Chapter : 18. DIFFERENTIAL EQUATIONS AND THEIR FORMATION

Exercise : 18A

Question: 1

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 2, 1.

Question: 2

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 2.

So the answer is 2, 2.

Question: 3

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 2 as we have $\frac{d^2s}{dt^2}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 2.

So the answer is 2, 2.

Question: 4

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 3 as we have $\frac{d^3y}{dx^3}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 2.

So the answer is 3, 2.

Question: 5

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 2, 1.

Question: 6

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 1 as we have $\frac{dy}{dx}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 1. Also, the equation has to be a polynomial, but here the exponential function does not take any derivative with this. Hence it is a polynomial.

So the answer is 1, 1.

Question: 7

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and the degree is the highest power to which a derivative is raised. But here when we open the series of e^x as $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+--------$. Also, the equation has to be polynomial. Therefore the degree is not defined. Also, the equation has to be a polynomial, but opening the exponential function will give undefined power to the highest derivative, so the degree of this function is not defined.

So the answer is 2, not defined .

Question: 8

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 1 as we have $\frac{dy}{dx}$ and the degree is the highest power to which a derivative is raised. But when we open Sin x as $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+-----$. Also, the equation has to be polynomial, and opening thus, Sin function will lead to an undefined power of the highest derivative. Therefore the degree is not defined.

So the answer is 1, not defined.

Question: 1

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 4 as we have $\frac{d^4y}{dx^4}$ and the degree is the highest power to which a derivative is raised. But when we open the Cos x series, we get $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+-----$. This leads to an undefined power on the highest derivative. Therefore the deg9ee of this function becomes undefined.

So the answer is 4, not defined.

Question: 10

Write order and d

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 1. Because the logarithm function is not at any derivative, so it doesn't destroy the polynomial.Hence degree is 1

So the answer is 2, 1.

Question: 11

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 1 as we have $\frac{dy}{dx}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 3. Because the Sine function is not at any derivative, so it doesn't destroy the polynomial.Hence the degree is 3.

So the answer is 1, 3.

Question: 12

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 3 as we have $\frac{d^3y}{dx^3}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 3, 1.

Question: 13

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 1 as we have

$$x\frac{dy}{dx}+\frac{2}{\left(\frac{dy}{dx}\right)}+9=y^2$$

$$x\left(\frac{dy}{dx}\right)^2+2+9\frac{dy}{dx}=y^2\frac{dy}{dx}$$

and the degree is the highest power to which a derivative is raised. So the power at this order is 2.

So the answer is 1, 2.

Question: 14

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 2/3 as we have $\sqrt{1-\left(\frac{dy}{dx}\right)^2}=\left(a\frac{d^2y}{dx^2}\right)^{1/3}$

and the degree is the highest power to which a derivative is raised. So the power at 2.

So the answer is 2/3, 2.

Question: 15

The order of a differential equation is the order of the highest derivative involved in the equation. So, the order comes out to be 1 as we have $\sqrt{1-y^2}dx+\sqrt{1-x^2}dy=0$

$$\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 1, 1.

Question: 16

Solution:

The order of a differential equation is the order of the highest derivative involved in the equation. So, the order comes out to be 3 as we have $(y'')^3+(y')^2+\sin y'+1=0$

and the degree is the highest power to which a derivative is raised. So the power at this order is 2.

So the answer is 3, 2.

Question: 17

The order of a differential equation is the order of the highest derivative involved in the equation. So, the order comes out to be 1 as we have $(3x + 5y)dy - 4x^2 dx = 0$

and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 1, 1.

Question: 18

Given: $y=\frac{dy}{dx}+\frac{5}{\left(\frac{dy}{dx}\right)}$

Solving, we get,

$$y \times \frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2 + 5$$

Now,

The order of a differential equation is the order of the highest derivative involved in the equation. So, the order comes out to be 2 as we have, $y \times \frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2 + 5$

and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 2, 1.

Exercise : 18B

Question: 1

Solution:

Given $x^2 = 2y^2 \log y$

On differentiating both sides with respect to x, we get

$$2x = 2(2y)\log y\left(\frac{dy}{dx}\right) + 2y^2\left(\frac{1}{y}\right)\frac{dy}{dx}$$

$$x = (2y)\log y\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right)$$

$$x = \left(\frac{dy}{dx}\right)((2y)\log y + y)$$

Multiply both sides with y

$$xy = (2y^2\log y + y^2)\frac{dy}{dx}$$

We know, $x^2 = 2y^2\log y$. So replace $2y^2\log y$ with x^2 in the above equation.

$$xy = (x^2 + y^2)\frac{dy}{dx}$$

$$(x^2 + y^2)\frac{dy}{dx} - xy = 0$$

Conclusion: Therefore $x^2 = 2y^2\log y$ is the solution of $(x^2 + y^2)\frac{dy}{dx} - xy = 0$

