Chapter : 14. SOME SPECIAL INTEGRALS

Exercise : 14A

Question: 1

Solution:

To find: $\int \frac{dx}{(1-9x)^2}$

Formula Used: $\int x^n = \frac{x^{n+1}}{n+1} + C$

Let y = (1 – 9x) ... (1)

Differentiating with respect to x,

$\frac{dy}{dx} = -9$

i.e., dy = -9 dx

Substituting in the equation to evaluate,

$\Rightarrow \int \frac{\frac{dy}{-9}}{y^2}$

$\Rightarrow \frac{-1}{9} \int \frac{dy}{y^2}$

$\Rightarrow \frac{-1}{9} \times \int y^{-2}\ dy$

$\Rightarrow \frac{-1}{9} \times \frac{y^{-2+1}}{-2+1} + C$

Simplifying and substituting the value of y from (1),

$\Rightarrow \frac{-1}{9} \times \frac{-1}{(1-9x)} + C$

$\Rightarrow \frac{1}{9(1-9x)} + C$

Therefore,

$\int \frac{dx}{(1-9x)^2} = \frac{1}{9(1-9x)} + C$

Question: 2

Solution:

To find: $\int \frac{dx}{(25-4x^2)}$

Formula Used: $\frac{dx}{(a^2-x^2)} = \frac{1}{2a} \times \log\left|\frac{a+x}{a-x}\right| + C$

Given equation = $\int \frac{dx}{4(\frac{25}{4}-x^2)}$

$\Rightarrow \frac{1}{4}\int \frac{dx}{\left(\left(\frac{5}{2}\right)^2 - x^2\right)}$ ... (1)

Here $a = \frac{5}{2}$

Therefore, (1) becomes

$$\Rightarrow \frac{1}{4} \times \frac{1}{5} \times \log\left|\frac{\frac{5}{2}+x}{\frac{5}{2}-x}\right| + C$$

$$\Rightarrow \frac{1}{20} \times \log\left|\frac{5+2x}{5-2x}\right| + C$$

Therefore,

$$\int \frac{dx}{(25-4x^2)} = \frac{1}{20} \times \log\left|\frac{5+2x}{5-2x}\right| + C$$

Question: 3

Solution:

To find: $\int \frac{dx}{(x^2+16)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{4^2 + x^2}$$

Here a = 4

$$\Rightarrow \frac{1}{4} \times \tan^{-1}\left(\frac{x}{4}\right) + C$$

Therefore,

$$\int \frac{dx}{(x^2+16)} = \frac{1}{4} \times \tan^{-1}\left(\frac{x}{4}\right) + C$$

Question: 4

Solution:

To find: $\int \frac{dx}{(4+9x^2)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{9}\int \frac{dx}{\left(\frac{4}{9}\right) + x^2}$$

$$\Rightarrow \frac{1}{9}\int \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2}$$

Here $a = \frac{2}{3}$

$$\Rightarrow \frac{1}{9} \times \frac{3}{2} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

$$\Rightarrow \frac{1}{6} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

Therefore,

$$\int \frac{dx}{(4 + 9x^2)} = \frac{1}{6} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

Question: 5

Solution:

To find: $\int \frac{dx}{(50+2x^2)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{2} \int \frac{dx}{25 + x^2}$$

$$\Rightarrow \frac{1}{2} \int \frac{dx}{5^2 + x^2}$$

Here a = 5

$$\Rightarrow \frac{1}{10} \times \tan^{-1}\left(\frac{x}{5}\right) + C$$

Therefore,

$$\int \frac{dx}{(x^2 + 16)} = \frac{1}{10} \times \tan^{-1}\left(\frac{x}{5}\right) + C$$

Question: 6

Solution:

To find: $\int \frac{dx}{(16x^2-25)}$

Formula Used: $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{16}\int \frac{dx}{x^2 - \left(\frac{25}{16}\right)}$$

$$\Rightarrow \frac{1}{16}\int \frac{dx}{x^2 - \left(\frac{5}{4}\right)^2}$$

Here $a = \frac{5}{4}$

$$\Rightarrow \frac{1}{16} \times \frac{2}{5} \times \ln\left|\frac{x - \frac{5}{4}}{x + \frac{5}{4}}\right| + C$$

$$\Rightarrow \frac{1}{40} \times ln\left|\frac{4x-5}{4x+5}\right| + C$$

Therefore,

$$\int \frac{dx}{(16x^2-25)} = \frac{1}{40} \times \log\left|\frac{4x-5}{4x+5}\right| + C$$

Question: 7

Solution:

To find: $\int \frac{(x^2-1)}{(x^2+4)}\, dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Given equation can be rewritten as the following:

$$\Rightarrow \int \frac{(x^2+4-5)}{(x^2+4)}\, dx$$

$$\Rightarrow \int \frac{(x^2+4)}{(x^2+4)} dx - \int \frac{5}{(x^2+4)}\, dx$$

$$\Rightarrow \int dx - 5\int \frac{1}{(x^2+2^2)}\, dx$$

Here a = 2,

$$\Rightarrow x - \frac{5}{2}\, tan^{-1}\frac{x}{2} + C$$

Therefore,

$$\int \frac{(x^2-1)}{(x^2+4)}\, dx = x - \frac{5}{2}\, tan^{-1}\frac{x}{2} + C$$

Question: 8

Solution:

To find: $\int \frac{x^2}{(9+4x^2)}\, dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Given equation can be rewritten as the following:

$$\Rightarrow \frac{1}{4}\int \frac{x^2}{(x^2+\frac{9}{4})}\, dx$$

$$\Rightarrow \frac{1}{4}\int \frac{x^2+\frac{9}{4}-\frac{9}{4}}{(x^2+\frac{9}{4})}\, dx$$

$$\Rightarrow \frac{1}{4}\int dx - \frac{9}{16}\int \frac{1}{\left(x^2+\left(\frac{3}{2}\right)^2\right)}\, dx$$

Here $a = \frac{3}{2}$,

$\Rightarrow \frac{x}{4} - \left(\frac{9}{16} \times \frac{2}{3} \, tan^{-1} \frac{2x}{3}\right) + C$

$\Rightarrow \frac{x}{4} - \frac{3}{8} \, tan^{-1}\left(\frac{2x}{3}\right) + C$

Therefore,

$$\int \frac{x^2}{(9 + 4x^2)} \, dx = \frac{x}{4} - \frac{3}{8} \, tan^{-1}\left(\frac{2x}{3}\right) + C$$

Question: 9

Solution:

To find: $\int \frac{e^x}{(e^{2x}+1)} \, dx$

Formula Used: $\int \frac{dx}{1 + x^2} = \tan^{-1} x$

Let y = e^x ... (1)

Differentiating both sides, we get

dy = e^x dx

Substituting in given equation,

$\Rightarrow \int \frac{dy}{y^2 + 1}$

$\Rightarrow \tan^{-1} y$

From (1),

$\Rightarrow \tan^{-1} (e^x)$

Therefore,

$$\int \frac{e^x}{(e^{2x} + 1)} \, dx = \tan^{-1}(e^x) + C$$

Question: 10

Solution:

To find: $\int \frac{\sin x}{(1 + \cos^2 x)} \, dx$

Formula Used: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Let y = cos x ... (1)

Differentiating both sides, we get

dy = –sin x dx

Substituting in given equation,

$\Rightarrow \int \frac{-dy}{1 + y^2}$

$\Rightarrow -\tan^{-1} y$

From (1),

$\Rightarrow -\tan^{-1} (\cos x)$

Therefore,

$$\int \frac{\sin x}{(1+\cos^2 x)}\,dx = -\tan^{-1}(\cos x) + C$$

Question: 11

Solution:

To find: $\int \frac{\cos x}{(1+\sin^2 x)}\,dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Let y = sin x ... (1)

Differentiating both sides, we get

dy = cos x dx

Substituting in given equation,

$$\Rightarrow \int \frac{dy}{1+y^2}$$

$\Rightarrow \tan^{-1} y$

From (1),

$\Rightarrow \tan^{-1}(\sin x)$

Therefore,

$$\int \frac{\cos x}{(1+\sin^2 x)}\,dx = \tan^{-1}(\sin x) + C$$

Question: 12

Solution:

To find: $\int \frac{3x^5}{(1+x^{12})}\,dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Let $y = x^6$ ... (1)

Differentiating both sides, we get

$dy = 6x^5\,dx$

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2}\,dy}{1+y^2}$$

$$\Rightarrow \frac{1}{2}\tan^{-1} y + C$$

From (1),

$$\Rightarrow \frac{1}{2}\tan^{-1}(x^6) + C$$

Therefore,

$$\int \frac{3x^5}{(1+x^{12})}\,dx = \frac{1}{2}\tan^{-1}(x^6) + C$$

Question: 13

Solution:

To find: $\int \frac{2x^3}{(4+x^8)}\,dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Let y = x^4 ... (1)

Differentiating both sides, we get

dy = $4x^3$ dx

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2}\,dy}{4+y^2}$$

$$\Rightarrow \frac{1}{2}\int \frac{1}{2^2+y^2}\,dy$$

$$\Rightarrow \frac{1}{4}\tan^{-1}\left(\frac{y}{2}\right) + C$$

From (1),

$$\Rightarrow \frac{1}{4}\tan^{-1}\left(\frac{x^4}{2}\right) + C$$

Therefore,

$$\int \frac{2x^3}{(4+x^8)}\,dx = \frac{1}{4}\tan^{-1}\left(\frac{x^4}{2}\right) + C$$

Question: 14

Solution:

To find: $\int \frac{dx}{(e^x+e^{-x})}$

Formula Used: $\int \frac{dx}{1+x^2} = \tan^{-1} x$

Given equation is:

$\int \frac{dx}{(e^x+e^{-x})} = \int \frac{e^x\,dx}{(e^{2x}+1)}$ ... (1)

Let y = e^x ... (1)

Differentiating both sides, we get

dy = e^x dx

Substituting in (1),

$$\Rightarrow \int \frac{dy}{y^2+1}$$

$\Rightarrow \tan^{-1} y$

From (1),

$\Rightarrow \tan^{-1}(e^x)$

Therefore,

$$\int \frac{dx}{(e^x + e^{-x})} = \tan^{-1}(e^x) + C$$

Question: 15

Solution:

To find: $\int \frac{x\,dx}{(1-x^4)}$

Formula Used: $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

Let y = x² ... (1)

Differentiating both sides, we get

dy = 2x dx

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2}\,dy}{1 - y^2}$$

Here a = 1,

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \log\left|\frac{1+y}{1-y}\right| + C$$

$$\Rightarrow \frac{1}{4}\log\left|\frac{1+y}{1-y}\right| + C$$

From (1),

$$\Rightarrow \frac{1}{4}\log\left|\frac{1+x^2}{1-x^2}\right| + C$$

Therefore,

$$\int \frac{x\,dx}{(1-x^4)} = \frac{1}{4}\log\left|\frac{1+x^2}{1-x^2}\right| + C$$

Question: 16

Solution:

To find: $\int \frac{x^2\,dx}{(a^6 - x^6)}$

Formula Used: $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

Let y = x³ ... (1)

Differentiating both sides, we get

dy = 3x² dx

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{3}\,dy}{a^6 - y^2}$$

$$\Rightarrow \frac{1}{3}\int \frac{1}{(a^3)^2 - y^2}\,dy$$

$$\Rightarrow \frac{1}{3} \times \frac{1}{2a^3} \times \log\left|\frac{a^3 + y}{a^3 - y}\right| + C$$

$$\Rightarrow \frac{1}{6a^3}\log\left|\frac{a^3 + y}{a^3 - y}\right| + C$$

From (1),

$$\Rightarrow \frac{1}{6a^3}\log\left|\frac{a^3 + x^3}{a^3 - x^3}\right| + C$$

Therefore,

$$\int \frac{x^2\,dx}{(a^6 - x^6)} = \frac{1}{6a^3}\log\left|\frac{a^3 + x^3}{a^3 - x^3}\right| + C$$

Question: 17

Solution:

To find: $\int \frac{dx}{(x^2+4x+8)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{((x+2)^2 + 4)}$$

$\Rightarrow \int \frac{dx}{((x+2)^2+2^2)}$ ... (1)

Let y = x + 2 ... (2)

Differentiating both sides,

dy = dx

Substituting in (1),

$$\Rightarrow \int \frac{dy}{(y^2 + 2^2)}$$

Here a = 2,

$$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{y}{2}\right) + C$$

From (2),

$$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right) + C$$

Therefore,

$$\int \frac{dx}{(x^2 + 4x + 8)} = \frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right) + C$$

Question: 18

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Solution:

To find: $\int \frac{dx}{(4x^2-4x+3)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$\Rightarrow \int \frac{dx}{((2x-1)^2+2)}$ ... (1)

Let y = 2x – 1 ... (2)

Differentiating both sides,

dy = 2dx

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{2}dy}{\left(y^2 + \left(\sqrt{2}\right)^2\right)}$$

Here a = √2,

$$\Rightarrow \frac{1}{2} \times \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y}{\sqrt{2}}\right) + C$$

From (2),

$$\Rightarrow \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$$

Therefore,

$$\int \frac{dx}{(4x^2-4x+3)} = \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$$

Question: 19

Solution:

To find: $\int \frac{dx}{(2x^2+x+3)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x + \frac{1}{2\sqrt{2}}\right)^2 + 3 - \frac{1}{8}\right)}$$

$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x+\frac{1}{2\sqrt{2}}\right)^2+\frac{23}{8}\right)}$... (1)

Let $y = \sqrt{2}x + \frac{1}{2\sqrt{2}}$ ... (2)

Differentiating both sides,

dy = √2 dx

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{\sqrt{2}}dy}{\left(y^2 + \left(\frac{\sqrt{23}}{2\sqrt{2}}\right)^2\right)}$$

Here $a = \frac{\sqrt{23}}{2\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{23}} \tan^{-1}\left(\frac{y \times 2\sqrt{2}}{\sqrt{23}}\right) + C$$

From (2),

$$\Rightarrow \frac{2}{\sqrt{23}} \tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right) + C$$

Therefore,

$$\int \frac{dx}{(2x^2 + x + 3)} = \frac{2}{\sqrt{23}} \tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right) + C$$

Question: 20

Solution:

To find: $\int \frac{dx}{(2x^2 - x - 1)}$

Formula Used: $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)^2 - 1 - \left(\frac{1}{2\sqrt{2}}\right)^2\right)}$$

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)^2 - 1 - \frac{1}{8}\right)}$$

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)^2 - \frac{9}{8}\right)}$$

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{3}{2\sqrt{2}}\right)^2\right)} \ldots (1)$$

Let $y = \sqrt{2}x - \frac{1}{2\sqrt{2}}$ ... (2)

Differentiating both sides,

dy = √2 dx

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{\sqrt{2}}dy}{\left(y^2 - \left(\frac{3}{2\sqrt{2}}\right)^2\right)}$$

Here $a = \frac{3}{2\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{3} \times \log \left| \frac{\frac{3}{2\sqrt{2}} + y}{\frac{3}{2\sqrt{2}} - y} \right| + C$$

$$\Rightarrow \frac{1}{3} \times \log \left| \frac{3 + 2\sqrt{2}y}{3 - 2\sqrt{2}y} \right| + C$$

From (2),

$$\Rightarrow \frac{1}{3} \times \log \left| \frac{3 + 4x - 1}{3 - 4x + 1} \right| + C$$

$$\Rightarrow \frac{1}{3} \log \left| \frac{1 + 2x}{2(1 - x)} \right| + C$$

$$\Rightarrow \frac{1}{3} \log \left| \frac{2(x - 1)}{2x + 1} \right| + C$$

Therefore,

$$\int \frac{dx}{(2x^2 - x - 1)} = \frac{1}{3} \log \left| \frac{2(x - 1)}{2x + 1} \right| + C$$

Question: 21

Solution:

To find: $\int \frac{dx}{(3-2x-x^2)}$

Formula Used: $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{-dx}{(x^2 + 2x - 3)}$$

$$\Rightarrow \int \frac{-dx}{(x + 1)^2 - 4}$$

$\Rightarrow \int \frac{-dx}{(x+1)^2 - 2^2}$ ... (1)

Let y = x + 1 ... (2)

