Chapter : 15. INTEGRATION USING PARTIAL FRACTIONS

Exercise : 15A

Question: 1

Solution:

Let $I = \int \frac{dx}{x(x+2)}$,

Putting $\frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} \ldots \ldots . (1)$

Which implies A(x+2) + Bx = 1, putting x+2=0

Therefore x=-2,

And B = -0.5

Now put x=0, A= �

From equation (1), we get

$$\frac{1}{x(x+2)} = \frac{1}{2} \times \frac{1}{x} - \frac{1}{2} \times \frac{1}{x+2}$$

$$\int \frac{1}{x(x+2)} dx = \frac{1}{2} \int \frac{1}{x} dx - \frac{1}{2} \int \frac{1}{x+2} dx$$

$$= \frac{1}{2} \log|x| - \frac{1}{2} \log|x+2| + c$$

$$= \frac{1}{2} [\log|x| - \log|x+2|] + c$$

$$= \frac{1}{2} \log\left|\frac{x}{x+2}\right| + c$$

Question: 2

Solution:

Let $I = \int \frac{(2x+1)}{(x+2)(x+3)} dx$,

Putting $\frac{2x+1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3} \ldots \ldots \ldots \ldots (1)$

Which implies 2x=1 = A(x-3) + B(x+2)

Now put x-3=0, x=3

2×3+1=A(0)+B 3+2)

So $B = \frac{7}{5}$

Now put x+2=0 ,x=-2

-4+1=A(-2-3) + B(0)

So $A = \frac{3}{5}$

From equation (1), we get ,

$$\frac{2x+1}{(x+2)(x-3)} = \frac{3}{5} \times \frac{1}{x+2} + \frac{7}{5} \times \frac{1}{x-3}$$

$$\int \frac{2x+1}{(x+2)(x-3)}dx = \frac{3}{5}\int \frac{1}{x+2}dx + \frac{7}{5}\int \frac{1}{x-3}dx$$

$$= \frac{3}{5}\log|x+2| + \frac{7}{5}\log|x-3| + c$$

Question: 3

Solution:

Let $I = \int \frac{x}{(x+2)(3-2x)}dx,$

Putting $\frac{x}{(x+2)(3-2x)} = \frac{A}{x+2} + \frac{B}{3-2x} \dots\dots\dots (1)$

Which implies A(3-2x)+B(x+2)=x

Now put 3-2x=0

Therefore, $x = \frac{3}{2}$

$$A(0) + B\left(\frac{3}{2}+2\right) = \frac{3}{2}$$

$$B\left(\frac{7}{2}\right) = \frac{3}{2}$$

$$B = \frac{3}{7}$$

Now put x+2=0

Therefore, x=-2

A(7)+B(0)=-2

$$A = \frac{-2}{7}$$

Now From equation (1) we get

$$\frac{x}{(x+2)(3-2x)} = \frac{-2}{7} \times \frac{1}{x+2} + \frac{3}{7} \times \frac{1}{3-2x}$$

$$\int \frac{x}{(x+2)(3-2x)}dx = \frac{-2}{7}\int \frac{1}{x+2}dx + \frac{3}{7}\int \frac{1}{3-2x}dx$$

$$= \frac{-2}{7}\log|x+2| + \frac{3}{7} \times \frac{1}{-2}\log|3-2x| + c$$

$$= \frac{-2}{7}\log|x+2| + \frac{3}{7} \times \frac{1}{-2}\log|3-2x| + c$$

$$= \frac{-2}{7}\log|x+2| - \frac{3}{14}\log|3-2x| + c$$

Question: 4

Solution:

Let $I = \int \frac{dx}{x(x-2)(x-4)},$

Putting $\frac{1}{x(x-2)(x-4)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-4} \dots\dots (1)$

Which implies,

A(x-2)(x-4)+Bx(x-4)+Cx(x-2)=1

Now put x-2=0

Therefore, x=2

A(0)+B×2(2-4)+C(0)=1

B×2(-2)=1

$B = -\frac{1}{4}$

Now put x-4=0

Therefore, x=4

A(0)+B×(0)+C×4(4-2)=1

C×4(2)=1

$C = \frac{1}{8}$

Now put x=0

A(0-2)(0-4)+B(0)+C(0)=1

$A = \frac{1}{8}$

Now From equation (1) we get

$$\frac{1}{x(x-2)(x-4)} = \frac{1}{8} \times \frac{1}{x} - \frac{1}{4} \times \frac{1}{x-2} + \frac{1}{8} \times \frac{1}{x-4}$$

$$\int \frac{dx}{x(x-2)(x-4)} = \frac{1}{8}\int \frac{1}{x}dx - \frac{1}{4}\int \frac{1}{x-2}dx + \frac{1}{8}\int \frac{1}{x-4}dx$$

$$= \frac{1}{8}\log|x| - \frac{1}{4}\log|x-2| + \frac{1}{8}\log|x-4| + c$$

Question: 5

Solution:

Let $I = \int \frac{(2x-1)}{(x-1)(x+2)(x-3)} dx$

Putting $\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3} \ldots \ldots (1)$

Which implies,

A(x+2)(x-2)+B(x-1)(x-3)+C(x-1)(x+2)=2x-1

Now put x+2=0

Therefore, x=-2

A(0)+B(-2-1)(-2-3)+C(0)=2x-2-1

B(-3)(-5)=-5

$B = -\frac{1}{3}$

Now put x-3=0

Therefore, x=3

A(0)+B(0)+C(2)(5)=5

$C = \frac{1}{2}$

Now put x-1=0

Therefore, x=1

A(3)(-2)+B(0)+C(0)=1

$A = -\frac{1}{6}$

Now From equation (1) we get,

$$\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{-1}{6} \times \frac{1}{x-1} - \frac{1}{3} \times \frac{1}{x+2} + \frac{1}{2} \times \frac{1}{x-3}$$

$$\int \frac{(2x-1)}{(x-1)(x+2)(x-3)} dx = \frac{-1}{6} \int \frac{1}{x-1} dx - \frac{1}{3} \int \frac{1}{x+2} dx + \frac{1}{2} \int \frac{1}{x-3} dx$$

$$= \frac{-1}{6} \log|x-1| - \frac{1}{3} \log|x+2| + \frac{1}{2} \log|x-3| + c$$

Question: 6

Solution:

Let $I = \int \frac{(2x-3)}{(x^2-1)(2x+3)} dx$

Putting $\frac{(2x-3)}{(x-1)(x+1)(2x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{2x+3} \ldots\ldots(1)$

Which implies,

A(x+1)(2x+3)+B(x-1)(2x+3)+C(x-1)(x+1)=2x-3

Now put x+1=0

Therefore, x=-1

A(0)+B(-1-1)(-2+3)+C(0)=-2-3

$B = -\frac{5}{2}$

Now put x-1=0

Therefore, x=1

A(2)(2+3)+B(0)+C(0)=-1

$A = -\frac{1}{10}$

Now put 2x+3=0

Therefore, $x = -\frac{3}{2}$

$$A(0) + B(0) + C\left(\frac{-3}{2} - 1\right)\left(\frac{-3}{2} + 1\right) = 2\left(\frac{-3}{2}\right) - 3$$

$$C(\frac{-5}{2})(\frac{-1}{2}) = -3 - 3$$

$$C = -\frac{24}{5}$$

.Now From equation (1) we get,

$$\frac{(2x-3)}{(x^2-1)(2x+3)} = \frac{-1}{10} \times \frac{1}{x-1} + \frac{5}{2} \times \frac{1}{x+1} - \frac{24}{5} \times \frac{1}{2x+3}$$

$$\int \frac{(2x-3)}{(x^2-1)(2x+3)} dx = \frac{-1}{10} \int \frac{1}{x-1} dx + \frac{5}{2} \int \frac{1}{x+1} dx - \frac{24}{5} \int \frac{1}{2x+3} dx$$

$$= \frac{-1}{10} \log|x-1| + \frac{5}{2} \log|x+1| - \frac{24}{5} \frac{\log|2x+3|}{2} + c$$

$$= \frac{-1}{10} \log|x-1| + \frac{5}{2} \log|x+1| - \frac{12}{5} \log|2x+3| + c$$

Question: 7

Solution:

Let $I = \int \frac{(2x+5)}{(x^2-x-2)} dx = \int \frac{(2x+5)}{(x-2)(x+1)} dx$

Putting $\frac{(2x+5)}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$ ……. (1)

Which implies,

A(x+1)+B(x-2)=2x+5

Now put x+1=0

Therefore, x=-1

A(0)+B(-1-2)=3

B=-1

Now put x-2=0

Therefore, x=2

A(2+1)+B(0)=2×2+5=9

A=3

Now From equation (1) we get,

$$\frac{(2x+5)}{(x-2)(x+1)} = \frac{3}{x-2} + \frac{-1}{x+1}$$

$$\int \frac{(2x+5)}{(x-2)(x+1)} dx = \int \frac{3}{x-2} + \int \frac{-1}{x+1}$$

$$= 3\log|x-2| - \log|x+1| + c$$

Question: 8

Solution:

Let $I = \int \frac{(x^2+5x+3)}{(x^2+3x+2)} dx = \int \frac{x^2+3x+2+2x+1}{(x^2+3x+2)} dx = \int \frac{x^2+3x+2}{(x^2+3x+2)} dx + \int \frac{2x+1}{(x^2+3x+2)} dx$

Which implies $I = \int dx + \int \frac{2x+1}{(x^2+3x+2)} dx$

Therefore, $I = x + I_1$

Where, $I_1 = \int \frac{2x+1}{(x^2+3x+2)} dx$

Putting $\frac{(2x+1)}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$ ……. (1)

Which implies,

A(x+2)+B(x+1)=2x+1

Now put x+2=0

Therefore, x=-2

A(0)+B(-1)=-4+1

B=3

Now put x+1=0

Therefore, x=-1

A(-1+2)+B(0)=-2+1

A=-1

Now From equation (1) we get,

$$\frac{(2x+1)}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{3}{x+2}$$

$$\int \frac{(2x+1)}{(x+1)(x+2)} dx = -\int \frac{1}{x+1} dx + \int \frac{3}{x+2} dx$$

$$= -\log|x+1| + 3\log|x+2| + c$$

Question: 9

Solution:

Let $I = \int \frac{x^2+1}{x^2-1} dx$

$$I = \int (1 + \frac{2}{x^2-1}) dx$$

$$I = \int dx + 2\int \frac{1}{x^2-1} dx$$

$$I = x + 2 \times \frac{1}{2} \log\left|\frac{x-1}{x+1}\right| + c$$

$$I = x + \log\left|\frac{x-1}{x+1}\right| + c$$

Question: 10

Solution:

Let $I = \int \frac{x^3}{x^2-4} dx$

$$I = \int x + \frac{4x}{x^2-4} dx$$

$$I = \int x\, dx + \int \frac{4x}{x^2-4} dx$$

$$= \frac{x^2}{2} + \int \frac{4x}{(x-2)(x+2)} dx$$

Let $I_1 = \int \frac{4x}{(x-2)(x+2)} dx$

So

$I = \frac{x^2}{2} + I_1$

Therefore $I_1 = \int \frac{4x}{x^2-4} dx$

Putting $x^2-4=t$

$2xdx = dt$

$I_1 = 2\int \frac{dt}{t}$

$I_1 = 2log|x^2 - 4| + c$

Putting the value of I_1 in I,

$I = \frac{x^2}{2} + 2log|x^2 - 4| + c$

Question: 11

Solution:

Let $I = \int \frac{3+4x-x^2}{(x+2)(x-1)} dx$

$= \int \left(-1 + \frac{5x + 1}{(x+2)(x-1)}\right) dx$

$= \int -dx + \int \frac{5x + 1}{(x+2)(x-1)} dx$

$= -x + I_1$

$I_1 = \int \frac{5x + 1}{(x+2)(x-1)} dx$

Put $\frac{5x+1}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$

A(x-1)+B(x+2)=5x+1

Now put x-1=0

Therefore, x=1

A(0)+B(1+2)=5+1=6

B=2

Now put x+2=0

Therefore, x=-2

A(-2-1)+B(0)=5×(-2)+1

A=3

Now From equation (1) we get,

$\frac{5x + 1}{(x+2)(x-1)} = \frac{3}{(x+2)} + \frac{2}{(x-1)}$

$\int \frac{5x + 1}{(x+2)(x-1)} dx = 3\int \frac{1}{(x+2)} dx + 2\int \frac{1}{(x-1)} dx$

$3\log|x + 2| + 2\log|x - 1| + c$

Therefore,

$I = -x + 3log|x+2| + 2log|x-1| + c$

Question: 12

Solution:

Let $I = \int \frac{x^3}{(x-1)(x-2)} dx$

$= \int \left\{(x+3) + \frac{7x-6}{(x-1)(x-2)}\right\} dx$

$= \frac{x^2}{2} + 3x + \int \frac{7x-6}{(x-1)(x-2)} dx$

$= \frac{x^2}{2} + 3x + I_1 \ldots\ldots..(1)$

Where,

$I_1 = \int \frac{7x-6}{(x-1)(x-2)} dx$

Putting $\frac{7x-6}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$..........(2)

A(x-2)+B(x-1)=7x-6

Now put x-2=0

Therefore, x=2

A(0)+B(2-1)=7×2-6

B=8

Now put x-1=0

Therefore, x=1

A(1-2)+B(0)=7-6=1

A=-1

Now From equation (2) we get,

$\frac{7x-6}{(x-1)(x-2)} = \frac{-1}{x-1} + \frac{8}{x-2}$

$I_1 = \int \frac{7x-6}{(x-1)(x-2)} dx = -\int \frac{1}{x-1} dx + 8\int \frac{1}{x-2} dx$

$= -\log|x-1| + 8log|x-2| + c$

Now From equation (1) we get,

$I = \frac{x^2}{2} + 3x - \log|x-1| + 8log|x-2| + c$

Question: 13

Solution:

Let $I = \int \frac{(x^3-x-2)}{(1-x^2)} dx$

$= \int \left(-x + \frac{-2}{1-x^2}\right) dx$

$$= \int -x dx + (-2)\int \frac{1}{1-x^2} dx$$

$$= \frac{-x^2}{2} - log\left|\frac{1+x}{1-x}\right| + c$$

$$= \frac{-x^2}{2} + log\left|\frac{1-x}{1+x}\right| + c$$

Question: 14

Solution:

Let $I = \int \frac{2x+1}{(4-3x-x^2)} dx$

$$= \int \frac{2x+1}{(1-x)(4+x)} dx$$

Putting $\frac{2x+1}{(1-x)(4+x)} = \frac{A}{1-x} + \frac{B}{4+x} \ldots\ldots\ldots (1)$

A(4+x)+B(1-x)=2x+1

Now put 1-x=0

Therefore, x=1

A(5)+B(0)=3

$$A = \frac{3}{5}$$

Now put 4+x=0

Therefore, x=-4

A(0)+B(5)=-8+1=-7

$$B = \frac{-7}{5}$$

Now From equation (1) we get,

$$\frac{2x+1}{(1-x)(4+x)} = \frac{3}{5} \times \frac{1}{1-x} + \frac{-7}{5} \times \frac{1}{4+x}$$

$$\int \frac{2x+1}{(1-x)(4+x)} dx = \frac{3}{5}\int \frac{1}{1-x} dx + \frac{-7}{5}\int \frac{1}{4+x} dx$$

$$= \frac{-3}{5} log|1-x| - \frac{7}{5} log|4+x| + c$$

$$= -\frac{1}{5}[3log|1-x| + 7log|4+x|] + c$$

Question: 15

Solution:

Put x^2=t

2xdx=dt

$$\int \frac{dt}{(1+t)(3+t)} = \frac{1}{2}\int \left(\frac{1}{1+t} - \frac{1}{3+t}\right) dt$$

$$\frac{1}{2}[log|1+t| - log|3+t|] + c = \frac{1}{2} log\left|\frac{1+t}{3+t}\right| + c$$

$= \frac{1}{2} log \left|\frac{1+x^2}{3+x^2}\right| + c$

Question: 16

Solution:

Let $I = \int \frac{cosx}{(1+sinx)(2+sinx)} dx$

Putting t=sin x

dt=cos x dx

$I = \int \frac{dt}{(1+t)(2+t)},$

Now putting, $\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t} \ldots\ldots\ldots (1)$

