Chapter : 21. LINEAR DIFFERENTIAL EQUATIONS

Exercise : 21

Question: 1

Solution:

Given Differential Equation :

$\frac{dy}{dx} + \frac{1}{x}.y = x^2$ ....eq(1)

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$I.F. = e^{\int P\, dx}$

Answer :

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and $Q = x^2$

Therefore, integrating factor is

$I.F. = e^{\int P\, dx}$

$= e^{\int \frac{1}{x} dx}$

$= e^{\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= x$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(x) = \int x^2.(x)dx + c$$

$\therefore xy = \int x^3 dx + c$

$\therefore xy = \frac{x^4}{4} + c \,\ldots\ldots\ldots \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$

$\therefore y = \frac{x^3}{4} + \frac{c}{x}$

Question: 2

Solution:

Given Differential Equation :

$$x\frac{dy}{dx} + 2y = x^2$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} + 2y = x^2$$

Dividing the above equation by x,

$\frac{dy}{dx} + \frac{2}{x}.y = x \,\ldots\ldots\ldots eq(1)$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2}{x}$ and $Q = x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$= e^{\int \frac{2}{x} dx}$

$= e^{2\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log x^2}$ ....$(\because a\log b = \log b^a)$

$= x^2$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(x^2) = \int x.(x^2)dx + c$$

$$\therefore x^2y = \int x^3 dx + c$$

$\therefore x^2y = \frac{x^4}{4} + c$ ....$\left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$

$$\therefore y = \frac{x^2}{4} + \frac{c}{x^2}$$

Question: 3

Solution:

Given Differential Equation :

$$2x\frac{dy}{dx} + y = 6x^3$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a\log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$I.F. = e^{\int p\, dx}$

Answer :

Given differential equation is

$$2x\frac{dy}{dx} + y = 6x^3$$

Dividing the above equation by 2x,

$\frac{dy}{dx} + \frac{1}{2x}.y = 3x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{2x}$ and $Q = 3x^2$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{2x}dx}$$

$$= e^{\frac{1}{2}\log x} \ldots\ldots\ldots \left(\because \int \frac{1}{x}dx = \log x\right)$$

$$= e^{\log\sqrt{x}} \ldots\ldots\ldots (\because a\log b = \log b^a)$$

$$= \sqrt{x} \ldots\ldots\ldots \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\sqrt{x}\right) = \int 3x^2.\left(\sqrt{x}\right)dx + c$$

$$\therefore \sqrt{x}.y = \int 3x^{5/2}dx + c$$

$$\therefore \sqrt{x}.y = 3\frac{x^{7/2}}{7/2} + c \ldots\ldots\ldots \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

Dividing the above equation by √x

$$\therefore y = \frac{6}{7}x^3 + \frac{c}{\sqrt{x}}$$

$$\therefore y = \frac{6}{7}x^3 + \frac{c}{\sqrt{x}}$$

Question: 4

Solution:

Given Differential Equation :

$$x\frac{dy}{dx} + y = 3x^2 - 2$$

Formula :

i) $\int \frac{1}{x}dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$x\frac{dy}{dx} + y = 3x^2 - 2$$

Dividing the above equation by x,

$$\frac{dy}{dx} + \frac{1}{x}.y = \frac{3x^2-2}{x} \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and $Q = \frac{3x^2-2}{x}$

Therefore, the integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{x}dx}$$

$$= e^{\log x} \text{ ....} \left(\because \int \frac{1}{x}dx = \log x\right)$$

$$= x\text{.........}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(x) = \int \left(\frac{3x^2-2}{x}\right).(x)dx + c$$

$$\therefore xy = \int (3x^2 - 2)dx + c$$

$$\therefore xy = 3\frac{x^3}{3} - 2x + c \text{ ....} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

Dividing the above equation by x

$$\therefore y = x^2 - 2 + \frac{c}{x}$$

$$\therefore y = x^2 - 2 + \frac{c}{x}$$

Question: 5

Solution:

Given Differential Equation :

$$x\frac{dy}{dx} - y = 2x^3$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = 2x^3$$

Dividing the above equation by x,

$$\frac{dy}{dx} - \frac{1}{x}.y = 2x^2 \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = 2x^2$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int \frac{-1}{x} dx}$$

$$= e^{-\log x} \text{ ....} \left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= e^{\log \frac{1}{x}} \text{ ....} (\because a \log b = \log b^a)$$

$= \frac{1}{x}$ ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int 2x^2.\left(\frac{1}{x}\right)dx + c$$

$$\therefore \frac{y}{x} = \int 2xdx + c$$

$$\therefore \frac{y}{x} = 2\frac{x^2}{2} + c \text{ ....} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

Multiplying above equation by x

$$\therefore y = x^3 + cx$$

$$\therefore y = x^3 + cx$$

Question: 6

Solution:

Given Differential Equation :

$$x\frac{dy}{dx} - y = x + 1$$

Formula :

i) $\int \frac{1}{x}dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = x + 1$$

Dividing above equation by x,

$\frac{dy}{dx} - \frac{1}{x}.y = \frac{x+1}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{x+1}{x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}dx}$$

$$= e^{-\log x} \text{ ....} \left(\because \int \frac{1}{x}dx = \log x\right)$$

$$= e^{\log \frac{1}{x}} \text{ ....} (\because a\log b = \log b^a)$$

$$= \frac{1}{x} \text{ ....} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int \left(\frac{x+1}{x}\right).\left(\frac{1}{x}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{x+1}{x^2}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{1}{x} + \frac{1}{x^2}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{1}{x} + x^{-2}\right)dx + c$$

$$\therefore \frac{y}{x} = \log x + \frac{x^{-1}}{-1} + c \text{ ....} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} \ \& \int \frac{1}{x}dx = \log x\right)$$

$$\therefore \frac{y}{x} = \log x - \frac{1}{x} + c$$

Multiplying above equation by x,

$$\therefore y = x\log x - 1 + cx$$

$$\therefore y = x\log x - 1 + cx$$

Question: 7

Solution:

Given Differential Equation :

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{(1+x^2)}$$

Formula :

i) $\int \frac{f'(x)}{f(x)}dx = \log f(x)$

ii) $\int \frac{1}{(1+x^2)} dx = \tan^{-1}x$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{(1+x^2)}$$

Dividing above equation by $(1+x^2)$,

$$\frac{dy}{dx} + \frac{2x}{(1+x^2)}.y = \frac{1}{(1+x^2)^2} \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2x}{(1+x^2)}$ and $Q = \frac{1}{(1+x^2)^2}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{2x}{(1+x^2)} dx}$$

Let, $f(x) = (1+x^2)$ & $f'(x) = 2x$

$$= e^{\log(1+x^2)} \text{ ....} \left(\because \int \frac{f'(x)}{f(x)} dx = \log f(x)\right)$$

$$= (1+x^2) \text{ ....} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(1+x^2) = \int \frac{1}{(1+x^2)^2}.(1+x^2)dx + c$$

$$\therefore y.(1+x^2) = \int \frac{1}{(1+x^2)} dx + c$$

$$\therefore y.(1+x^2) = \tan^{-1}x + c \text{ ....} \left(\because \int \frac{1}{(1+x^2)} dx = \tan^{-1}x\right)$$

Therefore, general solution is

$$y.(1+x^2) = tan^{-1}x + c$$

Question: 8

Solution:

Given Differential Equation :

$$(1-x^2)\frac{dy}{dx} + xy = x\sqrt{1-x^2}$$

Formula :

i) $\int \frac{f'(x)}{f(x)}dx = \log f(x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(1-x^2)\frac{dy}{dx} + xy = x\sqrt{1-x^2}$$

Dividing above equation by $(1-x^2)$,

$$\frac{dy}{dx} + \frac{x}{(1-x^2)}.y = \frac{x\sqrt{1-x^2}}{(1-x^2)}$$

$$\frac{dy}{dx} + \frac{x}{(1-x^2)}.y = \frac{x}{\sqrt{1-x^2}} \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{x}{(1-x^2)}$ and $Q = \frac{x}{\sqrt{1-x^2}}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{x}{(1-x^2)}dx}$$

$= e^{\frac{-1}{2}\int\frac{-2x}{(1-x^2)}dx}$

Let $(1 - x^2) = f(x)$

Therefore $f'(x) = -2x$

$\therefore \int \frac{f'(x)}{f(x)}dx = \int \frac{-2x}{(1-x^2)}dx = \log f(x) = \log(1-x^2)$ ……eq(2)

$\therefore I.F. = e^{\frac{-1}{2}\log(1-x^2)}$

$= e^{\log(1-x^2)^{-1/2}}$ ………$(\because a\log b = \log b^a)$

$= e^{\log\left(\frac{1}{\sqrt{1-x^2}}\right)}$

$= \frac{1}{\sqrt{1-x^2}}$ ………$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{\sqrt{1-x^2}}\right) = \int \left(\frac{x}{\sqrt{1-x^2}}\right).\left(\frac{1}{\sqrt{1-x^2}}\right)dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \int \frac{x}{(1-x^2)}dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{-1}{2}\int \frac{-2x}{(1-x^2)}dx + c$$

$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{-1}{2}\log(1-x^2) + c$ ………from eq(2)

Multiplying above equation by $\sqrt{1-x^2}$,

$$\therefore y = \frac{-1}{2}\sqrt{1-x^2}\log(1-x^2) + c\sqrt{1-x^2}$$

Question: 9

Solution:

<u>Given Differential Equation</u> :

$$(1-x^2)\frac{dy}{dx} + xy = ax$$

<u>Formula</u> :

i) $\int \frac{f'(x)}{f(x)}dx = \log f(x)$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(1 - x^2)\frac{dy}{dx} + xy = ax$$

