1}{t^2+1} dt$

$\Rightarrow I = 2\int\left(1 - \frac{1}{t^2+1}\right)dt$

$\Rightarrow I = 2\,[\,t - \tan^{-1} t] + c$

$\Rightarrow I = 2\,[\,\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1}] + c$

Ans) $2\,[\,\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1}] + c$

Question: 91

Solution:

To find: Value of $\int \frac{dx}{(x - \sqrt{x})}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(x - \sqrt{x})}$ ... (i)

$\Rightarrow I = \int \frac{dx}{\sqrt{x}(\sqrt{x} - 1)}$

Let $(\sqrt{x} - 1) = t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{\sqrt{x}}dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int \frac{1}{t}\frac{dt}{2}$

$I = \frac{1}{2}\log|t| + c$

$I = \frac{1}{2}\log\left|\sqrt{x} - 1\right| + c$

Ans) $\frac{1}{2}\log\left|\sqrt{x} - 1\right| + c$

Question: 92

Solution:

To find: Value of $\int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)}dx$

Formula used: $\int \sec^2 x\,dx = \tan x + c$

We have, $I = \int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)}dx$ ... (i)

Let $2\tan^{-1}x = t$

$\Rightarrow \frac{2}{1+x^2} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{1+x^2}dx = \frac{dt}{2}$

Putting this value in equation (i)

$I = \int \sec^2(t)\frac{dt}{2}$

$I = \frac{1}{2}\tan(t) + c$

$I = \frac{1}{2}\tan(2\tan^{-1}x) + c$

Ans) $\frac{1}{2}\tan(2\tan^{-1}x) + c$

Question: 93

Solution:

To find: Value of $\int \left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$ ... (i)

Let $x + \sin^2 x = t$

$\Rightarrow 1 + 2\sin x.\cos x = \frac{dt}{dx}$

$\Rightarrow (1 + \sin 2x)\, dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$

$I = \log|t| + c$

$I = \log\left|x + \sin^2 x\right| + c$

Ans) $\log\left|x + \sin^2 x\right| + c$

Question: 94

Solution:

To find: Value of $\int \left(\frac{1-\tan x}{x+\log(\cos x)}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \left(\frac{1-\tan x}{x+\log(\cos x)}\right)dx$ ... (i)

Let $x + \log(\cos x) = t$

$\Rightarrow 1 + \frac{1.(-\sin x)}{\cos x} = \frac{dt}{dx}$

$\Rightarrow 1 - \tan x = \frac{dt}{dx}$

$\Rightarrow (1 - \tan x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$

$I = \log|t| + c$

$I = \log|x + \log(\cos x)| + c$

Ans) $\log|x + \log(\cos x)| + c$

Question: 95

Solution:

To find: Value of $\int \left(\frac{1 + \cot x}{x + \log(\sin x)}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \left(\frac{1 + \cot x}{x + \log(\sin x)}\right)dx$ ... (i)

Let $x + \log(\sin x) = t$

$\Rightarrow 1 + \frac{1.(\cos x)}{\sin x} = \frac{dt}{dx}$

$\Rightarrow 1 + \cot x = \frac{dt}{dx}$

$\Rightarrow (1 + \cot x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{dt}{t}$

$I = \log|x + \log(\sin x)| + c$

$I = \log|x + \log(\sin x)| + c$

Ans) $\log|x + \log(\sin x)| + c$

Question: 96

Solution:

To find: Value of $\int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)}dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)}dx$ ... (i)

Let $1 - \tan^2 x = t$

$\Rightarrow 0 - 2.\tan x.\sec^2 x = \frac{dt}{dx}$

$\Rightarrow (\tan x.\sec^2 x)dx = \frac{dt}{-2}$

$\Rightarrow (1 + \cot x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{1}{t}\frac{dt}{(-2)}$

$I = \frac{1}{2}\log|t| + c$

$I = \frac{1}{2}\log|1 - \tan^2 x| + c$

Ans) $\frac{1}{2}\log|1 - \tan^2 x| + c$

Question: 97

Solution:

To find: Value of $\int \frac{\sin(2\tan^{-1} x)}{(1+x^2)}dx$

Formula used: $\int \sin x\, dx = \cos x + c$

We have, $I = \int \frac{\sin(2\tan^{-1} x)}{(1+x^2)}dx$ ... (i)

Let $2\tan^{-1} x = t$

$\Rightarrow 2\frac{1}{1+x^2} = \frac{dt}{dx}$

$\Rightarrow \frac{dx}{1+x^2} = \frac{dt}{2}$

$\Rightarrow (1 + \cot x)dx = dt$

Putting this value in equation (i)

$I = \int \sin(t)\frac{dt}{(2)}$

$I = -\frac{1}{2}\cos(t) + c$

$I = -\frac{1}{2}\cos(2\tan^{-1} x) + c$

Ans) $-\frac{1}{2}\cos(2\tan^{-1} x) + c$

Question: 98

Solution:

To find: Value of $\int \frac{dx}{\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)}$

Formula used: (i) $\int \frac{1}{x}dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{dx}{\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)}$ ... (i)

Let $x = t^6$

$\Rightarrow x^{\frac{1}{6}} = t$

$\Rightarrow 6t^5\, dt = dx$

Putting this value in equation (i)

$$I = \int \frac{6t^5\, dt}{(t^3 + t^2)}$$

$$I = \int \frac{6t^5\, dt}{t^2(t+1)}$$

$$I = 6\int \frac{t^3\, dt}{(t+1)}$$

$$I = 6\int \frac{t^3 + 1 - 1}{(t+1)} dt$$

$$I = 6\int \frac{(t+1)(t^2 - t + 1)}{(t+1)} dt - \int \frac{1}{(t+1)} dt$$

$$I = 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + c$$

$$I = [2t^3 - 3t^2 + 6t - 6\log|t+1|] + c$$

$$I = \left[2\left(x^{\frac{1}{6}}\right)^3 - 3\left(x^{\frac{1}{6}}\right)^2 + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

$$I = \left[2\sqrt{x} - 3\left(x^{\frac{1}{3}}\right) + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

Ans) $\left[2\sqrt{x} - 3\left(x^{\frac{1}{3}}\right) + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$

Question: 99

Solution:

To find: Value of $\int (\sin^{-1} x)^2 dx$

Formula used: $\int \sin x\, dx = \cos x + c$

We have, $I = \int (\sin^{-1} x)^2 dx$ ... (i)

Let $\sin^{-1} x = t$, $x = \sin t$,

$\Rightarrow \cos t = \sqrt{1 - x^2}$

$$\Rightarrow \frac{1}{\sqrt{1 - x^2}} = \frac{dt}{dx}$$

$\Rightarrow \sqrt{1 - x^2}\, dt = dx$

$\Rightarrow \sqrt{1 - (\sin t)^2}\, dt = dx$

$\Rightarrow \sqrt{1 - \sin^2 t}\, dt = dx$

$\Rightarrow \cos t\, dt = dx$

Putting this value in equation (i)

$$I = \int t^2 \cos t\ dt$$

$$I = \int t^2 \cos t\ dt - \int \left[\frac{d(t^2)}{dt} \int \cos t\ dt\right] dt$$

$$I = t^2 \sin t - \int [2t. \sin t\,] dt$$

$$I = t^2 \sin t\ - 2\left\{\int t\,[\sin t\,] dt - \int \left[\frac{dt}{dt} \int \sin t\, dt\right]\ dt\right\}$$

$$I = t^2 \sin t\ - 2\left[-t\cos t + \int 1.\cos t\, dt\right]$$

$$I = t^2 \sin t\ + 2t\cos t - 2\sin t + c$$

$$I = (\sin^{-1} x)^2 x + 2(\sin^{-1} x)\sqrt{1 - x^2} - 2x + c$$

Ans) $(\sin^{-1} x)^2 x + 2(\sin^{-1} x)\sqrt{1 - x^2} - 2x + c$

Question: 100

Solution:

To find: Value of $\int \frac{2x\tan^{-1}(x^2)}{(1 + x^4)} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{2x\tan^{-1}(x^2)}{(1 + x^4)} dx$ ... (i)

Let $\tan^{-1}(x^2) = t$

$$\Rightarrow \frac{1}{1 + (x^2)^2} . 2x = \frac{dt}{dx}$$

$$\Rightarrow \frac{2x}{1 + x^4} dx = dt$$

Putting this value in equation (i)

$$I = \int t.\, dt$$

$$I = \frac{t^2}{2} + c$$

$$I = \frac{\{\tan^{-1}(x^2)\}^2}{2} + c$$

Ans) $\frac{\{\tan^{-1}(x^2)\}^2}{2} + c$

Question: 101

Solution:

To find: Value of $\int \frac{(x^2+1)}{(x^4+1)} dx$

Formula used: $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$

We have, $I = \int \frac{(x^2+1)}{(x^4+1)} dx$ ... (i)

Dividing Numerator and Denominator by x^2,

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}+2-2\right)} dx$$

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2 - 2.x.\frac{1}{x} + \left(\frac{1}{x}\right)^2 + 2\right)} dx$$

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(\left(x-\frac{1}{x}\right)^2 + (\sqrt{2})^2\right)} dx$$

Let $x - \frac{1}{x} = t$

$$\Rightarrow \left(1+\frac{1}{x^2}\right) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{1}{(t)^2 + (\sqrt{2})^2} dt$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$$

Ans) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$

Question: 102

Solution:

To find: Value of $\int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx$

Formula used: $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + c$

We have, $I = \int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}}dx$ ... (i)

Let $(\sin x - \cos x) = t$

$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$

$\Rightarrow (\cos x + \sin x)\, dx = dt$

$\Rightarrow t^2 = \sin^2 x - 2\sin x.\cos x + \cos^2 x$

$\Rightarrow t^2 = 1 - 2\sin x.\cos x$

$\Rightarrow 2\sin x.\cos x = 1 - t^2$

$\Rightarrow \sin 2x = 1 - t^2$

Putting this value in equation (i)

$\Rightarrow I = \int \frac{dt}{\sqrt{1-t^2}}$

$I = \sin^{-1} t$

$I = \sin^{-1}(\sin x - \cos x)$

Let $\sin^{-1}(\sin x - \cos x) = \theta$

$\Rightarrow I = \sin^{-1}(\sin x - \cos x) = \theta$ ... (ii)

$\Rightarrow \sin\theta = \sin x - \cos x$

Now if $\sin\theta = \sin x - \cos x$

Then $\cos\theta = \sqrt{1 - (\sin x - \cos x)^2}$

$\Rightarrow \cos\theta = \sqrt{1 - (\sin^2 x - 2\sin x.\cos x + \cos^2 x)}$

$\Rightarrow \cos\theta = \sqrt{1 - (1 - 2\sin x.\cos x)}$

$\Rightarrow \cos\theta = \sqrt{2\sin x.\cos x)}$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Now $\tan\theta = \frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}\right)$

Comparing the value θ from eqn. (ii)

$I = \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}\right)$

Dividing Numerator and denominator from cosx

$I = \theta = \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x)}}\right)$

Ans.) $\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x)}}\right)$

Exercise : OBJECTIVE QUESTIONS I

Question: 1

Solution:

Given = $\int (2x+3)^5$

Let, 2x + 3 = z

⇒ 2dx = dz

So,

$\int (2x+3)^5 dx$

$= \int \frac{z^5}{2} dz$

$= \frac{1}{2}\frac{z^6}{6} + c$ where c is the integrating constant.

$= \frac{z^6}{12} + c$

$= \frac{(2x+3)^6}{12} + c$

Question: 2

Solution:

Given = $\int (3-5x)^7$

Let, 3 − 5x = z

⇒ -5dx = dz

So,

$\int (3-5x)^7 dx$

$= -\int \frac{z^7}{5} dz$

$= -\frac{1}{5}\frac{z^8}{8} + c$ where c is the integrating constant.

$= -\frac{z^8}{40} + c$

$= -\frac{(3-5x)^8}{40} + c$

Question: 3

Solution:

Given =

$$\int \frac{1}{(2-3x)^4}$$

Let, 2 − 3x = z

⇒ -3dx = dz

So,

$$\int \frac{1}{(2-3x)^4}dx$$

$$= \int \frac{1}{z^4}\left(\frac{dz}{-3}\right)$$

$$= -\frac{1}{3}\int \frac{dz}{z^4}$$

$$= -\frac{1}{3}\int z^{-4}dz$$

$$= -\frac{1}{3}\frac{z^{-3}}{-3} + c$$

$$= \frac{1}{9(2-3x)^3} + c$$

where c is the integrating constant.

Question: 4

Solution:

Given = $\int \sqrt{ax+b}$

Let, ax + b = z^2

⇒ adx = 2zdz

So,

$$\int \sqrt{ax+b}dx$$

$$= \int z\frac{2zdz}{a}$$

$$= \frac{2}{a}\int z^2dz$$

$$= \frac{2}{a}\frac{z^3}{3} + c$$

$$= \frac{2}{3a}z^3 + c$$

$$= \frac{2(ax+b)^{3/2}}{3a} + c$$

where c is the integrating constant.

Question: 5

Solution:

Given = $\int \sec^2(7-4x)$

Let, 7 − 4x = z

⇒ -4dx = dz

So,

CLASS24

$$\int \sec^2(7-4x)dx$$

$$= \int \sec^2 z \frac{dz}{-4}$$

$$= -\frac{1}{4}\int \sec^2 z dz$$ where c is the integrating constant.

$$= -\frac{1}{4}\tan z + c$$

$$= -\frac{1}{4}\tan(7-4x) + c$$

Question: 6

Solution:

Given = $\int \cos 3x$

So, $\int \cos 3x dx = \frac{\sin 3x}{3} + c$ where c is the integrating constant.

Question: 7

Solution:

Given = $\int e^{(5-3x)}$

Let, 5 – 3x = z

⇒ -3dx = dz

So,

$$\int e^{(5-3x)}dx$$

$$= \int e^z \frac{dz}{-3}$$

$$= -\frac{1}{3}\int e^z dz$$ where c is the integrating constant.

$$= -\frac{1}{3}e^z + c$$

$$= -\frac{1}{3}e^{(5-3x)} + c$$

Question: 8

Solution:

Given = $\int e^{(3x-4)}$

Let, 3x + 4 = z

⇒ 3dx = dz

So,

$$\int e^{(3x-4)}dx$$

$$= \int e^z \frac{dz}{3}$$

$$= \frac{1}{3}\int e^z dz$$

$$= \frac{1}{3}e^z + c$$

$$= \frac{1}{3}e^{(3x-4)} + c$$

where c is the integrating constant.

Question: 9

Solution:

Given = $\int \tan^2 \frac{x}{2}$

Let, $\frac{x}{2} = z$

$\Rightarrow dx = 2dz$

So,

$$\int \tan^2 \frac{x}{2}dx$$

$$= 2\int \tan^2 z dz$$

$$= 2\int \frac{\sin^2 z}{\cos^2 z}dz$$

$$= 2\int \frac{1-\cos^2 z}{\cos^2 z}dz$$

$$= 2\int (\sec^2 z - 1)dz$$

$$= 2[\tan z - z] + c$$

$$= 2[\tan\frac{x}{2} - \frac{x}{2}] + c$$

where c is the integrating constant.

Question: 10

Solution:

Given = $\int \sqrt{1-\cos x}$

So,

$\int \sqrt{1-\cos x}dx$

$= \int \sqrt{1-\cos x}\frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}dx$

$= \int \frac{\sqrt{1-\cos^2 x}}{\sqrt{1+\cos x}}dx$

$= \int \frac{\sin x}{\sqrt{1+\cos x}}dx$

Let 1 + cosx = u^2

So, -sinxdx = 2udu

$-\int \frac{2u}{u}du = -2\int du = -2u + c = -2\sqrt{1+\cos x} + c$

where c is the integrating constant.

Question: 11

Solution:

Given = $\int \sqrt{1+\sin x}$

So,

$\int \sqrt{1+\sin x}dx$

$= \int \sqrt{1+\sin x}\frac{\sqrt{1-\sin x}}{\sqrt{1-\sin x}}dx$

$= \int \frac{\sqrt{1-\sin^2 x}}{\sqrt{1-\sin x}}dx$

$= \int \frac{\cos x}{\sqrt{1-\sin x}}dx$

Let 1 - sinx = u^2

So, -cosxdx = 2udu

$-\int \frac{2u}{u}du = -2\int du = -2u + c = -2\sqrt{1-\sin x} + c$

where c is the integrating constant.

Question: 12

Solution:

Given = $\int \sin^3 xdx$

So,

$\int \sin^3 xdx$

$= \int \sin^2 x \sin xdx$

$= \int (1-\cos^2 x)\sin xdx$

Let cosx = u

So, -sinxdx = du

$$-\int(1-u^2)du$$

$$= -\int du + \int u^2 du$$

$$= -u + \frac{u^3}{3} + c$$

$$= -\cos x + \frac{\cos^3 x}{3} + c$$

where c is the integrating constant.

Question: 13

Solution:

Given = $\int \frac{\log x}{x}$

Let, logx = u

So, $\frac{1}{x}dx = du$

So,

$$\int \frac{\log x}{x} dx$$

$$= \int u du$$

$$= \frac{u^2}{2} + c$$

$$= \frac{(\log x)^2}{2} + c$$

where c is the integrating constant.

Question: 14

Solution:

Given = $\int \frac{\sec^2(\log x)}{x}$

Let, logx = z

$\Rightarrow \frac{dx}{x} = dz$

So,

$$\int \frac{\sec^2(\log x)}{x}dx$$
$$= \int \sec^2 z dz$$
$$= \tan z + c$$
$$= \tan(\log x) + c$$

where c is the integrating constant.

Question: 15

Solution:

Given = $\int \frac{1}{x(\log x)}$

Let, logx = z

$$\Rightarrow \frac{dx}{x} = dz$$

So,

$$\int \frac{1}{x(\log x)}dx$$
$$= \int \frac{1}{z}dz$$
$$= \log z + c$$
$$= \log(\log x) + c$$

where c is the integrating constant.

Question: 16

Solution:

Given = $\int e^{x^3} x^2$

Let, $x^3 = z$

$\Rightarrow 3x^2dx = dz$

$$\Rightarrow x^2 dx = \frac{dz}{3}$$

So,

$$\int e^{x^3} x^2 dx$$
$$= \frac{1}{3}\int e^z dz$$
$$= \frac{1}{3}e^z + c$$
$$= \frac{1}{3}e^{x^3} + c$$

where c is the integrating constant.

Question: 17

Solution:

Given = $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}$

Let, x = z²

$\Rightarrow dx = 2zdz$

So,

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$$

$$= \int \frac{e^z}{z} 2zdz$$

$$= 2\int e^z dz$$

$$= 2e^z + c$$

$$= 2e^{\sqrt{x}} + c$$

where c is the integrating constant.

Question: 18

Solution:

Given = $\int \frac{e^{\tan^{-1}x}}{(1+x^2)}$

Let, $\tan^{-1}x = z$

$$\Rightarrow \frac{1}{1+x^2}dx = dz$$

So,

$$\int \frac{e^{\tan^{-1}x}}{(1+x^2)}dx$$

$$= \int e^z dz$$

$$= e^z + c$$

$$= e^{\tan^{-1}x} + c$$

where c is the integrating constant.

Question: 19

Solution:

Given =

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}$$

Let, $x = z^2$

$\Rightarrow dx = 2zdz$

So,

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$$

$$= \int \frac{\sin z}{z} 2zdz$$

$$= 2\int \sin z dz$$

$$= -2\cos z + c$$

$$= -2\cos\sqrt{x} + c$$

where c is the integrating constant.

Question: 20

Solution:

Given = $\int \left(\sqrt{\sin x}\right)\cos x$

Let, $\sin x = z^2$

$\Rightarrow \cos x dx = 2zdz$

So,

$$\int \left(\sqrt{\sin x}\right)\cos x dx$$

$$= 2\int z^2 dz$$

$$= 2\frac{z^3}{3} + c$$

$$= \frac{2}{3}\sin^{3/2} x + c$$

where c is the integrating constant.

Question: 21

Solution:

Given = $\int \frac{1}{\left(1+x^2\right)\sqrt{\tan^{-1} x}}$

Let, $\tan^{-1}x = z^2$

$$\Rightarrow \frac{1}{1+x^2}dx = 2zdz$$

So,

$$\int \frac{1}{(1+x^2)\sqrt{\tan^{-1}x}}dx$$

$$= \int \frac{2z}{z}dz$$

$$= 2\int dz$$

$$= 2z + c$$

$$= 2\sqrt{\tan^{-1}x} + c$$

where c is the integrating constant.

Question: 22

Solution:

Given =

$$\int \frac{\cot x}{\log(\sin x)}$$

Let, sinx = z

$\Rightarrow$ cosxdx = dz

So,

$$\int \frac{\cot x}{\log(\sin x)}dx$$

$$= \int \frac{\cos x}{\sin x \log(\sin x)}dx$$

$$= \int \frac{dz}{z \log z}$$

Let, logz = u

$$\Rightarrow \frac{1}{z}dz = du$$

So,

$$\int \frac{dz}{z \log z}$$

$$= \int \frac{du}{u}$$

$$= \log u + c$$

$$= \log|\log z| + c$$

where c is the integrating constant.

Question: 23

Solution:

Given =

$$\int \frac{1}{x \cos^2(1+\log x)}$$

Let, 1 + logx = z

$\Rightarrow \frac{1}{x}dx = dz$

So,

$\int \frac{1}{x\cos^2(1+\log x)}dx$

$= \int \frac{dz}{\cos^2 z}$

$= \int \sec^2 z dz$

$= \tan z + c$

$= \tan(1+\log x) + c$

where c is the integrating constant.

Question: 24

Solution:

Given = $\int \frac{x^2 \tan^{-1} x^3}{(1+x^6)} dx$

Let, $\tan^{-1}x^3 = z$

$\Rightarrow \frac{1}{1+x^6} \times 3x^2 dx = dz$

$\Rightarrow \frac{x^2}{1+x^6} dx = \frac{dz}{3}$

So,

$\frac{1}{3}\int z dz$

$= \frac{1}{3}\frac{z^2}{2} + c$

$= \frac{z^2}{6} + c$

$= \frac{(\tan^{-1} x^3)^2}{6} + c$

where c is the integrating constant.

Question: 25

Solution:

Given = $\int \sec^5 x \tan x$

So, $\int \sec^5 \tan x dx = \int \sec^4 x(\sec x \tan x)dx$

Let, secx = z

$\Rightarrow \sec x \tan x dx = dz$

$$\int \sec^4 x(\sec x \tan x)dx$$
$$= \int z^4 dz$$
$$= \frac{z^5}{5} + c$$
$$= \frac{\sec^5 x}{5} + c$$

where c is the integrating constant.

Question: 26

Solution:

Given = $\int \operatorname{cosec}^3(2x+1)\cot(2x+1)$

So,

$$\int \operatorname{cosec}^3(2x+1)\cot(2x+1)dx$$
$$= \int \operatorname{cosec}^2(2x+1)\operatorname{cosec}(2x+1)\cot(2x+1)dx$$

Let, cosec(2x + 1) = z

⇒ -2cosec(2x + 1)cot(2x + 1)dx = dz

$$\int \operatorname{cosec}^2(2x+1)\operatorname{cosec}(2x+1)\cot(2x+1)dx$$
$$= \int z^2 \frac{dz}{-2} =$$
$$= -\frac{1}{2}\frac{z^3}{3} + c$$
$$= -\frac{\operatorname{cosec}^6(2x+1)}{6} + c$$

where c is the integrating constant.

Question: 27

Solution:

Given = $\int \frac{\tan(\sin^{-1} x)}{\sqrt{1-x^2}}$

Let, $\sin^{-1}x = z$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dz$$

So,

$$\int \frac{\tan\left(\sin^{-1} x\right)}{\sqrt{1-x^2}} dx$$

$$= \int \tan z dz$$

$$= \log\left|\sec z\right| + c$$

$$= \log\left|\sec\left(\sin^{-1} x\right)\right| + c$$

where c is the integrating constant.

Question: 28

Solution:

Given = $\int \frac{\tan\left(\log x\right)}{x}$

Let, logx = z

$$\Rightarrow \frac{1}{x} dx = dz$$

So,

$$\int \frac{\tan\left(\log x\right)}{x} dx$$

$$= \int \tan z dz$$

$$= \log\left|\sec z\right| + c$$

$$= \log\left|\sec\left(\log x\right)\right| + c$$

$$= -\log\left|\cos\left(\log x\right)\right| + c$$

where c is the integrating constant.

Question: 29

Solution:

Given = $\int e^x \cot\left(e^x\right) dx$

Let, $e^x = z$

$\Rightarrow e^x dx = dz$

So,

$$\int e^x \cot\left(e^x\right) dx$$

$$= \int \cot z dz$$

$$= \log\left|\sin z\right| + c$$

$$= \log\left|\sin\left(e^x\right)\right| + c$$

where c is the integrating constant.

Question: 30

Mark (√) against

Solution:

Given = $\int \frac{e^x}{\sqrt{1+e^x}}$

Let, $1 + e^x = z^2$

$\Rightarrow e^x dx = 2zdz$

So,

$$\int \frac{e^x}{\sqrt{1+e^x}}dx$$

$$= \int \frac{2zdz}{z}$$

$$= 2\int dz$$

$$= 2z + c$$

$$= 2\sqrt{1+e^x} + c$$

where c is the integrating constant.

Question: 31

Solution:

Given = $\int \frac{x}{\sqrt{1-x^2}}dx$

Let, $1 - x^2 = z^2$

$\Rightarrow -2xdx = 2zdz$

So,

$$\int \frac{x}{\sqrt{1-x^2}}dx$$

$$= -\int \frac{zdz}{z}$$

$$= -\int dz$$

$$= -z + c$$

$$= -\sqrt{1-x^2} + c$$

where c is the integrating constant.

Question: 32

Solution:

Given = $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$

Let, $xe^x = z$

$\Rightarrow e^x(1 + x)dx = dz$

So,

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$$

$$= \int \frac{dz}{\cos^2 z}$$

$$= \int \sec^2 z dz$$

$$= \tan z + c$$

$$= \tan(xe^x) + c$$

where c is the integrating constant.

Question: 33

Solution:

Given =

$$\int \frac{dx}{(e^x + e^{-x})}$$

$$= \int \frac{e^x}{(e^x + 1)}dx$$

Let, $e^x + 1 = z$

$\Rightarrow e^x dx = dz$

So,

$$\int \frac{e^x dx}{(e^x + 1)}$$

$$= \int \frac{dz}{z}$$

$$= \log|z| + c$$

$$= \tan|e^x + 1| + c$$

where c is the integrating constant.

Question: 34

Solution:

Given =

$$\int \frac{2^x dx}{1 - 4^x}$$

$$= \int \frac{2^x}{1 - (2^x)^2}dx$$

Let, $2^x = z$

$\Rightarrow 2^x(\log 2)dx = dz$

So,

$$\int \frac{2^x dx}{1-(2^x)^2}$$

$$= \frac{1}{\log 2}\int \frac{dz}{1-z^2}$$

$$= \frac{1}{\log 2}\sin^{-1} z + c$$

$$= \frac{\sin^{-1} 2x}{\log 2} + c$$

where c is the integrating constant.

Question: 35

Solution:

Given =

$$\int \frac{dx}{e^x - 1}$$

$$= -\int \frac{-1+e^x-e^x}{e^x-1}dx$$

$$= -\int \frac{e^x-1}{e^x-1}dx + \int \frac{e^x}{e^x-1}dx$$

$$= -\int dx + \int \frac{e^x}{e^x-1}dx$$

Let, $e^x - 1 = z$

$\Rightarrow e^x dx = dz$

So,

$$-\int dx + \int \frac{e^x}{e^x-1}dx$$

$$= -x + \int \frac{dz}{z}$$

$$= -x + \log z + c$$

$$= -x + \log\left|e^x - 1\right| + c$$

where c is the integrating constant.

Question: 36

Solution:

Given =

$$\int \frac{dx}{\left(\sqrt{x}+x\right)}$$

$$= \int \frac{1}{\sqrt{x}} \frac{1}{\left(1+\sqrt{x}\right)} dx$$

Let, $1+\sqrt{x}=z$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dz$$

So,

$$\int \frac{1}{\sqrt{x}} \frac{1}{\left(1+\sqrt{x}\right)} dx$$

$$= 2\int \frac{dz}{z}$$

$$= 2\log|z| + c$$

$$= 2\tan\left|1+\sqrt{x}\right| + c$$

where c is the integrating constant.

Question: 37

Solution:

Given

$$\int \frac{dx}{1+\sin x}$$

$$= \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \int \frac{dx}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$$

$$= \int \frac{\sec^2\frac{x}{2} dx}{\left(\tan\frac{x}{2} + 1\right)^2}$$

Let, $\tan\frac{x}{2} + 1 = z$

$$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2} dx = dz$$

So,

$$\int \frac{2dz}{z^2}$$

$$= -\frac{2}{z} + c$$

$$= -\frac{2}{\tan\frac{x}{2} + 1} + c$$

where c is the integrating constant.

Question: 38

Solution:

Given

$$\int \frac{\sin x}{1+\sin x} dx$$

$$= \int dx - \int \frac{dx}{1+\sin x}$$

$$= x - \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= x - \int \frac{dx}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$$

$$= x - \int \frac{\sec^2\frac{x}{2} dx}{\left(\tan\frac{x}{2} + 1\right)^2}$$

Let, $\tan\frac{x}{2} + 1 = z$

$$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2} dx = dz$$

So,

$$x - \int \frac{2dz}{z^2}$$

$$= x + \frac{2}{z} + c$$

$$= x + \frac{2}{\tan\frac{x}{2} + 1} + c$$

where c is the integrating constant.

Question: 39

Solution:

Given

CLASS24

$$\int \frac{\sin x}{1-\sin x}dx$$

$$= -\int dx + \int \frac{dx}{1-\sin x}$$

$$= -x + \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= -x + \int \frac{dx}{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2}$$

$$= -x + \int \frac{\sec^2\frac{x}{2}dx}{\left(\tan\frac{x}{2} - 1\right)^2}$$

Let, $\tan\frac{x}{2} - 1 = z$

$$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dz$$

So,

$$-x + \int \frac{2dz}{z^2}$$

$$= -x - \frac{2}{z} + c$$

$$= -x - \frac{2}{\tan\frac{x}{2} + 1} + c$$

where c is the integrating constant.