Differentiating both sides wrt x,

dy = dx

Substituting in (1),

$$\Rightarrow \int \frac{-dy}{y^2 - 2^2}$$

$$\Rightarrow \int \frac{dy}{2^2 - y^2}$$

Here a = 2,

$$\Rightarrow \frac{1}{4} \log \left| \frac{2 + y}{2 - y} \right| + C$$

From (2),

$\Rightarrow \frac{1}{4}\log\left|\frac{x+3}{1-x}\right| + C$

Therefore,

$$\int \frac{dx}{(3-2x-x^2)} = \frac{1}{4}\log\left|\frac{x+3}{1-x}\right| + C$$

Question: 22

Solution:

To find: $\int \frac{x\,dx}{(x^2+3x+2)}$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

Using partial fractions,

$$x = A\left(\frac{d}{dx}(x^2+3x+2)\right) + B$$

x = A (2x + 3) + B

Equating the coefficients of x,

1 = 2A

$$A = \frac{1}{2}$$

Also, 0 = 3A + B

$$B = \frac{-3}{2}$$

Therefore, the given equation becomes,

$$\Rightarrow \int \frac{\frac{1}{2}(2x+3) - \frac{3}{2}}{(x^2+3x+2)}\,dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\int \frac{1}{\left(\left(x+\frac{3}{2}\right)^2 + 2 - \left(\frac{3}{2}\right)^2\right)}dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\int \frac{1}{\left(\left(x+\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right)}dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\times\log\left|\frac{x+\frac{3}{2}-\frac{1}{2}}{x+\frac{3}{2}+\frac{1}{2}}\right| + C$$

$$\Rightarrow \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| + C$$

Therefore,

$$\int \frac{x\,dx}{(x^2+3x+2)} = \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| + C$$

Question: 23

Solution:

To find: $\int \frac{(x-3)\,dx}{(x^2+2x-4)}$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

Using partial fractions,

$$(x-3) = A\left(\frac{d}{dx}(x^2+2x-4)\right) + B$$

x – 3 = A (2x + 2) + B

Equating the coefficients of x,

1 = 2A

$$\Rightarrow A = \frac{1}{2}$$

Also, -3 = 2A + B

⇒ B = -4

Substituting in the given equation,

$$\Rightarrow \int \frac{\frac{1}{2}(2x+2)-4}{(x^2+2x-4)}dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+2x-4| - 4\int \frac{1}{(x+1)^2-(\sqrt{5})^2}dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+2x-4| - \left(4 \times \frac{1}{2\sqrt{5}} \times \log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|\right) + C$$

$$\Rightarrow \frac{1}{2}\log|x^2+2x-4| - \frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right| + C$$

Therefore,

$$\int \frac{(x-3)\,dx}{(x^2+2x-4)} = \frac{1}{2}\log|x^2+2x-4| - \frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right| + C$$

Question: 24

Solution:

To find: $\int \frac{(2x-3)}{(x^2+3x-18)}dx$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

Using partial fractions,

$$(2x-3) = A\left(\frac{d}{dx}(x^2+3x-18)\right) + B$$

2x – 3 = A(2x + 3) + B

Equating the coefficients of x,

2 = 2A

A = 1

Also, -3 = 3A + B

⇒ B = -6

Substituting in the given equation,

$$\Rightarrow \int \frac{(2x+3)-6}{(x^2+3x-18)}dx$$

$$\Rightarrow \log|x^2+3x-18| + C_1 - 6\int \frac{1}{\left(x+\frac{3}{2}\right)^2-18-\left(\frac{3}{2}\right)^2}dx \quad \dots (1)$$

Let I = $6\int \frac{1}{\left(x+\frac{3}{2}\right)^2-18-\left(\frac{3}{2}\right)^2}dx$

$$\Rightarrow 6\int \frac{1}{\left(x+\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)^2}dx$$

Here a = $\frac{9}{2}$

$$\Rightarrow \frac{6}{9}\times\log\left|\frac{x+\frac{3}{2}-\frac{9}{2}}{x+\frac{3}{2}+\frac{9}{2}}\right| + C_2$$

$$\Rightarrow \frac{2}{3}\times\log\left|\frac{x-3}{x+6}\right| + C_2 \quad \dots (2)$$

Substituting (2) in (1),

$$\Rightarrow \log|x^2+3x-18| - \frac{2}{3}\log\left|\frac{x-3}{x+6}\right| + C$$

Therefore,

$$\int \frac{(2x-3)}{(x^2+3x-18)}dx = \log|x^2+3x-18| - \frac{2}{3}\log\left|\frac{x-3}{x+6}\right| + C$$

Question: 25

Solution:

To find: $\int \frac{x^2}{(x^2+6x-3)}dx$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}dx = \log|f(x)| + C$

Given equation can be rewritten as following:

$$\Rightarrow \int \frac{x^2 + (6x-3) - (6x-3)}{(x^2 + 6x - 3)} dx$$

$$\Rightarrow \int \frac{(x^2 + 6x - 3) - (6x-3)}{(x^2 + 6x - 3)} dx$$

$$\Rightarrow x - \int \frac{6x-3}{x^2 + 6x - 3} dx$$

Let $I = \int \frac{6x-3}{x^2+6x-3} dx$ ... (2)

Using partial fractions,

$$(6x-3) = A\left(\frac{d}{dx}(x^2 + 6x - 3)\right) + B$$

6x – 3 = A (2x + 6) + B

Equating the coefficients of x,

6 = 2A

A = 3

Also, -3 = 6A + B

⇒ B = -21

Substituting in (1),

$$\Rightarrow \int \frac{3(2x+6) - 21}{(x^2 + 6x - 3)} dx$$

$$\Rightarrow 3 \times \log|x^2 + 6x - 3| + C_1 - 21 \int \frac{1}{(x+3)^2 - (\sqrt{12})^2} dx$$

$$\Rightarrow 3 \times \log|x^2 + 6x - 3| + C_1 - 21 \times \frac{1}{2\sqrt{12}} \times \log\left|\frac{x+3-\sqrt{12}}{x+3+\sqrt{12}}\right| + C_2$$

$$I = 3\log|x^2 + 6x - 3| - \frac{7\sqrt{3}}{4} \times \log\left|\frac{x+3-2\sqrt{3}}{x+3+2\sqrt{3}}\right| + C$$

Therefore,

$$\int \frac{x^2}{(x^2 + 6x - 3)} dx = x - 3\log|x^2 + 6x - 3| + \frac{7\sqrt{3}}{4} \times \log\left|\frac{x+3-2\sqrt{3}}{x+3+2\sqrt{3}}\right| + C$$

Question: 26

Solution:

To find: $\int \frac{2x-1}{(2x^2+2x+1)} dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$

Using partial fractions,

$$(2x-1) = A\left(\frac{d}{dx}(2x^2 + 2x + 1)\right) + B$$

2x – 1 = A (4x + 2) + B

Equating the coefficients of x,

2 = 4A

$A = \frac{1}{2}$

Also, -1 = 2A + B

⇒ B = -2

Substituting in the given equation,

$$\Rightarrow \int \frac{\frac{1}{2}(4x+2)-2}{(2x^2+2x+1)}dx$$

$$\Rightarrow \frac{1}{2}\log|2x^2+2x+1| - 2\int \frac{1}{2\left(x^2+x+\frac{1}{2}\right)}\,dx$$

Let I = $2\int \frac{1}{2\left(x^2+x+\frac{1}{2}\right)}\,dx$ ... (1)

$$\Rightarrow \int \frac{1}{\left(x^2+x+\frac{1}{2}\right)}dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{1}{2}\right)^2+\frac{1}{2}-\left(\frac{1}{2}\right)^2\right)}\,dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{1}{2}\right)^2+\frac{1}{2}-\frac{1}{4}\right)}\,dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2\right)}\,dx$$

Here $a = \frac{1}{2}$

$$\Rightarrow 2\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{1}{2}}\right)+C$$

⇒ $2\tan^{-1}(2x+1) + C$

Substituting in (1) and combining with original equation,

$$\Rightarrow \frac{1}{2}\log|2x^2+2x+1| - 2\tan^{-1}(2x+1)+C$$

Therefore,

$$\int \frac{2x-1}{(2x^2+2x+1)}dx = \frac{1}{2}\log|2x^2+2x+1| - 2\tan^{-1}(2x+1)+C$$

Question: 27

Solution:

To find: $\int \frac{1-3x}{(3x^2+4x+2)}dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\int \frac{f'(x)}{f(x)}\, dx = \log|f(x)| + C$

Rewriting the given equation,

$$\Rightarrow -\int \frac{3x-1}{(3x^2+4x+2)}\, dx$$

Using partial fractions,

$$(3x-1) = A\left(\frac{d}{dx}(3x^2+4x+2)\right) + B$$

3x – 1 = A (6x + 4) + B

Equating the coefficients of x,

3 = 6A

$A = \frac{1}{2}$

Also, -1 = 4A + B

⇒ B = -3

Substituting in the original equation,

$$\Rightarrow -\int \frac{\frac{1}{2}(6x+4)-3}{(3x^2+4x+2)}\, dx$$

$$\Rightarrow -\frac{1}{2}\log|3x^2+4x+2| + 3\int \frac{1}{3\left(x^2+\frac{4}{3}x+\frac{2}{3}\right)}\, dx$$

Let $I = 3\int \frac{1}{3\left(x^2+\frac{4}{3}x+\frac{2}{3}\right)}\, dx$

$$\Rightarrow \int \frac{1}{\left(x^2+\frac{4}{3}x+\frac{2}{3}\right)}\, dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{2}{3}\right)^2+\frac{2}{3}-\frac{4}{9}\right)}\, dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{2}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right)}\, dx$$

Here $a = \frac{\sqrt{2}}{3}$

$$\Rightarrow \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{x+\frac{2}{3}}{\frac{\sqrt{2}}{3}}\right) + C$$

$$\Rightarrow \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right) + C$$

Substituting in (1) and combining with original equation,

$$\Rightarrow -\frac{1}{2}\log|3x^2+4x+2| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right) + C$$

Therefore,

$$\int \frac{1-3x}{(3x^2+4x+2)}dx = -\frac{1}{2}\log|3x^2+4x+2| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right) + C$$

Question: 28

Solution:

To find: $\int \frac{2x}{(2+x-x^2)}dx$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

Rewriting the given equation,

$$\Rightarrow -2\int \frac{x}{(x^2-x-2)}\,dx$$

Using partial fractions,

$$x = A\left(\frac{d}{dx}(x^2-x-2)\right) + B$$

x = A (2x – 1) + B

Equating the coefficients of x,

1 = 2A

$A = \frac{1}{2}$

Also, 0 = -A + B

$B = \frac{1}{2}$

Substituting in the original equation,

$$\Rightarrow -2\int \frac{\frac{1}{2}(2x-1)+\frac{1}{2}}{(x^2-x-2)}\,dx$$

$$\Rightarrow -\log|x^2-x-2| - \int \frac{1}{(x^2-x-2)}dx$$

Let I = $\int \frac{1}{(x^2-x-2)}dx$

$$\Rightarrow \int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2-2-\frac{1}{4}\right)}dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2\right)}dx$$

Here $a = \frac{3}{2}$

$$\Rightarrow \frac{1}{3}\log\left|\frac{x-\frac{1}{2}-\frac{3}{2}}{x-\frac{1}{2}+\frac{3}{2}}\right| + C$$

$$\Rightarrow \frac{1}{3}\log\left|\frac{x-2}{x+1}\right| + C$$

Substituting for I and combining with the original equation,

$$\Rightarrow -\log|x^2 - x - 2| + \frac{1}{3}\log\left|\frac{x-2}{x+1}\right| + C$$

Therefore,

$$\int \frac{2x}{(2+x-x^2)}dx = -\log|x^2 - x - 2| + \frac{1}{3}\log\left|\frac{x-2}{x+1}\right| + C$$

or

$$\int \frac{2x}{(2+x-x^2)}dx = -\log|2 + x - x^2| + \frac{1}{3}\log\left|\frac{1+x}{2-x}\right| + C$$

Question: 29

Solution:

To find: $\int \frac{1}{(1+\cos^2 x)}dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\sec^2 x = 1 + \tan^2 x$

Dividing the given equation by $\cos^2 x$ in the numerator and denominator gives us,

$\Rightarrow \int \frac{\sec^2 x\, dx}{1+\sec^2 x}$ ... (1)

Let $y = \tan x$

$dy = \sec^2 x\, dx$ ... (2)

Also, $y^2 = \tan^2 x$

i.e., $y^2 = \sec^2 x - 1$

$\sec^2 x = y^2 + 1$ ... (3)

Substituting (2) and (3) in (1),

$$\Rightarrow \int \frac{dy}{1 + y^2 + 1}$$

$$\Rightarrow \int \frac{dy}{y^2 + (\sqrt{2})^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y}{\sqrt{2}}\right) + C$$

Since $y = \tan x$,

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + C$$

Therefore,

$$\int \frac{1}{(1+\cos^2 x)}dx = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + C$$

Question: 30

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Solution:

To find: $\int \frac{1}{(2+\sin^2 x)} dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\sec^2 x = 1 + \tan^2 x$

Dividing the given equation by $\cos^2 x$ in the numerator and denominator gives us,

$\Rightarrow \int \frac{\sec^2 x\, dx}{2\sec^2 x + \tan^2 x}$ ... (1)

Let y = tan x

dy = $\sec^2$ x dx ... (2)

Also, $y^2 = \tan^2 x$

i.e., $y^2 = \sec^2 x - 1$

$\sec^2 x = y^2 + 1$ ... (3)

Substituting (2) and (3) in (1),

$$\Rightarrow \int \frac{dy}{2y^2 + 2 + y^2}$$

$$\Rightarrow \int \frac{dy}{3y^2 + 2}$$

$$\Rightarrow \frac{1}{3}\int \frac{dy}{y^2 + \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{1}{3} \times \frac{\sqrt{3}}{\sqrt{2}}\tan^{-1}\left(\frac{y\sqrt{3}}{\sqrt{2}}\right) + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{2}}\right) + C$$

Therefore,

$$\int \frac{1}{(2 + \sin^2 x)} dx = \frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{2}}\right) + C$$

Question: 31

Solution:

To find: $\int \frac{dx}{(a^2\cos^2 x + b^2\sin^2 x)}$

Formula Used:

1. $\text{Sec}^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \;\, dx}{a^2 + b^2 \tan^2 x}$$

Let y = tan x

dy = sec² x dx

Therefore,

$$\Rightarrow \int \frac{dy}{a^2 + b^2 y^2}$$

$$\Rightarrow \frac{1}{b^2} \int \frac{dy}{\left(\frac{a}{b}\right)^2 + y^2}$$

$$\Rightarrow \frac{1}{b^2} \times \frac{b}{a} \tan^{-1} \frac{yb}{a} + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{ab} \tan^{-1}\left(\frac{b \tan x}{a}\right) + C$$

Therefore,

$$\int \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)} = \frac{1}{a^2} \tan^{-1}\left(\frac{b}{a} \tan x\right) + C$$

Question: 32

Solution:

To find: $\int \frac{dx}{(\cos^2 x - 3\sin^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right| + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \;\, dx}{1 - 3\tan^2 x}$$

Let y = tan x

dy = sec² x dx

Therefore,

$$\Rightarrow \int \frac{dy}{1 - 3y^2}$$

$$\Rightarrow \frac{1}{3} \int \frac{dy}{\left(\frac{1}{\sqrt{3}}\right)^2 - y^2}$$

$$\Rightarrow \frac{1}{3} \times \frac{\sqrt{3}}{2} \log \left|\frac{\frac{1}{\sqrt{3}} + y}{\frac{1}{\sqrt{3}} - y}\right| + C$$

$$\Rightarrow \frac{1}{2\sqrt{3}}\log\left|\frac{1+y\sqrt{3}}{1-y\sqrt{3}}\right| + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right| + C$$

Therefore,

$$\int \frac{dx}{(\cos^2 x\ -\ 3\sin^2 x)} = \frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right| + C$$

Question: 33

Solution:

To find: $\int \frac{dx}{(\sin^2 x - 4\cos^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x\ dx}{\tan^2 x - 4}$$

