

Exercise : 16A

Question: 1**Solution:**

$$\frac{242}{5}$$

Evaluation:

$$\int_1^3 x^4 dx = \left[\frac{x^5}{5} \right]$$

$$= \frac{3^5}{5} - \frac{1}{5}$$

$$= \frac{243 - 1}{5}$$

$$= \frac{242}{5}$$

Question: 2**Solution:**

$$\frac{14}{3}$$

Evaluation:

$$\int_1^4 \sqrt{x} dx = \left[\frac{2}{3} x^{\frac{3}{2}} \right]$$

$$= \frac{2}{3} \left[4^{\frac{3}{2}} - 1 \right]$$

$$= \frac{14}{3}$$

Question: 3**Solution:**

$$\frac{15}{64}$$

Evaluation:

$$\int_1^2 x^{-5} dx = \left[\frac{x^{-4}}{-4} \right]$$

$$= \frac{2^{-4}}{-4} - \frac{1}{-4}$$

$$= \frac{16 - 1}{64}$$

$$= \frac{15}{64}$$

Question: 4

Evaluate:

Solution:

$$\frac{512}{7}$$

Evaluation:

$$\int_0^{16} \frac{3}{x^4} dx = \left[\frac{4}{7} x^{-\frac{7}{4}} \right]$$

$$= \frac{4}{7} \left[16^{-\frac{7}{4}} - 1 \right]$$

$$= \frac{512}{7}$$

Question: 5

Solution:

$$-\log 4$$

Evaluation:

$$\int_{-4}^{-1} \frac{dx}{x} = -[\log x]$$

$$=[\log(-1) - \log(-4)]$$

$$=-[\log(-4) - \log(-1)]$$

$$=-\left[\log\left(\frac{-4}{-1}\right)\right]$$

$$=-\log 4$$

Question: 6

Solution:

$$2$$

Evaluation:

$$\int_1^4 \frac{dx}{\sqrt{x}} = [2\sqrt{x}]$$

$$=[2\sqrt{4} - 2]$$

$$=[4 - 2]$$

$$=2$$

Question: 7

Solution:

$$\frac{3}{2}$$

Evaluation:

$$\int_0^1 \frac{dx}{\sqrt[3]{x}} = \left[\frac{3}{2} x^{\frac{2}{3}} \right]$$

$$= \left[\frac{3}{2} 1^{\frac{4}{3}} - 0 \right]$$

$$= \frac{3}{2}$$

Question: 8

Solution:

3

Evaluation:

$$\int_1^8 \frac{dx}{x^{\frac{5}{3}}} = \left[\frac{3}{1} x^{\frac{1}{3}} \right]$$

$$= \left[3(8)^{\frac{1}{3}} - 3(1)^{\frac{1}{3}} \right]$$

$$= [6-3]$$

$$= 3$$

Question: 9

Solution:

6

Evaluation:

$$\int_2^4 3dx = 3[x]$$

$$= 3[4-2]$$

$$= 6$$

Question: 10

Solution:

$$\frac{\pi}{4}$$

Evaluation:

$$\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1}x]$$

$$= [\tan^{-1} 1 - \tan^{-1} 0]$$

$$= \pi/4$$

Question: 11

Solution:

$$\frac{\pi}{2}$$

Evaluation:

$$\int_0^{\infty} \frac{dx}{1+x^2} = [\tan^{-1}x]$$

$$=[\tan^{-1} \infty - \tan^{-1} 0]$$

$$=\pi/2$$

Question: 12

Solution:

$$\frac{\pi}{2}$$

Evaluation:

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = [\sin^{-1}x]$$

$$=[\sin^{-1} 1 - \sin^{-1} 0]$$

$$=\frac{\pi}{2}$$

Question: 13

Solution:

$$\frac{1}{\sqrt{3}}$$

Evaluation:

$$\int_0^{\frac{\pi}{6}} \sec^2 x \, dx = [\tan x]$$

$$=\left[\tan\left(\frac{\pi}{6}\right) - \tan 0\right]$$

$$=\frac{1}{\sqrt{3}}$$

Question: 14

Solution:

$$-2$$

Evaluation:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \operatorname{cosec}^2 x \, dx = [-\cot x]$$

$$=\left[-\cot\left(\frac{\pi}{4}\right) + \cot\left(-\frac{\pi}{4}\right)\right]$$

$$=\left[-\cot\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{4}\right)\right]$$

$$=-2$$

Question: 15

Solution:

$$\left(1 - \frac{\pi}{4}\right)$$

Evaluation:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^2 x dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\operatorname{cosec}^2 x - 1) dx$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\operatorname{cosec}^2 x - 1) dx = [-\cot x - x]$$

$$= \left[-\cot\left(\frac{\pi}{2}\right) - \frac{\pi}{2} + \cot\left(\frac{\pi}{4}\right) + \frac{\pi}{4}\right]$$

$$= \left[0 - \frac{\pi}{2} + 1 + \frac{\pi}{4}\right]$$

$$= \left[1 - \frac{\pi}{4}\right]$$

Question: 16

Solution:

$$\left(1 - \frac{\pi}{4}\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \tan^2 x dx = \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) dx$$

$$\int_0^{\frac{\pi}{4}} (\sec^2 x - 1) dx = [\tan x - x]$$

$$= \left[\tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4} - \tan(0) - 0\right]$$

$$= \left[1 - \frac{\pi}{4}\right]$$

Question: 17

Solution:

$$\frac{\pi}{4}$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} \sin^2 x dx = \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 - \cos 2x) dx$$

$$= \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right]$$

$$= \frac{1}{2} \left[\frac{\pi}{2} - \frac{\sin \pi}{2} - 0 + \frac{\sin 0}{2} \right]$$

$$= \frac{\pi}{4}$$

Question: 18

Evaluate:

Solution:

$$\left(\frac{\pi}{8} + \frac{1}{4} \right)$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \cos^2 x dx = \int_0^{\frac{\pi}{4}} \frac{1}{2} (1 + \cos 2x) dx$$

$$= \frac{1}{2} \left[x + \frac{\sin 2x}{2} \right]$$

$$= \frac{1}{2} \left[\frac{\pi}{4} + \frac{\sin(\frac{\pi}{2})}{2} - 0 - \frac{\sin 0}{2} \right]$$

$$= \frac{\pi}{8} + \frac{1}{4}$$

Question: 19

Solution:

$$\log 2$$

Evaluation:

$$\int_0^{\frac{\pi}{3}} \tan x dx = \log |\sec x|$$

$$= \log \left| \sec \left(\frac{\pi}{3} \right) \right| - \ln |\cos 0|$$

$$= \log |2| - \log |1|$$

$$= \log 2$$

Question: 20

Solution:

$$\log(\sqrt{2} - 1) + \log(2 + \sqrt{3})$$

Evaluation:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x dx = -\log |\operatorname{cosec} x + \cot x|$$

$$= -\log \left| \operatorname{cosec} \left(\frac{\pi}{4} \right) + \cot \left(\frac{\pi}{4} \right) \right| + \log \left| \operatorname{cosec} \left(\frac{\pi}{6} \right) + \cot \left(\frac{\pi}{6} \right) \right|$$

$$= -\log |\sqrt{2} + 1| + \log |2 + \sqrt{3}|$$

Question: 21

Evaluate:

Solution:

$$\frac{3\sqrt{3}}{8}$$

Evaluation:

$$\int_0^{\frac{\pi}{3}} \cos^3 x \, dx = \frac{1}{4} \int_0^{\frac{\pi}{3}} (3\cos x + \cos 3x) \, dx$$

$$\frac{1}{4} \int_0^{\frac{\pi}{3}} (3\cos x - \cos 3x) \, dx = \frac{1}{4} \left[3\sin x + \frac{\sin 3x}{3} \right]$$

$$= \frac{1}{4} \left[3\sin\left(\frac{\pi}{3}\right) + \frac{\sin \pi}{3} \right] - \frac{1}{4} \left[3\sin 0 + \frac{\sin 0}{3} \right]$$

$$= \frac{1}{4} \left[\frac{3\sqrt{3}}{2} \right]$$

$$= \frac{3\sqrt{3}}{8}$$

Question: 22

Solution:

$$\frac{2}{3}$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} \sin^3 x \, dx = \frac{1}{4} \int_0^{\frac{\pi}{2}} (3\sin x - \sin 3x) \, dx$$

$$\frac{1}{4} \int_0^{\frac{\pi}{2}} (3\sin x - \sin 3x) \, dx = \frac{1}{4} \left[-3\cos x + \frac{\cos 3x}{3} \right]$$

$$= \frac{1}{4} \left[-3\cos\left(\frac{\pi}{2}\right) + \frac{\cos\left(\frac{3\pi}{2}\right)}{3} \right] - \frac{1}{4} \left[-3\cos 0 + \frac{\cos 0}{3} \right]$$

$$= \frac{1}{4} \left[\frac{9-1}{3} \right]$$

$$= \frac{2}{3}$$

Question: 23

Solution:

$$(4 - 3\sqrt{2})$$

Evaluation:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(1 - 3\cos x)}{\sin^2 x} \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\operatorname{cosec}^2(x) - 3\operatorname{cosec}(x)\cot(x)) \, dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\operatorname{cosec}^2(x) - 3\operatorname{cosec}(x)\cot(x)) \, dx$$

Question: 24

Solution:

Evaluation:

$$\int_0^{\frac{\pi}{4}} \sqrt{1 + \cos 2x} \, dx = \int_0^{\frac{\pi}{4}} \sqrt{2\cos^2 x} \, dx$$

$$= \sqrt{2} [\sin x]$$

$$= \sqrt{2} \left[\sin\left(\frac{\pi}{4}\right) - \sin 0 \right]$$

$$= \sqrt{2} \left[\frac{1}{\sqrt{2}} \right]$$

$$= 1$$

Question: 25

Solution:

$$(\sqrt{2} - 1)$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \sqrt{1 - \sin 2x} \, dx = \int_0^{\frac{\pi}{4}} \sqrt{\sin^2 x + \cos^2 x - 2\sin x \cos x} \, dx$$

$$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x) \, dx$$

$$= [\sin x + \cos x]$$

$$= \left[\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right) - \cos 0 - \sin 0 \right]$$

$$= \left[+\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \right]$$

$$= [\sqrt{2} - 1]$$

Question: 26

Solution:

$$2$$

Evaluation:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1 + \sin x} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\left(\frac{x}{2}\right)}{\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^2} dx$$

$$\text{Let } u = \left(\tan\left(\frac{x}{2}\right) + 1 \right)$$

$$dx = \frac{2}{\sec^2\left(\frac{x}{2}\right)} du$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1 + \sin x} = 2 \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{u^2} du$$

$$= -\frac{2}{u}$$

$$= -\frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

$$= 2$$

Question: 27

Solution:

$$\frac{1}{2}$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \frac{dx}{1 + \cos 2x} = \int_0^{\frac{\pi}{4}} \frac{dx}{2\cos^2 x}$$

$$\int_0^{\frac{\pi}{4}} \frac{dx}{2\cos^2 x} = \int_0^{\frac{\pi}{4}} \frac{1}{2} \sec^2 x dx$$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2} \sec^2 x dx = \frac{1}{2} [\tan x]$$

$$= \frac{1}{2} \left[\tan\left(\frac{\pi}{4}\right) - \tan 0 \right]$$

$$= \frac{1}{2} [1]$$

$$= \frac{1}{2}$$

Question: 28

Solution:

$$\frac{1}{2}$$

Evaluation:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{1 - \cos 2x} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{2\sin^2 x}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{2\sin^2 x} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} \operatorname{cosec}^2 x dx$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} \operatorname{cosec}^2 x dx = \frac{1}{2} [\cot x]$$

$$= \frac{1}{2} \left[\cot\left(\frac{\pi}{2}\right) - \cot 0 \right]$$

$$= \frac{1}{2} [1]$$

$$= \frac{1}{2}$$

Question: 29

Evaluate:

Solution:

$$\frac{3}{5\sqrt{2}}$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \sin 2x \sin 3x dx = \frac{1}{2} \int_0^{\frac{\pi}{4}} (\cos x - \cos 5x) dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{4}} (\cos x - \cos 5x) dx$$

$$= \frac{1}{2} \left[\sin x - \frac{\sin 5x}{5} \right]$$

$$= \frac{1}{2} \left[\sin\left(\frac{\pi}{4}\right) - \frac{\sin\left(\frac{5\pi}{4}\right)}{5} \right] - \frac{1}{2} \left[\sin(0) - \frac{\sin(0)}{5} \right]$$

$$= \frac{1}{2} \left[\frac{1}{\sqrt{2}} + \frac{1}{5\sqrt{2}} \right]$$

$$= \frac{3}{5\sqrt{2}}$$

Question: 30

Solution:

$$\frac{5}{12}$$

Evaluation:

$$\int_0^{\frac{\pi}{6}} \cos x \cos 2x dx = \frac{1}{2} \int_0^{\frac{\pi}{6}} (\cos 3x + \cos x) dx$$

$$= \frac{1}{2} \left[\frac{\sin 3x}{3} + \sin x \right]$$

$$= \frac{1}{2} \left[\frac{\sin\left(\frac{\pi}{2}\right)}{3} + \sin\left(\frac{\pi}{6}\right) \right] - 0$$

$$= \frac{1}{2} \left[\frac{1}{3} + \frac{1}{2} \right]$$

$$= \frac{5}{12}$$

Question: 31

Solution:

$$\frac{-4}{5}$$

Evaluation:

$$\int_0^{\pi} \sin 2x \cos 3x dx = \frac{1}{2} \int_0^{\pi} (\sin 5x - \sin x) dx$$

$$\begin{aligned}
 &= \frac{1}{2} \left[\frac{-\cos 5x}{5} + \cos x \right] \\
 &= \frac{1}{2} \left[-\frac{\cos(5\pi)}{5} + \cos(\pi) \right] - \frac{1}{2} \left[-\frac{\cos(0)}{5} + \cos(0) \right] \\
 &= \frac{1}{2} \left[\frac{-(-1)}{5} - 1 \right] - \frac{1}{2} \left[-\frac{1}{5} + 1 \right] \\
 &= \frac{1}{2} \left[\frac{-4}{5} - \frac{4}{5} \right] \\
 &= \frac{1}{2} 2 \left(-\frac{4}{5} \right) \\
 &= -\frac{4}{5}
 \end{aligned}$$

Question: 32

Solution:

2

Explanation:

$$\begin{aligned}
 \int_0^{\frac{\pi}{2}} \sqrt{1 + \sin(x)} dx &= \int_0^{\frac{\pi}{2}} \sqrt{2} \cos\left(\frac{2x - \pi}{4}\right) dx \\
 &= 2^{\frac{3}{2}} \sin\left(\frac{2x - \pi}{4}\right) \\
 &= 2^{\frac{3}{2}} \left(0 - \sin\left(-\frac{\pi}{4}\right)\right) \\
 &= \frac{2\sqrt{2}}{\sqrt{2}} \\
 &= 2
 \end{aligned}$$

Question: 33

Solution:

2

Explanation:

$$\begin{aligned}
 \int_0^{\frac{\pi}{2}} \sqrt{1 + \cos(x)} dx &= \int_0^{\frac{\pi}{2}} \sqrt{2} \cos\left(\frac{x}{2}\right) dx \\
 &= 2^{\frac{3}{2}} \sin\left(\frac{x}{2}\right) \\
 &= 2^{\frac{3}{2}} \left(\sin\left(\frac{\pi}{4}\right) - 0\right) \\
 &= \frac{2\sqrt{2}}{\sqrt{2}} \\
 &= 2
 \end{aligned}$$

Question: 34

Solution:

$$\left(\frac{2}{3} + 2 \tan^{-1} 2 \right)$$

Explanation:

$$\begin{aligned} \int_0^2 \left\{ \frac{(x^4 + 1)}{x^2 + 1} \right\} dx &= \int_0^2 \frac{x^4 + 2 - 1}{x^2 + 1} dx \\ &= \int_0^2 \frac{x^4 - 1}{x^2 + 1} dx + \int_0^2 \frac{2}{x^2 + 1} dx \\ &= \int_0^2 \frac{(x^2 - 1)(x^2 + 1)}{x^2 + 1} dx + \int_0^2 \frac{2}{x^2 + 1} dx \\ &= \int_0^2 (x^2 - 1) dx + 2 \tan^{-1} x \\ &= \left[\frac{x^3}{3} - x + 2 \tan^{-1} x \right]_0^2 \\ &= \frac{2}{3} + 2 \tan^{-1} 2 \end{aligned}$$

Question: 35

Solution:

$$(2 \log 3 - 3 \log 2)$$

Explanation:

$$\begin{aligned} \int_1^2 \frac{dx}{(x+1)(x+2)} &= \int_1^2 \frac{(x+2) - (x+1)}{(x+1)(x+2)} dx \\ &= \int_1^2 \frac{1}{(x+1)} dx - \int_1^2 \frac{1}{(x+2)} dx \\ &= [\log(x+1) - \log(x+2)]_1^2 \\ &= 2 \log 3 - 3 \log 2 \end{aligned}$$

Question: 36

Solution:

$$\frac{1}{2} (\log 2 + \log 3)$$

Explanation:

$$\begin{aligned} \int_1^2 \frac{x+3}{x(x+2)} dx &= \int_1^2 \frac{3}{2x} dx - \int_1^2 \frac{1}{x+2} dx \\ &= \frac{3}{2} \log x - \log(x+2) \\ &= \frac{1}{2} (\log 2 + \log 3) \end{aligned}$$

Question: 37

Solution:

$$\frac{1}{4}(\log 5 - \log 3)$$

Evaluation:

$$\begin{aligned}\int_3^4 \frac{dx}{x^2 - 4} &= \int_3^4 \frac{1}{(x-2)(x+2)} dx \\&= \int_3^4 \frac{1}{4(x-2)} dx - \int_3^4 \frac{1}{4(x+2)} dx \\&= \frac{1}{4} \log(x-2) - \frac{1}{4} \log(x+2) \\&= \frac{1}{4} \log 3 - \frac{1}{4} \log 1 - \frac{1}{4} \log 6 + \frac{1}{4} \log 5 \\&= \frac{1}{4} \left(\log 5 - \log \left(\frac{6}{2} \right) \right) \\&= \frac{1}{4} (\log 5 - \log 3)\end{aligned}$$

Question: 38

Solution:

$$\log \left(\frac{5 + 3\sqrt{3}}{1 + \sqrt{3}} \right)$$

Evaluation:

$$\int \frac{dx}{\sqrt{x^2 + 2x + 3}} = \int \frac{dx}{\sqrt{(x+1)^2 + 2}}$$

Substitute:

$$\frac{x+1}{\sqrt{2}} = u$$

$$\therefore dx = \sqrt{2} du$$

$$= \int \frac{\sqrt{2} du}{\sqrt{2u^2 + 2}}$$

$$= \log(\sqrt{u^2 + 1} + u)$$

$$\text{Undo substitution: } u = \frac{x+1}{\sqrt{2}}$$

$$\begin{aligned}\therefore \int_0^4 \frac{dx}{\sqrt{x^2 + 4x + 3}} &= \log(\sqrt{(x+1)^2 + 2} + x + 1) \\&= \log(\sqrt{(4+1)^2 + 2} + 4 + 1) - \log(\sqrt{(0+1)^2 + 2} + 0 + 1) \\&= \log(5 + 3\sqrt{3}) - \log(1 + \sqrt{3}) \\&= \log \left(\frac{5 + 3\sqrt{3}}{1 + \sqrt{3}} \right)\end{aligned}$$

Question: 39

Solution:

$$\log(4 + \sqrt{15}) - \log(3 + \sqrt{8})$$

Evaluation:

$$\int \frac{dx}{\sqrt{x^2 + 4x + 3}} = \int \frac{dx}{\sqrt{(x+2)^2 - 1}}$$

Substitute:

$$x+2=u$$

$$\therefore dx=du$$

$$= \int \frac{du}{\sqrt{u^2 - 1}}$$

$$= \log(\sqrt{u^2 - 1} + u)$$

Undo substitution: $u = x + 2$

$$\therefore \int_1^2 \frac{dx}{\sqrt{x^2 + 4x + 3}} = \log(\sqrt{(x+2)^2 - 1} + x + 2)$$

$$= \log(\sqrt{(2+2)^2 - 1} + 2 + 2) - \log(\sqrt{(1+2)^2 - 1} + 1 + 2)$$

$$= \log(4 + \sqrt{15}) - \log(3 + \sqrt{8})$$

Question: 40

Evaluate:

Solution:

$$\frac{2}{\sqrt{7}} \left\{ \tan^{-1} \frac{5}{\sqrt{7}} - \tan^{-1} \frac{1}{\sqrt{7}} \right\}$$

Evaluation:

$$\int_0^1 \frac{1}{2x^2 + x + 1} dx = \int_0^1 \frac{1}{\left(\left(\sqrt{2x} + \frac{1}{\sqrt{2}} \right)^2 + \frac{7}{8} \right)} dx$$

Substitute $4x+1=\sqrt{7}u$

$$\therefore dx = \frac{\sqrt{7}}{4} du$$

Now solving:

$$\int \left(\frac{1}{u^2} + 1 \right) du = \tan^{-1} u$$

$$\frac{2}{\sqrt{7}} \int \frac{1}{u^2 + 1} du = \frac{2}{\sqrt{7}} \tan^{-1} u$$

$$\therefore \int_0^1 \frac{1}{2x^2 + x + 1} dx = \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{4x+1}{\sqrt{7}} \right)$$

$$= \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{4+1}{\sqrt{7}} \right) - \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{1}{\sqrt{7}} \right)$$

$$= \frac{2}{\sqrt{7}} \left\{ \tan^{-1} \left(\frac{5}{\sqrt{7}} \right) - \tan^{-1} \left(\frac{1}{\sqrt{7}} \right) \right\}$$

Question: 41

Evaluate:

Solution:

$$\frac{\pi}{4}(a+b)$$

Evaluation:

$$\begin{aligned}\int_0^{\frac{\pi}{2}} (a \cos^2 x + b \sin^2 x) dx &= \int_0^{\frac{\pi}{2}} \left[\frac{a}{2} (\cos 2x + 1) + \frac{b}{2} (1 - \cos 2x) \right] dx \\&= \left[\frac{a}{2} \left(\frac{\sin 2x}{2} + x \right) + \frac{b}{2} \left(x - \frac{\sin 2x}{2} \right) \right] \\&= \left[\frac{a}{2} \left(\frac{\sin \pi}{2} + \frac{\pi}{2} \right) + \frac{b}{2} \left(\frac{\pi}{2} - \frac{\sin \pi}{2} \right) - \frac{a}{2} \left(\frac{\sin 0}{2} + 0 \right) - \frac{b}{2} \left(0 - \frac{\sin 0}{2} \right) \right] \\&= \left[\frac{a}{2} \left(0 + \frac{\pi}{2} \right) + \frac{b}{2} \left(\frac{\pi}{2} - 0 \right) - \frac{a}{2} (0 + 0) - \frac{b}{2} (0 - 0) \right] \\&= \frac{\pi}{4} (a + b)\end{aligned}$$

Question: 42

Solution:

$$-\frac{2}{\sqrt{3}}$$

Evaluation:

$$\begin{aligned}\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx &= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\tan^2 x + 1}{\tan x} \right)^2 dx \\ \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\tan^2 x + 1}{\tan x} \right)^2 dx &= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sec^2 x (\tan^2 x + 1)}{\tan^2 x} dx\end{aligned}$$

Substitute:

$$\tan(x) = u$$

$$\therefore dx = \frac{1}{\sec^2(x)} du$$

$$\therefore \int \frac{(u^2 + 1)}{u^2} du$$

$$\therefore u - \frac{1}{u}$$

$$\begin{aligned}\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\tan^2 x + 1}{\tan x} \right)^2 dx &= [\tan(x) - \cot(x)] \\&= \left[\tan\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{3}\right) + \cot\left(\frac{\pi}{3}\right) \right] \\&= \left[1 - 1 - \sqrt{3} + \frac{1}{\sqrt{3}} \right] \\&= -\frac{2}{\sqrt{3}}\end{aligned}$$

Question: 43

Evaluate:

Solution:

$$\frac{3\pi}{16}$$

Evaluation:

By reduction formula:

$$\int_0^{\frac{\pi}{2}} \cos^4 x dx = \frac{\cos^3(x)\sin(x)}{4} + \frac{3}{4} \int \cos^2 x dx$$

We know that,

$$\int \cos^2 x dx = \frac{1}{2} \left[\frac{\sin 2x}{2} + x \right]$$

$$\int_0^{\frac{\pi}{2}} \cos^4 x dx = \frac{\cos^3(x)\sin(x)}{4} + \frac{3}{8} \left[\frac{\sin 2x}{2} + x \right]$$

$$= \frac{\cos^3\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)}{4} + \frac{3}{8} \left[\frac{\sin \pi}{2} + \frac{\pi}{2} \right] - \frac{\cos^3(0)\sin(0)}{4} - \frac{3}{8} \left[\frac{\sin 0}{2} + 0 \right]$$

$$= 0 + \frac{3}{8} \left[0 + \frac{\pi}{2} \right] - 0 - \frac{3}{8} [0 + 0]$$

$$= \frac{3\pi}{16}$$

Question: 44

Solution:

$$\frac{1}{\sqrt{5}a} \log \left\{ \frac{7+3\sqrt{5}}{2} \right\}$$

Evaluation:

Assume that $a \neq 0$.

$$\int_0^2 \frac{1}{-x^2 + ax + a^2} dx = - \int_0^2 \frac{1}{x^2 - ax - a^2} dx$$

$$= \int_0^2 \frac{4}{(2x + (-\sqrt{5} - 1)a)(2x + (\sqrt{5} - 1)a)} dx$$

$$= \int_0^2 \left(\frac{2}{\sqrt{5}a(2x + (-\sqrt{5} - 1)a)} - \frac{2}{\sqrt{5}a(2x + (\sqrt{5} - 1)a)} \right) dx$$

Now,

$$\int \frac{1}{2x + (-\sqrt{5} - 1)a} dx$$

Substitute:

$$u = 2x + (-\sqrt{5} - 1)a$$

$$\therefore dx = \frac{1}{2} du$$

$$= \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \log u$$

Undo substitution:

$$u = 2x + (-\sqrt{5} - 1)a$$

$$\therefore \int \frac{1}{2x + (-\sqrt{5} - 1)a} dx = \frac{1}{2} \log(2x + (-\sqrt{5} - 1)a)$$

Now,

$$\int \frac{1}{2x + (\sqrt{5} - 1)a} dx$$

Substitute:

$$u = 2x + (\sqrt{5} - 1)a$$

$$\therefore dx = \frac{1}{2} du$$

$$= \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \log u$$

Undo substitution:

$$u = 2x + (\sqrt{5} - 1)a$$

$$\therefore \int \frac{1}{2x + (\sqrt{5} - 1)a} dx = \frac{1}{2} \log(2x + (\sqrt{5} - 1)a)$$

$$\frac{2}{\sqrt{5}a} \int_0^2 \frac{1}{(2x + (-\sqrt{5} - 1)a)} dx - \frac{2}{\sqrt{5}a} \int_0^2 \frac{1}{2x + (\sqrt{5} - 1)a} dx$$

$$= \frac{\log(2x + (-\sqrt{5} - 1)a)}{\sqrt{5}a} - \frac{\log(2x + (\sqrt{5} - 1)a)}{\sqrt{5}a}$$

$$- \int_0^2 \frac{1}{x^2 - ax - a^2} dx = \frac{\log(2x + (\sqrt{5} - 1)a)}{\sqrt{5}a} - \frac{\log(2x + (-\sqrt{5} - 1)a)}{\sqrt{5}a}$$

$$= \frac{\log(4 + (\sqrt{5} - 1)a)}{\sqrt{5}a} - \frac{\log(4 + (-\sqrt{5} - 1)a)}{\sqrt{5}a} - \frac{\log(0 + (\sqrt{5} - 1)a)}{\sqrt{5}a} + \frac{\log(0 + (-\sqrt{5} - 1)a)}{\sqrt{5}a}$$

$$= \frac{1}{\sqrt{5}a} \log\left(\frac{7 + 3\sqrt{5}}{2}\right)$$

Question: 45

Solution:

$$\frac{\pi}{6}$$

Evaluation:

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{dx}{\sqrt{x - x^2}} = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{\frac{1}{4} - \left(x - \frac{1}{2}\right)^2}}$$

Substitute:

$$2x-1=u$$

$$\therefore dx = \frac{1}{2} du$$

$$\int \frac{1}{\sqrt{1-u^2}} du = \sin^{-1}(u)$$

Undo Substitution:

$$u=2x-1$$

$$\therefore = \sin^{-1}(2x-1)$$

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{dx}{\sqrt{x-x^2}} = \sin^{-1}(2x-1)$$

$$= \sin^{-1}(1-1) - \sin^{-1}\left(\frac{1}{2}-1\right)$$

$$= \frac{\pi}{6}$$

Question: 46

Solution:

$$\frac{\pi}{8}$$

Evaluation:

$$\begin{aligned} \int_0^1 \sqrt{x-x^2} dx &= \int_0^1 \sqrt{\frac{1}{4} - \left(x - \frac{1}{2}\right)^2} dx \\ &= \frac{1}{2} \int_0^1 \sqrt{1 - (2x-1)^2} dx \end{aligned}$$

Substitute:

$$2x-1=u$$

$$\therefore dx = \frac{1}{2} du$$

$$\therefore \frac{1}{2} \int \sqrt{1-u^2} du$$

Substitute:

$$u=\sin(v)$$

$$\therefore \sin^{-1}(u)=v$$

$$\therefore du = \cos(v) dv$$

$$= \int \cos(v) \sqrt{1 - \sin^2(v)} dv$$

$$= \int \cos^2(v) dv$$

We know that,

$$\int \cos^2(v) dv = \frac{1}{2} \left[\frac{\sin(2v)}{2} + v \right]$$

Undo Substitution:

$$v = \sin^{-1}(u) \sin(\sin^{-1}(u)) = u \cos(\sin^{-1}(u)) = \sqrt{1-u^2}$$

$$= \frac{\sin^{-1}(u)}{2} + \frac{u\sqrt{1-u^2}}{2}$$

Undo Substitution:

$$u = 2x - 1$$

$$\therefore = \frac{\sin^{-1}(2x-1)}{4} + \frac{(2x-1)\sqrt{1-(2x-1)^2}}{4}$$

$$\frac{1}{2} \int_0^1 \sqrt{1-(2x-1)^2} dx = \frac{\sin^{-1}(2x-1)}{8} + \frac{(2x-1)\sqrt{1-(2x-1)^2}}{8}$$

$$= \frac{\sin^{-1}(2-1)}{8} + \frac{(2-1)\sqrt{1-(2-1)^2}}{8} - \frac{\sin^{-1}(0-1)}{8} - \frac{(0-1)\sqrt{1-(0-1)^2}}{8}$$

$$= \frac{\pi}{16} + 0 - \frac{\pi}{8} - 0$$

$$= \frac{\pi}{8}$$

Question: 47

Evaluate:

Solution:

$$\log 2 - \log 3 + \frac{2}{3}$$

Evaluation:

$$\int_1^3 \frac{1}{x^2(x+1)} dx$$

Perform partial fraction decomposition:

$$\int_1^3 \frac{1}{x^2(x+1)} dx = \int_1^3 \left(\frac{1}{x+1} - \frac{1}{x} + \frac{1}{x^2} \right) dx$$

$$= \left[\log(x+1) - \log(x) - \frac{1}{x} \right]$$

$$= \left[\log(4) - \log(3) - \frac{1}{3} - \log(2) + \log(1) + \frac{1}{1} \right]$$

$$= \log(2) - \log(3) + \frac{2}{3}$$

Question: 48

Solution:

$$\log 6 - \log 5 - \frac{2}{15}$$

Evaluation:

$$\int_1^2 \frac{1}{x(2x+1)^2} dx = \int_1^2 \left(-\frac{2}{2x+1} - \frac{2}{(2x+1)^2} + \frac{1}{x} \right) dx$$

$$= -2 \int_1^2 \frac{1}{2x+1} dx - 2 \int_1^2 \frac{1}{(2x+1)^2} dx + \int_1^2 \frac{1}{x} dx$$

$$\begin{aligned}
 &= -2 \left[\frac{1}{2} \log(2x+1) \right] - 2 \left[\frac{-1}{2(2x+1)} \right] + [\log(x)] \\
 &= -[\log(5)] + \left[\frac{1}{(5)} \right] + [\log(2)] + [\log(3)] - \left[\frac{1}{(3)} \right] + [\log(1)] \\
 &= \log(6) - \log(5) - \frac{2}{15}
 \end{aligned}$$

Question: 49

Solution:

1

Evaluation:

$$\begin{aligned}
 \int_0^1 x e^x dx &= \int_0^1 (x-1+1) e^x dx \\
 &= [(x-1)e^x] \\
 &= [(1-1)e^1 - (0-1)e^0] \\
 &= 1
 \end{aligned}$$

Question: 50

Solution:

$$\left(\frac{\pi^2}{4} - 2 \right)$$

Evaluation:

$$\begin{aligned}
 \int_0^{\frac{\pi}{2}} x^2 \cos(x) dx &= x^2 \sin(x) - \int 2x \sin(x) dx \\
 \int_0^{\frac{\pi}{2}} x^2 \cos(x) dx &= [x^2 \sin(x) - 2 \sin(x) - 2x \cos(x)] \\
 &= \left[\left(\frac{\pi}{2} \right)^2 \sin\left(\frac{\pi}{2}\right) - 2 \sin\left(\frac{\pi}{2}\right) - \pi \cos\left(\frac{\pi}{2}\right) - (0)^2 \sin(0) + 2 \sin(0) + 0 \right] \\
 &= \left[\frac{\pi^2}{4} - 2 - 0 - 0 + 0 + 0 \right] \\
 &= \left(\frac{\pi^2}{4} - 2 \right)
 \end{aligned}$$

Question: 51

Solution:

$$\left(\sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2 \right)$$

Evaluation:

From integrate by parts:

$$\int_0^{\frac{\pi}{4}} x^2 \sin(x) dx = -x^2 \cos(x) - \int -2x \cos(x) dx$$

From integrate by parts:

$$\begin{aligned} \int_0^{\frac{\pi}{4}} x^2 \cos(x) dx &= [-x^2 \cos(x) + 2x \sin(x) + 2 \cos(x)] \\ &= [2x \sin(x) + (2-x^2) \cos(x)] \\ &= \left[\frac{\pi}{2} \sin\left(\frac{\pi}{4}\right) + \left(2 - \frac{\pi^2}{16}\right) \cos\left(\frac{\pi}{4}\right) - 2(0) \sin(0) - (2-0) \cos(0) \right] \\ &= \left[\frac{\pi}{2\sqrt{2}} + \frac{2}{\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} + 0 - 0 - 2 \right] \\ &= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2 \end{aligned}$$

Question: 52

Solution:

$$\frac{-\pi}{4}$$

Evaluation:

$$\begin{aligned} \int_0^{\frac{\pi}{2}} x^2 \cos(2x) dx &= \frac{x^2 \sin(2x)}{2} - \int x \sin(x) dx \\ \int_0^{\frac{\pi}{2}} x^2 \cos(x) dx &= \left[\frac{x^2 \sin(2x)}{2} - \frac{\sin(2x)}{4} + \frac{x \cos(2x)}{2} \right] \\ &= \left[\frac{\left(\frac{\pi}{2}\right)^2 \sin(\pi)}{2} - \frac{\sin(\pi)}{4} + \frac{\left(\frac{\pi}{2}\right) \cos(\pi)}{2} - \frac{(0)^2 \sin(0)}{2} + \frac{\sin(0)}{4} - \frac{(0) \cos(0)}{2} \right] \\ &= \left[0 - 0 - \frac{\pi}{4} - 0 + 0 - 0 \right] \\ &= -\frac{\pi}{4} \end{aligned}$$

Question: 53

Solution:

$$\left(\frac{2}{27} - \frac{\pi^2}{12} \right)$$

Evaluation:

$$\begin{aligned} \int_0^{\frac{\pi}{2}} x^3 \sin(3x) dx &= -\frac{x^3 \cos(3x)}{3} - \int -x^2 \cos(3x) dx \\ &= -\frac{x^3 \cos(3x)}{3} + \frac{x^2 \sin(3x)}{3} - \int \frac{2x \sin(3x)}{3} dx \\ &= -\frac{x^3 \cos(3x)}{3} + \frac{x^2 \sin(3x)}{3} + \frac{2x \cos(3x)}{9} + \frac{2}{3} \int -\frac{\cos(3x)}{3} dx \end{aligned}$$

$$\begin{aligned}
 &= -\frac{x^3 \cos(3x)}{3} + \frac{x^2 \sin(3x)}{3} + \frac{2x \cos(3x)}{9} - \frac{2 \sin(3x)}{27} \\
 &= -0 + \frac{\left(\frac{\pi}{2}\right)^2 \sin\left(\frac{3\pi}{2}\right)}{3} + 0 - \frac{2 \sin\left(\frac{3\pi}{2}\right)}{27} + 0 - 0 - 0 + 0 \\
 &= \left(\frac{2}{27} - \frac{\pi^2}{12}\right)
 \end{aligned}$$

Question: 54

Solution:

$$\left(\frac{\pi^3}{48} - \frac{\pi}{8}\right)$$

Evaluation:

$$\begin{aligned}
 \int_0^{\frac{\pi}{2}} x^2 \cos^2 x dx &= \int_0^{\frac{\pi}{2}} \frac{x^2}{2} (\cos(2x) + 1) dx \\
 &= \int_0^{\frac{\pi}{2}} \left(\frac{x^2}{2} \cos(2x) + \frac{x^2}{2}\right) dx \\
 \int_0^{\frac{\pi}{2}} \left(\frac{x^2}{2} \cos(2x) + \frac{x^2}{2}\right) dx &= \frac{x^2 \sin(2x)}{2} - \int x \sin(2x) dx + \frac{x^3}{6} \\
 &= \frac{x^2 \sin(2x)}{2} + \frac{x \cos(2x)}{4} + \int -\frac{\cos(2x)}{2} dx + \frac{x^3}{6} \\
 &= \frac{x^2 \sin(2x)}{2} + \frac{x \cos(2x)}{4} - \frac{\sin(2x)}{4} + \frac{x^3}{6} \\
 &= \frac{x^2 \sin(2x)}{2} + \frac{x \cos(2x)}{4} - \frac{\sin(2x)}{4} + \frac{x^3}{6} \\
 &= 0 + \frac{\frac{\pi}{2} \cos(\pi)}{4} - 0 + \frac{\left(\frac{\pi}{2}\right)^3}{6} - 0 - 0 + 0 - 0 \\
 &= \left(\frac{\pi^3}{48} - \frac{\pi}{8}\right)
 \end{aligned}$$

Question: 55

Solution:

$$(2 \log 2 - 1)$$

Evaluation:

$$\begin{aligned}
 \int_1^2 \log(x) dx &= x \log(x) - (x) \\
 &= 2 \log(2) - (2) - 1 \log(1) + (1) \\
 &= 2 \log(2) - 1
 \end{aligned}$$

Question: 56

Solution:

$$\frac{3}{4} \log 3 - \log 2$$

Evaluation:

$$\int_1^3 \frac{\log(x)}{(1+x)^2} dx = -\frac{\log(x)}{1+x} - \int \left(-\frac{1}{x(1+x)} \right) dx$$

Now,

$$\int \left(-\frac{1}{x(1+x)} \right) dx = - \int \left(\frac{1}{x^2 \left(\frac{1}{x} + 1 \right)} \right) dx$$

Let,

$$\frac{1}{x} + 1 = u$$

$$\therefore dx = -x^2 du$$

$$\therefore - \int \left(\frac{1}{x^2 \left(\frac{1}{x} + 1 \right)} \right) dx = \int \frac{1}{u} du$$

$$= \log(u)$$

Undo substitution:

$$u = \frac{1}{x} + 1$$

$$\int_1^3 \frac{\log(x)}{(1+x)^2} dx = -\frac{\log(x)}{1+x} + \log\left(\frac{1}{x} + 1\right)$$

$$= -\frac{\log(3)}{4} + \log\left(\frac{4}{3}\right) + \frac{\log(1)}{2} - \log(2)$$

$$= -\frac{\log(3)}{4} + \log(4) + \log(3) - \log 2$$

$$= \frac{3}{4} \log 3 - \log 2$$

Question: 57

Solution:

$$\left(\frac{e^2}{2} - e \right)$$

Correct answer is $\frac{e^2}{2}$

Evaluation:

Let,

$$\log(x) = u$$

$$\rightarrow x = e^u$$

$$\rightarrow dx = e^u du$$

$$\int \left(\frac{1}{u} - \frac{1}{u^2} \right) e^u du = \frac{e^u}{u}$$

Undo substitution:

$$u = \log(x)$$

$$\begin{aligned} \int_0^{e^2} \left\{ \frac{1}{\log(x)} - \frac{1}{\log(x)^2} \right\} dx &= \frac{x}{\log(x)} \\ &= \frac{e^2}{\log(e^2)} - 0 \\ &= \frac{e^2}{2} \end{aligned}$$

Question: 58

Solution:

$$e^e$$

Evaluation:

$$\begin{aligned} \int_1^e e^x \left(\frac{(1 + x \log(x))}{x} \right) dx &= \int_1^e e^x \left(\frac{1}{x} + \log(x) \right) dx \\ &= \log(x) e^x \\ &= \log(e) e^e - \log(1) e^1 \\ &= e^e \end{aligned}$$

Question: 59

Solution:

$$\left(\frac{e}{2} - 1 \right)$$

Evaluation:

$$\int_0^1 \frac{x e^x}{(1+x)^2} dx$$

From Integrates by parts:

$$\begin{aligned} &= -\frac{x e^x}{x+1} - \int \frac{-x e^x - e^x}{x+1} dx \\ \therefore \int \frac{-x e^x - e^x}{x+1} dx &= \int -e^x dx \\ &= -e^x \end{aligned}$$

$$\begin{aligned} \int_0^1 \frac{x e^x}{(1+x)^2} dx &= \left[-\frac{x e^x}{x+1} - e^x \right] \\ &= \left[-\frac{1 e^1}{1+1} - e^1 - \frac{0}{1+0} + e^0 \right] \\ &= \left[-\frac{e}{2} + e + 0 - 1 \right] \\ &= \left[\frac{e}{2} - 1 \right] \end{aligned}$$

Question: 60

Evaluate:

Solution:

$$(1 - \log 2)$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} 2 \tan^3 x dx = 2 \int_0^{\frac{\pi}{2}} \tan^2 x \tan x dx$$

$$= 2 \int_0^{\frac{\pi}{2}} \tan^2 x \tan x dx$$

$$= 2 \int_0^{\frac{\pi}{2}} (\sec^2 x - 1) \tan x dx$$

Substitute:

$$\sec(x) = u$$

$$\therefore dx = \frac{1}{\sec(x)\tan(x)} du$$

$$= 2 \int_0^{\frac{\pi}{2}} \frac{(u^2 - 1)}{u} du$$

$$= 2 \int_0^{\frac{\pi}{2}} \left(u - \frac{1}{u}\right) du$$

$$= 2 \int_0^{\frac{\pi}{2}} \left(u - \frac{1}{u}\right) du$$

$$= 2 \left[\frac{u^2}{2} - \log u \right]$$

Undo substitution:

$$u = \sec(x)$$

$$\therefore \int_0^{\frac{\pi}{2}} 2 \tan^3 x dx = 2 \left[\frac{\sec^2 x}{2} - \log(\sec x) \right]$$

$$= 2 \left[\frac{\sec^2\left(\frac{\pi}{2}\right)}{2} - \log\left(\sec\left(\frac{\pi}{2}\right)\right) - \frac{\sec^2(0)}{2} + \log(\sec(0)) \right]$$

$$= 2 \left[\frac{1}{2} - \log(1) \right]$$

$$= 1 - \log 2$$

Question: 61

Solution:

$$5 - \frac{5}{2} \left(9 \log \frac{5}{4} - \log \frac{3}{2} \right)$$

Explanation:

$$\int_1^2 \frac{5x^2}{(x^2 + 4x + 3)} dx = 5 \left[\int_1^2 \frac{x^2}{(x+3)(x+1)} dx \right]$$

$$= 5 \left[\int_1^2 \left(1 - \frac{9}{2(x+3)} + \frac{1}{2(x+1)} \right) dx \right]$$

$$\begin{aligned}
 &= 5 \left[x - \frac{9}{2} \log(x+3) + \frac{1}{2} \log(x+1) \right]_1^2 \\
 &= 5 \left[2 - \frac{9}{2} \log 5 + \frac{1}{2} \log 3 - 1 + \frac{9}{2} \log 4 - \frac{1}{2} \log 2 \right] \\
 &= 5 \left[1 - \frac{9}{2} \log \left(\frac{5}{4} \right) + \frac{1}{2} \log \left(\frac{3}{2} \right) \right] \\
 &= 5 - \frac{5}{2} \left(9 \log \left(\frac{5}{4} \right) - \log \left(\frac{3}{2} \right) \right)
 \end{aligned}$$

Exercise : 16B

Question: 1

Solution:

$$\text{Let } I = \int_0^1 \frac{1}{2x-3} dx$$

$$\text{Let } 2x-3=t$$

$$\Rightarrow 2dx=dt.$$

Hence,

$$I = \frac{1}{2} \int_0^1 \frac{1}{t} dt = \frac{1}{2} \log_e |t|$$

$$= \frac{1}{2} \log_e |2x-3| \Big|_0^1$$

$$\Rightarrow I = \frac{1}{2} \log_e 1 - \frac{1}{2} \log_e 3 = \frac{1}{2} \log_e \frac{1}{3}$$

$$= -\frac{1}{2} \log_e 3$$

$$(\text{Since } \log_a \frac{1}{b} = -\log_a b)$$

Question: 2

Solution:

$$\text{Let } I = \int_0^1 \frac{2x}{1+x^2} dx$$

$$\text{Let } 1+x^2=t$$

$$\Rightarrow 2xdx=dt.$$

Also,

$$\text{when } x=0, t=1$$

and

$$\text{when } x=1, t=2$$

$$\text{Hence, } I = \int_1^2 \frac{1}{t} dt = \log_e |t| \Big|_1^2$$

$$= \log_e 2 - \log_e 1$$

$$= \log_e 2$$

Question: 3

Evaluate the foll

Solution:

$$\text{Let } I = \int_1^2 \frac{3x}{9x^2-1} dx$$

$$\text{Let } 9x^2-1=t$$

$$\Rightarrow 18x dx = dt.$$

Also,

$$\text{when } x=1, t=8$$

and

$$\text{when } x=2, t=35.$$

Hence,

$$I = \frac{1}{6} \int_8^{35} \frac{1}{t} dt = \frac{1}{6} \log_e t \Big|_8^{35} = \frac{1}{6} (\log_e 35 - \log_e 8)$$

Question: 4

Solution:

$$\text{Let } I = \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$$

$$\text{Let } \tan^{-1} x = t$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt.$$

$$\text{Also, when } x=0, t=0$$

$$\text{and when } x=1, t = \frac{\pi}{4}$$

Hence,

$$I = \int_0^{\frac{\pi}{4}} t dt = \frac{1}{2} t^2 \Big|_0^{\frac{\pi}{4}} = \frac{\pi^2}{32}$$

Question: 5

Solution:

$$\text{Let } I = \int_0^1 \frac{e^x}{1+e^{2x}} dx$$

$$\text{Let } e^x = t$$

$$\Rightarrow e^x dx = dt.$$

Also,

$$\text{when } x=0, t=1$$

and

$$\text{when } x=1, t=e.$$

Hence,

$$\begin{aligned} I &= \int_1^e \frac{1}{1+t^2} dt = \tan^{-1} t \Big|_1^e \\ &= \tan^{-1} e - \frac{\pi}{4} \end{aligned}$$

Question: 6

Evaluate the foll

Solution:

$$\text{Let } I = \int_0^1 \frac{2x}{1+x^4} dx$$

$$\text{Let } x^2 = t$$

$$\Rightarrow 2x dx = dt.$$

Also,

$$\text{when } x=0, t=0$$

and

$$\text{when } x=1, t=1.$$

Hence,

$$I = \int_0^1 \frac{1}{1+t^2} dt$$

$$= \tan^{-1} t \Big|_0^1$$

$$= \frac{\pi}{4}$$

Question: 7**Solution:**

$$\text{Let } I = \int_0^1 x e^{x^2} dx$$

$$\text{Let } x^2 = t$$

$$\Rightarrow 2x dx = dt.$$

Also,

$$\text{when } x=0, t=0$$

and

$$\text{when } x=1, t=1.$$

Hence,

$$I = \frac{1}{2} \int_0^1 e^t dt$$

$$= \frac{1}{2} e^t \Big|_0^1$$

$$= \frac{1}{2} (e - 1)$$

Question: 8**Solution:**

$$\text{Let } I = \int_1^2 \frac{\frac{1}{x}}{\frac{1}{x^2}} dx$$

$$\text{Let } \frac{1}{x} = t$$

$$\Rightarrow \frac{-1}{x^2} dx = dt.$$

Also,

when $x=1, t=1$

and

when $x=2, t = \frac{1}{2}$.

Hence,

$$\begin{aligned} I &= - \int_1^{\frac{1}{2}} e^t dt \\ &= -e^t \Big|_1^{\frac{1}{2}} \\ &= e - \sqrt{e} \end{aligned}$$

Question: 9

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{6}} \frac{\cos x}{3+4\sin x} dx$$

$$\text{Let } 3+4\sin x = t$$

$$\Rightarrow 4\cos x dx = dt.$$

Also,

when $x=0, t=3$

and

when $x = \frac{\pi}{6}, t=5$.