Question: 40

Solution:

Given

$$\int \frac{dx}{1+\cos x}$$

$$= \int \frac{dx}{1+2\cos^2\frac{x}{2}-1}$$

$$= \frac{1}{2}\int \frac{dx}{\cos^2\frac{x}{2}}$$

$$= \frac{1}{2}\int \sec^2\frac{x}{2}dx$$

$$= \frac{1}{2}2\tan\frac{x}{2}+c$$

$$= \tan\frac{x}{2}+c$$

where c is the integrating constant.

Question: 41

Solution:

Given

$$\int \frac{dx}{1-\cos x}$$

$$= \int \frac{dx}{1-1+2\sin^2\frac{x}{2}}$$

$$= \frac{1}{2}\int \frac{dx}{\sin^2\frac{x}{2}}$$

$$= \frac{1}{2}\int \operatorname{cosec}^2\frac{x}{2}dx$$

$$= -\frac{1}{2}2\cot\frac{x}{2}+c$$

$$= -\cot\frac{x}{2}+c$$

where c is the integrating constant.

Question: 42

Solution:

Given

$$\int \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}dx$$

$$=\int \frac{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}dx$$

$$=\int \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}dx$$

Let, $\cos\frac{x}{2}+\sin\frac{x}{2}=z$

$$\Rightarrow\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)dx=dz$$

So,

$$\int \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}dx$$

$$=\int \frac{dz}{z}$$

$$=\log z+c$$

$$=\log\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+c$$

where c is the integrating constant.

Question: 43

Solution:

Given

$$\int \sqrt{e^x}dx$$

$$=\int (e^x)^{\frac{1}{2}}dx$$

$$=\int e^{\frac{1}{2}x}dx$$

$$=2e^{\frac{1}{2}x}+c$$

$$=2\sqrt{e^x}+c$$

where c is the integrating constant.

Question: 44

Solution:

Given

$$\int \frac{\cos x dx}{1+\cos x}$$

$$= \int \frac{1+\cos x - 1}{1+\cos x} dx$$

$$= \int dx - \int \frac{dx}{1+\cos x}$$

$$= x - \tan\frac{x}{2} + c$$

[From Question no. 40] where c is the integrating constant.

Question: 45

Solution:

Given

$$\int \sec^2 x \operatorname{cosec}^2 x dx$$

$$= \int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + c$$

where c is the integrating constant.

Question: 46

Solution:

Given

$$\int \frac{(1-\cos 2x)}{(1+\cos 2x)} dx$$

$$= \int \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx$$

$$= \int \tan^2 \frac{x}{2} dx$$

$$= \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= 2\tan \frac{x}{2} - x + c$$

where c is the integrating constant.

Question: 47

Solution:

Given

$$\int \frac{(1+\cos 2x)}{(1-\cos 2x)} dx$$

$$= \int \frac{2\cos^2 \frac{x}{2}}{2\sin^2 \frac{x}{2}} dx$$

$$= \int \cot^2 \frac{x}{2} dx$$

$$= \int \left(\operatorname{cosec}^2 \frac{x}{2} - 1 \right) dx$$

$$= -2\cot \frac{x}{2} - x + c$$

where c is the integrating constant.

Question: 48

Solution:

Given

$$\int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + c$$

where c is the integrating constant.

Question: 49

Solution:

Given

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\sin^2 x} dx - \int \frac{1}{\cos^2 x} dx$$

$$= \int \operatorname{cosec}^2 x dx - \int \sec^2 x dx$$

$$= -\tan x - \cot x + c$$

where c is the integrating constant.

Question: 50

Solution:

Given

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx$$

$$= \int \frac{-2\sin\left(\frac{2x+2\alpha}{2}\right)\sin\left(\frac{2x-2\alpha}{2}\right)}{-2\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \int \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \int \frac{2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\times 2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 2\int 2\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\int \cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\left(\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right)$$

$$= 2\int(\cos x + \cos\alpha)dx$$

$$= 2[\sin x + x\cos\alpha] + c$$

where c is the integrating constant.

Question: 51

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $1 + \cos 2x = 2\cos^2 x$; $1 - \cos 2x = 2\sin^2 x$

Therefore ,

$\Rightarrow \int \tan^{-1} \sqrt{\frac{1-\cos 2x}{1+\cos 2x}} dx = \int \tan^{-1} \sqrt{\frac{2\sin^2 x}{2\cos^2 x}} dx = \int \tan^{-1} \tan x \, dx$

$\Rightarrow \int x \, dx = \frac{x^2}{2} + c$

Question: 52

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $1 + \sin x = (\cos \frac{x}{2} + \sin \frac{x}{2})^2$

$\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$; $\tan (a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

Therefore ,

$\Rightarrow \int \tan^{-1}(\sec x + \tan x) \, dx = \int \tan^{-1} \left(\frac{1+\sin x}{\cos x}\right) dx$

$\Rightarrow \int \tan^{-1} \frac{(\cos \frac{x}{2}+\sin \frac{x}{2})^2}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}} dx = \int \tan^{-1} \frac{(\cos \frac{x}{2}+\sin \frac{x}{2})^2}{(\cos \frac{x}{2}+\sin \frac{x}{2})(\cos \frac{x}{2}-\sin \frac{x}{2})} dx$

$\Rightarrow \int \tan^{-1} \frac{(\cos \frac{x}{2}+\sin \frac{x}{2})^1}{(\cos \frac{x}{2}-\sin \frac{x}{2})} dx = \int \tan^{-1} \frac{1+\tan \frac{x}{2}}{1-\tan \frac{x}{2}} dx$

(Multiply by $\sec \frac{x}{2}$ in numerator and denominator)

$\Rightarrow \int \tan^{-1} \frac{1+\tan \frac{x}{2}}{1-\tan \frac{x}{2}} dx = \int \tan^{-1} \frac{\tan \frac{\pi}{4} + \tan \frac{x}{2}}{\tan \frac{\pi}{4} - \tan \frac{\pi}{4} \tan \frac{x}{2}} dx = \int \tan^{-1} \tan(\frac{\pi}{4} + \frac{x}{2}) \, dx$

$\Rightarrow \int (\frac{\pi}{4} + \frac{x}{2}) dx = \frac{\pi x}{4} + \frac{x^2}{4} + c$

Question: 53

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x dx = \tan x$

Therefore ,

$\Rightarrow \int \frac{1+\sin x(1+\sin x)}{1-\sin x(1+\sin x)} dx$

$\Rightarrow \int \frac{(1+\sin x)^2}{1-\sin^2 x} dx = \int \frac{1+\sin^2 x + 2\sin x}{\cos^2 x} dx$

$\Rightarrow \int \frac{1}{\cos^2 x} dx + 2\int \frac{\sin x}{\cos^2 x} dx + \int \frac{\sin^2 x}{\cos^2 x} dx$

$\Rightarrow \int \sec^2 x \, dx + 2\int \frac{\sin x}{\cos^2 x} dx + \int \tan^2 x \, dx$

$\Rightarrow \int \sec^2 x \, dx + 2\int \frac{\sin x}{\cos^2 x} dx + \int (-1 + \sec^2 x) \, dx$

$\Rightarrow 2\int \sec^2 x \, dx + 2\int \frac{\sin x}{\cos^2 x} dx - \int 1 \, dx$

Put cos x = t

Therefore -> sin x dx = - dt

$\Rightarrow 2\tan x - 2\int \frac{dt}{t^2} - x + c$

$\Rightarrow 2\tan x + 2\frac{1}{t} - x + c$

$\Rightarrow 2\tan x + 2\sec x - x + c$

Question: 54

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = tan\, x$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{x^4+1-1}{1+x^2} dx$

$\Rightarrow \int \frac{x^4-1}{1+x^2} dx + \int \frac{1}{1+x^2} dx = \int \frac{(1+x^2)(x^2-1)}{1+x^2} dx + \int \frac{1}{1+x^2} dx$

$\Rightarrow \int x^2 - 1\, dx + \int \frac{1}{1+x^2} dx$

$\Rightarrow \frac{x^3}{3} - x + \tan^{-1} x + c$

Question: 55

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int cot\, x = log\,(sin\, x) + c$

Therefore ,

$\Rightarrow \int \frac{\sin(x+\propto-2\propto)}{\sin(x+\propto)} dx$

$\Rightarrow \int \frac{\sin(x+\propto)\cos(-2\propto)+\cos(x+\propto)\sin(-2\propto)}{\sin(x+\propto)} dx$

$\Rightarrow \int \cos(2\propto)\, dx - \sin 2\propto \int \cot(x+\propto)\, dx$

$\Rightarrow \cos(2\propto)\, x - \sin 2\propto \log|\sin(x+\propto)| + c$

Question: 56

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int cot\, x = log\,(sin\, x) + c$

Therefore ,

$\Rightarrow \int \frac{(\sqrt{x+3}+\sqrt{x+2})}{(\sqrt{x+3}-\sqrt{x+2})(\sqrt{x+3}+\sqrt{x+2})} dx$ (Rationalizing the denominator)

$\Rightarrow \int (\sqrt{x+3} + \sqrt{x+2}) dx$

$\Rightarrow \int \sqrt{x+3}\, dx + \int \sqrt{x+2}\, dx$

$\Rightarrow \frac{2(x+3)^{\frac{3}{2}}}{3} + \frac{2(x+2)^{\frac{3}{2}}}{3} + c$

CLASS24

Question: 57

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$

Put cos x - sin x = t

(- sin x - cos x) dx = dt

(sin x + cos x) dx = -dt

$\Rightarrow \int \frac{-dt}{t} = -\log t + c$

$\Rightarrow -\log|\cos x - \sin x| + c$

Question: 59

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^3 = t$ $3x^2 dx = dt$

$\Rightarrow \int \frac{dt}{1+t^2}$

$\Rightarrow \tan^{-1} t + c$

$\Rightarrow \tan^{-1} x^3 + c$

Question: 59

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

Therefore ,

Put $x^3 = t$, $3x^2 dx = dt$

$\Rightarrow \int \frac{dt}{x \times 3x^2 \sqrt{t^2-1}} = \int \frac{dt}{3t\sqrt{t^2-1}}$

$\Rightarrow \frac{1}{3} \int \frac{dt}{t\sqrt{t^2-1}}$

$\Rightarrow \frac{1}{3} \sec^{-1} t + c$

$\Rightarrow \frac{1}{3}\sec^{-1}x^3 + c$

Question: 60

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}x + c$

Therefore ,

Put $x^2 + x + 1 = t$, $(2x + 1)dx = dt$

$\Rightarrow \int \sqrt{t}dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow \frac{2}{3}t^{\frac{3}{2}} + c$

$\Rightarrow \frac{2}{3}(x^2 + x + 1)^{\frac{3}{2}} + c$

Question: 61

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a + b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{(\sqrt{2x+3}-\sqrt{2x-3})}{(\sqrt{2x+3}+\sqrt{2x-3})(\sqrt{2x+3}-\sqrt{2x-3})} dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{6} dx$

$\Rightarrow \frac{1}{6}\int \sqrt{2x+3}\, dx - \frac{1}{6}\int \sqrt{2x-3}\, dx$

$\Rightarrow \frac{2(2x+3)^{\frac{3}{2}}}{3\times6\times2} - \frac{2(2x-3)^{\frac{3}{2}}}{3\times6\times2} + c$

$\Rightarrow \frac{(2x+3)^{\frac{3}{2}}}{18} - \frac{(2x-3)^{\frac{3}{2}}}{18} + c$

Question: 62

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a + b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{\sin x}{\cos x} dx$

Put cos x = t -sin x dx = dt

$\Rightarrow \int \frac{-dt}{t}$

$\Rightarrow -\log t + c$

$\Rightarrow -\log|\cos x| + c$

Question: 63

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx$

$\Rightarrow \int \frac{sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$

Put $\sec x + \tan x = t$, $(sec^2 x + \sec x \tan x)dx = dt$

$\Rightarrow \int \frac{dt}{t}$

$\Rightarrow \log t + c$

$\Rightarrow \log|\sec x + \tan x| + c$

Question: 64

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \operatorname{cosec} x \frac{\operatorname{cosec} x - \cot x}{\operatorname{cosec} x - \cot x} dx$

$$\Rightarrow \int \frac{cosec^2 x - \operatorname{cosec} x \cot x}{\operatorname{cosec} x - \cot x} dx$$

Put $\operatorname{cosec} x - \cot x = t$, $(cosec^2 x - \operatorname{cosec} x \cot x)dx = dt$

$$\Rightarrow \int \frac{dt}{t}$$

$\Rightarrow \log t + c$

$\Rightarrow \log|\operatorname{cosec} x - \cot x| + c$

Question: 65

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = \tan x$

Therefore ,

$\Rightarrow \int \frac{1+\sin x}{2cos^2\frac{x}{2}}dx$

CLASS24

$\Rightarrow \int \frac{1}{2cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2cos^2\frac{x}{2}}dx = \frac{1}{2}\int sec^2\frac{x}{2}dx + \int \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}dx$

$\Rightarrow \frac{1}{2}\tan\frac{x}{2} \times 2 + \int tan\ \frac{x}{2}dx$

$\Rightarrow \tan\frac{x}{2} + 2\left(-\log\cos\frac{x}{2}\right) + c$

$\Rightarrow \tan\frac{x}{2} - 2\log|\cos\frac{x}{2}| + c$

Question: 66

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = tan\ x$

Therefore ,

$\Rightarrow \int \frac{\sec x \tan x}{sec^2 x+1}dx$

Put sec x = t (sec x tan x) dx = dt

$\Rightarrow \int \frac{dt}{1+t^2} = \tan^{-1} t + c$

$\Rightarrow \tan^{-1}\sec x + c$

$\Rightarrow -\tan^{-1}(\cos x) + c$

Question: 67

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = tan\ x$

Therefore ,

$\Rightarrow \int \sqrt{\frac{(1+x)^2}{(1+x)(1-x)}}dx$

$\Rightarrow \int \frac{1+x}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}dx + \int \frac{x}{\sqrt{1-x^2}}dx$

Put $1 - x^2 = t$ -2x dx = dt

$\Rightarrow \sin^{-1} x - \frac{1}{2}\int \frac{1}{\sqrt{t}}dt + c$

$\Rightarrow \sin^{-1} x - \frac{1}{2}\frac{\sqrt{t}}{\frac{1}{2}} + c$

$\Rightarrow \sin^{-1} x - \sqrt{t} + c \quad = \quad \sin^{-1} x - \sqrt{1-x^2} + c$

Question: 68

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = tan\ x$

Therefore ,

Put $-\frac{1}{x}=t\ \frac{1}{x^2}dx=dt$

$\Rightarrow \int e^t dt$

$\Rightarrow e^t+c$

$\Rightarrow e^{-\frac{1}{x}}+c$

Question: 69

Solution:

Formula :- $\int x^n dx=\frac{x^{n+1}}{n+1}+c\ ;\ \int\frac{1}{1+x^2}dx=\tan^{-1}x+c$

Therefore ,

Put $x^4=t\ 4x^3dx=dt$

$\Rightarrow \frac{1}{4}\int\frac{1}{1+t^2}dt$

$\Rightarrow \frac{1}{4}\tan^{-1}t+c$

$\Rightarrow \frac{1}{4}\tan^{-1}x^4+c$

Question: 70

Solution:

Formula :- $\int x^n dx=\frac{x^{n+1}}{n+1}+c\ ;\ \int\frac{1}{1+x^2}dx=\tan^{-1}x+c$

Therefore ,

Put $x^1+\log x=t\ (1+\frac{1}{x})dx=dt\Rightarrow(\frac{x+1}{x})dx=dt$

$\Rightarrow \int t^2 dt$

$\Rightarrow \frac{t^3}{3}+c$

$\Rightarrow \frac{(x+\log x)^3}{3}+c$

Question: 71

Solution:

Formula :- $\int x^n dx=\frac{x^{n+1}}{n+1}+c\ ;\ \int\frac{1}{1+x^2}dx=\tan^{-1}x+c$

Therefore ,

Put $\tan^{-1}x^2=t\ (\frac{1}{1+(x^2)^2}\times 2x)dx=dt\Rightarrow(\frac{2x}{1+x^4})dx=dt$

$\Rightarrow \int t^1 dt$

$\Rightarrow \frac{t^2}{2}+c$

$\Rightarrow \frac{(\tan^{-1}x^2)^2}{2}+c$

Question: 72

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x^1} dx = \log x + c$

Therefore ,

Put $2 - 3x = t$ $-3dx = dt$

$\Rightarrow -\frac{1}{3}\int \frac{1}{t} dt$

$\Rightarrow -\frac{1}{3}\log t + c$

$\Rightarrow -\frac{1}{3}\log|2 - 3x| + c$

Question: 73

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x^1} dx = \log x + c$

Therefore ,

Put $x^2 - 1 = t$ $2xdx = dt$

$\Rightarrow \int \sqrt{t} dt$

$\Rightarrow \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c \Rightarrow \frac{t^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{(x^2-1)^{\frac{3}{2}}}{3} + c$

Question: 74

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int a^x dx = \frac{a^x}{\log a} + c$

Therefore ,

Put $5 - 3x = t$ $-3dx = dt$

$\Rightarrow -\frac{1}{3}\int 3^t dt$

$\Rightarrow -\frac{1}{3} \times \frac{3^t}{\log 3} + c \Rightarrow -\frac{1}{3} \times \frac{3^{(5-3x)}}{\log 3} + c$

$\Rightarrow -\frac{3^{(5-3x)}}{3\log 3} + c$

Question: 75

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\tan x = t$ $sec^2 x dx = dt$

$\Rightarrow \int e^t dt$

$\Rightarrow e^t + c \Rightarrow e^{\tan x} + c$

CLASS24

Question: 76

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $cos^2 x = t \Rightarrow 2\cos x\,(-\sin x)dx = dt \Rightarrow -\sin 2x\ dx = dt$

$\Rightarrow -\int e^t dt$

$\Rightarrow -e^t + c \Rightarrow -e^{cos^2 x} + c$

Question: 77

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\sin x^2 = t \Rightarrow 2x\cos x^2\, dx = dt$

$\Rightarrow \frac{1}{2}\int t^3 dt$

$\Rightarrow \frac{1}{2}\frac{t^4}{4} + c \Rightarrow \frac{t^4}{8} + c$

$\Rightarrow \frac{(\sin x^2)^4}{8} + c$

Question: 78

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $\sin e^{\sqrt{x}} = t \Rightarrow (\cos e^{\sqrt{x}}) \times (e^{\sqrt{x}}) \times (\frac{1}{2\sqrt{x}})dx = dt$

$\Rightarrow \int 2dt$

$\Rightarrow 2t + c \Rightarrow 2\sin e^{\sqrt{x}} + c$

Question: 79

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$\Rightarrow \frac{1}{3}\int \sin t\, dt$

$\Rightarrow -\frac{1}{3}\cos t + c \Rightarrow -\frac{1}{3}\cos x^3 + c$

Question: 80

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $xe^x = t \Rightarrow (e^x + xe^x)dx = dt \Rightarrow e^x(1+x)dx = dt$

$\Rightarrow \int \frac{dt}{cos^2 t} \Rightarrow \int sec^2 t\, dt = \tan t + c$

$\Rightarrow \tan(xe^x) + c$

Question: 81

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

Put $x^2 = t \Rightarrow 2xdx = dt$

$\Rightarrow \int \frac{1}{x\sqrt{t^2-1}} \times \frac{dt}{2x} \Rightarrow \frac{1}{2}\int \frac{1}{t\sqrt{t^2-1}} dt$

$\Rightarrow \frac{1}{2}\sec^{-1} t + c \Rightarrow \frac{1}{2}\sec^{-1} x^2 + c$

Question: 82

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

Put $x - 1 = t \Rightarrow x = t + 1 \Rightarrow dx = dt$

$\Rightarrow \int (t+1) \times \sqrt{t} dt \Rightarrow \int t^{\frac{3}{2}} dt + \int t^{\frac{1}{2}} dt$

$\Rightarrow \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c \Rightarrow \frac{2t^{\frac{5}{2}}}{5} + \frac{2t^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{2(x-1)^{\frac{5}{2}}}{5} + \frac{2(x-1)^{\frac{3}{2}}}{3} + c$

Question: 83

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

$\Rightarrow \int x\sqrt{x^2-1} dx$

Put $x^2 - 1 = t \Rightarrow 2xdx = dt$

$\Rightarrow \int \sqrt{t}\frac{dt}{2} \Rightarrow \frac{1}{2}\int \frac{t^{\frac{3}{2}}}{\frac{3}{2}}dt$

$\Rightarrow \frac{t^{\frac{3}{2}}}{3} + c \Rightarrow \frac{(x^2-1)^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{1}{3}(x^2-1)^{\frac{3}{2}} + c$

Question: 84

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}}dt = \sec^{-1} t + c$

Therefore ,

$\Rightarrow \int \frac{1}{1+\sqrt{x}}dx$

Put $x = t^2 \Rightarrow dx = 2tdt$

$\Rightarrow \int \frac{2t}{1+t}dt \Rightarrow 2\int \frac{t}{1+t}dt \Rightarrow 2\int \frac{t+1-1}{1+t}dt \Rightarrow 2\int dt - 2\int \frac{1}{1+t}dt$

$\Rightarrow 2t - 2\log(1+t) + c \Rightarrow 2\sqrt{x} - 2\log(1+\sqrt{x}) + c$

Question: 85

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\Rightarrow \int \sqrt{e^x - 1}dx$

Put $e^x - 1 = t \Rightarrow e^x dx = dt$

$\Rightarrow \int \sqrt{t}\frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t}dt$

Put $t = z^2$ dt = 2z dz

$\Rightarrow \int \frac{2z^2}{1+z^2}dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2}dz \Rightarrow 2\int \frac{1+z^2}{1+z^2}dz - 2\int \frac{1}{1+z^2}dz$

$\Rightarrow 2\int dz - 2\int \frac{1}{1+z^2}dz \Rightarrow 2z - 2\tan^{-1} z + c$

$\Rightarrow 2\sqrt{t} - 2\tan^{-1}\sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c$

Question: 86

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

We can write $\sin x = \frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]$

$\Rightarrow \int \frac{\frac{1}{2}[(\sin x - \cos x)+(\sin x + \cos x)]}{(\sin x - \cos x)}dx$

$\Rightarrow \frac{1}{2}\int\frac{(\sin x-\cos x)}{(\sin x-\cos x)}dx+\frac{1}{2}\int\frac{(\sin x+\cos x)}{(\sin x-\cos x)}dx$

$\Rightarrow \frac{1}{2}\int dx+\frac{1}{2}\int\frac{(\sin x+\cos x)}{(\sin x-\cos x)}dx \Rightarrow \frac{x}{2}+\frac{1}{2}\int\frac{(\sin x+\cos x)}{(\sin x-\cos x)}dx$

Put $(\sin x-\cos x)=t\ (\sin x+\cos x)\,dx=dt$

$\Rightarrow \frac{x}{2}+\frac{1}{2}\int\frac{1}{t}dt \Rightarrow \frac{x}{2}+\frac{1}{2}\log t+c \Rightarrow \frac{1}{2}x+\frac{1}{2}\log|\sin x-\cos x|+c$

Question: 87

Solution:

Formula :- $\int x^n dx=\frac{x^{n+1}}{n+1}+c\ ;\ \int e^x dx=e^x+c$

Therefore ,

$\Rightarrow \int\frac{1}{1-\frac{\sin x}{\cos x}}dx \Rightarrow \int\frac{\cos x}{\cos x-\sin x}dx$

We can write $\cos x=\frac{1}{2}[(\cos x-\sin x)+(\sin x+\cos x)]$

$\Rightarrow \int\frac{\frac{1}{2}[(\cos x-\sin x)+(\sin x+\cos x)]}{(\cos x-\sin x)}dx$

$\Rightarrow \frac{1}{2}\int\frac{(\cos x-\sin x)}{\cos x-\sin x}dx+\frac{1}{2}\int\frac{(\sin x+\cos x)}{\cos x-\sin x}dx$

$\Rightarrow \frac{1}{2}\int dx+\frac{1}{2}\int\frac{(\sin x+\cos x)}{\cos x-\sin x}dx \Rightarrow \frac{x}{2}+\frac{1}{2}\int\frac{(\sin x+\cos x)}{\cos x-\sin x}dx$

Put $(\cos x-\sin x)=t\ (\sin x+\cos x)\,dx=-dt$

$\Rightarrow \frac{x}{2}-\frac{1}{2}\int\frac{1}{t}dt \Rightarrow \frac{x}{2}-\frac{1}{2}\log t+c \Rightarrow \frac{1}{2}x-\frac{1}{2}\log|\cos x-\sin x|+c$

Question: 88

Solution:

Formula :- $\int x^n dx=\frac{x^{n+1}}{n+1}+c\ ;\ \int e^x dx=e^x+c$

Therefore ,

$\Rightarrow \int\frac{1}{1-\frac{\cos x}{\sin x}}dx \Rightarrow \int\frac{\sin x}{\sin x-\cos x}dx$

We can write $\sin x=\frac{1}{2}[(\sin x-\cos x)+(\sin x+\cos x)]$

$\Rightarrow \int\frac{\frac{1}{2}[(\sin x-\cos x)+(\sin x+\cos x)]}{(\sin x-\cos x)}dx$

$\Rightarrow \frac{1}{2}\int\frac{(\sin x-\cos x)}{(\sin x-\cos x)}dx+\frac{1}{2}\int\frac{(\sin x+\cos x)}{(\sin x-\cos x)}dx$

$\Rightarrow \frac{1}{2}\int dx+\frac{1}{2}\int\frac{(\sin x+\cos x)}{(\sin x-\cos x)}dx \Rightarrow \frac{x}{2}+\frac{1}{2}\int\frac{(\sin x+\cos x)}{(\sin x-\cos x)}dx$

Put $(\sin x-\cos x)=t\ (\sin x+\cos x)\,dx=dt$

$\Rightarrow \frac{x}{2}+\frac{1}{2}\int\frac{1}{t}dt \Rightarrow \frac{x}{2}+\frac{1}{2}\log t+c \Rightarrow \frac{1}{2}x+\frac{1}{2}\log|\sin x-\cos x|+c$

Question: 89

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $\tan x = t \Rightarrow sec^2 x \, dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{1-t^2}} dt \Rightarrow \sin^{-1} t + c$

$\Rightarrow \sin^{-1}(\tan x) + c$

Question: 90

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

Therefore ,

$\Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx \Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-2+2} dx \Rightarrow \int \frac{1+\frac{1}{x^2}}{(x-\frac{1}{x})^2+2} dx$

Put $x - \frac{1}{x} = t \Rightarrow (1 + \frac{1}{x^2}) \, dx = dt$

$\Rightarrow \int \frac{1}{t^2+2} dt \Rightarrow \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} + c$

$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}[\frac{1}{\sqrt{2}}(x - \frac{1}{x})] + c$

Question: 91

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{sin^6 x}{cos^6 x \, cos^2 x} dx \Rightarrow \int \frac{tan^6 x}{cos^2 x} dx \Rightarrow \int tan^6 x \, sec^2 x dx$

Put $\tan x = t \Rightarrow sec^2 x \, dx = dt$

$\Rightarrow \int t^6 dt \Rightarrow \frac{t^7}{7} + c$

$\Rightarrow \frac{(\tan x)^7}{7} + c$

Question: 92

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^4 x \sec x \tan x \, dx$

Put $\sec x = t \Rightarrow \sec x \tan x \, dx = dt$

$\Rightarrow \int t^4 dt \Rightarrow \frac{t^5}{5} + c$

$\Rightarrow \frac{(\sec x)^5}{5} + c$

Question: 93

Solution:

Formula :-

Therefore , $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

$\Rightarrow \int tan^3 x\, tan^2 x dx \Rightarrow \int tan^3 x(sec^2 x - 1)dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int tan^3 x dx \Rightarrow \int tan^3 x sec^2 x dx - \int tan^1 x\, tan^2 x dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int \tan x\,(sec^2 x - 1)dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int \tan x\, sec^2 x dx + \int \tan x\, dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int t^3 dt - \int t^1 dt + \log|\sec x| \Rightarrow \frac{t^4}{4} - \frac{t^2}{2} + \log|\sec x| + c$

$\Rightarrow \frac{(\tan x)^4}{4} - \frac{(\tan x)^2}{2} + \log|\sec x| + c$

Question: 94

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \cos x\,(cos^2 x\, sin^3 x)dx \Rightarrow \int \cos x\,((1 - sin^2 x)\, sin^3 x)dx$

$\Rightarrow \int \cos x\,(sin^3 x - sin^5 x)dx \Rightarrow \int sin^3 x cos\, x dx - \int sin^5 x \cos x\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

$\Rightarrow \int t^3 dt - \int t^5 dt \Rightarrow \frac{t^4}{4} - \frac{t^6}{6} + c$

$\Rightarrow \frac{(\sin x)^4}{4} - \frac{(\sin x)^6}{6} + c$

Question: 95

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^2 x sec^2 x \tan x\, dx \Rightarrow \int (1 + tan^2 x) sec^2 x \tan x\, dx$

$\Rightarrow \int sec^2 x \tan x\, dx + \int tan^3 x sec^2 x dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int t^1 dt + \int t^3 dt \Rightarrow \frac{t^2}{2} + \frac{t^4}{4} + c$

$\Rightarrow \frac{(\tan x)^2}{2} + \frac{(\tan x)^4}{4} + c$

Question: 96

Mark (√) against

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^2 x sec^2 x \tan x \, dx \Rightarrow \int (1 + tan^2 x) sec^2 x \tan x \, dx$

$\Rightarrow \int sec^2 x \tan x \, dx + \int tan^3 x sec^2 x dx$

Put $\log(\tan x) = t \Rightarrow \frac{1}{\tan x} sec^2 x dx = dt \Rightarrow \frac{1}{\sin x \cos x} dx = dt$

$\Rightarrow \int t^1 dt \Rightarrow \frac{t^2}{2} + c$

$\Rightarrow \frac{(\log|\tan x|)^2}{2} + c$

Question: 97

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sin^2(2x+1) \sin(2x+1) \, dx \Rightarrow \int (1 - cos^2(2x+1)) \sin(2x+1) \, dx$