Let y = tan x

$dy = \sec^2 x\ dx$

Therefore,

$$\Rightarrow \int \frac{dy}{y^2 - 2^2}$$

$$\Rightarrow \frac{1}{4}\log\left|\frac{y-2}{y+2}\right| + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{4}\log\left|\frac{\tan x - 2}{\tan x + 2}\right| + C$$

Therefore,

$$\int \frac{dx}{(\sin^2 x\ -\ 4\cos^2 x)} = \frac{1}{4}\log\left|\frac{\tan x - 2}{\tan x + 2}\right| + C$$

Question: 34

Solution:

To find: $\int \frac{dx}{(\sin x\cos x + 2\cos^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{x}dx = \log x + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \ dx}{\tan x + 2}$$

Let y = tan x

$dy = \sec^2 x \, dx$

Therefore,

$$\Rightarrow \int \frac{dy}{y + 2}$$

⇒ log |y + 2| + C

Since y = tan x,

⇒ log |tan x + 2| + C

Therefore,

$$\int \frac{dx}{(\sin x \cos x + 2\cos^2 x)} = \log|\tan x + 2| + C$$

Question: 35

Solution:

To find: $\int \frac{\sin 2x \, dx}{(\sin^4 x + \cos^4 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

3. sin 2x = 2 sin x cos x

Rewriting the given equation,

$$\Rightarrow \int \frac{2\sin x \cos x}{\sin^4 x + \cos^4 x} \ dx$$

Dividing by $\cos^4 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{2\tan x \sec^2 x \ dx}{\tan^4 x + 1}$$

Let y = tan x

$dy = \sec^2 x \, dx$

Therefore,

$$\Rightarrow \int \frac{2y}{y^4 + 1} \ dy$$

Let $z = y^2$

dz = 2y dy

$$\Rightarrow \int \frac{dz}{1 + z^2}$$

$\Rightarrow \tan^{-1} z + C$

Since $z = y^2$,

$\Rightarrow \tan^{-1}(y^2) + C$

Since y = tan x,

$\Rightarrow \tan^{-1}(\tan^2 x) + C$

Therefore,

$$\int \frac{\sin 2x \ dx}{(\sin^4 x + \cos^4 x)} = \tan^{-1}(\tan^2 x) + C$$

Question: 36

Solution:

To find: $\int \frac{(2\sin 2\phi - \cos\phi)}{(6 - \cos^2\phi - 4\sin\phi)} \, d\phi$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

3. $\sin 2x = 2 \sin x \cos x$

Rewriting the given equation,

$$\Rightarrow \int \frac{4\sin\phi\cos\phi - \cos\phi}{6 - \cos^2\phi - 4\sin\phi} \, d\phi$$

$$\Rightarrow \int \frac{\cos\phi \ (4\sin\phi - 1)}{6 - (1 - \sin^2\phi) - 4\sin\phi} \, d\phi$$

$$\Rightarrow \int \frac{\cos\phi \ (4\sin\phi - 1)}{5 + \sin^2\phi - 4\sin\phi} \, d\phi$$

Let $y = \sin \phi$

$dy = \cos \phi \, d\phi$

Substituting in the original equation,

$\Rightarrow \int \frac{4y-1}{y^2-4y+5} \, dy$ ... (1)

Using partial fraction,

$$4y - 1 = A\left(\frac{d}{dy}(y^2 - 4y + 5)\right) + B$$

$4y - 1 = A(2y - 4) + B$

Equating the coefficients of y,

$4 = 2A$

$A = 2$

Also, $-1 = -4A + B$

$B = 7$

Substituting in (1),

$$\Rightarrow \int \frac{2(2y - 4) + 7}{y^2 - 4y + 5} \, dy$$

$\Rightarrow 2\log|y^2 - 4y + 5| + 7\int \frac{1}{((y-2)^2 + 1)} dy$

$\Rightarrow 2 \log |y^2 - 4y + 5| + 7 \tan^{-1}(y - 2) + C$

But $y = \sin \phi$

$\Rightarrow 2 \log |\sin^2\phi - 4 \sin \phi + 5| + 7 \tan^{-1}(\sin \phi - 2) + C$

Therefore,

$$\int \frac{(2\sin 2\phi - cos\phi)}{(6 - \cos^2 \phi - 4\sin \phi)} d\phi$$
$$= 2 \log |\sin^2 \phi - 4\sin\phi + 5| + 7\tan^{-1}(\sin\phi - 2) + C$$

Question: 37

Solution:

To find: $\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{x} dx = \log x + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \ dx}{(\tan x - 2)(2\tan x + 1)}$$

Let $y = \tan x$

$dy = \sec^2 x \, dx$

Therefore,

$\Rightarrow \int \frac{dy}{(y-2)(2y+1)}$ ... (1)

Let

$$\frac{1}{(y-2)(2y+1)} = \frac{A}{(y-2)} + \frac{B}{(2y+1)}$$

$1 = A(2y + 1) + B(y - 2)$

When $y = 0$,

$1 = A - 2B$ ... (2)

When $y = 1$,

$1 = 3A - B \Rightarrow 2 = 6A - 2B$ ... (3)

Solving (2) and (3),

$1 = 5A$

$A = \frac{1}{5}$

So, $B = \frac{-2}{5}$

(1) becomes,

$$\Rightarrow \int \frac{\frac{1}{5}}{(y-2)} + \frac{\frac{-2}{5}}{(2y+1)}$$

$$\Rightarrow \frac{1}{5}\log|y-2| - \frac{2}{5}\log|2y+1| \times \frac{1}{2} + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{5}\log|\tan x - 2| - \frac{1}{5}\log|2\tan x + 1| + C$$

$$\Rightarrow \frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + C$$

Therefore,

$$\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)} = \frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + C$$

Question: 38

Solution:

To find: $\int \frac{(1-x^2)}{(1+x^4)}\,dx$

Formula used: $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

On dividing by x^2 in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + x^2} dx$$

$$\Rightarrow \int \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + x^2 + 2 - 2} dx$$

$$\Rightarrow \int \frac{-\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 - 2} dx$$

Let $y = x + \frac{1}{x}$

Differentiating wrt x,

$$dy = \left(1 - \frac{1}{x^2}\right)dx$$

Substituting in the original equation,

$$\Rightarrow \int \frac{-dy}{y^2 - (\sqrt{2})^2}$$

$$\Rightarrow \frac{-1}{2\sqrt{2}}\log\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right| + C$$

Substituting for $y = x + \frac{1}{x}$ and taking reciprocal of the value within logarithm, we get

$$\Rightarrow \frac{1}{2\sqrt{2}}\log\left|\frac{x+\frac{1}{x}+\sqrt{2}}{x+\frac{1}{x}-\sqrt{2}}\right|+C$$

$$\Rightarrow \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}x+x^2+1}{\sqrt{2}x-x^2+1}\right|+C$$

Therefore,

$$\int\frac{(1-x^2)}{(1+x^4)}\,dx = \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}x+x^2+1}{\sqrt{2}x-x^2+1}\right|+C$$

Question: 39

Solution:

To find: $\int\frac{(x^2+1)}{(x^4+x^2+1)}\,dx$

Formula used: $\int\frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$

On dividing by x² in the numerator and denominator of the given equation,

$$\Rightarrow \int\frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$

$$\Rightarrow \int\frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3}dx$$

Let $y = x-\frac{1}{x}$

Differentiating wrt x,

$$dy = \left(1+\frac{1}{x^2}\right)dx$$

Substituting in the original equation,

$$\Rightarrow \int\frac{dy}{y^2+\left(\sqrt{3}\right)^2}$$

$$\Rightarrow \frac{1}{\sqrt{3}}\tan^{-1}\frac{y}{\sqrt{3}}+C$$

Substituting for $y = x-\frac{1}{x}$

$$\Rightarrow \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right)+C$$

$$\Rightarrow \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+C$$

Therefore,

$$\int\frac{(x^2+1)}{(x^4+x^2+1)}\,dx = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+C$$

Question: 40

Solution:

To find: $\int \frac{dx}{(\sin^4 x+\cos^4 x)}$

Formula used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

Dividing by $\cos^4 x$ in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{\sec^4 x}{(\tan^4 x + 1)} dx$$

$$\Rightarrow \int \frac{\sec^2 x\,(1 + \tan^2 x)}{(1 + \tan^4 x)} dx$$

Let y = tan x

dy = $\sec^2$ x dx

Substituting in the original equation,

$$\Rightarrow \int \frac{1 + y^2}{1 + y^4} dy$$

Dividing by y^2 in the numerator and denominator,

$$\Rightarrow \int \frac{y^{-2} + 1}{y^{-2} + y^2} dy$$

$$\Rightarrow \int \frac{1 + y^{-2}}{y^2 + y^{-2} - 2 + 2} dy$$

$$\Rightarrow \int \frac{1 + y^{-2}}{\left(y - \frac{1}{y}\right)^2 + 2} dy$$

Let $z = y - \frac{1}{y}$

$$dz = \left(1 + \frac{1}{y^2}\right) dy$$

Therefore,

$$\Rightarrow \int \frac{dz}{z^2 + \left(\sqrt{2}\right)^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{z}{\sqrt{2}}\right) + C$$

Substituting for z,

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y - \frac{1}{y}}{\sqrt{2}}\right) + C$$

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y^2 - 1}{y\sqrt{2}}\right) + C$$

Substituting for y = tan x,

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2}\tan x}\right) + C$$

Therefore,

$$\int \frac{dx}{(\sin^4 x + \cos^4 x)} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2}\tan x}\right) + C$$

Exercise : 14B

Question: 1

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{16 - x^2}}$$

$$= \int \frac{dx}{\sqrt{4^2 - x^2}}$$

$= \sin^{-1}\frac{x}{4} + c$, c being the integrating constant

Question: 2

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 - 9x^2}}$$

$$= \int \frac{dx}{\sqrt{9\left\{\left(\frac{1}{9}\right) - x^2\right\}}}$$

$$= \frac{1}{3}\int \frac{dx}{\sqrt{1^2 - \left(\frac{x}{3}\right)^2}}$$

$$= \frac{1}{3}\sin^{-1}\frac{x}{\frac{1}{3}} + c$$

$= \frac{1}{3}\sin^{-1} 3x + c$, c being the integrating constant

Question: 3

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{15 - 8x^2}}$$

$$= \int \frac{dx}{\sqrt{15\left\{1-\left(\frac{\sqrt{8}}{\sqrt{15}}x\right)^2\right\}}}$$

$$= \frac{1}{\sqrt{15}}\int \frac{dx}{\sqrt{1^2-\left(\frac{\sqrt{8}}{\sqrt{15}}x\right)^2}}$$

$$= \frac{1}{\sqrt{15}}\sin^{-1}\frac{x}{\left(\frac{\sqrt{15}}{\sqrt{8}}\right)} + c$$

$= \frac{1}{\sqrt{15}}\sin^{-1}\frac{\sqrt{8}}{\sqrt{15}}x + c$, c being the integrating constant

Question: 4

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2-4}}$$

$$= \int \frac{dx}{\sqrt{x^2-2^2}}$$

$= \log\left|x + \sqrt{x^2-4}\right| + c$, c being the integrating constant

Question: 5

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{4x^2-1}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2-1^2}}$$

$= \frac{1}{2}\log\left|2x + \sqrt{4x^2-1}\right| + c$, c being the integrating constant

Question: 6

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{9x^2-7}}$$

$$= \int \frac{dx}{\sqrt{(3x)^2-\sqrt{7}^2}}$$

$= \log|3x + \sqrt{9x^2 - 7}| + c$, c being the integrating constant

Question: 7

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{x^2 - 9}}$

$= \int \frac{dx}{\sqrt{x^2 - 3^2}}$

$= \log|x + \sqrt{x^2 - 9}| + c$, c being the integrating constant

Question: 8

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{1 + 4x^2}}$

$= \int \frac{dx}{\sqrt{(2x)^2 + 1^2}}$

$= \frac{1}{2}\log|2x + \sqrt{4x^2 + 1}| + c$, c being the integrating constant

Question: 9

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{9 + 4x^2}}$

$= \int \frac{dx}{\sqrt{(2x)^2 + 3^2}}$

$= \frac{1}{2}\log|2x + \sqrt{4x^2 + 9}| + c$, c being the integrating constant

Question: 10

Solution:

Tip – $d(x^2) = 2xdx$ i.e. $xdx = (1/2)\times d(x^2)$

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$\therefore \int \frac{xdx}{\sqrt{9 - x^4}}$

$= \frac{1}{2}\int \frac{d(x^2)}{\sqrt{3^2 - (x^2)^2}}$

$= \frac{1}{2}\sin^{-1}\frac{x^2}{3} + c$, c being the integrating constant

Question: 11

Solution:

Tip – $d(x^3) = 3x^2dx$ so, $d(4x^3) = 4\times3x^2dx$ i.e $3x^2dx = (1/4)d(2x^3)$

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$\therefore \int \frac{3x^2dx}{\sqrt{9 - 16x^6}}$

$= \frac{1}{4}\int \frac{d(2x^3)}{\sqrt{3^2 - (4x^3)^2}}$

$= \frac{1}{4}\sin^{-1}\frac{4x^3}{3} + c$, c being the integrating constant

Question: 12

Solution:

Tip – $d(\tan x) = \sec^2 x dx$

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$\therefore \int \frac{\sec^2 x dx}{\sqrt{16 + \tan^2 x}}$

$= \int \frac{d(\tan x)}{\sqrt{4^2 + (\tan x)^2}}$

$= \log|\tan x + \sqrt{16 + \tan^2 x}| + c$, c being the integrating constant

Question: 13

Solution:

Tip – $d(\cos x) = -\sin x dx$ i.e. $\sin x dx = -d(\cos x)$

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$\therefore \int \frac{\sin x dx}{\sqrt{4 + \cos^2 x}}$

$= \int \frac{-d(\cos x)}{\sqrt{(\cos x)^2 + 2^2}}$

$= -\log|\cos x + \sqrt{4 + \cos^2 x}| + c$, c being the integrating constant

Question: 14

Solution:

Tip – d(sinx) = cosxdx so, d(3sinx) = 3cosxdx i.e. cosxdx = (1/3)d(3sinx)

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{\cos x dx}{\sqrt{9\sin^2 x - 1}}$$

$$= \frac{1}{3}\int \frac{d(3\sin x)}{\sqrt{(3\sin x)^2 - 1^2}}$$

$= \frac{1}{3}\log|\cos x + \sqrt{4 + \cos^2 x}| + c$, c being the integrating constant

Question: 15

Solution:

Tip – $d(e^x) = e^x dx$

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{e^x dx}{\sqrt{4 + e^{2x}}}$$

$$= \int \frac{d(e^x)}{\sqrt{2^2 + (e^x)^2}}$$

$= \log|e^x + \sqrt{4 + e^{2x}}| + c$, c being the integrating constant

Question: 16

Solution:

Tip – $d(e^x) = e^x dx$

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{2e^x dx}{\sqrt{4 - e^{2x}}}$$

$$= 2\int \frac{d(e^x)}{\sqrt{2^2 - (e^x)^2}}$$

$= 2\sin^{-1}\left(\frac{e^x}{2}\right) + c$, c being the integrating constant

Question: 17

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 - e^x}}$$

$$= \int \frac{dx}{\sqrt{e^x(e^{-x} - 1)}}$$

$$= \int \frac{e^{-\frac{x}{2}}dx}{\sqrt{e^{-x} - 1}}$$

$$= \int \frac{e^{-\frac{x}{2}}dx}{\sqrt{\left(e^{-\frac{x}{2}}\right)^2 - 1^2}}$$

Tip – Assuming $e^{-(x/2)} = a$, $-(1/2)\, e^{-(x/2)}dx = da$ i.e. $e^{-(x/2)}dx = -2da$

$$\therefore \int \frac{e^{-\frac{x}{2}}dx}{\sqrt{\left(e^{-\frac{x}{2}}\right)^2 - 1^2}}$$

$$= \int \frac{-2da}{\sqrt{a^2 - 1^2}}$$

$$= -2\log|a + \sqrt{a^2 - 1}| + c$$

$= -2\log|e^{-\frac{x}{2}} + \sqrt{e^{-x} - 1}| + c$, c being the integrating constant

Question: 18

Solution:

Tip – Taking $x = a\cos 2\theta$,

$dx = -2a\sin 2\theta\, d\theta$ and $\theta = \frac{1}{2}\cos^{-1}\frac{x}{a}$

$x = a\cos 2\theta$ i.e $\cos 2\theta = \frac{x}{a}$

$$\therefore \sin 2\theta = \sqrt{1 - \frac{x^2}{a^2}}$$

$$\therefore \int \sqrt{\frac{a - x}{a + x}}dx$$

$$= \int \sqrt{\frac{a - a\cos 2\theta}{a + a\cos 2\theta}} \times (-2a\sin 2\theta\, d\theta)$$

$$= \int \sqrt{\frac{a(1 - \cos 2\theta)}{a(1 + \cos 2\theta)}} \times (-2a\sin 2\theta\, d\theta)$$

Formula to be used - $\cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\therefore \int \sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}} \times (-2a\sin 2\theta\, d\theta)$$

$$= \int \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} \times (-2a\sin 2\theta\, d\theta)$$

$$= \int \frac{\sin\theta}{\cos\theta} \times (-2a \times 2\sin\theta\cos\theta\, d\theta)$$

$= -2a\int 2\sin^2\theta d\theta$

$= -2a\int 1 - \cos 2\theta\, d\theta$

$= -2a\left[\theta - \frac{\sin 2\theta}{2}\right]$

$= -2a\left[\theta - \frac{\sin 2\theta}{2}\right] + c$

$= -2a\left[\frac{1}{2}\cos^{-1}\frac{x}{a} - \frac{\sqrt{1-\frac{x^2}{a^2}}}{2}\right] + c$

$= -a\cos^{-1}\frac{x}{a} + a\sqrt{1-\frac{x^2}{a^2}} + c$

$= a\sin^{-1}\frac{x}{a} + \sqrt{a^2 - x^2} + c$, c being the integrating constant

Question: 19

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{x^2 + 6x + 5}}$

$= \int \frac{dx}{\sqrt{(x^2 + 2 \times x \times 3 + 3^2) + 5 - 3^2}}$

$= \int \frac{dx}{\sqrt{(x + 3)^2 - 2^2}}$

$= \log|(x + 3) + \sqrt{x^2 + 6x + 5}| + c$, c being the integrating constant

Question: 20

Solution:

Tip – d(2 - x) = - dx i.e. dx = - d(2 - x)

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{(2-x)^2 + 1}}$

$= \int \frac{-d(2-x)}{\sqrt{(2-x)^2 + 1}}$

$= -\log|(2-x) + \sqrt{(2-x)^2 + 1}| + c$

$= -\log|(2-x) + \sqrt{x^2 - 4x + 5}| + c$, c being the integrating constant

Question: 21

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Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{(x-3)^2 + 1}}$

$= \log|(x-3) + \sqrt{(x-3)^2 + 1}| + c$

$= \log|(x-3) + \sqrt{x^2 - 6x + 10}| + c$, c being the integrating constant

Question: 22

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{x^2 - 6x + 10}}$

$= \int \frac{dx}{\sqrt{(x-3)^2 + 1}}$

$= \log|(x-3) + \sqrt{(x-3)^2 + 1}| + c$

$= \log|(x-3) + \sqrt{x^2 - 6x + 10}| + c$, c being the integrating constant

Question: 23

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{2 + 2x - x^2}}$

$= \int \frac{dx}{\sqrt{3 - (x^2 - 2x + 1)}}$

$= \int \frac{dx}{\sqrt{\left(\sqrt{3}\right)^2 - (x-1)^2}}$

$= \sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + c$, c being the integrating constant

Question: 24

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{8 - 4x - 2x^2}}$

$= \int \frac{dx}{\sqrt{10 - 2(x^2 + 2x + 1)}}$

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$= \int \frac{dx}{\sqrt{\left(\sqrt{10}\right)^2 - 2(x+1)^2}}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\sqrt{5}\right)^2 - (x+1)^2}}$

$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x+1}{\sqrt{5}}\right) + c$, c being the integrating constant

Question: 25

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{16 - 6x - x^2}}$

$= \int \frac{dx}{\sqrt{25 - (x^2 + 6x + 9)}}$

$= \int \frac{dx}{\sqrt{(5)^2 - (x+3)^2}}$

$= \sin^{-1}\left(\frac{x+3}{5}\right) + c$, c being the integrating constant

Question: 26

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{7 - 6x - x^2}}$

$= \int \frac{dx}{\sqrt{16 - (x^2 + 6x + 9)}}$

$= \int \frac{dx}{\sqrt{(4)^2 - (x+3)^2}}$

$= \sin^{-1}\left(\frac{x+3}{4}\right) + c$, c being the integrating constant

Question: 27

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$\therefore \int \frac{dx}{\sqrt{x - x^2}}$

$= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x^2 - 2 \times x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2\right)}}$

$$= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - (x - \frac{1}{2})^2}}$$

$$= \sin^{-1}\left(\frac{x - \frac{1}{2}}{\frac{1}{2}}\right) + c$$

$= \sin^{-1}(2x - 1) + c$, c being the integrating constant

Question: 28

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{8 + 2x - x^2}}$$

$$= \int \frac{dx}{\sqrt{9 - (x^2 - 2x + 1)}}$$

$$= \int \frac{dx}{\sqrt{(3)^2 - (x - 1)^2}}$$

$= \sin^{-1}\left(\frac{x-1}{3}\right) + c$, c being the integrating constant

Question: 29

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 3x + 2}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 2 \times x \times \frac{3}{2} + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2}}$$

$$= \int \frac{dx}{\sqrt{(x - \frac{3}{2})^2 - \frac{1}{4}}}$$

$= \log|(x - \frac{3}{2}) + \sqrt{x^2 - 3x + 2}| + c$, c being the integrating constant

Question: 30

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{2x^2 + 3x - 2}}$$

$$= \int \frac{dx}{\sqrt{2\left(x^2 + 2 \times x \times \frac{3}{4} + \left(\frac{3}{4}\right)^2\right) - \frac{7}{8}}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x + \frac{3}{4})^2 - \left(\frac{\sqrt{7}}{4}\right)^2}}$$

$= \frac{1}{\sqrt{2}} \log|(x + \frac{3}{4}) + \sqrt{2x^2 + 3x - 2}| + c$, c being the integrating constant

Question: 31

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{2x^2 + 4x + 6}}$$

$$= \int \frac{dx}{\sqrt{2(x^2 + 2x + 1) + 4}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x + 1)^2 + (\sqrt{2})^2}}$$

$= \frac{1}{\sqrt{2}} \log|(x + 1) + \sqrt{2x^2 + 4x + 6}| + c$, c being the integrating constant

Question: 32

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 + 2x - 3x^2}}$$

$$= \int \frac{dx}{\sqrt{\left(1 - \frac{1}{3}\right) - 3\left(x^2 - 2 \times x \times \frac{1}{3} + \left(\frac{1}{3}\right)^2\right)}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 - 3\left(x - \frac{1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{2}}{3}\right)^2 - \left(x - \frac{1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{x - \frac{1}{3}}{\frac{\sqrt{2}}{3}}\right) + c$$

$= \frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{3x-1}{\sqrt{2}}\right) + c$, c being the integrating constant

Question: 33

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{5x - x^2}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{5}{2}\right)^2 - \left(x^2 - 2 \times x \times \frac{5}{2} + \left(\frac{5}{2}\right)^2\right)}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{5}{2}\right)^2 - \left(x - \frac{5}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{x - \frac{5}{2}}{\frac{5}{2}}\right) + c$$

$= \sin^{-1}\left(\frac{2x-5}{5}\right) + c$, c being the integrating constant

Question: 34

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{3 + 4x - 2x^2}}$$

$$= \int \frac{dx}{\sqrt{5 - 2(x^2 - 2x + 1)}}$$

$$= \int \frac{dx}{\sqrt{\left(\sqrt{5}\right)^2 - 2(x - 1)^2}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(\sqrt{\frac{5}{2}}\right)^2 - (x - 1)^2}}$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{x - 1}{\sqrt{\frac{5}{2}}}\right) + c$$

$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{\sqrt{2}(x-5)}{\sqrt{5}}\right) + c$, c being the integrating constant

Question: 35

Solution:

Tip – $d(x^3) = 3x^2dx$ i.e. $x^2dx = (1/3)d(x^3)$

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Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{x^2dx}{\sqrt{x^6 + 2x^3 + 3}}$$

$$= \int \frac{\frac{1}{3}d(x^3)}{\sqrt{(x^3)^2 + 2x^3 + 3}}$$

$$= \frac{1}{3}\int \frac{d(x^3)}{\sqrt{(x^3 + 1)^2 + (\sqrt{2})^2}}$$

$= \frac{1}{3}\log|(x^3 + 1) + \sqrt{x^6 + 2x^3 + 3}| + c$, c being the integrating constant

Question: 36

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{(2x + 3)}{\sqrt{x^2 + x + 1}}dx$$

$$= \int \frac{(2x + 1) + 2}{\sqrt{x^2 + x + 1}}dx$$

$$= \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}}dx + \int \frac{2}{\sqrt{x^2 + x + 1}}dx$$

Tip – Assuming $x^2 + x + 1 = a^2$, $(2x + 1)dx = 2ada$

$$\therefore \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}}dx$$

$$= \int \frac{2ada}{a}$$

$$= \int 2da$$

$$= 2a + c_1$$

$$= 2\sqrt{x^2 + x + 1} + c_1$$

$$\therefore \int \frac{2}{\sqrt{x^2 + x + 1}}dx$$

$$= 2\int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}}$$

$$= 2\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + 1}\right| + c_2$$

$$\therefore \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}}dx + \int \frac{2}{\sqrt{x^2 + x + 1}}dx$$

$= 2\sqrt{x^2 + x + 1} + 2\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + 1}\right| + c$, c is the integrating consta

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Question: 37

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{(5x + 3)}{\sqrt{x^2 + 4x + 10}} dx$$

$$= \int \frac{\frac{5}{2} \times (2x + 4) - 7}{\sqrt{x^2 + 4x + 10}} dx$$

$$= \frac{5}{2} \int \frac{(2x + 4)}{\sqrt{x^2 + 4x + 10}} dx - \int \frac{7}{\sqrt{x^2 + 4x + 10}} dx$$

Tip – Assuming $x^2 + 4x + 10 = a^2$, $(2x + 4)dx = 2ada$

$$\therefore \frac{5}{2} \int \frac{(2x + 4)}{\sqrt{x^2 + 4x + 10}} dx$$

$$= \frac{5}{2} \int \frac{2ada}{a}$$

$$= \frac{5}{2} \int 2da$$

$$= 5a + c_1$$

$$= 5\sqrt{x^2 + 4x + 10} + c_1$$

$$\therefore \int \frac{7}{\sqrt{x^2 + 4x + 10}} dx$$

$$= 7 \int \frac{dx}{\sqrt{(x + 2)^2 + (\sqrt{6})^2}}$$

$$= 7\log\left|(x + 2) + \sqrt{x^2 + 4x + 10}\right| + c_2$$

$$\therefore \frac{5}{2} \int \frac{(2x + 4)}{\sqrt{x^2 + 4x + 10}} dx - \int \frac{7}{\sqrt{x^2 + 4x + 10}} dx$$

$= 5\sqrt{x^2 + 4x + 10} - 7\log\left|(x + 2) + \sqrt{x^2 + 4x + 10}\right| + c$, c is the integrating constant

Question: 38

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{(4x + 3)}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \int \frac{(4x + 2) + 1}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx + \int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

Tip – Assuming $2x^2 + 2x - 3 = a^2$, $(4x + 2)dx = 2ada$

$$\therefore \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \int \frac{2ada}{a}$$

$$= \int 2da$$

$$= 2a + c_1$$

$$= 2\sqrt{2x^2 + 2x - 3} + c_1$$

$$\therefore \int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \int \frac{dx}{\sqrt{2\left(x + \frac{1}{2}\right)^2 - \left(\sqrt{\frac{7}{2}}\right)^2}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{7}}{2}\right)^2}}$$

$$= \frac{1}{\sqrt{2}} \log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x - \frac{3}{2}}\right| + c_2$$

$$\therefore \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx + \int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

$= 2\sqrt{2x^2 + 2x - 3} + \frac{1}{\sqrt{2}} \log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x - \frac{3}{2}}\right| + c$, c is the integrating constant

Question: 39

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{(3 - 2x)}{\sqrt{2 + x - x^2}} dx$$

$$= \int \frac{(1 - 2x) + 2}{\sqrt{2 + x - x^2}} dx$$

$$= \int \frac{(1 - 2x)}{\sqrt{2 + x - x^2}} dx + \int \frac{2}{\sqrt{2 + x - x^2}} dx$$

Tip – Assuming $2 + x - x^2 = a^2$, $(1 - 2x)dx = 2ada$

$$\therefore \int \frac{(1 - 2x)}{\sqrt{2 + x - x^2}} dx$$

$= \int \frac{2ada}{a}$

$= 2a + c_1$

$= 2\sqrt{2 + x - x^2} + c_1$

$\therefore \int \frac{2}{\sqrt{2 + x - x^2}} dx$

$= 2\int \frac{dx}{\sqrt{\left(\frac{3}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}}$

$= 2\sin^{-1}\frac{\left(x - \frac{1}{2}\right)}{\left(\frac{3}{2}\right)} + c_2$

$= 2\sin^{-1}\left(\frac{2x - 1}{3}\right) + c_2$

$\therefore \int \frac{(1 - 2x)}{\sqrt{2 + x - x^2}} dx + \int \frac{2}{\sqrt{2 + x - x^2}} dx$

$= 2\sqrt{2 + x - x^2} + 2\sin^{-1}\left(\frac{2x-1}{3}\right) + c$, c is the integrating constant

Question: 40

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$\therefore \int \frac{(x + 2)}{\sqrt{2x^2 + 2x - 3}} dx$

$= \int \frac{\frac{1}{4} \times (4x + 2) + \frac{3}{2}}{\sqrt{2x^2 + 2x - 3}} dx$

$= \frac{1}{4}\int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx + \frac{3}{2}\int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$

Tip – Assuming $2x^2 + 2x - 3 = a^2$, $(4x + 2)dx = 2ada$

$\therefore \frac{1}{4}\int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx$

$= \frac{1}{4}\int \frac{2ada}{a}$

$= \frac{1}{2}\int da$

$= \frac{a}{2} + c_1$

$= \frac{\sqrt{2x^2 + 2x - 3}}{2} + c_1$

$\therefore \frac{3}{2}\int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$

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$$= \frac{3}{2}\int \frac{dx}{\sqrt{2\left(x+\frac{1}{2}\right)^2-\left(\sqrt{\frac{7}{2}}\right)^2}}$$

$$= \frac{3}{2\sqrt{2}}\int \frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{7}}{2}\right)^2}}$$

$$= \frac{3}{2\sqrt{2}}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x-\frac{3}{2}}\right| + c_2$$

$$\therefore \frac{1}{4}\int \frac{(4x+2)}{\sqrt{2x^2+2x-3}}dx + \frac{3}{2}\int \frac{1}{\sqrt{2x^2+2x-3}}dx$$

$= \frac{\sqrt{2x^2+2x-3}}{2} + \frac{3}{2\sqrt{2}}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x-\frac{3}{2}}\right| + c$, c is the integrating constant

Question: 41

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{(3x+1)}{\sqrt{5-2x-x^2}}dx$$

$$= \int \frac{3(x+1)-2}{\sqrt{5-2x-x^2}}dx$$

$$= \int \frac{3(x+1)}{\sqrt{5-2x-x^2}}dx - \int \frac{2}{\sqrt{5-2x-x^2}}dx$$

Tip – Assuming $5 - 2x - x^2 = a^2$, $(-2-2x)dx = 2ada$ i.e. $(x+1)dx = -ada$

$$\therefore \int \frac{3(x+1)}{\sqrt{5-2x-x^2}}dx$$

$$= -3\int \frac{ada}{a}$$

$$= -3a + c_1$$

$$= -3\sqrt{5-2x-x^2} + c_1$$

$$\therefore \int \frac{2}{\sqrt{5-2x-x^2}}dx$$

$$= 2\int \frac{dx}{\sqrt{\left(\sqrt{6}\right)^2-(x+1)^2}}$$

$$= 2\sin^{-1}\frac{(x+1)}{\sqrt{6}} + c_2$$

$$\therefore \int \frac{3(x+1)}{\sqrt{5-2x-x^2}}dx - \int \frac{2}{\sqrt{5-2x-x^2}}dx$$