Hence,

$$\begin{aligned} I &= \frac{1}{4} \int_3^5 \frac{1}{t} dt \\ &= \frac{1}{4} \log_e t \Big|_3^5 \\ &= \frac{1}{4} (\log_e 5 - \log_e 3) \end{aligned}$$

Question: 10

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$$

$$\text{Let } \cos x = t$$

$$\Rightarrow -\sin x dx = dt.$$

Also,

when $x=0, t=1$

and

when $x = \frac{\pi}{2}, t=0$.

Hence,

$$I = -\int_1^0 \frac{1}{1+t^2} dt$$

$$= -\tan^{-1}t \Big|_1^0$$

$$= \frac{\pi}{4}$$

Question: 11

Solution:

$$\text{Let } I = \int_0^1 \frac{1}{e^x + e^{-x}} dx = \int_0^1 \frac{e^x}{1+e^{2x}} dx$$

$$\text{Let } e^x = t$$

$$\Rightarrow e^x dx = dt.$$

Also,

$$\text{when } x=0, t=1$$

and

$$\text{when } x=1, t=e.$$

Hence,

$$I = \int_1^e \frac{1}{1+t^2} dt$$

$$= \tan^{-1}t \Big|_1^e$$

$$= \tan^{-1}e - \frac{\pi}{4}$$

Question: 12

Solution:

$$\text{Let } I = \int_{\frac{1}{e}}^e \frac{1}{x(\log_e x)^{\frac{1}{2}}} dx$$

$$\text{Let } \log_e x = t$$

$$\Rightarrow \frac{1}{x} dx = dt.$$

Also,

$$\text{when } x = \frac{1}{e}, t=-1$$

and

$$\text{when } x=e, t=1.$$

Hence,

$$I = \int_{-1}^1 \frac{1}{t^{\frac{1}{2}}} dt$$

$$= \frac{3}{2} t^{\frac{2}{2}} \Big|_{-1}^1$$

$$= \frac{3}{2} (1 - 1)$$

=0

Question: 13

Solution:

$$\text{Let } I = \int_0^1 \frac{\sqrt{\tan^{-1}x}}{1+x^2} dx$$

$$\text{Let } \tan^{-1}x = t$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt.$$

Also,

$$\text{when } x=0, t=0$$

and

$$\text{when } x=1, t = \frac{\pi}{4}$$

Hence,

$$I = \int_0^{\frac{\pi}{4}} \sqrt{t} dt$$

$$= \frac{2}{3} t^{\frac{3}{2}} \Big|_0^{\frac{\pi}{4}}$$

$$= \frac{\pi^{\frac{3}{2}}}{12}$$

Question: 14

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1+\cos x}} dx$$

$$\text{Let } 1+\cos x = t$$

$$\Rightarrow -\sin x dx = dt.$$

$$\text{Also, when } x=0, t=2$$

and

$$\text{when } x = \frac{\pi}{2}, t=1$$

Hence,

$$I = - \int_2^1 \frac{1}{\sqrt{t}} dt$$

$$= -2\sqrt{t} \Big|_2^1$$

$$= 2(\sqrt{2}-1)$$

Question: 15

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sqrt{\sin x} \cos^5 x dx$$

$$\text{Let } \sin x = t$$

$$\Rightarrow \cos x \, dx = dt.$$

Also,

when $x=0$, $t=0$

and

when $x = \frac{\pi}{2}$, $t=1$.

Consider $\cos^5 x = \cos^4 x \times \cos x = (1 - \sin^2 x)^2 \times \cos x$ (Using $\sin^2 x + \cos^2 x = 1$)

Hence,

$$\begin{aligned} I &= \int_0^1 \sqrt{x} (1 - x^2)^2 dx \\ &= \int_0^1 \sqrt{x} dx + \int_0^1 x^{\frac{9}{2}} dx - 2 \int_0^1 x^{\frac{5}{2}} dx \\ \Rightarrow I &= \frac{2}{3} t^{\frac{3}{2}} \Big|_0^1 + \frac{2}{11} t^{\frac{11}{2}} \Big|_0^1 - \frac{4}{7} t^{\frac{7}{2}} \Big|_0^1 \\ &= \frac{2}{3} + \frac{2}{11} - \frac{4}{7} \\ &= \frac{64}{231} \end{aligned}$$

Question: 16

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1 + \sin^4 x} dx$$

Let $\sin^2 x = t$

$$\Rightarrow 2 \sin x \cos x = dt.$$

Also,

when $x=0$, $t=0$

and

when $x = \frac{\pi}{2}$, $t=1$.

Hence,

$$\begin{aligned} I &= \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{1}{1+t^2} dt \\ &= \frac{1}{2} \tan^{-1} t \Big|_0^1 \\ &= \frac{\pi}{8} \end{aligned}$$

Question: 17

Solution:

$$\text{Let } I = \int_0^a \sqrt{a^2 - x^2} dx$$

Let $x = a \sin t$

$$\Rightarrow a \cos t \, dt = dx.$$

Also,

when $x=0, t=0$

and

when $x=a, t = \frac{\pi}{2}$.

Hence,

$$I = \int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2 \sin^2 t} a \cos t \, dt = a^2 \int_0^{\frac{\pi}{2}} \cos^2 t \, dt$$

Using $\cos^2 t = \frac{1+\cos 2t}{2}$, we get

$$I = \frac{a^2}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2t) \, dt$$

$$= \frac{a^2}{2} \left(t + \frac{\sin 2t}{2} \right) \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi a^2}{4}$$

Question: 18

Solution:

$$\text{Let } I = \int_0^{\sqrt{2}} \sqrt{2-x^2} \, dx$$

$$\text{Consider, } I = \int_0^a \sqrt{a^2 - x^2} \, dx$$

Let $x=a \sin t$

$$\Rightarrow a \cos t \, dt = dx.$$

Also, when $x=0, t=0$

and when $x=a, t = \frac{\pi}{2}$.

Hence,

$$I = \int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2 \sin^2 t} a \cos t \, dt = a^2 \int_0^{\frac{\pi}{2}} \cos^2 t \, dt$$

Using $\cos^2 t = \frac{1+\cos 2t}{2}$, we get

$$I = \frac{a^2}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2t) \, dt$$

$$= \frac{a^2}{2} \left(t + \frac{\sin 2t}{2} \right) \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi a^2}{4}$$

Here $a = \sqrt{2}$, hence $I = \frac{\pi}{2}$

Question: 19

Solution:

$$\text{Let } I = \int_0^a \frac{x^4}{\sqrt{a^2 - x^2}} dx$$

$$\text{Let } x = a \sin t$$

$$\Rightarrow a \cos t \, dt = dx.$$

$$\text{Also, when } x=0, t=0$$

$$\text{and when } x=a, t = \frac{\pi}{2}.$$

Hence,

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \frac{a^4 \sin^4 t}{\sqrt{a^2 - a^2 \sin^2 t}} a \cos t \, dt \\ &= a^4 \int_0^{\frac{\pi}{2}} \sin^4 t \, dt \end{aligned}$$

$$\text{Using } \sin^2 t = \frac{1 - \cos 2t}{2}, \text{ we get}$$

$$\begin{aligned} I &= a^4 \int_0^{\frac{\pi}{2}} \left(\frac{1 - \cos 2t}{2} \right)^2 dt \\ &= \frac{a^4}{4} \int_0^{\frac{\pi}{2}} (1 + \cos^2 2t - 2\cos 2t) dt \\ \Rightarrow I &= \frac{a^4}{4} \left(t \Big|_0^{\frac{\pi}{2}} - \sin 2t \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \left(\frac{1 + \cos 4t}{2} \right) dt \right) \end{aligned}$$

$$\left(\text{Using } \cos^2 t = \frac{1 + \cos 2t}{2} \right)$$

Hence,

$$\begin{aligned} I &= \frac{\pi a^4}{8} + \frac{a^4}{4} \times \frac{t}{2} \Big|_0^{\frac{\pi}{2}} + \frac{a^4}{32} \sin 4t \Big|_0^{\frac{\pi}{2}} \\ &= \frac{3\pi a^4}{16} \end{aligned}$$

Question: 20

Solution:

$$\text{Let } I = \int_0^a \frac{x}{\sqrt{a^2 + x^2}} dx$$

$$\text{Let } a^2 + x^2 = t^2$$

$$\Rightarrow x \, dx = t \, dt.$$

$$\text{Also, when } x=0, t=a$$

$$\text{and when } x=a, t = \sqrt{2}a.$$

Hence,

$$\begin{aligned} I &= \int_a^{\sqrt{2}a} \frac{t}{\sqrt{t^2}} dt \\ &= t \Big|_a^{\sqrt{2}a} \\ &= a(\sqrt{2}-1) \end{aligned}$$

Question: 21

Solution:

$$\text{Let } I = \int_0^2 x\sqrt{2-x} dx$$

Using the property that $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$, we get

$$\begin{aligned} I &= \int_0^2 (2-x)\sqrt{x} dx \\ &= \int_0^2 2\sqrt{x} dx - \int_0^2 x^{\frac{3}{2}} dx \\ &= 2 \times \frac{2}{3} x^{\frac{3}{2}} \Big|_0^2 - \frac{2}{5} x^{\frac{5}{2}} \Big|_0^2 \end{aligned}$$

Hence,

$$\begin{aligned} I &= 2\sqrt{2} \left(\frac{4}{3} - \frac{4}{5} \right) \\ &= \frac{16}{15} \sqrt{2} \end{aligned}$$

Question: 22**Solution:**

$$\text{Let } I = \int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$$

$$\text{Let } f(x) = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$\text{Let } x = \tan \theta$$

$$\Rightarrow \theta = \tan^{-1} x$$

$$\Rightarrow f(x) = \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right)$$

$$= \sin^{-1} \left(\frac{2 \tan \theta}{\sec^2 \theta} \right)$$

$$= \sin^{-1} (2 \sin \theta \cos \theta)$$

$$= \sin^{-1} (\sin 2\theta)$$

$$\text{Hence } f(x) = 2\theta$$

$$= 2 \tan^{-1} x$$

Hence $I = 2 \int_0^1 1 \times \tan^{-1} x dx$ Using integration by parts, we get

$$I = 2x \tan^{-1} x \Big|_0^1 - \int_0^1 \frac{2x}{1+x^2} dx$$

$$= \frac{\pi}{2} - \int_0^1 \frac{2x}{1+x^2} dx \quad (1)$$

$$\text{Let } I' = \int_0^1 \frac{2x}{1+x^2} dx$$

$$\text{Let } 1+x^2 = t$$

$$\Rightarrow 2x dx = dt$$

Also, when $x=0$, $t=1$

and when $x=1$, $t=2$

Hence,

$$\begin{aligned} I' &= \int_1^2 \frac{1}{t} dt = \log_e |t| \Big|_1^2 \\ &= \log_e 2 - \log_e 1 \\ &= \log_e 2 - (2) \end{aligned}$$

Substituting value of (2) in (1), we get

$$I = \frac{\pi}{2} - \log_e 2$$

Question: 23

Solution:

$$\text{Let } I = \int_{\frac{\pi}{2}}^{\pi} \sqrt{1 + \cos x} \, dx$$

Using $1 + \cos x = 2\cos^2 \frac{x}{2}$, we get

$$\begin{aligned} I &= \sqrt{2} \int_0^{\frac{\pi}{2}} \cos\left(\frac{x}{2}\right) dx \\ &= 2\sqrt{2} \sin\left(\frac{x}{2}\right) \Big|_0^{\frac{\pi}{2}} \\ &= 2 \end{aligned}$$

Question: 24

Solution:

$$\text{Let } I = \int_{\frac{\pi}{2}}^{\pi} \sqrt{1 + \sin x} \, dx$$

Using $\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} = 1$ and $\sin x = 2\sin \frac{x}{2} \cos \frac{x}{2}$

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \sqrt{\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)^2} dx \\ &= \int_0^{\frac{\pi}{2}} \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) dx \\ &= -2 \cos\left(\frac{x}{2}\right) \Big|_0^{\frac{\pi}{2}} + 2 \sin\left(\frac{x}{2}\right) \Big|_0^{\frac{\pi}{2}} \\ &= -(\sqrt{2}-2) + (\sqrt{2}) \\ &= 2 \end{aligned}$$

Question: 25

Solution:

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2 \cos^2 x + b^2 \sin^2 x} dx$$

Dividing by $\cos^2 x$ in numerator and denominator, we get

$$I = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$$

Let $\tan x = t$

$$\Rightarrow \sec^2 x dx = dt$$

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2 + b^2 t^2} dt = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2} + t^2} dt$$

$$\text{Let } t = \frac{a}{b} \tan \theta = \tan x$$

$$I = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{\frac{a}{b} \sec^2 \theta}{\frac{a^2}{b^2} + \frac{a^2}{b^2} \tan^2 \theta} d\theta$$

$$= \frac{1}{ab} \theta$$

$$= \frac{1}{ab} \tan^{-1} \left(\frac{b}{a} \tan x \right) \Bigg|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2ab}$$

Question: 26

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{1}{\sec^2 x + \tan^2 x} dx$$

Dividing by $\cos^2 x$ in numerator and denominator, we get

$$I = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{\sec^2 x + \tan^2 x} dx = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{1 + 2 \tan^2 x} dx$$

$$\text{Consider } I = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$$

Let $\tan x = t$

$$\Rightarrow \sec^2 x dx = dt$$

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2 + b^2 t^2} dt$$

$$= \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2} + t^2} dt$$

$$\text{Let } t = \frac{a}{b} \tan \theta$$

$$= \tan x$$

$$I = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{\frac{a}{b} \sec^2 \theta}{\frac{a^2}{b^2} + \frac{a^2}{b^2} \tan^2 \theta} d\theta$$

$$= \frac{1}{ab} \theta = \frac{1}{ab} \tan^{-1} \left(\frac{b}{a} \tan x \right) \Bigg|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2ab}$$

Here, $a=1$ and $b=\sqrt{2}$

Hence,

$$I = \frac{\pi}{2\sqrt{2}}$$

Question: 27

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{1}{4 + 9 \tan^2 x} dx$$

Dividing by $\cos^2 x$ in numerator and denominator, we get

$$I = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{4 \sec^2 x + 9 \tan^2 x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{4 + 13 \tan^2 x} dx$$

$$\text{Consider } I = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$$

Let $\tan x = t$

$$\Rightarrow \sec^2 x dx = dt$$

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2 + b^2 t^2} dt$$

$$= \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2} + t^2} dt$$

$$\text{Let } t = \frac{a}{b} \tan \theta$$

$$= \tan x$$

$$I = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{\frac{a}{b} \sec^2 \theta}{\frac{a^2}{b^2} + \frac{a^2}{b^2} \tan^2 \theta} d\theta$$

$$= \frac{1}{ab} \theta$$

$$= \frac{1}{ab} \tan^{-1} \left(\frac{b}{a} \tan x \right) \Bigg|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2ab}$$

Here, $a=2$ and $b=\sqrt{13}$

Hence,

$$I = \frac{\pi}{4\sqrt{13}}$$

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{1}{5 + 4 \tan^2\left(\frac{x}{2}\right)} dx$$

Using $\sin x = \frac{2 \tan\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}$, we get

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \frac{1}{5 + 4 \frac{2 \tan\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}} dx \\ &= \int_0^{\frac{\pi}{2}} \frac{\sec^2\left(\frac{x}{2}\right)}{5 + 5 \tan^2\left(\frac{x}{2}\right) + 8 \tan\left(\frac{x}{2}\right)} dx \end{aligned}$$

$$\text{Let } \tan\left(\frac{x}{2}\right) = t$$

$$\Rightarrow \frac{1}{2} \sec^2\left(\frac{x}{2}\right) dx = dt,$$

when $x=0$, $t=0$ and when $x = \frac{\pi}{2}$, $t=1$.

$$\begin{aligned} \text{Hence, } I &= \int_0^1 \frac{2}{5 + 5t^2 + 8t} dt \\ &= \frac{2}{5} \int_0^1 \frac{1}{t^2 + \frac{8}{5}t + \frac{16}{25} + \frac{9}{25}} dt \\ &= \frac{2}{5} \int_0^1 \frac{1}{\left(t + \frac{4}{5}\right)^2 + \frac{9}{25}} dt \end{aligned}$$

$$\text{Let } t + \frac{4}{5} = u$$

$$\Rightarrow dt = du.$$

When $t=0$, $u = \frac{4}{5}$ and when $t=1$, $u = \frac{9}{5}$.

$$I = \frac{2}{5} \int_{\frac{4}{5}}^{\frac{9}{5}} \frac{1}{(u)^2 + \frac{9}{25}} du$$

$$= \frac{2}{5} \times \frac{5}{3} \tan^{-1}\left(\frac{5u}{3}\right) \Bigg|_{\frac{4}{5}}^{\frac{9}{5}}$$

$$= \frac{2}{3} \left(\tan^{-1} 3 - \tan^{-1}\left(\frac{4}{3}\right) \right)$$

$$= \frac{2}{3} \times \tan^{-1}\left(\frac{3 - \frac{4}{3}}{5}\right)$$

$$= \frac{2}{3} \tan^{-1}\left(\frac{1}{3}\right)$$

$$\left(\text{Using } \tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x - y}{1 + xy}\right) \right)$$

Question: 29**Solution:**

$$\text{Let } I = \int_0^{\pi} \frac{1}{5 + 4 \cos x} dx$$

Using $\cos x = \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}$, we get

$$I = \int_0^{\pi} \frac{1}{5 + 4 \times \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}} dx$$

$$= \int_0^{\pi} \frac{\sec^2\left(\frac{x}{2}\right)}{5 + 7 \tan^2\left(\frac{x}{2}\right)} dx$$

$$\text{Let } \tan\left(\frac{x}{2}\right) = t$$

$$\Rightarrow \frac{1}{2} \sec^2\left(\frac{x}{2}\right) dx = dt,$$

when $x=0$, $t=0$ and when $x=\pi$, $t=\infty$.

$$\text{Hence, } I = \int_0^{\infty} \frac{2}{5 + 7t^2} dt$$

$$= \frac{2}{7} \int_0^{\infty} \frac{1}{t^2 + \frac{5}{7}} dt$$

$$= \frac{2}{7} \times \sqrt{\frac{7}{5}} \tan^{-1} \left(\sqrt{\frac{7}{5}} x \right) \Big|_0^{\infty}$$

$$\Rightarrow I = \frac{2}{\sqrt{35}} \left(\frac{\pi}{2} - 0 \right)$$

$$= \frac{\pi}{\sqrt{35}}$$

Question: 30**Solution:**

$$\text{Let } I = \int_0^{\pi} \frac{1}{5 + 4 \cos x} dx$$

Using $\cos x = \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}$, we get

$$I = \int_0^{\pi} \frac{1}{5 + 4 \times \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}} dx$$

$$= \int_0^{\pi} \frac{\sec^2\left(\frac{x}{2}\right)}{9 + \tan^2\left(\frac{x}{2}\right)} dx$$

$$\text{Let } \tan\left(\frac{x}{2}\right) = t$$

$$\Rightarrow \frac{1}{2} \sec^2 \left(\frac{x}{2} \right) dx = dt,$$

when $x=0$, $t=0$ and when $x=\pi$, $t=\infty$.

$$\text{Hence, } I = \int_0^\infty \frac{2}{9+t^2} dt$$

$$= 2 \int_0^\infty \frac{1}{9+t^2} dt$$

$$= 2 \times \frac{1}{3} \tan^{-1} \left(\frac{t}{3} \right) \Big|_0^\infty$$

$$\Rightarrow I = \frac{2}{3} \left(\frac{\pi}{2} - 0 \right)$$

$$= \frac{\pi}{3}$$

Question: 31

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \tan^2 \left(\frac{x}{2} \right)} dx$$

$$\text{Using } \sin x = \frac{2 \tan \left(\frac{x}{2} \right)}{1 + \tan^2 \left(\frac{x}{2} \right)}$$

And

$$\cos x = \frac{1 - \tan^2 \left(\frac{x}{2} \right)}{1 + \tan^2 \left(\frac{x}{2} \right)},$$

we get

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{1}{\frac{1 - \tan^2 \left(\frac{x}{2} \right)}{1 + \tan^2 \left(\frac{x}{2} \right)} + 2 \frac{2 \tan \left(\frac{x}{2} \right)}{1 + \tan^2 \left(\frac{x}{2} \right)}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 \left(\frac{x}{2} \right)}{1 - \tan^2 \left(\frac{x}{2} \right) + 4 \tan \left(\frac{x}{2} \right)} dx$$

$$\text{Let } \tan \left(\frac{x}{2} \right) = t$$

$$\Rightarrow \frac{1}{2} \sec^2 \left(\frac{x}{2} \right) dx = dt,$$

when $x=0$, $t=0$

and when $x = \frac{\pi}{2}$, $t=1$.

Hence,

$$I = \int_0^1 \frac{2}{1-t^2+4t} dt$$

$$= -2 \int_0^1 \frac{1}{t^2 - 4t + 4 - 5} dt$$

$$= -2 \int_0^1 \frac{1}{(t-2)^2 - 5} dt$$

Let $t-2=u$

$$\Rightarrow dt=du.$$

Also, when $t=0$, $u=-2$

and when $t=1$, $u=-1$.

$$\Rightarrow I = -2 \int_{-2}^{-1} \frac{1}{u^2 - 5} dt$$

$$= -2 \times \frac{1}{2\sqrt{5}} \log_e \left| \frac{x-\sqrt{5}}{x+\sqrt{5}} \right| \Big|_{-2}^{-1}$$

$$\left(\text{Using } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log_e \left| \frac{x-a}{x+a} \right| \right)$$

Hence,

$$I = -\frac{1}{\sqrt{5}} \left(\log_e \left| \frac{-1-\sqrt{5}}{-1+\sqrt{5}} \right| - \log_e \left| \frac{-2-\sqrt{5}}{-2+\sqrt{5}} \right| \right)$$

$$= \frac{-1}{\sqrt{5}} \left(\log_e \left| \frac{\sqrt{5}+1}{\sqrt{5}-1} \right| \times \left| \frac{\sqrt{5}-2}{2+\sqrt{5}} \right| \right)$$

$$\left(\text{Using } \log_e a - \log_e b = \log_e \frac{a}{b} \right)$$

$$\Rightarrow I = \frac{-1}{\sqrt{5}} \left(\log_e \left| \frac{3-\sqrt{5}}{3+\sqrt{5}} \right| \right)$$

$$= \frac{-2}{\sqrt{5}} \left(\log_e \left(\frac{3-\sqrt{5}}{2} \right) \right)$$

$$\left(\text{Using } \log_e a^b = b \log_e a \right)$$

Question: 32

Solution:

$$\text{Let } I = \int_0^{\pi} \frac{1}{3 + \frac{1 - \tan^2(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})} + 2 \frac{2 \tan(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}} dx$$

$$\text{Using } \sin x = \frac{2 \tan(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}$$

And

$$\cos x = \frac{1 - \tan^2(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})},$$

we get

$$\Rightarrow I = \int_0^{\pi} \frac{1}{3 + \frac{1 - \tan^2(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})} + 2 \frac{2 \tan(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}} dx$$

$$= \int_0^{\pi} \frac{\sec^2(\frac{x}{2})}{4 + 2 \tan^2(\frac{x}{2}) + 4 \tan(\frac{x}{2})} dx$$

$$\text{Let } \tan\left(\frac{x}{2}\right) = t$$

$$\Rightarrow \frac{1}{2} \sec^2\left(\frac{x}{2}\right) dx = dt,$$

when $x=0$, $t=0$

and when $x = \pi$, $t = \infty$.

Hence,

$$I = \int_0^{\infty} \frac{1}{(t+1)^2+1} dt$$

Let $t+1=u$

$$\Rightarrow dt=du.$$

Also, when $t=0$, $u=1$

and when $t=\infty$, $u=\infty$.

$$I = \int_1^{\infty} \frac{1}{u^2+1} du$$

$$= \tan^{-1} u \Big|_1^{\infty}$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{4}$$

Question: 33

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1+\cos 2x} dx$$

Using $1+\cos 2x=2\cos^2 x$, we get

$$I = \frac{1}{2} \int_0^{\frac{\pi}{4}} \tan^3 x \sec^2 x dx$$

Let $\tan x=t$

$$\Rightarrow \sec^2 x dx = dt.$$

when $x=0$, $t=0$

and when $x = \frac{\pi}{4}$, $t=1$.

$$= \frac{1}{2} \int_0^1 t^3 dt = \frac{t^4}{8} \Big|_0^1$$

$$= \frac{1}{8}$$

Question: 34

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\cos^2 x + 3 \cos x + 2} dx$$

Let $\cos x=t$

$$\Rightarrow -\sin x dx = dt.$$

Also, when $x=0$, $t=1$

and when $x = \frac{\pi}{2}$, $t=0$.