$\Rightarrow \int \sin(2x+1) \, dx - \int cos^2(2x+1) \sin(2x+1) \, dx$

Put $\cos(2x+1) = t \Rightarrow -2\sin(2x+1) \, dx = dt$

$\Rightarrow -\int \frac{dt}{2} - (-\frac{1}{2}) \int t^2 dt \Rightarrow -\frac{1}{2} \int dt + \frac{1}{2} \int t^2 dt$

$\Rightarrow -\frac{1}{2} t + \frac{1}{2} \frac{t^3}{3} + c \Rightarrow -\frac{1}{2} t + \frac{t^3}{6} + c$

$\Rightarrow -\frac{1}{2} \cos(2x+1) + \frac{[\cos(2x+1)]^3}{6} + c$

Question: 98

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{\sqrt{\tan x}}{\sin x \times \cos x} dx \Rightarrow \int \frac{\sqrt{\tan x}}{\frac{\tan x}{\sec x} \times \frac{1}{\sec x}} dx \Rightarrow \int \frac{sec^2 x}{\sqrt{\tan x}} dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int \frac{dt}{\sqrt{t}} \Rightarrow \frac{\sqrt{t}}{\frac{1}{2}} + c \Rightarrow 2\sqrt{t} + c$

$\Rightarrow 2\sqrt{\tan x} + c$

Question: 99

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \frac{\cos x+\sin x}{\cos^2 x+\sin^2 x-\sin 2x} dx \Rightarrow \int \frac{\cos x+\sin x}{(\cos x-\sin x)^2} dx$

Put $\cos x - \sin x = t \Rightarrow (\cos x + \sin x)dx = -dt$

$\Rightarrow \int \frac{-dt}{t^2} \Rightarrow \frac{1}{t} + c \Rightarrow \frac{1}{\cos x-\sin x} + c$

Question: 100

Solution:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\Rightarrow \int \sqrt{e^x - 1} dx$

Put $e^x - 1 = t \Rightarrow e^x dx = dt$

$\Rightarrow \int \sqrt{t} \frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t} dt$

Put $t = z^2$ dt = 2z dz

$\Rightarrow \int \frac{2z^2}{1+z^2} dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2} dz \Rightarrow 2\int \frac{1+z^2}{1+z^2} dz - 2\int \frac{1}{1+z^2} dz$

$\Rightarrow 2\int dz - 2\int \frac{1}{1+z^2} dz \Rightarrow 2z - 2\tan^{-1} z + c$

$\Rightarrow 2\sqrt{t} - 2\tan^{-1} \sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2\tan^{-1} \sqrt{e^x - 1} + c$

Question: 101

Solution:

Let $I = \int \frac{dx}{\sqrt{\sin^3 x \times \cos x}}$

Now multiplying and dividing by $\cos^2 x$, we get,

$I = \int \frac{dx}{\sqrt{\sin^3 x \times \cos x}} \times \frac{1}{\cos^2 x} \times \cos^2 x$

$I = \int \frac{(\sec^2 x)}{\sqrt{\frac{\sin^3 x}{\cos^3 x}}} dx$

$I = \int \frac{\sec^2 x}{\sqrt{\tan^3 x}} dx$

Let tan x = t

Differentiating both sides, we get,

$\sec^2 x \, dx = dt$

Therefore,

$I = \int \frac{dt}{t^{3/2}}$

Integrating, we get,

$I = \frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C$

$I=\frac{t^{-\frac{1}{2}}}{-\frac{1}{2}}+C$

$I=-\frac{2}{\sqrt{t}}+C$

$I=-\frac{2}{\sqrt{\tan x}}+C$

Exercise : 13B

Question: 1

Solution:

i)

$\Rightarrow\int \sin^2 x dx$

Now, we know that $1-\cos 2x=2\sin^2 x$

So, applying this identity in the given integral, we get,

$\int \sin^2 x dx=\int \frac{(1-\cos 2x)dx}{2}$

$\Rightarrow\frac{1}{2}(\int dx-\int \cos 2x dx)$

$\Rightarrow\frac{x}{2}-\frac{\sin 2x}{2\times 2}+c$

$\Rightarrow\frac{x}{2}-\frac{\sin 2x}{4}+c$

Ans: $\int \sin^2 x dx=\frac{x}{2}-\frac{\sin 2x}{4}+c$

ii) $\int \cos^2 x\, dx$

$\Rightarrow\int \cos^2 x\, dx$

Now, we know that $1+\cos 2x=2\cos^2 x$

So, applying this identity in the given integral, we get,

$\int \cos^2 x\, dx=\int \frac{(1+\cos 2x)dx}{2}$

$\Rightarrow\frac{1}{2}(\int dx+\int \cos 2x dx)$

$\Rightarrow\frac{x}{2}+\frac{\sin 2x}{2\times 2}+c$

$\Rightarrow\frac{x}{2}+\frac{\sin 2x}{4}+c$

Ans: $\int \cos^2 x\, dx=\frac{x}{2}+\frac{\sin 2x}{4}+c$

Question: 2

Solution:

(i) $\int \cos^2 (x/2) dx$

$\Rightarrow\int \cos^2\left(\frac{x}{2}\right)dx$

Now, we know that $1+\cos x=2\cos^2(x/2)$

So, applying this identity in the given integral, we get,

$$\int \cos^2\left(\frac{x}{2}\right) dx = \int \frac{(1+\cos x)dx}{2}$$

$\Rightarrow \frac{1}{2}\left(\int dx + \int \cos x dx\right)$

$$\Rightarrow \frac{x}{2} + \frac{\sin 2x}{2} + c$$

$$\Rightarrow \frac{x}{2} + \frac{\sin 2x}{2} + c$$

Ans: $\frac{x}{2} + \frac{\sin 2x}{2} + c$

ii) $\int \cot^2(x/2)dx$

$\Rightarrow \int \cot^2\left(\frac{x}{2}\right)dx$

Now, we know that $\operatorname{cosec}^2 x-\cot^2 x=1$

So, applying this identity in the given integral we get,

$\Rightarrow \int \cot^2\left(\frac{x}{2}\right)dx = \int\left(\operatorname{cosec}^2\left(\frac{x}{2}\right)-1\right)dx$

$\Rightarrow \int\left(\operatorname{cosec}^2\left(\frac{x}{2}\right)-1\right)dx = \int \operatorname{cosec}^2\left(\frac{x}{2}\right)dx - \int 1dx$

$\Rightarrow \int \operatorname{cosec}^2\left(\frac{x}{2}\right)dx - \int 1dx = \frac{-\cot x}{\frac{1}{2}} - x + c$

$\Rightarrow -2\cot x - x + c$

$\Rightarrow \int \cot^2\left(\frac{x}{2}\right)dx = -2\cot x - x + c$

Ans: $-2\cot x - x + c$

Question: 3

Solution:

i)

$\Rightarrow \int \sin^2 nx dx$

Now, we know that $1-\cos 2nx=2\sin^2 nx$

So, applying this identity in the given integral, we get,

$$\int \sin^2 nx dx = \int \frac{(1-\cos 2nx)dx}{2}$$

$\Rightarrow \frac{1}{2}\left(\int dx - \int \cos 2nx dx\right)$

$$\Rightarrow \frac{x}{2} - \frac{\sin 2nx}{2n \times 2} + c$$

$$\Rightarrow \frac{x}{2} - \frac{\sin 2x}{4n} + c$$

Ans: $\int \sin^2 nx dx = \frac{x}{2} - \frac{\sin 2nx}{4n} + c$

(ii) $\int \sin^5 x\, dx$

We know that 1-$cos^2x=sin^2x$

⇒$\int sin^5xdx = \int (1-cos^2x)^2 sinxdx$

⇒Put cosx=t

⇒-sinxdx=dt

⇒$\int (1-cos^2x)^2 sinxdx = -\int (1-t^2)^2dt$

⇒$-\int (1-t^2)^2dt = -\int (1+t^4-2t^2)dt$

⇒$-\int dt + \int 2t^2dt - \int t^4dt$

⇒$-t + \frac{2t^3}{3} - \frac{t^5}{5} + c$

Resubstituting the value of t=cosx we get,

⇒$-cosx + \frac{2cos^3x}{3} - \frac{cos^5x}{5} + c$

Ans: $-cosx + \frac{2cos^3x}{3} - \frac{cos^5x}{5} + c$

Question: 4

Solution:

Substitute 3x+5=u

⇒3dx=du

⇒dx=du/3

⇒$\int cos^3(3x+5)dx = \frac{1}{3}\int cos^3(u)du$

Now We know that 1-$cos^2x=sin^2x$,

⇒$\frac{1}{3}\int cos^3(u)du = \frac{1}{3}\int (1-sin^2(u))cosu\,du$

⇒Substitute sinu=t

⇒cosu du=dt

⇒$\frac{1}{3}\int (1-sin^2(u))cosu\,du = \frac{1}{3}\int (1-t^2)\,dt$

⇒$\frac{1}{3}\int dt - \frac{1}{3}\int t^2dt$

⇒$\frac{t}{3} - \frac{t^3}{3\times 3} + c$

⇒$\frac{t}{3} - \frac{t^3}{9} + c$

Resubstituting the value of t=sinu and u=3x+5 we get,

⇒$\frac{\sin(3x+5)}{3} - \frac{sin^3(3x+5)}{9} + c$

Ans: $\frac{\sin(3x+5)}{3} - \frac{sin^3(3x+5)}{9} + c$

Question: 5

Solution:

⇒

Substitute 2x-3 $\int sin^7(2x-3)dx$

$\Rightarrow$ 2dx=du

$\Rightarrow$dx=du/2

$\Rightarrow -\left(\frac{1}{2}\right)\int \sin^7(u)du$

$\Rightarrow$ We know that 1-cos^{2}x=sin^{2}x

$\Rightarrow -\left(\frac{1}{2}\right)\int \left(1-\cos^2(u)\right)^3 \sin u\, du$

$\Rightarrow$Put cosu=t

$\Rightarrow$-sinxdu=dt

$\Rightarrow\left(\frac{1}{2}\right)\int(1-t^2)^3dt$

$\Rightarrow\left(\frac{1}{2}\right)\int(1-t^6-3t^2+3t^4)dt$

$\Rightarrow\left(\frac{1}{2}\right)[\int dt-\int t^6dt-\int 3t^2dt+\int 3t^4dt]$

$\Rightarrow\left(\frac{1}{2}\right)\left[t-\frac{t^7}{7}-\frac{3t^3}{3}+\frac{3t^5}{5}\right]+c$

$\Rightarrow\left(\frac{1}{2}\right)\left[t-\frac{t^7}{7}-t^3+\frac{3t^5}{5}\right]+c$

Resubstituting the value of t=cosu and u=2x-3 we get

$\Rightarrow\left(\frac{1}{2}\right)\left[\cos(2x-3)-\frac{\cos^7(2x-3)}{7}-\cos^3(2x-3)+\frac{3\cos^5(2x-3)}{5}\right]+c$

$\Rightarrow\frac{\cos(2x-3)}{2}-\frac{\cos^7(2x-3)}{14}-\frac{\cos^3(2x-3)}{2}+\frac{3\cos^5(2x-3)}{10}+c$

Now as we know cos(-x)=cosx

$\Rightarrow\frac{\cos(2x-3)}{2}-\frac{\cos^7(2x-3)}{14}-\frac{\cos^3(2x-3)}{2}+\frac{3\cos^5(2x-3)}{10}+c$

$=\frac{\cos(3-2x)}{2}-\frac{\cos^7(3-2x)}{14}-\frac{\cos^3(3-2x)}{2}+\frac{3\cos^5(3-2x)}{10}+c$

Ans: $\frac{\cos(3-2x)}{2}-\frac{\cos^7(3-2x)}{14}-\frac{\cos^3(3-2x)}{2}+\frac{3\cos^5(3-2x)}{10}+c$

Question: 6

Solution:

(i)

$\int\left(\frac{1-\cos 2x}{1+\cos 2x}\right)dx$

$\Rightarrow\int\frac{1-\cos 2x}{1+\cos 2x}dx$

1-cos2x=2sin^{2}x and 1+cos2x=2cos^{2}x

$\Rightarrow\int\frac{1-\cos 2x}{1+\cos 2x}dx=\int\frac{2\sin^2 x}{2\cos^2 x}dx$

$\Rightarrow\int \tan^2 x\, dx$

Now sec^{2}x-1=tan^{2}x

$\Rightarrow\int(\sec^2 x-1)dx$

$\Rightarrow\int \sec^2 x dx-\int dx$

$\Rightarrow$tanx-x+c

Ans: tanx-x+c

(ii) $\left(\frac{1+\cos 2x}{1-\cos 2x}\right)dx$

CLASS24

$\Rightarrow\int \frac{1+cos2x}{1-cos2x}dx$

1-cos2x=2sin^{2}x and 1+cos2x=2cos^{2}x

$\Rightarrow\int \frac{1+cos2x}{1-cos2x}dx = \int \frac{2cos^2 x}{2sin^2 x}dx$

$\Rightarrow\int cot^2 x\, dx$

Now cosec²x-1=cot²x

$\Rightarrow\int (cosec^2 x - 1)dx$

$\Rightarrow\int cosec^2 x dx - \int dx$

⇒-cotx-x+c

Ans: -cotx-x+c

Question: 7

Evaluate the foll

Solution:

i) $\int \frac{1-cosx}{1+cosx}dx$

$\Rightarrow\int \frac{1-cosx}{1+cosx}dx$

1-cosx=2sin^{2}x/2 and 1+cosx=2cos^{2}x/2

$\Rightarrow\int \frac{1-cosx}{1+cosx}dx = \int \frac{2sin^2(\frac{x}{2})}{2cos^2(\frac{x}{2})}dx$

$\Rightarrow\int tan^2(\frac{x}{2})dx$

Now sec²(x/2)-1=tan2(x/2)

$\Rightarrow\int \left(sec^2(\frac{x}{2}) - 1\right)dx$

$\Rightarrow\int sec^2(\frac{x}{2})dx - \int dx$

⇒2tan(x/2)-x+c

Ans: 2tan(x/2)-x+c

(ii) $\int \frac{1+cosx}{1-cosx}dx$

$\Rightarrow\int \frac{1+cosx}{1-cosx}dx$

1-cosx=2sin^{2}x/2 and 1+cosx=2cos^{2}x/2

$\Rightarrow\int \frac{1+cosx}{1-cosx}dx = \int \frac{2cos^2(\frac{x}{2})}{2sin^2(\frac{x}{2})}dx$

$\Rightarrow\int cot^2(\frac{x}{2})dx$

Now cosec^2 (x/2)-1=cot^2 (x/2)

$\Rightarrow\int \left(cosec^2(\frac{x}{2}) - 1\right)dx$

$\Rightarrow\int cosec^2(\frac{x}{2})dx - \int dx$

⇒-2cot(x/2)-x+c

Ans: ⇒-2cot(x/2)-x+c

Question: 8

Solution:

⇒ $\int sin3x \ cos4x \ dx$

Applying the formula: sinx×cosy=1/2(sin(x+y)-sin(y-x))

$⇒\frac{1}{2}\int(sin7x - sinx)dx$

$⇒\frac{1}{2}\int sin7x \ dx - \frac{1}{2}\int sinx \ dx$

$⇒\frac{-cos7x}{14} + \frac{cosx}{2} + c$

Ans: $\frac{-cos7x}{14} + \frac{cosx}{2} + c$

Question: 9

Solution:

⇒ $\int cos4x \ cos3x \ dx$

Applying the formula: cosx×cosy=1/2(cos(x+y)+cos(x-y))

$⇒\frac{1}{2}\int(cos7x + cosx)dx$

$⇒\frac{1}{2}\int cos7x \ dx + \frac{1}{2}\int cosx \ dx$

$⇒\frac{sin7x}{14} + \frac{sinx}{2} + c$

Ans: $\frac{sin7x}{14} + \frac{sinx}{2} + c$

Question: 10

Solution:

⇒ $\int sin4x \ sin8x \ dx$

Applying the formula: sinx×siny=1/2(cos(y-x)-cos(y+x))

$⇒\frac{1}{2}\int(cos4x - cos12x)dx$

$⇒\frac{1}{2}\int cos4x \ dx - \frac{1}{2}\int cos12x \ dx$

$⇒\frac{sin4x}{8} - \frac{sin12x}{24} + c$

Ans: $\frac{sin4x}{8} - \frac{sin12x}{24} + c$

Question: 11

Solution:

⇒ $\int sin6x \ cosx \ dx$

Applying the formula: sinx×cosy=1/2(sin(y+x)-sin(y-x))

$⇒\frac{1}{2}\int(sin7x - \sin(-5x))dx$

$\Rightarrow \frac{1}{2}\int \sin 7x\, dx + \frac{1}{2}\int \sin 5x\, dx$

$\Rightarrow \frac{-\cos 7x}{14} - \frac{\cos x}{10} + c$

Ans: $\frac{-\cos 7x}{14} - \frac{\cos x}{10} + c$

Question: 12

Evaluate the foll

Solution:

we know that $1+\cos 2x = 2\cos^2 x$

So, applying this identity in the given integral we get,

$\Rightarrow \int \sin x\sqrt{1+\cos 2x}\, dx$

$\Rightarrow \int \sin x\sqrt{(2\cos^2 x)}\, dx$

$\Rightarrow \sqrt{2}\int \sin x \cos x\, dx$

Let sinx =t

$\Rightarrow$ cosx dx=dt

$\Rightarrow \sqrt{2}\int t\, dt$

$\Rightarrow \sqrt{2}\frac{t^2}{2} + c = \frac{t^2}{\sqrt{2}} + c$

Resubstituting the value of t=sinx we get

$\Rightarrow \frac{\sin^2 x}{\sqrt{2}} + c$

Ans: $\frac{\sin^2 x}{\sqrt{2}} + c$

Question: 13

Solution:

$\Rightarrow$

$\Rightarrow \int (\frac{1+\cos 2x}{2})(\frac{1+\cos 2x}{2})\, dx$ ...$(\frac{1+\cos 2x}{2} = \cos^2 x)$

$\Rightarrow \frac{1}{4}\int (1+\cos 2x)^2\, dx$

$\Rightarrow \frac{1}{4}\int (1+\cos^2 2x + 2\cos 2x)\, dx$

$\Rightarrow \frac{1}{4}[\int 1\, dx + \int \cos^2 2x\, dx + \int 2\cos 2x\, dx$

$\Rightarrow \frac{1}{4}[x + \int \frac{(1+\cos 4x)dx}{2} + 2\frac{\sin 2x}{2}]$ ...$(1+\cos 4x = 2\cos^2 x)$

$\Rightarrow \frac{1}{4}[x + \frac{1}{2}(\int dx + \int \cos 4x\, dx) + \sin 2x] + c$

$\Rightarrow [\frac{x}{4} + \frac{1}{2}\times\frac{1}{4}\left(\int dx + \int \cos 4x\, dx\right) + \frac{\sin 2x}{4}\Big] + c$

$\Rightarrow [\frac{x}{4} + \left(\frac{x}{8} + \frac{\sin 4x}{32}\right) + \frac{\sin 2x}{4}\Big] + c$

$\Rightarrow \frac{3x}{8} + \frac{\sin 4x}{32} + \frac{\sin 2x}{4} + c$

Ans: $\frac{3x}{8} + \frac{\sin 4x}{32} + \frac{\sin 2x}{4} + c$

Question: 14

Evaluate the foll

Solution:

$\Rightarrow \int cos2x\, cos4x\, cos6x dx$

$\Rightarrow \frac{1}{2}\int (cos6x + cos2x) cos6x dx$

$\Rightarrow \frac{1}{2}\int cos^2 6x dx + \frac{1}{2}\int cos2x cos6x dx$

$$\Rightarrow \frac{1}{2}\int cos^2 6x dx + \frac{1}{4}\int (cos8x + cos4x) dx$$

$$\Rightarrow \frac{1}{2}\int cos^2 6x dx + \frac{1}{4}\int cos8x\, dx + \frac{1}{4}\int cos4x\, dx$$

$$\Rightarrow \frac{1}{2}\int \frac{(1 + cos12x)dx}{2} + \frac{1}{4}\frac{sin8x}{8} + \frac{1}{4}\frac{sin4x}{4} + c$$

$$\Rightarrow \frac{1}{4}\left(x + \frac{sin12x}{12}\right) + \frac{sin8x}{32} + \frac{sin4x}{16} + c$$

$$\Rightarrow \frac{x}{4} + \frac{sin12x}{48} + \frac{sin8x}{32} + \frac{sin4x}{16} + c$$

Ans: $\frac{x}{4} + \frac{sin12x}{48} + \frac{sin8x}{32} + \frac{sin4x}{16} + c$

Question: 15

Solution:

Let sinx =t

$\Rightarrow$ cosx dx =dt

$\Rightarrow \int sin^3 x\, cosx\, dx = \int t^3 dt$

$\Rightarrow \frac{t^4}{4} + c$

Resubstituting the value of t=sinx we get

$$\Rightarrow \frac{sin^4 x}{4} + c$$

Ans: $\frac{sin^4 x}{4} + c$

Question: 16

Solution:

$\Rightarrow$

$\Rightarrow \int sec^4 x\, dx = \int (1 + tan^2 x) sec^2 x dx$

$\Rightarrow$Put tanx=t $\Rightarrow$$sec^2$dx=dt

$\Rightarrow \int (1 + t^2) dt$

$\Rightarrow t + \frac{t^3}{3} + c$

Resubstituting the value of t=tanx we get

$\Rightarrow tanx + \frac{tan^3 x}{3} + c$

Ans: $tanx + \frac{tan^3 x}{3} + c$

Question: 17

Solution:

⇒

⇒$\int cos^3 x sin^4 x\, dx$ $\int cosx\, sin^4 x cos^2 x dx$

⇒$\int cosx\, sin^4 x(1 - sin^2 x)dx$

Put sinx=t

⇒cosxdx=dt

⇒$\int t^4(1 - t^2)dt$

⇒$\int t^4 dt - \int t^6\, dt$

⇒$\frac{t^5}{5} - \frac{t^7}{7} + c$

Resubstituting the value of t=sinx we get,

⇒$\frac{sin^5 x}{5} - \frac{sin^7 x}{7} + c$

Ans: $\frac{sin^5 x}{5} - \frac{sin^7 x}{7} + c$

Question: 18

Solution:

⇒

⇒$\int cos^4 x sin^3 x\, dx$ $\int sinx\, sin^2 x cos^4 x dx$

⇒$\int sinx\, cos^4 x(1 - cos^2 x)dx$

Put cosx=t

⇒-sinxdx=dt

⇒$\int t^4(t^2 - 1)dt$

⇒$\int t^6 dt - \int t^4\, dt$

⇒$\frac{t^7}{7} - \frac{t^5}{5} + c$

Resubstituting the value of t=sinx we get,

⇒$\frac{cos^7 x}{7} - \frac{cos^5 x}{5} + c$

Ans: $\frac{cos^7 x}{7} - \frac{cos^5 x}{5} + c$

Question: 19

Solution:

⇒

⇒$\int cos^3 x sin^{\frac{2}{3}} x\, dx$ $\int cosx\, cos^2 x sin^{\frac{2}{3}} x dx$

$\Rightarrow \int \cos x\,(1-\sin^2 x)\sin^{\frac{2}{3}} x\,dx$

Put sinx=t

⇒cosxdx=dt

$\Rightarrow \int t^{\frac{2}{3}}(1-t^2)dt$

$\Rightarrow \int t^{\frac{2}{3}}dt - \int t^{\frac{8}{3}}dt$

$\Rightarrow \frac{t^{\frac{5}{3}}}{\frac{5}{3}} - \frac{t^{\frac{11}{3}}}{\frac{11}{3}} + c$

Resubstituting the value of t=sinx we get

$\Rightarrow \frac{3\sin^{\frac{5}{3}} x}{5} - \frac{3\sin^{\frac{11}{3}} x}{11} + c$

Ans: $\frac{3\sin^{\frac{5}{3}} x}{5} - \frac{3\sin^{\frac{11}{3}} x}{11} + c$

Question: 20

Solution:

⇒

$\Rightarrow \int \sin^3 x \cos^{\frac{3}{5}} x\,dx$

$\int \sin x \sin^2 x \cos^{\frac{3}{5}} x\,dx$

$\Rightarrow \int \sin x\,(1-\cos^2 x)\cos^{\frac{3}{5}} x\,dx$

Put cosx=t

⇒-sinxdx=dt

$\Rightarrow \int t^{\frac{3}{5}}(t^2-1)dt$

$\Rightarrow \int t^{\frac{13}{5}}dt - \int t^{\frac{3}{5}}dt$

$\Rightarrow \frac{t^{\frac{18}{5}}}{\frac{18}{5}} - \frac{t^{\frac{8}{5}}}{\frac{8}{5}} + c$

Resubstituting the value of t=cosx we get

$\Rightarrow \frac{5\cos^{\frac{18}{5}} x}{18} - \frac{5\cos^{\frac{8}{5}} x}{8} + c$

Ans: $\frac{5\cos^{\frac{18}{5}} x}{18} - \frac{5\cos^{\frac{8}{5}} x}{8} + c$

Question: 21

Solution:

$\Rightarrow \int \operatorname{cosec}^4 2x\,dx$

$\Rightarrow \int \operatorname{cosec}^2 2x \operatorname{cosec}^2 2x\,dx$

$\Rightarrow \int cosec^2 2x\,(1 + cot^2 2x)dx$

$\Rightarrow$cot2x=t $\Rightarrow$-2cosec² 2xdx=dt

$\Rightarrow -1/2\int (1 + t^2)dt$

$\Rightarrow -1/2\int dt - 1/2\int t^2 dt$

$\Rightarrow -(\frac{1}{2})t - \frac{t^3}{6} + c$

Resubstituting the value of t=cotx we get

$\Rightarrow -\frac{\cot x}{2} - \frac{cot^3 x}{6} + c$

Ans: $-\frac{\cot x}{2} - \frac{cot^3 x}{6} + c$

Question: 22

Solution:

$\Rightarrow \int \frac{cos2x}{cosx}dx = \int \frac{2cos^2 x - 1}{cosx}dx$

$\Rightarrow \int \frac{2cos^2 x}{cosx}dx - \int \frac{1}{cosx}dx$

$\Rightarrow \int 2cosxdx - \int secxdx$

$\Rightarrow 2\sin x - \log|\sec x + \tan x| + c$

Ans: 2sinx-log|secx+tanx|+c

Question: 23

Solution:

$\Rightarrow \int \frac{cosx}{\cos(x+\alpha)}dx = \int \frac{\cos\left((x+\alpha) - \alpha\right)}{\cos(x+\alpha)}dx$

$\Rightarrow \int \frac{\cos(x+\alpha)cos\alpha + \sin(x+\alpha)\,sin\alpha}{\cos(x+\alpha)}dx$

$\Rightarrow \int cos\alpha dx + \int \tan(x+\alpha)\,sin\alpha\,dx$

Now α is a constant

$\Rightarrow x\cos\alpha - \sin\alpha\log|\cos(x+\alpha)| + c$

Ans:xcos α-sin αlog|cos(x+ α)|+c

Question: 24

Solution:

$\Rightarrow \int sin2x\,cos^3 xdx$

$\Rightarrow \int 2sinxcosxcos^3 xdx$

$\Rightarrow \int 2sinx\, cos^4 x dx$

CLASS24

Now put cosx=t

⇒-sinxdx=dt

$\Rightarrow -2\int t^4 dt$

$\Rightarrow -2\times\frac{t^5}{5}+c$

Resubstituting the value of t= cosx we get,

$\Rightarrow \frac{-2cos^5 x}{5}+c$

Ans: $\frac{-2cos^5 x}{5}+c$

Question: 25

Solution:

$\Rightarrow \int \frac{cos^9 x}{sinx}dx$

$\Rightarrow \int \frac{cos^9 x}{sin^2 x}sinx dx$

$\Rightarrow \int \frac{cos^9 x}{1-cos^2 x}sinx dx$

Put cosx =t

⇒ -sinxdx=dt

$\Rightarrow \int \frac{t^9}{t^2-1}dt$

Now put t^2-1=a

⇒2tdt=da

And $t^8=(a+1)^4$

$\Rightarrow \frac{1}{2}\int \frac{(a+1)^4}{a}da$

$\Rightarrow \frac{1}{2}\int (a^3+4a^2+6a+\frac{1}{a}+4)da$

$\Rightarrow \frac{1}{2}\left(\frac{a^4}{4}+\frac{4a^3}{3}+\frac{6a^2}{2}+lna+4a\right)+c$

$\Rightarrow \left(\frac{a^4}{8}+\frac{2a^3}{3}+\frac{3a^2}{2}+\frac{lna}{2}+2a\right)+c$

Resubstituting the value of a=t^2-1 and t=cosx ⇒a=cos^2x-1=-sin^2x we get

$\Rightarrow \left(\frac{(-sin^2 x)^4}{8}+\frac{2(-sin^2 x)^3}{3}+\frac{3(-sin^2 x)^2}{2}+\frac{\ln|(-sin^2 x)|}{2}+2(-sin^2 x)\right)+c$

$\Rightarrow \left(\frac{sin^8 x}{8}-\frac{2sin^6 x}{3}+\frac{3sin^4 x}{2}+\frac{2\ln|(-sinx)|}{2}-2sin^2 x\right)+c$

$\Rightarrow \left(\frac{sin^8 x}{8} - \frac{2sin^6 x}{3} + \frac{3sin^4 x}{2} + \ln(sinx) - 2sin^2 x\right) + c$

Ans: $\left(\frac{sin^8 x}{8} - \frac{2sin^6 x}{3} + \frac{3sin^4 x}{2} + \ln(sinx) - 2sin^2 x\right) + c$

Question: 26

Solution:

$\Rightarrow$

$\Rightarrow \int (\frac{1+cos4x}{2})(\frac{1+cos4x}{2})dx \ ...(\frac{1+cos4x}{2} = cos^2 2x)$

$\Rightarrow \frac{1}{4}\int (1 + cos4x)^2 dx$

$\Rightarrow \frac{1}{4}\int (1 + cos^2 4x + 2cos4x)dx$

$\Rightarrow \frac{1}{4}[\int 1dx \ + \int cos^2 4xdx + \int 2cos4x \ dx$

$\Rightarrow \frac{1}{4}[x + \int \frac{(1+cos8x)dx}{2} + 2\frac{sin4x}{4}]$ ...(1+cos8x=2cos² 4x)