$= -3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c$, c is the integrating constant

Question: 42

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Solution:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{(6x + 5)}{\sqrt{6 + x - 2x^2}} dx$$

$$= \int \frac{\frac{6}{4}(4x - 1) + \frac{13}{2}}{\sqrt{6 + x - 2x^2}} dx$$

$$= \frac{3}{2}\int \frac{(4x - 1)}{\sqrt{6 + x - 2x^2}} dx + \frac{13}{2}\int \frac{1}{\sqrt{6 + x - 2x^2}} dx$$

Tip – Assuming $6 + x - 2x^2 = a^2$, $(1 - 4x)dx = 2ada$ i.e. $(4x - 1)dx = - 2ada$

$$\therefore \frac{3}{2}\int \frac{(4x - 1)}{\sqrt{6 + x - 2x^2}} dx$$

$$= -\frac{3}{2}\int \frac{2ada}{a}$$

$$= -3a + c_1$$

$$= -3\sqrt{6 + x - 2x^2} + c_1$$

$$\therefore \frac{13}{2}\int \frac{1}{\sqrt{6 + x - 2x^2}} dx$$

$$= \frac{13}{2}\int \frac{dx}{\sqrt{\left(\frac{7}{2\sqrt{2}}\right)^2 - 2\left(x - \frac{1}{4}\right)^2}}$$

$$= \frac{13}{2\sqrt{2}}\int \frac{dx}{\sqrt{\left(\frac{7}{4}\right)^2 - \left(x - \frac{1}{4}\right)^2}}$$

$$= \frac{13}{2\sqrt{2}}\sin^{-1}\frac{\left(x - \frac{1}{4}\right)}{\left(\frac{7}{4}\right)} + c_2$$

$$= \frac{13}{2\sqrt{2}}\sin^{-1}\left(\frac{4x - 1}{7}\right) + c_2$$

$$\therefore \frac{3}{2}\int \frac{(4x - 1)}{\sqrt{6 + x - 2x^2}} dx + \frac{13}{2}\int \frac{1}{\sqrt{6 + x - 2x^2}} dx$$

$= -3\sqrt{6 + x - 2x^2} + \frac{13}{2\sqrt{2}}\sin^{-1}\left(\frac{4x-1}{7}\right) + c$, c is the integrating constant

Question: 43

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\int \sqrt{\frac{1 + x}{x}}dx$$

$$= \int \sqrt{\frac{(1 + x)^2}{x(1 + x)}}dx$$

$$= \int \frac{1 + x}{\sqrt{x^2 + x}}dx$$

$$= \int \frac{\frac{1}{2}(2x + 1) + \frac{1}{2}}{\sqrt{x^2 + x}}dx$$

$$= \frac{1}{2}\int \frac{2x + 1}{\sqrt{x^2 + x}}dx + \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + x}}$$

Tip – Taking $x^2 + x = a^2$, $(2x + 1)dx = 2ada$

$$\therefore \frac{1}{2}\int \frac{2x + 1}{\sqrt{x^2 + x}}dx$$

$$= \frac{1}{2}\int \frac{2ada}{a}$$

$$= a + c_1$$

$$= \sqrt{x^2 + x} + c_1$$

$$\therefore \frac{1}{2}\int \frac{1}{\sqrt{x^2 + x}}dx$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$= \frac{1}{2}\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x}\right| + c_2$$

$$\therefore \frac{1}{2}\int \frac{2x + 1}{\sqrt{x^2 + x}}dx + \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + x}}$$

$= \sqrt{x^2 + x} + \frac{1}{2}\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x}\right| + c$, c is the integrating constant

Question: 44

Solution:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\int \frac{(x + 2)}{\sqrt{x^2 + 5x + 6}}dx$$

$$= \int \frac{\frac{1}{2}(2x + 5) - \frac{1}{2}}{\sqrt{x^2 + 5x + 6}}dx$$

$$= \frac{1}{2}\int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}}dx - \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + 5x + 6}}$$

Tip – Taking $x^2 + 5x + 6 = a^2$, $(2x + 5)dx = 2ada$

$\therefore \frac{1}{2}\int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}}dx$

$= \frac{1}{2}\int \frac{2ada}{a}$

$= a + c_1$

$= \sqrt{x^2 + 5x + 6} + c_1$

$\therefore -\frac{1}{2}\int \frac{1}{\sqrt{x^2 + 5x + 6}}dx$

$= -\frac{1}{2}\int \frac{dx}{\sqrt{\left(x + \frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$

$= -\frac{1}{2}\log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6}\right| + c_2$

$\therefore \frac{1}{2}\int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}}dx - \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + 5x + 6}}$

$= \sqrt{x^2 + 5x + 6} - \frac{1}{2}\log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6}\right| + c$, c is the integrating constant

Exercise : 14C

Question: 1

Solution:

To Find : $\int \sqrt{4 - x^2}\,dx$

Now, $\int \sqrt{4 - x^2}\,dx$ can be written as $\int \sqrt{2^2 - x^2}\,dx$

Formula Used: $\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since $\int \sqrt{2^2 - x^2}\,dx$ is of the form $\int \sqrt{a^2 - x^2}\,dx$,

Hence, $\int \sqrt{2^2 - x^2}\,dx = \frac{1}{2}x\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\frac{x}{2} + C$

$= \frac{1}{2}x\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} + C$

$= \frac{1}{2}x\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2} + C$

Therefore, $\int \sqrt{4 - x^2}\,dx = \frac{1}{2}x\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2} + C$

Question: 2

Solution:

To Find : $\int \sqrt{4 - 9x^2}\,dx$

Now, $\int \sqrt{4 - 9x^2}\,dx$ can be written as $\int \sqrt{2^2 - (3x)^2}\,dx$

Formula Used: $\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since $\int \sqrt{2^2 - (3x)^2}\,dx$ is of the form $\int \sqrt{a^2 - x^2}\,dx$, Hence, $\int \sqrt{2^2-(3x)^2}\,dx = \frac{1}{2}(3x)\sqrt{2^2-(3x)^2} + \frac{2^2}{2}\sin^{-1}\frac{3x}{2} + C$

$= \frac{x}{2}\sqrt{4-9x^2} + \frac{4}{6}\sin^{-1}\frac{3x}{2} + C$

$= \frac{x}{2}\sqrt{4-9x^2} + \frac{2}{3}\sin^{-1}\frac{3x}{2} + C$

Therefore, $\int \sqrt{4-9x^2}\,dx = \frac{x}{2}\sqrt{4-9x^2} + \frac{2}{3}\sin^{-1}\frac{3x}{2} + C$

Question: 3

Solution:

To Find : $\int \sqrt{x^2-2}\,dx$

Now, $\int \sqrt{x^2-2}\,dx$ can be written as $\int \sqrt{x^2-(\sqrt{2})^2}\,dx$

Formula Used: $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}| + C$

Since $\int \sqrt{x^2-(\sqrt{2})^2}\,dx$ is of the form $\int \sqrt{x^2-a^2}\,dx$,

Hence, $\int \sqrt{x^2-(\sqrt{2})^2}\,dx = \frac{x}{2}\sqrt{x^2-(\sqrt{2})^2} - \frac{(\sqrt{2})^2}{2}\log|x+\sqrt{x^2-(\sqrt{2})^2}| + C$

$= \frac{x}{2}\sqrt{x^2-2} - \frac{2}{2}\log|x+\sqrt{x^2-2}| + C$

$= \frac{x}{2}\sqrt{x^2-2} - \log|x+\sqrt{x^2-2}| + C$

Therefore, $\int \sqrt{x^2-2}\,dx = \frac{x}{2}\sqrt{x^2-2} - \log|x+\sqrt{x^2-2}| + C$

Question: 4

Solution:

To Find : $\int\sqrt{2x^2-3}dx$

Now, $\int \sqrt{2x^2-3}\,dx$ can be written as $\int \sqrt{(\sqrt{2}x)^2-(\sqrt{3})^2}\,dx$

Formula Used: $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}| + C$

Since $\int \sqrt{(\sqrt{2}x)^2-(\sqrt{3})^2}\,dx$ is of the form $\int \sqrt{x^2-a^2}\,dx$,

Hence, $\int \sqrt{(\sqrt{2}x)^2-(\sqrt{3})^2}\,dx = \frac{\sqrt{2}x}{2}\sqrt{(\sqrt{2}x)^2-(\sqrt{3})^2} - \frac{(\sqrt{3})^2}{2}\log|\sqrt{2}x+\sqrt{(\sqrt{2}x)^2-(\sqrt{3})^2}| + C$

$= \frac{\sqrt{2}x}{2}\sqrt{2x^2-3} - \frac{3}{2}\log|\sqrt{2}x+\sqrt{2x^2-3}| + C$

$= \frac{x}{2}\sqrt{2x^2-3} - \frac{3}{2\sqrt{2}}\log|\sqrt{2}x+\sqrt{2x^2-3}| + C$

Therefore, $\int \sqrt{2x^2-3}\,dx = \frac{x}{2}\sqrt{2x^2-3} - \frac{3}{2\sqrt{2}}\log|\sqrt{2}x+\sqrt{2x^2-3}| + C$

Question: 5

Solution:

To Find : $\int \sqrt{x^2+5}\,dx$

Now, $\int \sqrt{x^2+5}\,dx$ can be written as $\int \sqrt{x^2+(\sqrt{5})^2}\,dx$

Formula Used: $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x+\sqrt{x^2+a^2}| + C$

Since $\int \sqrt{x^2+(\sqrt{5})^2}\,dx$ is of the form $\int \sqrt{x^2+a^2}\,dx$,

Hence, $\int \sqrt{x^2+(\sqrt{5})^2}\,dx = \frac{x}{2}\sqrt{x^2+(\sqrt{5})^2} + \frac{(\sqrt{5})^2}{2}\log|x+\sqrt{x^2+(\sqrt{5})^2}| + C$

$= \frac{x}{2}\sqrt{x^2+5} + \frac{5}{2}\log|x+\sqrt{x^2+5}| + C$

Therefore, $\int \sqrt{x^2+5}\,dx = \frac{x}{2}\sqrt{x^2+5} + \frac{5}{2}\log|x+\sqrt{x^2+5}| + C$

Question: 6

Solution:

To Find : $\int \sqrt{4x^2+9}\,dx$

Now, $\int \sqrt{4x^2+9}\,dx$ can be written as $\int \sqrt{(2x)^2+3^2}\,dx$

Formula Used: $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x+\sqrt{x^2+a^2}| + C$

Since $\int \sqrt{(2x)^2+3^2}\,dx$ is of the form $\int \sqrt{x^2+a^2}\,dx$,

Hence, $\int \sqrt{(2x)^2+3^2}\,dx = \frac{2x}{2}\sqrt{(2x)^2+3^2} + \frac{3^2}{2}\log|2x+\sqrt{(2x)^2+3^2}| + C$

$= \frac{2x}{2}\sqrt{4x^2+9} + \frac{9}{2}\log|2x+\sqrt{4x^2+9}| + C$

$= \frac{x}{2}\sqrt{4x^2+9} + \frac{9}{4}\log|2x+\sqrt{4x^2+9}| + C$

Therefore, $\int \sqrt{4x^2+9}\,dx = \frac{x}{2}\sqrt{4x^2+9} + \frac{9}{4}\log|2x+\sqrt{4x^2+9}| + C$

Question: 7

Solution:

To Find : $\int \sqrt{3x^2+4}\,dx$

Now, $\int \sqrt{3x^2+4}\,dx$ can be written as $\int \sqrt{(\sqrt{3}x)^2+2^2}\,dx$

Formula Used: $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x+\sqrt{x^2+a^2}| + C$

Since $\int \sqrt{(\sqrt{3}x)^2+2^2}\,dx$ is of the form $\int \sqrt{x^2+a^2}\,dx$,

Hence, $\int \sqrt{(\sqrt{3}x)^2+2^2}\,dx = \frac{\sqrt{3}x}{2}\sqrt{(\sqrt{3}x)^2+2^2} + \frac{2^2}{2}\log|\sqrt{3}x+\sqrt{(\sqrt{3}x)^2+2^2}| + C$

$= \frac{\sqrt{3}x}{2}\sqrt{3x^2+4} + \frac{4}{2}\log|\sqrt{3}x+\sqrt{3x^2+4}| + C$

$= \frac{x}{2}\sqrt{3x^2+4} + \frac{2}{\sqrt{3}} \log |\sqrt{3}x + \sqrt{3x^2+4}| + C$

Therefore, $\int \sqrt{3x^2+4}\,dx = \frac{x}{2}\sqrt{3x^2+4} + \frac{2}{\sqrt{3}} \log |\sqrt{3}x + \sqrt{3x^2+4}| + C$

Question: 8

Solution:

To Find : $\int cosx\sqrt{9-\sin^2 x}\,dx$

Now, let sin x = t

$\Rightarrow$cosx dx = dt

Therefore, $\int cosx\sqrt{9-\sin^2 x}\,dx$ can be written as $\int \sqrt{3^2-t^2}\,dt$

Formula Used: $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since , $\int \sqrt{3^2-t^2}\,dt$ is in the form of $\int \sqrt{a^2-x^2}\,dx$ with t as a variable instead of x .

$\Rightarrow \int \sqrt{3^2-t^2}\,dt = \frac{1}{2}t\sqrt{3^2-t^2} + \frac{3^2}{2}\sin^{-1}\frac{t}{3} + C$

$= \frac{t}{2}\sqrt{9-t^2} + \frac{9}{2}\sin^{-1}\frac{t}{3} + C$

Now since sin x = t and cosx dx = dt

$\Rightarrow \int cosx\sqrt{9-\sin^2 x}\,dx = \frac{\sin x}{2}\sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}(\frac{\sin x}{3}) + C$

Question: 9

Solution:

To Find : $\int \sqrt{x^2-4x+2}\,dx$

Now, $\int \sqrt{x^2-4x+2}\,dx$ can be written as $\int \sqrt{x^2-4x+2^2-2^2+2}\,dx$

i.e., $\int \sqrt{(x-2)^2-2}\,dx$

Here , let x – 2 = y $\Rightarrow$dx = dy

Therefore, $\int \sqrt{(x-2)^2-2}\,dx$ can be written as $\int \sqrt{y^2-(\sqrt{2})^2}\,dy$

Formula Used: $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log |x + \sqrt{x^2-a^2}| + C$

Since $\int \sqrt{y^2-(\sqrt{2})^2}\,dy$ is of the form $\int \sqrt{x^2-a^2}\,dx$ with change in variable.

$\Rightarrow \int \sqrt{y^2-(\sqrt{2})^2}\,dy = \frac{y}{2}\sqrt{y^2-(\sqrt{2})^2} - \frac{(\sqrt{2})^2}{2}\log |y + \sqrt{y^2-(\sqrt{2})^2}| + C$

$= \frac{y}{2}\sqrt{y^2-2} - \frac{4}{2}\log |y + \sqrt{y^2-2}| + C$

$= \frac{y}{2}\sqrt{y^2-2} - 2\log |y + \sqrt{y^2-2}| + C$

Since , x – 2 = y and dx = dy

$\Rightarrow \int \sqrt{(x-2)^2-2}\,dx = \frac{(x-2)}{2}\sqrt{(x-2)^2-2} - 2\log |(x-2) + \sqrt{(x-2)^2-2}| + C$ Therefore,

$\int \sqrt{x^2-4x+2}\,dx = \frac{(x-2)}{2}\sqrt{x^2-4x+2} - 2\log|(x-2)+\sqrt{x^2-4x+2}| + C$

Question: 10

Solution:

To Find : $\int \sqrt{x^2+6x-4}\,dx$

Now, $\int \sqrt{x^2+6x-4}\,dx$ can be written as $\int \sqrt{x^2+6x+3^2-3^2-4}\,dx$

i.e, $\int \sqrt{(x+3)^2-13}\,dx$

Here , let x + 3 = y $\Rightarrow$ dx = dy

Therefore, $\int \sqrt{(x+3)^2-13}\,dx$ can be written as $\int \sqrt{y^2-(\sqrt{13})^2}\,dy$

Formula Used: $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}| + C$

Since $\int \sqrt{y^2-(\sqrt{13})^2}\,dy$ is of the form $\int \sqrt{x^2-a^2}\,dx$ with change in variable.