A(2+t)+B(1+t)=1

Now put t+1=0

Therefore, t=-1

A(2-1)+B(0)=1

A=1

Now put t+2=0

Therefore, t=-2

A(0)+B(-2+1)=1

B=-1

Now From equation (1) we get,

$\frac{1}{(1+t)(2+t)} = \frac{1}{1+t} + \frac{-1}{2+t}$

$\int \frac{1}{(1+t)(2+t)} dt = \int \frac{1}{1+t} dt - \int \frac{1}{2+t} dt$

$= \log|1+t| - \log|t+2| + c$

$= log \left|\frac{t+1}{t+2}\right| + c$

So,

$I = \int \frac{cosx}{(1+sinx)(2+sinx)} dx = log \left|\frac{sinx+1}{sinx+2}\right| + c$

Question: 17

Solution:

Let $I = \int \frac{sec^2 x}{(2+tanx)(3+tanx)} dx$

Putting t=tanx

dt=sec^2xdx

$I = \int \frac{dt}{(2+t)(3+t)},$

Now putting, $\frac{1}{(3+t)(2+t)} = \frac{A}{2+t} + \frac{B}{3+t} \dots\dots\dots.(1)$

A(3+t)+B(2+t)=1

Now put t+2=0

Therefore, t=-2

A(3-2)+B(0)=1

A=1

Now put t+3=0

Therefore, t=-3

A(0)+B(2-3)=1

B=-1

Now From equation (1) we get,

$$\frac{1}{(2+t)(3+t)} = \frac{1}{2+t} + \frac{-1}{3+t}$$

$$\int \frac{1}{(2+t)(3+t)} dt = \int \frac{1}{2+t} dt - \int \frac{1}{3+t} dt$$

$$= \log|2+t| - \log|t+3| + c$$

$$= log\left|\frac{t+2}{t+3}\right| + c$$

So,

$$I = \int \frac{sec^2 x}{(2+tanx)(3+tanx)} dx = log\left|\frac{tanx+2}{tanx+3}\right| + c$$

Question: 18

Solution:

Let $I = \int \frac{sinx\, cosx}{cos^2x - cosx - 2} dx$

Putting t=cos x

dt=-sin x dx

$$I = \int \frac{(-dt)t}{t^2 - t - 2} =, - \int \frac{tdt}{(t+1)(t-2)},$$

Now putting, $\frac{-t}{(t+1)(t-2)} = \frac{A}{t+1} + \frac{B}{t-2} \dots\dots\dots.(1)$

A(t-2)+B(t+1)=-t

Now put t-2=0

Therefore, t=2

A(0)+B(2+1)=-2

$$B = \frac{-2}{3}$$

Now put t+1=0

Therefore, t=-1

A(-1-2)+B(0)=1

$A = \frac{-1}{3}$

Now From equation (1) we get,

$$\frac{-t}{(t+1)(t-2)} = \frac{-1}{3} \times \frac{1}{t+1} - \frac{2}{3} \times \frac{1}{t-2}$$

$$\int \frac{-t}{(t+1)(t-2)} dt = \frac{-1}{3} \int \frac{1}{t+1} - \frac{2}{3} \int \frac{1}{t-2}$$

$$= \frac{-1}{3} \log|t+1| - \frac{2}{3} \log|t-2| + c$$

So,

$$I = \int \frac{sinx\ cosx}{cos^2 x - cosx - 2} dx = \frac{-1}{3} \log|cosx + 1| - \frac{2}{3} \log|cosx - 2| + c$$

Question: 19

Solution:

Let $I = \int \frac{e^x}{e^{2x}+5e^x+6} dx$

Putting $t=e^x$

$dt=e^x\ dx$

$$I = \int \frac{dt}{(t^2+5t+6)},$$

Now putting, $\frac{1}{(t^2+5t+6)} = \frac{A}{2+t} + \frac{B}{3+t}$ ……….(1)

A(3+t)+B(2+t)=1

Now put t+2=0

Therefore, t=-2

A(3-2)+B(0)=1

A=1

Now put t+3=0

Therefore, t=-3

A(0)+B(2-3)=1

B=-1

Now From equation (1) we get,

$$\frac{1}{(2+t)(3+t)} = \frac{1}{2+t} + \frac{-1}{3+t}$$

$$\int \frac{1}{(2+t)(3+t)} dt = \int \frac{1}{2+t} dt - \int \frac{1}{3+t} dt$$

$$= \log|2+t| - \log|t+3| + c$$

$$= log\left|\frac{t+2}{t+3}\right| + c$$

$$= log\left|\frac{e^x+2}{e^x+3}\right| + c$$

Question: 20

Solution:

Let $I = \int \frac{e^x}{e^{3x}-3e^{2x}-e^x+3} dx$

Putting $t=e^x$

$dt=e^x\, dx$

$$I = \int \frac{dt}{(t^3-3t^2-t+3)} =, \int \frac{dt}{(t^2)(t-3)-(t-3)} = \int \frac{dt}{(t^2-1)(t-3)}$$

Now putting, $\frac{1}{(t-1)(t+1)(t-3)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{t-3} \dots\dots\dots (1)$

A(t+1)(t-3)+B(t-1)(t-3)+C(t-1)(t+1)=1

Now put t+1=0

Therefore, t=-1

A(0)+B(-1-1)(-1-3)+C(0)=1

B(-2)(-4)=1

$$B = \frac{1}{8}$$

Now put t-1=0

Therefore, t=1

A(1+1)(1-3)+B(0)+C(0)=1

$$A = \frac{-1}{4}$$

Now put t-3=0

Therefore, t=3

A(0)+B(0)+C(3-1)(3+1)=1

$$C = \frac{1}{8}$$

Now From equation (1) we get,

$$\frac{1}{(t-1)(t+1)(t-3)} = \frac{-1}{4} \times \frac{1}{t-1} + \frac{1}{8} \times \frac{1}{t+1} + \frac{1}{8} \times \frac{1}{t-3}$$

$$\int \frac{1}{(t-1)(t+1)(t-3)} = \frac{-1}{4}\int \frac{1}{t-1} + \frac{1}{8}\int \frac{1}{t+1} + \frac{1}{8}\int \frac{1}{t-3}$$

$$= \frac{-1}{4}\log|t-1| + \frac{1}{8}\log|t+1| + \frac{1}{8}\log|t-3| + c$$

$$\int \frac{e^x}{e^{3x}-3e^{2x}-e^x+3} dx = \frac{-1}{4}\log|e^x-1| + \frac{1}{8}\log|e^x+1| + \frac{1}{8}\log|e^x-3| + c$$

Question: 21

Solution:

Let $I = \int \frac{2\log x}{x[2(\log x)^2-\log x-3]} dx$

Putting t=log x

dt=dx/x

$I=\int \frac{2tdt}{(2t^2-t-3)},$

Now putting, $\frac{2t}{(2t^2-t-3)}=\frac{A}{2t-3}+\frac{B}{t+1}$ ……… ……. (1)

A(t+1)+B(2t-3)=2t

Now put 2t-3=0

Therefore, $t=\frac{3}{2}$

$A\left(\frac{3}{2}+1\right)+B(0)=2\times\frac{3}{2}=3$

$A=\frac{6}{5}$

Now put t+1=0

Therefore, t=-1

A(0)+B(-2-3)=-2

$B=\frac{2}{5}$

Now From equation (1) we get,

$\frac{2t}{(2t^2-t-3)}=\frac{6}{5}\times\frac{1}{2t-3}+\frac{2}{5}\times\frac{1}{t+1}$

$\int \frac{2t}{(2t^2-t-3)}dt=\frac{6}{5}\int\frac{1}{2t-3}dt+\frac{2}{5}\int\frac{1}{t+1}dt$

$=\frac{6}{5}\log\left|\frac{6}{5}\times\frac{log\ (2t-3)}{2}\right|+\frac{2}{5}\log|\log x+1|+c$

$\int \frac{2logx}{x[2(logx)^2-logx-3]}dx=\frac{3}{5}\log|2logx-3|+\frac{2}{5}\log|logx+1|+c$

Question: 22

Solution:

Let $I=\int\frac{cosec^2x}{(1-cot^2x)}dx$

Putting t=cot x

dt=-cosec²xdx

$I=\int\frac{-dt}{(1-t^2)}=-\int\frac{1}{(1-t^2)}dt$

$=\frac{-1}{2}log\left|\frac{1+cotx}{1-cotx}\right|+c$

Question: 23

Solution:

Let $I=\int\frac{sec^2x}{(tan^3x+4tanx)}dx$

Putting t=tan x

dt=sec²xdx

$$I=\int\frac{dt}{(t^3+4t)}=\int\frac{dt}{t(t^2+4)}$$

Now putting, $\frac{1}{t(t^2+4)}=\frac{A}{t}+\frac{Bt+C}{t^2+4}$ ……….. (1)

A(t^2+4)+(Bt + C)t=1

Putting t=0,

A(0+4)× B(0)=1

$$A=\frac{1}{4}$$

By equating the coefficients of t^2 and constant here,

A+B=0

$$\frac{1}{4}+B=0$$

$$B=-\frac{1}{4},C=0$$

Now From equation (1) we get,

$$\int\frac{1}{t(t^2+4)}dt=\frac{1}{4}\int\frac{dt}{t}-\frac{1}{4}\int\frac{t}{t^2+4}dt$$

$$=\frac{1}{4}logt-\frac{1}{4}\times\frac{1}{2}\log(t^2+4)+c$$

$$=\frac{1}{4}\log tanx-\frac{1}{8}\log(tan^2x+4)+c$$

Question: 41

Solution:

Let $I=\int\frac{dx}{x^3-1}$

Put $\frac{1}{x^3-1}=\frac{1}{(x-1)(x^2+x+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}$ ……….. (1)

A(x^2+x+1)+(Bx+C)(x-1)=1

Now putting x-1=0

X=1

A(1+1+1)+0=1

$$A=\frac{1}{3}$$

By equating the coefficient of x^2 and constant term, A+B=0

$$\frac{1}{3}+B=0$$

$$B=-\frac{1}{3}$$

A-C=1

$$\frac{1}{3}-C=1$$

$$C=\frac{1}{3}-1$$

$C = \frac{-2}{3}$

From the equation(1), we get,

$$\frac{1}{(x-1)(x^2+x+1)} = \frac{1}{3} \times \frac{1}{x-1} + \frac{-\frac{1}{3}x - \frac{2}{3}}{x^2+x+1}$$

$$I = \int \frac{1}{(x-1)(x^2+x+1)} dx$$

$$= \frac{1}{3}\int \frac{1}{x-1} dx - \frac{1}{3}\int \frac{x}{x^2+x+1} dx - \frac{2}{3}\int \frac{1}{x^2+x+1} dx$$

$$= \frac{1}{3} log|x-1| - \frac{1}{6}\int \frac{2x+1-1}{x^2+x+1} dx - \frac{2}{3}\int \frac{1}{x^2+x+1} dx$$

$$= \frac{1}{3} log|x-1| - \frac{1}{6}\int \frac{2x+1}{x^2+x+1} dx + \frac{1}{6}\int \frac{1}{x^2+x+1} dx - \frac{2}{3}\int \frac{1}{x^2+x+1} dx$$

Put $t=x^2+x+1$

$dt=(2x+1)dx$

$$I = \frac{1}{3} log|x-1| - \frac{1}{6}\int \frac{dt}{t} + \left(\frac{1}{6} - \frac{2}{3}\right)\int \frac{dx}{x^2+x+1}$$

$$= \frac{1}{3} log|x-1| - \frac{1}{6}\log t + \left(\frac{1-4}{6}\right)\int \frac{dx}{x^2 + 2 \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$= \frac{1}{3} log|x-1| - \frac{1}{6} log|x^2+x+1| - \frac{1}{2} \times \frac{1}{\sqrt{3}/2} tan^{-1}\frac{x+1/2}{\sqrt{3}/2} + c$$

$$= \frac{1}{3} log|x-1| - \frac{1}{6} log|x^2+x+1| - \frac{1}{\sqrt{3}} tan^{-1}\frac{2x+1}{\sqrt{3}} + c$$

Question: 42

Solution:

Let $I = \int \frac{dx}{x^3+1}$

Put $\frac{1}{x^3-1} = \frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \ldots\ldots\ldots.(1)$

$A(x^2-x+1)+(Bx+C)(x+1)=1$

Now putting x+1=0

X=-1

A(1+1+1)+C(0)=1

$A = \frac{1}{3}$

By equating the coefficient of x^2 and constant term, A+B=0

$\frac{1}{3} + B = 0$

$B = -\frac{1}{3}$

A+C+=1

$\frac{1}{3} + C = 1$

$C = 1 - \frac{1}{3}$

$C = \frac{2}{3}$

From the equation(1), we get,

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{1}{3} \times \frac{1}{x+1} + \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2-x+1}$$

$$I = \int \frac{1}{(x+1)(x^2-x+1)} dx$$

$$= \frac{1}{3}\int \frac{1}{x+1} dx - \frac{1}{3}\int \frac{x}{x^2-x+1} dx + \frac{2}{3}\int \frac{1}{x^2-x+1} dx$$

$$= \frac{1}{3} log|x+1| - \frac{1}{6}\int \frac{2x-1+1}{x^2-x+1} dx + \frac{2}{3}\int \frac{1}{x^2-x+1} dx$$

$$= \frac{1}{3} log|x+1| - \frac{1}{6}\int \frac{2x-1}{x^2-x+1} dx - \frac{1}{6}\int \frac{1}{x^2-x+1} dx + \frac{2}{3}\int \frac{1}{x^2-x+1} dx$$

$$= \frac{1}{3} log|x+1| - \frac{1}{6} log|x^2-x+1| - \frac{1}{2} \times \frac{1}{\sqrt{3}/2} tan^{-1} \frac{x-1/2}{\sqrt{3}/2} + c$$

$$= \frac{1}{3} log|x+1| - \frac{1}{6} log|x^2-x+1| + \frac{1}{\sqrt{3}} tan^{-1} \frac{2x-1}{\sqrt{3}} + c$$

Question: 24

Solution:

Let $I = \int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$

Putting t=sin x

dt=cos x dx

$$I = \int \frac{2t}{(1+t)(2+t)} dt$$

Now putting, $\frac{2t}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t} \ldots\ldots\ldots(1)$

A(2+t)+B(1+t)=2t

Now put t+2=0

Therefore, t=-2

A(0)+B(1-2)=-4

B=4

Now put t+1=0

Therefore, t=-1

A(2-)+B(0)=-2

A=-2

Now from equation (1), we get,

$$\frac{2t}{(1+t)(2+t)} = \frac{-2}{1+t} + \frac{4}{2+t}$$

$$\int \frac{2t}{(1+t)(2+t)}dt = -2\int \frac{1}{1+t}dt + 4\int \frac{1}{2+t}dt$$

$= 4\log|2+t| - 2\log|1+t| + c$

So,

$$\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)}dx = 4\log|2+t| - 2\log|1+t| + c$$

Question: 43

Solution:

Let $I = \int \frac{dx}{(x^2+1)(x+1)^2}$

Put $\frac{1}{(x^2+1)(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1} \ldots\ldots\ldots..(1)$

$A(x+1)(x^2+1)+B(x^2+1)+(Cx+D)(x+1)^2=1$

Put x+1=0

X=-1

A(0)+B(1+1)+0=1

$B = \frac{1}{2}$

By equating the coefficient of x^2 and constant term, A+C=0

A+B+2C=0……(2)

$A + 2C = \frac{-1}{2} \ldots\ldots\ldots.(3)$

A+B+D=1

Solving (2) and (3),we get,

$$\frac{1}{(x^2+1)(x+1)^2} = \frac{1}{2}\times\frac{1}{x+1} + \frac{1}{2}\times\frac{1}{(x+1)^2} + \frac{-\frac{1}{2}x+0}{x^2+1}$$

$$\int \frac{1}{(x^2+1)(x+1)^2}dx = \frac{1}{2}\int\frac{1}{x+1}dx + \frac{1}{2}\int\frac{1}{(x+1)^2}dx - \frac{1}{2}\int\frac{x}{x^2+1}dx$$

$$= \frac{1}{2}log|x+1| - \frac{1}{2}\times\frac{1}{x+1} - \frac{1}{4}\log|x^2+1| + c$$