Dividing above equation by $(1 - x^2)$,

$$\frac{dy}{dx} + \frac{x}{(1-x^2)}.y = \frac{ax}{(1-x^2)} \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{x}{(1-x^2)}$ and $Q = \frac{ax}{(1-x^2)}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{x}{(1-x^2)}dx}$$

$$= e^{\frac{-1}{2}\int \frac{-2x}{(1-x^2)}dx}$$

Let $(1 - x^2) = f(x)$

Therefore $f'(x) = -2x$

$$\therefore \int \frac{f'(x)}{f(x)}dx = \int \frac{-2x}{(1-x^2)}dx = \log f(x) = \log(1 - x^2)$$

$$\therefore I.F. = e^{\frac{-1}{2}\log(1-x^2)}$$

$$= e^{\log(1-x^2)^{-1/2}} \text{ ....}(\because a\log b = \log b^a)$$

$$= e^{\log\left(\frac{1}{\sqrt{1-x^2}}\right)}$$

$$= \frac{1}{\sqrt{1-x^2}} \text{ ....}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{\sqrt{1-x^2}}\right) = \int \left(\frac{ax}{(1-x^2)}\right).\left(\frac{1}{\sqrt{1-x^2}}\right)dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \int \frac{ax}{(1-x^2)^{3/2}}dx + c \text{ ......eq(2)}$$

Let

$I = \int \frac{ax}{(1-x^2)^{3/2}} dx$

Put $(1 - x^2) = t$

$\therefore -2x\,dx = dt$

$\therefore x\,dx = \frac{-dt}{2}$

$\therefore I = \int \frac{a}{t^{3/2}} . \frac{-dt}{2}$

$\therefore I = \frac{-a}{2} \int t^{-3/2} dt$

$\therefore I = \frac{-a}{2} . \frac{t^{-1/2}}{-1/2}$

$\therefore I = a . \frac{1}{\sqrt{t}}$

$\therefore I = \frac{a}{\sqrt{1-x^2}}$

Substituting I in eq(2)

$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{a}{\sqrt{1-x^2}} + c$

Multiplying above equation by $\sqrt{1-x^2}$,

$\therefore y = a + c\sqrt{1-x^2}$

Question: 10

Solution:

Given Differential Equation :

$(x^2 + 1)\frac{dy}{dx} - 2xy = (x^2 + 1)(x^2 + 2)$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log f(x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int 1 dx = x$

v) $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

vi) General solution :

For the differential equation in the form of

$\frac{dy}{dx} + Py = Q$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(x^2+1)\frac{dy}{dx} - 2xy = (x^2+1)(x^2+2)$$

Dividing above equation by $(1 + x^2)$,

$$\frac{dy}{dx} + \frac{-2x}{(1+x^2)}.y = (x^2+2) \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-2x}{(1+x^2)}$ and $Q = (x^2+2)$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-2x}{(1+x^2)}dx}$$

$$= e^{-\int \frac{2x}{(1+x^2)}dx}$$

Let $(1 + x^2) = f(x)$

Therefore $f'(x) = 2x$

$$\therefore \int \frac{f'(x)}{f(x)}dx = \int \frac{2x}{(1+x^2)}dx = \log f(x) = \log(1+x^2)$$

$$\therefore I.F. = e^{-\log(1+x^2)}$$

$$= e^{\log(1+x^2)^{-1}} \text{ ....}(\because a\log b = \log b^a)$$

$$= e^{\log\left(\frac{1}{(1+x^2)}\right)}$$

$$= \frac{1}{(1+x^2)} \text{ ....}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{(1+x^2)}\right) = \int (2+x^2).\left(\frac{1}{(1+x^2)}\right)dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \frac{2+x^2}{1+x^2}dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \frac{1+x^2+1}{1+x^2}dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int\left(\frac{1+x^2}{1+x^2} + \frac{1}{1+x^2}\right)dx \ + c$$

$$\therefore \frac{y}{(1+x^2)} = \int\left(1 + \frac{1}{1+x^2}\right)dx + c$$

$$\therefore \frac{y}{(1+x^2)} = x + \tan^{-1}x + c$$

$$\ldots\ldots\ldots\left(\because \int 1dx = x \ \& \int \frac{1}{1+x^2}dx = \tan^{-1}x\right)$$

$$\therefore y = (1+x^2)(x + \tan^{-1}x + c)$$

Therefore general solution is

$$y = (1+x^2)(x + tan^{-1}x + c)$$

Question: 11

Solution:

Given Differential Equation :

$$\frac{dy}{dx} + 2y = 6e^x$$

Formula :

i) $\int 1dx = x$

ii) $\int e^{kx}dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y = 6e^x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2$ and $Q = 6e^x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$= e^{\int 2dx}$