$\Rightarrow \int \sqrt{y^2-(\sqrt{13})^2}\,dy = \frac{y}{2}\sqrt{y^2-(\sqrt{13})^2} - \frac{(\sqrt{13})^2}{2}\log|y+\sqrt{y^2-(\sqrt{13})^2}| + C$

$= \frac{y}{2}\sqrt{y^2-13} - \frac{13}{2}\log|y+\sqrt{y^2-13}| + C$

Since , x + 3 = y and dx = dy

$\Rightarrow \int \sqrt{(x+3)^2-13}\,dx = \frac{(x+3)}{2}\sqrt{(x+3)^2-13} - \frac{13}{2}\log|(x+3)+\sqrt{(x+3)^2-13}| + C$

Therefore,

$\int \sqrt{x^2+6x-4}\,dx = \frac{(x+3)}{2}\sqrt{x^2+6x-4} - \frac{13}{2}\log|(x+3)+\sqrt{x^2+6x-4}| + C$

Question: 11

Solution:

To Find : $\int \sqrt{2x-x^2}\,dx$

Now, $\int \sqrt{2x-x^2}\,dx$ can be written as $\int \sqrt{2x-x^2-1^2+1^2}\,dx$

i.e, $\int \sqrt{1-(x-1)^2}\,dx$

Let x – 1 = y $\Rightarrow$ dx = dy

Therefore , $\int \sqrt{1-(x-1)^2}\,dx$ becomes $\int \sqrt{1^2-y^2}\,dy$

Formula Used: $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since $\int \sqrt{1^2-y^2}\,dy$ is of the form $\int \sqrt{a^2-x^2}\,dx$ with change in variable,

Hence $\int \sqrt{1^2-y^2}\,dy = \frac{1}{2}y\sqrt{1^2-y^2} + \frac{1^2}{2}\sin^{-1}\frac{y}{1} + C$

$= \frac{y}{2}\sqrt{1-y^2} + \frac{1}{2}\sin^{-1}\frac{y}{1} + C$

Here we have x – 1 = y and dx = dy

$\Rightarrow \int \sqrt{1-(x-1)^2}\,dx = \frac{(x-1)}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}\frac{(x-1)}{1} + C$

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Therefore, $\int \sqrt{2x-x^2}\,dx = \frac{(x-1)}{2}\sqrt{2x-x^2} + \frac{1}{2}\sin^{-1}(x-1) + C$

Question: 12

Solution:

To Find : $\int \sqrt{1-4x-x^2}$ dx

Now, $\int \sqrt{1-4x-x^2}\,dx$ can be written as $\int \sqrt{1-4x-x^2-2^2+2^2}\,dx$

i.e, $\int \sqrt{5-(x+2)^2}\,dx$

Let x + 2 = y ⇒ dx = dy

Therefore, $\int \sqrt{5-(x+2)^2}\,dx$ becomes $\int \sqrt{(\sqrt{5})^2-y^2}\,dy$

Formula Used: $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since $\int \sqrt{(\sqrt{5})^2-y^2}\,dy$ is of the form $\int \sqrt{a^2-x^2}\,dx$ with change in variable,

Hence $\int \sqrt{(\sqrt{5})^2-y^2}\,dy = \frac{1}{2}y\sqrt{(\sqrt{5})^2-y^2} + \frac{(\sqrt{5})^2}{2}\sin^{-1}\frac{y}{\sqrt{5}} + C$

$= \frac{y}{2}\sqrt{5-y^2} + \frac{5}{2}\sin^{-1}\frac{y}{\sqrt{5}} + C$

Here we have x + 2 = y and dx = dy

$\Rightarrow \int \sqrt{5-(x+2)^2}\,dx = \frac{(x+2)}{2}\sqrt{5-(x+2)^2} + \frac{5}{2}\sin^{-1}(\frac{x+2}{\sqrt{5}}) + C$

Therefore, $\int \sqrt{1-4x-x^2}\,dx = \frac{(x+2)}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}(\frac{x+2}{\sqrt{5}}) + C$

Question: 13

Solution:

To Find : $\int \sqrt{2ax-x^2}$ dx

Now, $\int \sqrt{2ax-x^2}\,dx$ can be written as $\int \sqrt{2ax-x^2-a^2+a^2}\,dx$

i.e, $\int \sqrt{a^2-(x-a)^2}\,dx$

Let x - a = y ⇒ dx = dy

Therefore, $\int \sqrt{a^2-(x-a)^2}\,dx$ becomes $\int \sqrt{a^2-y^2}\,dy$

Formula Used: $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since $\int \sqrt{a^2-y^2}\,dy$ is of the form $\int \sqrt{a^2-x^2}\,dx$ with change in variable,

Hence $\int \sqrt{a^2-y^2}\,dy = \frac{1}{2}y\sqrt{a^2-y^2} + \frac{a^2}{2}\sin^{-1}\frac{y}{a} + C$

$= \frac{y}{2}\sqrt{a^2-y^2} + \frac{a^2}{2}\sin^{-1}\frac{y}{a} + C$

Here we have x - a = y and dx = dy

$\Rightarrow \int \sqrt{a^2-(x-a)^2}\,dx = \frac{(x-a)}{2}\sqrt{a^2-(x-a)^2} + \frac{a^2}{2}\sin^{-1}(\frac{x-a}{a}) + C$

Therefore, $\int \sqrt{2ax-x^2}\,dx = \frac{(x-a)}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\sin^{-1}(\frac{x-a}{a}) + C$

Question: 14

Solution:

To Find : $\int \sqrt{2x^2+3x+4}dx$

Now, consider $\int \sqrt{2x^2+3x+4}dx = \int \sqrt{2[x^2+\frac{3}{2}x+2]}dx$

$= \sqrt{2}\int \sqrt{x^2+\frac{3}{2}x+2}dx$

$= \sqrt{2}\int \sqrt{x^2+\frac{3}{2}x+(\frac{3}{4})^2-\left(\frac{3}{4}\right)^2+2}dx$

$= \sqrt{2}\int \sqrt{(x+\frac{3}{4})^2+\frac{23}{16}}dx$

Let $x+\frac{3}{4} = y \Rightarrow dx = dy$

Hence $\sqrt{2}\int \sqrt{(x+\frac{3}{4})^2+\frac{23}{16}}dx$ becomes $\sqrt{2}\int \sqrt{y^2+(\frac{\sqrt{23}}{4})^2}dy$

Formula Used: $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x+\sqrt{x^2+a^2}| + C$

Now consider $\int \sqrt{y^2+(\frac{\sqrt{23}}{4})^2}dy$ which is in the form of $\int \sqrt{x^2+a^2}\,dx$ with change in variable.

$\Rightarrow \int \sqrt{y^2+(\frac{\sqrt{23}}{4})^2}dy = \frac{y}{2}\sqrt{y^2+(\frac{\sqrt{23}}{4})^2} + \frac{(\frac{\sqrt{23}}{4})^2}{2}\log|y+\sqrt{y^2+(\frac{\sqrt{23}}{4})^2}| + C$

$= \frac{y}{2}\sqrt{y^2+\frac{23}{16}} + \frac{23}{32}\log|y+\sqrt{y^2+\frac{23}{16}}| + C$

Since $x+\frac{3}{4} = y$ and $dx = dy$

$\Rightarrow \int \sqrt{(x+\frac{3}{4})^2+\frac{23}{16}}dx = \frac{1}{8}(4x+3)\sqrt{(x+\frac{3}{4})^2+\frac{23}{16}} + \frac{23}{32}\log|x+\frac{3}{4}+\sqrt{(x+\frac{3}{4})^2+\frac{23}{16}}| + C$

Now, $\sqrt{2}\int \sqrt{(x+\frac{3}{4})^2+\frac{23}{16}}dx = \frac{\sqrt{2}}{8}(4x+3)\sqrt{(x+\frac{3}{4})^2+\frac{23}{16}} + \frac{23\sqrt{2}}{32}\log|x+\frac{3}{4}+\sqrt{(x+\frac{3}{4})^2+\frac{23}{16}}| + C$

Therefore,

$\int \sqrt{2x^2+3x+4}dx = \frac{1}{8}(4x+3)\sqrt{2x^2+3x+4} + \frac{23}{32}\log|(x+\frac{3}{4})+\sqrt{2x^2+3x+4}| + C$

Question: 15

Solution:

To Find : $\int \sqrt{x^2+x}\,dx$

Now, $\int \sqrt{x^2+x}\,dx$ can be written as $\int \sqrt{x^2+x+(\frac{1}{2})^2-(\frac{1}{2})^2}dx$

i.e, $\int \sqrt{(x+\frac{1}{2})^2 - \frac{1}{4}}\, dx$

Here , let x + $\frac{1}{2}$ = y ⇒dx = dy

Therefore, $\int \sqrt{(x+\frac{1}{2})^2 - \frac{1}{4}}\, dx$ can be written as $\int \sqrt{y^2 - (\frac{1}{2})^2}\, dy$

Formula Used: $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log|x + \sqrt{x^2 - a^2}| + C$

Since $\int \sqrt{y^2 - (\frac{1}{2})^2}\, dy$ is of the form $\int \sqrt{x^2 - a^2}\, dx$ with change in variable.

$\Rightarrow \int \sqrt{y^2 - (\frac{1}{2})^2}\, dy = \frac{y}{2}\sqrt{y^2 - (\frac{1}{2})^2} - \frac{(\frac{1}{2})^2}{2}\log|y + \sqrt{y^2 - (\frac{1}{2})^2}| + C$

$= \frac{y}{2}\sqrt{y^2 - \frac{1}{4}} - \frac{1}{8}\log|y + \sqrt{y^2 - \frac{1}{4}}| + C$

Since , x + $\frac{1}{2}$ = y and dx = dy

$\Rightarrow \int \sqrt{(x+\frac{1}{2})^2 - \frac{1}{4}}\, dx = \frac{1}{4}(2x+1)\sqrt{(x+\frac{1}{2})^2 - \frac{1}{4}} - \frac{1}{8}\log|(x+\frac{1}{2}) + \sqrt{(x+\frac{1}{2})^2 - \frac{1}{4}}| + C$

Therefore,

$\int \sqrt{x^2 + x}\, dx = \frac{1}{4}(2x+1)\sqrt{x^2 + x} - \frac{1}{8}\log|x + \frac{1}{2} + \sqrt{x^2 + x}| + C$

Question: 16

Solution:

To Find :

$\int \sqrt{x^2 + x + 1}\, dx$

Now, $\int \sqrt{x^2 + x + 1}\, dx$ can be written as $\int \sqrt{x^2 + x + (\frac{1}{2})^2 - \left(\frac{1}{2}\right)^2 + 1}\, dx$

i.e, $\int \sqrt{(x+\frac{1}{2})^2 + \frac{3}{4}}\, dx$

Here , let x + $\frac{1}{2}$ = y ⇒dx = dy

Therefore, $\int \sqrt{(x+\frac{1}{2})^2 + \frac{3}{4}}\, dx$ can be written as $\int \sqrt{y^2 + (\frac{\sqrt{3}}{2})^2}\, dy$

Formula Used: $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log|x + \sqrt{x^2 + a^2}| + C$

Since $\int \sqrt{y^2 + (\frac{\sqrt{3}}{2})^2}\, dy$ is of the form $\int \sqrt{x^2 + a^2}\, dx$ with change in variable.

$\Rightarrow \int \sqrt{y^2 + (\frac{\sqrt{3}}{2})^2}\, dy = \frac{y}{2}\sqrt{y^2 + (\frac{\sqrt{3}}{2})^2} + \frac{(\frac{\sqrt{3}}{2})^2}{2}\log|y + \sqrt{y^2 + (\frac{\sqrt{3}}{2})^2}| + C$

$= \frac{y}{2}\sqrt{y^2 + \frac{3}{4}} + \frac{3}{8}\log|y + \sqrt{y^2 + \frac{3}{4}}| + C$

Since , x + $\frac{1}{2}$ = y and dx = dy

$\Rightarrow \int \sqrt{(x+\frac{1}{2})^2 + \frac{3}{4}}\, dx = \frac{1}{4}(2x+1)\sqrt{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}} + \frac{1}{8}\log|(x+\frac{1}{2}) + \sqrt{(x+\frac{1}{2})^2 + \frac{3}{4}}| + C$

Therefore,

$\int \sqrt{x^2+x+1}\,dx = \frac{1}{4}(2x+1)\sqrt{x^2+x+1} + \frac{3}{8}\log|x+\frac{1}{2}+\sqrt{x^2+x+1}|+C$

Question: 17

Solution:

To Find : $\int(2x-5)\sqrt{x^2-4x+3}\,dx$

Now, let 2x – 5 be written as (2x – 4) -1 and split

Therefore ,

$\int(2x-5)\sqrt{x^2-4x+3}\,dx = \int\{(2x-4)\sqrt{x^2-4x+3} - 1\sqrt{x^2-4x+3}\}dx$

$=\int(2x-4)\sqrt{x^2-4x+3}\,dx - \int\sqrt{x^2-4x+3}dx$

Now solving, $\int(2x-4)\sqrt{x^2-4x+3}\,dx$

Let x^2-4x+3 = u ⇒ dx = $\frac{du}{(2x-4)}$

Thus, $\int(2x-4)\sqrt{x^2-4x+3}\,dx$ becomes $\int\sqrt{u}\,du$

Now ,$\int\sqrt{u}\,du = \int u^{\frac{1}{2}}du = \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{2}{3}u^{\frac{3}{2}}$

$=\frac{2}{3}(x^2-4x+3)^{\frac{3}{2}}$

Now solving, $\int\sqrt{x^2-4x+3}dx$

$\int\sqrt{x^2-4x+3}dx = \int\sqrt{x^2-4x+2^2-2^2+3}dx$

$=\int\sqrt{(x-2)^2-1}dx$

Let x – 2 = y ⇒ dx = dy

Then $\int\sqrt{(x-2)^2-1}dx$ becomes $\int\sqrt{y^2-1^2}dy$

Formula Used: $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}|+ C$

Since $\int\sqrt{y^2-1^2}dy$ is in the form of $\int\sqrt{x^2-a^2}\,dx$ with change in variable.