Question: 25

Solution:

Let $I = \int \frac{e^x}{e^x(e^x-1)}dx$

Putting $t=e^x$

$dt=e^x dx$

$$I = \int \frac{dt}{t(t-1)}$$

Now putting, $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} \ldots\ldots..(1)$

A(t-1)+Bt=1

Now put t-1=0

Therefore, t=1

A(0)+B(1) =1

B=1

Now put t=0

A(0-1)+B(0)=1

A=-1

Now From equation (1) we get,

$$\frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\int \frac{1}{t(t-1)} dt = -\int \frac{1}{t} dt + \int \frac{1}{t-1} dt$$

$$= -\log t + \log|t-1| + c$$

$$= log\left|\frac{t-1}{t}\right| + c$$

$$= log\left|\frac{e^x - 1}{e^x}\right| + c$$

Question: 44

Solution:

Let $I = \int \frac{17}{(2x+1)(x^2+4)} dx$

Put $\frac{17}{(2x+1)(x^2+4)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+4} \ldots\ldots\ldots (1)$

$A(x^2+4)+(Bx+C)(2x+1) = 17$

Put 2x+1=0

$$x = -\frac{1}{2}$$

$$A\left(\frac{1}{4} + 4\right) + 0 = 17$$

$$A\left(\frac{17}{4}\right) = 17$$

A=4

By equating the coefficient of x^2 and constant term,

A+2B=0

4+2B=0

B=-2

4A+C=17

4×4+C=17

C=1

From the equation(1), we get,

$$\frac{17}{(2x+1)(x^2+4)} = \frac{4}{2x+1} + \frac{-2x+1}{x^2+4}$$

$$\int \frac{17}{(2x+1)(x^2+4)}dx = 4\int \frac{1}{2x+1}dx - 2\int \frac{2x}{x^2+4}dx + \int \frac{1}{x^2+2^2}dx$$

$$= \frac{4\log|2x+1|}{2} - \log|x^2+4| + \frac{1}{2}tan^{-1}\frac{x}{2} + c$$

$$= 2\log|2x+1| - \log|x^2+4| + \frac{1}{2}tan^{-1}\frac{x}{2} + c$$

Question: 26

Solution:

Let $I = \int \frac{dx}{x(x^4-1)}dx$

Putting $t=x^4$

$dt=4x^3dx$

$$I = \int \frac{x^3dx}{x^4(x^4-1)} = \frac{1}{4} \times \int \frac{dt}{t(t-1)}$$

Now putting, $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$ ……..(1)

A(t-1)+Bt=1

Now put t-1=0

Therefore, t=1

A(0)+B(1) =1

B=1

Now put t=0

A(0-1)+B(0)=1

A=-1

Now From equation (1) we get,

$$\frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\frac{1}{4}\int \frac{1}{t(t-1)}dt = -\frac{1}{4}\int \frac{1}{t}dt + \frac{1}{4}\int \frac{1}{t-1}dt$$

$$= -\frac{1}{4}\log t + \frac{1}{4}\log|t-1| + c$$

$$= -\frac{1}{4}\log x^4 + \frac{1}{4}\log|x^4-1| + c$$

$$= -\log|x| + \frac{1}{4}\log|x^4-1| + c$$

Question: 45

Solution:

Let $I = \int \frac{dx}{(x^2+2)(x^2+4)}dx$

Put $\frac{1}{(x^2+2)(x^2+4)} = \frac{1}{(t+2)(t+4)} = \frac{A}{t+2} + \frac{B}{t+4}$ ………..(1)

A(t+4)+B(t+2) = 1

Put t+4=0

t=-4

A(0)+B(-4+2)=1

$$B = -\frac{1}{2}$$

Put t+2=0

t=-2

A(-2+4)+B(0)=1

$$A = \frac{1}{2}$$

From equation(1),we get,

$$\frac{1}{(t+2)(t+4)} = \frac{1}{2} \times \frac{1}{t+2} - \frac{1}{2} \times \frac{1}{t+4}$$

$$\int \frac{1}{(x^2+2)(x^2+4)}dx = \frac{1}{2}\int \frac{1}{x^2+2}dx - \frac{1}{2}\int \frac{1}{x^2+4}dx$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} tan^{-1}\frac{x}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{2} tan^{-1}\frac{x}{2} + c$$

$$= \frac{1}{4} tan^{-1}\frac{x}{\sqrt{2}} - \frac{1}{4} tan^{-1}\frac{x}{2} + c$$

Question: 27

Solution:

Let $I = \int \frac{(x^2-1)}{x(2x-1)}dx = \int \left(\frac{1}{2} + \frac{(\frac{1}{2}x-1)}{x(2x-1)}\right)dx = \int \frac{1}{2}dx + \int \frac{x}{x(2x-1)}dx - \int \frac{1}{x(2x-1)}dx$

$$I = \frac{1}{2}x + \frac{1}{2} \times \frac{\log|2x-1|}{2} - I_1 \ldots \ldots (1)$$

Where $I_1 = \int \frac{1}{x(2x-1)}dx$ ....(2)

Now putting, $\frac{1}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1}$

A(2x-1)+Bx=1

Putting 2x-1=0

$$x = \frac{1}{2}$$

$$A(0) + B\left(\frac{1}{2}\right) = 1$$

B=2

Putting x=0,

A(0-1)+B(0)=1

A=-1

From equation (2), we get,

$$\frac{1}{x(2x-1)} = -\frac{1}{x} + \frac{2}{2x-1}$$

$$\int \frac{1}{x(2x-1)}dx = -\int \frac{1}{x}dx + 2\int \frac{1}{2x-1}dx$$

$$= -\log|x| + \frac{2\log|2x-1|}{2} + c$$

$$= \log|2x-1| - \log x + c$$

From equation (1),

$$I = \frac{1}{2}x + \frac{1}{4}\log|2x-1| - \log|2x-1| + logx + c$$

$$= \frac{1}{2}x - \frac{3}{4}\log|1-2x| + log|x| + c$$

Question: 46

Putting $\frac{x^2+1}{(x^2+4)(x^2+25)} = \frac{t+1}{(t+4)(t+25)} = \frac{A}{t+4} + \frac{B}{t+25} \ldots\ldots\ldots (1)$

Where t=x^2

(A+B)t+(25A+4B)=t+1

A+B=1 ....(1)

25A+4B=1............ (2)

Solving equation (1)and(2), we get,

$$A = \frac{-1}{7} \text{ and } B = \frac{8}{7}$$

Now,

$$\frac{t+1}{(t+4)(t+25)} = \frac{-1}{7} \times \frac{1}{t+4} + \frac{8}{7} \times \frac{1}{t+25}$$

$$\frac{x^2+1}{(x^2+4)(x^2+25)} = \frac{-1}{7} \times \frac{1}{x^2+4} + \frac{8}{7} \times \frac{1}{x^2+25}$$

$$\int \frac{x^2+1}{(x^2+4)(x^2+25)}dx = \frac{-1}{7}\int \frac{1}{x^2+2^2}dx + \frac{8}{7}\int \frac{1}{x^2+5^2}dx$$

$$= -\frac{1}{7} \times \frac{1}{2}tan^{-1}\left(\frac{x}{2}\right) + \frac{8}{7} \times \frac{1}{5}tan^{-1}\left(\frac{x}{5}\right) + c$$

$$= -\frac{1}{14}tan^{-1}\left(\frac{x}{2}\right) + \frac{8}{35}tan^{-1}\left(\frac{x}{5}\right) + c$$

Question: 28

Solution:

Let $I = \int \frac{x^2+x+1}{(x+2)(x+1)^2}dx$

Now putting, $\frac{x^2+x+1}{(x+2)(x+1)^2} = \frac{A}{(x+2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} \ldots\ldots\ldots (1)$

$A(x+1)^2+B(x+2)(x+1)+C(x+2)=x^2+x+1$

Now put x+1=0

Therefore, x=-1

A(0)+B(0)+C(-1+2) =1-1+1=1

C=1

Now put x+2=0

Therefore, x=-2

A(-2+1)²+B(0)+C(0) =4-2+1=3

A=3

Equating the coefficient of x^2,A+B=1

3+B=1

B=-2

Form equation (1),we get,

$$\frac{x^2+x+1}{(x+2)(x+1)^2} = \frac{3}{(x+2)} - \frac{2}{(x+1)} + \frac{1}{(x+1)^2}$$

So,

$$\int \frac{x^2+x+1}{(x+2)(x+1)^2}dx = \int \frac{3}{(x+2)}dx - \int \frac{2}{(x+1)}dx + \int \frac{1}{(x+1)^2}dx$$

$$= 3\log|x+2| - 2\log|x+1| - \frac{1}{1+x} + c$$

Question: 29

Solution:

Let $I = \int \frac{2x+9}{(x+2)(x-3)^2}dx$

Now putting, $\frac{2x+9}{(x+2)(x-3)^2} = \frac{A}{(x+2)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2} \ldots\ldots(1)$

$A(x-3)^2+B(x+2)(x-3)+C(x+2)=2x+9$

Now put x-3=0

Therefore, x=3

A(0)+B(0)+C(3+2) =6+9=15

C=3

Now put x+2=0

Therefore, x=-2

$A(-2-3)^2+B(0)+C(0) = -4+9=5$

$$A = \frac{1}{5}$$

Equating the coefficient of x^2,we get,

A+B=0

$$\frac{1}{5} + B = 0$$

$$B = -\frac{1}{5}$$

From equation (1), we get,

$$\frac{2x+9}{(x+2)(x-3)^2} = \frac{1}{5}\times\frac{1}{(x+2)} - \frac{1}{5}\times\frac{1}{(x-3)} + \frac{3}{(x-3)^2}$$

$$\int \frac{2x+9}{(x+2)(x-3)^2}dx = \frac{1}{5}\int \frac{1}{(x+2)}dx - \frac{1}{5}\int \frac{1}{(x-3)}dx + 3\int \frac{1}{(x-3)^2}dx$$

$$= \frac{1}{5}\log|x+2| - \frac{1}{5}\log|x-3| - \frac{3}{x-3} + c$$

Question: 47

Solution:

putting $t=e^x-1$

$e^x=t+1$

$dt= e^x\, dx$

$$\frac{dt}{e^x} = dx$$

$$\frac{dt}{t+1} = dx$$

Put $\frac{1}{(1+t)t^2} = \frac{A}{t+1} + \frac{Bt+C}{t^2}$ .... (1)

$A(t^2)+(Bt+C)(t+1)=1$

Put $t+1=0$

$t=-1$

$A=1$

Equating coefficients

$A+B=0$

$1+B=0$

$B=-1$

$C=1$

From equation (1),we get,

$$\frac{1}{(1+t)t^2} = \frac{1}{t+1} + \frac{-t+1}{t^2}$$

$$\int \frac{1}{(1+t)t^2}dt = \int \frac{1}{t+1}dt - \int \frac{t}{t^2}dt + \int \frac{1}{t^2}dt$$

$$= \log|t+1| - \int \frac{1}{t}dt + \int \frac{1}{t^2}dt$$

$$= \log|t+1| - log|t| - \frac{1}{t} + c$$

$$\int \frac{1}{(e^x-1)^2}dx = log\,|e^x| - \log|e^x-1| - \frac{1}{e^x-1} + c$$

Question: 48

Solution:

Let $I = \int \frac{dx}{x(x^5+1)}$

Put $t=x^5$

$dt=5x^4dx$

$$\int \frac{dt}{\frac{(5x^4)}{x(t+1)}} = \frac{1}{5}\int \frac{dt}{x^5(t+1)} = \frac{1}{5}\int \frac{dt}{t(t+1)}$$

Putting $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} \ldots\ldots..(1)$

A(t+1)+Bt=1

Now put t+1=0

t=-1

A(0)+B(-1)=1

B=-1

Now put t=0

A(0+1)+B(0)=1

A=1

$$\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$$

$$\int \frac{1}{t(t+1)}dt = \int \frac{1}{t}dt - \int \frac{1}{t+1}dt$$

$$= logt - \log|t+1| + c$$

$$= log\left|\frac{t}{t+1}\right| + c$$

$$\int \frac{dx}{x(x^5+1)} = \frac{1}{5}\int \frac{dt}{t(t+1)} = \frac{1}{5}log\left|\frac{x^5}{x^5+1}\right| + c$$

$$= logx - \frac{1}{5}log|x^5+1| + c$$

Question: 30

Solution:

Let $I = \int \frac{x^2+1}{(x+3)(x-1)^2}dx$

Now putting, $\frac{x^2+1}{(x+3)(x-1)^2} = \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \ldots\ldots..(1)$

$A(x-1)^2+B(x+3)(x-1)+C(x+3)=x^2+1$

Now put x-1=0

Therefore, x=1

A(0)+B(0)+C(4) =2

$$C = \frac{1}{2}$$

Now put x+3=0

Therefore, x=-3

$A(-3-1)^2+B(0)+C(0) =9+1=10$

$$A = \frac{5}{8}$$

By equating the coefficient of x^2, we get, A+B=1

$\frac{5}{8} + B = 1$

$B = 1 - \frac{5}{8} = \frac{3}{8}$

From equation (1), we get,

$$\frac{x^2+1}{(x+3)(x-2)^2} = \frac{5}{8} \times \frac{1}{(x+3)} + \frac{3}{8} \times \frac{1}{(x-2)} + \frac{1}{(x-2)^2}$$

$$\int \frac{x^2+1}{(x+3)(x-2)^2} dx = \frac{5}{8}\int \frac{1}{(x+3)} dx + \frac{3}{8}\int \frac{1}{(x-2)} dx + \int \frac{1}{(x-2)^2} dx$$

$$= \frac{5}{8}\log|x+3| + \frac{3}{8}\log|x-1| - \frac{1}{2(x-1)} + c$$

Question: 49

Solution:

Let $I = \int \frac{dx}{x(x^6+1)}$

Put $t=x^6$

$dt=6x^5dx$

$$\int \frac{dt}{\frac{(6x^5)}{x(t+1)}} = \frac{1}{6}\int \frac{dt}{x^6(t+1)} = \frac{1}{6}\int \frac{dt}{t(t+1)}$$

Putting $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$ ………(1)

A(t+1)+Bt=1

Now put t+1=0

t=-1

A(0)+B(-1)=1

B=-1

Now put t=0

A(0+1)+B(0)=1

A=1

$$\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$$

$$\int \frac{1}{t(t+1)} dt = \int \frac{1}{t} dt - \int \frac{1}{t+1} dt$$

$$= logt - \log|t+1| + c$$

$$= log\left|\frac{t}{t+1}\right| + c$$

$$\int \frac{dx}{x(x^6+1)} = \frac{1}{6}\int \frac{dt}{t(t+1)} = \frac{1}{6} log\left|\frac{x^6}{x^6+1}\right| + c$$

$$= logx - \frac{1}{6} log|x^6+1| + c$$

Question: 31

Solution:

Let $I = \int \frac{x^2+1}{(x-3)(x-1)^2} dx$

Now putting, $\frac{x^2+1}{(x-3)(x-1)^2} = \frac{A}{(x-3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \dots \dots . . (1)$

$A(x-1)^2+B(x-3)(x-1)+C(x-3)=x^2+1$

Putting x-1=0,

X=1

A(0)+B(0)+C(1-3)=1+1

C=-1

Putting x-3=0,

X=3

$A(3-1)^2+B(0)+C(0)=9+1$

A(4)=10

$A = \frac{5}{2}$

Equating the coefficient of x^2

A+B=1

$\frac{5}{2} + B = 1$

$B = 1 - \frac{5}{2} = \frac{-3}{2}$

From (i) $\int \frac{x^2+1}{(x-3)(x-1)^2} dx = \frac{5}{2} \int \frac{1}{x-3} dx - \frac{3}{2} \int \frac{1}{x-1} dx - \int \frac{1}{(x-1)^2} dx$

$= \frac{5}{2} log|x-3| - \frac{3}{2} \log|x-1| + \frac{1}{x-1} + C$

Question: 32

Solution:

Let $I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$

Now putting, $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$

$A(x^2+1)+(Bx+C)(x+2) = x^2+x+1$

$Ax^2+A+Bx^2+Cx+2Bx+2C = x^2+x+1$

$(A+B)x^2+(C+2B)x+(A+2C) = x^2+x+1$

Equating coefficients A+B=1…….(i)

A+2C=1

A=1-2C……(ii)

2B+C=1

2B=1-C

$B = \frac{1-C}{2} \dots\dots..(iii)$

$(1-2C) + \frac{1-C}{2} = 1$

2-4C+1-C=2

3-5C=2

-5C=-1

$C = \frac{1}{5}$

And $2B = 1 - \frac{1}{5} = \frac{4}{5}$

$B = \frac{2}{5}$

$A = 1 - 2 \times \frac{1}{5}$

$= 1 - \frac{2}{5}$

$= \frac{3}{5}$

$I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx = \int \frac{A}{(x+2)} dx + \int \frac{Bx+C}{(x^2+1)} dx$

$= \frac{3}{5} \times \int \frac{1}{(x+2)} dx + \frac{1}{5} \times \int \frac{2x+1}{(x^2+1)} dx$

$= \frac{3}{5} \log|x+2| + \frac{1}{5} I_1 + C_1$

$I_1 = \int \frac{2x+1}{(x^2+1)} dx = \int \frac{2x}{(x^2+1)} dx + \int \frac{1}{(x^2+1)} dx$

$= log|x^2+1| + tan^{-1}x + C_2$

$I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx = \frac{3}{5} \log|x+2| + \frac{1}{5} log|x^2+1| + \frac{1}{5} tan^{-1}x + C$

Question: 50

Solution:

let $I = \int \frac{dx}{sinx\,(3+2cosx)}$

Put t=cosx

dt=-sinxdx

$\frac{dt}{-sinx} = dx$

$I = \int \frac{\frac{dt}{-sinx}}{sinx\,(3+2t)}$

$$= -\int \frac{dt}{sin^2x(3+2t)} = -\int \frac{dt}{(1-cos^2x)(3+2t)}$$

$$= -\int \frac{dt}{(1-t^2)(3+2t)}$$

$$\frac{1}{(1-t^2)(3+2t)} = \frac{1}{(1-t)(1+t)(3+2t)}$$

Putting $\frac{1}{(1-t)(1+t)(3+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{3+2t} \dots \dots . (1)$

A(1+t)(3+2t)+B(1-t)(3+2t)+C(1+t)(1-t)=1

Now Putting 1+t=0

t=-1

A(0)+B(2)(3-2)+C(0)=1

$$B = \frac{1}{2}$$

Now Putting 1-t=0

t=1

A(2)(5)+B(0)+C(0)=1

$$A = \frac{1}{10}$$

Now Putting 3+2t=0

$$t = -\frac{3}{2}$$

$$A(0) + B(0) + C\left(1 - \frac{9}{4}\right) = 1$$

$$C = \frac{-4}{5}$$

$$\frac{1}{(1-t)(1+t)(3+2t)} = \frac{1}{10} \times \frac{1}{1-t} + \frac{1}{2} \times \frac{1}{1+t} - \frac{4}{5} \times \frac{1}{3+2t}$$

$$\int \frac{1}{(1-t)(1+t)(3+2t)} dt = \frac{1}{10} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{1+t} dt - \frac{4}{5} \int \frac{1}{3+2t} dt$$

$$= -\frac{1}{10} log|1-t| + \frac{1}{2} log|1+t| - \frac{4}{5} \times \frac{log|3+2t|}{2} + c$$

$$= -\frac{1}{10} log|1-cosx| + \frac{1}{2} log|1+cosx| - \frac{2}{5} log|3+2cosx| + c$$

Question: 33

Solution:

Let $I = \int \frac{2x}{(2x+1)^2} dx$

Now putting, $\frac{2x}{(2x+1)^2} = \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} \dots \dots \dots \dots . (1)$

A(2x+1)+B = 2x

Putting 2x+1=0,

$$x = \frac{-1}{2}$$

A(0)+B=-1

B=-1

By equating the coefficient of x,

2A=2

A=1

From equation (1),we get,

$$\frac{2x}{(2x+1)^2} = \frac{1}{(2x+1)} - \frac{1}{(2x+1)^2}$$

$$\int \frac{2x}{(2x+1)^2}dx = \int \frac{1}{(2x+1)}dx - \int \frac{1}{(2x+1)^2}dx$$

$$= \frac{\log|2x+1|}{2} + \frac{1}{2(2x+1)} + c$$

$$= \frac{1}{2}\left[\log|2x+1| + \frac{1}{2x+1}\right] + c$$

Question: 51

Solution:

let $I = \int \frac{dx}{cosx\,(5-4sinx)}$

Put t=sinx

dt=cosxdx

$$I = \int \frac{dt}{(1-sin^2x)(5-4t)} = \int \frac{dt}{(1-t^2)(5-4t)}$$

$$\frac{1}{(1-t^2)(5-4t)} = \frac{1}{(1-t)(1+t)(5-4t)}$$

Putting $\frac{1}{(1-t)(1+t)(5-4t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{5-4t} \ldots\ldots(1)$

A(1+t)(5-4t)+B(1-t)(5-4t)+C(1+t)(1-t)=1

Now Putting 1+t=0

t=-1

A(0)+B(2)(9)+C(0)=1

$$B = \frac{1}{18}$$

Now Putting 1-t=0

t=1

A(2) +B(0)+C(0)=1

$$A = \frac{1}{2}$$

Now Putting 5-4t=0

$$t = \frac{5}{4}$$

$$A(0) + B(0) + C\left(1 - \frac{25}{16}\right) = 1$$

$$C = \frac{-16}{9}$$

From equation(1),we get,

$$\frac{1}{(1-t)(1+t)(5-4t)} = \frac{1}{2} \times \frac{1}{1-t} + \frac{1}{18} \times \frac{1}{1+t} - \frac{16}{9} \times \frac{1}{5-4t}$$

$$\int \frac{1}{(1-t)(1+t)(5-4t)} dt = \frac{1}{2}\int \frac{1}{1-t} dt + \frac{1}{18}\int \frac{1}{1+t} dt - \frac{16}{9}\int \frac{1}{5-4t} dt$$

$$= -\frac{1}{2} log|1-t| + \frac{1}{18} log|1+t| - \frac{16}{9} \times \frac{log|5-4t|}{-4} + c$$

$$= -\frac{1}{2} log|1-sinx| + \frac{1}{18} log|1+sinx| + \frac{4}{9} log|5-4sinx| + c$$

Question: 34

Solution:

Let $I = \int \frac{3x+1}{(x+2)(x-2)^2} dx$

Now putting, $\frac{3x+1}{(x+2)(x-2)^2} = \frac{A}{(x+2)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} \ldots \ldots . . (1)$

$A(x-2)^2+B(x+2)(x-2)+C(x+2)=3x+1$

Putting x-2=0,

X=2

A(0)+B(0)+C(2+1)=3×2+1

$C = \frac{7}{4}$

Putting x+2=0,

X=-2

$A(-4)^2+B(0)+C(0)=-6+1=-5$

$A = \frac{-5}{16}$

By equation the coefficient of x^2, we get, A+B=0

$\frac{-5}{16} + B = 0$

$B = \frac{5}{16}$

$I = -\frac{5}{16} log|x+2| + \frac{5}{16} log|x-2| - \frac{7}{4(x-2)} + c$

Question: 52

Solution:

Let $I = \int \frac{1}{sinx \times cos^2 x} dx = \int \frac{sin^2 x + cos^2 x}{sinx \times cos^2 x} dx = \int \frac{sin^2 x}{sinx \times cos^2 x} dx + \int \frac{cos^2 x}{sinx \times cos^2 x} dx$

$$= \int \frac{sinx}{cos^2 x} dx + \int \frac{1}{sinx} dx$$

$$= \int (\tan x \sec x + cosec\, x) dx$$

$$= \sec x - \frac{1}{2} logcot^2 \frac{x}{2} = \sec x - \frac{1}{2} \log\left(\frac{1+cosx}{1-cosx}\right) + c$$

Question: 53

Solution:

let $I=\int\frac{\tan x}{(1-\sin x)}dx=\int\frac{\sin x}{\cos x(1-\sin x)}dx$

Put t=sinx

dt=cosxdx

$$I=\int\frac{\sin x\times\cos x}{\cos^2 x\,(1-\sin x)}dx=\int\frac{tdt}{(1-\sin^2 x)(1-t)}=\int\frac{tdt}{(1-t^2)(1-t)}$$

Putting $\frac{t}{(1-t)(1+t)(1-t)}=\frac{A}{1+t}+\frac{B}{1-t}+\frac{C}{(1-t)^2}\ldots\ldots(1)$

$A(1+t)^2+B(1-t)(1+t)+C(1+t)=t$

Now Putting 1-t=0

t=1

A(0)+B(0)+C(1+1)=1

$$C=\frac{1}{2}$$

Now Putting 1+t=0

t=-1

$A(2)^2+B(0)+C(0)=-1$

$$A=-\frac{1}{4}$$

By equating the coefficient of t^2,we get,A-B=0

$$\frac{-1}{4}-B=0$$

$$B=-\frac{1}{4}$$

From equation(1),we get,

$$\frac{t}{(1-t)(1+t)(1-t)}=\frac{-1}{4}\times\frac{1}{1+t}-\frac{1}{4}\times\frac{1}{1-t}+\frac{1}{2}\times\frac{1}{(1-t)^2}$$

$$\int\frac{t}{(1-t)(1+t)(1-t)}dt=\frac{-1}{4}\int\frac{1}{1+t}dt-\frac{1}{4}\int\frac{1}{1-t}dt+\frac{1}{2}\int\frac{1}{(1-t)^2}dt$$

$$=\frac{-1}{4}\int\frac{1}{1+t}dt-\frac{1}{4}\int\frac{1}{1-t}dt+\frac{1}{2}\int\frac{1}{(1-t)^2}dt$$

$$=-\frac{1}{4}log|1+t|-\frac{1}{4}log|1-t|-\frac{1}{2}\times\frac{1}{1-t}+c$$

$$=-\frac{1}{4}log|1+sinx|-\frac{1}{4}log|1-sinx|-\frac{1}{2}\times\frac{1}{1-sinx}+c$$

Question: 35

Solution:

Let $I=\int\frac{5x+8}{x^2(3x+8)}dx$

Now putting, $\frac{5x+8}{x^2(3x+8)}=\frac{A}{(3x+8)}+\frac{Bx+C}{x^2}\ldots\ldots(1)$

$Ax^2+(Bx + C)(3x+8) = 5x+8$

CLASS24

Putting 3x+8=0,

$x = -\frac{8}{3}$

$A\left(\frac{64}{9}\right) + B(0) = 5\left(-\frac{8}{3}\right) + 8$

$A\left(\frac{64}{9}\right) = \frac{-40 + 24}{3}$

$A\left(\frac{64}{9}\right) = \frac{-16}{3}$

$A = \frac{-3}{4}$

By equating the coefficient of x^2 and constant term,

A+3B=0

$\frac{-3}{4} + 3B = 0$

$3B = \frac{3}{4}$

$B = \frac{1}{4}$

8C=8

C=1

From equation (1), we get,

$\int \frac{5x + 8}{x^2(3x + 8)} dx = \frac{-3}{4} \times \int \frac{1}{(3x + 8)} dx + \frac{1}{4} \times \int \frac{x + 1}{x^2} dx$

$= \frac{-3}{4} \times \frac{\log(3x + 8)}{3} + \frac{1}{4} \int \frac{x}{x^2} dx + \int \frac{1}{x^2} dx$

$= -\frac{1}{4} \log|3x + 8| + \frac{1}{4} log x - \frac{1}{x} + c$

Putting x+2=0,

X=-2

$A(-4)^2+B(0)+C(0)=-6+1=-5$

$A = \frac{-5}{16}$

Question: 36

Solution:

Let $I = \int \frac{5x^2 18x+17}{(x-1)^2(2x-3)} dx$

Now putting, $\frac{5x^2 18x+17}{(x-1)^2(2x-3)} = \frac{A}{(2x-3)} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \ldots. (1)$

$A(x-1)^2+B(2x-3)(x-1)+C(2x-3) = 5x^2-18x+17$

Putting x-1=0,

X=1

A(0)+B(0)+C(2-3)=5-18+17

C(-1)=4

Putting 2x-3=0,

$$x = \frac{3}{2}$$

$$A\left(\frac{3}{2} - 1\right)^2 + B(0) + C(0) = 5\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + 17$$

$$A\left(\frac{1}{4}\right) + 0 = 5 \times \frac{9}{4} - 27 + 17$$

$$A\left(\frac{1}{4}\right) = \frac{45}{4} - 10 = \frac{5}{4}$$

A=5

By equating the coefficient of x^2, we get ,

A+2B=5

5+2B=5

2B=0

B=0

From equation (1), we get,

$$\frac{5x^2 18x + 17}{(x-1)^2(2x-3)} = 5 \times \frac{1}{(2x-3)} + 0 - 4 \times \frac{1}{(x-1)^2}$$

$$\int \frac{5x^2 18x + 17}{(x-1)^2(2x-3)} dx = \frac{5}{2}\log(2x-3) + \frac{4}{x-1} + c$$

Question: 54

Solution:

let $I = \int \frac{dx}{(sinx + sin2x)} = \int \frac{dx}{(sinx + 2sinxcosx)}$

Put t=cosx

dt=-sinxdx

$$\frac{-dt}{sinx} = dx$$

$$I = \int \frac{-dt}{sin^2 x\,(1+2t)} = \int \frac{dt}{(1 - cos^2 x)(1+2t)} = \int \frac{dt}{(1-t^2)(1+2t)}$$

Putting $\frac{t}{(1-t)(1+t)(1+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1+2t} \dots \dots .(1)$

A(1+t)(1+2t)+B(1-t)(1+2t)+C(1-t^2)=1

Putting 1+t=0

t=-1

A(0)+B(2)(1-2)+C(0)=1

$$B = -\frac{1}{2}$$

Putting 1-t=0

t=1

A(2)(3)+B(0)+C(0)=1

$A = \frac{1}{6}$

Putting 1+2t=0

$t = -\frac{1}{2}$

$A(0) + B(0) + C\left(1 - \frac{1}{4}\right) = 1$

$C = \frac{4}{3}$

From equation(1),we get,

$$\frac{1}{(1-t)(1+t)(1+2t)} = \frac{1}{6} \times \frac{1}{1-t} - \frac{1}{2} \times \frac{1}{1+t} + \frac{4}{3} \times \frac{1}{1+2t}$$

$$\int \frac{1}{(1-t)(1+t)(1+2t)} dt = \frac{1}{6}\int \frac{1}{1-t} dt - \frac{1}{2}\int \frac{1}{1+t} dt + \frac{4}{3}\int \frac{1}{1+2t} dt$$

$$= \frac{1}{6} log|1-t| - \frac{1}{2} log|1+t| + \frac{2}{3} log|1+2t| + c$$

$$= \frac{1}{6} log|1-cosx| - \frac{1}{2} log|1+cosx| + \frac{2}{3} log|1+2cosx| + c$$

Question: 55

Solution:

Let $I = \int \frac{x^2}{(x^4 - x^2 - 12)} dx$

Putting $\frac{x^2}{(x^4-x^2-12)} = \frac{t}{t^2-t-12} = \frac{t}{(t-4)(t+3)} = \frac{A}{t-4} + \frac{B}{t+3} \ldots\ldots\ldots (1)$