$= e^{2\int 1dx}$

$= e^{2x}$ ....$(\because \int 1dx = x)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(e^{2x}) = \int (6e^{x}).(e^{2x})dx + c$$

$$\therefore y.(e^{2x}) = 6\int e^{3x}dx + c$$

$$\therefore y.(e^{2x}) = 6\frac{e^{3x}}{3} + c \text{ ....} \left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$$

$$\therefore y.(e^{2x}) = 2e^{3x} + c$$

Dividing above equation by (e^{2x}),

$$\therefore y = \frac{2e^{3x}}{e^{2x}} + \frac{c}{e^{2x}}$$

$$\therefore y = 2e^{(3x-2x)} + ce^{-2x}$$

$$\therefore y = 2e^{x} + ce^{-2x}$$

Therefore general solution is

$$y = 2e^{x} + ce^{-2x}$$

Question: 12

Solution:

Given Differential Equation :

$$\frac{dy}{dx} + 3y = e^{-2x}$$

Formula :

i) $\int 1dx = x$

ii) $\int e^{kx}dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 3y = e^{-2x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 3$ and $Q = e^{-2x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int 3dx}$$

$$= e^{3\int 1dx}$$

$$= e^{3x} \text{ ....} (\because \int 1dx = x)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(e^{3x}) = \int (e^{-2x}).(e^{3x})dx + c$$

$$\therefore y.(e^{3x}) = \int e^{x}dx + c$$

$$\therefore y.(e^{3x}) = e^{x} + c \text{ ....} \left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$$

Dividing above equation by (e^{3x}),

$$\therefore y = \frac{e^{x}}{e^{3x}} + \frac{c}{e^{3x}}$$

$$\therefore y = e^{(x-3x)} + ce^{-3x}$$

$$\therefore y = e^{-2x} + ce^{-3x}$$

Therefore general solution is

$$y = e^{-2x} + ce^{-3x}$$

Question: 13

Solution:

Given Differential Equation :

$$\frac{dy}{dx} + 8y = 5e^{-3x}$$

Formula :

i) $\int 1dx = x$

ii) $\int e^{kx}dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$\frac{dy}{dx} + 8y = 5e^{-3x} \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 8$ and $Q = 5e^{-3x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 8dx}$$

$$= e^{8\int 1dx}$$

$$= e^{8x} \text{ ....} \left(\because \int 1dx = x\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(e^{8x}) = \int (5e^{-3x}).(e^{8x})dx + c$$

$$\therefore y.(e^{8x}) = 5\int e^{5x}dx + c$$

$$\therefore y.(e^{8x}) = 5\frac{e^{5x}}{5} + c \text{ ....} \left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$$

$$\therefore y.(e^{8x}) = e^{5x} + c$$

Dividing above equation by (e^{8x}),

$$\therefore y = \frac{e^{5x}}{e^{8x}} + \frac{c}{e^{8x}}$$

$$\therefore y = e^{(5x-8x)} + ce^{-8x}$$

$$\therefore y = e^{-3x} + ce^{-8x}$$

Therefore general solution is

$$y = e^{-3x} + ce^{-8x}$$

Question: 14

Find the general

Solution:

Given Differential Equation :

$$x\frac{dy}{dx} - y = (x-1)e^x$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int e^x (f(x) + f'(x)) dx = e^x . f(x)$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = (x-1)e^x$$

Dividing above equation by x,

$\frac{dy}{dx} - \frac{1}{x}y = \frac{(x-1)}{x} e^x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{(x-1)}{x} e^x$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int \frac{-1}{x} dx}$$

$= e^{-\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log x^{-1}}$ ....$(\because a \log b = \log b^a)$

$= \frac{1}{x}$.........$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int \left(\frac{(x-1)}{x}e^x\right).\left(\frac{1}{x}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{x-1}{x^2}e^x\right)dx + c \text{ ....eq(2)}$$

Let,

$$I = \int \left(\frac{x-1}{x^2}e^x\right)dx$$

$$\therefore I = \int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$$

Let $f(x) = \frac{1}{x}$ $\therefore f'(x) = \frac{-1}{x^2}$

$$\therefore I = e^x.\frac{1}{x} \text{ ....}\left(\because \int e^x\big(f(x) + f'(x)\big)dx = e^x.f(x)\right)$$

Substituting I in eq(2),

$$\therefore \frac{y}{x} = e^x.\frac{1}{x} + c$$

Multiplying above equation by x,

$$\therefore y = e^x + cx$$

Therefore general solution is

$$y = e^x + cx$$

Question: 15

Solution:

Given Differential Equation :

$$\frac{dy}{dx} - y\tan x = e^x \sec x$$

Formula :

i) $\int \tan x\, dx = \log(\sec x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int e^x dx = e^x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\text{I.F.} = e^{\int P\,dx}$

Answer :

Given differential equation is

$\frac{dy}{dx} - y\tan x = e^x \sec x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -\tan x$ and $Q = e^x \sec x$