Hence,

$$\begin{aligned}
 I &= -\int_1^0 \frac{t}{t^2+3t+2} dt \\
 &= -\int_1^0 \frac{2(t+1) - (t+2)}{(t+1)(t+2)} dt \\
 &= -\int_1^0 \frac{2}{(t+2)} dt + \int_1^0 \frac{1}{(t+1)} dt \\
 \Rightarrow I &= -2 \log_e(t+2) \Big|_1^0 + \log_e(t+1) \Big|_1^0 \\
 &= -2\log_e 2 + 2\log_e 3 - \log_e 2
 \end{aligned}$$

Hence $I = \log_e 9 - \log_e 8$

(Using $\log_e a = \log_e a^b$ and $\log_e a + \log_e b = \log_e ab$)

Question: 35

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\cos^4 x (\tan^4 x + 1)} dx$

Using $\sin 2x = 2 \sin x \cos x$, we get

$$\begin{aligned}
 I &= \int_0^{\frac{\pi}{2}} \frac{2 \sin x \cos x}{\cos^4 x (\tan^4 x + 1)} dx \\
 &= 2 \int_0^{\frac{\pi}{2}} \frac{\tan x \sec^2 x}{(\tan^4 x + 1)} dx
 \end{aligned}$$

Let $\tan x = t$

$\Rightarrow \sec^2 x dx = dt.$

Also, when $x=0$, $t=0$

and when $x = \frac{\pi}{2}$, $t=\infty$.

Hence, $2 \int_0^{\infty} \frac{t}{(t^4+1)} dt$

Let $x^2=t$

$\Rightarrow 2x dx = dt.$

Also, when $x=0$, $t=0$

and when $x=\infty$, $t=\infty$.

Hence, $I = \int_0^{\infty} \frac{1}{1+t^2} dt$

$$\begin{aligned}
 &= \tan^{-1} t \Big|_0^{\infty} \\
 &= \frac{\pi}{2}
 \end{aligned}$$

Question: 36

Solution:

Let $I = \int_{\frac{\pi}{2}}^{\pi} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{5}{2}}} dx$

Using $1 + \cos x = 2\cos^2\left(\frac{x}{2}\right)$

And

$$1 - \cos x = 2\sin^2\left(\frac{x}{2}\right),$$

we get

$$\begin{aligned} I &= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2} \cos\left(\frac{x}{2}\right)}{4\sqrt{2} \left(\sin\left(\frac{x}{2}\right)\right)^5} dx \\ &= \frac{1}{4} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot\left(\frac{x}{2}\right) \operatorname{cosec}^4\left(\frac{x}{2}\right) dx \end{aligned}$$

Let $\cot\left(\frac{x}{2}\right) = t$

$$\Rightarrow -\frac{1}{2} \operatorname{cosec}^2\left(\frac{x}{2}\right) dx = dt.$$

Also, when $x = \frac{\pi}{3}$, $t = \sqrt{3}$

and when $x = \frac{\pi}{2}$, $t = 1$

Hence,

$$\begin{aligned} I &= -\frac{1}{2} \int_{\sqrt{3}}^1 t(1+t^2) dt \\ &= -\frac{1}{2} \left[\frac{t^2}{2} \right]_{\sqrt{3}}^1 - \frac{1}{2} \left[\frac{t^4}{4} \right]_{\sqrt{3}}^1 \\ &= \frac{1}{2} + 1 \\ &= \frac{3}{2} \end{aligned}$$

Question: 37

Solution:

Let $I = \int_0^1 (\cos^{-1} x)^2 dx$

Let $x = \cos t \Rightarrow dx = -\sin t dt$.

Also, when $x=0$, $t = \frac{\pi}{2}$

and when $x=1$, $t=0$.

Hence, $I = -\int_{\frac{\pi}{2}}^0 t^2 \sin t dt$

Using integration by parts, we get

$$\begin{aligned} I &= -\left(t^2 \times -\cos t \right) \Big|_{\frac{\pi}{2}}^0 + 2 \int_{\frac{\pi}{2}}^0 t \cos t dt \\ &= -\left(0 - 0 + 2t \times \sin t \right) \Big|_{\frac{\pi}{2}}^0 - 2 \int_{\frac{\pi}{2}}^0 \sin t dt \\ &= -\left(-\pi + 2\cos t \right) \Big|_{\frac{\pi}{2}}^0 \end{aligned}$$

Hence, $I = \pi - 2$

Question: 38

Solution:

$$\text{Let } I = \int_0^1 x (\tan^{-1} x)^2 dx$$

Using integration by parts, we get

$$I = \frac{(\tan^{-1} x)^2 x^2}{2} \Big|_0^1 - \int_0^1 \frac{2 \tan^{-1} x}{1+x^2} \times \frac{x^2}{2} dx$$

$$= \frac{\pi^2}{32} - 0 - \int_0^1 \frac{\tan^{-1} x}{1+x^2} \times (1+x^2-1) dx$$

$$= \frac{\pi^2}{32} - \int_0^1 \tan^{-1} x dx + \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$$

$$\text{Let } \tan^{-1} x = t$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt.$$

$$\text{When } x=0, t=0 \text{ and when } x=1, t = \frac{\pi}{4}.$$

Hence

$$I = \frac{\pi^2}{32} - \tan^{-1} x \times x \Big|_0^1 + \int_0^1 \frac{x}{1+x^2} dx + \int_0^{\frac{\pi}{4}} t dt$$

$$= \frac{\pi^2}{32} - \frac{\pi}{4} + \frac{t^2}{2} \Big|_0^{\frac{\pi}{4}} + \int_0^1 \frac{x}{1+x^2} dx$$

$$\text{Let } 1+x^2=y$$

$$\Rightarrow 2x dx = dy.$$

$$\text{Also, when } x=0, y=1$$

$$\text{and when } x=1, y=2.$$

$$I = \frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2} \int_1^2 \frac{1}{y} dy$$

$$= \frac{\pi}{4} \left(\frac{\pi}{4} - 1 \right) + \frac{1}{2} \log_e y \Big|_1^2$$

$$= \frac{\pi}{4} \left(\frac{\pi}{4} - 1 \right) + \frac{1}{2} \log_e 2.$$

Question: 39

Solution:

$$\text{Let } I = \int_0^1 \sin^{-1} \sqrt{x} dx$$

$$\text{Let } \sqrt{x} = t$$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

or

$$dx = 2t dt.$$

$$\text{When, } x=0, t=0$$

and when $x=1$, $t=1$.

Hence,

$$I = 2 \int_0^1 t \sin^{-1} t \, dt$$

Using integration by parts, we get

$$\begin{aligned} I &= 2 \left(\sin^{-1} t \times \frac{t^2}{2} \Big|_0^1 - \int_0^1 \frac{1}{\sqrt{1-t^2}} \times \frac{t^2}{2} dt \right) \\ &= \frac{\pi}{2} - \int_0^1 \frac{t^2}{\sqrt{1-t^2}} dt \end{aligned}$$

Let $t = \sin y$

$\Rightarrow dt = \cos y \, dy$.

When $t=0$, $y=0$, when $t=1$, $y = \frac{\pi}{2}$.

$$I = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \sin^2 y \, dy \quad \dots (1)$$

Using, $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$, we get

$$I = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \cos^2 y \, dy \quad \dots (2)$$

Adding (1) and (2), we get

$$\begin{aligned} 2I &= \pi - \int_0^{\frac{\pi}{2}} dy \\ &= \pi - \frac{\pi}{2} \end{aligned}$$

Hence,

$$I = \frac{\pi}{4}$$

Question: 40

Solution:

$$\text{Let } I = \int_0^a \sin^{-1} \sqrt{\frac{x}{a+x}} dx$$

Let $x = a \tan^2 y$

$\Rightarrow dx = 2a \tan y \sec^2 y \, dy$.

Also, when $x=0$, $y=0$

and when $x=a$, $y = \frac{\pi}{4}$

$$\text{Hence } I = \int_0^{\frac{\pi}{4}} \sin^{-1} \left(\sqrt{\frac{a \tan^2 y}{a + a \tan^2 y}} \right) 2a \tan y \sec^2 y \, dy = 2a \int_0^{\frac{\pi}{4}} y \tan y \sec^2 y \, dy$$

Using integration by parts, we get

$$I = 2a \left(y \int_0^{\frac{\pi}{4}} \tan y \sec^2 y \, dy - \int_0^{\frac{\pi}{4}} \left(\int \tan y \sec^2 y \, dy \right) dy \right)$$

Let $\tan y = t$

$\Rightarrow \sec^2 y \, dy = dt$.

Also, when $y=0$, $t=0$

and when $y = \frac{\pi}{4}$, $t=1$.

Also, $y=\tan^{-1}t$

$$\Rightarrow dy = \frac{dt}{1+t^2}$$

$$I = 2a \left(\tan^{-1}t \int t dt \Big|_0^1 - \int_0^1 \left(\int t dt \right) \frac{dt}{1+t^2} \right)$$

$$= 2a \left(\frac{\tan^{-1}t \times t^2}{2} \Big|_0^1 \right) - 2a \int_0^1 \frac{t^2}{2(1+t^2)} dt$$

$$= \frac{a\pi}{4} - a \int_0^1 \frac{t^2}{1+t^2} dt$$

$$\text{Let } I' = \int_0^1 \frac{t^2}{1+t^2} dt$$

$$= \int_0^1 \frac{1+t^2-1}{1+t^2} dt$$

$$= \int_0^1 dt - \int_0^1 \frac{1}{1+t^2} dt$$

$$= t \Big|_0^1 - \tan^{-1}t \Big|_0^1$$

$$\text{Hence } I' = 1 - \frac{\pi}{4}$$

Substituting value of I' in I , we get

$$I = \frac{a\pi}{4} - a \left(1 - \frac{\pi}{4} \right)$$

$$= a \left(\frac{\pi}{2} - 1 \right)$$

Question: 41

Solution:

$$\text{Let } I = \int_0^9 \frac{1}{1+\sqrt{x}} dx$$

$$\text{Let } \sqrt{x}=u$$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = du$$

$$= \frac{1}{2u} dx \text{ or } dx=2u du.$$

Also, when $x=0$, $u=0$ and $x=9$, $u=3$.

Hence,

$$I = \int_0^3 \frac{2u}{1+u} du$$

$$= 2 \left(\int_0^3 \frac{u+1-1}{1+u} du \right)$$

$$= 2 \left(\int_0^3 du - \int_0^3 \frac{1}{1+u} du \right)$$

$$I = 2u \left|_0^3 - \log_e(1+u) \right|_0^3$$

$$= 6 - 2 \log_e 4$$

$$= 6 - 4 \log_e 2$$

(Using $\log_e a^b = b \log_e a$)

Question: 42

Solution:

$$\text{Let } I = \int_0^1 x^3 \sqrt{1+3x^4} dx$$

$$\text{Let } 1+3x^4=t$$

$$\Rightarrow 12x^3 dx = dt$$

Also, when $x=0$, $t=1$ and when $x=1$, $t=4$.

$$I = \frac{1}{12} \int_1^4 \sqrt{t} dt$$

$$= \frac{1}{12} \times \frac{2}{3} t^{\frac{3}{2}} \Big|_1^4$$

$$= \frac{7}{18}$$

Question: 43

Solution:

$$\text{Let } I = \int_0^1 \frac{1-x^2}{(1+x^2)^2} dx$$

$$\text{Let } I' = \int_0^1 \frac{1}{(1+x^2)^2} dx$$

$$\text{Let } x = \tan t$$

$$\Rightarrow dx = \sec^2 t dt$$

Also when $x=0$, $t=0$ and when $x=1$, $t = \frac{\pi}{4}$.

$$\text{Hence, } I' = \int_0^{\frac{\pi}{4}} \frac{\sec^2 t}{(1+\tan^2 t)^2} dt$$

$$= \int_0^{\frac{\pi}{4}} \cos^2 t dt$$

Using $\cos^2 t = \frac{1+\cos 2t}{2}$, we get

$$I' = \int_0^{\frac{\pi}{4}} \left(\frac{1+\cos 2t}{2} \right) dt$$

$$= \frac{t}{2} \Big|_0^{\frac{\pi}{4}} + \frac{\sin 2t}{4} \Big|_0^{\frac{\pi}{4}}$$

$$= \frac{\pi+2}{8}$$

$$\text{Let } I'' = \int_0^1 \frac{x^2}{(1+x^2)^2} dx$$

$$= \int_0^1 x \times \frac{x}{(1+x^2)^2} dx$$

$$= x \int_0^1 \frac{x}{(1+x^2)^2} dx - \int_0^1 \left(\int \frac{x}{(1+x^2)^2} dx \right) dx$$

$$\text{Let } 1+x^2=t \Rightarrow 2xdx=dt$$

When $x=0$, $t=1$ and when $x=1$, $t=2$.

$$I'' = \sqrt{t-1} \times \frac{1}{2} \int_1^2 \frac{1}{t^2} dt - \int_1^2 \left(\frac{1}{2} \int \frac{1}{t^2} dt \right) dt$$

$$= -\frac{\sqrt{t-1}}{2} \times \frac{1}{t} \Big|_1^2 + \int_1^2 \frac{dt}{4t\sqrt{t-1}}$$

$$= -\frac{1}{4} + \int_1^2 \frac{dt}{4t\sqrt{t-1}}$$

Substituting $t=1+x^2$

$$\Rightarrow 2xdx=dt.$$

When $t=1$, $x=0$ and when $t=2$, $x=1$.

$$I'' = -\frac{1}{4} + \int_0^1 \frac{2xdx}{4x(1+x^2)}$$

$$= -\frac{1}{4} + \frac{1}{2} \tan^{-1} x \Big|_0^1$$

$$= \frac{\pi - 2}{8}$$

Hence,

$$I = \frac{\pi + 2}{8} - \frac{\pi - 2}{8}$$

$$= \frac{1}{2}$$

Question: 44

Solution:

$$\text{Let } I = \int_1^2 \frac{1}{(x+1)\sqrt{x^2-1}} dx$$

$$\text{Let } x=\sec t$$

$$\Rightarrow dx=\sec t \tan t dt.$$

Also,

$$\text{when } x=1, t=0 \text{ and when } x=2, t = \frac{\pi}{3}$$

Hence,

$$I = \int_0^{\frac{\pi}{3}} \frac{\sec t \tan t}{(\sec t + 1)\sqrt{\sec^2 t - 1}} dt$$

$$= \int_0^{\frac{\pi}{3}} \frac{\sec t}{(\sec t + 1)} dt$$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{(1 + \cos t)} dt$$

Using $1 + \cos t = 2\cos^2\left(\frac{t}{2}\right)$, we get

$$\begin{aligned} I &= \frac{1}{2} \int_0^{\frac{\pi}{3}} \sec^2\left(\frac{t}{2}\right) dt \\ &= \tan\left(\frac{t}{2}\right) \Big|_0^{\frac{\pi}{3}} \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

Question: 45

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) dx = \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx$$

$$\text{Let } \sin x - \cos x = t$$

$$\Rightarrow (\cos x + \sin x) dx = dt.$$

$$\text{When } x=0, t=-1 \text{ and } x = \frac{\pi}{2}, t=1.$$

$$\text{Also, } t^2 = (\sin x - \cos x)^2$$

$$= \sin^2 x + \cos^2 x - 2\sin x \cos x$$

$$= 1 - 2\sin x \cos x$$

or

$$\sin x \cos x = \frac{1 - t^2}{2}$$

$$\text{Hence } I = \sqrt{2} \int_{-1}^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\text{Let } t = \sin y$$

$$\Rightarrow dt = \cos y dy.$$

$$\text{Also, when } t=-1, y = -\frac{\pi}{2}$$

$$\text{and when } t=1, y = \frac{\pi}{2}.$$

$$\begin{aligned} I &= \sqrt{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos y}{\sqrt{1 - \sin^2 y}} dy \\ &= \sqrt{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dy = \pi\sqrt{2} \end{aligned}$$

Question: 46

Solution:

$$\text{Let } I = \int_2^3 \frac{2-x}{\sqrt{5x-6-x^2}} dx$$

Let,

$$2-x = a \frac{d}{dx} (5x-6-x^2) + b$$

$$= -2ax + 5a + b$$

Hence $-2a = -1$ and $5a + b = 2$.

Solving these equations,

we get $a = \frac{1}{2}$ and $b = -\frac{1}{2}$.

We get,

$$I = \frac{1}{2} \int_2^3 \frac{-2x+5}{\sqrt{5x-6-x^2}} dx - \frac{1}{2} \int_2^3 \frac{1}{\sqrt{5x-6-x^2}} dx$$

$$\text{Let } I' = \int_2^3 \frac{-2x+5}{\sqrt{5x-6-x^2}} dx$$

$$\text{Let } 5x-6-x^2=t$$

$$\Rightarrow (5-2x) dx=dt.$$

When $x=2$, $t=0$ and when $x=3$, $y=0$.

$$\text{Hence } I' = \int_0^0 \frac{1}{\sqrt{t}} dt = 0$$

$$\left(\text{Since } \int_a^a f(x) dx = 0 \right)$$

Let,

$$I'' = \int_2^3 \frac{1}{\sqrt{5x-6-x^2}} dx$$

$$= \int_2^3 \frac{1}{\sqrt{\frac{1}{4} - \left(x - \frac{5}{2}\right)^2}} dx$$

$$= \sin^{-1} \left(\frac{x - \frac{5}{2}}{\frac{1}{2}} \right)$$

$$= \sin^{-1}(2x-5) \Big|_2^3$$

$$= \pi$$

Hence,

$$I = \frac{1}{2} \times 0 - \frac{1}{2} \times \pi$$

$$= -\frac{\pi}{2}$$

Question: 47

Solution:

$$\text{Let } I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{(\cos(x) + \sin(x))^2} dx$$

Using $\cos x = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$, we get

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}{\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)^2} dx$$

$$\text{Let } \cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right) = t$$

$$\Rightarrow \frac{1}{2} \left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right) \right) dx = dt.$$

Also, when $x = \frac{\pi}{4}$, $t = \cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right) = \alpha$ (Let)

and when $x = \frac{\pi}{2}$, $t = \sqrt{2}$

$$\begin{aligned} I &= \int_{\alpha}^{\sqrt{2}} \frac{2}{t^2} dt \\ &= -2 \times \frac{1}{t} \Big|_{\alpha}^{\sqrt{2}} \\ &= \frac{2}{\cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right)} - \sqrt{2} \end{aligned}$$

Question: 48

Solution:

Let $I = \int_0^{\left(\frac{\pi}{2}\right)^{\frac{1}{3}}} x^2 \sin(x^3) dx$

Let $x^3 = t$

$$\Rightarrow 3x^2 = dt.$$

Also, when $x=0$, $t=0$ and when $x = \left(\frac{\pi}{2}\right)^{\frac{1}{3}}$, $t = \frac{\pi}{2}$.

Hence, $I = \frac{1}{3} \int_0^{\frac{\pi}{2}} \sin(t) dt$

$$= -\frac{1}{3} \cos t \Big|_0^{\frac{\pi}{2}}$$

$$= -\frac{1}{3} (0 - 1)$$

$$= \frac{1}{3}$$

Question: 49

Solution:

Let $I = \int_1^2 \frac{1}{x(1+\log_e x)^2} dx$

Let $1 + \log_e x = t$

$$\Rightarrow \frac{1}{x} dx = dt.$$

Also, when $x=1$, $t=1$ and when $x=2$, $t = 1 + \log_e 2$

Hence $I = \int_1^{1+\log_e 2} \frac{1}{t^2} dt$

$$= -\frac{1}{t} \Big|_1^{1+\log_e 2}$$

$$= 1 - \frac{1}{1 + \log_e 2}$$

$$= \frac{\log_e 2}{1 + \log_e 2}$$

Question: 50

Solution:

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\csc x \cot x}{1 + \csc^2 x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} dx$$

Let $\sin x = t$

$$\Rightarrow \cos x \, dx = dt.$$

Also, when $x = \frac{\pi}{6}$, $t = \frac{1}{2}$ and when $x = \frac{\pi}{2}$, $t = 1$.

$$I = \int_{\frac{1}{2}}^1 \frac{1}{1 + t^2} dt$$

$$= \tan^{-1} t \Big|_{\frac{1}{2}}^1$$

$$= \tan^{-1} 1 - \tan^{-1} \left(\frac{1}{2} \right)$$

$$= \tan^{-1} \left(\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} \right)$$

$$= \tan^{-1} \left(\frac{1}{3} \right)$$

(Using $\tan^{-1} a - \tan^{-1} b = \tan^{-1} \left(\frac{a-b}{1+ab} \right)$)

Exercise : 16C

Question: 1

Solution:

$$y = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{2 \cos x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{\cos x + \cos x - \sin x + \sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} 1 + \frac{\cos x - \sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \left((x)_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{\sin x + \cos x} dx \right)$$

Let, $\sin x + \cos x = t$

$$\Rightarrow (\cos x - \sin x) \, dx = dt$$

At $x = 0$, $t = 1$

At $x = \pi/2$, $t = 1$

$$y = \frac{1}{2} \left(\frac{\pi}{2} + \int_1^1 \frac{1}{t} dt \right)$$

$$y = \frac{1}{2} \left(\frac{\pi}{2} + (\ln t)_1^1 \right)$$

$$y = \frac{\pi}{4}$$

Question: 2

Solution:

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx$$

$$= \int_0^{\pi/2} 1 dx$$

$$= (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 3 A

Solution:

$$y = \int_0^{\pi/2} \frac{\sin^2 x}{\sin^3\left(\frac{\pi}{2} - x\right) + \cos^3\left(\frac{\pi}{2} - x\right)} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$y = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin^3\left(\frac{\pi}{2} - x\right) + \cos^3\left(\frac{\pi}{2} - x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx + \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$= \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$= \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 3 B

Solution:

$$y = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\cos^3 \left(\frac{\pi}{2} - x \right)}{\sin^3 \left(\frac{\pi}{2} - x \right) + \cos^3 \left(\frac{\pi}{2} - x \right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx + \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$$

$$2y = \int_0^{\pi/2} \frac{\cos^3 x + \sin^3 x}{\sin^3 x + \cos^3 x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 4 A

Solution:

$$y = \int_0^{\pi/2} \frac{\sin^7 x}{\sin^7 x + \cos^7 x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\sin^7\left(\frac{\pi}{2}-x\right)}{\sin^7\left(\frac{\pi}{2}-x\right) + \cos^7\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos^7 x}{\sin^7 x + \cos^7 x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin^7 x}{\sin^7 x + \cos^7 x} dx + \int_0^{\pi/2} \frac{\cos^7 x}{\sin^7 x + \cos^7 x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin^7 x + \cos^7 x}{\sin^7 x + \cos^7 x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x) \Big|_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 4 B

Solution:

$$y = \int_0^{\pi/2} \frac{\cos^4 x}{\sin^4 x + \cos^4 x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\cos^4\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right) + \cos^4\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos^4 x}{\sin^4 x + \cos^4 x} dx + \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$$

$$2y = \int_0^{\pi/2} \frac{\cos^4 x + \sin^4 x}{\sin^4 x + \cos^4 x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x) \Big|_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 5

Prove that

Solution:

$$y = \int_0^{\pi/2} \frac{\cos^4 x}{\sin^4 x + \cos^4 x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\cos^4\left(\frac{\pi}{2} - x\right)}{\sin^4\left(\frac{\pi}{2} - x\right) + \cos^4\left(\frac{\pi}{2} - x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos^4 x}{\sin^4 x + \cos^4 x} dx + \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$$

$$2y = \int_0^{\pi/2} \frac{\cos^4 x + \sin^4 x}{\sin^4 x + \cos^4 x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 6

Solution:

$$y = \int_0^{\pi/2} \frac{\cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\cos^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx + \int_0^{\pi/2} \frac{\sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx$$

$$2y = \int_0^{\pi/2} \frac{\cos^{\frac{1}{4}} x + \sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 7

Solution:

$$y = \int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x + \cos^2 x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$y = \int_0^{\pi/2} \frac{\sin^2 \left(\frac{\pi}{2} - x \right)}{\sin^2 \left(\frac{\pi}{2} - x \right) + \cos^2 \left(\frac{\pi}{2} - x \right)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos^2 x}{\sin^2 x + \cos^2 x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x + \cos^2 x} dx + \int_0^{\pi/2} \frac{\cos^2 x}{\sin^2 x + \cos^2 x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin^2 x + \cos^2 x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_{\frac{\pi}{2}}$$

$$y = \frac{\pi}{4}$$

Question: 8

Solution:

$$y = \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$y = \int_0^{\pi/2} \frac{\sin^n \left(\frac{\pi}{2} - x \right)}{\sin^n \left(\frac{\pi}{2} - x \right) + \cos^n \left(\frac{\pi}{2} - x \right)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx + \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin^n x + \cos^n x}{\sin^n x + \cos^n x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 9

Solution:

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x} \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$y = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2} - x)}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx + \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 10

Solution:

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x} \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$y = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx \dots (1)$$

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx + \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 11

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \frac{\sin x}{\cos x}} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx + \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Solution:

$$y = \int_0^{\pi/2} \frac{1}{1 + \frac{\cos x}{\sin x}} dx$$

$$y = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$y = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx + \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 13