Question: 1

Verify that x

Solution:

Given $x^2 = 2y^2\log y$

On differentiating both sides with respect to x, we get

$$2x = 2(2y)\log y\left(\frac{dy}{dx}\right) + 2y^2\left(\frac{1}{y}\right)\frac{dy}{dx}$$

$$x = (2y)\log y\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right)$$

$$x = \left(\frac{dy}{dx}\right)((2y)\log y + y)$$

Multiply both sides with y

$$xy = (2y^2\log y + y^2)\frac{dy}{dx}$$

We know, $x^2 = 2y^2\log y$. So replace $2y^2\log y$ with x^2 in the above equation.

$$xy = (x^2 + y^2)\frac{dy}{dx}$$

$$(x^2 + y^2)\frac{dy}{dx} - xy = 0$$

Conclusion: Therefore $x^2 = 2y^2\log y$ is the solution of $(x^2 + y^2)\frac{dy}{dx} - xy = 0$

Question: 2

Verify that y = e

Solution:

Given $y = e^x\cos bx$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x \cos bx + e^x(-b \sin bx)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x \cos bx + e^x(-b \sin bx) + e^x(-b^2 \cos bx) + e^x(-b \sin bx)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$$
$$= e^x \cos bx + e^x(-b \sin bx) + e^x(-b^2 \cos bx) + e^x(-b \sin bx) - 2e^x \cos bx - 2e^x(-b \sin bx) + 2e^x \cos bx$$

$$= e^x \cos bx - e^x(b^2 \cos bx)$$

This is not a solution

Conclusion: Therefore, y = e^x cos bx is not the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

Question: 2

Solution:

Given $y = e^x \cos bx$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x \cos bx + e^x(-b \sin bx)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x \cos bx + e^x(-b \sin bx) + e^x(-b^2 \cos bx) + e^x(-b \sin bx)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$$
$$= e^x \cos bx + e^x(-b \sin bx) + e^x(-b^2 \cos bx) + e^x(-b \sin bx) - 2e^x \cos bx - 2e^x(-b \sin bx) + 2e^x \cos bx$$

$$= e^x \cos bx - e^x(b^2 \cos bx)$$

This is not a solution

Conclusion: Therefore, y = e^x cos bx is not the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

Question: 3

Solution:

Given $y = e^{(m)\cos^{-1}x}$

On differentiating with x, we get

$$\frac{dy}{dx} = e^{(m)\cos^{-1}x}(m)\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{-ym}{\sqrt{1-x^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{ym^2}{1-x^2} - \frac{mx}{(\sqrt{1-x^2})(1-x^2)}$$

We want to find $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

$$= ym^2 - \frac{mxy}{\sqrt{1-x^2}} + \frac{ymx}{\sqrt{1-x^2}} - m^2y$$

= 0

Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

Conclusion: Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of

$$(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$$

Question: 3

Verify that y = <

Solution:

Given $y = e^{(m)\cos^{-1}x}$

On differentiating with x, we get

$$\frac{dy}{dx} = e^{(m)\cos^{-1}x}(m)\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{-ym}{\sqrt{1-x^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{ym^2}{1-x^2} - \frac{mx}{(\sqrt{1-x^2})(1-x^2)}$$

We want to find $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

$$= ym^2 - \frac{mxy}{\sqrt{1-x^2}} + \frac{ymx}{\sqrt{1-x^2}} - m^2y$$

= 0

Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

Conclusion: Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of

$$(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$$

Question: 4

Solution:

Given $y = (a + bx)e^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = be^{2x} + 2(a + bx)e^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$

$= 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x} - 4be^{2x} - 8(a + bx)e^{2x} + 4(a + bx)e^{2x}$

= 0

Conclusion: Therefore, $y = (a + bx)e^{2x}$ is the solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

Question: 4

Solution:

Given $y = (a + bx)e^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = be^{2x} + 2(a + bx)e^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$

$= 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x} - 4be^{2x} - 8(a + bx)e^{2x} + 4(a + bx)e^{2x}$

= 0

Conclusion: Therefore, $y = (a + bx)e^{2x}$ is the solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

Question: 5

Solution:

Given $y = e^x(A\cos x + B\sin x)$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x)$$

$+ e^x(-A\cos x - B\sin x)$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

Question: 5

Solution:

Given

$$y = e^x(A\cos x + B\sin x)$$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x)$$

$$+ e^x(-A\cos x - B\sin x)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

Question: 6

Solution:

Given $y = A\cos 2x - B\sin 2x$

On differentiating with x, we get

$$\frac{dy}{dx} = -2A\sin 2x - 2B\cos 2x$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -4A\cos 2x + 4B\sin 2x$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + 4y$

$$= -4A\cos 2x + 4B\sin 2x + 4\cos 2x - 4B\sin 2x$$

= 0

Conclusion: Therefore, $y = A\cos 2x - B\sin 2x$ is the solution of $\frac{d^2y}{dx^2} + 4y = 0$

Question: 6

Solution:

Given $y = A\cos 2x - B\sin 2x$

On differentiating with x, we get

$$\frac{dy}{dx} = -2A\sin 2x - 2B\cos 2x$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -4A\cos 2x + 4B\sin 2x$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + 4y$

$$= -4A\cos 2x + 4B\sin 2x + 4\cos 2x - 4B\sin 2x$$

= 0

Conclusion: Therefore, $y = A\cos 2x - B\sin 2x$ is the solution of $\frac{d^2y}{dx^2} + 4y = 0$

Question: 7

Solution:

Given $y = ae^{2x} + be^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = 2ae^{2x} + 2be^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 4ae^{2x} + 4be^{2x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$

$= 4ae^{2x} + 4be^{2x} - 2ae^{2x} - 2be^{2x} - 2ae^{2x} - 2be^{2x}$

= 0

Conclusion : Therefore, $y = ae^{2x} + be^{2x}$ is the solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

Question: 7

Solution:

Given $y = ae^{2x} + be^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = 2ae^{2x} + 2be^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 4ae^{2x} + 4be^{2x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$

$= 4ae^{2x} + 4be^{2x} - 2ae^{2x} - 2be^{2x} - 2ae^{2x} - 2be^{2x}$

= 0

Conclusion : Therefore, $y = ae^{2x} + be^{2x}$ is the solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

Question: 8

Solution:

Given $y = e^x(A\cos x + B\sin x)$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x)$$

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Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Question: 8

Show that y = e

Solution:

Given $y = e^x(A\cos x + B\sin x)$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Question: 9

Verify that y

Solution:

Given, $y^2 = 4a(x + a)$

On differentiating with x, we get

$$2y\frac{dy}{dx} = 4a$$

Now let's see what is the value of $y(1 - (\frac{dy}{dx})^2)) - 2x\frac{dy}{dx}$

$$= y\left(1 - \left(\frac{2a}{y}\right)^2\right) - 4\frac{ax}{y}$$

$$= y - \frac{4a^2}{y} - 4\frac{ax}{y}$$

$$= \frac{y^2 - 4a(a + x)}{y}$$

$$= \frac{4a(a + x) - 4a(a + x)}{y}$$

= 0

Conclusion: Therefore, $y^2 = 4a(x + a)$is the solution of $y(1 - (\frac{dy}{dx})^2)) = 2x\frac{dy}{dx}$

Question: 9

Verify that y

Solution:

Given, $y^2 = 4a(x + a)$

On differentiating with x, we get

$$2y\frac{dy}{dx} = 4a$$

Now let's see what is the value of $y(1 - (\frac{dy}{dx})^2)) - 2x\frac{dy}{dx}$

$$= y\left(1 - \left(\frac{2a}{y}\right)^2\right) - 4\frac{ax}{y}$$

$$= y - \frac{4a^2}{y} - 4\frac{ax}{y}$$

$$= \frac{y^2 - 4a(a + x)}{y}$$

$$= \frac{4a(a + x) - 4a(a + x)}{y}$$

= 0

Conclusion: Therefore, $y^2 = 4a(x + a)$is the solution of $y(1 - (\frac{dy}{dx})^2)) = 2x\frac{dy}{dx}$

Question: 10

Verify that y = <

Solution:

Given $y = c\, e^{\tan^{-1}x}$

On differentiating with x, we get

$$\frac{dy}{dx} = c\tan^{-1}x\left(\frac{1}{1 + x^2}\right)e^{\tan^{-1}x} = y\tan^{-1}x\left(\frac{1}{1 + x^2}\right)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = c(\frac{1}{1 + x^2})^2 e^{\tan^{-1}x} + c\tan^{-1}x\left(\frac{-2x}{(1 + x^2)^2}\right)e^{\tan^{-1}x} + c\,(\tan^{-1}x)^2\left(\frac{1}{(1 + x^2)^2}\right)e^{\tan^{-1}x}$$

$$= y(\frac{1}{1 + x^2})^2 + y\tan^{-1}x\left(\frac{-2x}{(1 + x^2)^2}\right) + y\,(\tan^{-1}x)^2\left(\frac{1}{(1 + x^2)^2}\right)$$