Where t=x^2

A(t+3)+B(t-4)=t

Now put t+3=0

t=-3

A(0)+B(-7)=-3

$B = \frac{3}{7}$

Now put t-4=0

t=4

A(4+3)+B(0)=4

$A = \frac{4}{7}$

From equation(1)

$$\frac{t}{(t-4)(t+3)} = \frac{4}{7} \times \frac{1}{t-4} + \frac{3}{7} \times \frac{1}{t+3}$$

$$\frac{x^2}{(x^2-4)(x^2+3)} = \frac{4}{7} \times \frac{1}{x^2-2^2} + \frac{3}{7} \times \frac{1}{x^2+(\sqrt{3})^2}$$

$$\int \frac{x^2}{(x^2-4)(x^2+3)}dx = \frac{4}{7}\int \frac{1}{x^2-2^2}dx + \frac{3}{7}\int \frac{1}{x^2+(\sqrt{3})^2}dx$$

$$= \frac{4}{7} \times \frac{1}{2} \times \frac{1}{2} log\left|\frac{x-2}{x+2}\right| + \frac{3}{7} \times \frac{1}{\sqrt{3}} tan^{-1}\frac{x}{\sqrt{3}} + c$$

$$= \frac{1}{7} log\left|\frac{x-2}{x+2}\right| + \frac{\sqrt{3}}{7} tan^{-1}\frac{x}{\sqrt{3}} + c$$

Question: 37

Solution:

Let $I = \int \frac{8}{(x+2)(x^2+4)}dx$

Now putting, $\frac{8}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{(x^2+4)} \dots \dots (1)$

$A(x^2+4)+(Bx+C)(x+2)= 8$

Putting x+2=0,

X=-2

A(4+4)+0=8

A=1

By equating the coefficient of x^2 and constant term, A+B=0

1+B=0

B=-1

4A+2C=8

4×1+2C=8

2C=4

C=2

From equation (1),we get,

$$\frac{8}{(x+2)(x^2+4)} = \frac{1}{x+2} + \frac{-x+2}{(x^2+4)}$$

$$\int \frac{8}{(x+2)(x^2+4)}dx = \int \frac{1}{x+2}dx - \int \frac{x}{(x^2+4)}dx + 2\int \frac{1}{(x^2+4)}dx$$

$$= log|x+2| - \frac{1}{2}\log(x^2+4) + 2 \times \frac{1}{2} \times tan^{-1}\frac{x}{2} + c$$

$$= log|x+2| - \frac{1}{2}\log|x^2+4| + tan^{-1}\frac{x}{2} + c$$

Question: 56

Solution:

Let $I = \int \frac{x^4}{(x^2+1)(x^2+9)(x^2+16)}dx$

Putting $\frac{(x^2)^2}{(x^2+1)(x^2+9)(x^2+16)} = \frac{t^2}{(t+1)(t+9)(t+16)} = \frac{A}{t+1} + \frac{B}{t+9} + \frac{C}{t+16} \dots \dots \dots (1)$

Where $t=x^2$

$t^2=A(t+9)(t+16)+B(t+1)(t+16)+C(t+1)(t+9)$

Now put t+1=0

t=-1

A(8)(15)+B(0)+C(0)=1

$A = \frac{1}{120}$

Now put t+9=0

t=-9

$A(-9+9)(-9+16)+B(-9+1)(-9+16)+C(-9+1)(-9+9)=(-9)^2$

A(0)+B(-56)+C(0)=81

$B = -\frac{81}{56}$

Now put t+16=0

t=-16

$A(0)+B(0)+C(-15)(-7)=(-16)^2$

A(0)+B(0)+C(105)=256

$C = \frac{256}{105}$

From equation(1)

$$\frac{t^2}{(t+1)(t+9)(t+16)} = \frac{A}{t+1} + \frac{B}{t+9} + \frac{C}{t+16}$$

$$\int \frac{t^2}{(t+1)(t+9)(t+16)} dt = \int \left[\frac{\frac{1}{120}}{t+1} - \frac{\frac{81}{56}}{t+9} + \frac{\frac{256}{105}}{t+16}\right] dt$$

$$= \frac{1}{120}\int \frac{1}{t+1} dt - \frac{81}{56}\int \frac{1}{t+9} dt + \frac{256}{105}\int \frac{1}{t+16} dt$$

$$= \frac{1}{120}\int \frac{1}{x^2+1} dx - \frac{81}{56}\int \frac{1}{x^2+9} dx + \frac{256}{105}\int \frac{1}{x^2+16} dx$$

$$= \frac{1}{120}\int \frac{1}{x^2+1} dx - \frac{81}{56}\int \frac{1}{x^2+(3)^2} dx + \frac{256}{105}\int \frac{1}{x^2+(4)^2} dx$$

$$= \frac{1}{120} tan^{-1} x - \frac{81}{56} \times \frac{1}{3} tan^{-1}\left(\frac{x}{3}\right) + \frac{256}{105} \times \frac{1}{4} tan^{-1}\left(\frac{x}{4}\right) + c$$

$$= \frac{1}{120} tan^{-1} x - \frac{27}{56} tan^{-1}\left(\frac{x}{3}\right) + \frac{64}{105} tan^{-1}\left(\frac{x}{4}\right) + c$$

Question: 38

Solution:

Let $I = \int \frac{3x+5}{(x^3-x^2+x-1)} dx$

Now putting, $\frac{3x+5}{(x^3-x^2+x-1)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+1)} \ldots \ldots (1)$

$A(x^2+1)+(Bx+C)(x-1)=3x+5$

Putting x-1=0,

X=1

A(2)+B(0)=3+5=8

A=4

By equating the coefficient of x^2 and constant term, A+B=0

4+B=0

B=-4

A-C=5

4-C=5

C=-1

From equation (1),we get,

$$\frac{3x+5}{(x-1)(x^2+1)} = \frac{4}{x-1} + \frac{-4x-1}{(x^2+1)}$$

$$\int \frac{3x+5}{(x-1)(x^2+1)} dx = 4\int \frac{1}{x-1} dx - 4\int \frac{1}{(x^2+1)} dx - \int \frac{1}{(x^2+1)} dx$$

$$= 4\log(x-1) - \frac{4}{2}\log(x^2+1) - tan^{-1}x + c$$

$$= 4\log(x-1) - 2\log(x^2+1) - tan^{-1}x + c$$

Question: 57

Solution:

let $I = \int \frac{sin2x}{(1-cos2x)(2-cos2x)} dx$

Put t=cos2x

dt=-2sin2xdx

$$I = \int \frac{-dt/2}{(1-t)(2-t)} = \frac{1}{2}\int \frac{dt}{(t-2)(1-t)}$$

Putting $\frac{1}{(t-2)(1-t)} = \frac{A}{t-2} + \frac{B}{1-t} \dots\dots(1)$

A(1-t)+B(t-2)=1

Putting 1-t=0

t=1

A(0)+B(1-2) =1

B=-1

Putting t-2=0

t=2

A(1-2)+B(0) =1

A=-1

From equation (1), we get,

$$\frac{1}{(t-2)(1-t)} = \frac{-1}{t-2} + \frac{-1}{1-t}$$

$$\int \frac{1}{(t-2)(1-t)} dt = \int \frac{1}{2-t} dt + \int \frac{1}{t-1} dt$$

$$= -log|2-t| + log|t-1| + c$$

$= log|t-1| - log|2-t| + c$

$= log|cos2x - 1| - log|2 - cos2x| + c$

Question: 39

Solution:

Let $I = \int \frac{2x}{(x^2+1)(x^2+3)} dx$

Put t=x^2

dt=2xdx

Now putting, $\frac{1}{(t+1)(t+3)} = \frac{A}{t+1} + \frac{B}{t+3} \ldots\ldots(1)$

A(t+3) +B(t+1) = 1

Putting t+3=0,

X=-3

A(0) +B(-3+1)=1

$B = -\frac{1}{2}$

Putting t+1=0,

X=-1

A(-1+3)+B(0)=1

$A = \frac{1}{2}$

From equation(1),we get,

$$\frac{1}{(t+1)(t+3)} = \frac{1}{2} \times \frac{1}{t+1} - \frac{1}{2} \times \frac{1}{t+3}$$

$$\int \frac{1}{(t+1)(t+3)} dt = \frac{1}{2}\int \frac{1}{t+1} dt - \frac{1}{2}\int \frac{1}{t+3} dt$$

$$= \frac{1}{2}\log|t+1| - \frac{1}{2}\log|t+3| + c$$

$$\int \frac{2x}{(x^2+1)(x^2+3)} dx = \frac{1}{2}\log|x^2+1| - \frac{1}{2}\log|x^2+3| + c$$

Question: 58

Solution:

Let $I = \int \frac{2}{(1-x)(1+x^2)} dx$

Put $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{x^2+1} \ldots\ldots\ldots(1)$

A(1+x^2)+Bx(1-x)+C(1-x) =2

Put x=1

2=2A+0+0

A=1

Put x=0

2=A+C

C=2-A

C=2-1=1

Putting x=2

We have 2=5A-2B-C

2=5×1-2B-1

2B=2

B=1

$$\frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x}{1+x^2} + \frac{1}{1+x^2}$$

$$\int \frac{2}{(1-x)(1+x^2)}dx = \int \frac{1}{1-x}dx + \int \frac{x}{1+x^2}dx + \int \frac{1}{1+x^2}dx$$

$$-log|1-x| + \frac{1}{2}log|1+x^2| + tan^{-1}x + c$$

Question: 40

Solution:

Let $I = \int \frac{x^2}{(x^4-1)}dx$

Put t=x^2

dt=2xdx

Now putting, $\frac{x^2}{(x^4-1)} = \frac{t}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1} \ldots\ldots.(1)$

A(t+1)+B(t-1) = t

Putting t+1=0,

t=-1

A(0)+B(-1-1)=-1

$$B = \frac{1}{2}$$

Putting t-1=0,

t=1

A(1+1)+B(0)=1

$$A = \frac{1}{2}$$

From equation(1),we get,

$$\frac{t}{(t-1)(t+1)} = \frac{1}{2} \times \frac{1}{t-1} + \frac{1}{2} \times \frac{1}{t+1}$$

$$\int \frac{x^2}{(x^4-1)}dt = \frac{1}{2}\int \frac{1}{x^2-1}dt + \frac{1}{2}\int \frac{1}{x^2+1}dt$$

$$= \frac{1}{2} \times \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + \frac{1}{2}tan^{-1}x + c$$

$= \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| + \frac{1}{2}tan^{-1}x + c$

Question: 59

Solution:

Let $I = \int \frac{2x^2+1}{x^2(x^2+4)}dx$

Again let $x^2=t$

$$\frac{2t+1}{t(t+4)} = \frac{A}{t} + \frac{B}{(t+4)} \ldots\ldots..(1)$$

2t+1=A(t+4)+B(t)

Putting t=-4

2(-4)+1=A(-4+4)+B(-4)

-8+1=0-4B

-7=-4B

$B = \frac{7}{4}$

Putting t=0

2(0)+1=A(0+4)+B(0)

1=4A

$A = \frac{1}{4}$

$$\frac{2t+1}{t(t+4)} = \frac{\frac{1}{4}}{t} + \frac{\frac{7}{4}}{(t+4)}$$

$$\int \frac{2t+1}{t(t+4)}dt = \int \frac{2x^2+1}{x^2(x^2+4)}dx = \frac{1}{4}\int \frac{1}{x^2}dx + \frac{7}{4}\int \frac{1}{(x^2+2^2)}dx$$

$$= \frac{1}{4} \times \frac{(-1)}{x} + \frac{7}{4} \times \frac{1}{2}tan^{-1}\left(\frac{x}{2}\right) + c$$

$$I = \frac{-1}{4x} + \frac{7}{8}tan^{-1}\left(\frac{x}{2}\right) + c$$

Exercise : 15B

Question: 1

Solution:

$\int x^{-6}dx = \frac{x^{-6+1}}{-6+1} + c$

$$\because \left\{\int x^n = \frac{x^{n+1}}{n+1} + c\right\}$$

$$= \frac{x^{-5}}{-5} + c$$

$$\int x^{-6}dx = -\frac{1}{5x^5} + c$$

Question: 2

Solution:

$\int (\sqrt{x} + 1/\sqrt{x})dx \ = \int \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)dx$

$\left\{\int x^n = \frac{x^{n+1}}{n+1} + c\right\}$

$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx \ = \int \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \ dx$

$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx \ = \int \frac{2}{3}x^{\frac{3}{2}} + 2\sqrt{x} + c$

Question: 3

Solution:

$\int si\,n\,3x\,dx = \frac{-1}{3}cos\,3x + c$

$\left\{\int \sin ax\,dx = \frac{-1}{a}\cos ax\right\}$

Question: 4

Solution:

Let $x^3 + 1 = t$

$3x^2dx = dt$

$\frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}l\,n\,t + c$

$\int \frac{x^2}{1+x^3}dx = \frac{1}{3}l\,n(x^3+1) + c$

Question: 5

Solution:

Let sin x=t

cos x dx=dt

$\int \frac{2\cos x}{\;} dx = \int \frac{2}{\;} dt \ = -\frac{2}{\;} + c$

$\int \frac{2\cos x}{\;} dx = \frac{-2}{\;} cs\,c\,x + c$

Question: 6

Solution:

$\frac{(3\sin Ø - 6 + 4)\cos Ø}{(4 + 1 - cos^2 Ø - 4\sin Ø)} = \frac{3(\sin Ø - 2)\cos Ø + 4\cos Ø}{(\sin Ø - 2)^2}$

$$= \frac{3\cos\emptyset}{(\sin\emptyset - 2)} + \frac{4\cos\emptyset}{(\sin\emptyset - 2)^2}$$

$$\int \left(\frac{3\cos\emptyset}{(\sin\emptyset - 2)} + \frac{4\cos\emptyset}{(\sin\emptyset - 2)^2}\right) d\,\emptyset$$

Let (sin ∅-2)= t

cos ∅ d∅=dt

$$\int \frac{3dt}{t} + \frac{4dt}{t^2} = 3\ln t - \frac{4}{t} + c$$

$$\int \frac{(3\sin\phi - 2)\cos\phi}{(5 - \cos^2\phi - 4\sin\phi)} d\phi = 3\ln|\sin\phi - 2| - \frac{4}{(\sin\phi - 2)} + c$$

Question: 7

Solution:

$$\int \sin^2 x\, dx = \int \frac{1}{2} - \frac{\cos 2x}{2}\, dx$$

{1-cos 2x=2 sin² x}

$$\int \sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$\left\{\int \cos ax\, dx = \frac{1}{a}\sin ax\right\}$$

Question: 8

Solution:

Let log x = t

$$\frac{1}{x}dx = dt$$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$\int \frac{(\log x)^2}{x} dx = \frac{(\log x)^3}{3} + c$$

Question: 9

Solution:

$$\int \frac{(x+1)(x+\log x)^2}{x} = \int \left(1 + \frac{1}{x}\right)(x + \log x)^2 dx$$

Let $x + \log x = t$

$$\left(1 + \frac{1}{x}\right) dx = dt$$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$\int \frac{(x+1)(x + \log x)^2}{x} = \frac{(x + \log x)^3}{3} + c$$

Question: 10

Solution:

Let 1+cosx=t

-sin x dx=dt

$$\int \frac{-dt}{t} = -\ln t + c$$

$$\int \frac{\sin x}{(1+\cos x)} dx = -\ln|1+\cos x| + c$$

Question: 11

Solution:

$$\frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$\int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Let cos x-sin x=t

-(sin x + cos x)dx=dt

$$\int \frac{-dt}{t} = -\ln t + c$$

$$\int \frac{1+\tan x}{1-\tan x} dx = -\ln|\cos x - \sin x| + c$$

Question: 12

Solution:

$$\frac{1-\cot x}{1+\cot x} = \frac{\sin x - \cos x}{\sin x + \cos x}$$

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

Let sin x + cos x=t

(cos x-sin x)dx=dt

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx = \int \frac{-dt}{t} = -\ln|\sin x + \cos x| + c$$

$$\int \frac{1-\cot x}{1+\cot x} dx = -\ln|\sin x + \cos x| + c$$

Question: 13

Solution:

Let (x + log (sin x))=t

(1+cot x) dx=dt

$$\int \frac{dt}{t} = \ln t + c$$

$$\int \frac{(1+\cot x)}{(x+\log \sin x)} = \ln|x + \log(\sin x)| + c$$

Question: 14

Solution:

Let (x + cos² x)=t

(1-sin 2x) dx=dt

$$\int \frac{dt}{t} = \ln t + c$$

$$\int \frac{1 - \sin 2x}{x + \cos^2 x} = \ln(|x + \cos^2 x|) + c$$

Question: 15

Solution:

Let $\log x = t$

$$\frac{1}{x}dx = dt$$

$$\int sec^2 t dt = \tan t + c$$

$$\int \frac{sec^2 (\log x)}{x} dx = \tan(\log x) + c$$

Question: 16

Solution:

Let $\tan^{-1} x = t$

$$\frac{1}{1 + x^2} dx = dt$$

$$\int \sin 2t = -\frac{\cos 2t}{2} + c$$

$$\int \frac{\sin(2 \tan^{-1} x)}{(1 + x^2)} dx = \frac{-1}{2} \cos(2 \tan^{-1} x) + c$$

Question: 17

Solution:

Let $1-\tan^2 x=t$

$-2 \tan x. \sec^2 x\, dx=dt$

$$\frac{-1}{2} \int \frac{dt}{t} = \frac{-1}{2} \log t + c$$

$$\int \frac{\tan x \sec^2 x}{} dx = \frac{-1}{} \log |1 - \tan^2 x| + c$$

Question: 18

Solution:

$$\frac{x^4+1}{x^2+1} = \frac{x^4-1+2}{x^2+1}$$

$$= x^2 - 1 + \frac{2}{x^2+1}$$

$$\int \left(x^2 - 1 + \frac{2}{x^2+1}\right) dx = \frac{x^3}{3} - x + 2tan^{-1} x + c$$

Question: 19

Solution:

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$

$$tan^{-1}\sqrt{\frac{1-cos\left(\frac{\pi}{2}-x\right)}{1+cos\left(\frac{\pi}{2}-x\right)}} = tan^{-1}\sqrt{\frac{2sin^2(\frac{\pi}{4}-\frac{x}{2})}{2cos^2(\frac{\pi}{4}-\frac{x}{2})}}$$

$$= tan^{-1}\left((tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$$

$$\int \left(\frac{\pi}{4}-\frac{x}{2}\right) dx = \frac{\pi}{4}x - \frac{x^2}{4} + c$$

Question: 20

Solution:

Using Integration by Parts

$$\int u_{II} v_I dx = u\int vdx - \int u' \int vdx\, dx + c$$

Here 1 is the first function and $\log(x^2+1)$ is second function

$$\int log(1+x^2)dx = (log(1+x^2))x - \int \frac{2x}{1+x^2} x\,dx$$

$$= (log(1+x^2))x - 2\int \frac{x^2+1-1}{x^2+1} dx$$

$= (log(1 + x\text{^}2\ ))x - 2x + 2\ \ ⟦tan⟧\ \text{^}(-1)\, x + c$

Question: 21

Solution:

$$\frac{1}{2}\int 2\cos x \cos 3x\, dx$$

{2 cos A cos B=cos(A+B)+cos(A-B)}

$$\frac{1}{2}\int (\cos 4x + \cos 2x)\, dx = \frac{\sin 4x}{8} + \frac{\sin 2x}{4} + c$$

Question: 22 Evaluate

Solution:

$$\frac{1}{2}\int 2\sin 3x \sin x\, dx$$

{ 2 sin A sin B=cos(A-B)-cos(A+B) }

$$\frac{1}{2}\int (cos\,2x - cos\,4x)dx \;=\; \frac{sin\,2x}{4} - \frac{sin\,4x}{8} + c$$

CLASS24

Question: 23

Solution:

$$\frac{e^x(x+1-1)}{(x+1)^2} = e^x\left(\frac{1}{x+1} - \frac{1}{(x+1)^2}\right)$$

$$\{\int\left(e^x(f(x)+f'(x)\right)dx = e^x f(x) + c\}$$

$$\int \frac{xe^x}{(x+1)^2}dx \;=\; \frac{e^x}{x+1} + c$$

Question: 24

Solution:

$$\int\left(e^x(f(x)+f'(x)\right)dx = e^x f(x) + c$$

Here f(x)=-log cos x

$\int e^x$ (tan x- log cos x)dx=-e^x (log cos x)+c

Question: 25

Solution:

Multiplying Num^r and Den^r with (1+sinx)

$$\int \frac{1+sin\,x}{cos^2 x}dx \;=\; \int \sec^2 x \; + \sec x \tan x \;\; dx$$

= tan x + sec x + c

Question: 26

Solution:

Let x^2 =t

2xdx=dt

$$\frac{1}{2}\int \cos t\,dt = \frac{1}{2}\sin t + c$$

$$\int x\cos x^2 \;\, dx = \frac{1}{2}\sin x^2 + c$$

Question: 27

Solution:

$$\frac{\cot x}{\sqrt{\sin x}} = \frac{\cos x}{(\sin x)^{3/2}}$$

Let sin x=t

cos x dx=dt

$$\int \frac{dt}{t^{3/2}} = \frac{-2}{\sqrt{t}} + c$$

$$\int \frac{\cot x}{\sqrt{\sin x}} dx = \frac{-2}{\sqrt{\sin x}} + c$$

Question: 28

Solution:

$$\frac{sec^2 x}{cosec^2 x} = tan^2 x$$

$$\int tan^2 x\, dx = \int (sec^2 x - 1)\, dx$$

$$\int \frac{sec^2 x}{cosec^2 x} dx = \tan x - x + c$$

Question: 29

Solution:

$$\int sin^{-1}(cos\, x)\, dx = \int \left(\frac{\pi}{2} - \cos^{-1}(cosx)\right) dx$$

$$\int (\frac{\pi}{2} - x)\, dx = \frac{\pi}{2} x - \frac{x^2}{2} + c$$

Question: 30

Solution:

On rationalizing

$$\int \frac{dx}{(\sqrt{x+2} + \sqrt{x+1})} = \int \frac{\sqrt{x+2} - \sqrt{x+1}}{(\sqrt{x+2} + \sqrt{x+1})\sqrt{x+2} - \sqrt{x+1}} dx$$

$$= \int \frac{\sqrt{x+2} - \sqrt{x+1}}{(x+2-x-1)} dx$$

$$\int \frac{\sqrt{x+2} - \sqrt{x+1}}{1} dx = \frac{2}{3}(x+2)^{3/2} - \frac{2}{3}(x+1)^{\frac{3}{2}} + c$$

Question: 31

Solution:

We know that,

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int 2^x dx = \frac{2^x}{\ln 2} + c$$

.

Question: 32

Solution:

Let (x + log (sec x))=t

(1+tan x) dx=dt

$\int \frac{dt}{t} = ln\, t + c$

$\int \frac{(1 + tan\, x)}{(x + log\, sec\, x)} = ln|x + log(sec\, x)| + c$

Question: 33

Solution:

Let $\log x = t$

$\frac{1}{x} dx = dt$

$\int sec^2 t dt = tan\, t + c$

$\int \frac{sec^2(\log x)}{x} dx = \tan(\log x) + c$

Question: 34

Solution:

Let $x^2+x+1=t$

$(2x+1)dx=dt$

$\int \sqrt{t} dt = \frac{2}{3} t^{3/2} + c = \frac{2}{3}(x^2 + x + 1)^{3/2} + c$

Question: 35

Solution:

We know that,

$\int \frac{dx}{\sqrt{(ax)^2 + b^2}} = \frac{1}{a} \log\left|ax + \sqrt{(ax)^2 + b^2}\right| + c$

$\int \frac{dx}{\sqrt{(3x)^2 + 4^2}} = \frac{1}{3} \log\left|3x + \sqrt{9x^2 + 16}\right| + c$

Question: 36

Solution:

We know that,

$\int \frac{dx}{\sqrt{b^2 - (ax)^2}} = \frac{1}{a} \sin^{-1} \frac{ax}{b} + c$

$\int \frac{dx}{\sqrt{2^2 - (3x)^2}} = \frac{1}{3} \sin^{-1} \frac{3x}{2} + c$

Question: 37

Solution:

We know that,

$$\int \frac{dx}{\sqrt{(ax)^2 - b^2}} = \frac{1}{a}\log\left|ax + \sqrt{(ax)^2 - b^2}\right| + c$$

$$\int \frac{dx}{\sqrt{(2x)^2 - 5^2}} = \frac{1}{2}\log\left|2x + \sqrt{4x^2 - 25}\right| + c$$

Question: 38

Solution:

We know that,

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$$

$$\int \sqrt{2^2 - x^2}\,dx = \frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2} + c$$

Question: 39

Solution:

We know that,

$$\int \sqrt{a^2 + x^2}\,dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left|x + \sqrt{a^2 + x^2}\right| + c$$

$$\int \sqrt{3^2 + x^2}\,dx = \frac{x}{2}\sqrt{9 + x^2} + \frac{9}{2}\log\left|x + \sqrt{9 + x^2}\right| + c$$

Question: 40

Solution:

We know that,

$$\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$$

$$\int \sqrt{x^2 - 4^2}\,dx = \frac{x}{2}\sqrt{x^2 - 16} - 8\log\left|x + \sqrt{x^2 - 16}\right| + c$$

Exercise : OBJECTIVE QUESTIONS I

Question: 1

Solution:

$$= \int \frac{dx}{x^2 + 3^2}$$

We know, $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{3}\tan^{-1}\frac{x}{3} + c$$

Question: 2

Solution:

$$= \int \frac{dx}{(4x)^2 + 2^2}$$

4x=t

4dx=dt

$$dx = \frac{dt}{4}$$

$$= \frac{1}{4}\int \frac{dt}{t^2 + 2^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{8}\tan^{-1}\frac{t}{2} + c$$

put t=4x

$$= \frac{1}{8}\tan^{-1}\frac{4x}{2} + c$$

$$= \frac{1}{8}\tan^{-1}2x + c$$

Question: 3

Solution:

$$\int \frac{dx}{(2x)^2 + 3^2}$$

2x=t

2dx=dt

$$dx = \frac{dt}{2}$$

$$= \frac{1}{2}\int \frac{dt}{t^2 + 3^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{6}\tan^{-1}\frac{t}{3} + c$$

put t=2x

$$= \frac{1}{6}\tan^{-1}\frac{2x}{3} + c$$

Question: 4

Solution:

$$\int \frac{\sin x}{(\cos x)^2 + 1^2}\,dx$$

cos x=t

-sin x dx=dt

$$= -\int \frac{dt}{t^2 + 1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= -\tan^{-1}\frac{t}{1} + c$

put t=cos x

=-tan^{-1} (cos x)+c

Question: 5

Solution:

$$\int \frac{\cos x}{(\sin x)^2 + 1^2}\, dx$$

sin x=t

cos x dx=dt

$$= \int \frac{dt}{t^2 + 1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \tan^{-1}\frac{t}{1} + c$

put t=sin x

= tan^{-1} (sin x)+c

Question: 6

Solution:

$$= \int \frac{e^x}{(e^x)^2 + 1^2}\, dx$$

e^x =t

e^x dx=dt

$$= \int \frac{dt}{t^2 + 1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \tan^{-1}\frac{t}{1} + c$

put t=e^x

tan^{-1} e^x +c

Question: 7

Solution:

$$= \int \frac{3x^5}{(x^6)^2 + 1^2}\, dx$$

Let x^6 =t

$6x^5$ dx=dt

$3x^5\ dx = \frac{dt}{2}$

$= \frac{1}{2}\int \frac{dt}{t^2 + 1^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{1}{2}\tan^{-1}\frac{t}{1} + c$

put t=x^6

$= \frac{1}{2}\tan^{-1}\frac{x^6}{1} + c$

$= \frac{1}{2}\tan^{-1}x^6 + c$

Question: 8

Solution:

$= \int \frac{2x^3}{(x^4)^2 + 2^2}\ dx$

Let x^4=t

$4x^3$ dx=dt

$2x^3\ dx = \frac{dt}{2}$

$= \frac{1}{2}\int \frac{dt}{t^2 + 2^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{1}{4}\tan^{-1}\frac{t}{2} + c$

put t=x^4

$= \frac{1}{4}\tan^{-1}\frac{x^4}{2} + c$

Question: 9

Solution:

$= \int \frac{dx}{x^2 + 4x + 8}$

Completing the square

$x^2 + 4x + 8 = x^2 + 4x + 8\ (+4-4)$

$= x^2 + 4x + 4 + 4$

$= (x+2)^2 + 2^2$

$= \int \frac{dx}{(x+2)^2 + 2^2}$

Let x+2=t

dx=dt

$= \int \frac{dt}{t^2 + 2^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{1}{2}\tan^{-1}\frac{t}{2} + c$

put t=x+2

$= \frac{1}{2}\tan^{-1}\frac{x+2}{2} + c$

Question: 10

Solution:

$= \int \frac{dx}{2x^2 + x + 3}$

Completing the square

$\Rightarrow 2x^2 + x + 3 \; = 2x^2 + \frac{1}{2}x + \frac{3}{2})$

$= 2(x^2 + \frac{1}{2}x + \frac{3}{2} + \frac{1}{16} - \frac{1}{16})$

$= 2(\,(x + \frac{1}{4})^2 + \frac{23}{16})$

$= \frac{1}{2}\int \frac{dx}{(\,(x+\frac{1}{4})^2 + \frac{23}{16})}$

$Let\ x + \frac{1}{4} = t$

dx=dt

$= \int \frac{dt}{t^2 + \frac{\sqrt{23}}{4}^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{4}{2\sqrt{23}}\tan^{-1}\frac{t}{\frac{\sqrt{23}}{4}} + c$

$put\ t = x + \frac{1}{4}$

$= \frac{2}{\sqrt{23}}\tan^{-1}\frac{x+\frac{1}{4}}{\frac{\sqrt{23}}{4}} + c$

$= \frac{2}{\sqrt{23}}\tan^{-1}\frac{4x+1}{\sqrt{23}} + c$

Question: 11

Solution:

$= \int \frac{1}{e^x + e^{-x}}\,dx$

$$= \int \frac{e^x}{(e^x)^2 + 1^2}\, dx$$

$e^x = t$ e^x

$e^x\, dx = dt$

$$= \int \frac{dt}{t^2 + 1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \tan^{-1}\frac{t}{1} + c$$

put $t = e^x$

$= \tan^{-1} e^x + c$

Question: 12

Solution:

$$\int \frac{x^2}{4x^2+9} = \frac{1}{4}\int \frac{4x^2+9-9}{4x^2+9}\, dx$$

$$= \frac{1}{4}\int 1 + \frac{1}{4}\int \frac{-9}{4x^2+9}\, dx$$

$$= \frac{x}{4} - \frac{9}{4}\int \frac{1}{(2x)^2+3^2}\, dx$$

Let $2x = t$

$2\, dx = dt$

$$= \frac{x}{4} - \frac{9}{8}\int \frac{1}{(t)^2+3^2}\, dx$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{x}{4} - \frac{9}{4.2.3}\tan^{-1}\frac{t}{3} + c$$

put $t = 2x$

$$= \frac{x}{4} - \frac{3}{8}\tan^{-1}\frac{2x}{3} + c$$

Question: 13

Solution:

$$\int \frac{x^2-1}{x^2+4} = \int \frac{x^2}{x^2+4} - \int \frac{1}{x^2+4}$$

$$= \int \frac{x^2}{x^2+4} - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= \int \frac{x^2+4-4}{x^2+4} - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= \int (1 - \frac{4}{x^2+4}) - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= x - 2\tan^{-1}\frac{x}{2} - \frac{1}{2}\tan^{-1}\frac{x}{2} + c$$

$$= x - \frac{5}{2}\tan^{-1}\frac{x}{2} + c$$

Question: 14

Solution:

Consider $\int \frac{dx}{(3x)^2+2^2}$,

3x=t

3dx=dt

$$dx = \frac{dt}{3}$$

$$= \frac{1}{3}\int \frac{dt}{t^2 + 2^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{6}\tan^{-1}\frac{t}{2} + c$$

put t=3x

$$= \frac{1}{6}\tan^{-1}\frac{3x}{2} + c$$

Question: 15

Solution:

Consider $\int \frac{dx}{4x^2-4x+3}$,

Completing the square

$$4x^2 - 4x + 3 = 4(x^2 - x + \frac{3}{4})$$

$$= 4(x^2 - x + \frac{3}{4} + \frac{1}{4} - \frac{1}{4})$$

$$= 4(\ (x - \frac{1}{2})^2 + \frac{1}{2})$$

$$= \frac{1}{4}\int \frac{dx}{(\ (x - \frac{1}{2})^2 + \frac{1}{2})}$$

$$Let\ x - \frac{1}{2} = t$$

dx=dt

$$= \frac{1}{4}\int \frac{dt}{t^2 + \frac{1}{\sqrt{2}}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{\sqrt{2}}{4}\tan^{-1}\frac{t}{\frac{1}{\sqrt{2}}} + c$$

