Chapter : 19. DIFFERENTIAL EQUATIONS WITH VARIABLE SEPARABLE

Exercise : 19A

Question: 1

Solution:

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

Rearranging the terms,we get:

$$\Rightarrow \frac{dy}{1 + y^2} = (1 + x^2)dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{1 + y^2} = \int (1 + x^2)dx + c$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + c \ ...(\int \frac{dy}{1 + y^2} = \tan^{-1} y, \int x^n = \frac{x^{n+1}}{n+1})$$

Ans: $\tan^{-1} y = x + \frac{x^3}{3} + c$

Question: 2 Ans:

Find the general

Solution:

$$x^4 \frac{dy}{dx} = -y^4$$

$$\Rightarrow \frac{dy}{-y^4} = \frac{dx}{x^4}$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{-y^4} = \int \frac{dx}{x^4} + c'$$

$$\Rightarrow \frac{-y^{-4+1}}{-4 + 1} = \frac{x^{-4+1}}{-4 + 1} + c'$$

$$\Rightarrow \frac{1}{3y^3} = -\frac{1}{3x^3} + c'$$

$$\Rightarrow \frac{1}{y^3} + \frac{1}{x^3} = 3c'$$

$$\Rightarrow \frac{1}{x^3} + \frac{1}{y^3} = c \ ...(3c' = c)$$

Question: 3

Solution:

$$\frac{dy}{dx} = 1 + x + y + xy = 1 + y + x(1 + y)$$

$\Rightarrow \frac{dy}{dx} = (1 + y)(1 + x)$

Rearranging the terms we get:

$\Rightarrow \frac{dy}{1 + y} = (1 + x)dx$

Integrating both the sides we get,

$\Rightarrow \int \frac{dy}{1 + y} = \int (1 + x)dx + c$

$\Rightarrow \log|1 + y| = x + \frac{x^2}{2} + c \ldots(\int \frac{dy}{1 + y} = \log|1 + y|)$

Ans: $\log|1 + y| = x + \frac{x^2}{2} + c$

Question: 4

Solution:

$\Rightarrow \frac{dy}{dx} = 1 - x + y - xy = 1 + y - x(1 + y)$

$\Rightarrow \frac{dy}{dx} = (1 + y)(1 - x)$

Rearranging the terms we get:

$\Rightarrow \frac{dy}{1 + y} = (1 - x)dx$

Integrating both the sides we get,

$\Rightarrow \int \frac{dy}{1 + y} = \int (1 - x)dx + c$

$\Rightarrow \log|1 + y| = x - \frac{x^2}{2} + c \ldots(\int \frac{dy}{1 + y} = \log|1 + y|)$

Ans: $\log|1 + y| = x - \frac{x^2}{2} + c$

Question: 5

Solution:

$(x - 1)\frac{dy}{dx} = 2x^3y$

Separating the variables we get:

$\Rightarrow \frac{dy}{y} = 2x^3\frac{dx}{(x - 1)}$

$\Rightarrow \frac{dy}{y} = \frac{2\left((x - 1)(x^2 + x + 1) + 1\right)}{(x - 1)}dx$

$\Rightarrow \frac{dy}{y} = 2\left(x^2 + x + 1 + \frac{1}{x - 1}\right)dx$

Integrating both the sides we get,

$\Rightarrow \int \frac{dy}{y} = \int 2\left(x^2 + x + 1 + \frac{1}{x - 1}\right)dx + c$

$\Rightarrow \log|y| = \frac{2x^3}{3} + \frac{2x^2}{2} + 2x + 2\log|x-1| + c$

$\Rightarrow \log|y| = \frac{2x^3}{3} + x^2 + 2x + 2\log|x-1| + c$

Ans: $\log|y| = \frac{2x^3}{3} + x^2 + 2x + 2\log|x-1| + c$

Question: 6

Solution:

$\frac{dy}{dx} = e^x e^y$

Rearringing the terms we get:

$\Rightarrow \frac{dy}{e^y} = e^x dx$

Integrating both the sides we get,

$\Rightarrow \int \frac{dy}{e^y} = \int e^x dx + c$

$\Rightarrow \frac{e^{-y}}{-1} = e^x + c$

$\Rightarrow e^x + e^{-y} = c$

Ans: $e^x + e^{-y} = c$

Question: 7

Solution:

$(e^{x} + e^{-x})dy - (e^{x} - e^{-x})dx = 0$

$\Rightarrow dy = \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

Integrating both the sides we get,

$\Rightarrow \int dy = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx + c$

$\Rightarrow y = \log|e^x + e^{-x}| + c$ ...($\frac{d}{dx}(e^x + e^{-x}) = e^x - e^{-x}$)

Ans: $y = \log|e^x + e^{-x}| + c$

Question: 8

Solution:

Given:

$\frac{dy}{dx} = e^x e^{-y} + x^2 e^{-y}$

$\Rightarrow \frac{dy}{dx} = e^{-y}(e^x + x^2)$

$\Rightarrow \frac{dy}{e^{-y}} = (e^x + x^2)dx$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dy}{e^{-y}} = \int (e^x + x^2)dx + c$$

CLASS24

$$\Rightarrow e^y = e^x + \frac{x^3}{3} + c$$

Ans:$e^y = e^x + \frac{x^3}{3} + c$

Question: 9

Solution:

$e^{2x}e^{-3y}dx + e^{2y}e^{-3x}dy = 0$

Rearringing the terms we get:

$$\Rightarrow \frac{e^{2x}dx}{e^{-3x}} = -\frac{e^{2y}dy}{e^{-3y}}$$

$\Rightarrow e^{2x+3x}dx = -e^{2y+3y}dy$

$\Rightarrow e^{5x}dx = -e^{5y}dy$

Integrating both the sides we get:

$$\Rightarrow \int e^{5x}dx = -\int e^{5y}dy + c'$$

$$\Rightarrow \frac{e^{5x}}{5} = -\frac{e^{5y}}{5} + c'$$

$\Rightarrow e^{5x} + e^{5y} = 5c' = c$

Ans: $e^{5x} + e^{5y} = c$

Question: 10

Find the general

Solution:

Rearranging all the terms we get:

$$\frac{e^x dx}{1 - e^x} = -\frac{sec^2 y\, dy}{tany}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{e^x dx}{1 - e^x} = -\int \frac{sec^2 y\, dy}{tany} + c$$

$$\Rightarrow \frac{\log|1 - e^x|}{-1} = -\log|tany| + logc$$

$\Rightarrow \log|1 - e^x| = \log|tany| - logc$

$\Rightarrow \log|1 - e^x| + logc = \log|tany|$

$\Rightarrow tany = c(1 - e^x)$

Ans: $tany = c(1 - e^x)$

Question: 11

Find the general

Solution:

Rearranging the terms we get:

$$\frac{sec^2 x\,dx}{tanx} = -\frac{sec^2 y\,dy}{tany}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{sec^2 x\,dx}{tanx} = -\int \frac{sec^2 y\,dy}{tany} + c$$

⇒log|tanx| = - log|tany| + logc

⇒ log|tanx| + log|tany| = logc

⇒tanx.tany = c

Ans: tanx.tany = c

Question: 12

Solution:

Rearranging the terms we get:

$$\frac{\cos x\,dx}{(1 + \sin x)} = \frac{siny\,dy}{(1 + cosy)}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{\cos x\,dx}{(1 + \sin x)} = \int \frac{siny\,dy}{(1 + cosy)} + c$$