$\Rightarrow \frac{1}{4}[x + \frac{1}{2}\left(\int dx + \int cos8xdx\right) + (\frac{sin4x}{2})\Big] + c$

$\Rightarrow [\frac{x}{4} + \frac{1}{2} \times \frac{1}{4}\left(\int dx + \int cos8xdx\right) + \frac{sin4x}{8}\Big] + c$

$\Rightarrow [\frac{x}{4} + \left(\frac{x}{8} + \frac{sin8x}{64}\right) + \frac{sin4x}{8}\Big] + c$

$\Rightarrow \frac{3x}{8} + \frac{sin8x}{64} + \frac{sin4x}{8} + c$

Ans: $\frac{3x}{8} + \frac{sin8x}{64} + \frac{sin4x}{8} + c$

Question: 27

Solution:

Doing tangent half angle substitution we get,

$$\Rightarrow \int \frac{sin^2 x}{(1 + cos^2 x)} dx = \int \frac{\left(\frac{2tan\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right)^2}{[1 + (\frac{1 - tan^2\frac{x}{2}}{1 - tan^2\frac{x}{2}})]^2}$$

Substitute u=tan(x/2)

⇒2du=sec²(x/2)dx

⇒dx=$\frac{2du}{u^2+1}$

$\Rightarrow 2\int \frac{u^2}{1+u^2} du$

$\Rightarrow 2\int \frac{1+u^2}{1+u^2} du - 2\int \frac{1}{1+u^2} du$

$\Rightarrow 2\int du - \tan^{-1} u + c$

$\Rightarrow 2u - \tan^{-1} u + c$

Resubstituting the values we get,

$\Rightarrow 2\tan\frac{x}{2} - \tan^{-1}\tan\frac{x}{2} + c$

$\Rightarrow 2\tan\frac{x}{2} - \frac{x}{2} + c$

Ans: $2\tan\frac{x}{2} - \frac{x}{2} + c$

Question: 28

Solution:

$$\int \frac{dx}{3cosx + 4sinx} = \int \frac{dx}{3\left(\frac{1 - tan^2\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) + 4\left(\frac{2tan\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{3 + 8tan\frac{x}{2} - 3tan^2\frac{x}{2}}$$

Let $\tan\frac{x}{2}$=t

$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$

$$\Rightarrow \int \frac{2dt}{3 + 8t - 3t^2} = \frac{2}{3}\int \frac{dt}{1 + \frac{8}{3}t - t^2} = \frac{2}{3}\int \frac{dt}{1 - \left(t - \frac{4}{3}\right)^2 + \frac{16}{9}}$$

$$\Rightarrow \frac{2}{3}\int \frac{dt}{\frac{25}{9} - \left(t - \frac{4}{3}\right)^2} = \frac{2}{3}\int \frac{dt}{\left(\frac{5}{3}\right)^2 - \left(t - \frac{4}{3}\right)^2}$$

$$\Rightarrow \frac{2}{3} \times \frac{1}{2 \times \frac{5}{3}} \ln\left|\frac{\frac{5}{3} + \left(t - \frac{4}{3}\right)}{\frac{5}{3} - \left(t - \frac{4}{3}\right)}\right| + c = \frac{1}{5}\ln\left|\frac{1 + 3t}{9 - 3t}\right| + c$$

Resubstituting the value of t we get

$$\Rightarrow \frac{1}{5}\ln\left|\frac{1 + 3tan\frac{x}{2}}{9 - 3tan\frac{x}{2}}\right| + c$$

Ans: $\frac{1}{5}\ln\left|\frac{1+3tan\frac{x}{2}}{9-3tan\frac{x}{2}}\right| + c$

Question: 29

Solution:

$$\int \frac{dx}{(acosx + bsinx)^2}$$

Taking bcosx common from the denominator we get,

$$\int \frac{dx}{b^2cos^2x(\frac{a}{b} + tanx)^2}$$

$$\Rightarrow \frac{1}{b^2}\int \frac{sec^2xdx}{(\frac{a}{b} + tanx)^2}$$

Let (a/b)+tanx=t

$\therefore sec^2xdx = dt$

$$\Rightarrow \frac{1}{b^2}\int \frac{dt}{t^2} = \frac{-1}{b^2}\times\frac{1}{t} = \frac{-1}{b^2 t} + c$$

Resubstituting the value of t = (a/b)+tanx we get

$$\Rightarrow \frac{-1}{b^2(\frac{a}{b} + tanx)} + c = \frac{-1}{ab + b^2 tanx} + c$$

Ans: $\frac{-1}{ab+b^2 tanx} + c$

Question: 30

Solution:

$$\int \frac{dx}{cosx - sinx} = \int \frac{dx}{\left(\frac{1 - tan^2\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) - \left(\frac{2tan\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{1 - 2tan\frac{x}{2} - tan^2\frac{x}{2}}$$

Let $tan\frac{x}{2}$=t

$$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$$

$$\Rightarrow \int \frac{2dt}{1 - 2t - t^2} = -2\int \frac{dt}{t^2 + 2t - 1} = -2\int \frac{dt}{(t+1)^2 - 2}$$
$$= -2\int \frac{dt}{(t+1)^2 - (\sqrt{2})^2}$$

$\Rightarrow -2 \times \frac{1}{2\times\sqrt{2}}\ln\left|\frac{t+1-\sqrt{2}}{t+1+\sqrt{2}}\right| + c$ resubstituting the value of t we get

$$\Rightarrow \frac{-1}{\sqrt{2}}\ln\left|\frac{tan\frac{x}{2} + 1 - \sqrt{2}}{tan\frac{x}{2} + 1 + \sqrt{2}}\right| + c = \frac{-1}{\sqrt{2}}\ln\left|\tan\left(\frac{\pi}{8} - \frac{x}{2}\right)\right| + c$$

Ans: $\frac{-1}{\sqrt{2}}\ln\left|\tan\left(\frac{\pi}{8} - \frac{x}{2}\right)\right| + c$

Question: 31

Solution:

$$\int (2tanx - 3cotx)^2 dx$$

$$\Rightarrow \int (4tan^2x + 9cot^2x - 12tanxcotx)dx$$

$$\Rightarrow \int (4(sec^2x - 1) + 9(cosec^2x - 1) - 12)dx$$

$$\Rightarrow \int 4sec^2x\,dx + \int 9cosec^2x dx - \int 25dx$$

$\Rightarrow$ 4tanx – 9cotx – 25x + c

Ans: 4tanx-9cotx-25x+c

Question: 32

Evaluate the foll

Solution:

$\Rightarrow\int sinx\ sin2x sin3x\ dx$

Applying the formula: sinx×siny=1/2(cos(y-x)-cos(y+x))

$\Rightarrow\frac{1}{2}\int(cos2x - cos4x)sin2xdx$

$\Rightarrow\frac{1}{2}\int sin2xcos2x\ dx - \frac{1}{2}\int sin2xcos4x\ dx$

$\Rightarrow\frac{1}{4}\int sin4x\ dx - \frac{1}{4}\int(sin6x - sin2x)dx$

$\Rightarrow\frac{-cos4x}{16} + \frac{cos6x}{24} - \frac{cos2x}{8} + c$

Ans: $\frac{-cos4x}{16} + \frac{cos6x}{24} - \frac{cos2x}{8} + c$

Question: 33

Solution:

$$\Rightarrow \int \frac{1 - cotx}{1 + cotx}dx = \int \frac{1 - \frac{cosx}{sinx}}{1 + \frac{cosx}{sinx}}dx$$

$$\Rightarrow \int \frac{sinx - cosx}{sinx + cosx}dx = -\int \frac{cosx - sinx}{sinx + cosx}dx$$

$$\Rightarrow -\int \frac{d(sinx + cosx)}{sinx + cosx}$$

$\Rightarrow -\log|\text{sinx} + \text{cosx}| + \text{c}$

Ans: -log(sinx+cosx)+c

Question: 34

Solution:

$$\int \frac{dx}{cosx + 2sinx + 3} = \int \frac{dx}{\left(\frac{1 - tan^2\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) + 2\left(\frac{2tan\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) + 3}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{3 + 1 + 3tan^2\frac{x}{2} + 4tan\frac{x}{2} - tan^2\frac{x}{2}}$$

Let $\tan\frac{x}{2}$=t

$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$

$$\Rightarrow \int \frac{2dt}{4 + 4t + 2t^2} = \int \frac{dt}{2 + 2t + t^2} = \frac{2}{3}\int \frac{dt}{(t + 1)^2 + 2 - 1}$$

$$\Rightarrow \int \frac{dt}{(t + 1)^2 + 1} = \int \frac{dt}{(1)^2 + (t + 1)^2}$$

$\Rightarrow \tan^{-1}(t + 1) + c$

Resubstituting the value of t we get

$\Rightarrow \tan^{-1}(tan\frac{x}{2} + 1) + c$

Ans: $\tan^{-1}(tan\frac{x}{2} + 1) + c$

Exercise : 13C

Question: 1

Solution:

Using BY PART METHOD.

Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x.e^x dx = x\int e^x - \int\frac{dx}{dx}.\int e^x dx$$

$$= xe^x - \int 1.e^x dx$$

$$= xe^x - e^x + c$$

$$= e^x(x-1) + c$$

Question: 2

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and cos x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\cos xdx = x\int \cos x - \int\left[\frac{dx}{dx}.\int \cos xdx\right]dx$$

$$= x\sin x - \int 1.\sin xdx$$

$$= x\sin x + \cos x + c$$

Question: 3

Solution:

Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^{2x} is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int xe^{2x}dx = x\int e^{2x}dx - \int\left[\frac{dx}{dx}.\int e^{2x}dx\right]dx$$

$$= x\frac{e^{2x}}{2} - \int 1.\frac{e^{2x}}{2}dx$$

$$= x\frac{e^{2x}}{2} - \frac{e^{2x}}{2\times 2} + c$$

$$= x\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + c$$

Question: 4

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Sin 3x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\sin 3xdx = x\int \sin 3xdx - \int\left[\frac{dx}{dx}.\int \sin 3xdx\right]dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) - \int 1.\left(\frac{-\cos 3x}{3}\right)dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) + \left(\frac{\sin 3x}{3\times 3}\right) + c$$

$$= x\left(\frac{-\cos 3x}{3}\right) + \left(\frac{\sin 3x}{9}\right) + c$$

Question: 5

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Cos 2x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\cos 2x dx = x\int \cos 2x dx - \int\left[\frac{dx}{dx}.\int \cos 2x dx\right]dx$$

$$= x\left(\frac{\sin 2x}{2}\right) - \int 1.\left(\frac{\sin 2x}{2}\right)dx$$

$$= x\left(\frac{\sin 2x}{2}\right) + \left(\frac{\cos 2x}{2\times 2}\right) + c$$

$$= x\left(\frac{\sin 2x}{2}\right) + \left(\frac{\cos 2x}{4}\right) + c$$

Question: 6

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log 2x is the first function, and x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\log 2x dx = \log 2x\int xdx - \int\left[\frac{d\log 2x}{dx}.\int xdx\right]dx$$

$$= \log 2x.\frac{x^2}{2} - \int\left[\frac{1\times 2}{2x}\frac{x^2}{2}\right]dx$$

$$= \frac{x^2}{2}\log 2x - \int\frac{x}{2}dx$$

$$= \frac{x^2}{2}\log 2x - \frac{x^2}{2\times 2} + c$$

$$= \frac{x^2}{2}\log 2x - \frac{x^2}{4} + c$$

Question: 7

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and $\text{cosec}^2 x$ is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\,\text{cosec}^2x\,dx = x\int \text{cosec}^2x - \int\left[\frac{dx}{dx}.\int \text{cosec}^2x\,dx\right]dx$$

$$= x(-\cot x) - \int 1.(-\cot x)\,dx$$

$$= -x\cot x + \int \cot x\,dx$$

$$= -x\cot x + \ln|\sin x| + c$$

Question: 8

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and cos x is the second function.

Using Integration by part

$$\int a.b.dx = a\int b\,dx - \int\left[\frac{da}{dx}.\int b\,dx\right]dx$$

$$\Rightarrow \int x^2\cos x\,dx = x^2\int \cos x\,dx - \int\left[\frac{dx^2}{dx}.\int \cos x\,dx\right]dx$$

$$= x^2\sin x - \int[2x\times\sin x]dx$$

$$= x^2\sin x - 2\left[\int x\sin x\,dx\right]$$

Again applying by the part method in the second half, we get

$$x^2\sin x - 2\int x\sin x\,dx$$

$$= x^2\sin x - 2\left[x\int \sin x\,dx - \int\left(\frac{dx}{dx}.\int \sin x\,dx\right)dx\right]$$

$$= x^2\sin x - 2\left[x(-\cos x) - \int 1.(-\cos x)dx\right]$$

$$= x^2\sin x - 2[-x\cos x + \sin x] + c$$

$$= x^2\sin x + 2x\cos x - 2\sin x + c$$

Question: 9

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a\int b\,dx - \int\left[\frac{da}{dx}.\int b\,dx\right]dx$$

Writing $\text{Sin}^2x = \frac{1+\cos 2x}{2}$

We have

$$\int x\sin^2 x dx = \int x\left(\frac{1-\cos 2x}{2}\right)dx$$

$$= \int\left(\frac{x}{2} - \frac{x\cos 2x}{2}\right)dx$$

$$= \int \frac{x}{2}dx - \int \frac{x\cos 2x}{2}dx$$

$$= \frac{x^2}{2\times 2} - \frac{1}{2}\int x\cos 2x dx$$

Taking X as first function and Cos 2x as the second function.

$$= \frac{x^2}{4} - \frac{1}{2}\left\{x\int \cos 2x dx - \int\left(\frac{dx}{dx}.\int \cos 2x dx\right)dx\right\}$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{x.\frac{\sin 2x}{2} - \int\left(1.\frac{\sin 2x}{2}\right)dx\right\}$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{\frac{x\sin 2x}{2} - \left(\frac{-\cos 2x}{2\times 2}\right)\right\} + c$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right\} + c$$

$$= \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c$$

Question: 10

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

Writing $\tan^2 x = \sec^2 x - 1$

We have

$$\int x\tan^2 x dx = \int x\left(\sec^2 x - 1\right)dx$$

$$= \int x\sec^2 x dx - \int x dx$$

Using x as the first function and Sec^2x as the second function

$$\int x\sec^2 x dx - \int x dx$$

$$= \left\{x\int \sec^2 x dx - \int\left(\frac{dx}{dx}.\int \sec^2 x dx\right)dx\right\} - \frac{x^2}{2}$$

$$= \left\{x.\tan x - \int 1.\tan x dx\right\} - \frac{x^2}{2}$$

$$= x\tan x - \ln|\sec x| - \frac{x^2}{2} + c$$

$$= x\tan x - \ln\left|\frac{1}{\cos x}\right| - \frac{x^2}{2} + c$$

$$x\tan x + \ln|\cos x| - \frac{x^2}{2} + c$$

Question: 11

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^x is the second function.

Using Integration by part

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\int x^2 e^x dx = \left[x^2\int e^x dx - \int\left(\frac{dx^2}{dx}.\int e^x dx\right)dx\right]$$

$$= x^2e^x - \int 2x.e^x dx$$

$$= x^2e^x - 2\int xe^x dx$$

$$= x^2e^x - 2\left[x\int e^x dx - \int\left(\frac{dx}{dx}.\int e^x dx\right)dx\right]$$

$$= x^2e^x - 2\left[xe^x - \int 1.e^x dx\right]$$

$$= x^2e^x - 2\left[xe^x - e^x\right] + c$$

$$= x^2e^x - 2xe^x + 2e^x + c$$

$$= e^x\left(x^2 - 2x + 2\right) + c$$

Question: 12

Solution:

We know that $\text{Cos}3x = 4\text{Cos}^3x - 3\text{Cos}x$

$$\text{Cos}^3x = \frac{\cos 3x + 3\cos x}{4}$$

$$\int x^2\cos^3 x dx = \int x^2\left(\frac{\cos 3x + 3\cos x}{4}\right)dx$$

$$= \frac{1}{4}\left(\int x^2\cos 3x dx + 3\int x^2\cos x dx\right)$$

Taking X² as the first function and cos 3x and cos x as the second function and applying By part method.

$$\frac{1}{4}\left(\int x^2\cos 3x dx + 3\int x^2\cos x dx\right)$$

$$= \frac{1}{4}\left\{\left(x^2\int\cos 3x dx - \int\left[\frac{dx^2}{dx}.\int\cos 3x dx\right]dx\right) + 3\left(x^2\int\cos x dx - \int\left[\frac{dx^2}{dx}.\int\cos x dx\right]dx\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\frac{x^2\sin 3x}{3} - \int 2x.\frac{\sin 3x}{3}dx\right) + 3\left(x^2\sin x - \int 2x.\sin x dx\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\frac{x^2\sin 3x}{3} - \frac{2}{3}\int x\sin 3x dx\right) + 3\left(x^2\sin x - 2\int x\sin x dx\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\frac{x^2\sin 3x}{3} - \frac{2}{3}\left[x\int\sin 3x dx - \int\left(\frac{dx}{dx}.\int\sin 3x dx\right)dx\right]\right) + 3\left(x^2\sin x - 2\left[x\int\sin x dx - \int\left(\frac{dx}{dx}.\int\sin x dx\right)dx\right]\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\frac{x^2\sin 3x}{3} - \frac{2}{3}\left[x\frac{-\cos 3x}{3} - \int 1.\frac{-\cos 3x}{3}dx\right]\right) + 3\left(x^2\sin x - 2\left[-x\cos x - \int -\cos x dx\right]\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\frac{x^2\sin 3x}{3} - \frac{2}{3}\left[\frac{-x\cos 3x}{3} + \frac{\sin 3x}{9}\right]\right) + 3\left(x^2\sin x + 2x\cos x - 2\sin x\right)\right\} + c$$

$$= \frac{1}{4}\left\{\frac{x^2\sin 3x}{3} + \frac{2x\cos 3x}{9} - \frac{2\sin 3x}{27} + 3x^2\sin x + 6x\cos x - 6\sin x\right\} + c$$

$$= \frac{x^2\sin 3x}{12} + \frac{x\cos 3x}{18} - \frac{\sin 3x}{54} + \frac{3x^2\sin x}{4} + \frac{3x\cos x}{2} - \frac{3}{2}\sin x + c$$

Question: 13

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^{3x} is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x^2 e^{3x} dx = x^2 \int e^{3x} dx - \int \left(\frac{dx^2}{dx} . \int e^{3x} dx \right) dx$$

$$= x^2 \frac{e^{3x}}{3} - \int 2x. \frac{e^{3x}}{3} dx$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \int xe^{3x} dx$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \left(x \int e^{3x} dx - \int \left[\frac{dx}{dx} . \int e^{3x} dx \right] dx \right)$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \left(x \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} dx \right)$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3} \left(x \frac{e^{3x}}{3} - \frac{e^{3x}}{9} \right) + c$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{27} + c$$

$$= e^{3x} \left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27} \right) + c$$

Question: 14

Solution:

We can write $\sin^2 x = \frac{1 - \cos 2x}{2}$

We have

$$\int x^2 \left(\frac{1 - \cos 2x}{2} \right) dx = \int \frac{x^2}{2} - \frac{x^2 \cos 2x}{2} dx$$

$$= \int \frac{x^2}{2} dx - \int \frac{x^2 \cos 2x}{2} dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and Cos 2x is the second function.

Using Integration by part

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} . \int b dx \right] dx$$

$$=\frac{x^3}{3\times 2}-\frac{1}{2}\int x^2\cos 2x dx$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2\int\cos 2x dx-\int\left[\frac{dx^2}{dx}.\int\cos 2x dx\right]dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\int 2x.\frac{\sin 2x}{2}dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\int x.\sin 2x dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\left[x\int\sin 2x dx-\int\left(\frac{dx}{dx}.\int\sin 2x dx\right)dx\right]\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\left[x\frac{-\cos 2x}{2}-\int 1.\frac{-\cos 2x}{2}dx\right]\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}+\frac{x\cos 2x}{2}-\frac{\sin 2x}{4}\right)+c$$

$$=\frac{x^3}{6}-\frac{x^2\sin 2x}{4}-\frac{x\cos 2x}{4}+\frac{\sin 2x}{8}+c$$

Question: 15

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log2x is the first function, and x^3 is the second function.

Using Integration by part

$$\int a.b.dx = a\int b dx-\int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\int x^3\log 2x dx=\log 2x\int x^3 dx-\int\left(\frac{d\log 2x}{dx}.\int x^3 dx\right)dx$$

$$=\log 2x\frac{x^4}{4}-\int\frac{1.2}{2x}.\frac{x^4}{4}dx$$

$$=\log 2x\frac{x^4}{4}-\frac{1}{4}\int x^3 dx$$

$$=\log 2x\frac{x^4}{4}-\frac{1}{4}.\frac{x^4}{4}+c$$

$$=\log 2x\frac{x^4}{4}-\frac{x^4}{16}+c$$

Question: 16

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(x + 1) is first function and x is second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x\log(x+1) = \log(x+1)\int xdx - \int\left(\frac{d\log(x+1)}{dx}.\int xdx\right)dx$$

$$= \log(x+1)\frac{x^2}{2} - \int\frac{1}{x+1}\times\frac{x^2}{2}dx$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\int\frac{x^2-1+1}{x+1}dx$$

Adding and subtracting 1 in the numerator,

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int\frac{x^2-1}{x+1} + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int\frac{(x+1)(x-1)}{x+1} + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int(x-1) + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\frac{x^2}{2} - x + \log(x+1)\right] + c$$

$$= \log(x+1)\frac{x^2}{2} - \frac{x^2}{4} + \frac{x}{2} - \frac{\log(x+1)}{2} + c$$

$$= \log(x+1)\frac{x^2-1}{2} - \frac{x^2}{4} + \frac{x}{2} + c$$

Question: 17

Solution:

We can write it as $\int x^{-n}.\log xdx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logx is the first function, and x^{-n} is the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x^{-n} \log x dx = \log x \int x^{-n} dx - \int\left(\frac{d \log x}{dx}.\int x^{-n} dx\right)dx$$

$$= \log x\left(\frac{x^{-n-1}}{-n+1}\right) - \int \frac{1}{x}.\frac{x^{-n-1}}{-n+1}dx$$

$$= \frac{x^{-n-1}\log x}{1-n} + \frac{1}{1-n}\int \frac{x^{-n}.x}{x}dx$$

$$= \frac{x^{-n-1}\log x}{1-n} + \frac{1}{1-n} \times \frac{x^{-n-1}}{-n+1} + c$$

$$= \frac{x^{-n-1}\log x}{1-n} - \frac{x^{-n-1}}{(1-n)^2} + c$$

Question: 18

Solution:

We can write it as $\int 2.x.x^2.e^{x^2} dx$

Let $x^2 = t$

$2xdx = dt$

Using the relation in the above condition, we get

$$\int 2x.x^2.e^{x^2} dx = \int t.e^t dt$$

Integrating with respect to t

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function, and e^t is the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int te^t dt = t\int e^t dt - \int\left(\frac{dt}{dt}.\int e^t dt\right)dt$$

$$= te^t - \int 1.e^t dt$$

$$= te^t - e^t + c$$

Replacing t with x^2,we get

$$x^2 e^{x^2} - e^{x^2} + c$$

$$= e^{x^2}\left(x^2 - 1\right) + c$$

Question: 19

Solution:

We know that $\sin 3x = 3\sin x - 4\sin^3 x$

$\sin^3 x = (3\sin x - \sin 3x)/4$

$$\int x\sin^3 x dx = \int x\left(\frac{3\sin x - \sin 3x}{4}\right)dx$$

$$= \frac{1}{4}\int 3x\sin x - x\sin 3x dx$$

$$= \frac{3}{4}\int x\sin x dx - \frac{1}{4}\int x\sin 3x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and sinx and sin3x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{3}{4}\int x\sin x dx - \frac{1}{4}\int x\sin 3x dx$$

$$= \frac{3}{4}\left(x\int \sin x dx - \int\left[\frac{dx}{dx}.\int \sin x dx\right]dx\right) - \frac{1}{4}\left(s\int \sin 3x dx - \int\left[\frac{dx}{dx}.\int \sin 3x dx\right]dx\right)$$

$$= \frac{3}{4}\left(-x\cos x + \int \cos x dx\right) - \frac{1}{4}\left(\frac{-x\cos 3x}{3} + \int\frac{\cos 3x}{3}dx\right)$$

$$= \frac{3}{4}\left(-x\cos x + \sin x\right) - \frac{1}{4}\left(\frac{-x\cos 3x}{3} + \frac{\sin 3x}{9}\right) + c$$

$$= \frac{-3x\cos x}{4} + \frac{3\sin x}{4} + \frac{x\cos 3x}{12} - \frac{\sin 3x}{36} + c$$

Question: 20

Solution:

We can write $\cos^3 x = (\cos 3x + 3\cos x)/4$, we have

$$\int x\cos^3 x dx = \int x\left(\frac{\cos 3x + 3\cos x}{4}\right)dx$$

$$= \frac{1}{4}\int x\cos 3x dx + \frac{3}{4}\int x\cos x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and cosx and cos3x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{1}{4}\left(x\int \cos 3x dx - \int\left[\frac{dx}{dx}.\int \cos 3x dx\right]dx\right) + \frac{3}{4}\left(x\int \cos x dx - \int\left[\frac{dx}{dx}.\int \cos x dx\right]dx\right)$$

$$= \frac{1}{4}\left(x\frac{\sin 3x}{3} - \int\frac{\sin 3x}{3}dx\right) + \frac{3}{4}\left(x\sin x - \int \sin x dx\right)$$

$$= \frac{1}{4}\left(\frac{x\sin 3x}{3} + \frac{\cos 3x}{9}\right) + \frac{3}{4}\left(x\sin x + \cos x\right) + c$$

$$= \frac{x\sin 3x}{12} + \frac{\cos 3x}{36} + \frac{3x\sin x}{4} + \frac{3\cos x}{4} + c$$

Question: 21

Solution:

We can write it as

$$\int x.x^2 \cos x^2 dx$$

Now let $x^2 = t$

$2xdx = dt$

$Xdx = dt/2$

Now

$$\frac{1}{2}\int t\cos tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and cost as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int t\cos tdt = \frac{1}{2}\left(t\int \cos tdt - \int\left[\frac{dt}{dt}.\int \cos tdt\right]dt\right)$$

$$= \frac{1}{2}\left(t\sin t - \int \sin tdt\right)$$

$$= \frac{1}{2}\left(t\sin t + \cos t\right) + c$$

Replacing t with x^2

$$= \frac{1}{2}x^2 \sin x^2 + \frac{1}{2}\cos x^2 + c$$

Question: 22

Solution:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(cosx) is the first function and sinx as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \sin x \log(\cos x)dx = \log(\cos x)\int \sin xdx - \int\left(\frac{d\log(\cos x)}{dx}.\int \sin xdx\right)dx$$

$$= -\cos x\log(\cos x) + \int \frac{-\sin x}{\cos x}.\cos xdx$$

$$= -\cos x\log(\cos x) - \int \sin xdx$$

$$= -\cos x\log(\cos x) + \cos x + c$$

Question: 23

CLASS24

Solution:

We know that Sin2x = 2Sinxcosx

$$\int x\sin x\cos x dx = \frac{1}{2}\int x\sin 2x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and sin2x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int x\sin 2x dx = \frac{1}{2}\left(x\int \sin 2x dx - \int\left[\frac{dx}{dx}.\int \sin 2x dx\right]dx\right)$$

$$= \frac{1}{2}\left(x\frac{-\cos 2x}{2} + \int\frac{\cos 2x}{2}dx\right)$$

$$= \frac{1}{2}\left(\frac{-x\cos 2x}{2} + \frac{\sin 2x}{4}\right) + c$$

$$= \frac{-x\cos 2x}{4} + \frac{\sin 2x}{8} + c$$

Question: 24

Solution:

Let √x = t

$$\frac{1}{2\sqrt{x}}dx = dt$$

$$\Rightarrow dx = 2\sqrt{x}dt$$

$$\Rightarrow dx = 2tdt$$

We can write it as

$$\int \cos\sqrt{x}dx = 2\int t\cos tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is first function and cos t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow 2\int t\cos tdt = 2\left(t\int \cos tdt - \int\left[\frac{dt}{dt}\right]\int \cos tdt\right)dt$$

$$= 2\left(t\sin t - \int \sin tdt\right)$$

$$= 2t\sin t + 2\cos t + c$$

Replacing t with √x

$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + c$

$= 2(\cos\sqrt{x} + \sqrt{x}\sin\sqrt{x}) + c$

Question: 25

Solution:

We can write it as

$$\int \operatorname{cosec}^3 dx = \int \operatorname{cosec}x.\operatorname{cosec}^2 x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here cosecx is first function and $\operatorname{cosec}^2 x$ as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \operatorname{cosec}x.\operatorname{cosec}^2 xdx = \operatorname{cosec}x\int .\operatorname{cosec}^2 xdx - \int\left(\frac{d\operatorname{cosec}x}{dx}.\int .\operatorname{cosec}^2 xdx\right)dx$$

$$= \operatorname{cosec}x(-\cot x) - \int(-\operatorname{cosec}x.\cot x)(-\cot x)dx$$

$$= -\operatorname{cosec}x.\cot x - \int \operatorname{cosec}x.\cot^2 xdx$$

We know that $\operatorname{Cot}^2 x = \operatorname{Cosec}^2 x - 1$

$$-\operatorname{cosec}x.\cot x - \int \operatorname{cosec}x(\operatorname{cosec}^2 x - 1)dx$$

$$= -\operatorname{cosec}x.\cot x - \int \operatorname{cosec}^3 xdx - \int \operatorname{cosec}xdx$$

We can write $\int \operatorname{cosec}^3 xdx = I$

$$\Rightarrow \int \operatorname{cosec}^3 xdx - \operatorname{cosec}x.\cot x - \int \operatorname{cosec}^3 xdx + \int \operatorname{cosec}xdx$$

$$\Rightarrow 2\int \operatorname{cosec}^3 xdx = -\operatorname{cosec}x.\cot x + \int \operatorname{cosec}xdx$$

$$\Rightarrow 2\int \operatorname{cosec}^3 xdx = -\operatorname{cosec}x.\cot x + \ln|\sec x + \tan x| + c_1$$

$$\Rightarrow \int \operatorname{cosec}^3 xdx = \frac{-\operatorname{cosec}x.\cot x + \ln|\sec x + \tan x|}{2} + c$$

Question: 26

Solution:

We can write it as $\int x\sin^2 x\sin x\cos xdx$

We also know that 2sinx.cosx = sin2x

$$\int x\sin^2 x\sin x\cos xdx = \frac{1}{2}\int x\sin^2 x\sin 2xdx$$

We also know that $\sin^2 x = \frac{1-\cos 2x}{2}$

$$\frac{1}{2}\int x\sin^2 x\sin 2x dx = \frac{1}{2}\int x.\left(\frac{1-\cos 2x}{2}\right)\sin 2x dx$$

$$= \frac{1}{2}\left[\left(\int \frac{x\sin 2x}{2}dx - \int \frac{x\cos 2x\sin 2x}{2}dx\right)\right]$$

Here Sin4x = 2sin2x.cos2x

$$= \frac{1}{2}\left[\left(\int \frac{x\sin 2x}{2}dx - \frac{1}{4}\int x\sin 4x dx\right)\right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and Sin2x and sin4x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{x\int \sin 2x dx - \int\left(\frac{dx}{dx}.\int \sin 2x dx\right)dx\right\}\right) - \left(\frac{1}{4}\left\{x\int \sin 4x - \int\left(\frac{dx}{dx}.\int \sin 4x dx\right)dx\right\}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2} + \int\frac{\cos 2x}{2}dx\right\}\right) - \left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4} + \int\frac{\cos 4x}{4}dx\right\}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2} + \frac{\sin 2x}{4}\right\}\right) - \left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4} + \frac{\sin 4x}{16}\right\}\right)\right] + c$$

$$= \frac{-x\cos 2x}{8} + \frac{\sin 2x}{16} + \frac{x\cos 4x}{32} - \frac{\sin 4x}{128} + c$$

Question: 27

Solution:

Let cosx = t

- sinxdx = dt

Now the integral we have is

$$\int \sin x\log(\cos x)dx = -\int \log tdt$$

$$= -\int 1.\log tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logt is first function and 1 as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$-\int 1.\log t dt = \log t \int 1 dt - \int \left(\frac{d \log t}{dt} . \int 1.dt \right) dt$$

$$= -\log t.t + \int \frac{1}{t}.tdt$$

$$= -t \log t + t + c$$

Replacing t with cosx

$$t(-\log t + 1) + c$$

$$= \cos x(1 - \log(\cos x)) + c$$

Question: 28

Solution:

Let logx = t

1/x dx = dt

$$\int \frac{\log(\log x)}{x} dx = \int \log t dt = \int 1.\log t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logt is first function and 1 as the second function.