$= \frac{1}{2\sqrt{2}} \tan^{-1} \sqrt{2}t + c$

put t=x- �

$= \frac{1}{2\sqrt{2}} \tan^{-1} \frac{2x-1}{\sqrt{2}} + c$

Question: 16

Solution:

$\int \frac{dx}{sin^4 x + cos^4 x} = \int \frac{1}{cos^4 x(tan^4 x + 1)} dx$

$= \int \frac{sec^4 x}{tan^4 x + 1} dx$

$= \int \frac{sec^2 x \, sec^2 x}{tan^4 x + 1} dx$

$= \int \frac{sec^2 x \,(1 + tan^2 x)}{tan^4 x + 1} dx$

tan x=t

sec² x dx=dt

$= \int \frac{1 + t^2}{t^4 + 1} dt$

$= \int \frac{t^2 + 1}{t^4 + 1} dt$

$= \int \frac{1 + t^{-2}}{t^2 + t^{-2}} dt$

$= \int \frac{1 + t^{-2}}{t^2 + t^{-2} + 2 - 2} dt$

$= \int \frac{1 + t^{-2}}{(t - t^{-1})^2 + 2} dt$

Let t-t^{-1} =u

1+x^{-2} dt=du

$= \int \frac{du}{(u)^2 + \sqrt{2}^2}$

We know, $\int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{u}{\sqrt{2}} + c$

put u=t-t^{-1}

$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{t - t^{-1}}{\sqrt{2}} + c$

$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{t^2 - 1}{\sqrt{2}t} + c$

put t=tan x

$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{tan^2 x - 1}{\sqrt{2} \tan x} + c$

Question: 17

Solution:

$$\int \frac{(x^2+1)}{(x^4+x^2+1)} dx = \int \frac{1+x^{-2}}{x^2+1+x^{-2}} dx$$

$$= \int \frac{1+x^{-2}}{x^2+1+x^{-2}+2-2} dx$$

$$= \int \frac{1+x^{-2}}{(x-x^{-1})^2+3} dx$$

Let $x\text{-}x^{-1}=t$

$1+x^{-2}\,dx=dt$

$$= \int \frac{dt}{(t)^2+\sqrt{3}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{t}{\sqrt{3}} + c$$

put $t=x\text{-}x^{-1}$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{x-x^{-1}}{\sqrt{3}} + c$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{x^2-1}{\sqrt{3}x} + c$$

Question: 18

Solution:

$$\int \frac{\sin 2x}{sin^4 x + cos^4 x} dx = \int \frac{2 \sin x \cos x}{cos^4 x (tan^4 x + 1)} dx$$

$$= \int \frac{2 \tan x \; sec^2 x}{(tan^2 x)^2 + 1} dx$$

$$= \int \frac{2 \tan x \; sec^2 x}{(sec^2 x - 1)^2 + 1} dx$$

Let $sec^2\,x\text{-}1=t$

2 sec x sec x tan x dx=dt

$$= \int \frac{dt}{(t)^2+1}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$=\tan^{-1} t + c$

put $t=sec^2\,x\text{-}1$

$= \tan^{-1} sec^2 x - 1 + c$

$= \tan^{-1} \tan^2 x + c$

CLASS24

Question: 19

Solution:

Consider

3x=t $\int \frac{dx}{(1)^2-(3x)^2}$

3dx=dt

$dx = \frac{dt}{3}$

$= \frac{1}{3}\int \frac{dt}{1^2-(t)^2}$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$= \frac{1}{6}\log\frac{1+t}{1-t} + c$

put t=3x

$\frac{1}{6}\tan^{-1}\frac{1+3x}{1-3x} + c$

Question: 20

Solution:

Consider $\int \frac{dx}{(4)^2-(2x)^2}$

2x=t

2dx=dt

$dx = \frac{dt}{2}$

$= \frac{1}{2}\int \frac{dt}{4^2-(t)^2}$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$= \frac{1}{16}\log\frac{4+t}{4-t} + c$

put t=2x

$= \frac{1}{16}\tan^{-1}\frac{4+2x}{4-2x} + c$

$= \frac{1}{16}\tan^{-1}\frac{2+x}{2-x} + c$

Question: 21

Solution:

$= \int \frac{x^2}{(1)^2-(x^3)^2}\,dx$

Let x^3 =t

$3x^2\,dx = dt$

$$x^2\,dx = \frac{dt}{3}$$

$$= \frac{1}{3}\int \frac{dt}{1^2 - t^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$$= \frac{1}{6}\log\frac{1+t}{1-t} + c$$

put $t = x^3$

$$= \frac{1}{6}\log\frac{1+x^3}{1-x^3} + c$$

Question: 22

Solution:

$$= \int \frac{x}{(1)^2 - (x^2)^2}\,dx$$

Let $x^2 = t$

$2x\,dx = dt$

$$x\,dx = \frac{dt}{2}$$

$$= \frac{1}{2}\int \frac{dt}{1^2 - t^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$$= \frac{1}{4}\log\frac{1+t}{1-t} + c$$

put $t = x^2$

$$= \frac{1}{4}\log\frac{1+x^2}{1-x^2} + c$$

Question: 23

Solution:

$$= \int \frac{x^2}{(a^3)^2 - (x^3)^2}\,dx$$

Let $x^3 = t$

$3x^2\,dx = dt$

$$x^2\,dx = \frac{dt}{3}$$

$$= \frac{1}{3}\int \frac{dt}{(a^3)^2 - t^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$$= \frac{1}{6a^3}\log\frac{a^3+t}{a^3-t} + c$$

put t=x³

$$= \frac{1}{6a^3} \log \frac{a^3 + x^3}{a^3 - x^3} + c$$

Question: 24

Solution:

$$= -\int \frac{dx}{x^2 + 2x - 3}$$

Completing the square

$x^2 + 2x - 3 = x^2 + 2x - 3 + 1 - 1$

$(x+1)^2 - 4$

$$= -\int \frac{dx}{(x+1)^2 - 4}$$

Let x+1=t

dx=dt

$$= -\int \frac{dt}{t^2 - 2^2}$$

$$= -\int \frac{dt}{2^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{1}{4} \log \frac{2+t}{2-t} + c$$

put t=x+1

$$= \frac{1}{4} \log \frac{2+x+1}{2-x-1} + c$$

$$= \frac{1}{4} \log \frac{x+3}{1-x} + c$$

Question: 25

Solution:

$$\int \frac{1}{\cos^2 x - 3\sin^2 x} dx = \int \frac{1}{\cos^2 x(1 - 3\tan^2 x)} dx$$

$$= \int \frac{\sec^2 x}{(1 - (\sqrt{3}\tan x)^2)} dx$$

Let $\sqrt{3}$ tan x=t

$\sqrt{3}\sec^2 x\, dx = dt$

$$= \frac{1}{\sqrt{3}} \int \frac{dt}{1^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{1}{2\sqrt{3}} \log \frac{1+t}{1-t} + c$$

put t=$\sqrt{3}$ tan x

$$=\frac{1}{2\sqrt{3}}\log\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}+c$$

Question: 26

Solution:

$$\int\frac{cosec^2 x}{1-cot^2 x}dx$$

Let cot x=t

-cosec² x dx=dt

$$=-\int\frac{dt}{1^2-t^2}$$

We know, $\int\frac{1}{a^2-x^2}=\frac{1}{2a}\log\frac{a+x}{a-x}+c$

$$=\frac{-1}{2}\log\frac{1+t}{1-t}+c$$

put t=cot x

$$=\frac{-1}{2}\log\frac{1+\cot x}{1-\cot x}+c$$

Question: 27

Solution:

Consider

$$\int\frac{dx}{(2x)^2-1^2}$$

2x=t

2dx=dt

$$dx=\frac{dt}{2}$$

$$=\frac{1}{2}\int\frac{dt}{t^2-1^2}$$

We know, $\int\frac{1}{x^2-a^2}=\frac{1}{2a}\log\frac{x-a}{x+a}+c$

$$=\frac{1}{4}\log\frac{t-1}{t+1}+c$$

put t=2x

$$=\frac{1}{4}\log\frac{2x-1}{2x+1}+c$$

Question: 28

Solution:

$$=\int\frac{x}{(x^2)^2-(4)^2}dx$$

Let x^2 =t

2x dx=dt

$x\,dx = \frac{dt}{2}$

$= \frac{1}{2}\int \frac{1}{(t)^2 - (4)^2}\,dt$

We know, $\int \frac{1}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$

$= \frac{1}{16}\log\frac{t-4}{t+4} + c$

put $t=x^2$

$= \frac{1}{16}\log\frac{x^2-4}{x^2+4} + c$

Question: 29

Solution:

$\int \frac{1}{sin^2 x - 4cos^2 x}dx = \int \frac{1}{cos^2 x(tan^2 x - 4)}dx$

$= \int \frac{sec^2 x}{((\tan x)^2 - 2^2)}dx$

Let $\tan x = t$

$\sec^2 x\,dx = dt$

$= \int \frac{dt}{t^2 - 2^2}$

We know, $\int \frac{1}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$

$= \frac{1}{4}\log\frac{t-2}{t+2} + c$

put $t = \tan x$

$= \frac{1}{4}\log\frac{\tan x - 2}{\tan x + 2} + c$

Question: 30

Solution:

$\int \frac{1}{4sin^2 x + 5cos^2 x}dx = \int \frac{1}{cos^2 x(4tan^2 x + 5)}dx$

$\int \frac{sec^2 x}{((2\tan x)^2 + \sqrt{5}^2)}dx$

Let $2\tan x = t$

$2\sec^2 x\,dx = dt$

$= \frac{1}{2}\int \frac{dt}{t^2 + \sqrt{5}^2}$

We know,

$= \frac{1}{2\sqrt{5}}\tan^{-1}\frac{t}{\sqrt{5}} + c$

$\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

put t=2 tan x

$$=\frac{1}{2\sqrt{5}}\tan^{-1}\frac{2\tan x}{\sqrt{5}}+c$$

Question: 31

Solution:

$$\int\frac{\sin x}{\sin 3x}dx=\int\frac{\sin x}{3\sin x-4sin^3 x}dx$$

$$=\int\frac{1}{3-4sin^2 x}dx$$

$$=\int\frac{1}{cos^2 x(3sec^2 x-4tan^2 x)}dx$$

$$=\int\frac{sec^2 x}{3(1+tan^2 x)-4tan^2 x}dx$$

$$=\int\frac{sec^2 x}{3-tan^2 x}dx$$

Let tan x=t

sec^2 x dx=dt

$$=\int\frac{dt}{\sqrt{3}^2-t^2}$$

We know, $\int\frac{1}{a^2-x^2}=\frac{1}{2a}\log\frac{a+x}{a-x}+c$

$$=\frac{1}{2\sqrt{3}}\log\frac{\sqrt{3}+t}{\sqrt{3}-t}+c$$

put t= tan x

$$=\frac{1}{2\sqrt{3}}\log\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}+c$$

Question: 32

Solution:

$$\int\frac{(x^2+1)}{(x^4+1)}dx=\int\frac{1+x^{-2}}{x^2+x^{-2}}dx$$

$$=\int\frac{1+x^{-2}}{x^2+x^{-2}+2-2}dx$$

$$=\int\frac{1+x^{-2}}{(x-x^{-1})^2+2}dx$$

Let $x-x^{-1}$=t

$1+x^{-2}$ dx=dt

$$=\int\frac{dt}{(t)^2+\sqrt{2}^2}$$

We know, $\int\frac{1}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} + c$

put t=x-x^{-1}

$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{x - x^{-1}}{\sqrt{2}} + c$

$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{x^2 - 1}{\sqrt{2}x} + c$

Exercise : OBJECTIVE QUESTIONS II

Question: 1

Solution:

$\int \frac{dx}{\sqrt{4 - 9x^2}} = \int \frac{1}{3} \frac{dx}{\sqrt{\frac{4}{9} - x^2}}$

$= \int \frac{1}{3} \frac{dx}{\sqrt{\left(\frac{2}{3}\right)^2 - x^2}}$

$= \frac{1}{3} \sin^{-1} \frac{x}{\frac{2}{3}} + c$

$= \frac{1}{3} \sin^{-1} \frac{3x}{2} + c.$

Question: 2

Solution:

$\int \frac{dx}{\sqrt{16 - 4x^2}} = \int \frac{1}{2} \frac{dx}{\sqrt{\frac{16}{4} - x^2}}$

$= \int \frac{1}{2} \frac{dx}{\sqrt{(2)^2 - x^2}}$

$= \frac{1}{2} \sin^{-1} \frac{x}{2} + c$

Question: 3

Solution:

Put sin x =t

⇒ cos x dx = dt

∴ The given equation becomes

$\int \frac{dt}{\sqrt{4 - t^2}}$

$= \sin^{-1} \frac{t}{2} + c$

But t = sin x

$= \sin^{-1}\left(\frac{\sin x}{2}\right) + c$

Question: 4

Solution:

⇒ Let t=2^x

dt = log 2. 2^x.dx

$\Rightarrow \frac{dt}{log 2} = 2^x . dx$

$= \int \frac{dt}{\log 2 \sqrt{1-t^2}}$

$= \frac{1}{\log 2} \int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{1}{\log 2} \sin^{-1} t$

But t =2^x

$= \frac{1}{\log 2} \sin^{-1}(2^x)$

Question: 5

Solution:

$\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{2x-x^2+1-1}}$

$= \int \frac{dx}{\sqrt{-x^2+2x-1+1}}$

$= \int \frac{dx}{\sqrt{1-(x-1)^2}}$

=$\sin^{-1}$ (x-1)+c

Question: 6

Solution:

$\int \frac{dx}{\sqrt{x-2x^2}} = \int \frac{dx}{\sqrt{2}\sqrt{-x^2+\frac{1}{2}x}}$

$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x\right)}}$

$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x\right)+\frac{1}{16}-\frac{1}{16}}}$

$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x+\frac{1}{16}\right)+\frac{1}{16}}}$

$$= \int \frac{dx}{\sqrt{2}\sqrt{\frac{1}{16} - \left(x - \frac{1}{4}\right)^2}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{\left(\frac{1}{4}\right)^2 - \left(\frac{4x-1}{4}\right)^2}}$$

$$= \frac{1}{\sqrt{2}}(\sin^{-1}\left(\frac{\frac{4x-1}{4}}{\frac{1}{4}}\right)$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}(4x-1)$$

Question: 7

Solution:

$\Rightarrow \int \frac{3x^2 dx}{\sqrt{9-16x^6}}$

Let $x^3 = t$

$\therefore 3x^2 dx = dt$

$\therefore x^6 = t^2$

$\Rightarrow \int \frac{1}{4}\frac{dt}{\sqrt{\frac{9}{16}-t^2}}$

$\Rightarrow \frac{1}{4}\sin^{-1}\left(\frac{4t}{3}\right) + c$

But $t = x^3$

$\Rightarrow \frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right) + c$

Question: 8

Solution:

$\Rightarrow \int \frac{dx}{\sqrt{2+2x-x^2}} = \int \frac{dx}{\sqrt{2x-x^2+1+2-3}}$

$\Rightarrow \int \frac{dx}{\sqrt{-((x^2-2x+1)-3)}}$

$\Rightarrow \int \frac{dx}{\sqrt{3-(x-1)^2}}$

$\Rightarrow \sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + c.$

Question: 9

Solution:

$$\int \frac{dx}{\sqrt{16-6x-x^2}} = \int \frac{dx}{\sqrt{-x^2-6x-9+16+9}}$$

$$= \int \frac{dx}{\sqrt{25-(x+3)^2}}$$

$$= \sin^{-1}\left(\frac{x+3}{5}\right) + c.$$

Question: 10

Solution:

$$\int \frac{dx}{\sqrt{x-x^2}} = \int \frac{dx}{\sqrt{-x^2+x-}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-x)+\frac{1}{4}-\frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{-\left(x^2-x+\frac{1}{4}\right)+\frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{\frac{2x-1}{2}}{\frac{1}{2}}\right)+c$$