Hence $\int\sqrt{y^2-1^2}dy = \frac{y}{2}\sqrt{y^2-1^2} - \frac{1^2}{2}\log|y+\sqrt{y^2-1^2}|+ C$

$= \frac{y}{2}\sqrt{y^2-1} - \frac{1}{2}\log|y+\sqrt{y^2-1}|+ C$

Now, since x – 2 = y and dx = dy

$\int\sqrt{(x-2)^2-1}dx = \frac{(x-2)}{2}\sqrt{(x-2)^2-1} - \frac{1}{2}\log|(x-2)+\sqrt{(x-2)^2-1}|+ C$

Hence $\int\sqrt{x^2-4x+3}dx = \frac{(x-2)}{2}\sqrt{x^2-4x+3} - \frac{1}{2}\log|(x-2)+\sqrt{x^2-4x+3}|+ C$

Therefore , $\int(2x-4)\sqrt{x^2-4x+3}\,dx - \int\sqrt{x^2-4x+3}dx = \frac{2}{3}(x^2-4x+3)^{\frac{3}{2}}$

$-\frac{(x-2)}{2}\sqrt{x^2-4x+3} + \frac{1}{2}\log|(x-2)+\sqrt{x^2-4x+3}|+ C$

$i.e, \int(2x-5)\sqrt{x^2-4x+3}\,dx = \frac{2}{3}(x^2-4x+3)^{\frac{3}{2}}$

$-\frac{(x-2)}{2}\sqrt{x^2-4x+3}+\frac{1}{2}\log|x-2+\sqrt{x^2-4x+3}|+C$

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Question: 18

Solution:

To Find :

$\int(x+2)\sqrt{x^2+x+1}\,dx$

Now, let x + 2 be written as $\frac{1}{2}(2x+1)+\frac{3}{2}$ and split

Therefore ,

$\int(x+2)\sqrt{x^2+x+1}\,dx = \int\{\frac{(2x+1)\sqrt{x^2+x+1}}{2}+\frac{3}{2}\sqrt{x^2+x+1}\}dx$

$=\frac{1}{2}\int(2x+1)\sqrt{x^2+x+1}\,dx+\frac{3}{2}\int\sqrt{x^2+x+1}\,dx$

Now solving, $\frac{1}{2}\int(2x+1)\sqrt{x^2+x+1}\,dx$

Let $x^2+x+1=u \Rightarrow dx=\frac{du}{(2x+1)}$

Thus, $\frac{1}{2}\int(2x+1)\sqrt{x^2+x+1}\,dx$ becomes $\frac{1}{2}\int\sqrt{u}\,du$

Now , $\frac{1}{2}\int\sqrt{u}\,du = \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{2}\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right)=\frac{1}{3}u^{\frac{3}{2}}$

$=\frac{1}{3}(x^2+x+1)^{\frac{3}{2}}$

Now solving , $\int\sqrt{x^2+x+1}\,dx$

Now, $\int\sqrt{x^2+x+1}\,dx$ can be written as $\int\sqrt{x^2+x+(\frac{1}{2})^2-\left(\frac{1}{2}\right)^2+1}\,dx$

i.e, $\int\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$

Here , let $x+\frac{1}{2}=y \Rightarrow dx=dy$

Therefore, $\int\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$ can be written as $\int\sqrt{y^2+(\frac{\sqrt{3}}{2})^2}\,dy$

Formula Used: $\int\sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log|x+\sqrt{x^2+a^2}|+C$

Since $\int\sqrt{y^2+(\frac{\sqrt{3}}{2})^2}\,dy$ is of the form $\int\sqrt{x^2+a^2}\,dx$ with change in variable.

$\Rightarrow \int\sqrt{y^2+(\frac{\sqrt{3}}{2})^2}\,dy = \frac{y}{2}\sqrt{y^2+(\frac{\sqrt{3}}{2})^2}+\frac{(\frac{\sqrt{3}}{2})^2}{2}\log|y+\sqrt{y^2+(\frac{\sqrt{3}}{2})^2}|+C$

$=\frac{y}{2}\sqrt{y^2+\frac{3}{4}}+\frac{3}{8}\log|y+\sqrt{y^2+\frac{3}{4}}|+C$

Since , $x+\frac{1}{2}=y$ and $dx=dy$

$\Rightarrow \int\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx = \frac{1}{4}(2x+1)\sqrt{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}+\frac{1}{8}\log|(x+\frac{1}{2})+\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}|+C$

Therefore,

$\int\sqrt{x^2+x+1}\,dx = \frac{1}{4}(2x+1)\sqrt{x^2+x+1}+\frac{3}{8}\log|x+\frac{1}{2}+\sqrt{x^2+x+1}|+C$

Hence ,

$\frac{1}{2}\int(2x + 1)\sqrt{x^2+x+1}dx + \frac{3}{2}\int\sqrt{x^2+x+1}dx = \frac{1}{3}(x^2+x+1)^{\frac{3}{2}} + \frac{3}{8}(2x+1)\sqrt{x^2+x+1} + \frac{9}{16}\log|(x$
$+\frac{1}{2})+\sqrt{x^2+x+1}|+C$

Therefore , $\int(x+2)\sqrt{x^2+x+1}\,dx = \frac{1}{3}(x^2+x+1)^{\frac{3}{2}} + \frac{3}{8}(2x+1)\sqrt{x^2+x+1} + \frac{9}{16}\log|(x+\frac{1}{2})+$
$\sqrt{x^2+x+1}|+C$

Question: 19

Solution:

To Find : $\int(x-5)\sqrt{x^2+x}\,dx$

Now, let x - 5 be written as $\frac{1}{2}(2x + 1) - \frac{11}{2}$ and split

Therefore ,

$\int(x-5)\sqrt{x^2+x}\,dx = \int\{\frac{(2x+1)\sqrt{x^2+x}}{2} - \frac{11}{2}\sqrt{x^2+x}\}dx$

$=\frac{1}{2}\int(2x + 1)\sqrt{x^2+x}dx - \frac{11}{2}\int\sqrt{x^2+x}dx$

Now solving, $\frac{1}{2}\int(2x + 1)\sqrt{x^2+x}\,dx$

Let $x^2+x = u \Rightarrow dx = \frac{du}{(2x+1)}$

Thus, $\frac{1}{2}\int(2x + 1)\sqrt{x^2+x}dx$ becomes $\frac{1}{2}\int\sqrt{u}\,du$

Now , $\frac{1}{2}\int\sqrt{u}\,du = \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{2}(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}) = \frac{1}{3}u^{\frac{3}{2}}$

$=\frac{1}{3}(x^2+x)^{\frac{3}{2}}$

Now solving, $\int\sqrt{x^2+x}\,dx$

Now, $\int\sqrt{x^2+x}\,dx$ can be written as $\int\sqrt{x^2+x+(\frac{1}{2})^2-(\frac{1}{2})^2}\,dx$

i.e, $\int\sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}\,dx$

Here , let x + $\frac{1}{2}$ = y ⇒dx = dy

Therefore, $\int\sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}\,dx$ can be written as $\int\sqrt{y^2-(\frac{1}{2})^2}\,dy$

Formula Used: $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}|+ C$

Since $\int\sqrt{y^2-(\frac{1}{2})^2}\,dy$ is of the form $\int\sqrt{x^2-a^2}\,dx$ with change in variable.

$\Rightarrow \int\sqrt{y^2-(\frac{1}{2})^2}\,dy = \frac{y}{2}\sqrt{y^2-(\frac{1}{2})^2} - \frac{(\frac{1}{2})^2}{2}\log|y+\sqrt{y^2-(\frac{1}{2})^2}|+ C$

$= \frac{y}{2}\sqrt{y^2-\frac{1}{4}} - \frac{1}{8}\log|y+\sqrt{y^2-\frac{1}{4}}|+ C$

Since , x + $\frac{1}{2}$ = y and dx = dy

$\Rightarrow \int \sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}dx = \frac{1}{4}(2x+1)\sqrt{(x+\frac{1}{2})^2-\frac{1}{4}} - \frac{1}{8}\log|(x+\frac{1}{2}) + \sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}|+C$

Therefore,

$\int \sqrt{x^2+x}\,dx = \frac{1}{4}(2x+1)\sqrt{x^2+x} - \frac{1}{8}\log|x+\frac{1}{2}+\sqrt{x^2+x}|+C$

Now ,

$\frac{1}{2}\int(2x+1)\sqrt{x^2+x}dx - \frac{11}{2}\int\sqrt{x^2+x}dx = \frac{1}{3}(x^2+x)^{\frac{3}{2}} - \frac{11}{8}(2x+1)\sqrt{x^2+x} + \frac{11}{16}\log|x+\frac{1}{2}+\sqrt{x^2+x}|+C$

Therefore ,

$\int(x-5)\sqrt{x^2+x}\,dx = \frac{1}{3}(x^2+x)^{\frac{3}{2}} - \frac{11}{8}(2x+1)\sqrt{x^2+x} + \frac{11}{16}\log|x+\frac{1}{2}+\sqrt{x^2+x}|+C$

Question: 20

Solution:

To Find : $\int(4x+1)\sqrt{x^2-x-2}\,dx$

Now, let 4x + 1 be written as 2(2x - 1) + 3 and split

Therefore ,

$\int(4x+1)\sqrt{x^2-x-2}\,dx = \int\{2(2x-1)\sqrt{x^2-x-2} + 3\sqrt{x^2-x-2}\}\,dx$

$= 2\int(2x-1)\sqrt{x^2-x-2}dx + 3\int\sqrt{x^2-x-2}dx$

Now solving, $2\int(2x-1)\sqrt{x^2-x-2}dx$

Let $x^2-x-2 = u \Rightarrow dx = \frac{du}{(2x-1)}$

Thus, $2\int(2x-1)\sqrt{x^2-x-2}dx$ becomes $2\int\sqrt{u}\,du$

Now , $2\int\sqrt{u}\,du = 2\int u^{\frac{1}{2}}du = 2\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) = \frac{4}{3}u^{\frac{3}{2}}$

$= \frac{4}{3}(x^2-x-2)^{\frac{3}{2}}$

Now solving, $\int\sqrt{x^2-x-2}dx$

Now, $\int\sqrt{x^2-x-2}\,dx$ can be written as $\int\sqrt{x^2-x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-2}\,dx$

i.e, $\int\sqrt{(x-\frac{1}{2})^2-\frac{9}{4}}\,dx$

Here , let x - $\frac{1}{2}$ = y $\Rightarrow$ dx = dy

Therefore, $\int\sqrt{(x-\frac{1}{2})^2-\frac{9}{4}}\,dx$ can be written as $\int\sqrt{y^2-(\frac{3}{2})^2}\,dy$

Formula Used: $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}|+C$

Since $\int\sqrt{y^2-(\frac{3}{2})^2}\,dy$ is of the form $\int\sqrt{x^2-a^2}\,dx$ with change in variable.

$\Rightarrow \int\sqrt{y^2-(\frac{3}{2})^2}\,dy = \frac{y}{2}\sqrt{y^2-(\frac{3}{2})^2} - \frac{(\frac{3}{2})^2}{2}\log|y+\sqrt{y^2-(\frac{3}{2})^2}|+C$

$= \frac{y}{2}\sqrt{y^2 - \frac{9}{4}} - \frac{9}{8}\log|y + \sqrt{y^2 - \frac{9}{4}}| + C$

Since , $x - \frac{1}{2} = y$ and $dx = dy$

$\Rightarrow \int \sqrt{(x-\frac{1}{2})^2 - \frac{9}{4}}\,dx = \frac{1}{4}(2x-1)\sqrt{(x-\frac{1}{2})^2 - \frac{9}{4}} - \frac{9}{8}\log|(x - \frac{1}{2}) + \sqrt{(x-\frac{1}{2})^2 - \frac{9}{4}}| + C$

Therefore,

$\int \sqrt{x^2 - x - 2}\,dx = \frac{1}{4}(2x-1)\sqrt{x^2-x-2} - \frac{9}{8}\log|x - \frac{1}{2} + \sqrt{x^2-x-2}| + C$

Hence ,

$2\int(2x-1)\sqrt{x^2-x-2}dx + 3\int\sqrt{x^2-x-2}dx = \frac{4}{3}(x^2-x-2)^{\frac{3}{2}} + \frac{3}{4}(2x-1)\sqrt{x^2-x-2} - \frac{27}{8}\log|x - \frac{1}{2} + \sqrt{x^2-x-2}| + C$

Therefore ,

$\int(4x+1)\sqrt{x^2-x-2}\,dx = \frac{4}{3}(x^2-x-2)^{\frac{3}{2}} + \frac{3}{4}(2x-1)\sqrt{x^2-x-2} - \frac{27}{8}\log|x - \frac{1}{2} + \sqrt{x^2-x-2}| + C$

Question: 21

Solution:

To Find :

Now, $\int(x+1)\sqrt{2x^2+3}\,dx$ can be written as

$\int(x+1)\sqrt{2x^2+3}\,dx = \int\{x\sqrt{2x^2+3} + \sqrt{2x^2+3}\}\,dx$

$= \int x\sqrt{2x^2+3}dx + \int\sqrt{2x^2+3}dx$

Now solving, $\int x\sqrt{2x^2+3}dx$

Let $2x^2 + 3 = u \Rightarrow dx = \frac{1du}{4x}$

Thus, $\int x\sqrt{2x^2+3}dx$ becomes $\frac{1}{4}\int\sqrt{u}\,du$

Now , $\frac{1}{4}\int\sqrt{u}\,du = \frac{1}{4}\int u^{\frac{1}{2}}du = \frac{1}{4}\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) = \frac{1}{6}u^{\frac{3}{2}}$

$= \frac{1}{6}(2x^2+3)^{\frac{3}{2}}$

Now solving, $\int\sqrt{2x^2+3}dx$

Now, $\int\sqrt{2x^2+3}\,dx$ can be written as $\int\sqrt{(\sqrt{2}x)^2 + (\sqrt{3})^2}\,dx$

Formula Used: $\int\sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x + \sqrt{x^2+a^2}| + C$

Since $\int\sqrt{2x^2+3}\,dx$ is of the form $\int\sqrt{x^2+a^2}\,dx$.

$\Rightarrow \int\sqrt{2x^2+3}\,dx = \frac{\sqrt{2}x}{2}\sqrt{(\sqrt{2}x)^2 + (\sqrt{3})^2} + \frac{(\sqrt{3})^2}{2}\log|\sqrt{2}x + \sqrt{(\sqrt{2}x)^2 + (\sqrt{3})^2}| + C$

$= \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log|\sqrt{2}x + \sqrt{2x^2+3}| + C$

Therefore,

$\int x\sqrt{2x^2+3}dx + \int\sqrt{2x^2+3}dx = \frac{1}{6}(2x^2+3)^{\frac{3}{2}} + \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log|\sqrt{2}x + \sqrt{2x^2+3}|$ CLASS24

Hence ,

$\int(x+1)\sqrt{2x^2+3}\,dx = \frac{1}{6}(2x^2+3)^{\frac{3}{2}} + \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log|\sqrt{2}x + \sqrt{2x^2+3}| + C$

Question: 22

Solution:

To Find :

$\int x\sqrt{1+x-x^2}\,dx$

Now, let x be written as $\frac{1}{2}$ - $\frac{1}{2}$(1 - 2x) and split

Therefore ,

$\int x\sqrt{1+x-x^2}\,dx = \int\{\frac{\sqrt{-x^2+x+1}}{2} - \frac{(1-2x)\sqrt{-x^2+x+1}}{2}\}dx$

$= \frac{1}{2}\int(2x-1)\sqrt{-x^2+x+1}\,dx + \frac{1}{2}\int\sqrt{-x^2+x+1}\,dx$

Now solving, $\frac{1}{2}\int(2x-1)\sqrt{-x^2+x+1}\,dx$

Let $-x^2+x+1 = u \Rightarrow dx = \frac{du}{(1-2x)}$

Thus, $\frac{1}{2}\int(2x-1)\sqrt{-x^2+x+1}\,dx$ becomes $-\frac{1}{2}\int\sqrt{u}\,du$

Now , $-\frac{1}{2}\int\sqrt{u}\,du = -\frac{1}{2}\int u^{\frac{1}{2}}du = -\frac{1}{2}\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) = -\frac{1}{3}u^{\frac{3}{2}}$

$= -\frac{1}{3}(-x^2+x+1)^{\frac{3}{2}}$

Now solving, $\int\sqrt{-x^2+x+1}\,dx$

$\int\sqrt{-x^2+x+1}\,dx$ can be written as $\int\sqrt{-x^2+x-\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+1}\,dx$

i.e, $\int\sqrt{\frac{5}{4}-(x-\frac{1}{2})^2}\,dx = \frac{1}{2}\int\sqrt{5-(2x-1)^2}\,dx$

let $2x-1 = y \Rightarrow dx = \frac{1dy}{2}$

Therefore , $\frac{1}{4}\int\sqrt{5-(2x-1)^2}\,dx$ becomes $\frac{1}{4}\int\sqrt{(\sqrt{5})^2-y^2}\,dy$

Formula Used: $\int\sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since $\int\sqrt{(\sqrt{5})^2-y^2}\,dy$ is of the form $\int\sqrt{a^2-x^2}\,dx$ with change in variable .