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} . \int b dx \right] dx$$

$$\int 1.\log t dt = \log t \int 1 dt - \int \left(\frac{d \log t}{dt} . \int 1.dt \right) dt$$

$$= t.\log t - \int \frac{1}{t} t dt$$

$$= t \log t - t + c$$

Now replacing t with logx

$$\log x.\log(\log x) - \log x + c$$

$$= \log x(\log(\log x) - 1) + c$$

Question: 29

Solution:

$$= \int 1.\log(2 + x^2) dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log(2 + x^2)$ is the first function and 1 as the second function.

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} . \int b dx \right] dx$$

$$\int 1.\log\left(2+x^2\right)dx = \log\left(2+x^2\right)\int 1dx - \int\left(\frac{d\log\left(2+x^2\right)}{dx}.\int 1dx\right)dx$$

$$= \log\left(2+x^2\right).x - \int \frac{1.2x}{2+x^2}.xdx$$

$$= x\log\left(2+x^2\right) - \int \frac{2x^2}{2+x^2}dx$$

$$= x\log\left(2+x^2\right) - 2\int \frac{x^2+2-2}{2+x^2}dx$$

$$= x\log\left(2+x^2\right) - 2\left[\left(\int 1dx\right) - \int \frac{2}{2+x^2}dx\right]$$

$$= x\log\left(2+x^2\right) - 2\left[x - \left(2\int \frac{1}{2+x}\right)dx\right]$$

$$= x\log\left(2+x^2\right) - 2\left[x - 2\left(\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}\right)\right] + c$$

$$= x\log\left(2+x^2\right) - 2x + 2\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}} + c$$

Question: 30

Solution:

$$\int \frac{x}{1+\sin x}dx = \int \frac{x(1-\sin x)}{(1+\sin x).(1-\sin x)}dx$$

We can write it as

$$= \int \frac{x(1-\sin x)}{1-\sin^2 x}dx$$

$$= \int \frac{x(1-\sin x)}{\cos^2 x}dx$$

$$= \int x\sec^2 xdx - \int x\tan x\sec xdx$$

Using by part and ILATE

Taking x as first function and $\sec^2 x$ and secxtanx as the second function, we have

$$\int x\sec^2 xdx - \int x\sec x\tan xdx = \left(x\int \sec^2 xdx - \int\left(\frac{dx}{dx}.\int \sec^2 xdx\right)dx\right)$$

$$-\left(x\int \sec x\tan xdx - \int\left(\frac{dx}{dx}.\int \sec x\tan xdx\right)dx\right)$$

$$= \left(x\tan x - \int 1.\tan xdx\right) - \left(x.\sec x - \int 1.\sec xdx\right)$$

$$= x\tan x - \ln|\sec x| - x\sec x + \ln|\sec x + \tan x| + c$$

$$= x(\tan x - \sec x) + \ln\left|\frac{\sec x + \tan x}{\sec x}\right| + c$$

$$= x(\tan x - \sec x) + \ln|1+\sin x| + c$$

Question: 31

Solution:

Let us assume logx = t

$X = e^t$

$dx = e^t dt$

Now we have

$$\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx=\int\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt$$

Considering $f(x) = 1/t$; $f'(x) = -1/t^2$

$$\frac{d}{dt}\left(\frac{1}{t}\right)=-\frac{1}{t^2}$$

By the integral property of $\int\{f(x)+f'(x)\}e^x dx = e^x.f(x)+c$

So the solution of the integral is

$$\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx=e^t\times\frac{1}{t}+c$$

Substituting the value of t as logx

$$=e^{\log x}\times\frac{1}{\log x}+c$$

$$=\frac{x}{\log x}+c$$

Question: 32

Solution:

$$\cos A.\cos B=\frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$$

We know that $\Rightarrow \cos 4x.\cos 2x=\frac{1}{2}\left[\cos(4x+2x)+\cos(4x-2x)\right]$

$$=\frac{1}{2}\left[\cos 6x+\cos 2x\right]$$

Putting in the original equation

$$\int e^{-x}\cos 2x.\cos 4x dx=\int e^{-x}\left(\frac{1}{2}\left[\cos 6x+\cos 2x\right]\right)$$

$$=\frac{1}{2}\left[\left(\int e^{-x}\cos 6x dx\right)+\left(\int e^{-x}\cos 2x dx\right)\right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here cos6x and cos2x is first function and e^{-x} as the second function.

$$\int a.b.dx=a\int b dx-\int\left[\frac{da}{dx}.\int b dx\right]dx$$

Solving both parts individually

$$I=\int e^{-x}\cos 6x dx=\cos 6x\int e^{-x}dx-\int\left(\frac{d\cos 6x}{dx}.\int e^{-x}dx\right)dx$$

$$I=\cos 6x.\left(-e^{-x}\right)-\int\left(-6\sin 6x\right).\left(-e^{-x}\right)dt$$

$$I=-\cos 6x.e^{-x}-6\int \sin 6x.e^{-x}dx$$

$$I=-e^{-x}\cos 6x-6\left[\sin 6x\int e^{-x}dx-\int\left(\frac{d\sin 6x}{dx}.\int e^{-x}dx\right)dx\right]$$

$$I=-e^{-x}\cos 6x-6\left[\sin 6x\left(-e^{-x}\right)-\int\left(6\cos 6x\right).\left(-e^{-x}\right)dt\right]$$

$$I=-e^{-x}\cos 6x-6\left[-e^{-x}\sin 6x+6\int e^{-x}\cos 6x dx\right]$$

$$I=-e^{-x}\cos 6x-6\left[-e^{-x}\sin 6x+6I\right]$$

$$I=-e^{-x}\cos 6x+6e^{-x}\sin 6x-36I$$

$$37I=e^{-x}\left(6\sin 6x-\cos 6x\right)$$

$$I=\frac{e^{-x}\left(6\sin 6x-\cos 6x\right)}{37}$$

Solving the second part,

$$I=\int e^{-x}\cos 2x dx=\cos 2x\int e^{-x}dx-\int\left(\frac{d\cos 2x}{dx}.\int e^{-x}dx\right)dx$$

$$J=\cos 2x.\left(-e^{-x}\right)-\int\left(-2\sin 2x\right).\left(-e^{-x}\right)dt$$

$$J=-\cos 2x.e^{-x}-2\int \sin 2x.e^{-x}dx$$

$$J=-e^{-x}\cos 2x-2\left[\sin 2x\int e^{-x}dx-\int\left(\frac{d\sin 2x}{dx}.\int e^{-x}dx\right)dx\right]$$

$$J=-e^{-x}\cos 2x-2\left[\sin 2x\left(-e^{-x}\right)-\int\left(2\cos 2x\right).\left(-e^{-x}\right)dt\right]$$

$$J=-e^{-x}\cos 2x-2\left[-e^{-x}\sin 2x+2\int e^{-x}\cos 2x dx\right]$$

$$J=-e^{-x}\cos 2x-2\left[-e^{-x}\sin 2x+2J\right]$$

$$J=-e^{-x}\cos 2x+2e^{-x}\sin 2x-4J$$

$$5J=e^{-x}\left(2\sin 2x-\cos 2x\right)$$

$$J=\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{5}$$

Putting in the obtained equation

$$=\frac{1}{2}\left[\frac{e^{-x}\left(6\sin 6x-\cos 6x\right)}{37}+\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{5}\right]+c$$

$$=\frac{e^{-x}\left(6\sin 6x-\cos 6x\right)}{74}+\frac{e^{-x}\left(2\sin 2x-\cos 2x\right)}{10}+c$$

$$=e^{-x}\left(\frac{\left(6\sin 6x-\cos 6x\right)}{74}+\frac{\left(2\sin 2x-\cos 2x\right)}{10}\right)+c$$

Question: 33

Evaluate the foll

Solution:

Let √x = t

$$\frac{1}{2\sqrt{x}}dx = dt$$

$$dx = 2\sqrt{x}dt$$

$$\Rightarrow dx = 2tdt$$

Replacing in the original equation , we get

$$\int e^{\sqrt{x}}dx = \int e^{t}.2tdt$$

$$= 2\int te^{t}dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$2\int te^{t}dt = 2\left[t\int e^{t}dt - \int\left(\frac{dt}{dt}.\int e^{t}dt\right)dt\right]$$

$$= 2\left[te^{t} - \int 1.e^{t}dt\right]$$

$$= 2\left[te^{t} - e^{t}\right] + c$$

$$= 2e^{t}(t-1) + c$$

Replacing t with √x

$= 2e^{\sqrt{x}}(\sqrt{x} - 1) + c$

Question: 34

Solution:

We can write Sin2x = 2sinx.cosx

$$\int e^{\sin x}\sin 2xdx = 2\int e^{\sin x}.\sin x\cos xdx$$

Let Sinx = t

Cosxdx = dt

$$2\int e^{\sin x}\sin x\cos xdx = 2\int e^{t}.t.dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$2\int e^t.tdt = 2\left[t\int e^t dt - \int\left(\frac{dt}{dt}.\int e^t dt\right)dt\right]$$

$$= 2\left[t.e^t - \int 1.e^t dt\right]$$

$$= 2\left[t.e^t - e^t\right] + c$$

$$= 2e^t(t-1) + c$$

Replacing t with sin x

= $2e^{\sin x}(\sin x - 1) + c$

Question: 35

Solution:

Let $\sin^{-1}x = t$

$X = \sin t$

$$\frac{1}{\sqrt{1-x^2}}dx = dt$$

Putting this in the original equation, we get

$$\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = \int t.\sin t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and sin t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int t.\sin tdt = t\int \sin tdt - \int\left(\frac{dt}{dt}.\int \sin tdt\right)dt$$

$$= t(-\cos t) - \int 1.(-\cos t)dt$$

$$= -t\cos t + \sin t + c$$

We can write $\cos t = \sqrt{1 - \sin^2 t}$

$= -t(\sqrt{1 - \sin^2 t}) + \sin t + c$

Now replacing $\sin^{-1}x = t$

$= -\sin^{-1}x(\sqrt{1 - x^2}) + x + c$

Question: 36

Solution:

Let $\tan^{-1} x = t$ and $x = \tan t$

Differentiating both sides, we get

$$\frac{1}{1+x^2}dx = dt$$

Now we have

$$\int \frac{x^2 \tan^{-1} x}{\left(1+x^2\right)} dx = \int \tan^2 t.tdt$$

$$\int t.\tan^2 tdt = \int t\left(\sec^2 t - 1\right)dt$$

$$= \int t\sec^2 tdt - \int tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and $\sec^2 t$ as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int t\sec^2 tdt - \int tdt = t\int \sec^2 tdt - \int\left(\frac{dt}{dt}.\int \sec^2 tdt\right)dt - \frac{t^2}{2}$$

$$= t.\tan t - \int \tan tdt - \frac{t^2}{2}$$

$$= t.\tan t - \ln|\sec t| - \frac{t^2}{2} + c$$

We know that $\sec t = \sqrt{\tan^2 t + 1}$

$$= \tan^{-1} x.x - \ln\left|\sqrt{\tan^2 t + 1}\right| - \frac{\tan^2 x}{2} + c$$

$$= x\tan^{-1} x - \ln\left|\sqrt{x^2 + 1}\right| - \frac{\tan^2 x}{2} + c$$

Question: 37

Solution:

We can write it as $\int \log(x+2).\frac{1}{(x+2)^2}dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log(x+2)$ is first function and $(x+2)^{-2}$ as second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \log(x+2).\frac{1}{(x+2)^2}dx = \log(x+2)$$

$$\int \frac{1}{(x+2)^2}dx - \int\left(\frac{d\log(x+2)}{dx}.\int\frac{1}{(x+2)^2}dx\right)dx$$

$$= \log(x+2).\frac{-1}{(x+2)} - \int \frac{1}{x+2}.\frac{-1}{(x+2)}dx$$

$$= -\log(x+2)\frac{1}{(x+2)} + \int\frac{1}{(x+2)^2}dx$$

$$= -\log(x+2)\frac{1}{(x+2)} - \frac{1}{(x+2)} + c$$

Question: 38

Solution:

Let x = sin t ; t = $\sin^{-1}x$

dx = cos t dt

$$\Rightarrow \int x\sin^{-1}x dx = \int \sin t.\sin^{-1}(\sin t)\cos t dt$$

$$= \int \sin t.t.\cos t dt$$

We know that sin 2t = 2 sint×cost

We have $\int t\cos t\sin t dt = \frac{1}{2}\int t\sin 2t dt$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and sin 2t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int t\sin 2t dt = \frac{1}{2}\left(t\int \sin 2t dt - \int\left[\frac{dt}{dt}.\int \sin 2t dt\right]dt\right)$$

$$= \frac{1}{2}\left(t.\frac{-\cos 2t}{2} + \int\frac{\cos 2t}{2}dt\right)$$

$$= \frac{1}{2}\left(\frac{-t\cos 2t}{2} + \frac{\sin 2t}{4}\right) + c$$

$$= \frac{-t\cos 2t}{4} + \frac{\sin 2t}{8} + c$$

We know that cos2t = 1 - $2\sin^2 t$, sin2t = 2sint×cost and cos t = $\sqrt{1 - \sin^2 t}$

Replacing in above equation

$$= \frac{-t\left(1-2\sin^2 t\right)}{4} + \frac{2\sin t \times \cos t}{8} + c$$

$$= \frac{-t\left(1-2\sin^2 t\right)}{4} + \frac{\sqrt{1-\sin^2 t}}{4}.\sin t + c$$

$$= \frac{-\sin^{-1}x\left(1-2x^2\right)}{4} + \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \frac{1}{2}x^2\sin^{-1}x - \frac{\sin^{-1}x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c$$

$$= \frac{1}{2}x^2\sin^{-1}x - \frac{\sin^{-1}x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c$$

Question: 39

Solution:

Let $x = \cos t$; $t = \cos^{-1}x$

$dx = -\sin t\,dt$

$$\int x\cos^{-1}x\,dx = -\int \cos t.\cos^{-1}(\cos t)\sin t\,dt$$

$$= -\int \cos t.t.\sin t.dt$$

We know that $\sin 2t = 2\sin t \times \cos t$

We have $-\int t\cos t\sin t\,dt = \frac{-1}{2}\int t\sin 2t\,dt$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking first function to the one which comes first in the list.

Here t is first function and sin 2t as second function.

$$\int a.b.dx = a\int b\,dx - \int\left[\frac{da}{dx}.\int b\,dx\right]dx$$

$$\frac{-1}{2}\int t\sin 2t\,dt = \frac{-1}{2}\left(t\int \sin 2t\,dt - \int\left[\frac{dt}{dt}.\int \sin 2t\,dt\right]dt\right)$$

$$= \frac{-1}{2}\left(t.\frac{-\cos 2t}{2} + \int \frac{\cos 2t}{2}dt\right)$$

$$= \frac{-1}{2}\left(\frac{-t\cos 2t}{2} + \frac{\sin 2t}{4}\right) + c$$

$$= \frac{t\cos 2t}{4} - \frac{\sin 2t}{8} + c$$

We know that $\cos 2t = 2\cos^2 t - 1$ and $\sin 2t = 2\sin t \times \cos t$ and $\sin t = \sqrt{1} - \cos^2 t$

Replacing in above equation

$$=\frac{t\left(2\cos^2 t-1\right)}{4}-\frac{2\sin t\times\cos t}{8}+c$$

$$=\frac{t\left(2\cos^2 t-1\right)}{4}-\frac{\sqrt{1-\cos^2 t}}{4}.\cos t+c$$

$$=\frac{\cos^{-1}x\left(2x^2-1\right)}{4}-\frac{x\sqrt{1-x^2}}{4}+c$$

$$=\frac{1}{2}x^2\cos^{-1}x-\frac{\cos^{-1}x}{4}-\frac{1}{4}x\sqrt{1-x^2}+c$$

$$=\frac{1}{2}x^2\cos^{-1}x+\frac{\sin^{-1}x}{4}-\frac{1}{4}x\sqrt{1-x^2}+c$$

Question: 40

Solution:

We can write it as $\int\cot^{-1}x.1dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\cot^{-1}x$ is first function and 1 as the second function.

$$\int a.b.dx=a\int bdx-\int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int\cot^{-1}x.1dx=\cot^{-1}x\int 1dx-\int\left(\frac{d\cot^{-1}x}{dx}.\int 1dx\right)dx$$

$$=\cot^{-1}x.x-\int\frac{-1}{1+x^2}.x.dx$$

$$=x\cot^{-1}x+\int\frac{x}{1+x^2}dx$$

Let $1+x^2=t$

$2xdx=dt$

$Xdx=dt/2$

$$\Rightarrow\int\cot^{-1}xdx=x\cot^{-1}x+\int\frac{dt}{2t}$$

$$=x\cot^{-1}x+\frac{\log t}{2}+c$$

Now replacing t with $1+x^2$

$=x\cot^{-1}x+\log(1+x^2)/2+c$

Question: 41

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second f

respectively.

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x$,

$$\therefore \int x\cot^{-1}x\,dx$$

$$= \cot^{-1}x\int xdx - \int\left\{\frac{d}{dx}(\cot^{-1}x)\int xdx\right\}dx$$

$$= \frac{x^2\cot^{-1}x}{2} - \int\frac{1}{(1+x^2)}\times\frac{x^2}{2}dx$$

$$= \frac{x^2\cot^{-1}x}{2} - \frac{1}{2}\int\frac{x^2}{(1+x^2)}dx$$

$$= \frac{x^2\cot^{-1}x}{2} - \frac{1}{2}\int\frac{1+x^2-x^2}{(1+x^2)}dx$$

$$= \frac{x^2\cot^{-1}x}{2} - \frac{1}{2}\int 1 - \frac{1}{(1+x^2)}dx$$

$= \frac{x^2\cot^{-1}x}{2} - \frac{1}{2}[x - \tan^{-1}x] + c$, where c is the integrating constant

Question: 42

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x^2$,

$$\therefore \int x^2\cot^{-1}x\,dx$$

$$= \cot^{-1}x\int x^2dx - \int\left\{\frac{d}{dx}(\cot^{-1}x)\int x^2dx\right\}dx$$

$$= \frac{x^3\cot^{-1}x}{3} - \int\frac{1}{(1+x^2)}\times\frac{x^3}{3}dx$$

$$= \frac{x^3\cot^{-1}x}{3} - \frac{1}{3}\int\frac{x^3}{(1+x^2)}dx$$

Taking $(1+x^2)=a$,

$2xdx=da$ i.e. $xdx=da/2$

Again, $x^2=a-1$

$$\therefore \frac{1}{3}\int\frac{x^2\times xdx}{(1+x^2)}$$

$$= \frac{1}{3}\int\frac{(a-1)da}{2a}$$

$$= \frac{1}{6}\int\left(1-\frac{1}{a}\right)da$$

$$= \frac{1}{6}(a - \ln a)$$

Replacing the value of a, we get,

$$\therefore \frac{1}{6}(a - \ln a)$$

$$= \frac{1}{6}[(1 + x^2) - \ln|x^2 + 1| + c_1$$

$$= \frac{x^2}{6} - \frac{\ln|x^2 + 1|}{6} + \left(c_1 + \frac{1}{6}\right)$$

$$= \frac{x^2}{6} - \frac{\ln|x^2 + 1|}{6} + c$$

The total integration yields as

$= \frac{x^3 \cot^{-1} x}{3} + \frac{x^2}{6} - \frac{\ln|x^2+1|}{6} + c$, where c is the integrating constant

Question: 43

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \sin^{-1}\sqrt{x}\,dx$$

$$= \sin^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}(\sin^{-1}\sqrt{x})\int dx\right\}dx$$

$$= x\sin^{-1}\sqrt{x} - \int \frac{1}{2\sqrt{x}\sqrt{1-x}} \times x\,dx$$

$$= x\sin^{-1}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}\,da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= \mathbf{x \sin^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c}$, where c is the integrating constant

Question: 44

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \cos^{-1}\sqrt{x}\,dx$$

$$= \cos^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\cos^{-1}\sqrt{x}\right)\int dx\right\}dx$$

$$= x\cos^{-1}\sqrt{x} - \int \frac{-1}{2\sqrt{x}\sqrt{1-x}} \times x dx$$

$$= x\cos^{-1}\sqrt{x} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= \mathbf{x \cos^{-1}\sqrt{x} - \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c}$, where c is the integrating constant

Question: 45

Solution:

Formula to be used – We know , $\cos 3x = 4\cos^3 x - 3\cos x$

$$\therefore \int \cos^{-1}(4x^3 - 3x)\,dx$$

Assuming x = cosa, $4\cos^3 a - 3\cos a = \cos 3a$

And, dx = -sinada

Hence, $a = \cos^{-1}x$

Again, $\sin a = \sqrt{(1-x^2)}$

$$\therefore \int \cos^{-1}(4x^3 - 3x)\,dx$$

$$= \int \cos^{-1}(\cos 3a)\{-\sin a\,da\}$$

$$= -3\int a\sin a\,da$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = a$ and $f_2(x) = \sin a$,

$$\therefore -3\int a\sin a\,da$$

$$= -3\left[a\int \sin a\,da - \int\left\{\frac{d}{dx}a\int \sin a\,da\right\}da\right]$$

$$= 3a\cos a - \int \cos a\,da$$

$$= 3a\cos a - \sin a + c$$

Replacing the value of a we get,

$$\therefore 3a\cos a - \sin a + c$$

$= \mathbf{3x\cos^{-1}x - \sqrt{1-x^2} + c}$, where c is the integrating constant

Question: 46

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and $f_2(x) = 1$,

$$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$$

$$= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\int dx - \int\left[\frac{d}{dx}\left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}\int dx\right]dx$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \int\left[\frac{\frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}}\right]dx$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \int \frac{-4x^2 dx}{(1+x^2)^2 \times \frac{1}{1+x^2} \times 2x}$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) - \int \frac{2x dx}{1+x^2}$$

Now,

$$\int \frac{2x dx}{1+x^2}$$

$$= \int \frac{d(1+x^2)}{1+x^2}$$

$$= \ln(1+x^2) + c$$

Again, we know,

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

$$\Rightarrow 2x = \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$$

Replacing x by tanx, it is obtained that,

$$2\tan x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

So, the final integral yielded is

$2x\tan x - \ln(1+x^2) + c$, where c is the integrating constant

Question: 47

Solution:

Formula to be used – We know, $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$$

Assuming x = tana,

$$\frac{2\tan a}{1-\tan^2 a} = \tan 2a$$

And, $dx = \sec^2 a\, da$

Hence, $a = \tan^{-1} x$

Now, $\sec^2 a - \tan^2 a = 1$, so, $\sec a = \sqrt{(1+x^2)}$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$$

$$= \int \tan^{-1}(\tan 2a)\{\sec^2 a\, da\}$$

$$= 2\int a \sec^2 a\, da$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = a$ and $f_2(x) = sec^2a$,

$$\therefore 2\int asec^2ada$$

$$= 2\left[a\int sec^2ada - \int\left\{\frac{d}{dx}a\int sec^2ada\right\}da\right]$$

$$= 2atana - \int tanada$$

$$= 2atana - \ln|seca| + c$$

Replacing the value of a we get,

$$\therefore 2atana - \ln|seca| + c$$

$= \mathbf{2x\tan^{-1}x - \ln\sqrt{1+x^2} + c}$, where c is the integrating constant

Question: 48

Solution:

Formula to be used – We know, $\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

$$\therefore \int \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)dx$$

Assuming x = tana,

$$\frac{3tana - tan^3a}{1-3tan^2a} = tan3a$$

And, $dx = sec^2ada$

Hence, $a=tan^{-1}x$

Now, $sec^2a-tan^2a=1$, so, $seca=\sqrt{(1+x^2)}$

$$\therefore \int \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)dx$$

$$= \int \tan^{-1}(tan3a)\{sec^2ada\}$$

$$= 3\int asec^2ada$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = a$ and $f_2(x) = sec^2a$,

$$\therefore 3\int asec^2ada$$

$$= 3\left[a\int sec^2ada - \int\left\{\frac{d}{dx}a\int sec^2ada\right\}da\right]$$

$$= 3atana - \frac{3}{2}\int tanada$$

$$= 3atana - \frac{3}{2}\ln|seca| + c$$

Replacing the value of a we get,

$\therefore 3a\tan a - \frac{3}{2}\ln|\sec a| + c$

$= \mathbf{3x\tan^{-1}x - \frac{3}{2}\ln\sqrt{1+x^2} + c}$ **, where c is the integrating constant**

Question: 49

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}x$ and $f_2(x) = 1/x^2$,

$\therefore \int \frac{\sin^{-1}x}{x^2}dx$

$= \sin^{-1}x\int\frac{1}{x^2}dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\int\frac{1}{x^2}dx\right\}dx$

$= \frac{-\sin^{-1}x}{x} - \int\frac{1}{\sqrt{1-x^2}}\times\left(-\frac{1}{x}\right)dx$

$= \frac{-\sin^{-1}x}{x} + \int\frac{1}{x\sqrt{1-x^2}}dx$

Taking x= sina, dx = cosada

Hence, coseca=1/x

Now, $\mathrm{cosec}^2a - \cot^2a = 1$ so $\cot a = \sqrt{(1-x^2)}/x$

$\therefore \int\frac{1}{x\sqrt{1-x^2}}dx$

$= \int\frac{1}{\sin a\cos a}(\cos a\,da)$

$= \int \mathrm{cosec}\,a\,da$

$= \ln|\mathrm{cosec}\,a - \cot a| + c$

Replacing the value of a, we get,

$\therefore \ln|\mathrm{cosec}\,a - \cot a| + c$

$= \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + c$

The total integration yields as

$= \mathbf{\frac{-\sin^{-1}x}{x} + \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + c}$ **, where c is the integrating constant**

Question: 50

Solution:

Say, tanx = a

Hence, $\sec^2 x dx = da$

$$\therefore \int \frac{\tan x \sec^2 x}{1-\tan^2 x} dx$$

$$= \int \frac{a da}{1-a^2}$$

Now, taking $1-a^2 = k$, $-2ada=dk$ i.e. $ada=-dk/2$

$$\therefore \int \frac{a da}{1-a^2}$$

$$= \int \frac{-dk}{2k}$$

$$= -\frac{1}{2}\ln|k| + c$$

Replacing the value of k,

$$-\frac{1}{2}\ln|k| + c$$

$$= -\frac{1}{2}\ln|1-a^2| + c$$

Replacing the value of a,

$$-\frac{1}{2}\ln|1-a^2| + c$$

$= -\frac{1}{2}\ln|1-\tan^2 x| + c$, where c is the integrating constant

Question: 51

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin 4x$ and $f_2(x) = e^{3x}$,

$$\therefore \int e^{3x}\sin 4x dx$$

$$= \sin 4x \int e^{3x}dx - \int\left\{\frac{d}{dx}(\sin 4x)\int e^{3x}dx\right\}dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \int 4\cos 4x \times \frac{e^{3x}}{3}dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4}{3}\int e^{3x}\cos 4x dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4}{3}\left[\cos 4x \int e^{3x}dx - \int\left\{\frac{d}{dx}(\cos 4x)\int e^{3x}dx\right\}dx\right]$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4e^{3x}\cos 4x}{9} - \frac{4}{3}\int 4\sin 4x \times \frac{e^{3x}}{3}dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4e^{3x}\cos 4x}{9} - \frac{16}{9}\int e^{3x}\sin 4x dx$$

$\therefore \left(1+\frac{16}{9}\right)\int e^{3x}\sin4x dx = \frac{e^{3x}\sin4x}{3} - \frac{4e^{3x}\cos4x}{9} + c_1$

$\Rightarrow \frac{25}{9}\int e^{3x}\sin4x dx = \frac{3e^{3x}\sin4x - 4e^{3x}\cos4x}{9} + c_1$

$\Rightarrow \int e^{3x}\sin4x dx = \frac{e^{3x}}{25}(3\sin4x - 4\cos4x) + c$, where c is the integrating constant

Question: 52

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin x$ and $f_2(x) = e^{2x}$,

$\therefore \int e^{2x}\sin x dx$

$= \sin x\int e^{2x}dx - \int\left\{\frac{d}{dx}(\sin x)\int e^{2x}dx\right\}dx$

$= \frac{e^{2x}\sin x}{2} - \int \cos x \times \frac{e^{2x}}{2}dx$

$= \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x}\cos x dx$

$= \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x\int e^{2x}dx - \int\left\{\frac{d}{dx}(\cos x)\int e^{2x}dx\right\}dx\right]$

$= \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{2}\int \sin x \times \frac{e^{2x}}{2}dx$

$= \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}\int e^{2x}\sin x dx$

$\therefore \left(1+\frac{1}{4}\right)\int e^{2x}\sin x dx = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} + c_1$

$\Rightarrow \frac{5}{4}\int e^{2x}\sin x dx = \frac{2e^{2x}\sin x - e^{2x}\cos x}{4} + c_1$

$\Rightarrow \int e^{2x}\sin x dx = \frac{e^{2x}}{5}(2\sin x - \cos x) + c$, where c is the integrating constant

Question: 53

Solution:

$\int e^{2x}\sin x\cos x dx$

$= \frac{1}{2}\int e^{2x} \times 2\sin x\cos x dx$

$= \frac{1}{2}\int e^{2x}\sin2x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where f₁(x) and f₂(x) are the first and second f respectively.