= $\sin^{-1}(2x-1)+c$

Question: 11

Solution:

$$\int \frac{dx}{\sqrt{1+2x-3x^2}} = \int \frac{dx}{\sqrt{3}\sqrt{-x^2+\frac{2}{3}x+\frac{1}{3}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2-\frac{2}{3}x-\frac{1}{3}\right)}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2-\frac{2}{3}x-\frac{1}{3}\right)+\frac{1}{9}-\frac{1}{9}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2-\frac{2}{3}x+\frac{1}{9}\right)+\frac{1}{3}+\frac{1}{9}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{\frac{4}{9}-\left(x-\frac{1}{3}\right)^2}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{\left(\frac{2}{3}\right)^2-\left(\frac{3x-1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}}(\sin^{-1}\left(\frac{\frac{3x-1}{3}}{\frac{2}{3}}\right)$$

$$= \frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{3x-1}{2}\right)$$

Question: 12

Mark (√) against

Solution:

We know

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right|$$

$$\int \frac{dx}{\sqrt{x^2 - 4^2}} = \log\left|x + \sqrt{x^2 - 16}\right|$$

Question: 13

Solution:

$$\int \frac{dx}{\sqrt{(2x)^2 - (3)^2}}$$

Put t =2x

dt =2 dx

$$\Rightarrow dx = \frac{dt}{2}$$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t^2 - 9}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$= \frac{1}{2}\log|t + \sqrt{t^2 - 9}$$

But t = 2x

$$= \frac{1}{2}\log|2x + \sqrt{4x^2 - 9}|$$

Question: 14

Solution:

$$\Rightarrow \int \frac{x^2\,dx}{\sqrt{(x^3)^2 - (1)^2}}$$

Put t =x^3

dt =$3x^2$dx

$$\Rightarrow dx = \frac{dt}{3x^2}$$

$$\Rightarrow \frac{1}{3}\int \frac{1}{x^2}\frac{x^2\,dt}{\sqrt{t^2 - 1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$= \frac{1}{3}\log|t + \sqrt{t^2 - 1}$$

But t = x^3

$$= \frac{1}{3}\log|x^3 + \sqrt{x^6 - 1}|$$

CLASS24

Question: 15

Solution:

$$\Rightarrow \int \frac{\sin x dx}{\sqrt{(2\cos x)^2-(1)^2}}$$

Put t =2cos x

dt =-2sinxdx

$$\Rightarrow dx = -\frac{dt}{2\sin x}$$

$$= -\frac{1}{2}\int \frac{dt}{\sqrt{t^2-1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$$

$$= -\frac{1}{2}\log|t+\sqrt{t^2-1}$$

But t = 2cosx

$$\Rightarrow -\frac{1}{2}\log|2\cos x + \sqrt{4\cos^2 x - 1}|$$

Question: 16

Solution:

$$\int \frac{\sec^2 x\, dx}{\sqrt{(\tan x)^2-(1)^2}}$$

Put t =tanx

dt = sec²x

$$\Rightarrow dx = -\frac{dt}{sec^2 x}$$

$$= \int \frac{1}{sec^2 x}\frac{sec^2 x\, dt}{\sqrt{t^2-1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$$

$$= \log|t+\sqrt{t^2-1}\,|$$

But t = tanx

$$= \log|\tan x + \sqrt{4\tan^2 x - 1}|$$

Question: 17

Solution:

Differentiating both side with respect to t

$$-2e^{2x}\frac{dx}{dt} = 1 \Rightarrow dx = -\frac{1}{2}\frac{dt}{1-t}$$

$$y = -\frac{1}{2}\int \frac{1}{(1-t)t}dt$$

$$y = -\frac{1}{2}\int \frac{t+(1-t)}{(1-t)t}dt$$

$$y = -\frac{1}{2}\int \frac{1}{(1-t)} + \frac{1}{t}dt$$

$$y = -\frac{1}{2}(-\log(1-t) + \log t) + c$$

Again put, t = 1 – e^{2x}

$$y = -\frac{1}{2}(-\log e^{2x} + \log(1-e^{2x})) + c$$

$$y = -\log\sqrt{\frac{1-e^{2x}}{e^{2x}}} + c$$

$$y = -\log\sqrt{e^{-2x}-1} + c$$

Question: 18

Solution:

$$\int \frac{dx}{\sqrt{x^2-3x+2}} = \int \frac{dx}{\sqrt{x^2-3x+2+\frac{9}{4}-\frac{9}{4}}}$$

$$= \int \frac{dx}{\sqrt{x^2-3x+\frac{9}{4}-\frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{\left(x-\frac{3}{2}\right)^2-\frac{1}{4}}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$$

$$= \log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right.$$

Question: 19

Solution:

$$\Rightarrow \int \frac{\cos x}{\sqrt{\sin^2 x-2\sin x-3}}dx$$

Let t =sin x

dt =cos x dx

$$\Rightarrow dx = \frac{dt}{cos\,x}$$

$$= \frac{cos\,x\,dt}{cos\,x\sqrt{t^2-2t-3+2-2}}$$

$$= \frac{dt}{\sqrt{(t^2-2t+2)-5}}$$

$$= \frac{dt}{\sqrt{(t-1)^2-5}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$\Rightarrow \int \frac{dt}{\sqrt{(t-1)^2 - 5}} = log\left|t - 1 + \sqrt{t^2 - 2t - 3}\right|$$

But t =sin x

$$\therefore\ log\left|\sin x - 1 + \sqrt{\sin^2 x - 2\sin x - 3}\right|$$

Question: 20

Solution:

$$\int \frac{dx}{\sqrt{x^2 - 4x + 2}} = \int \frac{dx}{\sqrt{x^2 - 4x + 2 + 4 - 4}}$$

$$= \int \frac{dx}{\sqrt{(x-2)^2 - 2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-2)^2 - 2}} = log\left|x - 2 + \sqrt{x^2 - 4x + 2}\right|$$

Question: 21

Solution:

$$\int \frac{dx}{\sqrt{x^2 + 6x + 5}} = \int \frac{dx}{\sqrt{x^2 + 6x + 5 + 9 - 9}}$$

$$= \int \frac{dx}{\sqrt{(x+3)^2 - 4}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x+3)^2 - 4}} = log\left|x + 3 + \sqrt{x^2 + 6x + 5}\right|$$

Question: 22

Solution:

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2 - 1}} = log\left|x - 3 + \sqrt{x^2 - 6x + 9 - 1}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2 - 1}} = log\left|x - 3 + \sqrt{x^2 - 6x + 8}\right|$$

Question: 23

Solution:

$$\int \frac{dx}{\sqrt{x^2-6x+10}} = \int \frac{dx}{\sqrt{x^2-6x+10+9-9}}$$

$$= \int \frac{dx}{\sqrt{(x-3)^2+1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2+1}} = log\left|x+3+\sqrt{x^2-6x+10}\right|$$

Question: 24

Solution:

$$\int \frac{x^2dx}{\sqrt{(x^3)^2+(a)^6}}$$

Put t =x³

dt =3x²dx

$$\Rightarrow dx = \frac{dt}{3x^2}$$

$$= \frac{1}{3}\int \frac{1}{x^2}\frac{x^2dt}{\sqrt{t^2+a^6}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$= \frac{1}{3}\log|t+\sqrt{t^2+a^6}$$

But t = x³

$$= \frac{1}{3}log|x^3+\sqrt{x^6+a^6}|+c.$$

Question: 25

Solution:

$$\int \frac{\sec^2 x\, dx}{\sqrt{(\tan x)^2+(4)^2}}$$

Put t = tan x

dt = sec²x

$$\Rightarrow dx = \frac{dt}{sec^2 x}$$

$$= \int \frac{1}{\sec^2 x}\frac{\sec^2 x\, dt}{\sqrt{t^2+16}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$= \log|t+\sqrt{t^2+16}$$

But t = tan x

$$= \log|\tan x + \sqrt{\tan^2 x+16}|$$

Question: 26

Solution:

$$\int \frac{dx}{\sqrt{3x^2+6x+12}} = \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{x^2+2x+4}}$$

$$= \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{x^2+2x+3+1}}$$

$$= \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{(x+1)^2+3}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{(x+1)^2+3}} = log\left|x+1+\sqrt{x^2+2x+4}\right|$$

Question: 27

Solution:

$$\int \frac{dx}{\sqrt{2x^2+4x+6}} = \int \frac{1}{\sqrt{2}} \frac{dx}{\sqrt{x^2+2x+3}}$$

$$= \int \frac{1}{\sqrt{2}} \frac{dx}{\sqrt{x^2+2x+1+2}}$$

$$= \int \frac{1}{\sqrt{2}} \frac{dx}{\sqrt{(x+1)^2+2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{(x+1)^2+2}} = log\left|x+1+\sqrt{x^2+2x+3}\right|$$

Question: 28

Solution:

$$\int \frac{x^2 dx}{\sqrt{x^6+2x^3+3}}$$

Let $x^3 = t$

$\Rightarrow 3x^2dx = dt$

$$\Rightarrow \frac{dt}{3x^2} = dx$$

$$\int \frac{x^2 dt}{3x^2\sqrt{t^2+2t+3}} = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+2t+3}}$$

$$= \int \frac{1}{3} \frac{dx}{\sqrt{t^2+2t+1+2}}$$

$$= \int \frac{1}{3} \frac{dx}{\sqrt{(t+1)^2+2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \frac{1}{3}\int \frac{dx}{\sqrt{(t+1)^2+2}} = log\left|t+1+\sqrt{t^2+2t+3}\right|$$

But t =x³

$$= \log\left|x^3+1+\sqrt{x^6+2x^3+3}\right|$$

Question: 29

Solution:

We know

$$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)+C$$

$$\Rightarrow \int \sqrt{2^2-x^2} = \frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}sin^{-1}\left(\frac{x}{2}\right)+C$$

$$\Rightarrow \int \sqrt{4-x^2} = \frac{x}{2}\sqrt{4-x^2}+2\,sin^{-1}\left(\frac{x}{2}\right)+C$$

Question: 30

Solution:

We know

$$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)+C$$

$$\Rightarrow \sqrt{1^2-(3x)^2} = 3\sqrt{\frac{1}{9}-x^2}$$

$$\Rightarrow 3\sqrt{\frac{1}{9}-x^2} = \frac{3x}{2}\sqrt{\frac{1}{9}-x^2}+\frac{\frac{1}{9}}{2}sin^{-1}\left(\frac{x}{\frac{1}{3}}\right)+C$$

$$\Rightarrow \sqrt{1^2-(3x)^2} = \frac{x}{2}\sqrt{1-9x^2}+\frac{3}{18}sin^{-1}(3x)+C$$

$$\Rightarrow \sqrt{1^2-(3x)^2} = \frac{x}{2}\sqrt{1-9x^2}+\frac{1}{6}sin^{-1}(3x)+C$$

Question: 31

Solution:

We know

$$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)+C$$

$$\Rightarrow \sqrt{3^2-(2x)^2} = 2\sqrt{\frac{9}{4}-x^2}$$

$$\Rightarrow 2\sqrt{\frac{9}{4}-x^2}. = \frac{x}{2}\sqrt{\frac{9}{4}-x^2}+\frac{\frac{9}{4}}{2}sin^{-1}\left(\frac{x}{\frac{3}{2}}\right)+C$$

$\Rightarrow \sqrt{9-4x^2} = \frac{x}{2}\sqrt{9-4x^2} + \frac{2.9}{8} sin^{-1}(2x) + C$

$\Rightarrow \sqrt{9-4x^2} = \frac{x}{2}\sqrt{9-4x^2} + \frac{9}{4} sin^{-1}(2x) + C$

Question: 32

Solution:

Given: $\int \cos x \sqrt{9-\sin^2 x}\, dx$

Let sin x =t

cos x dx =dt

$\Rightarrow \frac{dt}{\cos x} = dx$

$= \frac{dt}{\cos x}\sqrt{9-sin^2 x}\cos x$

$= \sqrt{9-t^2}\, dt$

$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} sin^{-1}\left(\frac{x}{a}\right) + C$

$\Rightarrow \int \sqrt{3^2-t^2} = \frac{t}{2}\sqrt{9-t^2} + \frac{9}{2} sin^{-1}\left(\frac{x}{3}\right) + C$

But t =sin x

$\Rightarrow \int \cos x \sqrt{9-sin^2 x} = \frac{\sin x}{2}\sqrt{9-sin^2 x} + \frac{9}{2} sin^{-1}\left(\frac{\sin x}{3}\right) + C$

Question: 33

Solution:

We know

$\Rightarrow \int \sqrt{x^2-a^2} = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} log\left|x + \sqrt{x^2-a^2}\right| + C$

$\Rightarrow \int \sqrt{x^2-4^2} = \frac{x}{2}\sqrt{x^2-4^2} - \frac{4^2}{2} log\left|x + \sqrt{x^2-4^2}\right| + C$

$\Rightarrow \int \sqrt{x^2-16} = \frac{x}{2}\sqrt{x^2-16} - 8\, log\left|x + \sqrt{x^2-16}\right| + C$

Question: 34

Solution:

$\sqrt{x^2-4x+2dx}$

It can be written as

$\Rightarrow \sqrt{x^2-4x+2+2-2} = \sqrt{x^2-4x+4-2}$

$= \sqrt{(x-2)^2-2}$

We know

$\Rightarrow \int \sqrt{x^2-a^2} = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} log\left|x + \sqrt{x^2-a^2}\right| + C$

$$\Rightarrow \int \sqrt{(x-2)^2-2} = \frac{(x-2)}{2}\sqrt{(x-2)^2-2} - \frac{\left(\sqrt{2}\right)^2}{2} log\left|\sqrt{(x-2)^2-2}\right| + C$$

$$\Rightarrow \int \sqrt{x^2-4x+2} = \frac{x-2}{2}\sqrt{x^2-4x+2} - log|x^2-4x+2| + C$$

Question: 35

Solution:

$$\Rightarrow \int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} log\left|x+\sqrt{x^2+a^2}\right| + C$$

$$\Rightarrow 3\int \sqrt{x^2+\left(\frac{4}{3}\right)^2} = 3\left(\frac{x}{2}\sqrt{x^2+\left(\frac{4}{3}\right)^2} + \frac{\frac{16}{9}}{2} log\left|x+\sqrt{x^2+\left(\frac{4}{3}\right)^2}\right|\right)$$

$$\Rightarrow \int \sqrt{9x^2+16}dx = \frac{x}{2}\sqrt{9x^2+16} + \frac{8}{3} log\left|3x+\sqrt{9x^2+16}\right|$$

Question: 36

Solution:

$$\int e^x\sqrt{e^{2x}+4}dx$$

Let $e^x = t$

e^x dx =dt

$$= \int \sqrt{t^2+2^2}dt$$

$$\Rightarrow \int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} log\left|x+\sqrt{x^2+a^2}\right| + C$$

$$\Rightarrow \int \sqrt{t^2+2^2} = \frac{t}{2}\sqrt{t^2+2^2} + \frac{2^2}{2} log\left|t+\sqrt{t^2+2^2}\right| + C$$

But t =e^x

$$\Rightarrow \int e^x\sqrt{e^{2x}+4}dx = \frac{e^x}{2}\sqrt{e^{2x}+4} + 2\,log\left|e^x+\sqrt{e^{2x}+4}\right| + C$$

Question: 37

Solution:

$$\int \frac{\sqrt{16+(\log x)^2}}{x}dx$$

Let log x =t

$$\Rightarrow \frac{1}{x}dx = dt$$

$$= \int \sqrt{t^2+4^2}\,dt$$

$$\Rightarrow \int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} log\left|x+\sqrt{x^2+a^2}\right| + C$$

$$\Rightarrow \int \sqrt{t^2 + 4^2 dt} = \frac{t}{2}\sqrt{t^2 + 4^2} + \frac{4^2}{2} log \left|t + \sqrt{t^2 + 4^2}\right| + C$$

But t =log x

$$\Rightarrow \int \frac{\sqrt{16 + (log\, x)^2}}{x} dx$$
$$= \frac{log\, x}{2}\sqrt{log^2 x + 16} + 8\, log \left|log\, x + \sqrt{log^2 x + 16}\right| + C$$