Now let's see what is the value of $(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx}$

$$= y\left(\frac{1}{1 + x^2}\right) + y\tan^{-1}x\left(\frac{-2x}{1 + x^2}\right) + y(\tan^{-1}x)^2\left(\frac{1}{1 + x^2}\right) + \left(\frac{2xy}{1 + x^2}\right)\tan^{-1}x - \tan^{-1}x\,(\frac{y}{1 + x^2})$$

$= \left(\frac{1}{1+x^2}\right)y + y(\tan^{-1}x)^2\left(\frac{1}{1+x^2}\right) - \tan^{-1}x\left(\frac{y}{1+x^2}\right)$

Conclusion: Therefore, $y = c\,e^{\tan^{-1}x}$ is not the solution of

$(1 + x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx}$

Question: 10

Verify that y = <

Solution:

Given $y = c\,e^{\tan^{-1}x}$

On differentiating with x, we get

$\frac{dy}{dx} = c\tan^{-1}x\left(\frac{1}{1+x^2}\right)e^{\tan^{-1}x} = y\tan^{-1}x\left(\frac{1}{1+x^2}\right)$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = c\left(\frac{1}{1+x^2}\right)^2 e^{\tan^{-1}x} + c\tan^{-1}x\left(\frac{-2x}{(1+x^2)^2}\right)e^{\tan^{-1}x} + c(\tan^{-1}x)^2\left(\frac{1}{(1+x^2)^2}\right)e^{\tan^{-1}x}$$

$= y\left(\frac{1}{1+x^2}\right)^2 + y\tan^{-1}x\left(\frac{-2x}{(1+x^2)^2}\right) + y(\tan^{-1}x)^2\left(\frac{1}{(1+x^2)^2}\right)$

Now let's see what is the value of $(1 + x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx}$

$$= y\left(\frac{1}{1+x^2}\right) + y\tan^{-1}x\left(\frac{-2x}{1+x^2}\right) + y(\tan^{-1}x)^2\left(\frac{1}{1+x^2}\right) + \left(\frac{2xy}{1+x^2}\right)\tan^{-1}x - \tan^{-1}x\left(\frac{y}{1+x^2}\right)$$

$= \left(\frac{1}{1+x^2}\right)y + y(\tan^{-1}x)^2\left(\frac{1}{1+x^2}\right) - \tan^{-1}x\left(\frac{y}{1+x^2}\right)$

Conclusion: Therefore, $y = c\,e^{\tan^{-1}x}$ is not the solution of

$(1 + x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx}$

Question: 11

Verify that y = <

Solution:

Given $y = a\,e^{bx}$

On differentiating with x, we get

$\frac{dy}{dx} = ab\,e^{bx}$

On differentiating again with x, we get

$\frac{d^2y}{dx^2} = ab^2\,e^{bx}$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

$= ab^2\,e^{bx} - \left(\frac{1}{y}\right)(abe^{bx})^2$

$= ab^2\,e^{bx} - ab^2\,e^{bx}$

$= 0$

Conclusion: Therefore, $y = a\,e^{bx}$is the solution of $\frac{d^2y}{dx^2} = \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

Question: 11

Verify that y = <

Solution:

Given $y = a\,e^{bx}$

On differentiating with x, we get

$$\frac{dy}{dx} = ab\,e^{bx}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = ab^2\,e^{bx}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

$$= ab^2\,e^{bx} - \left(\frac{1}{y}\right)(abe^{bx})^2$$

$$= ab^2\,e^{bx} - ab^2\,e^{bx}$$

= 0

Conclusion: Therefore, $y = a\,e^{bx}$is the solution of $\frac{d^2y}{dx^2} = \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

Question: 12

Verify that y = <

Solution:

Given $y = \frac{a}{x} + b$

On differentiating with x, we get

$$\frac{dy}{dx} = -\frac{a}{x^2}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{2a}{x^3}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right)$

$$= \frac{2a}{x^3} - \frac{2a}{x^3}$$

= 0

Conclusion: Therefore, $y = \frac{a}{x} + b$ is the solution of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right) = 0$

Question: 12

Verify that y = <

Solution:

Given $y = \frac{a}{x} + b$

On differentiating with x, we get

$$\frac{dy}{dx} = -\frac{a}{x^2}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{2a}{x^3}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right)$

$$= \frac{2a}{x^3} - \frac{2a}{x^3}$$

= 0

Conclusion: Therefore, $y = \frac{a}{x} + b$ is the solution of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right) = 0$

Question: 13

Solution:

Given $y = e^{-x} + Ax + B$

On differentiating with x, we get

$$\frac{dy}{dx} = -e^{-x} + A$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^{-x}$$

Now let's see what is the value of $e^x\left(\frac{d^2y}{dx^2}\right)$

$$= e^x(e^{-x})$$

= 1

Conclusion: Therefore, $y = e^{-x} + Ax + B$ is the solution of $e^x\left(\frac{d^2y}{dx^2}\right) = 1$