Taking $f_1(x) = \sin 2x$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\sin 2x dx$$

$$= \sin 2x \int e^{2x}dx - \int\left\{\frac{d}{dx}(\sin 2x)\int e^{2x}dx\right\}dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \int 2\cos 2x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \int e^{2x}\cos 2x dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \left[\cos 2x\int e^{2x}dx - \int\left\{\frac{d}{dx}(\cos 2x)\int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} - \int 2\sin 2x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} - \int e^{2x}\sin x dx$$

$$\therefore (1+1)\int e^{2x}\sin 2x dx = \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} + c_1$$

$$\Rightarrow 2\int e^{2x}\sin 2x dx = \frac{e^{2x}\sin 2x - e^{2x}\cos 2x}{2} + c_1$$

$$\Rightarrow \int e^{2x}\sin 2x dx = \frac{e^{2x}}{4}(\sin 2x - \cos 2x) + c'$$

$$\therefore \frac{1}{2}\int e^{2x}\sin 2x dx$$

$$= \frac{1}{2} \times \left[\frac{e^{2x}}{4}(\sin 2x - \cos 2x) + c'\right]$$

$= \frac{e^{2x}}{8}(\sin 2x - \cos 2x) + c$, where c is the integrating constant

Question: 54

Solution:

Tip – If f₁(x) and f₂(x) are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where f₁(x) and f₂(x) are the first and second functions respectively.

Taking $f_1(x) = \cos(3x+4)$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\cos(3x+4)\,dx$$

$$= \cos(3x+4)\int e^{2x}dx - \int\left\{\frac{d}{dx}\cos(3x+4)\int e^{2x}dx\right\}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \int 3\sin(3x+4) \times \frac{e^{2x}}{2}dx$$

CLASS24

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{2}\int e^{2x}\sin(3x+4)dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{2}\left[\sin(3x+4)\int e^{2x}dx - \int\left\{\frac{d}{dx}\sin(3x+4)\int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} - \frac{3}{2}\int 3\cos(3x+4)\times\frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} - \frac{9}{4}\int e^{2x}\cos(3x+4)dx$$

$$\therefore \left(1+\frac{9}{4}\right)\int e^{2x}\cos(3x+4)dx = \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} + c_1$$

$$\Rightarrow \frac{13}{4}\int e^{2x}\cos(3x+4)\,dx = \frac{2e^{2x}\cos(3x+4) + 3e^{2x}\sin(3x+4)}{4} + c_1$$

$\Rightarrow \int e^{2x}\cos(3x+4)\,dx = \frac{e^{2x}}{13}\left(2\cos(3x+4) + 3\sin(3x+4)\right) + c$, where c is the integrating constant

Question: 55

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos x$ and $f_2(x) = e^{-x}$,

$$\therefore \int e^{-x}\cos x\,dx$$

$$= \cos x\int e^{-x}dx - \int\left\{\frac{d}{dx}\cos x\int e^{-x}dx\right\}dx$$

$$= -e^{-x}\cos x - \int e^{-x}\sin x\,dx$$

$$= -e^{-x}\cos x - \left[\sin x\int e^{-x}dx - \int\left\{\frac{d}{dx}\sin x\int e^{-x}dx\right\}dx\right]$$

$$= -e^{-x}\cos x - \left[-e^{-x}\sin x + \int e^{-x}\cos x dx\right]$$

$$= -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x dx$$

$$\therefore (1+1)\int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$$

$$\Rightarrow 2\int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$$

$\Rightarrow \int e^{-x}\cos x dx = \frac{e^{-x}}{2}(\sin x - \cos x) + c$, where c is the integrating constant

Question: 56

Solution:

$\int e^x(\sin x + \cos x)dx$

$= \int e^x \sin x dx + \int e^x \cos x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \sin x dx + \int e^x \cos x dx$

$= \sin x \int e^x dx - \int\left[\frac{d}{dx}(\sin x)\int e^x dx\right]dx + \int e^x \cos x dx$

$= e^x \sin x - \int e^x \cos x dx + \int e^x \cos x dx + c$

$= \mathbf{e^x \sin x + c}$, where c is the integrating constant

Question: 57

Solution:

$\int e^x(\cot x - \operatorname{cosec}^2 x)dx$

$= \int e^x \cot x dx + \int e^x \operatorname{cosec}^2 x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \cot x dx + \int e^x \operatorname{cosec}^2 x dx$

$= \cot x \int e^x dx - \int\left[\frac{d}{dx}(\cot x)\int e^x dx\right]dx + \int e^x \operatorname{cosec}^2 x dx$

$= e^x \cot x - \int e^x \operatorname{cosec}^2 x dx + \int e^x \operatorname{cosec}^2 x dx + c$

$= \mathbf{e^x \cot x + c}$, where c is the integrating constant

Question: 58

Solution:

$\int e^x \sec x(1 + \tan x)dx$

$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second f

respectively.

Taking $f_1(x)$ = secx and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \sec x dx + \int e^x \sec x \tan x dx$$

$$= \sec x \int e^x dx - \int\left[\frac{d}{dx}(\sec x)\int e^x dx\right]dx + \int e^x \sec x \tan x dx$$

$$= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx + c$$

$= \mathbf{e^x \sec x + c}$, where c is the integrating constant

Question: 59

Solution:

$$\int e^x\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx$$

$$= \int e^x \tan^{-1}x\, dx + \int \frac{e^x}{1+x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan^{-1}x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \tan^{-1}x\, dx + \int \frac{e^x}{1+x^2}dx$$

$$= \tan^{-1}x \int e^x dx - \int\left[\frac{d}{dx}(\tan^{-1}x)\int e^x dx\right]dx + \int \frac{e^x}{1+x^2}dx$$

$$= e^x \tan^{-1}x - \int \frac{e^x}{1+x^2}dx + \int \frac{e^x}{1+x^2}dx + c$$

$= \mathbf{e^x \tan^{-1}x + c}$, where c is the integrating constant

Question: 60

Solution:

$$\int e^x(\cot x + \log\sin x)dx$$

$$= \int e^x \cot x\, dx + \int e^x \log\sin x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logsinx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \cot x\, dx + \int e^x \log\sin x dx$$

$= \int e^x \cot x\, dx + \text{logsinx} \int e^x dx - \int \left[\frac{d}{dx}(\text{logsinx}) \int e^x dx\right]$

$= \int e^x \cot x\, dx + e^x \text{logsinx} - \int e^x \cot x\, dx + c$

$=$ **$e^x \log|\sin x| + c$** , where c is the integrating constant

Question: 61

Solution:

$\int e^x(\tan x + \text{logcosx})dx$

$= \int e^x \tan x\, dx + \int e^x \text{logcosxdx}$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logcosx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$\int e^x \tan x\, dx - \int e^x \text{logcosxdx}$

$= \int e^x \tan x\, dx - \text{logcosx} \int e^x dx + \int \left[\frac{d}{dx}(\text{logcosx}) \int e^x dx\right]$

$= \int e^x \tan x\, dx - e^x \text{logcosx} - \int e^x \tan x\, dx + c$

$=$ **$e^x \log|\sec x| + c$** , where c is the integrating constant

Question: 62

Solution:

$\int e^x[\sec x + \log(\sec x + \tan x)]dx$

$= \int e^x \sec x\, dx + \int e^x \log(\sec x + \tan x)dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logcosx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$\int e^x \sec x\, dx + \int e^x \log(\sec x + \tan x)dx$

$$= \int e^x \sec x dx + \log(\sec x + \tan x) \int e^x dx - \int \left[\frac{d}{dx}(\log(\sec x + \tan x)) \int e^x dx\right]$$

CLASS24

$$= \int e^x \sec x\, dx + e^x \log(\sec x + \tan x) - \int \frac{e^x \tan x \times (\sec^2 x + \sec x \tan x)\, dx}{\sec x + \tan x} + c$$

$$= \int e^x \sec x\, dx + e^x \log(\sec x + \tan x) - \int e^x \sec x\, dx + c$$

$= \mathbf{e^x \log|\sec x + \tan x| + c}$, where c is the integrating constant

Question: 63

Solution:

$$\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x}\right) dx$$

$$= \int e^x (\sec^2 x + \tan x) dx$$

$$= \int e^x \sec^2 x dx + \int e^x \tan x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \sec^2 x dx + \int e^x \tan x dx$$

$$= \int e^x \sec^2 x dx + \tan x \int e^x dx - \int \left[\frac{d}{dx}(\tan x) \int e^x dx\right]$$

$$= \int e^x \sec^2 x\, dx + e^x \tan x - \int e^x \sec^2 x\, dx + c$$

$= \mathbf{e^x \tan x + c}$, where c is the integrating constant

Question: 64

Solution:

$$\int e^x \left(\frac{\sin x \cos x - 1}{\sin^2 x}\right) dx$$

$$= \int e^x (\cot x - \mathrm{cosec}^2 x) dx$$

$$= \int e^x \cot x dx - \int e^x \mathrm{cosec}^2 x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \cot x dx - \int e^x \mathrm{cosec}^2 x dx$$

$= \cot x \int e^x dx - \int \left\{\frac{d}{dx}(\cot x) \int e^x dx\right\} dx - \int e^x \text{cosec}^2 x dx$

$= e^x \cot x + \int e^x \text{cosec}^2 x dx - \int e^x \text{cosec}^2 x dx + c$

$= \mathbf{e^x \cot x + c}$, where c is the integrating constant

Question: 65

Solution:

$\int e^x \left(\frac{\cos x + \sin x}{\cos^2 x}\right) dx$

$= \int e^x (\sec x + \sec x \tan x) dx$

$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sec x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \sec x dx + \int e^x \sec x \tan x dx$

$= \sec x \int e^x dx - \int \left[\frac{d}{dx}(\sec x) \int e^x dx\right] dx + \int e^x \sec x \tan x dx$

$= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx + c$

$= \mathbf{e^x \sec x + c}$, where c is the integrating constant

Question: 66

Solution:

$\int e^x \left(\frac{2 - \sin 2x}{1 - \cos 2x}\right) dx$

$= \int e^x \left(\frac{1 - \sin x \cos x}{\sin^2 x}\right) dx$

$= \int e^x (\text{cosec}^2 x - \cot x) dx$

$= \int e^x \text{cosec}^2 x dx - \int e^x \cot x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$\int e^x \text{cosec}^2 x dx - \int e^x \cot x dx$

CLASS24

$$= \int e^x \text{cosec}^2 x dx - \cot x \int e^x dx + \int \left\{\frac{d}{dx}(\cot x) \int e^x dx\right\} dx$$

$$= \int e^x \text{cosec}^2 x dx - e^x \cot x - \int e^x \text{cosec}^2 x dx$$

$= \mathbf{-e^x \cot x + c}$, where c is the integrating constant

Question: 67

Evaluate the foll

Solution:

$$\left(\frac{1 + \sin x}{1 + \cos x}\right)$$

$$= \left(\frac{1 + \frac{2\tan {}^x/_2}{1 + \tan^2 ({}^x/_2)}}{1 + \frac{1 - \tan^2 ({}^x/_2)}{1 + \tan^2 ({}^x/_2)}}\right)$$

$$= \frac{\left(1 + \tan {}^x/_2\right)^2}{2}$$

$$\therefore \int e^x \left(\frac{1 + \sin x}{1 + \cos x}\right) dx$$

$$= \int e^x \times \frac{\left(1 + \tan {}^x/_2\right)^2}{2}$$

$$= \int \frac{e^x \left(1 + \tan^2 {}^x/_2 + 2\tan {}^x/_2\right)}{2} dx$$

$$= \int \frac{e^x \left(\sec^2 {}^x/_2 + 2\tan {}^x/_2\right)}{2} dx$$

$$= \int \frac{e^x \sec^2 {}^x/_2 \, dx}{2} + \int e^x \tan {}^x/_2 \, dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan(x/2)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x \sec^2 {}^x/_2 \, dx}{2} + \int e^x \tan {}^x/_2 \, dx$$

$$= \int \frac{e^x \sec^2 {}^x/_2 \, dx}{2} + \tan {}^x/_2 \int e^x dx - \int \left[\frac{d}{dx}\left(\tan {}^x/_2\right) \int e^x dx\right] dx$$

$$= \int \frac{e^x \sec^2 {}^x/_2 \, dx}{2} + e^x \tan {}^x/_2 - \int \frac{e^x \sec^2 {}^x/_2 \, dx}{2} + c$$

$= \mathbf{e^x \tan {}^x/_2 + c}$, where c is the integrating constant

Question: 68

Solution:

$$\int e^x \left(\frac{\sin 4x - 1}{1 - \cos 4x}\right) dx$$

$= \int e^x\left(\frac{2\sin 2x\cos 2x - 4}{2\sin^2 2x}\right)dx$

$= \int e^x(\cot 2x - 2\operatorname{cosec}^2 2x)dx$

$= \int e^x\cot 2x dx - \int 2e^x\operatorname{cosec}^2 2x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot 2x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x\cot 2x dx - \int 2e^x\operatorname{cosec}^2 2x dx$

$= \cot 2x\int e^x dx - \int\left\{\frac{d}{dx}(\cot 2x)\int e^x dx\right\}dx - \int 2e^x\operatorname{cosec}^2 2x dx$

$= e^x\cot 2x + \int 2e^x\operatorname{cosec}^2 2x dx - \int 2e^x\operatorname{cosec}^2 2x dx + c$

$= \mathbf{e^x\cot 2x + c}$, where c is the integrating constant

Question: 69

Evaluate the foll

Solution:

$\int \frac{e^x\left[\sqrt{1-x^2}\sin^{-1}x + 1\right]}{\sqrt{1-x^2}}dx$

$= \int e^x\left(\sin^{-1}x + \frac{1}{\sqrt{1-x^2}}\right)dx$

$= \int e^x\sin^{-1}x\, dx + \int \frac{e^x}{\sqrt{1-x^2}}dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x\sin^{-1}x\, dx + \int \frac{e^x}{\sqrt{1-x^2}}dx$

$= \sin^{-1}x\int e^x dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\int e^x dx\right\}dx + \int \frac{e^x}{\sqrt{1-x^2}}dx$

$= e^x\sin^{-1}x - \int \frac{e^x}{\sqrt{1-x^2}}dx + \int \frac{e^x}{\sqrt{1-x^2}}dx + c$

$= \mathbf{e^x\sin^{-1}x + c}$, where c is the integrating constant

Question: 70

Solution:

$\int e^x\left(\frac{1+x\log x}{x}\right)dx$

$= \int e^x\left(\frac{1}{x}+\log x\right)dx$

$= \int \frac{e^x}{x}dx + \int e^x \log x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$\int \frac{e^x}{x}dx + \int e^x \log x dx$

$= \int \frac{e^x}{x}dx + \log x\int e^x dx - \int\left[\frac{d}{dx}(\log x)\int e^x dx\right]dx$

$= \int \frac{e^x}{x}dx + e^x \log x - \int \frac{e^x}{x}dx + c$

$= \mathbf{e^x \log x + c}$, where c is the integrating constant

Question: 71

Solution:

$\frac{x}{(1+x)^2} = \frac{A}{(1+x)} + \frac{B}{(1+x)^2}$

$\Rightarrow x = A(1+x) + B$

For x=-1, equation: -1 = B i.e. B = -1

For x=0, equation: 0 = A-1 i.e. A = 1

$\therefore \frac{x}{(1+x)^2}$

$= \frac{1}{(1+x)} - \frac{1}{(1+x)^2}$

The given equation becomes

$\int e^x\left[\frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right]dx$

$= \int e^x \times \frac{1}{(1+x)}dx - \int e^x \times \frac{1}{(1+x)^2}dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int \frac{e^x}{(1+x)}dx - \int \frac{e^x}{(1+x)^2}dx$

$= \frac{1}{(1+x)}\int e^x dx - \int\left[\frac{d}{dx}\left(\frac{1}{1+x}\right)\int e^x dx\right]dx - \int \frac{e^x}{(1+x)^2}dx$

$$= \frac{e^x}{(1+x)} + \int \frac{e^x}{(1+x)^2}dx - \int \frac{e^x}{(1+x)^2}dx + c$$

$= \frac{e^x}{(1+x)} + c$, where c is the integrating constant

Question: 72

Solution:

$$\frac{x-1}{(x+1)^3} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$\Rightarrow x - 1 = A(x+1)^2 + B(x+1) + C$

For x=-1, equation: -2 = C i.e. C = -2

For x=0, equation: -1 = A+B-2 i.e. A+B = 1

For x=1, equation: 0 = 4A+2B-2

i.e. 2(A+B+A) = 2

⇨1+A = 1

⇨A = 0

And, B = 1

$\therefore \frac{x-1}{(x+1)^3}$

$$= \frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}$$

The given equation becomes

$$\int e^x \left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right]dx$$

$$= \int e^x \times \frac{1}{(x+1)^2}dx - \int e^x \times \frac{2}{(x+1)^3}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(x+1)^2}dx - \int \frac{2e^x}{(x+1)^3}dx$$

$$= \frac{1}{(x+1)^2}\int e^x dx - \int\left[\frac{d}{dx}\left(\frac{1}{(x+1)^2}\right)\int e^x dx\right]dx - \int \frac{2e^x}{(x+1)^3}dx$$

$$= \frac{e^x}{(x+1)^2} + \int \frac{2e^x}{(x+1)^3}dx - \int \frac{2e^x}{(x+1)^3}dx + c$$

$= \frac{e^x}{(x+1)^2} + c$, where c is the integrating constant

Question: 73

Solution:

$$\frac{2-x}{(1-x)^2}=\frac{A}{(1-x)}+\frac{B}{(1-x)^2}$$

$\Rightarrow 2-x=A(1-x)+B$

For x=1, equation: 1 = B i.e. B = 1

For x=2, equation: 0 = -A+1 i.e. A = 1

$\therefore \frac{2-x}{(1-x)^2}$

$=\frac{1}{(1-x)}+\frac{1}{(1-x)^2}$

The given equation becomes

$$\int e^x\left[\frac{1}{(1-x)}+\frac{1}{(1-x)^2}\right]dx$$

$$=\int e^x\times\frac{1}{(1-x)^2}dx+\int e^x\times\frac{1}{1-x}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = 1/(1-x) and $f_2(x)$ = e^x in the second integral and keeping the first integral intact,

$$\int\frac{e^x}{(1-x)^2}dx+\int\frac{e^x}{1-x}dx$$

$$=\int\frac{e^x}{(1-x)^2}dx+\frac{1}{1-x}\int e^x dx-\int\left[\frac{d}{dx}\left(\frac{1}{1-x}\right)\int e^x dx\right]dx$$

$$=\int\frac{e^x}{(1-x)^2}dx+\frac{e^x}{1-x}-\int\frac{e^x}{(1-x)^2}dx+c$$

$=\frac{e^x}{1-x}+c$, where c is the integrating constant

Question: 74

Solution:

$$\frac{x-3}{(x-1)^3}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}$$

$\Rightarrow x-3=A(x-1)^2+B(x-1)+C$

For x=1, equation: -2 = C i.e. C = -2

For x=0, equation: -3 = A-B-2 i.e. B = A+1

For x=3, equation: 0 = 4A+2B-2

i.e. 2(A+B+A) = 2

⇨1+3A = 1

⇨A = 0

And, B = 1

$\therefore \frac{x-3}{(x-1)^3}$

$= \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}$

CLASS24

The given equation becomes

$$\int e^x \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right] dx$$

$$= \int e^x \times \frac{1}{(x-1)^2} dx - \int e^x \times \frac{2}{(x-1)^3} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1-x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(x-1)^2} dx - \int \frac{2e^x}{(x-1)^3} dx$$

$$= \frac{1}{(x-1)^2} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{(x-1)^2}\right) \int e^x dx\right] dx - \int \frac{2e^x}{(x-1)^3} dx$$

$$= \frac{e^x}{(x-1)^2} + \int \frac{2e^x}{(x-1)^3} dx - \int \frac{2e^x}{(x-1)^3} dx + c$$

$= \frac{e^x}{(x-1)^2} + c$, where c is the integrating constant

Question: 75

Solution:

$$\int e^{3x} \left(\frac{3x-1}{9x^2}\right) dx$$

$$= \int \frac{e^{3x}}{3x} dx - \int \frac{e^{3x}}{9x^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/3x$ and $f_2(x) = e^{3x}$ in the first integral and keeping the second integral intact,

$$\int \frac{e^{3x}}{3x} dx - \int \frac{e^{3x}}{9x^2} dx$$

$$= \frac{1}{3x} \int e^{3x} dx - \int \left[\frac{d}{dx}\left(\frac{1}{3x}\right) \int e^{3x} dx\right] dx - \int \frac{e^{3x}}{9x^2} dx$$

$$= \frac{e^{3x}}{9x} + \int \frac{e^{3x}}{9x^2} dx - \int \frac{e^{3x}}{9x^2} dx + c$$

$= \frac{e^{3x}}{9x} + c$, where c is the integrating constant

Question: 76

Solution:

$$\frac{x+1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$$

$\Rightarrow x + 1 = A(x + 2) + B$

For x=-2, equation: -1 = B i.e. B = -1

For x=-1, equation: 0 = A-1 i.e. A = 1

$$\therefore \frac{x+1}{(x+2)^2}$$

$$= \frac{1}{(x+2)} - \frac{1}{(x+2)^2}$$

The given equation becomes

$$\int e^x \left[\frac{1}{(x+2)} - \frac{1}{(x+2)^2}\right] dx$$

$$= \int e^x \times \frac{1}{x+2} dx - \int e^x \times \frac{1}{(x+2)^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(x+2)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{x+2} dx - \int \frac{e^x}{(x+2)^2} dx$$

$$= \frac{1}{x+2} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{x+2}\right) \int e^x dx\right] dx - \int \frac{e^x}{(x+2)^2} dx$$

$$= \frac{e^x}{x+2} + \int \frac{e^x}{(x+2)^2} dx - \int \frac{e^x}{(x+2)^2} dx + c$$

$= \frac{e^x}{x+2} + c$, where c is the integrating constant

Question: 77

Solution:

$$\frac{x}{(1+2x)^2} = \frac{A}{(1+2x)} + \frac{B}{(1+2x)^2}$$

$\Rightarrow x = A(1 + 2x) + B$

For x=-1/2, equation: -1/2 = B i.e. B = -1/2

For x=0, equation: 0 = A-1/2 i.e. A = 1/2

$$\therefore \frac{x}{(1+2x)^2}$$

$$= \frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}$$

The given equation becomes

$$\int e^{2x} \left[\frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}\right] dx$$

$$= \int e^{2x} \times \frac{1}{2(1+2x)} dx - \int e^{2x} \times \frac{1}{2(1+2x)^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ c INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+2x)$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\int e^{2x}\times\frac{1}{2(1+2x)}dx - \int e^{2x}\times\frac{1}{2(1+2x)^2}dx$$

$$= \frac{1}{2}\left[\frac{1}{1+2x}\int e^{2x}dx - \int\left[\frac{d}{dx}\left(\frac{1}{1+2x}\right)\int e^{2x}dx\right]dx - \int\frac{e^{2x}}{(1+2x)^2}dx\right]$$

$$= \frac{1}{2}\left[\frac{e^{2x}}{2(2x+1)} + \int\frac{e^{2x}}{(2x+1)^2}dx - \int\frac{e^{2x}}{(2x+1)^2}dx\right]$$

$= \frac{e^{2x}}{4(2x+1)} + c$, where c is the integrating constant

Question: 78

Solution:

$$\int e^{2x}\left(\frac{2x-1}{4x^2}\right)dx$$

$$= \int\frac{e^{2x}}{2x}dx - \int\frac{e^{2x}}{4x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/2x$ and $f_2(x) = e^{2x}$ in the first integral and keeping the second integral intact,

$$\int\frac{e^{2x}}{2x}dx - \int\frac{e^{2x}}{4x^2}dx$$

$$= \frac{1}{2x}\int e^{2x}dx - \int\left[\frac{d}{dx}\left(\frac{1}{2x}\right)\int e^{2x}dx\right]dx - \int\frac{e^{2x}}{4x^2}dx$$

$$= \frac{e^{2x}}{4x} + \int\frac{e^{2x}}{4x^2}dx - \int\frac{e^{2x}}{4x^2}dx + c$$

$= \frac{e^{2x}}{4x} + c$, where c is the integrating constant

Question: 79

Solution:

$$\int e^x\left(\log x + \frac{1}{x^2}\right)dx$$

$$= \int e^x \log x dx - \int\frac{e^x}{x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log x$ and $f_2(x) = e^x$ in the first integral and keeping the second integra

$$\int e^x \log x dx - \int \frac{e^x}{x^2} dx$$

$$= \log x \int e^x dx - \int \left[\frac{d}{dx}(\log x) \int e^x dx\right] dx - \int \frac{e^x}{x^2} dx$$

$$= e^x \log x - \int \frac{e^x}{x} dx - \int \frac{e^x}{x^2} dx$$

$$= e^x \log x - \left[\frac{1}{x} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{x}\right) \int e^x dx\right] dx\right] - \int \frac{e^x}{x^2} dx$$

$$= e^x \log x - \frac{e^x}{x} + \int \frac{e^x}{x^2} dx - \int \frac{e^x}{x^2} dx + c$$

$= \mathbf{e^x \left(\log x - \frac{1}{x}\right) + c}$, where c is the integrating constant

Question: 80

Solution:

$$\frac{\log x}{(1 + \log x)^2} = \frac{A}{(1 + \log x)} + \frac{B}{(1 + \log x)^2}$$

$\Rightarrow \log x = A(1 + \log x) + B$

For x=1, equation: 0 = A+B

For x=1/e, equation: -1 = B i.e. B = -1

So, A = 1

$$\therefore \frac{\log x}{(1 + \log x)^2}$$

$$= \frac{1}{(1 + \log x)} - \frac{1}{(1 + \log x)^2}$$

The given equation becomes

$$\int \left[\frac{1}{(1 + \log x)} - \frac{1}{(1 + \log x)^2}\right] dx$$

$$= \int \frac{1}{(1 + \log x)} dx - \int \frac{1}{(1 + \log x)^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+\log x)$ and $f_2(x) = 1$in the second integral and keeping the first integral intact,

$$\int \frac{1}{(1 + \log x)} dx - \int \frac{1}{(1 + \log x)^2} dx$$

$$= \frac{1}{(1 + \log x)} \int dx - \int \left[\frac{d}{dx}\left(\frac{1}{(1 + \log x)}\right) \int dx\right] dx - \int \frac{1}{(1 + \log x)^2} dx$$

$$= \frac{x}{(1 + \log x)} + \int \frac{1}{(1 + \log x)^2} dx - \int \frac{1}{(1 + \log x)^2} dx + c$$

$= \mathbf{\frac{x}{(1+\log x)} + c}$, where c is the integrating constant

Question: 81

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin(\log x)$ and $f_2(x) = 1$in the first integral and keeping the second integral intact,

$$\int \sin(\log x)dx + \int \cos(\log x)dx$$

$$= \sin(\log x)\int dx - \int\left[\frac{d}{dx}(\sin(\log x))\int dx\right]dx + \int \cos(\log x)dx$$

$$= x\sin(\log x) - \int \cos(\log x)dx + \int \cos(\log x)dx + c$$

$= \mathbf{e^{\log x}\sin(\log x) + c}$, where c is the integrating constant

Question: 82

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(\log x)$ and $f_2(x) = 1$in the first integral and keeping the second integral intact,

$$\int \frac{1}{\log x}dx - \int \frac{1}{(\log x)^2}dx$$

$$= \frac{1}{\log x}\int dx - \int\left[\frac{d}{dx}\left(\frac{1}{\log x}\right)\int dx\right]dx - \int \frac{1}{(\log x)^2}dx$$

$$= \frac{x}{\log x} + \int \frac{1}{(\log x)^2}dx - \int \frac{1}{(\log x)^2}dx + c$$

$= \mathbf{\frac{x}{\log x} + c}$, where c is the integrating constant

Question: 83

Solution:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log(\log x)$ and $f_2(x) = 1$in the first integral and keeping the second integral intact,

$$\int \log(\log x)dx + \int \frac{1}{(\log x)^2}dx$$

$$= \log(\log x)\int dx - \int\left[\frac{d}{dx}(\log(\log x))\int dx\right]dx + \int \frac{1}{(\log x)^2}dx$$

CLASS24

$= x\log(\log x) - \int \frac{1}{\log x}dx + \int \frac{1}{(\log x)^2}dx$

$= x\log(\log x) - \left[\frac{1}{\log x}\int dx - \int\left[\frac{d}{dx}\left(\frac{1}{\log x}\right)\int dx\right]dx\right] + \int \frac{1}{(\log x)^2}dx$

$= x\log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2}dx + \int \frac{1}{(\log x)^2}dx + c$

$= \mathbf{x\left[\log(\log x) - \frac{1}{\log x}\right] + c}$ **, where c is the integrating constant**

Question: 84

Solution:

It is know that $\sin^{-1}x+\cos^{-1}x = \pi/2$

$\therefore \left(\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\right)$

$= \frac{2}{\pi}\left(\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}\right)$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Now, for the first term,

Taking $f_1(x) = \sin^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$\therefore \int \sin^{-1}\sqrt{x}\,dx$

$= \sin^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\sin^{-1}\sqrt{x}\right)\int dx\right\}dx$

$= x\sin^{-1}\sqrt{x} - \int \frac{1}{2\sqrt{x}\sqrt{1-x}} \times x dx$

$= x\sin^{-1}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$

$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$

$= -\int \sqrt{1-a^2}\,da$

$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$

Replacing the value of a, we get,

$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2}+\frac{1}{2}\sin^{-1}a\right]$

$= -\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c$

The total integration yields as

$= x\sin^{-1}\sqrt{x}+\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c'$, where c' is the integrating constant

For the second term,

Taking $f_1(x) = \cos^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$\therefore \int \cos^{-1}\sqrt{x}\,dx$

$= \cos^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\cos^{-1}\sqrt{x}\right)\int dx\right\}dx$

$= x\cos^{-1}\sqrt{x} - \int \frac{-1}{2\sqrt{x}\sqrt{1-x}} \times x dx$

$= x\cos^{-1}\sqrt{x} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$

$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$

$= -\int \sqrt{1-a^2}da$

$= -\left[\frac{1}{2}a\sqrt{1-a^2}+\frac{1}{2}\sin^{-1}a\right]$

Replacing the value of a, we get,

$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2}+\frac{1}{2}\sin^{-1}a\right]$

$= -\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c$

The total integration yields as

$= x\cos^{-1}\sqrt{x}-\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c''$, where c'' is the integrating constant

$\therefore \int\left(\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}\right)dx$

$= \frac{2}{\pi}\int\left(\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}\right)dx$

$$= \frac{2}{\pi}\left[x\sin^{-1}\sqrt{x}+\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]-x\cos^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]\right]+c$$

$= \frac{2}{\pi}\left[\sqrt{x - x^2} + x\left(\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}\right) + \sin^{-1}\sqrt{1 - x}\right] + c$ where c is the integrating c

Question: 85

Solution:

Tip – 5^x is to be replaced by a

$\therefore 5^x = a$

$\Rightarrow 5^x \log 5 dx = da$

$\Rightarrow 5^x dx = \frac{da}{\log 5}$

The equation becomes as follows:

$$\int 5^{5^a} \times 5^a \times \frac{1}{\log 5} da$$

Tip – 5^a is to be replaced by k

$\therefore 5^a = k$

$\Rightarrow 5^a \log 5 da = dk$

$\Rightarrow 5^a da = \frac{dk}{\log 5}$

The equation becomes as follows:

$$\int 5^k \times \frac{1}{(\log 5)^2} dk$$

$$= \frac{1}{(\log 5)^2} \int 5^k dk$$

$$= \frac{5^k}{(\log 5)^3} + c$$

Re-replacing the value of k,

$$\frac{5^{5^a}}{(\log 5)^3} + c$$

Re-replacing the value of a,

$\frac{5^{5^{5^x}}}{(\log 5)^3} + c$, where c is the integrating constant

Question: 86

Solution:

$$\left(\frac{1 + \sin 2x}{1 + \cos 2x}\right)$$

$$= \left(\frac{1 + \frac{2\tan x}{1 + \tan^2 x}}{1 + \frac{1 - \tan^2 x}{1 + \tan^2 x}}\right)$$

$$= \frac{(1 + \tan x)^2}{2}$$

$$\therefore \int e^{2x}\left(\frac{1 + \sin 2x}{1 + \cos 2x}\right) dx$$

$= \int e^{2x} \times \frac{(1+\tan x)^2}{2}$

$= \int \frac{e^{2x}(1+\tan^2 x + 2\tan x)}{2} dx$

$= \int \frac{e^{2x}(\sec^2 x + 2\tan x)}{2} dx$

$= \int \frac{e^{2x}\sec^2 x dx}{2} + \int e^{2x}\tan x\, dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$\int \frac{e^{2x}\sec^2 x dx}{2} + \int e^{2x}\tan x dx$

$= \int \frac{e^{2x}\sec^2 x dx}{2} + \tan x \int e^{2x}dx - \int\left[\frac{d}{dx}(\tan x)\int e^{2x}dx\right]dx$

$= \int \frac{e^{2x}\sec^2 x dx}{2} + \frac{1}{2}e^{2x}\tan x - \int \frac{e^{2x}\sec^2 x dx}{2} + c$

$= \frac{1}{2}e^x \tan {}^x\!/_2 + c$, where c is the integrating constant

Question: 87

Solution:

$\left(\frac{1-\sin 2x}{1-\cos 2x}\right)$

$= \left(\frac{1 - \frac{2\tan x}{1+\tan^2 x}}{1 - \frac{1-\tan^2 x}{1+\tan^2 x}}\right)$

$= \frac{(1-\tan x)^2}{2}$

$\therefore \int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

$= \int e^{2x} \times \frac{(1-\tan x)^2}{2}$

$= \int \frac{e^{2x}(1+\tan^2 x - 2\tan x)}{2} dx$

$= \int \frac{e^{2x}(\sec^2 x - 2\tan x)}{2} dx$

$= \int \frac{e^{2x}\sec^2 x dx}{2} - \int e^{2x}\tan x\, dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integ

$$\int \frac{e^{2x}\sec^2 x dx}{2} - \int e^{2x}\tan x dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} - \tan x \int e^{2x}dx + \int \left[\frac{d}{dx}(\tan x)\int e^{2x}dx\right]dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} - \frac{1}{2}e^{2x}\tan x + \int \frac{e^{2x}\sec^2 x dx}{2} + c$$

$= -\frac{1}{2}e^x \tan {}^x/_2 + c$, where c is the integrating constant

Exercise : OBJECTIVE QUESTIONS II

Question: 1

Solution:

To find: Value of $\int x\, e^x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\, e^x dx$ ... (i)

$I = \int x\, e^x dx$

$\Rightarrow x\int e^x dx - \int \left[\frac{d(x)}{x}\int e^x dx\right]dx$

$\Rightarrow I = x\, e^x - \int 1.e^x dx$

$\Rightarrow I = x\, e^x - e^x + c$

$\therefore I = e^x (x-1) + c$

Ans) c $e^x (x-1) + c$

Question: 2

Mark (√) against

Solution:

To find: Value of $\int x\, e^{2x} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\, e^{2x} dx$ ... (i)

$I = \int x\, e^{2x} dx$

$\Rightarrow x\int e^{2x} dx - \int \left[\frac{d(x)}{x}\int e^{2x} dx\right]dx$

$\Rightarrow I = x\frac{e^{2x}}{2} - \int 1.\frac{e^{2x}}{2}dx$

$\Rightarrow I = x\frac{e^{2x}}{2} - \frac{1}{2}\int \frac{e^{2x}}{2}dx$

$\Rightarrow I = \frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}dx$

$\Rightarrow I = \frac{x}{2}e^{2x} - \frac{1}{2}\frac{e^{2x}}{2} + c$

$\Rightarrow I = \frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

$\therefore I = \frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

Ans) B $\frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

Question: 3

Solution:

To find: Value of $\int x\cos 2x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\cos 2x dx$ ... (i)

Let $2x = t$

$\Rightarrow x = \frac{t}{2}$

$\Rightarrow 2 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{2}$

$I = \int \frac{t}{2}\cos t \frac{dt}{2}$

$I = \frac{1}{4}\int t\cos t\, dt$

Taking 1st function as t and second function as $\cos t$

$\Rightarrow I = \frac{1}{4}\left[t\int \cos t\, dt - \int\left(\frac{dt}{dt}\int \cos t\, dt\right)dt\right]$

$\Rightarrow I = \frac{1}{4}\left[t(\sin t) - \int(1(\sin t))dt\right]$

$\Rightarrow I = \frac{1}{4}[t(\sin t) - (-\cos t)] + c$

$\Rightarrow I = \frac{1}{4}[t\sin t + \cos t] + c$

$\Rightarrow I = \frac{1}{4}[2x \sin 2x + \cos 2x] + c$

$\Rightarrow I = \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + c$

Ans) A $\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + c$

Question: 4

Solution:

To find: Value of $\int x \sec^2 x \, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int x \sec^2 x \, dx$ ... (i)

Taking 1st function as x and second function as $\sec^2 x$

$\Rightarrow I = \left[x \int \sec^2 x \, dx - \int \left(\frac{dx}{dx} \int \sec^2 x \, dx \right) dx \right]$

$\Rightarrow I = \left[x \tan x - \int (1 \tan x) dx \right]$

$\Rightarrow I = \left[x \tan x - \int \tan x dx \right]$

$\Rightarrow I = [x \tan x - (-\log|\cos x|)] + c$

$\Rightarrow I = x \tan x + \log|\cos x| + c$

Ans) B $x \tan x + \log|\cos x| + c$

Question: 5

Solution:

To find: Value of $\int x \sin 2x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int x \sin 2x dx$ ... (i)

Let $2x = t$

$\Rightarrow x = \frac{t}{2}$

$\Rightarrow 2 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{2}$

$I = \int \frac{t}{2} \sin t \frac{dt}{2}$

$I = \frac{1}{4}\int t\sin t\,dt$

Taking 1st function as t and second function as $\sin t$

$\Rightarrow I = \frac{1}{4}\left[t\int \sin t\,dt - \int\left(\frac{dt}{dt}\int \sin t\,dt\right)dt\right]$

$\Rightarrow I = \frac{1}{4}\left[t(-\cos t) - \int(1(-\cos t))\,dt\right]$

$\Rightarrow I = \frac{1}{4}\left[-t\cos t - \int -\cos t\,dt\right]$

$\Rightarrow I = \frac{1}{4}[-t\cos t + \sin t]+c$

$\Rightarrow I = \frac{1}{4}[-2x\cos 2x + \sin 2x]+c$

$\Rightarrow I = -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x+c$

Ans) C $-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x+c$

Question: 6

Solution:

To find: Value of $\int x\log x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\log x\,dx$ ... (i)

Taking 1st function as $\log x$ and second function as x

$\Rightarrow I = \left[\log x\int x\,dx - \int\left(\frac{d\log x}{dx}\int x\,dx\right)dx\right]$

$\Rightarrow I = \left[\log x\frac{x^2}{2} - \int\left(\frac{1}{x}\frac{x^2}{2}\right)dx\right]$

$\Rightarrow I = \left[\log x\frac{x^2}{2} - \int\left(\frac{x}{2}\right)dx\right]$

$\Rightarrow I = \left[\log x\frac{x^2}{2} - \frac{1}{2}\int x\,dx\right]$

$\Rightarrow I = \left[\log x\frac{x^2}{2} - \frac{1}{2}\frac{x^2}{2}\right]+c$

$\Rightarrow I = \frac{1}{2}x^2\log x - \frac{1}{4}x^2+c$

Ans) C $\frac{1}{2}x^2\log x - \frac{1}{4}x^2+c$

Question: 7

Mark (√) against

Solution:

To find: Value of $\int x\, cosec^2 x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\, cosec^2 x dx$ ... (i)

$I = \int x\, cosec^2 x dx$

$\Rightarrow x\int cosec^2 x dx - \int\left[\frac{d(x)}{x}\int cosec^2 x dx\right]dx$

$\Rightarrow I = x(-\cot x) - \int 1.(-\cot x)dx$

$\Rightarrow I = -x(\cot x) + \log|\sin x| + c$

Ans) D None of these

Question: 8

Solution:

To find: Value of $\int x \sin x \cos x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x \sin x \cos x dx$ ... (i)

$I = \frac{1}{2}\int x\, 2\sin x \cos x dx$

$I = \frac{1}{2}\int x \sin 2x dx$

$\Rightarrow \frac{1}{2}\left[x\int \sin 2x dx - \int\left[\frac{d(x)}{x}\int \sin 2x dx\right]dx\right]$

$\Rightarrow \frac{1}{2}\left[\frac{-x\cos 2x}{2} - \int\left[1\ \frac{-\cos 2x}{2}\right]dx\right]$

$\Rightarrow \frac{1}{2}\left[\frac{-x\cos 2x}{2} + \frac{\sin 2x}{4}\right] + c$

$\Rightarrow \frac{-x\cos 2x}{4} + \frac{\sin 2x}{8} + c$

Ans) D $\frac{-x\cos 2x}{4} + \frac{\sin 2x}{8} + c$

Question: 9

Solution:

To find: Value of $\int x \cos^2 x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\cos^2 x\, dx$ ... (i)

$I = \int x\frac{1}{2}(1+\cos2x)dx$

$I = \frac{1}{2}\int x\, dx + \frac{1}{2}\int x\cos2x dx$

$I = \frac{1}{2}\frac{x^2}{2} + \frac{1}{2}\left[x\int \cos2x dx - \int\left[\frac{d(x)}{x}\int \cos2x dx\right]dx\right]$

$I = \frac{1}{2}\frac{x^2}{2} + \frac{1}{2}\left[x\frac{\sin2x}{2} - \int 1.\frac{\sin2x}{2}dx\right]$

$I = \frac{1}{2}\frac{x^2}{2} + \frac{1}{2}\left[x\frac{\sin2x}{2} - \frac{1}{2}\int \sin2x dx\right]$

$I = \frac{1}{2}\frac{x^2}{2} + \frac{1}{2}\left[x\frac{\sin2x}{2} - \frac{1}{2}\left(-\frac{\cos2x}{2}\right) + c\right]$

$I = \frac{1}{2}\frac{x^2}{2} + \frac{1}{2}\left[\frac{x\sin2x}{2} + \frac{\cos2x}{4} + c\right]$

$I = \frac{x^2}{4} + \frac{x\sin2x}{4} + \frac{\cos2x}{8} + c$

Ans) D $\frac{x^2}{4} + \frac{x\sin2x}{4} + \frac{\cos2x}{8} + c$

Question: 10

Solution:

To find: Value of $\int \frac{\log x}{x^2}dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \frac{\log x}{x^2}dx$ ... (i)

$I = \int x^{-2}\log x dx$

$\Rightarrow \log x\int x^{-2}dx - \int\left[\frac{d(\log x)}{x}\int x^{-2}dx\right]dx$

$\Rightarrow \log x\frac{x^{-1}}{-1} - \int\left(\frac{1}{-x^2}\right)dx$

$\Rightarrow -\frac{\log x}{x} + \left(-\frac{1}{x}\right) + c$

$\Rightarrow -\frac{1}{x}(\log x+1) + c$

Ans) A $-\frac{1}{x}(\log x+1) + c$

Question: 11

Solution:

To find: Value of $\int \log x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \log x \,.\, 1.\, dx$ ... (i)

Taking 1^{st} function as $\log x$ and second function as 1

$\Rightarrow I = \left[\log x \int 1\, dx - \int\left(\frac{d\log x}{dx}\int 1\, dx\right)dx\right]$

$\Rightarrow I = \left[\log x \,.\, x - \int\left(\frac{1}{x}\int 1\, dx\right)dx\right]$

$\Rightarrow I = \left[\log x \,.\, x - \int\left(\frac{1}{x}\, x\right)dx\right]$

$\Rightarrow I = \left[\log x \,.\, x - \int 1dx\right]$

$\Rightarrow I = [\log x \,.\, x - x] + c$

$\Rightarrow I = x(\log x - 1) + c$

Ans) D $x(\log x - 1) + c$

Question: 12

Solution:

To find: Value of $\int \log_{10} x\, dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \log_{10} x\, dx$ ... (i)

$I = \int \log_{10} x\, dx = \int \frac{\log x}{\log 10}\, dx$

$I = \frac{1}{\log_e 10}\int \log x \,.\, 1\, dx$

Taking 1^{st} function as $\log x$ and second function as 1

$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \int 1\, dx - \int\left(\frac{d\log x}{dx}\int 1\, dx\right)dx\right]$

$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int\left(\frac{1}{x}\int 1\, dx\right)dx\right]$

$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int\left(\frac{1}{x}\, x\right)dx\right]$

$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int 1dx\right]$

$\Rightarrow I = \frac{1}{\log_e 10}[\log x \,.\, x - x] + c$

$\Rightarrow I = x(\log x - 1)\log_{10} e + c$

Ans) D $x(\log x - 1)\log_{10} e + c$

Question: 13

Solution:

To find: Value of $\int (\log x)^2\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int (\log x)^2 \,.\, 1 \,.\, dx$ ... (i)

Taking 1^{st} function as $(\log x)^2$ and second function as 1

$\Rightarrow I = \left[(\log x)^2\int 1\, dx - \int\left(\frac{d(\log x)^2}{dx}\int 1\, dx\right)dx\right]$

$\Rightarrow I = \left[(\log x)^2\int 1\, dx - \int\left(\frac{2(\log x)}{x}\int 1\, dx\right)dx\right]$

$\Rightarrow I = \left[(\log x)^2 .\, x - 2\int \log x dx\right]$

$\Rightarrow I = [(\log x)^2 .\, x - 2(x\log x - x)] + c$

$\Rightarrow I = [(\log x)^2 .\, x - 2x\log x + 2x] + c$

$\Rightarrow I = x(\log x)^2 - 2x\log x + 2x + c$

Ans) C $x(\log x)^2 - 2x\log x + 2x + c$

Question: 14

Solution:

To find: Value of $\int e^{\sqrt{x}} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int e^{\sqrt{x}} dx$ ... (i)

Putting $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow dx = 2\sqrt{x}\, dt$

$\Rightarrow dx = 2t\, dt$

$\Rightarrow I = 2\int t.e^t\, dt$

$\Rightarrow I = 2\left[t\int e^t\, dt - \int\left[\frac{d(t)}{dt}\int e^t dt\right]dt\right]$

$\Rightarrow I = 2\left[te^t - \int [1\ e^t]dt\right]$

$\Rightarrow I = 2[te^t - e^t]$

$\Rightarrow I = e^t.\ 2(t-1)+c$

$\therefore I = 2\ e^{\sqrt{x}}\ (\sqrt{x}-1)+c$

Ans) C $2\ e^{\sqrt{x}}\ (\sqrt{x}-1)+c$

Question: 15

Solution:

To find: Value of $\int \cos\sqrt{x}\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \cos\sqrt{x}\ dx$ ... (i)

Putting $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow dx = 2\sqrt{x}\ dt$

$\Rightarrow dx = 2t\ dt$

$\Rightarrow\ I = \int \cos t.\ 2t\ dt$

$\Rightarrow I = 2\int t.\ \cos t\ dt$

$\Rightarrow I = 2\left[t\int \cos t\ \ dt - \int\left[\frac{d(t)}{dt}\ \int \cos t\ dt\right]dt\right]$

$\Rightarrow I = 2\left[te^t - \int [1\ e^t]dt\right]$

$\Rightarrow I = 2[te^t - e^t]$

$\Rightarrow I = e^t.\ 2(t-1)+c$

$\therefore I = 2\ e^{\sqrt{x}}\ (\sqrt{x}-1)+c$

Ans) C $2\ e^{\sqrt{x}}\ (\sqrt{x}-1)+c$

Question: 16

Solution:

To find: Value of $\int \cos(log x)\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \cos(\log x)\ dx$ ... (i)

$I = \int 1.\ \cos(\log x)\, dx$

Taking cos(logx) as first function and 1 as second function.

$\Rightarrow I = \left[\cos\log x \int 1\, dx - \int \left[\frac{d[\cos(\log x)]}{dx} \int 1 dx\right] dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) - \int \left[-\sin(\log x)\frac{1}{x}\, x\right] dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \int [\sin(\log x)] dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \int [1.\ \sin(\log x)] dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \left\{\sin(\log x) \int 1\, dx - \left(\frac{d\sin(\log x)}{dx} \int 1.dx\right) dx\right\}\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \left\{x.\ \sin(\log x) - \left(\cos(\log x)\frac{1}{x}\, x\right) dx\right\}\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \left\{x.\ \sin(\log x) - \left(\cos(\log x)\frac{1}{x}\, x\right) dx\right\}\right]$

$\Rightarrow I = [x.\cos(\log x) + \{x.\ \sin(\log x) - (\cos(\log x))dx\}]$

$\Rightarrow I = [x.\cos(\log x) + x.\ \sin(\log x) - I]$

$\Rightarrow 2I = [x.\cos(\log x) + x.\ \sin(\log x)]$

$\Rightarrow I = \frac{x}{2}[\cos(\log x) + \sin(\log x)] + c$

Ans) B $\frac{x}{2}[\cos(\log x) + \sin(\log x)] + c$

Question: 17

Solution:

To find: Value of $\int \sec^3 x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int \sec^3 x\, dx$ ... (i)

$I = \int \sec x \sec^2 x\, dx$

Taking secx as first function and $\sec^2 x$ as second function.

$\Rightarrow I = \left[\sec x \int \sec^2 x\, dx - \int \left[\frac{d[\sec x]}{dx} \int \sec^2 x\, dx\right] dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int [\sec x \tan x \tan x] dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int [\sec x \tan^2 x] dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int [\sec x(\sec^2 x - 1)]dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int (\sec^3 x - \sec x)\, dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int \sec^3 x\, dx + \int \sec x dx\right]$

$\Rightarrow I = [\sec x \tan x - I + \log|\sec x + \tan x| + c]$

$\Rightarrow 2I = [\sec x \tan x + \log|\sec x + \tan x| + c]$

$\Rightarrow I = \frac{1}{2}[\sec x \tan x + \log|\sec x + \tan x| + c]$

Ans) B $\frac{1}{2}[\sec x \tan x + \log|\sec x + \tan x| + c]$

Question: 18

Solution:

To find: Value of $\int \left\{\frac{1}{(\log x)} - \frac{1}{(\log x)^2}\right\} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \left\{\frac{1}{(\log x)} - \frac{1}{(\log x)^2}\right\} dx$ ... (i)

Put $t = \log x$

$e^t = e^{\log x} = x$

$\frac{dx}{dt} = e^t$

$\Rightarrow dx = e^t\, dt$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\} dx$

We know $\int e^x \left(f(x) + f'(x)\right) dx = e^x f(x)$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\} dx = e^t \frac{1}{t}$

$\Rightarrow \frac{x}{\log x} + c$

Ans) B $\frac{x}{\log x} + c$

Question: 19

Solution:

To find: Value of $\int \left\{\frac{1}{(\log x)} - \frac{1}{(\log x)^2}\right\} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int\left\{\frac{1}{(\log x)} - \frac{1}{(\log x)^2}\right\}dx$ ... (i)

Put $t = \log x$

$e^t = e^{\log x} = x$

$\frac{dx}{dt} = e^t$

$\Rightarrow dx = e^t\,dt$

$\Rightarrow I = \int\left\{\frac{1}{t} - \frac{1}{t^2}\right\}dx$

We know $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)$

$\Rightarrow I = \int\left\{\frac{1}{t} - \frac{1}{t^2}\right\}dx = e^t\frac{1}{t}$

$\Rightarrow \frac{x}{\log x} + c$

Ans) B $\frac{x}{\log x} + c$

Question: 20

Solution:

To find: Value of $\int(x2^x)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int(x2^x)dx$ ... (i)

$\Rightarrow I = x\int 2^x dx - \int\left(\frac{dx}{dx}\int 2^x dx\right)dx$

$\Rightarrow I = x\frac{2^x}{\log 2} - \int\left(\frac{2^x}{\log 2}\right)dx$

$\Rightarrow I = x\frac{2^x}{\log 2} - \frac{1}{\log 2}\int 2^x dx$

$\Rightarrow I = x\frac{2^x}{\log 2} - \frac{1}{\log 2}\frac{2^x}{\log 2}$

$\Rightarrow I = \frac{x.\,2^x}{\log 2} - \frac{2^x}{(\log 2)^2} + c$

$\Rightarrow I = \frac{2^x}{(\log 2)^2}(x\log 2 - 1) + c$

Ans) D

Question: 21

Mark (√) against

Solution:

To find: Value of $\int x \cot^2 x \, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x \cot^2 x \, dx$ ... (i)

$\Rightarrow I = x\int \cot^2 x \, dx - \int\left(\frac{dx}{dx}\int \cot^2 x \, dx\right)dx$

$\Rightarrow I = x\int(\mathrm{cosec}^2 x - 1)\,dx - \int\left(1.\int(\mathrm{cosec}^2 x - 1)dx\right)dx$

$\Rightarrow I = x(-\cot x - x) - \int(-\cot x - x)dx$

$\Rightarrow I = -x\cot x - x^2 + \log|\sin x| + \frac{x^2}{2}$

$\Rightarrow I = -x\cot x - \frac{x^2}{2} + \log|\sin x| + c$

Ans) B $-x\cot x - \frac{x^2}{2} + \log|\sin x| + c$

Question: 22

Solution:

To find: Value of $\int \sin\sqrt{x} \, dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \sin\sqrt{x} \, dx$ ... (i)

$\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow dx = 2\sqrt{x}dt$

$\Rightarrow dx = 2t dt$

$I = \int \sin t.\ 2t \, dt$

$I = 2\int t.\ \sin t \, dt$

$\Rightarrow I = 2t\int \sin t \, dt - \int\left(\frac{dt}{dt}\int \sin t \, dt\right)dt$

$\Rightarrow I = 2t(-\cos t) - \int 1(-\cos t)dt$

$\Rightarrow I = 2t(-\cos t) + \int \cos t \, dt$

$\Rightarrow I = 2t(-\cos t) + \sin t + c$

$\Rightarrow I = -2\sqrt{x}\cos\sqrt{x}+\sin\sqrt{x}+c$

Ans) C $2\sqrt{x}\cos\sqrt{x}+\sin\sqrt{x}+c$

Question: 23

Solution:

To find: Value of $\int e^{sinx}\sin 2x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^{\sin x}\sin 2x\, dx$ ... (i)

$I = \int e^{\sin x} 2\sin x\cos x\, dx$

Put sinx = t

$\cos x = \frac{dt}{dx}$

$\Rightarrow \cos x\, dx = dt$

$I = 2\int e^t . t . dt$

$\Rightarrow I = 2\left[t\int e^t dt - \int\left(\frac{dt}{dt}\int e^t dt\right)dt\right]$

$\Rightarrow I = 2\left[t e^t - \int 1\, e^t dt\right]$

$\Rightarrow I = 2t e^t - 2e^t + c$

$\Rightarrow I = 2 e^t (t-1)+c$

$\Rightarrow I = 2 e^{\sin x}(\sin x-1)+c$

Ans) D $2 e^{\sin x}(\sin x-1)+c$

Question: 24

Solution:

To find: Value of $\int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx$ ... (i)

$I = \int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx$

$$I = \int \frac{\sin^{-1} x}{\sqrt{1-x^2}\,(1-x^2)} dx$$

Putting $\sin^{-1}x = t$, $x = \sin t$

$\Rightarrow \cos t = \sqrt{1-x^2}$

$\Rightarrow \tan t = \frac{x}{\sqrt{1-x^2}}$

$\frac{1}{\sqrt{1-x^2}} dx = dt$

$$I = \int \frac{t}{\left(1-\sin^2 t\right)} dt$$

$$I = \int \frac{t}{\cos^2 t} dt$$

$$I = \int t.\sec^2 t\ dt$$

$$\Rightarrow I = \left[t\int \sec^2 t\ \ dt - \int\left(\frac{dt}{dt}\int \sec^2 t\ \ dt\right)dt\right]$$

$$\Rightarrow I = \left[t\tan t - \int 1\tan t\, dt\right]$$

$$\Rightarrow I = \left[\sin^{-1}x \frac{x}{\sqrt{1-x^2}} - \log\left|\sqrt{1-x^2}\right| + c\right]$$

$$\Rightarrow I = [t\tan t - \log|\cos t| + c]$$

$$\Rightarrow I = 2t\,e^t - 2e^t + c$$

$$\Rightarrow I = 2\,e^t\,(t-1) + c$$

$$\Rightarrow I = 2\,e^{\sin x}\,(\sin x - 1) + c$$

Ans) D $2\,e^{\sin x}\,(\sin x-1)+c$

Question: 25

Solution:

To find: Value of $\int \frac{x\tan^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{x\tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx$ ... (i)

$$I = \int \frac{x\tan^{-1} x}{\sqrt{1+x^2}\,(1+x^2)} dx$$

Putting $\tan^{-1}x = t$, $x = \tan t$

$dx = \sec^2 t\ dt$

When $x = \tan t$

CLASS24

$\Rightarrow 1+x^2=1+\tan^2 t$

$\Rightarrow 1+x^2=\sec^2 t$

$\Rightarrow \sqrt{1+x^2}=\sec t$

$\Rightarrow \sqrt{1+x^2}=\sec t$

$\Rightarrow \frac{1}{\sqrt{1+x^2}}=\cos t$

$\Rightarrow \frac{1}{1+x^2}=\cos^2 t$

$\Rightarrow 1-\frac{1}{1+x^2}=1-\cos^2 t$

$\Rightarrow \frac{1+x^2-1}{1+x^2}=\sin^2 t$

$\Rightarrow \frac{x}{\sqrt{1+x^2}}=\sin t$

$I=\int \frac{\tan t\, t}{\sec t \sec^2 t}\sec^2 t\, dt$

$I=\int t \sin t dt$

Taking 1^{st} function as t and second function as sint

$\Rightarrow I=\left[t\int \sin t\, dt-\int\left(\frac{dt}{dt}\int \sin t\, dt\right)dt\right]$

$\Rightarrow I=\left[t(-\cos t)-\int(1\,(-\cos t))dt\right]$

$\Rightarrow I=\left[t(-\cos t)+\int \cos t dt\right]$

$\Rightarrow I=-t\cos t+\sin t+c$

$\Rightarrow I=-\tan^{-1}x\frac{1}{\sqrt{1+x^2}}+\frac{x}{\sqrt{1+x^2}}+c$

$\Rightarrow I=\frac{-\tan^{-1}x\,1}{\sqrt{1+x^2}}+\frac{x}{\sqrt{1+x^2}}+c$

Ans) B $\frac{-\tan^{-1}x\,1}{\sqrt{1+x^2}}+\frac{x}{\sqrt{1+x^2}}+c$

Question: 26

Solution:

To find: Value of $\int x\ \tan^{-1}x\ dx$

Formula used:

(i) $\int f(x)g(x)dx=f(x)\int g(x)dx-\int\left[f'(x)\int g(x)dx\right]dx$

We have, $I=\int x\ \tan^{-1}x\ dx$ ... (i)