Solution:

$$y = \int_0^{\pi/2} \frac{1}{1 + \frac{\sin^2 x}{\cos^2 x}} dx$$

$$y = \int_0^{\pi/2} \frac{\cos^2 x}{\sin^2 x + \cos^2 x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$y = \int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2} - x\right)}{\sin^2\left(\frac{\pi}{2} - x\right) + \cos^2\left(\frac{\pi}{2} - x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x + \cos^2 x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\cos^2 x}{\sin^2 x + \cos^2 x} dx + \int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x + \cos^2 x} dx$$

$$2y = \int_0^{\pi/2} \frac{\cos^3 x + \sin^3 x}{\sin^3 x + \cos^3 x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 14

Solution:

$$y = \int_0^{\pi/2} \frac{1}{1 + \frac{\cos^2 x}{\sin^2 x}} dx$$

$$y = \int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x + \cos^2 x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\sin^2(\frac{\pi}{2} - x)}{\sin^2(\frac{\pi}{2} - x) + \cos^2(\frac{\pi}{2} - x)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos^2 x}{\sin^2 x + \cos^2 x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x + \cos^2 x} dx + \int_0^{\pi/2} \frac{\cos^2 x}{\sin^2 x + \cos^2 x} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin^2 x + \cos^2 x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 15

Solution:

$$y = \int_0^{\pi/2} \frac{1}{1 + \frac{\sin x}{\cos x}} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx \dots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx$$

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 16

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{\frac{\sqrt{\cos x}}{\sqrt{\sin x}}}{1 + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx \dots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx \dots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx$$

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 17**Solution:**

$$y = \int_0^{\frac{\pi}{2}} \frac{\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}{1 + \frac{\sin x}{\sqrt{\cos x}}} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2}-x\right)} + \sqrt{\cos\left(\frac{\pi}{2}-x\right)}\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx$$

$$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 18**Solution:**

$$y = \int_{\frac{\pi}{2}}^{\pi} \frac{\sin x - \cos x}{1 + \cos x \sin x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right) - \cos\left(\frac{\pi}{2}-x\right)}{1 + \sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx$$

$$2y = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x + \cos x - \sin x}{1 + \cos x \sin x} dx$$

$$2y = \int_0^{\frac{\pi}{2}} 0 dx$$

$$y = 0$$

Question: 19

Solution:

$$y = \int_0^1 x(1-x)^5 dx$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^1 (1-x)x^5 dx$$

$$y = \int_0^1 x^5 - x^6 dx$$

$$y = \left(\frac{x^6}{6} - \frac{x^7}{7} \right)_0^1$$

$$y = \frac{1}{6} - \frac{1}{7}$$

$$= \frac{1}{42}$$

Question: 20

Solution:

$$y = \int_0^2 x\sqrt{2-x} dx$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^2 (2-x)\sqrt{x} dx$$

$$y = \int_0^2 2x^{\frac{1}{2}} - x^{\frac{3}{2}} dx$$

$$y = \left(2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right)_0$$

$$y = \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5} = \frac{16\sqrt{2}}{15}$$

Question: 21

Solution:

$$y = \int_0^{\pi} x \cos^2 x \, dx \dots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$$

$$y = \int_0^{\pi} (\pi-x) \cos^2(\pi-x) \, dx$$

$$y = \int_0^{\pi} \pi \cos^2 x - x \cos^2 x \, dx \dots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} x \cos^2 x \, dx + \int_0^{\pi} \pi \cos^2 x - x \cos^2 x \, dx$$

$$2y = \int_0^{\pi} \pi \cos^2 x \, dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{1 + \cos 2x}{2} \, dx$$

$$y = \frac{\pi}{2} \left(\frac{x}{2} + \frac{\sin 2x}{4} \right)_0^{\pi}$$

$$y = \frac{\pi}{2} \left(\frac{\pi}{2} + \frac{\sin 2\pi}{4} \right) = \frac{\pi^2}{4}$$

Question: 22

Solution:

$$y = \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} \, dx \dots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$$

$$y = \int_0^{\pi} \frac{(\pi-x) \tan(\pi-x)}{\sec(\pi-x) \operatorname{cosec}(\pi-x)} \, dx$$

$$y = \int_0^{\pi} \frac{-(\pi-x) \tan x}{-\sec x \operatorname{cosec} x} \, dx$$

$$y = \int_0^{\pi} \frac{\pi \tan x - x \tan x}{\sec x \operatorname{cosec} x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx + \int_0^{\pi} \frac{\pi \tan x - x \tan x}{\sec x \operatorname{cosec} x} dx$$

$$2y = \int_0^{\pi} \frac{\pi \tan x}{\sec x \operatorname{cosec} x} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} \times \frac{1}{\sin x}} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{1 - \cos 2x}{2} dx$$

$$y = \frac{\pi}{2} \left(\frac{x}{2} - \frac{\sin 2x}{4} \right)_0^{\pi}$$

$$y = \frac{\pi}{2} \left(\frac{\pi}{2} - \frac{\sin 2\pi}{4} \right) = \frac{\pi^2}{4}$$

Question: 23

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin x + \cos x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\cos^2 \left(\frac{\pi}{2} - x \right)}{\sin \left(\frac{\pi}{2} - x \right) + \cos \left(\frac{\pi}{2} - x \right)} dx$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin x + \cos x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$$

$$2y = \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx$$

$$2y = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x} dx$$

$$2y = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{1}{\sin \left(x + \frac{\pi}{4} \right)} dx$$

$$y = \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} \operatorname{cosec} \left(x + \frac{\pi}{4} \right) dx$$

$$y = \frac{1}{2\sqrt{2}} \left(\ln \left(\operatorname{cosec} \left(x + \frac{\pi}{4} \right) - \cot \left(x + \frac{\pi}{4} \right) \right) \right)_0^{\frac{\pi}{2}}$$

$$y = \frac{1}{2\sqrt{2}} \left(\ln \left(\operatorname{cosec} \frac{3\pi}{4} - \cot \frac{3\pi}{4} \right) - \ln \left(\operatorname{cosec} \frac{\pi}{4} - \cot \frac{\pi}{4} \right) \right)$$

$$y = \frac{1}{2\sqrt{2}} \ln \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$$

$$y = \frac{1}{2\sqrt{2}} \ln (\sqrt{2} + 1)^2 = \frac{1}{\sqrt{2}} \ln (\sqrt{2} + 1)$$

Question: 24

Solution:

$$y = \int_0^{\pi} \frac{\frac{x \sin x}{\cos x}}{\frac{1}{\cos x} + \cos x} dx$$

$$y = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$y = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx$$

$$y = \int_0^{\pi} \frac{\pi \sin x - x \sin x}{1 + \cos^2 x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx + \int_0^{\pi} \frac{\pi \sin x - x \sin x}{1 + \cos^2 x} dx$$

$$2y = \int_0^{\pi} \frac{\pi \sin x}{1 + \cos^2 x} dx$$

Let, $\cos x = t$

$$\Rightarrow -\sin x dx = dt$$

$$\text{At } x = 0, t = 1$$

$$\text{At } x = \pi, t = -1$$

$$y = -\frac{\pi}{2} \int_1^{-1} \frac{1}{1 + t^2} dt$$

$$y = -\frac{\pi}{2} (\tan^{-1} t)_1^{-1}$$

$$y = -\frac{\pi}{2} (\tan^{-1}(-1) - \tan^{-1} 1)$$

$$y = \frac{\pi^2}{4}$$

Solution:

$$\dots(1)$$

Use King Theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi} \frac{(\pi-x) \sin(\pi-x)}{1+\sin(\pi-x)} dx$$

$$y = \int_0^{\pi} \frac{\pi \sin x}{1+\sin x} - \frac{x \sin x}{1+\sin x} dx \dots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx + \int_0^{\pi} \frac{\pi \sin x}{1+\sin x} - \frac{x \sin x}{1+\sin x} dx$$

$$2y = \int_0^{\pi} \frac{\pi(\sin x + 1 - 1)}{1+\sin x} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} 1 - \frac{1}{1+\sin x} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} 1 - \frac{1-\sin x}{\cos^2 x} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} 1 - \sec^2 x + \frac{\sin x}{\cos^2 x} dx$$

Let, $\cos x = t$

$$\Rightarrow -\sin x dx = dt$$

$$\text{At } x = 0, t = 1$$

$$\text{At } x = \pi, t = -1$$

$$y = \frac{\pi}{2} \left((x - \tan x)_0^{\pi} - \int_1^{-1} \frac{1}{t^2} dt \right)$$

$$y = \frac{\pi}{2} \left(\pi - \tan \pi - \left(\frac{-1}{t} \right)_1^{-1} \right)$$

$$y = \frac{\pi}{2} (\pi - 2) = \pi \left(\frac{\pi}{2} - 1 \right)$$

Question: 26

Solution:

$$y = \int_0^{\pi} \frac{x}{1+\cos 2x} dx \dots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi} \frac{(\pi-x)}{1+\sin^2(\pi-x)} dx$$

$$y = \int_0^{\pi} \frac{\pi}{1+\sin^2 x} - \frac{x}{1+\sin^2 x} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \frac{x}{1+\sin^2 x} dx + \int_0^{\pi} \frac{\pi}{1+\sin^2 x} - \frac{x}{1+\sin^2 x} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{1}{1+\sin^2 x} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{\frac{1}{\cos^2 x}}{\frac{1+\sin^2 x}{\cos^2 x}} dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{\sec^2 x}{\sec^2 x + \tan^2 x} dx$$

We break it in two parts

$$y = \frac{\pi}{2} \int_0^{\pi} \frac{\sec^2 x}{\sec^2 x + \tan^2 x} dx$$

Let, $\tan x = t$

$$\Rightarrow \sec^2 x dx = dt$$

At $x = 0$, $t = 0$

At $x = \pi$, $t = 0$

$$y = \frac{\pi}{2} \int_0^0 \frac{1}{1+t^2} dt$$

We know that when upper and lower limit is same in definite integral then value of integration is 0.

So, $y = 0$

Question: 27

Solution:

$$y = \int_0^{\frac{\pi}{2}} \log \frac{\cos^2 x}{\sin 2x} dx$$

$$y = \int_0^{\frac{\pi}{2}} \log \frac{\cos^2 x}{2 \sin x \cos x} dx$$

$$y = \int_0^{\frac{\pi}{2}} \log \left(\frac{1}{2} \cot x \right) dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2} \cot\left(\frac{\pi}{2} - x\right)\right) dx$$

$$y = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2} \tan x\right) dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2} \cot x\right) dx + \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2} \tan x\right) dx$$

$$y = \frac{1}{2} \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{4} \cot x \tan x\right) dx \text{ [Use } \cot x \tan x = 1]$$

$$y = \frac{1}{2} \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{4}\right) dx$$

$$y = \frac{1}{2} \log\left(\frac{1}{4}\right) (x)_0^{\frac{\pi}{2}}$$

$$y = -\frac{\pi}{4} \log 4$$

Question: 28

Solution:

$$y = \int_0^{\infty} \frac{x}{(1+x)(1+x^2)} dx$$

Let, $x = \tan t$

$$\Rightarrow dx = \sec^2 t dt$$

$$\text{At } x = 0, t = 0$$

$$\text{At } x = \infty, t = \pi/2$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\tan t}{(1 + \tan t)(1 + \tan^2 t)} \sec^2 t dt$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\tan t}{(1 + \tan t)} dt$$

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin t}{(\cos t + \sin t)} dt \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - t\right)}{\sin\left(\frac{\pi}{2} - t\right) + \cos\left(\frac{\pi}{2} - t\right)} dt$$

$$y = \int_0^{\pi/2} \frac{\cos t}{\sin t + \cos t} dt \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/2} \frac{\sin t}{\sin t + \cos t} dx + \int_0^{\pi/2} \frac{\cos t}{\sin t + \cos t} dx$$

$$2y = \int_0^{\pi/2} \frac{\sin t + \cos t}{\sin t + \cos t} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

Question: 29

Solution:

Let, $x = a \sin t$

$$\Rightarrow dx = a \cos t dt$$

At $x = 0$, $t = 0$

At $x = a$, $t = \pi/2$

$$y = \int_0^{\pi/2} \frac{a \cos t}{a \sin t + \sqrt{a^2 - a^2 \sin^2 t}} dt$$

$$y = \int_0^{\pi/2} \frac{\cos t}{\sin t + \cos t} dt$$

$$y = \frac{1}{2} \int_0^{\pi/2} \frac{\cos t + \cos t - \sin t + \sin t}{\sin t + \cos t} dt$$

$$y = \frac{1}{2} \int_0^{\pi/2} 1 + \frac{\cos t - \sin t}{\sin t + \cos t} dt$$

$$y = \frac{1}{2} \left((t)_0^{\pi/2} + \int_0^{\pi/2} \frac{\cos t - \sin t}{\sin t + \cos t} dt \right)$$

Again, $\sin t + \cos t = z$

$$\Rightarrow (\cos t - \sin t) dt = dz$$

At $t = 0$, $z = 1$

At $t = \pi/2$, $z = 1$

$$y = \frac{1}{2} \left(\frac{\pi}{2} + \int_1^1 \frac{1}{z} dz \right)$$

$$y = \frac{1}{2} \left(\frac{\pi}{2} + (\ln z)_1^1 \right)$$

$$y = \frac{\pi}{4}$$

Question: 30

$$y = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx \dots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx \dots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx + \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx$$

$$2y = \int_0^a \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx$$

$$y = \frac{1}{2} \int_0^a dx$$

$$y = \frac{1}{2} (x)_0^a$$

$$y = \frac{a}{2}$$

Question: 31

Solution:

$$y = \int_0^\pi \sin^2 x \cos^3 x dx \dots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^\pi \sin^2(\pi-x) \cos^3(\pi-x) dx$$

$$y = - \int_0^\pi \sin^2 x \cos^3 x dx \dots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^\pi \sin^2 x \cos^3 x dx + \left(- \int_0^\pi \sin^2 x \cos^3 x dx \right)$$

$$y = 0$$

Question: 32

Solution:

$$y = \int_0^{\pi} \sin^{2m} x \cos^{2m+1} x dx \dots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi} \sin^{2m}(\pi-x) \cos^{2m+1}(\pi-x) dx$$

$$y = -\int_0^{\pi} \sin^{2m} x \cos^{2m+1} x dx \dots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \sin^{2m} x \cos^{2m+1} x dx + \left(-\int_0^{\pi} \sin^{2m} x \cos^{2m+1} x dx \right)$$

$$y = 0$$

Question: 33

Solution:

$$\text{Let, } \sin x + \cos x = t$$

$$\Rightarrow \cos x - \sin x dx = dt$$

$$\text{At } x = 0, t = 1$$

$$\text{At } x = \pi/2, t = 1$$

$$y = \int_1^1 -\log t dt$$

We know that when upper and lower limit in definite integral is equal then value of integration is zero.

$$\text{So, } y = 0$$

Question: 34

Solution:

$$y = \int_0^{\frac{\pi}{2}} \log(2 \sin x \cos x) dx$$

$$y = \int_0^{\frac{\pi}{2}} \log 2 + \log \sin x + \log \cos x dx$$

$$\text{Let, } I = \int_0^{\frac{\pi}{2}} \log \sin x dx \dots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx$$

$$I = \int_0^{\frac{\pi}{2}} \log \cos x \, dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx + \int_0^{\frac{\pi}{2}} \log \cos x \, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log \frac{2 \sin x \cos x}{2} \, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log \sin 2x - \log 2 \, dx$$

Let, $2x = t$

$$\Rightarrow 2 \, dx = dt$$

At $x = 0$, $t = 0$

At $x = \pi/2$, $t = \pi$

$$2I = \frac{1}{2} \int_0^{\pi} \log \sin t \, dt - \frac{\pi}{2} \log 2$$

$$2I = \frac{2}{2} \int_0^{\frac{\pi}{2}} \log \sin x \, dx - \frac{\pi}{2} \log 2$$

$$2I = I - \frac{\pi}{2} \log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx = -\frac{\pi}{2} \log 2$$

$$\text{Similarly, } \int_0^{\frac{\pi}{2}} \log \cos x \, dx = -\frac{\pi}{2} \log 2$$

$$y = \int_0^{\frac{\pi}{2}} \log 2 \, dx + \int_0^{\frac{\pi}{2}} \log \sin x \, dx + \int_0^{\frac{\pi}{2}} \log \cos x \, dx$$

$$y = \frac{\pi}{2} \log 2 - \frac{\pi}{2} \log 2 - \frac{\pi}{2} \log 2$$

$$y = -\frac{\pi}{2} \log 2$$

Question: 35

Solution:

$$y = \int_a^{\pi} x \log \sin x \, dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$$

$$y = \int_0^{\pi} (\pi - x) \log \sin(\pi - x) dx$$

$$y = \int_0^{\pi} \pi \log \sin x - x \log \sin x dx \dots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} x \log \sin x dx + \int_0^{\pi} \pi \log \sin x - x \log \sin x dx$$

$$y = \frac{\pi}{2} \int_0^{\pi} \log \sin x dx$$

$$y = \frac{2\pi}{2} \int_0^{\frac{\pi}{2}} \log \sin x dx \dots(3)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$y = \pi \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx$$

$$y = \pi \int_0^{\frac{\pi}{2}} \log \cos x dx \dots(4)$$

Adding eq.(3) and eq.(4)

$$2y = \pi \left(\int_0^{\frac{\pi}{2}} \log \sin x dx + \int_0^{\frac{\pi}{2}} \log \cos x dx \right)$$

$$2y = \pi \left(\int_0^{\frac{\pi}{2}} \log \frac{2 \sin x \cos x}{2} dx \right)$$

$$2y = \pi \left(\int_0^{\frac{\pi}{2}} \log \sin 2x - \log 2 dx \right)$$

Let, $2x = t$

$$\Rightarrow 2 dx = dt$$

At $x = 0$, $t = 0$

At $x = \pi/2$, $t = \pi$

$$2y = \frac{\pi}{2} \int_0^{\pi} \log \sin t dt - \frac{\pi^2}{2} \log 2$$

$$2y = \frac{2\pi}{2} \int_0^{\frac{\pi}{2}} \log \sin x dx - \frac{\pi^2}{2} \log 2$$

$$2y = y - \frac{\pi^2}{2} \log 2$$

$$y = -\frac{\pi^2}{2} \log 2$$

Question: 36

Solution:

...(1)

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_0^{\pi} \log(1 + \cos(\pi - x)) dx$$

$$y = \int_0^{\pi} \log(1 - \cos x) dx \dots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi} \log(1 + \cos x) dx + \int_0^{\pi} \log(1 - \cos x) dx$$

$$2y = \int_0^{\pi} \log \sin^2 x dx$$

$$y = 2 \int_0^{\frac{\pi}{2}} \log \sin x dx \dots(3)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = 2 \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx$$

$$y = 2 \int_0^{\frac{\pi}{2}} \log \cos x dx \dots(4)$$

Adding eq.(3) and eq.(4)

$$2y = 2 \left(\int_0^{\frac{\pi}{2}} \log \sin x dx + \int_0^{\frac{\pi}{2}} \log \cos x dx \right)$$

$$2y = 2 \left(\int_0^{\frac{\pi}{2}} \log \frac{2 \sin x \cos x}{2} dx \right)$$

$$2y = 2 \left(\int_0^{\frac{\pi}{2}} \log \sin 2x - \log 2 dx \right)$$

Let, $2x = t$

$$\Rightarrow 2 dx = dt$$

At $x = 0, t = 0$

At $x = \pi/2, t = \pi$

$$2y = \frac{2}{2} \int_0^{\pi} \log \sin t \, dt - \frac{2\pi}{2} \log 2$$

$$2y = \frac{4}{2} \int_0^{\frac{\pi}{2}} \log \sin x \, dx - \frac{2\pi}{2} \log 2$$

$$2y = y - \pi \log 2$$

$$y = -\pi \log 2$$

Question: 37

Solution:

$$y = \int_0^{\frac{\pi}{2}} \log \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) dx$$

$$y = \int_0^{\frac{\pi}{2}} \log \frac{1}{\sin x \cos x} dx$$

$$y = - \left(\int_0^{\frac{\pi}{2}} \log \sin x \, dx + \int_0^{\frac{\pi}{2}} \log \cos x \, dx \right)$$

$$\text{Let, } I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin \left(\frac{\pi}{2} - x \right) dx$$

$$I = \int_0^{\frac{\pi}{2}} \log \cos x \, dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx + \int_0^{\frac{\pi}{2}} \log \cos x \, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log \frac{2 \sin x \cos x}{2} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log \sin 2x - \log 2 \, dx$$

Let, $2x = t$

$$\Rightarrow 2 \, dx = dt$$

At $x = 0, t = 0$

At $x = \pi/2, t = \pi$

$$2I = \frac{1}{2} \int_0^{\pi} \log \sin t \, dt - \frac{\pi}{2} \log 2$$

$$2I = \frac{2}{2} \int_0^{\frac{\pi}{2}} \log \sin x \, dx - \frac{\pi}{2} \log 2$$

$$2I = I - \frac{\pi}{2} \log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx = -\frac{\pi}{2} \log 2$$

Similarly, $\int_0^{\frac{\pi}{2}} \log \cos x \, dx = -\frac{\pi}{2} \log 2$

$$y = - \left(\int_0^{\frac{\pi}{2}} \log \sin x \, dx + \int_0^{\frac{\pi}{2}} \log \cos x \, dx \right)$$

$$y = \frac{\pi}{2} \log 2 + \frac{\pi}{2} \log 2$$

$$y = \pi \log 2$$

Question: 38

Solution:

$$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\cos x}{\sin x + \cos x} \, dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$$

$$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\cos \left(\frac{3\pi}{8} + \frac{\pi}{8} - x \right)}{\sin \left(\frac{3\pi}{8} + \frac{\pi}{8} - x \right) + \cos \left(\frac{3\pi}{8} + \frac{\pi}{8} - x \right)} \, dx$$

$$y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sin x}{\sin x + \cos x} \, dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\cos x}{\sin x + \cos x} \, dx + \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sin x}{\sin x + \cos x} \, dx$$

$$2y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sin x + \cos x}{\sin x + \cos x} \, dx$$

$$2y = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} 1 \, dx$$

$$2y = (x)^{\frac{3\pi}{8}}$$

$$2y = \frac{3\pi}{8} - \frac{\pi}{8}$$

$$y = \frac{\pi}{8}$$

Question: 39

Solution:

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)} + \sqrt{\cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}\right)} dx$$

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx$$

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx$$

$$2y = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 dx$$

$$2y = (x)^{\frac{\pi}{6}}$$

$$y = \frac{\pi}{12}$$

Question: 40

Solution:

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{2\cos^2 \frac{x}{2}} dx$$

$$y = \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sec^2 \frac{x}{2} dx$$

$$y = \frac{1}{2} \left(\frac{\tan \frac{x}{2}}{\frac{1}{2}} \right)_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$y = \tan \frac{3\pi}{8} - \tan \frac{\pi}{8}$$

$$y = (\sqrt{2} + 1) - (\sqrt{2} - 1) = 2$$

Question: 41

Solution:

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx \dots(1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\left(\frac{3\pi}{4} + \frac{\pi}{4} - x\right)}{1 + \sin\left(\frac{3\pi}{4} + \frac{\pi}{4} - x\right)} dx$$

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{1 + \sin x} dx \dots(2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1 + \sin x} dx + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{1 + \sin x} dx$$

$$y = \frac{\pi}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1 + \sin x} dx$$

$$y = \frac{\pi}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} dx$$

$$y = \frac{\pi}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1 - \sin x}{\cos^2 x} dx$$

$$y = \frac{\pi}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sec^2 x - \frac{\sin x}{\cos^2 x} dx$$