⇒log|1 + sinx| = - log|1 + cosy| + logc

⇒log|1 + sinx| + log|1 + cosy| = logc

⇒(1 + sinx)(1 + cosy) = c

Ans: (1 + sinx)(1 + cosy) = c

Question: 13

For each of the f

Solution:

$$\cos\left(\frac{dy}{dx}\right) = a$$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$$

⇒dy = $\cos^{-1}$a dx

Integrating both the sides we get:

$$\Rightarrow \int dy = \int \cos^{-1} a\ dx + c$$

⇒y = x$\cos^{-1}$a + c

when x = 0, y = 2

∴.2 = 0 + c

∴.c = 2

∴.y = x$\cos^{-1}$a + 2

$$\Rightarrow \frac{y-2}{x} = \cos^{-1} a$$

$$\Rightarrow \cos\left(\frac{y-2}{x}\right) = a$$

Ans: $\cos\left(\frac{y-2}{x}\right) = a$

Question: 14

For each of the f

Solution:

Rearranging the terms we get:

$$\frac{dy}{y^2} = -4xdx$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dy}{y^2} = -\int 4xdx + c$$

$$\Rightarrow \frac{y^{-1}}{-1} = -\frac{4x^2}{2} + c$$

$\Rightarrow y^{-1} = 2x^2 + c$

y = 1 when x = 0

$\Rightarrow (1)^{-1} = 2(0)^2 + c$

$\Rightarrow c = 1$

$$\Rightarrow \frac{1}{y} = 2x^2 + 1$$

$$\Rightarrow \frac{1}{2x^2 + 1} = y$$

Ans: $y = \frac{1}{2x^2+1}$

Question: 15

For each of the f

Solution:

Rearranging the terms we get:

$$dy = \frac{2x^2 + 1}{x}dx$$

$$\Rightarrow dy = 2x\,dx + \frac{1}{x}\,dx$$

Integrating both the sides we get:

$$\Rightarrow \int dy = \int 2x\,dx + \int \frac{1}{x}\,dx + c$$

$\Rightarrow y = x^2 + \log|x| + c$

y = 1 when x = 1

$\therefore 1 = 1^2 + \log 1 + c$

$\therefore 1 - 1 = 0 + c$ ...($\log 1 = 0$)

$\Rightarrow c = 0$

$\therefore y = x^2 + \log|x|$

Ans: $y = x^2 + \log|x|$

Question: 16

For each of the f

Solution:

Rearranging the terms we get:

$$\frac{dy}{y} = \tan x\, dx$$

$$\Rightarrow \int \frac{dy}{y} = \int \tan x\, dx + c$$

$\Rightarrow \log|y| = \log|\sec x| + \log c$

$\Rightarrow \log|y| - \log|\sec x| = \log c$

$\Rightarrow \log|y| + \log|\cos x| = \log c$

$\Rightarrow y\cos x = c$

$y = 1$ when $x = 0$

$\therefore 1 \times \cos 0 = c$

$\therefore c = 1$

$\Rightarrow y\cos x = 1$

$\Rightarrow y = 1/\cos x$

$\Rightarrow y = \sec x$

Ans: $y = \sec x$

Exercise : 19B

Question: 1

Solution:

$(y + 2)dy = (x - 1)dx$

Integrating on both sides,

$$\int (y + 2)dy = \int (x - 1)dx$$

$$\frac{y^2}{2} + 2y = \frac{x^2}{2} - x + C$$

$$y^2 + 4y - x^2 + 2x = C$$

Question: 2

Solution:

$$dy = \frac{x}{x^2 + 1}dx$$

Multiply and divide 2 in numerator and denominator of RHS,

$$y = \frac{1}{2}.\left(\frac{2x}{x^2 + 1}dx\right)$$

Integrating on both sides

$$y = \frac{1}{2}.\log(x^2 + 1) + C$$

Question: 3

Find the general

Solution:

$$\frac{1}{1 + y^2} dy = (1 + x)dx$$

Integrating on both sides

$$\int \frac{1}{1 + y^2} dy = \int (1 + x)dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + C$$

Question: 4

Solution:

$$\frac{1}{y}. dy = \frac{x}{x^2 + 1} dx$$

Multiply and divide 2 in numerator and denominator of RHS,

$$\frac{1}{y}. dy = \frac{1}{2}. \left(\frac{2x}{x^2 + 1} dx\right)$$

Integrating on both sides

$$\log y = \frac{1}{2}. \log(1 + x^2) + \log C$$

$$\log y = \log \sqrt{1 + x^2} + \log C$$

$$\Rightarrow y = \sqrt{1 + x^2}. C_1$$

Question: 5

Solution:

$$\frac{dy}{dx} = 1 - y$$

$$\frac{1}{1 - y} dy = dx$$

Integrating on both sides

$$\int \frac{1}{1 - y} dy = \int dx$$

$$\Rightarrow \log|1 - y| = x + C$$

Question: 6

Solution:

$$\frac{dy}{dx} = -\sqrt{\frac{1 - y^2}{1 - x^2}}$$

$$\frac{1}{\sqrt{1 - y^2}} dy = -\frac{1}{\sqrt{1 - x^2}} dx$$

Integrating on both sides

$$\int \frac{1}{\sqrt{1-y^2}}dy = \int -\frac{1}{\sqrt{1-x^2}}dx$$

$$\sin^{-1} y = \sin^{-1} x + C$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = C$$

Question: 7

Solution:

$$\Longrightarrow x.\frac{dy}{dx} + y = y^2$$

$$x.\frac{dy}{dx} = y^2 - y$$

$$\frac{1}{y^2 - y}dy = \frac{1}{x}dx$$

$$\frac{1}{y(y-1)}dy = \frac{1}{x}dx$$

Integrating on both the sides,

$$\int \frac{1}{y(y-1)}dy = \int \frac{1}{x}dx$$

LHS:

Let $\frac{1}{y(y-1)}dy = \frac{A}{y} + \frac{B}{(y-1)}$

$$\frac{1}{y(y-1)}dy = \frac{A(y-1)}{y} + \frac{By}{(y-1)}$$

$$1 = A(y-1) + By$$

$$1 = Ay + By - A$$

Comparing coefficients in both the sides,

A = - 1, B = 1

$$\frac{1}{y(y-1)}dy = -\frac{1}{y} + \frac{1}{(y-1)}$$

$$\int \frac{1}{y(y-1)}dy = \int \left[-\frac{1}{y} + \frac{1}{(y-1)}\right]dy$$

$$\int -\frac{1}{y}dy + \int \frac{1}{(y-1)}dy$$

$$-\log y + \log(y-1)$$

$$\Longrightarrow \log\left(\frac{y-1}{y}\right)$$

RHS:

$$\int \frac{1}{x}dx$$

$$\int \frac{1}{x}dx = \log x + \log C$$

Therefore the solution of the given differential equation is

$$\log\left(\frac{y-1}{y}\right) = \log x + \log C$$

$$\frac{y-1}{y} = x.C$$

$$y - 1 = yxC$$

$$\Rightarrow y = 1 + xyC$$

Question: 8

Solution:

$$x^2(y + 1)dx + y^2(x - 1)dy = 0$$

$$x^2(y + 1)dx = -y^2(x - 1)dy$$

$$x^2(y + 1)dx = y^2(1 - x)dy$$

$$\frac{x^2}{(1-x)}dx = \frac{y^2}{y+1}dy$$

Add and subtract 1 in numerators of both LHS and RHS,

$$\frac{x^2 - 1 + 1}{(1-x)}dx = \frac{y^2 - 1 + 1}{y+1}dy$$

$$\frac{(x^2 - 1) + 1}{(1-x)}dx = \frac{(y^2 - 1) + 1}{y+1}dy$$

$$(a^2 - b^2) = (a + b).(a - b)$$

By the identity,

$$\frac{(x + 1)(x - 1) + 1}{(1-x)}dx = \frac{(y + 1)(y - 1) + 1}{(y + 1)}dy$$

Splitting the terms,

$$-(x + 1)dx + \frac{1}{(1-x)}dx = (y - 1)dy + \frac{1}{(y+1)}dy$$

Integrating,

$$\int -(x + 1)dx + \int \frac{1}{(x-1)}dx = \int (y - 1)dy + \int \frac{1}{(y+1)}dy$$

$$-\left(\frac{x^2}{2} + x\right) + \log|x - 1| = \left(\frac{y^2}{2} - y\right) + \log|1 + y| + C$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} + x - y + \log|x - 1| + \log|1 + y| = C$$