Therefore, integrating factor is

$\text{I.F.} = e^{\int P\,dx}$

$= e^{\int -\tan x\,dx}$

$= e^{-\log(\sec x)}$ ....$(\because \int \tan x\,dx = \log(\sec x))$

$= e^{\log(\sec x)^{-1}}$ ....$(\because a\log b = \log b^a)$

$= e^{\log(\cos x)}$

$= \cos x$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(\text{I.F.}) = \int Q.(\text{I.F.})dx + c$$

$$\therefore y.(\cos x) = \int (e^x \sec x).(\cos x)dx + c$$

$$\therefore y.(\cos x) = \int \left(e^x.\frac{1}{\cos x}\right).(\cos x)dx + c$$

$$\therefore y.(\cos x) = \int e^x dx + c$$

$\therefore y.(\cos x) = e^x + c$ ....$(\because \int e^x dx = e^x)$

Therefore general solution is

$y.(\cos x) = e^x + c$

Question: 16

Solution:

Given Differential Equation :

$$(x\log x)\frac{dy}{dx} + y = 2\log x$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log(f(x))$

ii) $a^{\log_a b} = b$

iii) $\int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx$

iv) $\frac{d}{dx}(\log x) = \frac{1}{x}$

v) $\int \frac{1}{x}dx = \log x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(x\log x)\frac{dy}{dx} + y = 2\log x$$

Dividing above equation by (x.log x),

$\frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x\log x}$ and $Q = \frac{2}{x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{x\log x}dx}$$

$$= e^{\int \frac{1/x}{\log x}dx}$$

Let, $f(x) = \log x \therefore f'(x) = 1/x$

$\therefore I.F. = e^{\log(\log x)}$ ....$\left(\because \int \frac{f'(x)}{f(x)}dx = \log(f(x))\right)$

$= \log x$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\log x) = \int \left(\frac{2}{x}\log x\right)dx + c$$

$\therefore y.(\log x) = 2\int\left(\frac{1}{x}\log x\right)dx + c$ ....eq(2)

Let,

$$I = \int \frac{1}{x}.\log x\,dx$$

Let, $u = \log x$ & $v = \frac{1}{x}$

$$\therefore I = \log x \int \frac{1}{x}dx - \int \left(\frac{d}{dx}(\log x).\int \frac{1}{x}dx\right)dx$$

$$.........\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$\therefore I = \log x.\log x - \int \left(\frac{1}{x}.\log x\right)d$$

$$.........\left(\because \frac{d}{dx}(\log x) = \frac{1}{x}\ \&\ \int \frac{1}{x}dx = \log x\right)$$

$$\therefore I = (\log x)^2 - I$$

$$\therefore 2I = (\log x)^2$$

$$\therefore I = \frac{1}{2}(\log x)^2$$

Substituting I in eq(2),

$$\therefore y.(\log x) = 2.\frac{1}{2}(\log x)^2 + c$$

$$y.(\log x) = (\log x)^2 + c$$

Question: 17

Solution:

Given Differential Equation :

$$x\frac{dy}{dx} + y = x\log x$$

Formula :

i) $\int \frac{1}{x}dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx$

iv) $\frac{d}{dx}(\log x) = \frac{1}{x}$

v) $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$I.F. = e^{\int P\,dx}$

<u>Answer</u> :

Given differential equation is

$$x\frac{dy}{dx} + y = x\log x$$

Dividing above equation by x,

$\frac{dy}{dx} + \frac{1}{x}y = \log x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and $Q = \log x$

Therefore, integrating factor is

$I.F. = e^{\int P\,dx}$

$= e^{\int \frac{1}{x} dx}$

$= e^{\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= x$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.(x) = \int (x\log x)dx + c$ ....eq(2)

Let,

$$I = \int (x\log x)dx$$

Let, $u = \log x$ & $v = x$

$$\therefore I = \log x \int x\,dx - \int \left(\frac{d}{dx}(\log x).\int x\,dx\right)dx$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$$\therefore I = \log x.\frac{x^2}{2} - \int \left(\frac{1}{x}.\frac{x^2}{2}\right)dx$$

.........$\left(\because \frac{d}{dx}(\log x) = \frac{1}{x}\ \&\ \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

$$\therefore I = \log x.\frac{x^2}{2} - \frac{1}{2}\int (x)\,dx$$

$\therefore I = \log x.\frac{x^2}{2} - \frac{1}{2}\left(\frac{x^2}{2}\right)$ ....$\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

$$\therefore I = \frac{x^2}{2}.\log x - \frac{x^2}{4}$$

Substituting I in eq(2),

$$\therefore xy = \frac{x^2}{2}.\log x - \frac{x^2}{4} + c$$

Multiplying above equation by 4,

$$\therefore 4xy = 2x^2.\log x - x^2 + 4c$$

Therefore general equation is

$$4xy = 2x^2.\log x - x^2 + 4c$$

Question: 18

Solution:

Given Differential Equation :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

v) $\frac{d}{dx}(\log x) = \frac{1}{x}$

vi) $\int x^n\, dx = \frac{x^{n+1}}{n+1}$

vii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Dividing above equation by x,

$$\frac{dy}{dx} + \frac{2}{x}y = x \log x \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2}{x}$ and $Q = x \log x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{2}{x} dx}$$

$$= e^{2\int \frac{1}{x} dx}$$

$$= e^{2 \log x} \text{.........}\left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= e^{\log x^2} \text{.........} = x^2 \left(\because a \log b = \log b^a\right)$$

$\text{.........}\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(x^2) = \int (x^2.x \log x)dx + c$$

$$\therefore y.(x^2) = \int (x^3 \log x)dx + c \text{ ....eq(2)}$$

Let,

$$I = \int (x^3 \log x)dx$$

Let, $u = \log x$ & $v = x^3$

$$\therefore I = \log x \int x^3\,dx - \int \left(\frac{d}{dx}(\log x).\int x^3\,dx\right)dx$$

$$\text{.........}\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$\therefore I = \log x.\frac{x^4}{4} - \int \left(\frac{1}{x}.\frac{x^4}{4}\right)dx$$

$$\text{.........}\left(\because \frac{d}{dx}(\log x) = \frac{1}{x} \ \& \ \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore I = \log x.\frac{x^4}{4} - \frac{1}{4}\int (x^3)dx$$

$$\therefore I = \log x.\frac{x^4}{4} - \frac{1}{4}\left(\frac{x^4}{4}\right) \text{.........}\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore I = \frac{x^4}{4}.\log x - \frac{x^4}{16}$$

Substituting I in eq(2),

$$\therefore x^2 y = \frac{x^4}{4}.\log x - \frac{x^4}{16} + c$$

Dividing above equation by x^2,

$$\therefore y = \frac{x^2}{4}.\log x - \frac{x^2}{16} + \frac{c}{x^2}$$

$$\therefore y = \frac{x^2}{16}(4\log x - 1) + \frac{c}{x^2}$$

Therefore general equation is

$$y = \frac{x^2}{16}(4\log x - 1) + \frac{c}{x^2}$$

Question: 19

Solution:

Given Differential Equation :

$$(1+x)\frac{dy}{dx} - y = e^{3x}(1+x)^2$$

Formula :

i) $\int \frac{1}{px+q} dx = \frac{1}{p}\log(px+q)$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int e^{kx} dx = \frac{1}{k} e^{kx}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(1+x)\frac{dy}{dx} - y = e^{3x}(1+x)^2$$

Dividing above equation by (1+x),

$\frac{dy}{dx} - \frac{1}{(1+x)} y = e^{3x}(1+x)$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{(1+x)}$ and $Q = e^{3x}(1+x)$

Therefore, integrating factor is

$\text{I. F.} = e^{\int P\,dx}$

$= e^{\int \frac{-1}{(1+x)}dx}$

$= e^{-\int \frac{1}{(1+x)}dx}$

$= e^{-\log(1+x)}$ ....$\left(\because \int \frac{1}{px+q}dx = \frac{1}{p}\log(px+q)\right)$

$= e^{\log\frac{1}{(1+x)}}$ ....$(\because a\log b = \log b^a)$

$= \frac{1}{(1+x)}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.\left(\frac{1}{(1+x)}\right) = \int e^{3x}(1+x)\left(\frac{1}{(1+x)}\right)dx + c$$

$$\therefore y.\left(\frac{1}{(1+x)}\right) = \int e^{3x}dx + c$$

$\therefore y.\left(\frac{1}{(1+x)}\right) = \frac{1}{3}e^{3x} + c$ ....$\left(\because \int e^{kx}dx = \frac{1}{k}e^{kx}\right)$

Multiplying above equation by (1+x),

$$\therefore y = \frac{1}{3}(1+x)e^{3x} + c(1+x)$$

Therefore general equation is

$$y = \frac{1}{3}(1+x)e^{3x} + c(1+x)$$

Question: 20

Solution:

Given Differential Equation :

$$\frac{dy}{dx} + \frac{4x}{(x^2+1)}y + \frac{1}{(1+x^2)^2} = 0$$

Formula :

i) $\int \frac{f'(x)}{f(x)}dx = \log(f(x))$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int 1\,dx = x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$\frac{dy}{dx} + \frac{4x}{(x^2+1)}y + \frac{1}{(1+x^2)^2} = 0$$

$$\therefore \frac{dy}{dx} + \frac{4x}{(x^2+1)}y = \frac{-1}{(1+x^2)^2} \text{.........eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{4x}{(x^2+1)}$ and $Q = \frac{-1}{(1+x^2)^2}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{4x}{(x^2+1)}dx}$$

$$= e^{2\int \frac{2x}{(x^2+1)}dx}$$

Let, $f(x) = (x^2+1)$ & $f'(x) = 2x$

$$\therefore I.F. = e^{2\log(x^2+1)} \text{.........} \left(\because \int \frac{f'(x)}{f(x)}dx = \log(f(x))\right)$$

$$= e^{\log(1+x^2)^2} \text{.........} (\because a\log b = \log b^a)$$

$$= (1+x^2)^2 \text{.........} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(1+x^2)^2 = \int \frac{-1}{(1+x^2)^2}(1+x^2)^2 dx + c$$

$$\therefore y.(1+x^2)^2 = \int -1dx + c$$

$$\therefore y.(1+x^2)^2 = -x + c \text{.........} \left(\because \int 1\,dx = x\right)$$