Let, $\cos x = t$

$$\Rightarrow -\sin x dx = dt$$

$$\text{At } x = \pi/4, t = \frac{1}{\sqrt{2}}$$

$$\text{At } x = 3\pi/4, t = -\frac{1}{\sqrt{2}}$$

$$y = \frac{\pi}{2} \left((\tan x)^{\frac{3\pi}{4}} + \int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \frac{1}{t^2} dt \right)$$

$$y = \frac{\pi}{2} \left(\tan \frac{3\pi}{4} - \tan \frac{\pi}{4} + \left(\frac{-1}{t} \right)_{\frac{1}{\sqrt{2}}}^{\frac{-1}{\sqrt{2}}} \right)$$

$$y = \frac{\pi}{2} (-1 - 1 + \sqrt{2} + \sqrt{2}) = \pi(\sqrt{2} - 1)$$

Question: 42

Solution:

$$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{\frac{3a}{4} + \frac{a}{4} - x}}{\sqrt{\frac{3a}{4} + \frac{a}{4} - x} + \sqrt{x}} dx$$

$$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx$$

Adding eq.(1) and eq.(2)

$$2y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx + \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx$$

$$2y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx$$

$$y = \frac{1}{2} \int_{\frac{a}{4}}^{\frac{3a}{4}} 1 dx$$

$$y = \frac{1}{2} (x)_{\frac{a}{4}}^{\frac{3a}{4}}$$

$$y = \frac{a}{4}$$

Question: 43

Solution:

$$y = \int_1^4 \frac{\sqrt{x}}{\sqrt{5-x+\sqrt{x}}} dx$$

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$y = \int_1^4 \frac{\sqrt{4+1-x}}{\sqrt{4+1-x} + \sqrt{x}} dx$$

$$y = \int_1^4 \frac{\sqrt{5-x}}{\sqrt{5-x} + \sqrt{x}} dx$$

Adding eq.(1) and eq.(2)

$$2y = \int_1^4 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{5-x}} dx + \int_1^4 \frac{\sqrt{5-x}}{\sqrt{5-x} + \sqrt{x}} dx$$

$$2y = \int_1^4 \frac{\sqrt{x} + \sqrt{5-x}}{\sqrt{5-x} + \sqrt{x}} dx$$

$$y = \frac{1}{2} \int_1^4 1 dx$$

$$y = \frac{1}{2} (x)_1^4$$

$$y = \frac{3}{2}$$

Question: 44**Solution:**

Use integration by parts

$$\int I \times II dx = I \int II dx - \int \frac{d}{dx} I \left(\int II dx \right) dx$$

$$y = x \int \cot x dx - \int \frac{d}{dx} x \left(\int \cot x dx \right) dx$$

$$y = (x \log \sin x)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log \sin x dx$$

$$\text{Let, } I = \int_0^{\frac{\pi}{2}} \log \sin x dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin \left(\frac{\pi}{2} - x \right) dx$$

$$I = \int_0^{\frac{\pi}{2}} \log \cos x dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx + \int_0^{\frac{\pi}{2}} \log \cos x \, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log \frac{2 \sin x \cos x}{2} \, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log \sin 2x - \log 2 \, dx$$

Let, $2x = t$

$$\Rightarrow 2 \, dx = dt$$

At $x = 0$, $t = 0$

At $x = \pi/2$, $t = \pi$

$$2I = \frac{1}{2} \int_0^{\pi} \log \sin t \, dt - \frac{\pi}{2} \log 2$$

$$2I = \frac{2}{2} \int_0^{\frac{\pi}{2}} \log \sin x \, dx - \frac{\pi}{2} \log 2$$

$$2I = I - \frac{\pi}{2} \log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx = -\frac{\pi}{2} \log 2$$

$$y = (x \log \sin x) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log \sin x \, dx$$

$$y = \frac{\pi}{2} \log \sin \frac{\pi}{2} - \left(-\frac{\pi}{2} \log 2 \right)$$

$$y = \frac{\pi}{2} \log 2$$

Question: 45

Solution:

Let, $x = \sin t$

$$\Rightarrow dx = \cos t \, dt$$

At $x = 0$, $t = 0$

At $x = 1$, $t = \pi/2$

$$y = \int_0^{\frac{\pi}{2}} \frac{\sin^{-1} \sin t}{\sin t} \cos t \, dt$$

$$y = \int_0^{\frac{\pi}{2}} \frac{t \cos t}{\sin t} \, dt$$

$$y = \int_0^{\frac{\pi}{2}} t \cot t \, dt$$

Use integration by parts

$$\int I \times II \, dt = I \int II \, dt - \int \frac{d}{dt} I \left(\int II \, dt \right) dt$$

$$y = t \int \cot t \, dt - \int \frac{d}{dt} t \left(\int \cot t \, dt \right) dt$$

$$y = (t \log \sin t) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log \sin t \, dt$$

$$\text{Let, } I = \int_0^{\frac{\pi}{2}} \log \sin t \, dt \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(t) \, dt = \int_a^b f(a+b-t) \, dt$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin \left(\frac{\pi}{2} - t \right) dt$$

$$I = \int_0^{\frac{\pi}{2}} \log \cos t \, dt \dots (2)$$

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log \sin t \, dt + \int_0^{\frac{\pi}{2}} \log \cos t \, dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log \frac{2 \sin t \cos t}{2} dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log \sin 2t - \log 2 \, dt$$

$$\text{Let, } 2t = z$$

$$\Rightarrow 2 \, dt = dz$$

$$\text{At } t = 0, z = 0$$

$$\text{At } t = \pi/2, z = \pi$$

$$2I = \frac{1}{2} \int_0^{\pi} \log \sin z \, dz - \frac{\pi}{2} \log 2$$

$$2I = \frac{2}{2} \int_0^{\frac{\pi}{2}} \log \sin z \, dz - \frac{\pi}{2} \log 2$$

$$2I = I - \frac{\pi}{2} \log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin z \, dz = -\frac{\pi}{2} \log 2$$

$$y = (t \log \sin t) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log t \, dt$$

$$y = \frac{\pi}{2} \log \sin \frac{\pi}{2} - \left(-\frac{\pi}{2} \log 2 \right)$$

$$y = \frac{\pi}{2} \log 2$$

Question: 46

Solution:

Use integration by parts

$$\int I \times II \, dx = I \int II \, dx - \int \frac{d}{dx} I \left(\int II \, dx \right) dx$$

$$y = \log x \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{d}{dx} \log x \left(\int \frac{1}{\sqrt{1-x^2}} dx \right) dx$$

$$y = (\log x \sin^{-1} x) \Big|_0^1 - \int_0^1 \frac{\sin^{-1} x}{x} dx$$

$$y = - \int_0^1 \frac{\sin^{-1} x}{x} dx$$

Let, $x = \sin t$

$\Rightarrow dx = \cos t \, dt$

At $x = 0$, $t = 0$

At $x = 1$, $t = \pi/2$

$$y = - \int_0^{\frac{\pi}{2}} \frac{\sin^{-1} \sin t}{\sin t} \cos t \, dt$$

$$y = - \int_0^{\frac{\pi}{2}} \frac{t \cos t}{\sin t} dt$$

$$y = - \int_0^{\frac{\pi}{2}} t \cot t \, dt$$

Use integration by parts

$$\int I \times II \, dt = I \int II \, dt - \int \frac{d}{dt} I \left(\int II \, dt \right) dt$$

$$y = - \left(t \int \cot t \, dt - \int \frac{d}{dt} t \left(\int \cot t \, dt \right) dt \right)$$

$$y = - \left((t \log \sin t) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log \sin t \, dt \right)$$

Let, $I = \int_0^{\frac{\pi}{2}} \log \sin t \, dt \dots (1)$

Use King theorem of definite integral

$$\int_a^b f(t) \, dt = \int_a^b f(a+b-t) \, dt$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin \left(\frac{\pi}{2} - t \right) \, dt$$

$$I = \int_0^{\frac{\pi}{2}} \log \cos t \, dt \dots (2)$$

Adding eq.(1) and eq.(2)

$$2I = \int_0^{\frac{\pi}{2}} \log \sin t \, dt + \int_0^{\frac{\pi}{2}} \log \cos t \, dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log \frac{2 \sin t \cos t}{2} \, dt$$

$$2I = \int_0^{\frac{\pi}{2}} \log \sin 2t - \log 2 \, dt$$

Let, $2t = z$

$$\Rightarrow 2 \, dt = dz$$

At $t = 0, z = 0$

At $t = \pi/2, z = \pi$

$$2I = \frac{1}{2} \int_0^{\pi} \log \sin z \, dz - \frac{\pi}{2} \log 2$$

$$2I = \frac{2}{2} \int_0^{\frac{\pi}{2}} \log \sin z \, dz - \frac{\pi}{2} \log 2$$

$$2I = I - \frac{\pi}{2} \log 2$$

$$I = \int_0^{\frac{\pi}{2}} \log \sin z \, dz = -\frac{\pi}{2} \log 2$$

$$y = - \left((t \log \sin t) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log t \, dt \right)$$

$$y = \frac{-\pi}{2} \log \sin \frac{\pi}{2} + \left(-\frac{\pi}{2} \log 2 \right)$$

$$y = \frac{-\pi}{2} \log 2$$

Question: 47

Solution:

$$\text{Let } x = \tan t$$

$$\Rightarrow dx = \sec^2 t \, dt$$

$$\text{At } x = 0, t = 0$$

$$\text{At } x = 1, t = \pi/4$$

$$y = \int_0^{\pi/4} \frac{\log(1 + \tan t)}{1 + \tan^2 t} \sec^2 t \, dt$$

$$y = \int_0^{\pi/4} \log(1 + \tan t) \, dt \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(t) \, dt = \int_a^b f(a + b - t) \, dt$$

$$y = \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - t\right)\right) \, dt$$

$$y = \int_0^{\pi/4} \log\left(1 + \frac{1 - \tan t}{1 + \tan t}\right) \, dt$$

$$y = \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan t}\right) \, dt \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_0^{\pi/4} \log(1 + \tan t) \, dt + \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan t}\right) \, dt$$

$$2y = \int_0^{\pi/4} \log(1 + \tan t) \left(\frac{2}{1 + \tan t}\right) \, dt$$

$$2y = \int_0^{\pi/4} \log 2 \, dt$$

$$y = \frac{\pi}{8} \log 2$$

Question: 48

Solution:

$$y = \int_{-a}^a x^3 \sqrt{a^2 - x^2} \, dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(t) dt = \int_a^b f(a+b-t) dt$$

$$y = \int_{-a}^a (a-a-x)^3 \sqrt{a^2 - (a-a-x)^2} dx$$

$$y = \int_{-a}^a -x^3 \sqrt{a^2 - x^2} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{-a}^a x^3 \sqrt{a^2 - x^2} dx + \left(- \int_{-a}^a x^3 \sqrt{a^2 - x^2} dx \right)$$

$$y = 0$$

Question: 49

Solution:

$$y = \int_{-\pi}^{\pi} \sin^{75} x + x^{125} dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(t) dt = \int_a^b f(a+b-t) dt$$

$$y = \int_{-\pi}^{\pi} \sin^{75}(\pi - \pi - x) + (\pi - \pi - x)^{125} dx$$

$$y = \int_{-\pi}^{\pi} -\sin^{75} x - x^{125} dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{-\pi}^{\pi} \sin^{75} x + x^{125} dx + \left(- \int_{-\pi}^{\pi} \sin^{75} x + x^{125} dx \right)$$

$$y = 0$$

Question: 50

Solution:

$$y = \int_{-\pi}^{\pi} x^{12} \sin^9 x dx \dots (1)$$

Use King theorem of definite integral

$$\int_a^b f(t) dt = \int_a^b f(a+b-t) dt$$

$$y = \int_{-\pi}^{\pi} (\pi - \pi - x)^{12} \sin^9(\pi - \pi - x) dx$$

$$y = \int_{-\pi}^{\pi} -x^{12} \sin^9 x dx \dots (2)$$

Adding eq.(1) and eq.(2)

$$2y = \int_{-\pi}^{\pi} x^{12} \sin^9 x \, dx + \left(- \int_{-\pi}^{\pi} x^{12} \sin^9 x \, dx \right)$$

$$y = 0$$

Question: 51

Solution:

We know that

$$|x| = -x \text{ in } [-1, 0)$$

$$|x| = x \text{ in } [0, 1]$$

$$y = \int_{-1}^0 e^{|x|} \, dx + \int_0^1 e^{|x|} \, dx$$

$$y = \int_{-1}^0 e^{-x} \, dx + \int_0^1 e^x \, dx$$

$$y = (-e^{-x})_{-1}^0 + (e^x)_0^1$$

$$y = -(1-e) + (e-1)$$

$$y = 2(e-1)$$

Question: 52

$$|x+1| = -(x+1) \text{ in } [-2, -1]$$

$$|x+1| = (x+1) \text{ in } [-1, 2]$$

$$y = \int_{-2}^{-1} |x+1| \, dx + \int_{-1}^2 |x+1| \, dx$$

$$= - \int_{-2}^{-1} (x+1) \, dx + \int_{-1}^2 (x+1) \, dx$$

$$= - \left(\frac{x^2}{2} + x \right)_{-2}^{-1} + \left(\frac{x^2}{2} + x \right)_{-1}^2$$

$$= - \left(\frac{1}{2} - 1 - 2 + 2 \right) + \left(2 + 2 - \frac{1}{2} + 1 \right)$$

$$= 5$$

Question: 53

Solution:

We know that

$$|x-5| = -(x-5) \text{ in } [0, 5]$$

$$|x-5| = (x-5) \text{ in } [5, 8]$$

$$y = \int_0^5 |x-5| \, dx + \int_5^8 |x-5| \, dx$$

$$y = -\int_0^5 (x-5) dx + \int_5^8 (x-5) dx$$

$$y = -\left(\frac{x^2}{2} - 5x\right)_0^5 + \left(\frac{x^2}{2} - 5x\right)_5^8$$

$$y = -\left(\frac{25}{2} - 25\right) + \left(32 - 40 - \frac{25}{2} + 25\right)$$

$$= 17$$

Question: 54

Solution:

We know that

$$|\cos x| = \cos x \text{ in } [0, \pi/2]$$

$$|\cos x| = -\cos x \text{ in } [\pi/2, 3\pi/2]$$

$$|\cos x| = \cos x \text{ in } [3\pi/2, 2\pi]$$

$$y = \int_0^{\frac{\pi}{2}} |\cos x| dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} |\cos x| dx + \int_{\frac{3\pi}{2}}^{2\pi} |\cos x| dx$$

$$y = \int_0^{\frac{\pi}{2}} \cos x dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x dx$$

$$y = (\sin x)_0^{\frac{\pi}{2}} - (\sin x)_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + (\sin x)_{\frac{3\pi}{2}}^{2\pi}$$

$$y = (1-0) - 1 - 1 + (0+1)$$

$$= 4$$

Question: 55

Solution:

We know that

$$|\sin x| = -\sin x \text{ in } [-\pi/4, 0]$$

$$|\sin x| = \sin x \text{ in } [0, \pi/4]$$

$$y = \int_{-\frac{\pi}{4}}^0 |\sin x| dx + \int_0^{\frac{\pi}{4}} |\sin x| dx$$

$$y = -\int_{-\frac{\pi}{4}}^0 \sin x dx + \int_0^{\frac{\pi}{4}} \sin x dx$$

$$y = -(-\cos x)_{-\frac{\pi}{4}}^0 + (-\cos x)_0^{\frac{\pi}{4}}$$

$$y = \left(1 - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} - 1\right)$$

$$= 2 - \frac{1}{\sqrt{2}}$$

Question: 56

Solution:

$$y = \int_1^3 f(x) dx$$

$$y = \int_1^2 f(x) dx + \int_2^3 f(x) dx$$

$$y = \int_1^2 2x + 1 dx + \int_2^3 x^2 + 1 dx$$

$$y = (x^2 + x)_1^2 + \left(\frac{x^3}{3} + x\right)_2^3$$

$$y = (4 + 2 - 1 - 1) + \left(9 + 3 - \frac{8}{3} - 2\right)$$

$$= \frac{34}{3}$$

Question: 57

Solution:

$$y = \int_0^4 f(x) dx$$

$$y = \int_0^2 f(x) dx + \int_2^4 f(x) dx$$

$$y = \int_0^2 3x^2 + 4 dx + \int_2^4 9x - 2 dx$$

$$y = (x^3 + 4x)_0^2 + \left(\frac{9x^2}{2} - 2x\right)_2^4$$

$$y = (8 + 8) + (72 - 8 - 18 + 4)$$

$$= 66$$

Question: 58

Solution:

$$y = \int_0^4 |x| + |x - 2| + |x - 4| dx$$

$$y = \int_0^2 |x| + |x - 2| + |x - 4| dx + \int_2^4 |x| + |x - 2| + |x - 4| dx$$

$$y = \int_0^2 x - (x - 2) - (x - 4) dx + \int_2^4 x + (x - 2) - (x - 4) dx$$

$$y = \left(-\frac{x^2}{2} + 6x \right)_0^2 + \left(\frac{x^2}{2} + 2x \right)_2^4$$

$$y = (-2 + 12) + (8 + 8 - 2 - 4)$$

$$= 20$$

Exercise : 16D

Question: 1

Solution:

$f(x)$ is continuous in $[0, 2]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h = 2/n$

$$\int_0^2 (x + 4) dx = \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} f(2r/n)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(\frac{2r}{n} + 4 \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \left(\frac{(n-1)(n)}{n} + 4(n-1) \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2n^2 - n + 4n^2 - 4n}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{25n^2 - 5n}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{10n^2 - 10n}{n^2}$$

$$= \lim_{n \rightarrow \infty} 10 - (10/n)$$

$$= 10$$

Question: 2

Solution:

$f(x)$ is continuous in $[1, 2]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h = 1/n$

$$\int_1^2 (3x - 2) dx = \lim_{n \rightarrow \infty} \left(\frac{1}{n} \right) \sum_{r=0}^{n-1} f \left(1 + \left(\frac{r}{n} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{1}{n} \right) \sum_{r=0}^{n-1} \left(3 + 3 \frac{r}{n} - 2 \right)$$

$$\lim_{n \rightarrow \infty} \left(\frac{1}{n} \right) \left(n + \frac{3(n-1)(n)}{2n} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{1}{n} \right) \left(\frac{2n^2 + 3n^2 - 3n}{2n} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{5n^2 - 3n}{2n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{5}{2} \right) - \left(\frac{3}{2n} \right)$$

$$= 5/2$$

Question: 3

Solution:

$f(x)$ is continuous in $[1, 3]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h = 2/n$

$$\int_1^3 (x^2) dx = \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} f \left(1 + \left(\frac{2r}{n} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(1 + \left(\frac{2r}{n} \right) \right)^2$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(\frac{4r^2}{n^2} + 1 + \frac{4r}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{4(n-1)(n)(2n-1)}{6n^2} + n + \frac{4(n-1)(n)}{2n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{4(2n^3 - 2n^2 - n^2 + n)}{6n^2} + n + \frac{2(n^2 - n)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{(8n^3 - 12n^2 + 4n) + (6n^3) + (12n^3 - 12n^2)}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{26n^3 - 24n^2 + 4n}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{52n^3 - 48n^2 + 8n}{6n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{52}{6} \right) - \left(\frac{26}{6n} \right) + \left(\frac{8}{6n^2} \right)$$

$$= 26/3$$

Question: 4

Solution:

$f(x)$ is continuous in $[0,3]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b-a)/n$$

here $h=3/n$

$$\int_0^3 (x^2 + 1) dx = \lim_{n \rightarrow \infty} \left(\frac{3}{n} \right) \sum_{r=0}^{n-1} f\left(\frac{3r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n} \right) \sum_{r=0}^{n-1} \left(\left(\frac{3r}{n} \right)^2 + 1 \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n} \right) \sum_{r=0}^{n-1} \left(\frac{9r^2}{n^2} + 1 \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{9(n-1)(n)(2n-1)}{6n^2} + n \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{9(n^2 - n)(2n-1)}{6n^2} + n \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{9(2n^3 - 2n^2 - n^2 + n)}{6n^2} + n \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{(18n^3 - 27n^2 + 9n) + (6n^3)}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{24n^3 - 27n^2 + 9n}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{72n^3 - 81n^2 + 27n}{6n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{72}{6} \right) - \left(\frac{81}{6n} \right) + \left(\frac{27}{6n^2} \right)$$

$$= 12$$

Question: 5

Solution:

$f(x)$ is continuous in $[2, 5]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h = 3/n$

$$\int_2^5 (3x^2 - 5) dx = \lim_{n \rightarrow \infty} \left(\frac{3}{n} \right) \sum_{r=0}^{n-1} f\left(2 + \frac{3r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n} \right) \sum_{r=0}^{n-1} \left(3 \left(2 + \frac{3r}{n} \right)^2 - 5 \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n} \right) \sum_{r=0}^{n-1} 3 \left(\frac{9r^2}{n^2} + 4 + \frac{12r}{n} \right) - 5$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{27(n-1)(n)(2n-1)}{6n^2} + 12n + \frac{18n(n-1)}{n} - 5n \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{27(n^2 - n)(2n - 1)}{6n^2} + 12n + \frac{18n(n-1)}{n} - 5n \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{27(2n^3 - 2n^2 - n^2 + n)}{6n^2} + 12n + \frac{18n(n-1)}{n} - 5n \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{(54n^3 - 81n^2 + 27n) + (42n^3) + (108n^3 - 108n^2)}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{204n^3 - 189n^2 + 27n}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{612n^3 - 567n^2 + 27n}{6n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{612}{6} \right) - \left(\frac{567}{6n} \right) + \left(\frac{27}{6n^2} \right)$$

$$= 102$$

Question: 6

Evaluate each of

Solution:

$f(x)$ is continuous in $[2,5]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h=3/n$

$$\int_0^3 (x^2 + 2x) dx = \lim_{n \rightarrow \infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} f\left(\frac{3r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} \left(\left(\frac{3r}{n}\right)^2 + \frac{6r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} \left(\frac{9r^2}{n^2} + \frac{6r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{9(n-1)(n)(2n-1)}{6n^2} + \frac{3n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{9(n^2 - n)(2n - 1)}{6n^2} + \frac{3n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{9(2n^3 - 2n^2 - n^2 + n)}{6n^2} + \frac{3n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{(18n^3 - 27n^2 + 9n) + (18n^3 - 18n^2)}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{36n^3 - 45n^2 + 9n}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{108n^3 - 135n^2 + 27n}{6n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{108}{6} \right) - \left(\frac{135}{6n} \right) + \left(\frac{27}{6n^2} \right)$$

$$= 18$$

Question: 7**Solution:**

$f(x)$ is continuous in $[1,4]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h=3/n$

$$\int_1^4 (3x^2 + 2x) dx = \lim_{n \rightarrow \infty} \left(\frac{3}{n} \right) \sum_{r=0}^{n-1} f \left(\left(1 + \frac{3r}{n} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n} \right) \sum_{r=0}^{n-1} \left(3 \left(1 + \frac{3r}{n} \right)^2 + 2 \left(1 + \frac{3r}{n} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n} \right) \sum_{r=0}^{n-1} 3 \left(\frac{9r^2}{n^2} + 1 + \frac{6r}{n} \right) + 2 \left(1 + \frac{3r}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{27(n-1)(n)(2n-1)}{6n^2} + 3n + \frac{9n(n-1)}{n} + 2n + \frac{3n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{27(n^2 - n)(2n - 1)}{6n^2} + 5n + \frac{12n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{27(2n^3 - 2n^2 - n^2 + n)}{6n^2} + 5n + \frac{12n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{(54n^3 - 81n^2 + 27n) + (30n^3) + (72n^3 - 72n^2)}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{156n^3 - 153n^2 + 27n}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{468n^3 - 459n^2 + 81n}{6n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{468}{6} \right) - \left(\frac{459}{6n} \right) + \left(\frac{81}{6n^2} \right)$$

$$= 78$$

Question: 8

Solution:

$f(x)$ is continuous in $[1, 3]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h = 3/n$

$$\int_1^3 (x^2 + 5x) dx = \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} f \left(\left(1 + \frac{2r}{n} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(\left(1 + \frac{2r}{n} \right)^2 + 5 \left(1 + \frac{2r}{n} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(1 + \frac{4r^2}{n^2} + \frac{4r}{n} + 5 + \frac{10r}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(1 + \frac{4r^2}{n^2} + \frac{4r}{n} + 5 + \frac{10r}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{4(n-1)(n)(2n-1)}{6n^2} + 6n + \frac{7n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{4(n^2 - n)(2n - 1)}{6n^2} + 6n + \frac{7n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{4(2n^3 - 2n^2 - n^2 + n)}{6n^2} + 6n + \frac{7n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{(8n^3 - 12n^2 + 4n) + (42n^3 - 42n^2) + (36n^3)}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{86n^3 - 54n^2 + 4n}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{172n^3 - 108n^2 + 8n}{6n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{172}{6} - \frac{108}{6n} + \frac{8}{6n^2} \right)$$

$$= 86/3$$

Question: 9

Solution:

$f(x)$ is continuous in $[1, 3]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h = 2/n$

$$\int_1^3 (2x^2 + 5x) dx = \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} f \left(1 + \frac{2r}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(2 \left(1 + \frac{2r}{n} \right)^2 + 5 \left(1 + \frac{2r}{n} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(2 + \frac{8r^2}{n^2} + \frac{8r}{n} + 5 + \frac{10r}{n} \right)$$

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \sum_{r=0}^{n-1} \left(7 + \frac{8r^2}{n^2} + \frac{18r}{n} \right) \right) \\
 &= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{8(n-1)(n)(2n-1)}{6n^2} + 7n + \frac{9n(n-1)}{n} \right) \\
 &= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{8(n^2-n)(2n-1)}{6n^2} + 7n + \frac{9n(n-1)}{n} \right) \\
 &= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{8(2n^3 - 2n^2 - n^2 + n)}{6n^2} + 7n + \frac{9n(n-1)}{n} \right) \\
 &= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{(16n^3 - 24n^2 + 8n) + (54n^3 - 54n^2) + (42n^3)}{6n^2} \right) \\
 &= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{112n^3 - 78n^2 + 8n}{6n^2} \right) \\
 &= \lim_{n \rightarrow \infty} \left(\frac{224n^3 - 156n^2 + 8n}{6n^3} \right) \\
 &= \lim_{n \rightarrow \infty} \left(\frac{224}{6} - \frac{156}{6n} + \frac{8}{6n^2} \right) \\
 &= 112/3
 \end{aligned}$$