Taking 1st function as $\tan^{-1} x$ and second function as x

$$\Rightarrow I = \left[\tan^{-1} x \int x\,dx - \int \left(\frac{d(\tan^{-1} x)}{dx} \int x\,dx\right)dx\right]$$

$$\Rightarrow I = \left[\tan^{-1} x \frac{x^2}{2} - \int \left(\frac{1}{1+x^2}\ \frac{x^2}{2}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \left(\frac{x^2+1-1}{1+x^2}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\left[\int 1dx - \int \frac{1}{1+x^2}dx\ \right]\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}[x - \tan^{-1} x]\right] + c$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}x + \frac{1}{2}\tan^{-1} x\right] + c$$

$$\Rightarrow I = \frac{1}{2}(1+x^2)\tan^{-1} x - \frac{1}{2}x + c$$

Ans) C $\frac{1}{2}(1+x^2)\tan^{-1} x - \frac{1}{2}x + c$

Question: 27

Solution:

To find: Value of $\int \tan^{-1}\sqrt{x}\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \tan^{-1}\sqrt{x}\ dx$ ... (i)

Let $\sqrt{x} = t$,

$$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$$

$\Rightarrow dx = 2t\ dt$

$$I = \int \tan^{-1}\sqrt{x}\ dx$$

$$\Rightarrow I = \int \tan^{-1} t\ 2t\,dt$$

$$\Rightarrow I = 2\int \tan^{-1} t\ t\,dt$$

Taking 1st function as $\tan^{-1} t$ and second function as t

$$\Rightarrow I = 2\left[\tan^{-1} t \int t\,dt - \int \left(\frac{d(\tan^{-1} t)}{dt} \int t\,dt\right)dt\right]$$

$\Rightarrow I = 2\left[\tan^{-1} t \frac{t^2}{2} - \int \left(\frac{1}{1+t^2}\ \frac{t^2}{2}\right) dt\right]$

$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\int \left(\frac{t^2+1-1}{1+t^2}\right) dt\right]$

$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\left[\int 1dt - \int \frac{1}{1+t^2} dt\ \right]\right]$

$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}[t - \tan^{-1} t]\right] + c$

$\Rightarrow I = 2\left[\frac{x}{2}\tan^{-1}\sqrt{x} - \frac{1}{2}\sqrt{x} + \frac{1}{2}\tan^{-1}\sqrt{x}\right] + c$

$\Rightarrow I = x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + c$

$\Rightarrow I = (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$

Ans) B $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$

Question: 28

Solution:

To find: Value of $\int \cos^{-1} x\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int \cos^{-1} x\ dx$ ... (i)

Let $\cos^{-1} x = \theta, \Rightarrow x = \cos\theta$

$\Rightarrow dx = -\sin\theta\ d\theta$

If $x = \cos\theta$,

Then $\sqrt{1-x^2} = \sin\theta$

$I = \int \cos^{-1} x\ dx$

$\Rightarrow I = -\int \theta\sin\theta\ d\theta$

Taking 1^{st} function as θ and second function as $\sin\theta$

$\Rightarrow I = -\left[\theta\int \sin\theta\ d\theta - \int \left(\frac{d\theta}{d\theta}\int \sin\theta\ d\theta\right) d\theta\right]$

$\Rightarrow I = -\left[\theta(-\cos\theta) - \int (-\cos\theta) d\theta\right] + c$

$\Rightarrow I = -[\theta(-\cos\theta) - (-\sin\theta)] + c$

$\Rightarrow I = -[\theta(-\cos\theta) + \sin\theta] + c$

$\Rightarrow I = \theta\cos\theta - \sin\theta + c$

$\Rightarrow I = x \cdot \cos^{-1}x - \sqrt{1-x^2} + c$

Ans) A $x \cdot \cos^{-1}x - \sqrt{1-x^2} + c$

Question: 29

Solution:

To find: Value of $\int \tan^{-1}x \; dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \tan^{-1}x \; dx$ ... (i)

Let $\tan^{-1}x = \theta$, $\Rightarrow x = \tan\theta$

$\Rightarrow dx = \sec^2\theta \; d\theta$

If $x = \tan\theta$,

Then $1 + x^2 = \sec^2\theta$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$$I = \int \tan^{-1}x \; dx$$

$$\Rightarrow I = \int \theta \sec^2\theta \; d\theta$$

Taking 1^{st} function as θ and second function as $\sec^2\theta$

$$\Rightarrow I = \left[\theta\int \sec^2\theta \; d\theta - \int\left(\frac{d\theta}{d\theta}\int \sec^2\theta \; d\theta\right)d\theta\right]$$

$$\Rightarrow I = \left[\theta(\tan\theta) - \int(1\,(\tan\theta))d\theta\right] + c$$

$$\Rightarrow I = [\theta(\tan\theta) - (\log|\sec\theta|)] + c$$

$$\Rightarrow I = \left[\tan^{-1}x\,(x) - \log\left|\sec\left(\sec^{-1}\sqrt{1+x^2}\right)\right|\right] + c$$

$$\Rightarrow I = \left[x \cdot \tan^{-1}x - (\log\left|\sqrt{1+x^2}\right|)\right] + c$$

$$\Rightarrow I = x \cdot \tan^{-1}x - \frac{1}{2}\log|1+x^2| + c$$

Ans) B $x \cdot \tan^{-1}x - \frac{1}{2}\log|1+x^2| + c$

Question: 30

Solution:

To find: Value of $\int \sec^{-1}x \; dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sec^{-1} x \, dx$ ... (i)

Let $\sec^{-1} x = \theta$, $\Rightarrow x = \sec\theta$

$\Rightarrow dx = \sec\theta \tan\theta \, d\theta$

If $x = \sec\theta$,

Then $\sqrt{x^2 - 1} = \tan\theta$

$I = \int \sec^{-1} x \, dx$

$\Rightarrow I = \int \theta \sec\theta \tan\theta \, d\theta$

Taking 1^{st} function as θ and second function as $\sec\theta \tan\theta$

$\Rightarrow I = \left[\theta \int \sec\theta \tan\theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sec\theta \tan\theta \, d\theta\right) d\theta\right]$

$\Rightarrow I = \left[\theta(\sec\theta) - \int (1 (\sec\theta)) d\theta\right] + c$

$\Rightarrow I = [\theta(\sec\theta) - (\log|\sec\theta + \tan\theta|)] + c$

$\Rightarrow I = \left[\sec^{-1} x (x) - (\log\left|x + \sqrt{x^2 - 1}\right|)\right] + c$

$\Rightarrow I = x . \sec^{-1} x - \log\left|x + \sqrt{x^2 - 1}\right| + c$

Ans) B $x . \sec^{-1} x - \log\left|x + \sqrt{x^2 - 1}\right| + c$

Question: 31

Solution:

To find: Value of $\int \sin^{-1}(3x - 4x^3) \, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int \sin^{-1}(3x - 4x^3) \, dx$ ... (i)

Let $x = \sin\theta$, $\Rightarrow \theta = \sin^{-1} x$

$\Rightarrow dx = \cos\theta \, d\theta$

If $x = \sin\theta$,

Then $\sqrt{1 - x^2} = \cos\theta$

$I = \int \sin^{-1}(3x - 4x^3) \, dx$

$\Rightarrow I = \int \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right) \cos\theta \, d\theta$

$\Rightarrow I = \int \sin^{-1}(\sin 3\theta) \cos\theta \, d\theta$

$\Rightarrow I = \int 3\theta \cos\theta \, d\theta$

$\Rightarrow I = 3\int \theta\cos\theta\, d\theta$

Taking 1st function as θ and second function as $\cos\theta$

$\Rightarrow I = 3\left[\theta\int \cos\theta\, d\theta - \int\left(\frac{d\theta}{d\theta}\int \cos\theta\, d\theta\right)d\theta\right]$

$\Rightarrow I = 3\left[\theta(\sin\theta) - \int(1(\sin\theta))d\theta\right]$

$\Rightarrow I = 3[\theta(\sin\theta)-(-\cos\theta)]+c$

$\Rightarrow I = 3[\theta(\sin\theta)+\cos\theta]+c$

$\Rightarrow I = 3\sin^{-1}x(x)+3\sqrt{1-x^2}+c$

$\Rightarrow I = 3x\sin^{-1}x+3\sqrt{1-x^2}+c$

$\Rightarrow I = 3\left[x\sin^{-1}x+\sqrt{1-x^2}\right]+c$

Ans) A $3\left[x\sin^{-1}x+\sqrt{1-x^2}\right]+c$

Question: 32

Solution:

To find: Value of $\int \sin^{-1}\frac{2x}{1+x^2}\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sin^{-1}\frac{2x}{1+x^2}\,dx$ ... (i)

Let $x = \tan\theta$, $\Rightarrow \theta = \tan^{-1}x$

$\Rightarrow dx = \sec^2\theta\, d\theta$

If $x = \tan\theta$,

Then $1 + x^2 = \sec^2\theta$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$I = \int \sin^{-1}\frac{2x}{1+x^2}\,dx$

$\Rightarrow I = \int \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta\, d\theta$

$\Rightarrow I = \int \sin^{-1}(\sin 2\theta)\sec^2\theta\, d\theta$

$\Rightarrow I = \int 2\theta\sec^2\theta\, d\theta$

$\Rightarrow I = 2\int \theta\sec^2\theta\, d\theta$

Taking 1st function as θ and second function as $\sec^2\theta$

$\Rightarrow I = 2\left[\theta \int \sec^2\theta\, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sec^2\theta\, d\theta\right) d\theta\right]$

$\Rightarrow I = 2\left[\theta(\tan\theta) - \int (1\,(\tan\theta))d\theta\right]$

$\Rightarrow I = 2[\theta(\tan\theta)-(\log(\sec\theta)]+c$

$\Rightarrow I = 2\left[\tan^{-1}x\,(x)-(\log(\sec\left(\sec^{-1}\sqrt{1+x^2}\right))\right]+c$

$\Rightarrow I = 2\left[\tan^{-1}x\,(x)-(\log\sqrt{1+x^2})\right]+c$

$\Rightarrow I = 2\left[x\,.\,\tan^{-1}x - \frac{1}{2}(\log 1+x^2)\right]+c$

$\Rightarrow I = 2x\,.\,\tan^{-1}x - (\log 1+x^2)+c$

Ans) B $2x\,.\,\tan^{-1}x - (\log 1+x^2)+c$

Question: 33

Solution:

To find: Value of $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$

Formula used: $\int \frac{1}{x}dx = \log|x|+c$

We have, $I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$ ... (i)

Let $x = \cos\theta$, $\Rightarrow \theta = \cos^{-1}x$

$\Rightarrow dx = -\sin\theta\, d\theta$

If $x = \cos\theta$,

Then $\sqrt{1-x^2} = \sin\theta$

$I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$

$\Rightarrow I = \int \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\,.\,-\sin\theta\, d\theta$

$\Rightarrow I = \int \tan^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\,.\,-\sin\theta\, d\theta$

$\Rightarrow I = \int \tan^{-1}\sqrt{\tan^2\frac{\theta}{2}}\,.\,-\sin\theta\, d\theta$

$\Rightarrow I = \int \tan^{-1}\left(\tan\frac{\theta}{2}\right)\,.\,-\sin\theta\, d\theta$

$\Rightarrow I = \int \frac{\theta}{2} \cdot -\sin\theta \, d\theta$

$\Rightarrow I = -\frac{1}{2}\int \theta . \sin\theta \, d\theta$

Taking 1^{st} function as θ and second function as sinθ

$\Rightarrow I = -\frac{1}{2}\left[\theta \int \sin\theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sin\theta \, d\theta\right) d\theta\right]$

$\Rightarrow I = -\frac{1}{2}\left[\theta(-\cos\theta) - \int (1\,(-\cos\theta))d\theta\right]$

$\Rightarrow I = -\frac{1}{2}\left[\theta(-\cos\theta) + \int (\cos\theta)d\theta\right]$

$\Rightarrow I = -\frac{1}{2}[\theta(-\cos\theta) + \sin\theta] + c$

$\Rightarrow I = \frac{1}{2}\cos^{-1}x\,(x) - \frac{1}{2}\sqrt{1-x^2} + c$

$\Rightarrow I = \frac{1}{2}x.\ \cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + c$

Ans) C $\frac{1}{2}x.\ \cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + c$

Question: 34

Solution:

To find: Value of $\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx$ ... (i)

Let $x = \tan\theta$, $\Rightarrow \theta = \tan^{-1}x$

$\Rightarrow dx = \sec^2\theta \, d\theta$

If $x = \tan\theta$,

Then $1 + x^2 = \sec^2\theta$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$I = \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx$

$\Rightarrow I = \int \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)\sec^2\theta \, d\theta$

$\Rightarrow I = \int \tan^{-1}(\tan 3\theta)\sec^2\theta \, d\theta$

$\Rightarrow I = \int 3\theta \sec^2\theta \, d\theta$

$\Rightarrow I = 3\int \theta \sec^2\theta \, d\theta$

Taking 1^{st} function as θ and second function as $\sec^2\theta$

$\Rightarrow I = 3\left[\theta\int \sec^2\theta \, d\theta - \int\left(\frac{d\theta}{d\theta}\int \sec^2\theta \, d\theta\right)d\theta\right]$

$\Rightarrow I = 3\left[\theta\tan\theta - \int(\tan\theta)d\theta\right]$

$\Rightarrow I = 3[\theta\tan\theta-(\log\sec\theta)]+c$

$\Rightarrow I = 3\theta\tan\theta-3\log(\sec\theta)+c$

$\Rightarrow I = 3\tan^{-1}x\tan(\tan^{-1}x)-3\log\left\{\sec\left(\sec^{-1}\sqrt{1+x^2}\right)\right\}+c$

$\Rightarrow I = 3x.\tan^{-1}x-3\log\left\{\sqrt{1+x^2}\right\}+c$

$\Rightarrow I = 3x.\tan^{-1}x-\frac{3}{2}\log\{1+x^2\}+c$

Ans) B $3x.\tan^{-1}x-\frac{3}{2}\log\{1+x^2\}+c$

Question: 35

Solution:

To find: Value of $\int x^2\cos x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x^2\cos x\,dx$ ... (i)

Taking 1^{st} function as x^2 and second function as $\cos x$

$\Rightarrow I = \left[x^2\int \cos x\,dx - \int\left(\frac{dx^2}{dx}\int \cos x\,dx\right)dx\right]$

$\Rightarrow I = \left[x^2\sin x - \int(2x\sin x)dx\right]$

$\Rightarrow I = \left[x^2\sin x - 2\int(x\sin x)dx\right]$

Taking 1^{st} function as x and second function as $\sin x$

$\Rightarrow I = x^2\sin x - 2\left[x\int \sin x\,dx - \int\left(\frac{dx}{dx}\int \sin x\,dx\right)dx\right]$

$\Rightarrow I = x^2\sin x - 2\left[x(-\cos x) - \int(1(-\cos x)dx\right]$

$\Rightarrow I = x^2\sin x - 2[x(-\cos x)-(-\sin x)]+c$

$\Rightarrow I = x^2\sin x - 2[x(-\cos x)+\sin x]+c$

$\Rightarrow I = x^2\sin x + 2x\cos x-2\sin x+c$

Ans) A $x^2\sin x + 2x\cos x-2\sin x+c$

Question: 36

Solution:

To find: Value of $\int sinx \log(cosx)\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sin x \log(\cos x)\ dx$ ... (i)

Let cosx = t

-sinx dx = dt

$I = \int \sin x \log(\cos x)\ dx$

$I = -\int \log t\, dt$

$I = -\int \log t \cdot 1 \cdot dt$

Taking 1^{st} function as $\log t$ and second function as 1

$\Rightarrow I = -\left[\log t\int 1\, dt - \int\left(\frac{d\log t}{dt}\int 1\, dt\right)dt\right]$

$\Rightarrow I = -\left[\log t \cdot t - \int\left(\frac{1}{t}t\right)dt\right]$

$\Rightarrow I = -\left[\log t \cdot t - \int 1dt\right]$

$\Rightarrow I = -[\log t \cdot t - t] + c$

$\Rightarrow I = -\log t \cdot t + t + c$

$\Rightarrow I = -\cos x \cdot \log(\cos x) + \cos x + c$

Ans) B $-\cos x \cdot \log(\cos x) + \cos x + c$

Question: 37

Solution:

To find: Value of $\int x \sin x \cos x\, dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int x \sin x \cos x\, dx$ ... (i)

$I = \frac{1}{2}\int x\, 2\sin x \cos x\, dx$

$I = \frac{1}{2}\int x \sin 2x\, dx$

Let 2x = t

2dx = dt

$dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{t}{2} \sin t \frac{dt}{2}$

$I = \frac{1}{8}\int t \sin t\, dt$

Taking 1st function as t and second function as sint

$\Rightarrow I = \frac{1}{8}\left[t\int \sin t\ dt - \int\left(\frac{dt}{dt}\int \sin t\ dt\right)dt\right]$

$\Rightarrow I = \frac{1}{8}\left[t.(-\cos t) - \int(-\cos t)\, dt\right]$

$\Rightarrow I = \frac{1}{8}[-t.\cos t-(-\sin t)]+c$

$\Rightarrow I = \frac{1}{8}[-t.\cos t+\sin t]+c$

$\Rightarrow I = -\frac{1}{8}2x.\cos 2x+\frac{1}{8}\sin 2x+c$

$\Rightarrow I = -\frac{1}{4}x.\cos 2x+\frac{1}{8}\sin 2x+c$

Ans) A $-\frac{1}{4}x.\cos 2x+\frac{1}{8}\sin 2x+c$

Question: 38

Solution:

To find: Value of $\int x^3 \cos x^2 dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x^3 \cos x^2 dx$ ... (i)

Let $x^2 = t$

$\Rightarrow xdx = \frac{1}{2}dt$

$I = \int x^3 \cos x^2 dx$

$I = \int x.\ x^2 \cos x^2 dx$

$I = \int t \cos t \frac{1}{2}\, dt$

$I = \frac{1}{2}\int t \cos t\, dt$

Taking 1st function as t and second function as cos t

$\Rightarrow I = \frac{1}{2}\left[t\int \cos t\ dt - \int\left(\frac{dt}{dt}\int \cos t\ dt\right)dt\right]$

$\Rightarrow I = \frac{1}{2}\left[t . \sin t - \int \sin t\, dt\right]$

$\Rightarrow I = \frac{1}{2}[t . \sin t - (-\cos t) + c]$

$\Rightarrow I = \frac{1}{2}[t . \sin t + \cos t + c]$

$\Rightarrow I = \frac{1}{2}x^2 . \sin x^2 + \frac{1}{2}\cos x^2 + c$

Ans) B $\frac{1}{2}x^2 . \sin x^2 + \frac{1}{2}\cos x^2 + c$

Question: 39

Solution:

To find: Value of $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$ ... (i)

Let $x = \tan t$, $t = \tan^{-1}x$

$\Rightarrow dx = \sec^2 t\ dt$

If $\tan t = x$,

$\sec t = 1 + x^2$

$I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

$I = \int \cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right)\sec^2 t\, dt$

$I = \int \cos^{-1}(\cos 2t)\sec^2 t\ dt$

$I = \int 2t\sec^2 t\ dt$

$I = 2\int t\sec^2 t\ dt$

Taking 1st function as t and second function as $\sec^2 t$

$\Rightarrow I = 2\left[t\int \sec^2 t\ dt - \int\left(\frac{dt}{dt}\int \sec^2 t\ dt\right)dt\right]$

$\Rightarrow I = 2\left[t\tan t - \int \tan t\, dt\right]$

$\Rightarrow I = 2[t\tan t - \log|\sec t| + c]$

$\Rightarrow I = 2[\tan^{-1}x\ x - \log|1+x^2| + c]$

$\Rightarrow I = 2x\tan^{-1}x - 2\log|1+x^2| + c$

Ans) D None of these

Question: 40

Solution:

To find: Value of $\int x\tan^{-1}x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\tan^{-1}x\,dx$ ... (i)

Taking 1^{st} function as $\tan^{-1}x$ and second function as x

$$\Rightarrow I = \left[\tan^{-1}x\int x\,dx - \int\left(\frac{d\tan^{-1}x}{dx}\int x\,dx\right)dx\right]$$

$$\Rightarrow I = \left[\tan^{-1}x\frac{x^2}{2} - \int\left(\frac{1}{(1+x^2)}\frac{x^2}{2}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(\frac{x^2}{(1+x^2)}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(1 - \frac{1}{(1+x^2)}\right)dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int 1.\,dx + \frac{1}{2}\int\frac{1}{(1+x^2)}.\,dx\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x\right] + c$$

$$\Rightarrow I = \left[\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x\right] + c$$

$$\Rightarrow I = \frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + c$$

Ans) A $\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + c$

Question: 41

Solution:

To find: Value of $\int \sin(logx)\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sin(\log x)\,dx$ ... (i)

$I = \int \sin(\log x) . 1 . dx$

Taking 1^{st} function as sin(logx) and second function as 1

$\Rightarrow I = \left[\sin(\log x)\int 1\, dx - \int\left(\frac{d\sin(\log x)}{dx}\int 1\, dx\right)dx\right]$

$\Rightarrow I = \left[\sin(\log x) . x - \int \frac{\cos(\log x). x}{x}dx\right]$

$\Rightarrow I = \left[\sin(\log x) . x - \int \cos(\log x)\, dx\right]$

Taking 1^{st} function as cos(logx) and second function as 1

$\Rightarrow I = \sin(\log x) .x - \left[\cos(\log x)\int 1\, dx - \int\left(\frac{d\cos(\log x)}{dx}\int 1\, dx\right)dx\right]$

$\Rightarrow I = \sin(\log x) . x - \left[\cos(\log x) .x- \int - \frac{\sin(\log x). x}{x}dx\right]$

$\Rightarrow I = \sin(\log x) . x - \left[\cos(\log x) .x+ \int \sin(\log x)\, dx\right]$

$\Rightarrow I = \sin(\log x) . x - [\cos(\log x) .x+ I]+c$

$\Rightarrow I = \sin(\log x) . x - \cos(\log x) .x- I+c$

$\Rightarrow 2I = \sin(\log x) . x - \cos(\log x) .x+c$

$\Rightarrow I = \frac{\sin(\log x) . x - \cos(\log x) .x}{2}+c$

$\Rightarrow I = \frac{1}{2}x. \sin(\log x) - x . \frac{1}{2}\cos(\log x)+c$

Ans) B $\frac{1}{2}x. \sin(\log x) - x . \frac{1}{2}\cos(\log x)+c$

Question: 42

Solution:

To find: Value of $\int(\sin^{-1} x)^2 dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int(\sin^{-1} x)^2 dx$ ... (i)

Putting sint = x, $\Rightarrow t = \sin^{-1} x$

$\Rightarrow$ dx = cost dt

When x = sint then $\sqrt{1-x^2} = \cos t$

$I = \int(\sin^{-1} x)^2 dx$

$\Rightarrow I = \int(\sin^{-1}(\sin t))^2 \cos t\, dt$

$\Rightarrow I = \int t^2 \cos t \, dt$

Taking 1st function as t^2 and second function as cost

$\Rightarrow I = \left[t^2 \int \cos t \, dt - \int \left(\frac{dt^2}{dt} \int \cos t \, dt\right) dt\right]$

$\Rightarrow I = \left[t^2 \sin t - \int (2t \sin t) dt\right]$

$\Rightarrow I = \left[t^2 \sin t - 2\int (t \sin t) dt\right]$

Taking 1st function as t and second function as sint

$\Rightarrow I = t^2 \sin t - 2\left[\int (t \sin t) dt\right]$

$\Rightarrow I = t^2 \sin t - 2\left[t \int \sin t \, dt - \int \left(\frac{dt}{dt} \int \sin t \, dt\right) dt\right]$

$\Rightarrow I = t^2 \sin t - 2\left[t(-\cos t) - \int (-\cos t) dt\right]$

$\Rightarrow I = t^2 \sin t - 2[-t\cos t - (-\sin t) + c]$

$\Rightarrow I = t^2 \sin t + 2t\cos t - 2\sin t + c$

$\Rightarrow I = x\left(\sin^{-1} x\right)^2 + 2 \sin^{-1} x \sqrt{1-x^2} - 2x + c$

Ans) D $x\left(\sin^{-1} x\right)^2 + 2 \sin^{-1} x \sqrt{1-x^2} - 2x + c$

Question: 43

Solution:

To find: Value of $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$ ... (i)

Here $f(x) = \frac{1}{x}$

$\Rightarrow f'(x) = -\frac{1}{x^2}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \frac{1}{x} + c$

Ans) C $e^x \frac{1}{x} + c$

Question: 44

Mark (√) against

Solution:

To find: Value of $\int e^x\left(\frac{1}{x^2}-\frac{2}{x^3}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\frac{1}{x^2}-\frac{2}{x^3}\right)dx$ ... (i)

Here $f(x) = \frac{1}{x^2}$

$\Rightarrow f'(x) = -\frac{2}{x^3}$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x\frac{1}{x^2} + c$

Ans) B $e^x\frac{1}{x^2} + c$

Question: 45

Solution:

To find: Value of $\int e^x\left(\sin^{-1}x + \frac{1}{\sqrt{1-x^2}}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\sin^{-1}x + \frac{1}{\sqrt{1-x^2}}\right)dx$ ... (i)

Here $f(x) = \sin^{-1}x$

$\Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x\sin^{-1}x + c$

Ans) B $e^x\sin^{-1}x + c$

Question: 46

Solution:

To find: Value of $\int e^x(\tan x + \log(\sec x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\tan x + \log(\sec x))dx$ ... (i)

$\Rightarrow I = \int e^x(\tan x - \log(\cos x))dx$

Here $f(x) = -\log(\cos x)$

$\Rightarrow f'(x) = \tan x$

$\Rightarrow I = \int e^x\left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = -e^x\log(\cos x) + c$

$\Rightarrow I = e^x\log(\sec x) + c$

Ans) A $e^x\log(\sec x)+c$

Question: 47

Solution:

To find: Value of $\int e^x(\tan x + \log(\sec x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\tan x + \log(\sec x))dx$ ... (i)

$\Rightarrow I = \int e^x(\tan x - \log(\cos x))dx$

Here $f(x) = -\log(\cos x)$

$\Rightarrow f'(x) = \tan x$

$\Rightarrow I = \int e^x\left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = -e^x\log(\cos x) + c$

$\Rightarrow I = e^x\log(\sec x) + c$

Ans) A $e^x\log(\sec x)+c$

Question: 48

Solution:

To find: Value of $\int e^x(\cot x + \log(\sin x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\cot x+\log(\sin x))dx$ ... (i)

Here $f(x) = \log(\sin x)$

$\Rightarrow f'(x) = \cot x$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = e^x\log(\sin x)+c$

Ans) D None of these

Question: 49

Solution:

To find: Value of $\int e^x\left(\tan^{-1}x + \frac{1}{(1+x)^2}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\tan^{-1}x+ \frac{1}{(1+x)^2}\right)dx$ ... (i)

Here $f(x) = \tan^{-1}x$

$\Rightarrow f'(x) = \frac{1}{(1+x)^2}$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = e^x(\tan^{-1}x)+c$

Ans) B $e^x(\tan^{-1}x)+c$

Question: 50

Solution:

To find: Value of $\int e^x(\tan x - \log(\cos x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\tan x-\log(\cos x))dx$ ... (i)

Here $f(x) = -\log(\cos x)$

$\Rightarrow f'(x) = \tan x$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = -e^x \log(\cos x) + c$

$\Rightarrow I = e^x \log(\sec x) + c$

Ans) C $e^x \log(\sec x) + c$

Question: 51

Solution:

To find: Value of $\int e^x(\cot x - \operatorname{cosec}^2 x)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\cot x - \operatorname{cosec}^2 x)dx$ ... (i)

Here $f(x) = \cot x$

$\Rightarrow f'(x) = -\operatorname{cosec}^2 x$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \cot x + c$

Ans) B $e^x \cot x + c$

Question: 52

Solution:

To find: Value of $\int e^x(\sin x + \cos x)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\sin x + \cos x)dx$ ... (i)

Here $f(x) = \sin x$

$\Rightarrow f'(x) = \cos x$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \sin x + c$

Ans) A $e^x \sin x + c$

Question: 53

Solution:

To find: Value of $\int e^x \sec x\,(1 + \tan x)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x \sec x\,(1+\tan x)dx$ ... (i)

$I = \int e^x(\sec x+\sec x\tan x)dx$

Here $f(x) = \sec x$

$\Rightarrow f'(x) = \sec x\tan x$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = e^x\sec x+c$

Ans) B $e^x\sec x+c$

Question: 54

Solution:

To find: Value of $\int e^x\left(\frac{1+x\log x}{x}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\frac{1+x\log x}{x}\right)dx$ ... (i)

$I = \int e^x\left(\frac{1}{x}+\log x\right)dx$

Here $f(x) = \log x$

$\Rightarrow f'(x) = \frac{1}{x}$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x)+c$

$\Rightarrow I = e^x\log x+c$

Ans) B $e^x\log x+c$

Question: 55

Solution:

To find: Value of $\int e^x\frac{x}{(1+x)^2}dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\frac{x}{(1+x)^2}dx$ ... (i)

$I = \int e^x \left(\frac{x+1-1}{(1+x)^2}\right) dx$

$\Rightarrow I = \int e^x \left(\frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right) dx$

Here $f(x) = \frac{1}{(1+x)}$

$\Rightarrow f'(x) = -\frac{1}{(1+x)^2}$

$\Rightarrow I = \int e^x \left(f(x)+f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \frac{1}{(1+x)} + c$

Ans) A $e^x \frac{1}{(1+x)} + c$

Question: 56

Solution:

To find: Value of $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$ ... (i)

$I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

$\Rightarrow I = \int e^x \left(\frac{1}{1+\cos x} + \frac{\sin x}{1+\cos x}\right) dx$

$\Rightarrow I = \int e^x \left(\frac{1}{2\cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) dx$

$\Rightarrow I = \int e^x \left(\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right) dx$

Here $f(x) = \tan\frac{x}{2}$

$\Rightarrow f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$

$\Rightarrow I = \int e^x \left(f(x)+f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \tan\frac{x}{2} + c$

Ans) C $e^x \tan\frac{x}{2} + c$