Dividing above equation by $(1+x^2)^2$,

$$\therefore y = \frac{-x}{(1+x^2)^2} + \frac{c}{(1+x^2)^2}$$

Therefore general equation is

$$y = \frac{-x}{(1+x^2)^2} + \frac{c}{(1+x^2)^2}$$

Question: 21

Solution:

Given Differential Equation :

$(y + 3x^2)\frac{dx}{dy} = x$

Formula :

i) $\int \frac{1}{x}dx = \log x$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int 1\,dx = x$

v) General solution :

For the differential equation in the form of

$\frac{dy}{dx} + Py = Q$

General solution is given by,

$y.(I.F.) = \int Q.(I.F.)dx + c$

Where, integrating factor,

$I.F. = e^{\int P\,dx}$

Answer :

Given differential equation is

$(y + 3x^2)\frac{dx}{dy} = x$

$\therefore \frac{dy}{dx} = \frac{(y + 3x^2)}{x}$

$\therefore \frac{dy}{dx} = \frac{y}{x} + 3x$

$\therefore \frac{dy}{dx} - \frac{y}{x} = 3x$ ....eq(1)

Equation (1) is of the form

$\frac{dy}{dx} + Py = Q$

Where, $P = \frac{-1}{x}$ and $Q = 3x$

Therefore, integrating factor is

$I.F. = e^{\int P\,dx}$

$= e^{\int \frac{-1}{x}dx}$

$= e^{-\log x}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= e^{\log\left(\frac{1}{x}\right)}$ ....($\because$ $a\log b = \log b^a$)

$= \frac{1}{x}$ ....($\because$ $a^{\log_a b} = b$)

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int 3x.\left(\frac{1}{x}\right)dx + c$$

$$\therefore \frac{y}{x} = \int 3dx + c$$

$$\therefore \frac{y}{x} = 3\int 1dx + c$$

$\therefore \frac{y}{x} = 3x + c$ ....($\because$ $\int 1\,dx = x$)

Multiplying above equation by x,

$$\therefore y = 3x^2 + cx$$

Therefore general equation is

$$y = 3x^2 + cx$$

Question: 22

Solution:

Given Differential Equation :

$$xdy - (y + 2x^2)dx = 0$$

Formula :

i) $\int \frac{1}{x}dx = \log x$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int 1\,dx = x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$xdy - (y + 2x^2)dx = 0$

$\therefore xdy = (y + 2x^2)dx$

$\therefore \frac{dy}{dx} = \frac{(y + 2x^2)}{x}$

$\therefore \frac{dy}{dx} = \frac{y}{x} + 2x$

$\therefore \frac{dy}{dx} - \frac{y}{x} = 2x$ ....eq(1)

Equation (1) is of the form

$\frac{dy}{dx} + Py = Q$

Where, $P = \frac{-1}{x}$ and $Q = 2x$

Therefore, integrating factor is

$I.F. = e^{\int P\,dx}$

$= e^{\int \frac{-1}{x} dx}$

$= e^{-\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log\left(\frac{1}{x}\right)}$ ....$(\because a \log b = \log b^a)$

$= \frac{1}{x}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$y.(I.F.) = \int Q.(I.F.)dx + c$

$\therefore y.\left(\frac{1}{x}\right) = \int 2x.\left(\frac{1}{x}\right) dx + c$

$\therefore \frac{y}{x} = \int 2dx + c$

$\therefore \frac{y}{x} = 2\int 1dx + c$

$\therefore \frac{y}{x} = 2x + c$ ....$(\because \int 1\,dx = x)$

Multiplying above equation by x,

$\therefore y = 2x^2 + cx$

Therefore general equation is

$y = 2x^2 + cx$

Question: 23

Solution:

Given Differential Equation :

$xdy + (y - x^3)dx = 0$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$$xdy + (y - x^3)dx = 0$$

$$\therefore xdy = -(y - x^3)dx$$

$$\therefore xdy = (x^3 - y)dx$$

$$\therefore \frac{dy}{dx} = \frac{(x^3 - y)}{x}$$

$$\therefore \frac{dy}{dx} = x^2 - \frac{y}{x}$$

$$\therefore \frac{dy}{dx} + \frac{y}{x} = x^2 \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and $Q = x^2$