Question: 10

Solution:

$f(x)$ is continuous in $[0,2]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b-a)/n$$

here $h=2/n$

$$\int_0^2 (x^3) dx = \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(\frac{2r}{n} \right)^3$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(\frac{8r^3}{n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{8(n-1)^2(n)^2}{4n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{8(n^2 - 2n + 1)(n^2)}{4n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{8(n^4 - 2n^3 + n^2)}{4n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{16n^4 - 32n^3 + 16n^2}{4n^4} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{16}{4} \right) - \left(\frac{32}{4n} \right) + \left(\frac{16}{4n^2} \right)$$

$$= 4$$

Question: 11

Solution:

$f(x)$ is continuous in $[2, 4]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h = 3/n$

$$\int_2^4 (x^2 - 3x + 2) dx = \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} f\left(2 + \frac{2r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(\left(2 + \frac{2r}{n}\right)^2 - 3\left(2 + \frac{2r}{n}\right) + 2 \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(\frac{4r^2}{n^2} + \frac{8r}{n} + 4 - 6 - \frac{6r}{n} + 2 \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{4(n-1)(n)(2n-1)}{6n^2} + \frac{n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{4(n^2 - n)(2n - 1)}{6n^2} + \frac{n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{4(2n^3 - 2n^2 - n^2 + n)}{6n^2} + \frac{n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{(8n^3 - 12n^2 + 4n) + (6n^3 - 6n^2)}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{14n^3 - 18n^2 + 4n}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{28n^3 - 36n^2 + 8n}{6n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{28}{6} \right) - \left(\frac{36}{6n} \right) + \left(\frac{8}{6n^2} \right)$$

$$= 14/3$$

Question: 12

Solution:

$f(x)$ is continuous in $[0,2]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h = 2/n$

$$\int_0^2 (x^2 + x) dx = \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(\left(\frac{2r}{n} \right)^2 + \left(\frac{2r}{n} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n} \right) \sum_{r=0}^{n-1} \left(\frac{4r^2}{n^2} + \frac{2r}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{4(n-1)(n)(2n-1)}{6n^2} + \frac{n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{4(n^2 - n)(2n-1)}{6n^2} + \frac{n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{4(2n^3 - 2n^2 - n^2 + n)}{6n^2} + \frac{n(n-1)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{(8n^3 - 12n^2 + 4n) + (6n^3 - 6n^2)}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{14n^3 - 18n^2 + 4n}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{28n^3 - 36n^2 + 8n}{6n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{28}{6} \right) - \left(\frac{36}{6n} \right) + \left(\frac{8}{6n^2} \right)$$

$$= 14/3$$

Question: 13

Solution:

$f(x)$ is continuous in $[0,3]$

$$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a+rh), \text{ where } h = (b-a)/n$$

here $h=3/n$

$$\int_0^3 (2x^2 + 3x + 5)dx = \lim_{n \rightarrow \infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} f\left(\frac{3r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} \left(2\left(\frac{3r}{n}\right)^2 + 3\left(\frac{3r}{n}\right) + 5\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} \left(\frac{18r^2}{n^2} + \frac{9r}{n} + 5\right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{18(n-1)(n)(2n-1)}{6n^2} + \frac{9n(n-1)}{2n} + 5n \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{18(n^2-n)(2n-1)}{6n^2} + \frac{9n(n-1)}{2n} + 5n \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{18(2n^3 - 2n^2 - n^2 + n)}{6n^2} + \frac{9n(n-1)}{2n} + 5n \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{(36n^3 - 54n^2 + 18n) + (27n^3 - 27n^2) + 30n^3}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(\frac{93n^3 - 81n^2 + 18n}{6n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{279n^3 - 243n^2 + 54n}{6n^3} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{279}{6} \right) - \left(\frac{243}{6n} \right) + \left(\frac{54}{6n^2} \right)$$

$$= 93/2$$

Question: 14

Solution:

Since it is modulus function so we need to break the function and then solve it

$$f(x) = \int_0^{\frac{1}{3}} (1-3x)dx + \int_{\frac{1}{3}}^1 (3x-1)dx$$

it is continuous in $[0,1]$

$$\text{let } g(x) = \int_0^{\frac{1}{3}} (1-3x)dx \text{ and } h(x) = \int_{\frac{1}{3}}^1 (3x-1)dx$$

$$g(x) = \int_0^{\frac{1}{3}} (1 - 3x) dx$$

here $h=1/3n$

$$\int_0^{\frac{1}{3}} (1 - 3x) dx = \lim_{n \rightarrow \infty} \left(\frac{1}{3n} \right) \sum_{r=0}^{n-1} f(r/3n)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{1}{3n} \right) \sum_{r=0}^{n-1} \left(1 - 3 \left(\frac{r}{3n} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{1}{3n} \right) \left(n - \frac{3(n-1)(n)}{6n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{3n} \frac{6n^2 - 3n^2 + 3n}{3n}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{3n} \frac{3n^2 + 3n}{3n}$$

$$= \lim_{n \rightarrow \infty} \frac{3n^2 + 3n}{9n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{3} + \left(\frac{3}{9n} \right)$$

$$= 1/3$$

$$h(x) = \int_{\frac{1}{3}}^1 (3x - 1) dx$$

here $h=2/3n$

$$\int_{\frac{1}{3}}^1 (3x - 1) dx = \lim_{n \rightarrow \infty} \left(\frac{2}{3n} \right) \sum_{r=0}^{n-1} f\left(\left(\frac{1}{3} \right) + \left(\frac{2r}{3n} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{3n} \right) \sum_{r=0}^{n-1} \left(3 \left(\frac{1}{3} + \frac{2r}{3n} \right) - 1 \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{3n} \right) \left(\frac{(n-1)(n)}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{2}{3n} \cdot \frac{n^2 - n}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{2}{3n} \cdot \frac{n^2 - n}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{2n^2 - 2n}{3n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{2}{3} - \left(\frac{2}{3n}\right)$$

$$= 2/3$$

$$f(x) = g(x) + h(x)$$

$$= (1/3) + (2/3)$$

$$= 3/3$$

$$= 1$$

Question: 15

Solution:

$f(x)$ is continuous in $[0, 2]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h = 2/n$

$$\int_0^2 (e^x) dx = \lim_{n \rightarrow \infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} e^{\frac{2r}{n}}$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n}\right) (e^0 + e^h + e^{2h} + \dots + e^{nh})$$

sum of $e^0 + e^h + e^{2h} + \dots + e^{nh}$

Which is g.p with common ratio $e^{1/n}$

$$\text{Whose sum is } = \frac{e^h(1 - e^{nh})}{1 - e^h}$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n}\right) \left(\frac{e^h(1 - e^{nh})}{1 - e^h}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n}\right) \left(\frac{e^h(1 - e^{nh})}{\frac{1 - e^h}{h}}\right)$$

$$\lim_{h \rightarrow 0} \frac{1 - e^h}{h} = -1$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n}\right) \cdot \frac{e^h(1 - e^{nh})}{-h}$$

As $h = 2/n$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n}\right) \cdot \frac{e^{(2/n)}(1 - e^{n \cdot (2/n)})}{-2/n}$$

$$= e^2 \cdot 1$$

Question: 16**Solution:** $f(x)$ is continuous in $[1,3]$

$$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h=2/n$

$$\int_1^3 (e^{-x})dx = \lim_{n \rightarrow \infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} f\left(1 + \left(\frac{2r}{n}\right)\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} e^{-(1 + \frac{2r}{n})}$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} e^{-1} \cdot e^{-\frac{2r}{n}}$$

Common ratio is $h = -2/n$

$$\text{sum} = e^{-1}(e^0 + e^h + e^{2h} + \dots + e^{nh})$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2e^{-1}}{n}\right)(e^0 + e^h + e^{2h} + \dots + e^{nh})$$

$$\text{sum of} = e^0 + e^h + e^{2h} + \dots + e^{nh}$$

Which is g.p. with common ratio $e^{1/n}$

$$\text{Whose sum is} = \frac{e^h(1 - e^{nh})}{1 - e^h}$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2e^{-1}}{n}\right) \left(\frac{e^h(1 - e^{nh})}{1 - e^h}\right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2e^{-1}}{n}\right) \left(\frac{e^h(1 - e^{nh})}{1 - e^h}\right)$$

$$\lim_{h \rightarrow 0} \frac{1 - e^h}{h} = -1$$

$$= \lim_{n \rightarrow \infty} \left(\frac{2e^{-1}}{n}\right) \left(\frac{e^h(1 - e^{nh})}{-h}\right)$$

As $h=-2/n$

$$= \lim_{n \rightarrow \infty} \left(\frac{2e^{-1}}{n}\right) \left(\frac{e^{(-\frac{2}{n})}(1 - e^{n \cdot (-2/n)})}{2/n}\right)$$

$$= \frac{(1 - e^{-2})}{e}$$

$$= \frac{(e^2 - 1)}{e^3}$$

Question: 17

Evaluate each of

$f(x)$ is continuous in $[a, b]$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \text{ where } h = (b - a)/n$$

here $h = (b - a)/n$

$$\int_a^b (\cos x) dx = \lim_{n \rightarrow \infty} \left(\frac{b - a}{n} \right) \sum_{r=0}^{n-1} f(a + rh)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{b - a}{n} \right) \sum_{r=0}^{n-1} \cos(a + rh)$$

$$S = \cos(a) + \cos(a+h) + \cos(a+2h) + \cos(a+3h) + \dots + \cos(a+(n-1)h) = \frac{\sin\left(\frac{nh}{2}\right) \cos\left(a + \frac{(n-1)h}{2}\right)}{\sin\left(\frac{h}{2}\right)}$$

Putting $h = (b - a)/n$

$$= \lim_{n \rightarrow \infty} \left(\frac{b - a}{n} \right) \frac{\sin\left(\frac{n(b-a)}{2n}\right) \cos\left(a + \frac{(n-1)(b-a)}{2n}\right)}{\frac{\sin\left(\frac{b-a}{2n}\right)}{\frac{b-a}{2n}}}$$

As we know

$$\lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \right) = 1$$

$$= \lim_{n \rightarrow \infty} 2 \sin\left(\frac{(b-a)}{2}\right) \cos\left(a + \left(\frac{1}{2} - \frac{1}{2n}\right)(b-a)\right)$$

$$= 2 \sin\left(\frac{b-a}{2}\right) \cos\left(\frac{b+a}{2}\right)$$

Which is trigonometry formula of $\sin(b) - \sin(a)$

Final answer is $\sin(b) - \sin(a)$

Exercise : OBJECTIVE QUESTIONS

Question: 1

Solution:

$$y = \int_1^4 x \sqrt{x} dx$$

$$= \int_1^4 x^{\frac{3}{2}} dx$$

$$= \left(\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} \right)_1^4$$

$$= \frac{2}{5} \left(4^{\frac{5}{2}} - 1^{\frac{5}{2}} \right)$$

$$= \frac{2}{5}(32 - 1)$$

$$= \frac{62}{5}$$

$$= 12.4$$

Question: 2

Solution:

$$y = \int_0^2 \sqrt{6x+4} \, dx$$

$$= \left(\frac{(6x+4)^{\frac{1}{2}+1}}{6\left(\frac{1}{2}+1\right)} \right)_0^2$$

$$= \frac{2}{6 \times 3} \left(16^{\frac{3}{2}} - 4^{\frac{3}{2}} \right)$$

$$= \frac{2}{6 \times 3} (64 - 8)$$

$$= \frac{56}{9}$$

Question: 3

Solution:

$$y = \int_0^1 \frac{dx}{\sqrt{5x+3}}$$

$$= \left(\frac{(5x+3)^{-\frac{1}{2}+1}}{5\left(-\frac{1}{2}+1\right)} \right)_0^1$$

$$= \frac{2}{5} \left(8^{\frac{1}{2}} - 3^{\frac{1}{2}} \right)$$

$$= \frac{2}{5} (\sqrt{8} - \sqrt{3})$$

Question: 4

Solution:

$$y = \int_0^1 \frac{1}{1+x^2} \, dx$$

$$= (\tan^{-1} x)_0^1$$

$$= \tan^{-1} 1 - \tan^{-1} 0$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4}$$

Question: 5

Solution:

$$y = \int_0^2 \frac{dx}{\sqrt{4-x^2}}$$

Use formula $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a}$

$$y = \left(\sin^{-1} \frac{x}{2} \right)_0^2$$

$$= \sin^{-1} 1 - \sin^{-1} 0$$

$$= \frac{\pi}{2}$$

Question: 6

Solution:

$$y = \int_{\sqrt{3}}^{\sqrt{8}} x \sqrt{1+x^2} dx$$

Let, $x^2 = t$

Differentiating both side with respect to t

$$2x \frac{dx}{dt} = 1$$

$$\Rightarrow x dx = \frac{1}{2} dt$$

$$\text{At } x = \sqrt{3}, t = 3$$

$$\text{At } x = \sqrt{8}, t = 8$$

$$y = \frac{1}{2} \int_3^8 \sqrt{1+t} dt$$

$$= \frac{1}{2} \left(\frac{(1+t)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)} \right)_3^8$$

$$= \frac{1}{3} \left(9^{\frac{3}{2}} - 4^{\frac{3}{2}} \right)$$

$$= \frac{1}{3} (27 - 8)$$

$$= \frac{19}{3}$$

Question: 7

Solution:

Let, $x^4 = t$

Differentiating both side with respect to t

$$4x^3 \frac{dx}{dt} = 1$$

$$\Rightarrow x^3 dx = \frac{1}{4} dt$$

$$\text{At } x = 0, t = 0$$

$$\text{At } x = 1, t = 1$$

$$y = \frac{1}{4} \int_0^1 \frac{1}{1+t^2} dt$$

$$= \frac{1}{4} (\tan^{-1} t)_0^1$$

$$= \frac{1}{4} (\tan^{-1} 1 - \tan^{-1} 0)$$

$$= \frac{\pi}{16}$$

Question: 8

Solution:

Let, $\log x = t$

Differentiating both side with respect to t

$$\frac{1}{x} \frac{dx}{dt} = 1$$

$$\Rightarrow \frac{1}{x} dx = dt$$

At $x = 1$, $t = 0$

At $x = e$, $t = 1$

$$y = \int_0^1 t^2 dt$$

$$= \left(\frac{t^3}{3} \right)_0^1$$

$$= \frac{1}{3}$$

Question: 9

Solution:

$$y = (\ln(\sin x))_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \ln\left(\sin \frac{\pi}{2}\right) - \ln\left(\sin \frac{\pi}{4}\right)$$

$$= \ln 1 - \ln \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2} \ln 2$$

Question: 10

Solution:

$$y = \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) dx$$

$$= (\tan x - x)_0^{\frac{\pi}{4}}$$

$$= \left(\tan \frac{\pi}{4} - \frac{\pi}{4} \right) - (\tan 0 - 0)$$

$$= 1 - \frac{\pi}{4}$$

Question: 11

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2x}{2} dx$$

$$= \left(\frac{x}{2} + \frac{\sin 2x}{4} \right)_0^{\frac{\pi}{2}}$$

$$= \left(\frac{\frac{\pi}{2}}{2} + \frac{\sin \pi}{4} \right) - \left(\frac{0}{2} + \frac{\sin 0}{4} \right)$$

$$= \frac{\pi}{4}$$

Question: 12

Solution:

$$y = (\ln(\operatorname{cosec} x - \cot x))_{\frac{\pi}{2}}^{\frac{\pi}{3}}$$

$$= \ln \left(\operatorname{cosec} \frac{\pi}{3} - \cot \frac{\pi}{3} \right) - \ln \left(\operatorname{cosec} \frac{\pi}{2} - \cot \frac{\pi}{2} \right)$$

$$= \ln(1 - 0) - \ln \left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} \right)$$

$$= \frac{1}{2} \log 3$$

Question: 13

Solution:

$$y = \int_0^{\frac{\pi}{2}} \cos x (1 - \sin^2 x) dx$$

Let, $\sin x = t$

Differentiating both side with respect to t

$$\cos x \frac{dx}{dt} = 1$$

$$\Rightarrow \cos x dx = dt$$

$$\text{At } x = 0, t = 0$$

$$\text{At } x = \frac{\pi}{2}, t = 1$$

$$y = \int_0^1 1 - t^2 dt$$

$$= \left(t - \frac{t^3}{3} \right)_0^1$$

$$= 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

Question: 14

Solution:

$$y = \int_0^{\frac{\pi}{4}} e^{\tan x} \sec^2 x \, dx$$

Let, $\tan x = t$

Differentiating both side with respect to t

$$\sec^2 x \frac{dx}{dt} = 1$$

$$\Rightarrow \sec^2 x \, dx = dt$$

At $x = 0$, $t = 0$

At $x = \frac{\pi}{4}$, $t = 1$

$$y = \int_0^1 e^t \, dt$$

$$= e^t \Big|_0^1$$

$$= e^1 - e^0$$

$$= e - 1$$

Question: 15

Solution:

Let, $\sin x = t$

Differentiating both side with respect to t

$$\cos x \frac{dx}{dt} = 1$$

$$\Rightarrow \cos x \, dx = dt$$

At $x = 0$, $t = 0$

At $x = \frac{\pi}{2}$, $t = 1$

$$y = \int_0^1 \frac{1}{1+t^2} \, dt$$

$$= (\tan^{-1} t) \Big|_0^1$$

$$= \tan^{-1} 1 - \tan^{-1} 0$$

$$= \pi/4$$

Question: 16

Solution:

Let, $1/x = t$

Differentiating both side with respect to t

$$\frac{-1}{x^2} \frac{dx}{dt} = 1$$

$$\Rightarrow \frac{1}{x^2} dx = -dt$$

At $x = 1/\pi$, $t = \pi$

At $x = 2/\pi$, $t = \pi/2$

$$y = \int_{\pi}^{\frac{\pi}{2}} \sin t \, dt$$

$$= (-\cos t)_{\pi}^{\frac{\pi}{2}}$$

$$= 1$$

Question: 17

Solution:

$$\begin{aligned} y &= \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx \\ &= \int_0^{\pi} \frac{1-\sin x}{\cos^2 x} dx \\ &= \int_0^{\pi} \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} dx \\ &= \int_0^{\pi} \sec^2 x \, dx - \int_0^{\pi} \frac{\sin x}{\cos^2 x} dx \end{aligned}$$

Let, $\cos x = t$

Differentiating both side with respect to t

$$-\sin x \frac{dx}{dt} = 1$$

$$\Rightarrow \sin x \, dx = -dt$$

At $x = 0$, $t = 1$

At $x = \pi$, $t = -1$

$$y = (\tan x)_0^{\pi} + \int_1^{-1} \frac{1}{t^2} dt$$

$$= (\tan \pi - \tan 0) + \left(\frac{t^{-1}}{-1} \right)_1^{-1}$$

$$= 2$$

Question: 18

Solution:

$$y = \int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \, dx$$

$$y = \int_0^{\frac{\pi}{2}} \sin^2 x \cos x (1 - \sin^2 x) \, dx$$

Let, $\sin x = t$

Differentiating both side with respect to t

$$\cos x \frac{dx}{dt} = 1$$

$$\Rightarrow \cos x \, dx = dt$$

$$\text{At } x = 0, t = 0$$

$$\text{At } x = \pi/2, t = 1$$

$$y = \int_0^1 t^3 - t^7 \, dt$$

$$= \left(\frac{t^4}{4} - \frac{t^8}{8} \right)_0^1$$

$$= \frac{1}{4} - \frac{1}{8}$$

$$= \frac{1}{8}$$

Question: 19

Solution:

$$y = \int_0^1 \frac{e^x (x+1-1)}{(1+x)^2} \, dx$$

$$= \int_0^1 e^x \left(\frac{1}{1+x} - \frac{1}{(1+x)^2} \right) \, dx$$

Use formula $\int e^x (f(x) + f'(x)) \, dx = e^x f(x)$

$$\text{If } f(x) = \frac{1}{1+x}$$

$$\text{then } f'(x) = -\frac{1}{(1+x)^2}$$

$$y = \left(\frac{e^x}{1+x} \right)_0^1$$

$$y = \frac{e}{2} - 1$$

Question: 20

Solution:

$$y = \int_0^{\frac{\pi}{2}} e^x \left(\frac{1+\sin x}{2\cos^2 \frac{x}{2}} \right) \, dx$$

$$\begin{aligned}
 &= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2\cos^2 \frac{x}{2}} + \frac{\sin x}{2\cos^2 \frac{x}{2}} \right) dx \\
 &= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2\cos^2 \frac{x}{2}} + \frac{2\sin \frac{x}{2} \cos \frac{x}{2}}{2\cos^2 \frac{x}{2}} \right) dx \\
 &= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right) dx
 \end{aligned}$$

Use formula $\int e^x (f(x) + f'(x)) dx = e^x f(x)$

If $f(x) = \tan \frac{x}{2}$ then $f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$

$$\begin{aligned}
 &= e^{\frac{\pi}{2}} \tan \frac{\frac{\pi}{2}}{2} - e^0 \tan \frac{0}{2} \\
 &= e^{\frac{\pi}{2}}
 \end{aligned}$$

Question: 21

Solution:

$$\begin{aligned}
 y &= \int_0^{\frac{\pi}{4}} \sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x} dx \\
 &= \int_0^{\frac{\pi}{4}} \sin x + \cos x dx \\
 &= (-\cos x + \sin x) \Big|_0^{\frac{\pi}{4}} \\
 &= \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (-1 + 0) \\
 y &= 1
 \end{aligned}$$

Question: 22

Solution:

$$\begin{aligned}
 y &= \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2 x} dx \\
 &= \int_0^{\frac{\pi}{2}} \sqrt{2} \cos x dx \\
 &= \sqrt{2} (\sin x) \Big|_0^{\frac{\pi}{2}} \\
 &= \sqrt{2}
 \end{aligned}$$

Question: 23

Solution:

$$y = \int_0^1 \frac{1-x-1+1}{1+x} dx$$

$$= \int_0^1 \frac{2}{1+x} - 1 dx$$

$$= (2 \ln(1+x) - x)_0^1$$

$$= 2 \ln 2 - 1$$

Question: 24

Solution:

$$y = \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx$$

$$= \left(\frac{x}{2} - \frac{\sin 2x}{4} \right)_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{4} - \frac{\sin \pi}{4}$$

$$= \frac{\pi}{4}$$

Question: 25

Solution:

$$y = \int_0^{\frac{\pi}{6}} \cos x (1 - 2\sin^2 x) dx$$

$$= \int_0^{\frac{\pi}{6}} \cos x - 2 \cos x \sin^2 x dx$$

$$= (\sin x)_0^{\frac{\pi}{6}} - 2 \int_0^{\frac{\pi}{6}} \cos x \sin^2 x dx$$

Let, $\sin x = t$

Differentiating both side with respect to t

$$\cos x \frac{dx}{dt} = 1$$

$$\Rightarrow \cos x dx = dt$$

$$\text{At } x = 0, t = 0$$

$$\text{At } x = \pi/6, t = 1/2$$

$$y = \sin \frac{\pi}{6} - \sin 0 - 2 \int_0^{\frac{1}{2}} t^2 dt$$

$$= \frac{1}{2} - 2 \left(\frac{t^3}{3} \right)_0^{\frac{1}{2}}$$

$$= \frac{1}{2} - \frac{1}{12}$$

$$= \frac{5}{12}$$

Question: 26

Solution:

$$y = \int_0^{\frac{\pi}{2}} \sin x (2 \sin x \cos x) dx$$

$$= 2 \int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx$$

Let, $\sin x = t$

Differentiating both side with respect to t

$$\cos x \frac{dx}{dt} = 1$$

$$\Rightarrow \cos x dx = dt$$

$$\text{At } x = 0, t = 0$$

$$\text{At } x = \pi/2, t = 1$$

$$y = 2 \int_0^1 t^2 dt$$

$$= 2 \left(\frac{t^3}{3} \right)_0^1$$

$$= \frac{2}{3}$$

Question: 27

Solution:

$$y = \int_0^{\pi} (2 \sin x \cos x)(4 \cos^3 x - 3 \cos x) dx$$

Let, $\cos x = t$

Differentiating both side with respect to t

$$-\sin x \frac{dx}{dt} = 1$$

$$\Rightarrow \sin x dx = -dt$$

$$\text{At } x = 0, t = 1$$

$$\text{At } x = \pi, t = -1$$

$$y = - \int_1^{-1} 8t^4 - 6t^2 dt$$

$$= - \left(8 \frac{t^5}{5} - 6 \frac{t^3}{3} \right)_1^{-1}$$

$$= - \left[\left(\frac{-8}{5} + 2 \right) - \left(\frac{8}{5} - 2 \right) \right]$$

$$= -\frac{4}{5}$$

Question: 28

Solution:

$$y = \int_0^1 \frac{e^x}{1+e^{2x}} dx$$

$$\text{Let } e^x = t$$

Differentiating both side with respect to t

$$e^x \frac{dx}{dt} = 1$$

$$\Rightarrow e^x dx = dt$$

$$\text{At } x = 0, t = 1$$

$$\text{At } x = 1, t = e$$

$$y = \int_1^e \frac{1}{1+t^2} dt$$

$$= (\tan^{-1} t)_1^e$$

$$= \tan^{-1} e - \tan^{-1} 1$$

$$= \tan^{-1} e - \pi/4$$

Question: 29

Solution:

$$\text{Let, } x = t^2$$

Differentiating both side with respect to t

$$\frac{dx}{dt} = 2t$$

$$\Rightarrow dx = 2t dt$$

$$\text{At } x = 0, t = 0$$

$$\text{At } x = 9, t = 3$$

$$y = \int_0^3 \frac{2t}{1+t} dt$$

$$= 2 \int_0^3 \frac{t+1-1}{1+t} dt$$

$$= 2 \int_0^3 1 - \frac{1}{1+t} dt$$

$$= 2(t - \ln(1+t))_0^3$$

$$y = 2[(3 - \ln 4) - (0 - \ln 1)]$$

$$= 6 - 2 \log 4$$

Question: 30

Solution:

Use integration by parts

$$\int I \times II \, dx = I \times \int II \, dx - \int \frac{d}{dx} I \left(\int II \, dx \right) dx$$

$$y = x \int_0^{\frac{\pi}{2}} \cos x \, dx - \int_0^{\frac{\pi}{2}} \frac{d}{dx} x \left(\int \cos x \, dx \right) dx$$

$$= (x \sin x)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \, dx$$

$$= \frac{\pi}{2} - (-\cos x)_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} + (0 - 1)$$

$$= \frac{\pi}{2} - 1$$

Question: 31**Solution:**

We have to convert denominator into perfect square

$$1 + x + x^2 = x^2 + 2(x) \left(\frac{1}{2} \right) + \frac{1}{4} - \frac{1}{4} + 1$$

$$= \left(x + \frac{1}{2} \right)^2 + \frac{3}{4}$$

$$= \left(x + \frac{1}{2} \right)^2 + \left(\frac{\sqrt{3}}{2} \right)^2$$

$$y = \int_0^1 \frac{1}{\left(x + \frac{1}{2} \right)^2 + \left(\frac{\sqrt{3}}{2} \right)^2} dx$$

Use formula $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$

$$y = \left(\frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right)_0^1$$

$$= \frac{2}{\sqrt{3}} \left(\tan^{-1} \frac{2}{\sqrt{3}} \left(\frac{3}{2} \right) - \tan^{-1} \frac{2}{\sqrt{3}} \left(\frac{1}{2} \right) \right)$$

$$= \frac{2}{\sqrt{3}} \left(\frac{\pi}{3} - \frac{\pi}{6} \right)$$

$$= \frac{\pi}{3\sqrt{3}}$$

Question: 32**Solution:**

Let, $x = \sin t$

Differentiating both side with respect to t

$$\frac{dx}{dt} = \cos t \Rightarrow dx = \cos t \, dt$$

At $x = 0$, $t = 0$

At $x = 1$, $t = \pi/2$

$$\begin{aligned} y &= \int_0^{\pi/2} \sqrt{\frac{1 - \sin t}{1 + \sin t}} \cos t \, dt \\ &= \int_0^{\pi/2} \sqrt{\frac{1 - \sin t}{1 + \sin t}} \times \frac{1 - \sin t}{1 - \sin t} \cos t \, dt \\ &= \int_0^{\pi/2} \frac{1 - \sin t}{\cos t} \cos t \, dt \\ &= \int_0^{\pi/2} 1 - \sin t \, dt \\ &= (t + \cos t) \Big|_0^{\pi/2} \\ &= \left(\frac{\pi}{2} + 0\right) - (0 + 1) \\ &= \frac{\pi}{2} - 1 \end{aligned}$$

Question: 33

Solution:

$$\begin{aligned} y &= \int_0^1 \frac{1-x+1-1}{1+x} dx \\ &= \int_0^1 \frac{2}{1+x} - 1 \, dx \\ &= (2 \ln(1+x) - x) \Big|_0^1 \\ &= 2 \log 2 - 1 \end{aligned}$$

Question: 34

Solution:

Let, $x = a \sin t$

Differentiating both side with respect to t

$$\frac{dx}{dt} = a \cos t \Rightarrow dx = a \cos t \, dt$$

At $x = -a$, $t = -\pi/2$

At $x = a$, $t = \pi/2$

$$\begin{aligned}
 y &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{a - a \sin t}{a + a \sin t}} a \cos t \, dt \\
 &= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{1 - \sin t}{1 + \sin t} \times \frac{1 - \sin t}{1 - \sin t}} \cos t \, dt \\
 &= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1 - \sin t}{\cos t} \cos t \, dt \\
 &= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 - \sin t \, dt \\
 &= a(t + \cos t) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= a \left[\left(\frac{\pi}{2} + 0 \right) - \left(-\frac{\pi}{2} + 0 \right) \right] \\
 &= a\pi
 \end{aligned}$$

Question: 35

Solution:

Use formula $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$

$$\begin{aligned}
 y &= \int_0^{\sqrt{2}} \sqrt{(\sqrt{2})^2 - x^2} \, dx \\
 &= \left(\frac{x}{2} \sqrt{2 - x^2} + \frac{2}{2} \sin^{-1} \frac{x}{\sqrt{2}} \right) \Big|_0^{\sqrt{2}} \\
 &= \left(\frac{\sqrt{2}}{2} \sqrt{2 - 2} + \sin^{-1} \frac{\sqrt{2}}{\sqrt{2}} \right) - (0 + \sin^{-1} 0) \\
 &= \frac{\pi}{2}
 \end{aligned}$$

Question: 36

Solution:

We know that

$$|x| = -x \text{ in } [-2, 0]$$

$$|x| = x \text{ in } [0, 2]$$

$$\begin{aligned}
 y &= \int_{-2}^0 |x| \, dx + \int_0^2 |x| \, dx \\
 &= \int_{-2}^0 -x \, dx + \int_0^2 x \, dx
 \end{aligned}$$

$$= \left(-\frac{x^2}{2}\right)_{-2}^0 + \left(\frac{x^2}{2}\right)_0^2$$

$$y = 0 - (-2) + 2 - 0$$

$$= 4$$

Question: 37

Solution:

We know that

$$|2x - 1| = -(2x - 1) \text{ in } [0, 1/2]$$

$$|2x - 1| = (2x - 1) \text{ in } [1/2, 1]$$

$$y = \int_0^{\frac{1}{2}} |2x - 1| dx + \int_{\frac{1}{2}}^1 |2x - 1| dx$$

$$= \int_0^{\frac{1}{2}} -(2x - 1) dx + \int_{\frac{1}{2}}^1 2x - 1 dx$$

$$= -(x^2 - x)_0^{\frac{1}{2}} + (x^2 - x)_{\frac{1}{2}}^1$$

$$= -\left[\left(\frac{1}{4} - \frac{1}{2}\right) - (0 - 0)\right] + \left[(1 - 1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$y = \frac{1}{2}$$

Question: 38

Solution:

We know that

$$|2x + 1| = -(2x + 1) \text{ in } [-2, -1/2]$$

$$|2x + 1| = (2x + 1) \text{ in } [-1/2, 1]$$

$$y = \int_{-2}^{-\frac{1}{2}} |2x + 1| dx + \int_{-\frac{1}{2}}^1 |2x + 1| dx$$

$$= \int_{-2}^{-\frac{1}{2}} -(2x + 1) dx + \int_{-\frac{1}{2}}^1 2x + 1 dx$$

$$= -(x^2 + x)_{-2}^{-\frac{1}{2}} + (x^2 + x)_{-\frac{1}{2}}^1$$

$$= -\left[\left(\frac{1}{4} - \frac{1}{2}\right) - (4 - 2)\right] + \left[(1 + 1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$y = \frac{9}{2}$$

Question: 39

Mark (✓) against

Solution:

We know that

$$|x| = -x \text{ in } [-2, 0)$$

$$|x| = x \text{ in } [0, 1]$$

$$y = \int_{-2}^0 \frac{|x|}{x} dx + \int_0^1 \frac{|x|}{x} dx$$

$$= \int_{-2}^0 \frac{-x}{x} dx + \int_0^1 \frac{x}{x} dx$$

$$= \int_{-2}^0 -1 dx + \int_0^1 1 dx$$

$$= (-x)_{-2}^0 + (x)_0^1$$

$$= -(0 - (-2)) + (1 - 0)$$

$$= -1$$

Question: 40

Solution:

We know that

$$|x| = -x \text{ in } [-a, 0) \text{ where } a > 0$$

$$|x| = x \text{ in } [0, a] \text{ where } a > 0$$

$$y = \int_{-a}^0 x|x| dx + \int_0^a x|x| dx$$

$$= \int_{-a}^0 x(-x) dx + \int_0^a x(x) dx$$

$$= - \int_{-a}^0 x^2 dx + \int_0^a x^2 dx$$

$$= - \left(\frac{x^3}{3} \right)_{-a}^0 + \left(\frac{x^3}{3} \right)_0^a$$

$$= - \left(0 - \left(\frac{-a^3}{3} \right) \right) + \left(\frac{a^3}{3} - 0 \right)$$

$$= 0$$

Question: 41

Solution:

Find the equivalent expression to $|\cos x|$ at $0 \leq x$

$$\text{In } 0 \leq x \leq \frac{\pi}{2} \leq \pi$$

$$= \cos x$$

$$\ln \frac{\pi}{2} \leq x \leq \pi$$

$$= -\cos x$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \cos x \, dx + \int_{\frac{\pi}{2}}^{\pi} -\cos x \, dx$$

$$\Rightarrow \sin \frac{\pi}{2} - \sin 0 - \cos \pi + \cos \frac{\pi}{2}$$

$$\Rightarrow 1 - 0 - (-1) + 0 = 2$$

Question: 42

Solution:

Find the equivalent expression to $|\sin x|$ at $0 \leq x$

$$\ln 0 \leq x \leq \pi \leq 2\pi$$

$$|\sin x| = \sin x$$

$$\ln \pi \leq x \leq 2\pi$$

$$|\sin x| = -\sin x$$

$$\Rightarrow \int_0^{\pi} \sin x \, dx + \int_{\pi}^{2\pi} -\sin x \, dx = -\cos \pi - (-\cos 0) + \cos 2\pi - \cos \pi$$

$$= -(-1) + 1 + 1 - (-1)$$

$$= 2 + 2$$

$$= 4$$

Question: 43

Solution:

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = 1 \dots (\text{let})$$

$$\therefore \text{Here, } a = \frac{\pi}{2}$$

$$f(x) = \frac{\sin x}{(\sin x + \cos x)}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$\frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} = \frac{\cos x}{\cos x + \sin x}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question: 44

Solution:

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$$

$\therefore$ Here, ...[let]

$$a = \frac{\pi}{2};$$

$$f(x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$\frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} = \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question: 45

Solution:

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$$

$\therefore$ Here,

...[let]

$$a = \frac{\pi}{2};$$

$$f(x) = \frac{\sin^4 x}{\sin^4 x + \cos^4 x}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$\frac{\sin^4\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}=\frac{\cos^4 x}{\sin^4 x+\cos^4 x}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2 \cdot 2}$$

$$= \frac{\pi}{4}$$

Question: 46

Solution:

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I \dots (\text{let})$$

$\therefore$ Here,

$$a = \frac{\pi}{2};$$

$$f(x) = \frac{\cos^4 x}{\sin^4 x + \cos^4 x}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2}-x\right)$$

$$\frac{\cos^4\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)} = \frac{\sin^4 x}{\sin^4 x + \cos^4 x}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2 \cdot 2}$$

$$= \frac{\pi}{4}$$

Question: 47

Mark (✓) against

Solution:

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = 1 \dots(\text{let})$$

$\therefore$ Here,

$$a = \frac{\pi}{2};$$

$$f(x) = \frac{\sin^n x}{\cos^n x + \sin^n x}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\cos^n x}{\cos^n x + \sin^n x}$$

$$\therefore 2I = \int_0^{\frac{\pi}{2}} 1 \, dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2 \cdot 2}$$

$$= \frac{\pi}{4}$$

Question: 48

Solution:

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = 1$$

$\therefore$ Here,

$$a = \frac{\pi}{2};$$

$$f(x) = \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\sqrt{\tan x}}{\sqrt{\cot x} + \sqrt{\tan x}}$$

$$\therefore 2I = \int_0^{\frac{\pi}{2}} 1 \, dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2 \cdot 2}$$

$$= \frac{\pi}{4}$$

Question: 49

Mark (✓) against

Solution:

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I \dots (\text{let})$$

$$= \frac{\sqrt[3]{\tan x}}{\sqrt[3]{\cot x} + \sqrt[3]{\tan x}}$$

$$= \frac{\sqrt[3]{\frac{\sin x}{\cos x}}}{\sqrt[3]{\frac{\cos x}{\sin x}} + \sqrt[3]{\frac{\sin x}{\cos x}}}$$

$$= \frac{\sqrt[3]{\frac{\sin x}{\cos x}} * (\sqrt[3]{\sin x} \sqrt[3]{\cos x})}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$= \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$\therefore$ Here,

$$a = \frac{\pi}{2};$$

$$f(x) = \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$\therefore 2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2 \cdot 2}$$

$$= \frac{\pi}{4}$$

Question: 50

Solution:

$$\frac{1}{1 + \tan x} = \frac{1}{1 + \frac{\sin x}{\cos x}}$$

$$= \frac{1}{(\cos x + \sin x) \frac{1}{\cos x}}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

So our integral becomes, $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = 1 \dots (\text{let})$$

$\therefore$ Here,

$$a = \frac{\pi}{2}$$

$$f(x) = \frac{\sin x}{(\sin x + \cos x)}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2 \cdot 2}$$

$$= \frac{\pi}{4}$$

Question: 51

Solution:

So our integral becomes

$$\frac{1}{\sqrt{\cot x + 1}} = \frac{1}{\sqrt{\frac{\cos x}{\sin x} + 1}}$$

$$= \frac{\sqrt{\sin x}}{\sqrt{\cos x + \sin x}}$$

$\therefore$ Here,

$$a = \frac{\pi}{2};$$

$$f(x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x + \sin x}}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right) + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}}}$$

$$= \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2 \cdot 2}$$

$$= \frac{\pi}{4}$$

Question: 52

Solution:

$$\frac{1}{1 + \tan^3 x} = \frac{\cos^3 x}{\sin^3 x + \cos^3 x}$$

$\therefore$ Here,

$$a = \frac{\pi}{2};$$

$$f(x) = \frac{\cos^3 x}{\sin^3 x + \cos^3 x}$$

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I \dots (\text{let})$$

$$f(a-x) = \frac{\sin^3 x}{\sin^3 x + \cos^3 x}$$

$$\therefore 2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2 \cdot 2}$$

$$= \frac{\pi}{4}$$

Question: 53

Solution:

so our integral becomes,

$$\frac{\sec^5 x}{\sec^5 x + \operatorname{cosec}^5 x} = \frac{\frac{1}{\cos^5 x}}{\frac{1}{\cos^5 x} + \frac{1}{\sin^5 x}}$$

$$= \frac{\sin^5 x}{\sin^5 x + \cos^5 x}$$

Here $a = \frac{\pi}{2}$ and $f(x) = \frac{\sin^5 x}{\sin^5 x + \cos^5 x}$

$$f(a - x) = \frac{\cos^5 x}{\sin^5 x + \cos^5 x}$$

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a - x) = I \dots (\text{let})$$

$$\therefore 2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2 \cdot 2}$$

$$= \frac{\pi}{4}$$

Question: 54

Solution:

So our integral becomes,

$$\frac{\sqrt{\cot x}}{1 + \sqrt{\cot x}} = \frac{\frac{\sqrt{\cos x}}{\sqrt{\sin x}}}{1 + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}}$$

$$= \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a - x) = I$$

so, we know that, $\dots (\text{let})$

$\therefore$ Here,

$$a = \frac{\pi}{2};$$

$$f(a - x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$$

$$\therefore f(x) = \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a - x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2 \cdot 2}$$

$$= \frac{\pi}{4}$$

Question: 55

Solution:

So our integral becomes,

$$\frac{\tan x}{1 + \tan x} = \frac{\sin x}{\cos x} \left(\frac{1}{1 + \frac{\sin x}{\cos x}} \right)$$

$$= \frac{\sin x}{\sin x + \cos x}$$

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I \dots (\text{let})$$

$\therefore$ Here,

$$a = \frac{\pi}{2}$$

$$f(x) = \frac{\sin x}{(\sin x + \cos x)}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2 \cdot 2}$$

$$= \frac{\pi}{4}$$

Question: 56

Mark (✓) against

Solution:

If f is an odd function,

$$\int_{-a}^a f(x) dx = 0$$

$$\text{as, } \int_0^a f(x) dx = - \int_{-a}^0 f(x) dx$$

here $f(x) = x^4 \sin x$

we will see $f(-x) = (-x)^4 \sin(-x)$

$$= -x^4 \sin x$$

Therefore, $f(x)$ is an odd function,

$$\int_{-\pi}^{\pi} x^4 \sin x dx = 0$$

Question: 57

Solution:

If f is an odd function,

$$\int_{-a}^a f(x) dx = 0$$

$$\text{as, } \int_0^a f(x) dx = - \int_{-a}^0 f(x) dx$$

here $f(x) = x^3 \cos^3 x$

we will see $f(-x) = (-x)^3 \cos^3(-x)$

$$= -x^3 \cos^3 x$$

Therefore, $f(x)$ is an odd function,

$$\int_{-\pi}^{\pi} x^3 \cos^3 x dx = 0$$

Question: 58

Solution:

If f is an odd function,

$$\int_{-a}^a f(x) dx = 0$$

$$\text{as, } \int_0^a f(x) dx = - \int_{-a}^0 f(x) dx$$

$f(x) = \sin^5 x$

$f(-x) = \sin^5(-x)$

$$= -\sin^5 x$$

Therefore, $f(x)$ is an odd function,

$$\int_{-\pi}^{\pi} \sin^5 x dx = 0$$

Question: 59

Mark (✓) against

Solution:

$$\begin{aligned}\int_{-1}^{-2} x^3(1-x^2)dx &= \int_{-1}^{-2} (x^3 - x^5)dx \\&= \left[\frac{x^4}{4} - \frac{x^6}{6} \right] \\&= \left[\frac{2^4}{4} - \frac{1^6}{4} - \frac{2^6}{6} + \frac{1^6}{6} \right] \\&= -\frac{27}{4}\end{aligned}$$

Question: 60

Solution:

If f is an odd function,

$$\begin{aligned}\int_{-a}^a f(x)dx &= 0 \\ \text{as, } \int_0^a f(x)dx &= -\int_{-a}^0 f(x)dx \\ f(x) &= \log\left(\frac{a-x}{a+x}\right) \\ f(-x) &= \log\frac{a-(-x)}{a-x} \\ &= \log\frac{a+x}{a-x} \\ &= -\log\frac{a-x}{a+x}\end{aligned}$$

Hence it is a odd function

$$\int_{-a}^a \log\frac{a-x}{a+x} = 0$$

Question: 61

Solution:

If f is an odd function,

$$\begin{aligned}\int_{-a}^a f(x)dx &= 0 \\ \text{as, } \int_0^a f(x)dx &= -\int_{-a}^0 f(x)dx\end{aligned}$$

$\sin^{61}x$ and x^{123} is an odd function,

so there integral is zero.

Question: 62

Solution:

$$f(x) = \tan x$$

$$f(-x) = \tan(-x)$$

$$= -\tan x$$

hence the function is odd,

therefore, $I=0$

Question: 63

Solution:

By by parts,

$$\begin{aligned}\int \log(x + \sqrt{x^2 + 1}) &= x \log(x + \sqrt{x^2 + 1}) - \int \frac{x}{(x + \sqrt{x^2 + 1}) \left(1 + \frac{x}{\sqrt{x^2 + 1}}\right)} \\&= x \log(x + \sqrt{x^2 + 1}) - \int \frac{x}{\sqrt{x^2 + 1}} = x \log(x + \sqrt{x^2 + 1}) - \sqrt{x^2 + 1}\end{aligned}$$

Question: 64

Solution:

$\cos x$ is an even function so,

$$\begin{aligned}\int_{-a}^a f(x) dx &= 2 \int_0^a f(x) dx \\ \therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx &= 2 \int_0^{\frac{\pi}{2}} \cos x dx \\ &= 2(1-0) \\ &= 2\end{aligned}$$

Question: 65

Solution:

Here,

$$f(x) = \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}}$$

$$f(a-x) = \frac{\sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}}$$

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I \dots (\text{let})$$

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}} dx$$

$$= \int_0^a dx$$

$$I = \frac{a}{2}$$

Question: 66

Solution:

$$\text{let } I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$$

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$$

$$\therefore f(a-x) = \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right)$$

$$= \log\left(1 + \frac{\left(\tan\frac{\pi}{4} - \tan x\right)}{1 + \tan\frac{\pi}{4} \tan x}\right) = \log(1 + 1(1 - \tan x)) \frac{1}{1 + \tan x}$$

$$= \log \frac{2}{1 + \tan x}$$

$$\therefore \int_0^a f(a-x) = I$$

$$= \int_0^{\frac{\pi}{4}} \log \frac{2}{1 + \tan x} dx$$

$$= \int_0^{\frac{\pi}{4}} \log 2 dx - \int_0^{\frac{\pi}{4}} (1 + \tan x) dx$$

$$\therefore I = \int_0^{\frac{\pi}{4}} \log 2 dx - I$$

$$\therefore 2I = \frac{\pi}{4} \log 2$$

$$\therefore I = \frac{\pi}{8} \log 2$$

Question: 67

Solution:

$$\therefore \int_{-a}^a f(x) dx$$

$$\therefore \int_{-a}^0 f(x) dx + \int_0^a f(x) dx$$

$$\therefore \int_0^a f(-x) dx = \int_{-a}^0 f(x) dx$$

$$\therefore \int_0^a f(-x) dx + \int_0^a f(x) dx$$

Question: 68

Solution:

$$\therefore \int_0^{1.5} [x] dx$$

$$= \int_0^1 [x] dx + \int_1^{1.5} [x] dx$$

$$= \int_0^1 0 dx + \int_1^{1.5} 1 dx$$

$$= \frac{3}{2} - 1$$

$$= \frac{1}{2}$$

Question: 69

Solution:

$$\int_{-1}^1 [x] dx = \int_{-1}^0 [x] dx + \int_0^1 [x] dx$$

$$= \int_{-1}^0 -1 dx + \int_0^1 0 dx$$

$$= -1 - 0 + 0$$

$$= -1$$

Question: 70

Solution:

$$\int_1^2 |x^2 - 3x + 2| dx$$

$$\therefore x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

so, 2, and 1 itself are the limits so no breaking points for the integral,

$$\therefore \int_1^2 (-x^2 + 3x - 2) dx$$

$$= \left[\frac{-x^3}{3} + \frac{3x^2}{2} - 2x \right] (1 \text{ to } 2)$$

$$\therefore = \frac{1}{6}$$

Question: 71

Solution:

$$\therefore \sin x = 0$$

$$\therefore x = 0, \pi, 2\pi, \dots$$

So $\pi, 2\pi$ are the limits so no breaking points for the integral,

$$\therefore \int_{\pi}^{2\pi} -\sin x dx = -\cos x (\pi \text{ to } 2\pi)$$

$$= 2$$

Question: 72

Solution:

put

$$\sin^{-1} x = t;$$

$$dt = \frac{dx}{\sqrt{1-x^2}};$$

$$x = \sin t$$

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$= t;$$

$$\text{and } \sin^{-1} 0 = 0$$

$$= t$$

Limit changes to,

$$\int_0^{\frac{\pi}{4}} \frac{tdt}{1 - \sin^2 t} = \int_0^{\frac{\pi}{4}} t \sec^2 t dt$$

$$= t \tan t - \int_0^{\frac{\pi}{4}} \tan t dt$$

$$= [t \tan t + \log \cos t] \left(0 \text{ to } \frac{\pi}{4}\right)$$

$$= \frac{\pi}{4} - \frac{1}{2} \log 2$$

Question: 73

Solution:

$$\text{put } x = \tan y$$

$$dx = \sec^2 y dy$$

$$\int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2y) \sec^2 y dy$$

$$= 2 \int_0^{\frac{\pi}{4}} y \sec^2 y dy$$

$$= 2[y \tan y - \int_0^{\frac{\pi}{4}} \tan y dy]$$

$$= 2[y \tan y + \log \cos y] \left(0 \text{ to } \frac{\pi}{4}\right)$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2} \log 2\right]$$

$$= \frac{\pi}{2} - \log 2$$