Question: 13

Solution:

Given $y = e^{-x} + Ax + B$

On differentiating with x, we get

$$\frac{dy}{dx} = -e^{-x} + A$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^{-x}$$

Now let's see what is the value of $e^x\left(\frac{d^2y}{dx^2}\right)$

$$= e^x(e^{-x})$$

= 1

Conclusion: Therefore, $y = e^{-x} + Ax + B$ is the solution of $e^x\left(\frac{d^2y}{dx^2}\right) = 1$

Question: 14

Verify that Ax

Solution:

Given $Ax^2 + By^2 = 1$

On differentiating with x, we get

$$2Ax + 2By\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{Ax}{By}$$

On differentiating again with x, we get

$$2A + 2B\left(\frac{dy}{dx}\right)^2 + 2By\left(\frac{d^2y}{dx^2}\right) = 0$$

$$y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = -\frac{A}{B}$$

Now let's see what is the value of $x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) - y\frac{dy}{dx}$

$$= x\left(-\frac{A}{B}\right) - y\left(-\frac{Ax}{By}\right)$$

$$= \left(-\frac{Ax}{B}\right) + \left(\frac{Ax}{B}\right)$$

= 0

Conclusion: Therefore, $Ax^2 + By^2 = 1$ is the solution of

$$x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) = y\frac{dy}{dx}$$

Question: 14

Solution:

Given $Ax^2 + By^2 = 1$

On differentiating with x, we get

$$2Ax + 2By\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{Ax}{By}$$

On differentiating again with x, we get

$$2A + 2B\left(\frac{dy}{dx}\right)^2 + 2By\left(\frac{d^2y}{dx^2}\right) = 0$$

$$y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = -\frac{A}{B}$$

Now let's see what is the value of $x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) - y\frac{dy}{dx}$

$$= x\left(-\frac{A}{B}\right) - y\left(-\frac{Ax}{By}\right)$$

$$= \left(-\frac{Ax}{B}\right) + \left(\frac{Ax}{B}\right)$$

= 0

Conclusion: Therefore, $Ax^2 + By^2 = 1$ is the solution of

$$x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) = y\frac{dy}{dx}$$

Question: 15

Solution:

Given $y = \frac{c-x}{1+cx}$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{-1-c^2}{(1 + cx)^2}$$

Now let's see what is the value of $(1 + x^2)\frac{dy}{dx} + (1 + y^2)$

$$= -\frac{(1 + x^2)(1 + c^2)}{(1 + cx)^2} + (1 + \left(\frac{c-x}{1 + cx}\right)^2)$$

$$= \frac{(-1-c^2-x^2-x^2c^2) + (1 + c^2x^2 + 2cx + c^2 + x^2 - 2cx)}{(1 + cx)^2}$$

= 0

Conclusion: Therefore, $y = \frac{c-x}{1+cx}$ is the solution of $(1 + x^2)\frac{dy}{dx} + (1 + y^2) = 0$

Question: 15

Solution:

Given $y = \frac{c-x}{1+cx}$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{-1-c^2}{(1 + cx)^2}$$

Now let's see what is the value of $(1 + x^2)\frac{dy}{dx} + (1 + y^2)$

$$= -\frac{(1 + x^2)(1 + c^2)}{(1 + cx)^2} + (1 + \left(\frac{c-x}{1 + cx}\right)^2)$$

$$= \frac{(-1-c^2-x^2-x^2c^2) + (1 + c^2x^2 + 2cx + c^2 + x^2 - 2cx)}{(1 + cx)^2}$$

= 0

Conclusion: Therefore, $y = \frac{c-x}{1+cx}$ is the solution of $(1 + x^2)\frac{dy}{dx} + (1 + y^2) = 0$

Question: 16

Solution:

Given $y = \log\left(x + \sqrt{x^2 + a^2}\right)$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}}(1 + \frac{x}{\sqrt{x^2 + a^2}})$$

$$= \frac{1}{\sqrt{x^2 + a^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

$$= -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{x^2 + a^2}}$$

Conclusion: Therefore, $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ is not the solution of

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$$

Question: 16

Solution:

Given $y = \log\left(x + \sqrt{x^2 + a^2}\right)$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}}\left(1 + \frac{x}{\sqrt{x^2 + a^2}}\right)$$

$$= \frac{1}{\sqrt{x^2 + a^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

$$= -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{x^2 + a^2}}$$

Conclusion: Therefore, $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ is not the solution of

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$$

Question: 17

Solution:

Given, $y = e^{-3x}$

On differentiating with x, we get

$$\frac{dy}{dx} = -3e^{-3x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 9\,e^{-3x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y$