Hence, $\int\sqrt{(\sqrt{5})^2-y^2}\,dy = \frac{1}{2}y\sqrt{(\sqrt{5})^2-y^2} + \frac{(\sqrt{5})^2}{2}\sin^{-1}\frac{y}{\sqrt{5}} + C$

$= \frac{1}{2}y\sqrt{5-y^2} + \frac{5}{2}\sin^{-1}\frac{y}{\sqrt{5}} + C$

Since , $2x-1 = y$ and $dx = \frac{1dy}{2}$

Therefore,

$\frac{1}{4}\int\sqrt{5-(2x-1)^2}\,dx = \frac{1}{8}(2x-1)\sqrt{5-(2x-1)^2} + \frac{5}{8}\sin^{-1}\frac{(2x-1)}{\sqrt{5}} + C$

i.e, $\int\sqrt{-x^2+x+1}\,dx = \frac{1}{8}(2x-1)\sqrt{-x^2+x+1} + \frac{5}{8}\sin^{-1}\frac{(2x-1)}{\sqrt{5}} + C$

hence , $\int x\sqrt{1+x-x^2}\,dx = \frac{1}{2}\int(2x-1)\sqrt{-x^2+x+1}\,dx + \frac{1}{2}\int\sqrt{-x^2+x+1}\,dx = =$
$-\frac{1}{3}(-x^2+x+1)^{\frac{3}{2}} + \frac{1}{16}(2x-1)\sqrt{-x^2+x+1} + \frac{5}{16}\sin^{-1}(\frac{2x-1}{\sqrt{5}}) + C$

Question: 23

Solution:

To Find : $\int(2x-5)\sqrt{2+3x-x^2}\,dx$ $\int(2x-5)\sqrt{2+3x-x^2}dx$

Now, let 2x - 5 be written as (2x – 3) -2 and split

Therefore ,

$\int(2x-5)\sqrt{2+3x-x^2}\,dx = \int\{(2x-3)\sqrt{-x^2+3x+2} - 2\sqrt{-x^2+3x+2}\}\,dx$

$= \int(2x-3)\sqrt{-x^2+3x+2}\,dx$ - 2 $\int\sqrt{-x^2+3x+2}\,dx$

Now solving, $\int(2x-3)\sqrt{-x^2+3x+2}\,dx$

Let $-x^2+3x+2 = u \Rightarrow dx = \frac{du}{(3-2x)}$

Thus, $\int(2x-3)\sqrt{-x^2+3x+2}\,dx$ becomes $-\int\sqrt{u}\,du$

Now , $-\int\sqrt{u}\,du = -\int u^{\frac{1}{2}}\,du = -(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}) = -\frac{2}{3}u^{\frac{3}{2}}$

$= -\frac{2}{3}(-x^2+3x+2)^{\frac{3}{2}}$

Now solving, $\int\sqrt{-x^2+3x+2}\,dx$

$\int\sqrt{-x^2+3x+2}\,dx$ can be written as $\int\sqrt{-x^2+3x-\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)^2+2}\,dx$

i.e, $\int\sqrt{\frac{17}{4}-(x-\frac{3}{2})^2}\,dx$

let $x-\frac{3}{2} = y \Rightarrow dx = dy$

Therefore , $\int\sqrt{\frac{17}{4}-(x-\frac{3}{2})^2}\,dx$ becomes $\int\sqrt{(\frac{\sqrt{17}}{2})^2-y^2}\,dy$

Formula Used: $\int\sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since $\int\sqrt{(\frac{\sqrt{17}}{2})^2-y^2}\,dy$ is of the form $\int\sqrt{a^2-x^2}\,dx$ with change in variable .

Hence, $\int\sqrt{(\frac{\sqrt{17}}{2})^2-y^2}\,dy = \frac{1}{2}y\sqrt{(\frac{\sqrt{17}}{2})^2-y^2} + \frac{(\frac{\sqrt{17}}{2})^2}{2}\sin^{-1}\frac{y}{\frac{\sqrt{17}}{2}} + C$

$= \frac{1}{2}y\sqrt{\frac{17}{4}-y^2} + \frac{17}{8}\sin^{-1}\frac{y}{\frac{\sqrt{17}}{2}} + C$

Since , $x-\frac{3}{2} = y$ and $dx = dy$

Therefore,

$\int \sqrt{\frac{17}{4} - (x - \frac{3}{2})^2}\, dx = \frac{1}{4}(2x-3)\sqrt{\frac{17}{4} - (x-\frac{3}{2})^2} + \frac{17}{8}\sin^{-1}(\frac{2x-3}{\sqrt{17}}) + C$

i.e, $\int \sqrt{-x^2+3x+2}\, dx = \frac{1}{4}(2x-3)\sqrt{-x^2+3x+2} + \frac{17}{8}\sin^{-1}(\frac{2x-3}{\sqrt{17}}) + C$

hence ,

$\int (2x-5)\sqrt{2+3x-x^2}\, dx = \int (2x-3)\sqrt{-x^2+3x+2}\, dx - 2\int \sqrt{-x^2+3x+2}\, dx =$
$-\frac{2}{3}(-x^2+3x+2)^{\frac{3}{2}} - \frac{1}{2}(2x-3)\sqrt{-x^2+3x+2} - \frac{17}{4}\sin^{-1}(\frac{2x-3}{\sqrt{17}}) + C$

Question: 24

Solution:

To Find :

$\int (6x+5)\sqrt{6+x-2x^2}\, dx$

Now, let 6x + 5 be written as $\frac{13}{2} - \frac{3}{2}$(1 - 4x)and split

Therefore ,

$\int (6x+5)\sqrt{6+x-2x^2}\, dx = \int \{\frac{13\sqrt{-2x^2+x+6}}{2} - \frac{3(1-4x)\sqrt{-2x^2+x+6}}{2}\} dx$

$= \frac{3}{2}\int (4x-1)\sqrt{-2x^2+x+6}\, dx + \frac{13}{2}\int \sqrt{-2x^2+x+6}\, dx$

Now solving, $\int (4x-1)\sqrt{-2x^2+x+6}\, dx$

Let $-2x^2+x+6$ = u ⇒ dx = $\frac{du}{(1-4x)}$

Thus, $\int (4x-1)\sqrt{-2x^2+x+6}\, dx$ becomes $-\int \sqrt{u}\, du$

Now , $-\int \sqrt{u}\, du = -\int u^{\frac{1}{2}}\, du = -(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}) = -\frac{2}{3}u^{\frac{3}{2}}$

$= -\frac{2}{3}(-2x^2+x+6)^{\frac{3}{2}}$

Now solving, $\int \sqrt{-2x^2+x+6}\, dx$

$\int \sqrt{-2x^2+x+6}\, dx$ can be written as $\int \sqrt{-(\sqrt{2}x)^2 + x - \left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2 + 6}\, dx$

i.e, $\int \sqrt{\frac{49}{8} - (\sqrt{2}x - \frac{1}{2\sqrt{2}})^2}\, dx$

let $\sqrt{2}x - \frac{1}{2\sqrt{2}}$= y ⇒ dx = $\frac{dy}{\sqrt{2}}$

Therefore , $\int \sqrt{\frac{49}{8} - (\sqrt{2}x - \frac{1}{2\sqrt{2}})^2}\, dx$ becomes $\int \sqrt{(\frac{7}{2\sqrt{2}})^2 - y^2}\, dy$

Formula Used: $\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since $\int \sqrt{(\frac{7}{2\sqrt{2}})^2 - y^2}\, dy$ is of the form $\int \sqrt{a^2-x^2}\, dx$ with change in variable .

Hence, $\int \sqrt{(\frac{7}{2\sqrt{2}})^2 - y^2}\, dy = \frac{1}{2}y\sqrt{(\frac{7}{2\sqrt{2}})^2 - y^2} + \frac{(\frac{7}{2\sqrt{2}})^2}{2}\sin^{-1}\frac{y}{\frac{7}{2\sqrt{2}}} + C$

$= \frac{1}{2}y\sqrt{\frac{49}{8} - y^2} + \frac{7}{16}\sin^{-1}\frac{y}{\frac{\sqrt{17}}{2}} + C$

Since, $\sqrt{2}x - \frac{1}{2\sqrt{2}} = y$ and $dx = \frac{dy}{\sqrt{2}}$

Therefore,

$\int \sqrt{\frac{49}{8} - (\sqrt{2}x - \frac{1}{2\sqrt{2}})^2}\,dx = \frac{1}{4\sqrt{2}}(4x-1)\sqrt{\frac{49}{8} - (\sqrt{2}x - \frac{1}{2\sqrt{2}})^2} + \frac{49}{16}\sin^{-1}(\frac{4x-1}{7}) + C$

i.e, $\int \sqrt{-2x^2 + x + 6}\,dx = \frac{1}{4\sqrt{2}}(4x-1)\sqrt{-2x^2+x+6} + \frac{49}{16}\sin^{-1}(\frac{4x-1}{7}) + C$

hence,

$\int (6x+5)\sqrt{6+x-2x^2}\,dx = \frac{3}{2}\int (4x-1)\sqrt{-2x^2+x+6}\,dx + \frac{13}{2}\int \sqrt{-2x^2+x+6}\,dx =$
$-(-2x^2+x+6)^{\frac{3}{2}} + \frac{13}{16}(4x-1)\sqrt{-2x^2+x+6} + \frac{637}{32\sqrt{2}}\sin^{-1}(\frac{4x-1}{7}) + C$

Question: 25

Solution:

To Find :

$\int (x+1)\sqrt{1-x-x^2}\,dx$

Now, let x + 1 be written as $\frac{1}{2} - \frac{1}{2}$(-2x - 1) and split

Therefore ,

$\int (x+1)\sqrt{1-x-x^2}\,dx = \int \{\frac{\sqrt{-x^2-x+1}}{2} - \frac{(-2x-1)\sqrt{-x^2-x+1}}{2}\}\,dx$

$= \frac{1}{2}\int (2x-1)\sqrt{-x^2-x+1}\,dx + \frac{1}{2}\int \sqrt{-x^2-x+1}\,dx$

Now solving, $\int (2x-1)\sqrt{-x^2-x+1}\,dx$

Let $-x^2 - x + 1 = u \Rightarrow dx = \frac{du}{-2x-1}$

Thus, $\int (2x-1)\sqrt{-x^2-x+1}\,dx$ becomes $-\int \sqrt{u}\,du$

Now, $-\int \sqrt{u}\,du = -\int u^{\frac{1}{2}}\,du = -(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}) = -\frac{2}{3}u^{\frac{3}{2}}$

$= -\frac{2}{3}(-x^2 - x + 1)^{\frac{3}{2}}$

Now solving, $\int \sqrt{-x^2-x+1}\,dx$

$\int \sqrt{-x^2-x+1}\,dx$ can be written as $\int \sqrt{-x^2 - x - \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 1}\,dx$

i.e, $\int \sqrt{\frac{5}{4} - (x+\frac{1}{2})^2}\,dx$

let $x + \frac{1}{2} = y \Rightarrow dx = dy$

Therefore, $\int \sqrt{\frac{5}{4} - (x+\frac{1}{2})^2}\,dx$ becomes $\int \sqrt{(\frac{\sqrt{5}}{2})^2 - y^2}\,dy$

Formula Used: $\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since $\int \sqrt{(\frac{\sqrt{5}}{2})^2 - y^2}\,dy$ is of the form $\int \sqrt{a^2 - x^2}\,dx$ with change in variable.

Hence, $\int \sqrt{(\frac{\sqrt{5}}{2})^2 - y^2}\, dy = \frac{1}{2} y \sqrt{(\frac{\sqrt{5}}{2})^2 - y^2} + \frac{(\frac{\sqrt{5}}{2})^2}{2} \sin^{-1} \frac{y}{\frac{\sqrt{5}}{2}} + C$

$= \frac{1}{2} y \sqrt{\frac{5}{4} - y^2} + \frac{5}{8} \sin^{-1} \frac{y}{\frac{\sqrt{5}}{2}} + C$

Since , $x + \frac{1}{2} = y$ and $dx = dy$

Therefore,

$\int \sqrt{\frac{5}{4} - (x + \frac{1}{2})^2}\, dx = \frac{1}{4}(2x+1)\sqrt{\frac{5}{4} - (x+\frac{1}{2})^2} + \frac{5}{8} \sin^{-1}(\frac{2x+1}{\sqrt{5}}) + C$

i.e, $\int \sqrt{-x^2 - x + 1}\, dx = \frac{1}{4}(2x+1)\sqrt{-x^2 - x + 1} + \frac{5}{8} \sin^{-1}(\frac{2x+1}{\sqrt{5}}) + C$

hence ,

$\int (x+1)\sqrt{1 - x - x^2}\, dx = \frac{1}{2}\int (2x-1)\sqrt{-x^2 - x + 1}\, dx + \frac{1}{2}\int \sqrt{-x^2 - x + 1}\, dx = -\frac{1}{3}(-x^2 - x + 1)^{\frac{3}{2}} + \frac{1}{8}(2x+1)\sqrt{-x^2 - x + 1} + \frac{5}{16} \sin^{-1}(\frac{2x+1}{\sqrt{5}}) + C$

Question: 26

Solution:

To Find : $\int (x-3)\sqrt{x^2 + 3x - 18}\, dx$

Now, let x - 3 be written as $\frac{1}{2}(2x + 3) - \frac{9}{2}$ and split

Therefore ,

$\int (x-3)\sqrt{x^2 + 3x - 18}\, dx = \int \{\frac{(2x+3)\sqrt{x^2+3x-18}}{2} - \frac{9\sqrt{x^2+3x-18}}{2}\} dx$

$= \frac{1}{2}\int (2x+3)\sqrt{x^2 + 3x - 18}\, dx - \frac{9}{2} \int \sqrt{x^2 + 3x - 18}\, dx$

Now solving, $\int (2x+3)\sqrt{x^2 + 3x - 18}\, dx$

Let $x^2 + 3x - 18 = u \Rightarrow dx = \frac{du}{2x+3}$

Thus, $\int (2x+3)\sqrt{x^2 + 3x - 18}\, dx$ becomes $\int \sqrt{u}\, du$

Now , $\int \sqrt{u}\, du = \int u^{\frac{1}{2}} du = (\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}) = \frac{2}{3} u^{\frac{3}{2}}$

$= \frac{2}{3}(x^2 + 3x - 18)^{\frac{3}{2}}$

Now solving, $\int \sqrt{x^2 + 3x - 18}\, dx$

$\int \sqrt{x^2 + 3x - 18}\, dx$ can be written as $\int \sqrt{x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 18}\, dx$

i.e, $\int \sqrt{(x + \frac{3}{2})^2 - \frac{81}{4}}\, dx$

let $x + \frac{3}{2} = y \Rightarrow dx = dy$

Therefore, $\int \sqrt{(x+\frac{3}{2})^2 - \frac{81}{4}}\, dx$ can be written as $\int \sqrt{y^2 - (\frac{9}{2})^2}\, dy$

Formula Used: $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$

Since $\int \sqrt{y^2 - (\frac{9}{2})^2}\, dy$ is of the form $\int \sqrt{x^2 - a^2}\, dx$ with change in variable.

$\Rightarrow \int \sqrt{y^2 - (\frac{9}{2})^2}\, dy = \frac{y}{2}\sqrt{y^2 - (\frac{9}{2})^2} - \frac{(\frac{9}{2})^2}{2} \log |y + \sqrt{y^2 - (\frac{9}{2})^2}| + C$

$= \frac{y}{2}\sqrt{y^2 - \frac{81}{4}} - \frac{81}{8} \log |y + \sqrt{y^2 - \frac{81}{4}}| + C$

Since , $x + \frac{3}{2} = y$ and $dx = dy$

$\Rightarrow \int \sqrt{(x + \frac{3}{2})^2 - \frac{81}{4}}\, dx = \frac{1}{4}(2x + 3)\sqrt{(x + \frac{3}{2})^2 - \frac{81}{4}} - \frac{81}{8} \log |(x + \frac{3}{2}) + \sqrt{(x + \frac{3}{2})^2 - \frac{81}{4}}| + C$

Therefore,

$\int \sqrt{x^2 + 3x - 18}\, dx = \frac{1}{4}(2x + 3)\sqrt{x^2 + 3x - 18} - \frac{81}{8} \log |x + \frac{3}{2} + \sqrt{x^2 + 3x - 18}| + C$

Hence ,

$\int (x - 3)\sqrt{x^2 + 3x - 18}\, dx = \frac{1}{2}\int (2x + 3)\sqrt{x^2 + 3x - 18}\, dx - \frac{9}{2} \int \sqrt{x^2 + 3x - 18}\, dx =$
$\frac{1}{3}(x^2 + 3x - 18)^{\frac{3}{2}} - \frac{9}{8}(2x + 3)\sqrt{x^2 + 3x - 18} + \frac{726}{16} \log |x + \frac{3}{2} + \sqrt{x^2 + 3x - 18}| + C$