Question: 9

Solution:

$$\frac{y}{1 + y^2}dy = \frac{x}{1 - x^2}dx$$

Multiply 2 in both LHS and RHS,

$$\frac{2y}{1 + y^2}dy = \frac{2x}{1 - x^2}dx$$

Integrating on both the sides,

$$\int \frac{2y}{1 + y^2}dy = \int \frac{2x}{1 - x^2}dx$$

$$\log(1 + y^2) = -\log(1 - x^2) + \log C$$

$$\log(1 + y^2) + \log(1 - x^2) = \log C$$

$$= (1 + y^2).(1 - x^2) = C$$

Question: 10

Solution:

$$y.\log y\, dx = x dx$$

$$\frac{1}{x}dx = \frac{1}{y.\log y}dy$$

Integrating on both the sides,

$$\int \frac{1}{x}dx = \int \frac{1}{y.\log y}dy$$

LHS:

$$\int \frac{1}{x}dx = \log x$$

RHS:

$$\int \frac{1}{y.\log y}dy$$

Let $\log y = t$

So, $\frac{1}{y}dy = dt$

$$\int \frac{1}{y.\log y}dy = \int \frac{1}{t}dt$$

$$= \log(\log y)$$

Therefore the solution of the given differential equation is

$$\log x = \log(\log y) + \log C$$

$$x = \log y . C$$

Question: 11

Solution:

$$x.x^2(1 - y^2)dy + y.y^2(1 + x^2)dx = 0$$

$$x^3(1 - y^2)dy + y^3(1 + x^2)dx = 0$$

$$\frac{1 + x^2}{x^3}dx + \frac{1 - y^2}{y^3}dy = 0$$

$$\frac{1}{x^3}dx + \frac{1}{x}dx + \frac{1}{y^3}dy - \frac{1}{y}dy = 0$$

Integrating,

$$\int \frac{1}{x^3}dx + \int \frac{1}{x}dx + \int \frac{1}{y^3}dy - \int \frac{1}{y}dy = C$$

$$\frac{x^{-3+1}}{-3+1} + \log x - \log y + \frac{y^{-3+1}}{-3+1} = C$$

$$-\frac{1}{2x^2} + -\frac{1}{2y^2} + \log x - \log y = C$$

$$-\frac{1}{2x^2} + -\frac{1}{2y^2} + \log\left(\frac{x}{y}\right) = C$$

Question: 12

Solution:

$$(1-x^2)dy = -xy(1-y)dx$$

$$(1-x^2)dy = xy(y-1)dx$$

$$\frac{1}{y(y-1)}dy = \frac{x}{1-x^2}dx$$

Integrating on both the sides,

$$\int \frac{1}{y(y-1)}dy = \int \frac{x}{1-x^2}dx$$

LHS:

Let $\frac{1}{y(y-1)}dy = \frac{A}{y} + \frac{B}{(y-1)}$

$$\frac{1}{y(y-1)}dy = \frac{A(y-1)}{y} + \frac{By}{(y-1)}$$

$$1 = A(y-1) + By$$

$$\Rightarrow 1 = Ay + By - A$$

Comparing coefficients in both the sides,

A = - 1, B = 1

$$\frac{1}{y(y-1)}dy = -\frac{1}{y} + \frac{1}{(y-1)}$$

$$\int \frac{1}{y(y-1)}dy = \int \left[-\frac{1}{y} + \frac{1}{(y-1)}\right]dy$$

$$\int -\frac{1}{y}dy + \int \frac{1}{(y-1)}dy$$

$$-\log y + \log(y-1)$$

$$= \log\left(\frac{y-1}{y}\right)$$

RHS:

$$\int \frac{x}{1-x^2}dx$$

Multiply and divide 2

$$\frac{1}{2}\cdot\int \frac{2x}{1-x^2}dx$$

$-\frac{1}{2}.\log(1-x^2) + \log C$

$-\log\sqrt{1-x^2} + \log C$

Therefore the solution of the given differential equation is

$\log\left(\frac{y-1}{y}\right) = -\log\sqrt{1-x^2} + \log C$

$-\log\left(\frac{y-1}{y}\right) = \log\sqrt{1-x^2} + \log C$

$\log\left(\frac{y}{y-1}\right) = \log\sqrt{1-x^2} + \log C$

$\frac{y}{y-1} = \sqrt{1-x^2}.C$

$= y = (y-1).\sqrt{1-x^2}.C$

Question: 13

Solution:

$\frac{1-x^2}{x}dx = \frac{y(1+y)}{(1-y)}dy$

$\left[\frac{1}{x}-x\right]dx = \left[\frac{y+y^2}{1-y}\right]dy$

$\left[\frac{1}{x}-x\right]dx = \left[\frac{y}{1-y} + \frac{y^2}{1-y}\right]dy$

Integrating on both the sides,

$\int\left[\frac{1}{x}-x\right]dx = \int\left[\frac{y}{1-y} + \frac{y^2}{1-y}\right]dy$

LHS:

$\int\left[\frac{1}{x}-x\right]dx = \log x - \frac{x^2}{2}$

RHS:

$\int\frac{y}{1-y}dy = \int\frac{y-1+1}{1-y}dy$

$\int\frac{y-`1}{1-y}dy + \int\frac{1}{1-y}dy$

$\int -1.dy + \int\frac{1}{1-y}dy$

$-y + \log|1-y|$

$\int\frac{y^2}{1-y}dy$

Add and subtract 1 in numerators of both LHS and RHS,

$\frac{y^2-1+1}{(1-y)}dy$

$$\frac{(y^2-1)+1}{(1-y)}dy$$

By the identity, $(a^2-b^2) = (a+b).(a-b)$

$$\frac{(y+1)(y-1)+1}{(1-y)}dy$$

Splitting the terms,

$$-(y+1)dy + \frac{1}{(1-y)}dy$$

Integrating,

$$\int -(y+1)dy - \int \frac{1}{(y-1)}dy$$

$$-\left(\frac{y^2}{2}+y\right) + \log|y-1|$$

Therefore the solution of the given differential equation is

$$\log x - \frac{x^2}{2} = -y + \log|1-y| - \left(\frac{y^2}{2}+y\right) + \log|y-1|$$

$$= \log|x.(1-y)^2| = \frac{x^2}{2} - \frac{y^2}{2} - 2y + C$$

Question: 14

Solution:

$$y(1+x)dx + x(1-y^2)dy = 0$$

$$\frac{1+x}{x}dx + \frac{1-y^2}{y}dy = 0$$

$$\frac{1}{x}dx + 1.dx + \frac{1}{y}dy - ydy = 0$$

Integrating,

$$\int \frac{1}{x}dx + \int 1.dx + \int \frac{1}{y}dy - \int ydy = C$$

$$\log|x| + x + \log|y| - \frac{y^2}{2} = C$$

$$= \log|xy| + x - \frac{y^2}{2} = C$$

Question: 15

Solution:

$$x^2(1-y)dy + y^2(1+x)dx = 0$$

$$\frac{1+x}{x^2}dx + \frac{1-y}{y^2}dy = 0$$

$$\frac{1}{x^2}dx + \frac{1}{x}dx + \frac{1}{y^2}dy - \frac{1}{y}dy = 0$$

Integrating,

$$\int \frac{1}{x^2}dx + \int \frac{1}{x}dx + \int \frac{1}{y^2}dy - \int \frac{1}{y}dy = C$$

$$-\frac{1}{x} + \log|x| - \frac{1}{y} - \log|y| = C$$

$$\log\left|\frac{x}{y}\right| = \frac{1}{x} + \frac{1}{y} + C$$

Question: 16

Solution:

$$x^2(y-1)dx + y^2(x-1)dy = 0$$

$$\frac{x^2}{x-1}dx + \frac{y^2}{y-1}dy = 0$$

Add and subtract 1 in numerators ,

$$\frac{x^2 - 1 + 1}{(x-1)}dx + \frac{y^2 - 1 + 1}{(y-1)}dy$$

$$\frac{(x^2-1) + 1}{(x-1)}dx + \frac{(y^2-1) + 1}{(y-1)}dy$$

$$(a^2 - b^2) = (a+b).(a-b)$$

By the identity,

$$\frac{(x+1)(x-1) + 1}{(x-1)}dx + \frac{(y+1)(y-1) + 1}{(y-1)}dy$$

Splitting the terms,

$$(x+1)dx + \frac{1}{(x-1)}dx + (y+1)dy + \frac{1}{(y-1)}dy$$

Integrating,

$$\int (x+1)dx + \int \frac{1}{(x-1)}dx + \int (y+1)dy + \int \frac{1}{(y-1)}dy = C$$

$$\frac{x^2}{2} + x + \log|x-1| + \frac{y^2}{2} + y + \log|y-1|$$

$$\frac{1}{2}.(x^2 + y^2) + (x+y) + \log|(x-1)(y-1)|$$

Question: 17

Solution:

$$\frac{x}{\sqrt{1+x^2}}dx + \frac{y}{\sqrt{1+y^2}}dy = 0$$

Integrating,

$$\int \frac{x}{\sqrt{1+x^2}}dx + \int \frac{y}{\sqrt{1+y^2}}dy$$

$$= C\ formula{:}\left\{\frac{d}{dx}\left(\sqrt{1+x^2}\right) = \frac{2x}{2.\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}\right\}$$

$$\sqrt{1+x^2} + \sqrt{1+y^2} = C$$

Question: 18

Solution:

$$\frac{dy}{dx} = e^x . e^y + x^2 . e^y$$

$$\frac{dy}{dx} = e^y(e^x + x^2)$$

$$\frac{1}{e^y} dy = (e^x + x^2)dx$$

Integrating on both the sides,

$$\int \frac{1}{e^y} dy = \int (e^x + x^2)dx$$

$$-e^{-y} = e^x + \frac{x^3}{3} + C$$

$$e^x + e^{-y} + \frac{x^3}{3} = C$$

Question: 19

Solution:

Considering 'd' as exponential 'e'

$$\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$$

$$\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + \frac{1}{e^x}}$$

$$\frac{dy}{dx} = \frac{(3e^{2x} + 3e^{4x}).e^x}{e^{2x} + 1}$$

$$\frac{dy}{dx} = \frac{3.e^{2x}(1 + e^{2x}).e^x}{e^{2x} + 1}$$

$$\frac{dy}{dx} = 3.e^{3x}$$

$$dy = 3.e^{3x}dx$$

Integrating on both the sides,

$$\int dy = \int 3.e^{3x}dx$$

$$y = 3.\frac{e^{3x}}{3} + C$$

$$y = e^{3x} + C$$

Question: 31

Solution:

Given: $\frac{dy}{dx} + \frac{cos2x}{cosx} = \frac{cos3x}{cosx}$

$\Rightarrow \frac{dy}{dx} = \frac{cos(x+2x) - cos2x}{cosx}$

$\Rightarrow \frac{dy}{dx} = \frac{(cosxcos2x - sinxsin2x) - (2cos^2x - 1)}{cosx}$

$\Rightarrow \frac{dy}{dx} = cos2x - \frac{2sinxcosxsinx}{cosx} - 2cosx + secx$

$\Rightarrow \frac{dy}{dx} = cos2x - 2sin^2x - 2cosx + secx$

$\Rightarrow y = \int(cos2x - 2sin^2x - 2cosx + secx)dx$

$\Rightarrow y = \int cos2xdx - \int 2sin^2xdx - \int 2cosxdx + \int secxdx$

$\Rightarrow y = \int cos2xdx - \int(1 - cos2x)dx - \int 2cosxdx + \int secxdx$

$\Rightarrow y = \frac{sin2x}{2} - 2sinx - x + log|secx + tanx| + c$

Question: 20

Solution:

$\Rightarrow$

$3.e^x.\tan y\, dx = (e^x - 1)\sec^2 y\, dy$

$3.\frac{e^x}{e^x - 1}dx = \frac{\sec^2 y}{\tan y}dy$

$3.\left[\frac{1}{\frac{e^x - 1}{e^x}}\right]dx = \frac{\sec^2 y}{\tan y}dy$

$3.\left[\frac{1}{1 - e^{-x}}\right]dx = \frac{\sec^2 y}{\tan y}dy$

Integrating on both the sides,

$\int 3.\left[\frac{1}{1 - e^{-x}}\right]dx = \int \frac{\sec^2 y}{\tan y}dy$

$3.\log|1 - e^{-x}| = \log|\tan y| + \log C$ formula:$\left\{\frac{d}{dy}\tan y = \frac{1}{\tan y}.\sec^2 y\right\}$

$\log(1 - e^{-x})^3 = \log|\tan y| + \log C$

$\tan y = (1 - e^{-x})^3.C$

Question: 32

Solution:

Given:$\frac{dy}{dx} + \frac{1+cos2y}{1-cos2x} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{2cos^2y}{2sin^2x}$

$\Rightarrow sec^2y\frac{dy}{dx} = -cosec^2x$

$\Rightarrow \int sec^2ydy = -\int cosec^2xdx$

$\Rightarrow tany = cotx + c$

Question: 21

Solution:

$e^y(1 + x^2)dy = \frac{x}{y}dx$

$e^y . y\, dy = \frac{x}{1 + x^2} dx$

Integrating on both the sides,

$\int e^y . y\, dy = \int \frac{x}{1 + x^2} dx$

LHS:

$\int e^y . y\, dy$

By ILATE rule,

$\int e^y . y\, dy = y . \int e^y dy - \int \left[\frac{d}{dy}(y) . \int e^y dy\right] dy$

$y . e^y - \int e^y\, dy$

$y . e^y - e^y$

$e^y(y - 1)$

RHS:

$\int \frac{x}{1 + x^2} dx$

Multiply and divide by 2

$\frac{1}{2} \int \frac{2x}{1 + x^2} dx$

$\frac{1}{2} . \log|1 + x^2|$

$\log\sqrt{1 + x^2}$

Therefore the solution of the given differential equation is

$\Rightarrow e^y(y - 1) = \log\sqrt{1 + x^2} + C$

Question: 33

Solution:

Given: $\frac{dy}{dx} = -\frac{cosxsiny}{cosy}$

$\Rightarrow \frac{dy}{dx} = -cosxtany$

$\Rightarrow \int cotydy = -\int cosxdx$

$\Rightarrow \log|siny| = -sinx + c$

Question: 22

Solution:

$\frac{dy}{dx} = e^x . e^y + e^x . e^{-y}$

$\frac{dy}{dx} = e^x(e^y + e^{-y})$

$$\frac{1}{e^y + e^{-y}} dy = e^x dx$$

$$\frac{1}{e^y + \frac{1}{e^y}} dy = e^x dx$$

$$\frac{e^y}{(e^y)^2 + 1} dy = e^x dx$$

Integrating on both the sides,

$\int \frac{e^y}{(e^y)^2+1} dy = \int e^x dx$ formula: $\left\{\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}\right\}$

$= \Rightarrow \tan^{-1} e^{-y} = e^x + C$

Question: 34

Solution:

Given: cosx(1+cosy)dx-siny(1+sinx)dy=0

Dividing the whole equation by (1+sinx)(1+cosy), we get,

$\Rightarrow \frac{\int \cos x dx}{1+\sin x} = \frac{\int \sin y dy}{1+\cos y}$

$\Rightarrow$ log|1+sinx|+log|1+cosy|=logc

$\Rightarrow$ (1+sinx)(1+cosy)=c

Question: 35

Solution:

Using $\sin^3 x = \frac{3\sin x - \sin 3x}{4}$

We have,

$\Rightarrow \frac{3\sin x - \sin 3x}{4} dx - \sin y dy = 0$

$\Rightarrow \frac{3}{4}\sin x dx - \frac{\sin 3x}{4} dx - \sin y dy = 0$

$\Rightarrow \int \frac{3}{4}\sin x dx - \int \frac{\sin 3x}{4} dx - \int \sin y dy = 0$

$\Rightarrow \frac{3}{4}(-\cos x) + \frac{1}{12}\cos 3x + \cos y = k$

$\Rightarrow 12\cos y + \cos 3x - 9\cos x = c$

Question: 23

Solution:

$$\frac{\cos x}{\sin x} dx + \frac{e^y}{e^y + 1} dy = 0$$

$$\cot x\, dx + \frac{e^y}{e^y + 1} dy = 0$$

Integrating,

$$\int \cot x\, dx + \int \frac{e^y}{e^y + 1} dy = C$$

$\log|\sin x| + \log|e^y + 1| = \log C$

$\log|\sin x.(e^y + 1)| = \log C$

$\Rightarrow \sin x.(e^y + 1) = C$

Question: 36

Solution:

$\frac{dy}{dx} + \sin(x + y) = \sin(x - y)$

$\Rightarrow \frac{dy}{dx} = \sin(x - y) - \sin(x + y)$

$\Rightarrow \frac{dy}{dx} = -2 sin y cos x$ (Using sin(A+B)-sin(A-B)=2sinBcosA)

$\Rightarrow -cosec y dy = cos x dx$

$\Rightarrow -\int cosec y dy = \int cos x dx$

$\Rightarrow -\log|cosec y - cot y| = sin x + c$

$\Rightarrow$ sinx+log|cosecy-coty|+c=0

Question: 24

Solution:

$\frac{dy}{dx} + \frac{y(1 + x)}{x(1 + y)} = 0$

$\frac{1 + y}{y} dy + \frac{1 + x}{x} dx = 0$

$\frac{1}{y} dy + 1.dy + \frac{1}{x} dx + 1.dx = 0$

Integrating,

$\int \frac{1}{y} dy + \int 1.dy + \int \frac{1}{x} dx + \int 1.dx = C$

$\log|y| + y + \log|x| + x = C$

$\Rightarrow \log|xy| + x + y = C$

Question: 25

Solution:

$dy = \frac{x}{\sqrt{1 - x^4}} dx$

Multiply and divide by 2,

$dy = \frac{1}{2}.\frac{2x}{\sqrt{1 - x^4}} dx$

$dy = \frac{1}{2}.\frac{2x}{\sqrt{1 - (x^2)^2}} dx$

Integrating on both the sides,

$\int dy = \frac{1}{2} . \int dy = \frac{1}{2} . \frac{2x}{\sqrt{1-x^4}} dx$ formula: $\left\{\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}\right\}$

$\Rightarrow y = \frac{1}{2} . \sin^{-1} x^2 + c$

Question: 37

Solution:

Given:

$\Rightarrow y cos^2 y dy + x cos^2 x dx = 0$ $\frac{1}{2} cos^2 y dy + \frac{1}{2} cos^2 x dx = 0$

$\Rightarrow \frac{y}{2}(1 + cos^2)dy + \frac{x}{2}(1 + cos^2)dx = 0$ (Using, 2cos²a=1+cos2a)

$\Rightarrow ydy + ycos^2 ydy + xdx + xcos^2 xdx = 0$

$\Rightarrow \frac{y^2}{2} + \frac{y}{2} sin^2 y - \int \frac{sin^2 y}{2} dy$

$\Rightarrow \frac{y^2}{2} + \frac{y}{2} sin^2 y + \frac{cos^2}{4} + \frac{x^2}{2} + \frac{x}{2} sin^2 x + \frac{sin^2}{4} = c$

Question: 26

Solution:

$$\frac{\log y}{y} dy + \frac{x^2}{\csc x} dx = 0$$

$$\frac{\log y}{y} dy + x^2 . \sin x \, dx = 0$$

Integrating,

$$\int \frac{\log y}{y} dy + \int x^2 . \sin x \, dx = C$$

Consider the integral $\int \frac{\log y}{y} dy$

Let $\log y = t$

So, $\frac{1}{y} dy = dt$

$$\int \frac{\log y}{y} dy = \int t . dt$$

$$\frac{t^2}{2}$$

$$\frac{(\log y)^2}{2}$$

Consider the integral $\int x^2 . \sin x \, dx$

By ILATE rule,

$$\int x^2 . \sin x \, dx = x^2 \int \sin x \, dx - \int \left[\frac{d}{dx}(x^2) \int \sin x \, dx .\right] dx$$

$$-x^2 . \cos x - \int \left[2x . \int \sin x \, dx\right] dx$$

$$-x^2 \cos x + 2 \int [x . \cos x] dx$$

Again by ILATE rule,

$$-x^2 \cos x \ + \ 2\left[x.\int \cos x\, dx - \int \left\{\frac{d}{dx}x.\int \cos x\, dx\right\} dx\right]$$

$$-x^2 \cos x \ + \ 2\left[x \sin x - \int \sin x\, dx\right]$$

$$-x^2 \cos x \ + \ 2\left[x \sin x \ + \ \cos x\right]$$

$$-x^2 \cos x \ + \ 2\, x \sin x \ + \ 2 \cos x$$

$$\cos x\,(2 - x^2) \ + \ 2x \sin x$$

Therefore the solution of the given differential equation is,

$$\frac{(\log y)^2}{2} \ + \ \cos x\,(2 - x^2) \ + \ 2x \sin x \ = \ C$$

Question: 38

Solution:

Here we have, $y = \int(\sin^3 x \cos^2 x + xe^x)\, dx$

$\Rightarrow \int \cos^2 x(1 - \cos^2 x)\sin x dx + \int xe^x dx$

Taking cosx as t we have,

$\Rightarrow cosx = t,$

$\Rightarrow -sinxdx = dt,$

So we have,

$\Rightarrow y = \int cos^2 xsinxdx - \int cos^4 xsinxdx + \int xe^x dx$

$\Rightarrow y = -\int t^2 dt - \int t^4(-dt) + \int xe^x dx$

$\Rightarrow y = -\frac{cos^3 x}{3} + \frac{cos^5 x}{5} + xe^x - e^x + c$

Question: 27

Solution:

$$\frac{1}{\tan^{-1} x.(1 \ + \ x^2)} dx \ + \ \frac{1}{y} dy \ = \ 0$$

Integrating,

$$\int \frac{1}{\tan^{-1} x.(1 \ + \ x^2)} dx \ + \ \int \frac{1}{y} dy \ = \ C$$

Consider the integral $\int \frac{1}{\tan^{-1} x.(1 + x^2)} dx$

Let $\tan^{-1} x \ = \ t$

So, $\frac{1}{1 + x^2} dx \ = \ dt$

$$\int \frac{1}{\tan^{-1} x.(1 \ + \ x^2)} dx \ = \ \int \frac{1}{t} dt$$

$\log t$

$\log(\tan^{-1} x)$

Consider the integral $\int \frac{1}{y} dy$

$\log y$

Therefore the solution of the differential equation is

$\log(\tan^{-1} x) + \log y = \log C$

$\tan^{-1} x . y = C$

Question: 39

Find the particul

Solution:

Given:

$$\frac{dy}{dx} = (1 + x)(1 + y)$$

$\Rightarrow \frac{dy}{1+y} = (1 + x)dx$

$\Rightarrow log|y + 1| = (x + \frac{x^2}{2} + c)$

$\Rightarrow now, for\ y = 0\ and\ x = 1,$

We have,

$\Rightarrow 0 = 1 + \frac{1}{2} + c$

$\Rightarrow c = -\frac{3}{2}$

$\Rightarrow \log|y + 1| = \frac{x^2}{2} + x - \frac{3}{2}$

Question: 28

Solution:

$dy = x . \tan^{-1} x\, dx$

Integrating on both the sides,

$$\int dy = \int x . \tan^{-1} x\, dx$$

$$y = \tan^{-1} x \int x\, dx - \int \left[\frac{d}{dx}(\tan^{-1} x) . \int x\, dx\right] dx \ \langle by\ ILATE\ rule\rangle$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \int \left[\frac{1}{1 + x^2}\right] . \frac{x^2}{2} dx$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} . \int \frac{x^2}{x^2 + 1} dx$$

$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} \int \left[\frac{x^2 - 1 + 1}{x^2 + 1}\right]$ (adding and subtracting 1)

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} \int \left[1 - \frac{1}{x^2 + 1}\right] dx$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} [x - \tan^{-1} x] + C$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} x + \frac{\tan^{-1} x}{2} + C$$

$y = \frac{1}{2}.\tan^{-1} x\,(x^2 + 1) - \frac{1}{2}x + C$

Question: 40

Find the particul

Solution:

$\frac{2xdx}{1+x^2} - \frac{2ydy}{1+y^2} = 0$

$\Rightarrow \frac{\log(1+x^2)}{1+y^2} = 0$

$\Rightarrow (1 + x^2) = c(1 + y^2)$

$\Rightarrow y = 1, x = 0$

$\Rightarrow 1 = c(2)$

$\Rightarrow c = \frac{1}{2}$

$\Rightarrow 2(1 + x^2) = 1 + y^2$

$\Rightarrow 2 + 2x^2 - 1 = y^2$

$\Rightarrow 2x^2 + 1 = y^2$

$\Rightarrow y = \sqrt{2x^2 + 1}$

Question: 29

Solution:

$e^x.x\,dx + \frac{y}{\sqrt{1-y^2}}dy = 0$

Integrating,

$\int e^x.x\,dx + \int \frac{y}{\sqrt{1-y^2}}dy = C$

Consider the integral $\int e^x.x\,dx$

By ILATE rule,

$\int e^x.x\,dx = x.\int e^x\,dx - \int\left[\frac{d}{dx}(x).\int e^x\,dx\right]dx$

$x.e^x - \int e^x\,dx$

$x.e^x - e^x$

$e^x(x-1)$

Consider the integral $\int \frac{y}{\sqrt{1-y^2}}dy$

Its value is - $\sqrt{1-y^2}$ as $\frac{d}{dx}\left(\sqrt{1-y^2}\right) = \frac{-2y}{2.\sqrt{1-y^2}} = \frac{-y}{\sqrt{1-y^2}}$

Therefore the solution of the given differential equation is

$e^x(x-1)$ - $\sqrt{1-y^2} = C$

Question: 41

Find the particul

Solution:

$\log\left(\frac{dy}{dx}\right) = 3x + 4y$

$\Rightarrow y = 0$

$\Rightarrow x = 0$

$\Rightarrow \frac{dy}{dx} = e^{3x}e^{4y}$

$\Rightarrow e^{-4y}dy = e^{3x}dx$

$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$

$\Rightarrow$ For $y = 0, x = 0$, we have

$\Rightarrow -\frac{1}{4} = \frac{1}{3} + c$

$\Rightarrow c = -\frac{7}{12}$

$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$

Hence, the particular solution is:

$\Rightarrow 4e^{3x} + 3e^{-4x} = 7$

Question: 30

Solution:

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

$$dy = \frac{1 - \cos x}{1 + \cos x}dx$$

can be written as $\cos x = \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}$

$\cos x$

$$dy = \frac{1 - \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}}{1 + \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}}dx$$

$$dy = \frac{\left[\frac{1 + \tan^2\left(\frac{x}{2}\right) - \left(1 - \tan^2\left(\frac{x}{2}\right)\right)}{1 + \tan^2\left(\frac{x}{2}\right)}\right]}{\frac{1 + \tan^2\left(\frac{x}{2}\right) + \left(1 - \tan^2\left(\frac{x}{2}\right)\right)}{1 + \tan^2\left(\frac{x}{2}\right)}}dx$$

$$dy = \frac{1 + \tan^2\left(\frac{x}{2}\right) - 1 + \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right) + 1 - \tan^2\left(\frac{x}{2}\right)}dx$$

$$dy = \frac{2\tan^2\left(\frac{x}{2}\right)}{2}dx$$

$$dy = \tan^2\left(\frac{x}{2}\right)dx$$

Integrating on both the sides,

$\int dy = \int \tan^2\left(\frac{x}{2}\right)dx$

$y = \int\left[\sec^2\left(\frac{x}{2}\right) - 1\right]dx$ formula: $\{\sec^2 x - \tan^2 x = 1\}$

$y = 2.\tan\left(\frac{x}{2}\right) - x + C$ formula: $\left\{\frac{d}{dx}\tan\left(\frac{x}{2}\right) = \sec^2\left(\frac{x}{2}\right).\frac{1}{2}\right\}$

Question: 42

Solution:

$x^2(1-y)dy + y^2(1+x^2)dx = 0$

$\Rightarrow \frac{(1-y)}{y^2}dy + \frac{(1+x^2)}{x^2}dx = 0$

$\Rightarrow \int\frac{(1-y)}{y^2}dy + \int\frac{(1+x^2)}{x^2}dx = 0$

$\Rightarrow -\frac{1}{y} - logy - \frac{1}{x} + x = c$

For y=1,x=1, we have,

$\Rightarrow -1 - 0 - 1 + 1 = c$

$\Rightarrow c = -1$

Hence, the required solution is:

$\Rightarrow \frac{1}{y} + logy + \frac{1}{x} - x = 1$

Question: 43

Solution:

Given: $e^x\sqrt{1-y^2}dx + \frac{y}{x}dy = 0$ Separating the variables we get,

$\Rightarrow xe^x dx + \frac{y}{\sqrt{1-y^2}}dy = 0$

$\Rightarrow \int xe^x dx + \int\frac{y}{\sqrt{1-y^2}}dy = 0$ Substituting $\sqrt{1-y^2} = t, 1-y^2 = t^2, -2ydy = 2tdt$, we have,

$\Rightarrow xe^x - e^x - \frac{1}{2}\log\left|\sqrt{1-y^2}\right| = c$

For y=1 and x=0, we have,

$\Rightarrow 0 - 1 - 0 = c$

$\Rightarrow c = -1$

$\Rightarrow$ Hence, the particular solution will be:-

$\Rightarrow xe^x - e^x - \frac{1}{2}\log\left|\sqrt{1-y^2}\right| + 1 = 0$

Question: 44

Solution:

Given: $\frac{dy}{dx} = \frac{x(2logx+1)}{(siny+ycosy)}$

$\Rightarrow \int sinydy + \int ycosydy = \int 2xlogxdx + \int xdx$

Let $\int ycosydy = I$ Then,

$\int ycosydy = (\int cosydy)\, y - \int ((\int ycosydy)\,.\frac{d}{dx} y)dy$

And $\int xlogx = (\int xdx)logx - \int ((\int xdx)\frac{d}{dx}logx)dx$

We have,

$\Rightarrow -cosy + ysiny + cosy = x^2logx - \frac{x^2}{2} + \frac{x^2}{2} + c$

For $y = \frac{\pi}{2}, x = 1$ we have,

$$0 + \frac{\pi}{2} + 0 = 0 + c$$

$$c = \frac{\pi}{2}$$

$\Rightarrow ysiny = x^2logx + \frac{\pi}{2}$

Question: 45

Solution:

We have,

$$\frac{dy}{dx} = ysin2x$$

$\Rightarrow \frac{dy}{y} = sin2xdx$

$\Rightarrow logy = -\frac{cos2x}{2} + c$

For y=1, x=0, we have,

$c = \frac{1}{2}$

$\Rightarrow logy = \frac{1}{2}(1 - cos2x)$

$\Rightarrow logy = sin^2 x$

Thus,

The particular solution is:

$y = e^{sin^2 x}$

Question: 46

Solution:

Given: $(x + 1)\frac{dy}{dx} = 2xy$

$\Rightarrow \frac{dy}{y} = 2\frac{x}{x+1}dx$

$\Rightarrow logy = \int 2 - \frac{2}{x+1}dx$

$\Rightarrow logy = 2x - 2\log(x + 1) + c$

For x=2 and y=3, we have,

c = 3log3 – 4

Hence, the particular solution is,

$\Rightarrow y(x + 1)^2 = 27e^{2x-4}$

Question: 47

Solution:

we have, $\frac{dy}{dx} = 2xlogx + x$, Integrating we get,

$y = \int(2xlogx + x)dx$,

$y = \int 2xlogxdx + xdx$

$y = \left(\int 2xdx\right)logx - \int\left[\left(\int 2xdx\right)\left(\frac{d}{dx}logx\right)\right]dx + \frac{x^2}{2} + c$

given that y=0 when x=2

$\Rightarrow y = x^2logx - \frac{x^2}{2} + \frac{x^2}{2} + c$

now putting x=2 and y=0,

$\Rightarrow 0 = 4log2 + c$

$\Rightarrow c = -4log2$

Thus, the solution is:

$y = x^2logx - 4log2$

Question: 48

Solution:

we have, $\left(x^3 + x^2 + x + 1\right)\frac{dy}{dx} = 2x^2 + x$,

$\left\{ \quad \right\}$Given that: y=1 when x=0,

$\Rightarrow (x^2 + 1)(x + 1)\frac{dy}{dx} = 2x^2 + x$

$\Rightarrow \frac{dy}{dx} = \frac{x^2+x+x^2}{(x^2+1)(x+1)}dx$

$\Rightarrow \frac{dy}{dx} = \frac{x(x+1)+x^2+1-1}{(x^2+1)(x+1)}dx$

$\Rightarrow \frac{dy}{dx} = \frac{x}{x^2+1}dx + \frac{x^2+1-1}{(x^2+1)(x+1)}dx$

$\Rightarrow dy = \frac{xdx}{x^2+1} + \frac{dx}{x+1} - \frac{dx}{(x^2+1)(x+1)}$

$\Rightarrow \int dy = \int\frac{xdx}{x^2+1} + \int\frac{dx}{x+1} - \int\frac{\frac{-1}{2}x+\frac{1}{2}}{x^2+1}dx + \int\frac{\frac{1}{2}}{x+1}dx$

$\Rightarrow y = \frac{1}{2}\log|x^2 + 1| + \log|x + 1| + \frac{1}{4}\log|x^2 + 1| - \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x + 1| + c$

$\Rightarrow y = \frac{3}{4}\log|x^2 + 1| + \frac{1}{2}\log|x + 1| - \frac{1}{2}\tan^{-1}x + c$

For y=1, when x=0, we have,

$1 = 0 + 0 - 0 + c$

$\Rightarrow c = 1$

$y=\frac{1}{2}\left\{\log|x+1|+\frac{3}{2}\log\left(x^2+1\right)-\tan^{-1}x\right\}+1$

CLASS24

Question: 49

Solution:

we have, $\frac{dy}{dx}=y\tan x.$

given that: y=1 when x=0

$\Rightarrow \frac{dy}{dx}=ytanx$

$\Rightarrow \frac{dy}{y}=tanx\,dx$

$\Rightarrow logy=losecx+c$

$\Rightarrow 0=0+c$

$\Rightarrow ycosx=1$ is the particular solution...

Question: 50

Solution:

we have: $\frac{dy}{dx}=y^2\tan 2x.$

Given that, y=2 when x=0

$\Rightarrow \frac{dy}{y^2}=tan2xdx$

$\Rightarrow \int\frac{dy}{y^2}=\int tan2xdx$ ...integrating both sides

$\Rightarrow -\frac{1}{y}=\frac{\log(sec2x)}{2}$

$\Rightarrow -\frac{1}{2}=0+c$

$\Rightarrow c=-\frac{1}{2}$

$\Rightarrow y(1+logcos2x)=2$ ...is the particular solution

Question: 51

Solution:

we have $\frac{dy}{dx}=y\cot 2x.$

Given that, y=2 when x=$\frac{\pi}{2}$

$\Rightarrow \frac{dy}{y}=ycot2x$

$\Rightarrow \frac{dy}{y}=cot2xdx$

$\Rightarrow \int\frac{dy}{y}=\int cot2xdx$

$\Rightarrow logy = -\frac{\log(sin2x)}{2} + c$

$\Rightarrow log2 = 0$+c

$\Rightarrow$ Thus, $c = log2$

The particular solution is :- $log\frac{y}{\sqrt{sin2x}} = log2$

$\therefore y = 2\sqrt{sin2x}$

Question: 52

Solution:

we have, $(1 + x^2)$ sec2 y dy + 2x tan y dx = 0,

Given that, $y = \frac{\pi}{4}$ when x=1

$\Rightarrow (1 + x^2)sec2ydy + 2xtanydx = 0$

$\Rightarrow \frac{sec^2 y}{tany}dy + \frac{2x}{1+x^2}dx = 0$

$\Rightarrow \int\frac{sec^2 y}{tany}dy + \int\frac{2x}{1+x^2}dx = 0$

$\Rightarrow logtany + \log(1 + x^2) = logc$

For $y = \frac{\pi}{4}, x = 1$

We have, $0 + log2 = logc,$

c = 2,

Hence the required particular solution is:-

$\therefore tany(1 + x^2) = 2$

Question: 53

Solution:

we have, sin x cos y dx + cos x sin y dy = 0

$\Rightarrow sinx\ cosy\ dx + cosx\ siny\ dy = 0$

$\Rightarrow tanx\ dx + tany\ dy = 0$

$\Rightarrow \log secx + \log secy = logc$

$\Rightarrow secx\ secy = c$

Given that, coordinates of point, $(0,\frac{\pi}{4})$

$\Rightarrow c = \sqrt{2}$

$\Rightarrow secy = \sqrt{2}cosx$

$\therefore y = cos^{-1}(\frac{1}{\sqrt{2}}secx)$ ...is the required particular solution