$= 9e^{-3x} - 3e^{-3x} - 6e^{-3x}$

= 0

Conclusion: Therefore, $y = e^{-3x}$ is the solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

Question: 17

Solution:

Given, $y = e^{-3x}$

On differentiating with x, we get

$$\frac{dy}{dx} = -3e^{-3x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 9e^{-3x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y$

$= 9e^{-3x} - 3e^{-3x} - 6e^{-3x}$

= 0

Conclusion: Therefore, $y = e^{-3x}$ is the solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

Exercise : 18C

Question: 1

Y = mx + c

Differentiating the above equation with respect to x,

$$\frac{dy}{dx} = m$$

Differentiating the above equation with respect to x,

$$\frac{d^2y}{dx^2} = 0$$

This is the differential equation of the family of straight lines y=mx+c, where m and c are arbitrary constants

Question: 2

Solution:

Now, in the general equation of of the family of concentric circles $x^2+y^2=a^2$, where a>0, 'a' represents the radius of the circle and is an arbitrary constant.

The given equation represents a family of concentric circles centered at the origin.

$x^2+y^2=a^2$

Differentiating the above equation with respect to x on both sides, we have,

$2x + 2y\frac{dy}{dx} = 0$ (As a>0, derivative of a with respect to x is 0.)

$$x + y\frac{dy}{dx} = 0$$

Question: 3

Solution:

Equation of the family of curves, y=a sin (bx+c), Where a and c are parameters.

Differentiating the above equation with respect to x on both sides, we have,

$y = a\sin(bx + c)$ (1)

$\frac{dy}{dx} = ab\cos(bx + c)$

$\frac{d^2y}{dx^2} = -ab^2\sin(bx + c)$ (Substituting equation 1 in this equation)

$\frac{d^2y}{dx^2} = -b^2y$

$\frac{d^2y}{dx^2} + b^2y = 0$

This is the required differential equation.

Question: 4

Solution:

Equation of the family of curves, x=A cos nt+ Bsin nt, where A and B are arbitrary constants.

Differentiating the above equation with respect to t on both sides, we have,

$x = A\cos(nt) + B\sin(nt)$ (1)

$\frac{dx}{dt} = -An\sin(nt) + Bn\cos(nt)$

$\frac{d^2x}{dt^2} = -An^2\cos(nt) - Bn^2\sin(nt)$

$\frac{d^2x}{dt^2} = -n^2\left(A\cos(nt) + B\sin(nt)\right)$ (Substituting equation 1 in this equation)

$\frac{d^2x}{dt^2} = -n^2x$

$\frac{d^2x}{dt^2} + n^2x = 0$

This is the required differential equation.

Question: 5

Solution:

Equation of the family of curves, $y=ae^{bx}$, where a and b are arbitrary constants.

Differentiating the above equation with respect to x on both sides, we have,

$y = ae^{bx}$ (1)

$\frac{dy}{dx} = abe^{bx}$ (2)

$\frac{d^2y}{dx^2} = ab^2e^{bx}$

$y\frac{d^2y}{dx^2} = ab^2e^{bx}(ae^{bx})$ (Multiplying both sides of the equation by y)

$y\frac{d^2y}{dx^2} = \left(abe^{bx}\right)^2$ (Substituting equation 2 in this equation)

$$y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

This is the required differential equation.

Question: 6

Solution:

Equation of the family of curves, $y^2=m(a^2-x^2)$, where a and m are parameters.

Differentiating the above equation with respect to x on both sides, we have,

$$2y\frac{dy}{dx} = m(-2x)$$

$$y\frac{dy}{dx} = -mx$$

$m = -\frac{y}{x}\frac{dy}{dx}$ (1)

Differentiating the above equation with respect to x on both sides,

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -m$ (2)

From equations (1) and (2),

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

This is the required differential equation.

Question: 7

Solution:

Equation of the family of curves, $(x-a)^2+2y^2=a^2$, where a is an arbitrary constant.

$$x^2 - 2ax + a^2 + 2y^2 = a^2$$

$x^2 - 2ax + 2y^2 = 0$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$2x - 2a + 4y\frac{dy}{dx} = 0$$

$$x - a + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{a-x}{2y}$$

$$\frac{dy}{dx} = \frac{a-x}{2y}\left(\frac{2x}{2x}\right)$$

$\frac{dy}{dx} = \frac{2ax-2x^2}{4xy}$ (Substituting 2ax from equation 1)

$$\frac{dy}{dx} = \frac{x^2 + 2y^2 - 2x^2}{4xy}$$

$$\frac{dy}{dx} = \frac{2y^2 - x^2}{4xy}$$

This is the required differential equation.

Question: 8

Solution:

Equation of the family of curves, $x^2+y^2-2ay=a^2$, where a is an arbitrary constant.

$$x^2 - 2ax + a^2 + 2y^2 = a^2$$

$$x^2 - 2ax + 2y^2 = 0 \ (1)$$

Differentiating the above equation with respect to x on both sides, we have,

$$2x - 2a + 4y\frac{dy}{dx} = 0$$

$$x - a + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{a - x}{2y}$$

$$\frac{dy}{dx} = \frac{a - x}{2y}\left(\frac{2x}{2x}\right)$$

$$\frac{dy}{dx} = \frac{2ax - 2x^2}{4xy}$$ (Substituting 2ax from equation 1)

$$\frac{dy}{dx} = \frac{x^2 + 2y^2 - 2x^2}{4xy}$$

$$\frac{dy}{dx} = \frac{2y^2 - x^2}{4xy}$$

This is the required differential equation.

Question: 9

Solution:

Equation of the family of all circles touching the y-axis at the origin can be represented by

$(x-a)^2+y^2=a^2$, where a is an arbitrary constants.

$$\mathbf{(x - a)^2 + y^2 = a^2} \ (1)$$

Differentiating the above equation with respect to x on both sides, we have,

$$2(x - a) + 2y\frac{dy}{dx} = 0$$

$$x - a + y\frac{dy}{dx} = 0$$

$$a = x + y\frac{dy}{dx}$$

Substituting the value of a in equation (1)

$$\left(y\frac{dy}{dx}\right)^2 + y^2 = \left(x + y\frac{dy}{dx}\right)^2$$

$$\left(y\frac{dy}{dx}\right)^2 + y^2 = x^2 + xy\frac{dy}{dx} + \left(y\frac{dy}{dx}\right)^2$$

Rearranging the above equation

$$y^2 - x^2 - xy\frac{dy}{dx} = 0$$

This is the required differential equation.

Question: 10

From the differen

Solution:

Equation of the family of circles having centers on y-axis and radius 2 units can be represented by

$(x)^2+(y - a)^2= 4$, where a is an arbitrary constant.

$$(y - a)^2 + x^2 = 4 \ (1)$$

Differentiating the above equation with respect to x on both sides, we have,

$$2(x) + 2(y - a)\frac{dy}{dx} = 0$$

$$x - a\frac{dy}{dx} + y\frac{dy}{dx} = 0$$

$$a = \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}$$

Substituting the value of a in equation (1)

$$x^2 + \left(y - \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2 = 4$$

$$x^2 + \left(\frac{y\frac{dy}{dx} - x - y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2 = 4$$

$$x^2 + \left(\frac{x}{\frac{dy}{dx}}\right)^2 = 4$$

Rearranging the above equation

$$x^2\left(1 + \frac{1}{\left(\frac{dy}{dx}\right)^2}\right) = 4$$

This is the required differential equation.

Question: 11

Solution:

Equation of the family of circles in the second quadrant and touching the coordinate axes

can be represented by

$(x - (-a))^2+(y - a)^2= a^2$, where a is an arbitrary constants.

$$(x + a)^2 + (y - a)^2 = a^2 \ (1)$$

Differentiating the above equation with respect to x on both sides, we have,

$$2(x + a) + 2(y - a)\frac{dy}{dx} = 0$$

$$x + a - a\frac{dy}{dx} + y\frac{dy}{dx} = 0$$

$$a = \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}$$

Substituting the value of a in equation (1)

$$\left(x + \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2 + \left(y - \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2 = \left(\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2$$

$$\left(\frac{x\frac{dy}{dx} - x + x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2 + \left(\frac{y\frac{dy}{dx} - y - x - y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2 = \left(\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx} - 1}\right)^2$$

$$\left(x\frac{dy}{dx} - x + x + y\frac{dy}{dx}\right)^2 + \left(y\frac{dy}{dx} - y - x - y\frac{dy}{dx}\right)^2 = \left(x + y\frac{dy}{dx}\right)^2$$

$$\left(\frac{dy}{dx}\right)^2 (x + y)^2 + (-y - x)^2 = \left(x + y\frac{dy}{dx}\right)^2$$

$$\left(\frac{dy}{dx}\right)^2 (x + y)^2 + (y + x)^2 = \left(x + y\frac{dy}{dx}\right)^2$$

Rearranging the above equation

$$(x + y)^2\{\left(\frac{dy}{dx}\right)^2 + 1\} = \left(x + y\frac{dy}{dx}\right)^2$$

This is the required differential equation.