Question: 54

Solution:

Given, $\frac{dy}{dx} = e^x sinx$

$dy = e^x sinx dx$

$\Rightarrow \int dy = \int e^x sinx dx$

Let $I = \int e^x sinx dx$

$\Rightarrow I = \int e^x dx\ sinx\ - \int (\int e^x dx\ ).(\frac{d}{dx} sinx)\ dx$

$\Rightarrow I = e^x sinx - \int e^x cosx dx$

$\Rightarrow I = e^x sinx - \int e^x dx\ cosx\ - \int (\int e^x dx\ ).(\frac{d}{dx} cosx)\ dx$

$\Rightarrow I = e^x sinx - e^x cosx - \int e^x sinx dx$

$\Rightarrow 2I = e^x sinx - e^x cosx$

$\Rightarrow I = \frac{e^x sinx - e^x cosx}{2} + c$

$\therefore\ y = \frac{e^x sinx - e^x cosx}{2} + c$

For the curve passes through (0,0)

We have, c = $\frac{1}{2}$

$\therefore\ 2y - e^x sinx + e^x cosx = 1$

Question: 55

Solution:

Given that the product of slope of tangent and y coordinate equals the x-coordinate i.e., $y\frac{dy}{dx} = x$

We have, $ydy = xdx$

$\Rightarrow \int ydy = \int xdx$

$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c$

For the curve passes through (0, -2), we get c = 2,

Thus, the required particular solution is:-

$\therefore\ y^2 = x^2 + 4$

Question: 56

Solution:

Given : $\frac{dy}{dx} = \frac{2(y+3)}{x+4}$

$\Rightarrow \frac{dy}{y+3} = \frac{2dx}{x+4}$

$\Rightarrow \int \frac{dy}{y+3} = 2\int \frac{dx}{x+4}$

$\Rightarrow \log(y+3) = 2\log(x+4) + c$

The curve passes through (-2, 1)we have,

c = 0,

$\therefore\ y + 3 = (x+4)^2$

Question: 57

Solution:

Given:

$$\frac{dp}{dt} = \left(\frac{r}{100}\right) \times p$$

Here, p is the principal, r is the rate of interest per annum and t is the time in years.

Solving the differential equation we get,

$$\frac{dp}{p} = \left(\frac{r}{100}\right) dt$$

$\Rightarrow \int \frac{dp}{p} = \int \frac{r}{100} dt$

$\Rightarrow logp = \frac{rt}{100} + c$

$\Rightarrow p = e^{\frac{rt}{100}+c}$

As it is given that the principal doubles itself in 10 years, so

Let the initial interest be p1 (for t = 0), after 10 years p1 becomes 2p1.

Thus, $p1 = e^{c}$ for (t = 0) ...(i)

$p = 2p1 = e^{\frac{r.(10)}{100}}.e^{c}$ ...(ii)

Substituting (i) in (ii), we get,

$\Rightarrow 2\text{p}1 = e^{\frac{r}{10}}.p1$

$\Rightarrow 2 = e^{\frac{r}{10}}$

$\Rightarrow log2 = \frac{r}{10}$

$\Rightarrow r = 10log2$

$\Rightarrow r = 6.931$

∴ Rate of interest = 6.931

Question: 58

Solution:

Given: rate of interest = 5%

P(initial) = Rs 1000

And,

$$\frac{dp}{dt} = \frac{5}{100} \times p$$

$\Rightarrow \frac{\text{dp}}{\text{p}} = \frac{5}{100}\text{dt}$

$\Rightarrow \int \frac{\text{dp}}{\text{p}} = \int \frac{5}{100}\text{dt}$

$\Rightarrow \text{logp} = \frac{5\text{t}}{100} + \text{c}$

$\Rightarrow \text{p} = \text{e}^{\frac{5\text{t}}{100}+\text{c}}$

For t = 0, we have p = 1000

$1000 = e^c$

For t = 10 years we have, $p = e^{\frac{50}{100}}.1000$

$p = 1000e^{1/2}$

$p = 1648$

Thus, principal is Rs1648 for t = 10 years.

Question: 59

The volume of a s

Solution:

Given:

Volume V $= \frac{4\pi r^3}{3}$

$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$

$\Rightarrow \frac{dV}{dt} = k$ (constant)

$4\pi r^2 \frac{dr}{dt} = k$

$\Rightarrow 4\pi r^2 dr = kdt$

$\Rightarrow \int 4\pi r^2 dr = \int kdt$

$\Rightarrow \frac{4\pi r^3}{3} = kt + c$

For t = 0, r = 3 and for t = 3, r = 6, So, we have,

$\Rightarrow \frac{4\pi(3)^3}{3} = 0 + c$

$\Rightarrow c = 36\pi$

$\frac{4\pi(6)^3}{3} = k.(3) + 36\pi$

$\Rightarrow$ k $= 84\pi$

So after t seconds the radius of the balloon will be,

$\Rightarrow \frac{4\pi r^3}{3} = 84\pi t + 36\pi$

$\Rightarrow 4\pi r^3 = 252\pi t + 108\pi$

$\Rightarrow r^3 = \frac{252\pi t + 108\pi}{4\pi}$

$\Rightarrow r^3 = 63t + 27$

$\Rightarrow r = \sqrt[3]{63t + 27}$

Hence, radius of the balloon as a function of time is

$\therefore\ r = (63t + 27)^{1/3}$

Question: 60

Solution:

Let y be the bacteria count, then, we have,

rate of growth of bacteria is proportional to the number present

So, $\frac{dy}{dt} = cy$

Where c is a constant,

Then, solving the equation we have,

$\frac{dy}{y} = cdt$

$\int \frac{dy}{y} = \int cdt$

$logy = ct + k$

Where k is constant of integration

$y = e^{ct+k}$

And we have for t = 0, y = 10000,

$10000 = e^{k}$ ...(i)

For t = 2hrs, y is increased by 10% i. e. y = 110000

$110000 = e^{c(2)}.e^{k}$

$\Rightarrow 110000 = e^{2c}.(100000)$ from (i)

$\Rightarrow e^{2c} = 1.1$

$\Rightarrow e^{c} = \sqrt{1.1}$

$\Rightarrow c = \frac{1}{2}\log(\frac{11}{10})$

When y = 200000, we have,

$200000 = e^{ct}.100000$

$\Rightarrow e^{ct} = 2$

$\Rightarrow (e^{c})^{t} = 2$

$\Rightarrow tc = log2$

$\Rightarrow t = \frac{2log2}{log\frac{11}{10}}$

Hence, t = $\frac{2\log 2}{\log\frac{11}{10}}$