Question: 12

Solution:

Equation of the family of circles having centers on the x-axis and radius unity can be represented by

$(x - a)^2+(y)^2= 1$, where a is an arbitrary constants.

$$(\mathbf{x} - \mathbf{a})^2 + \mathbf{y}^2 = \mathbf{1} \quad (1)$$

Differentiating the above equation with respect to x on both sides, we have,

$$2(x - a) + 2(y)\frac{dy}{dx} = 0$$

$$x - a + y\frac{dy}{dx} = 0$$

$$a = x + y\frac{dy}{dx}$$

Substituting the value of a in equation (1)

$$\left(x - x - y\frac{dy}{dx}\right)^2 + y^2 = 1$$

$$\left(y\frac{dy}{dx}\right)^2 + y^2 = 1$$

This is the required differential equation.

Question: 13

Form the differen

Solution:

Now, it is not necessary that the centre of the circle will lie on origin in this case. Hence let us assume the coordinates of the centre of the circle be (0, h). Here, h is an arbitrary constant.

Also, the radius as calculated by the Pythagoras theorem will be $a^2 + h^2$.

Hence, the equation of the family of circles passing through the fixed point (a,0) and (-a,0), where a is the parameter can be represented by

$(x)^2+(y - h)^2= a^2 + h^2$, where a is an arbitrary constants.

$$x^2 + (y - h)^2 = a^2 + h^2 \quad (1)$$

Differentiating the above equation with respect to x on both sides, we have,

$$2(x) + 2(y - h)\frac{dy}{dx} = 0$$

$$x - h\frac{dy}{dx} + y\frac{dy}{dx} = 0$$

$$h = \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}$$

Substituting the value of a in equation (1)

$$x^2 + \left(y - \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2 = a^2 + \left(\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2$$

$$x^2 + \left(\frac{y\frac{dy}{dx} - x - y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2 = a^2 + \left(\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}\right)^2$$

$$x^2\left(\frac{dy}{dx}\right)^2 + (x)^2 = a^2\left(\frac{dy}{dx}\right)^2 + \left(x + y\frac{dy}{dx}\right)^2$$

$$x^2\left(\frac{dy}{dx}\right)^2 + (x)^2 = a^2\left(\frac{dy}{dx}\right)^2 + x^2 + 2xy\frac{dy}{dx} + \left(y\frac{dy}{dx}\right)^2$$

$$x^2\left(\frac{dy}{dx}\right)^2 = a^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} + \left(y\frac{dy}{dx}\right)^2$$

$$(x^2 - a^2 - y^2)\left(\frac{dy}{dx}\right) = 2xy$$

This is the required differential equation.

Question: 14

Solution:

Equation of the family of parabolas having a vertex at the origin and axis along positive y-axis can be represented by

$(x)^2 = 4ay$, where a is an arbitrary constants.

$$x^2 = 4ay \quad (1)$$

Differentiating the above equation with respect to x on both sides, we have,

$$2(x) = 4(a)\frac{dy}{dx}$$

$$x = 2a\frac{dy}{dx}$$

$$a = \frac{x}{2\frac{dy}{dx}}$$

Substituting the value of a in equation (1)

$$x^2 = 4\frac{x}{2\frac{dy}{dx}}y$$

$$x\frac{dy}{dx} = 2y$$

This is the required differential equation.

Question: 15

Solution:

Equation of the family of an ellipse having foci on the y-axis and centers at the origin can be represented by

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$\frac{2x}{b^2} + \frac{2y}{a^2}\frac{dy}{dx} = 0$$

$$\frac{x}{b^2} + \frac{y}{a^2}\frac{dy}{dx} = 0$$

$$\frac{y}{a^2}\frac{dy}{dx} = -\frac{x}{b^2}$$

$$\frac{y}{x}\frac{dy}{dx} = -\frac{a^2}{b^2}$$

Again differentiating the above equation with respect to x on both sides, we have,

$$\frac{y}{x}\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{\frac{dy}{dx}x - y\frac{dx}{dx}}{x^2}\right) = 0$$

$$xy\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{dy}{dx}x - y\frac{dx}{dx}\right) = 0$$

Rearranging the above equation

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

This is the required differential equation.

Question: 16

Solution:

Equation of the family of an ellipse having foci on the y-axis and centers at the origin can be represented by

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (1)

Differentiating the above equation with respect to x on both sides, we have,

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{x}{a^2} - \frac{y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{y}{b^2}\frac{dy}{dx} = \frac{x}{a^2}$$

$$\frac{y}{x}\frac{dy}{dx} = \frac{b^2}{a^2}$$

Again differentiating the above equation with respect to x on both sides, we have,

$$\frac{y}{x}\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{\frac{dy}{dx}x - y\frac{dx}{dx}}{x^2}\right) = 0$$

$$xy\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{dy}{dx}x - y\frac{dx}{dx}\right) = 0$$

Rearranging the above equation

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

This is the required differential equation.