Therefore, integrating factor is

$$I.F. = e^{\int P \, dx}$$

$$= e^{\int \frac{1}{x} dx}$$

$$= e^{\log x} \text{ ....} \left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= x \text{ ....} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(x) = \int x^2.(x)dx + c$$

$$\therefore xy = \int x^3 dx + c$$

$$\therefore xy = \frac{x^4}{4} + c \,\ldots\ldots\ldots \left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

Dividing above equation by x,

$$\therefore y = \frac{x^3}{4} + \frac{c}{x}$$

Therefore general equation is

$$y = \frac{x^3}{4} + \frac{c}{x}$$

Question: 24

Solution:

Given Differential Equation :

$$\frac{dy}{dx} + 2y = \sin x$$

Formula :

i) $\int 1\,dx = x$

ii) $\int u.v\,dx = u.\int v dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx$

iii) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(\sin x) = \cos x$

v) $\frac{d}{dx}(\cos x) = \sin x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y = \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=2$ and $Q=\sin x$

Therefore, integrating factor is

$$I.F.= e^{\int P\,dx}$$

$$=e^{\int 2\,dx}$$

$$=e^{2\int 1\,dx}$$

$$=e^{2x}\ \ldots\ldots\ldots(\because \int 1\,dx=x)$$

General solution is

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

$$\therefore y.(e^{2x})=\int \sin x.(e^{2x})dx+c\ \ldots\ldots\ldots eq(2)$$

Let,

$$I=\int \sin x.(e^{2x})dx$$

Let, u=sin x and v=e^{2x}

$$I=\sin x.\int e^{2x}dx-\int\left(\frac{d}{dx}(\sin x).\int e^{2x}\,dx\right)dx$$

$$\ldots\ldots\ldots\left(\because \int u.v\,dx=u.\int vdx-\int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$=\sin x.\frac{e^{2x}}{2}-\int\left(\cos x.\frac{e^{2x}}{2}\right)dx$$

$$\ldots\ldots\ldots\left(\because \int e^{kx}\,dx=\frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(\sin x)=\cos x\right)$$

$$=\sin x.\frac{e^{2x}}{2}-\frac{1}{2}\int(\cos x.e^{2x})dx$$

Again, let u=cos x and v=e^{2x}

$$\therefore I=\sin x.\frac{e^{2x}}{2}-\frac{1}{2}\left\{\cos x.\int e^{2x}dx-\int\left(\frac{d}{dx}(\cos x).\int e^{2x}\,dx\right)dx\right\}$$

$$\ldots\ldots\ldots\left(\because \int u.v\,dx=u.\int vdx-\int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$\therefore I=\sin x.\frac{e^{2x}}{2}-\frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2}-\int\left((-\sin x).\frac{e^{2x}}{2}\right)dx\right\}$$

$$\ldots\ldots\ldots\left(\because \int e^{kx}\,dx=\frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(\cos x)=\sin x\right)$$

$$\therefore I=\sin x.\frac{e^{2x}}{2}-\frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2}+\int\left(\sin x.\frac{e^{2x}}{2}\right)dx\right\}$$

$$\therefore I=\sin x.\frac{e^{2x}}{2}-\frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2}+\frac{1}{2}\int(\sin x.e^{2x})dx\right\}$$

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2} + \frac{I}{2}\right\}$$

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \cos x.\frac{e^{2x}}{4} - \frac{I}{4}$$

$$\therefore I + \frac{I}{4} = \sin x.\frac{e^{2x}}{2} - \cos x.\frac{e^{2x}}{4}$$

$$\therefore \frac{5I}{4} = \sin x.\frac{e^{2x}}{2} - \cos x.\frac{e^{2x}}{4}$$

Multiplying above equation by 4,

$$\therefore 5I = 2\sin x.e^{2x} - \cos x.e^{2x}$$

$$\therefore 5I = e^{2x}(2\sin x - \cos x)$$

$$\therefore I = \frac{e^{2x}}{5}(2\sin x - \cos x)$$

Substituting I in eq(2),

$$\therefore y.(e^{2x}) = \frac{e^{2x}}{10}(2\sin x - \cos x) + c$$

Dividing above equation by e^{2x},

$$\therefore y = \frac{1}{5}(2\sin x - \cos x) + ce^{-2x}$$

Therefore general equation is

$$y = \frac{1}{5}(2\sin x - \cos x) + ce^{-2x}$$

Question: 25

Solution:

Given Differential Equation :

$$\frac{dy}{dx} + y = \cos x - \sin x$$

Formula :

i) $\int 1\,dx = x$

ii) $\int e^x.\big(f(x) + f'(x)\big)dx = e^x.f(x)$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

CLASS24

Answer :

Given differential equation is

$\frac{dy}{dx} + y = \cos x - \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 1$ and $Q = \cos x - \sin x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 1\,dx}$$

$$= e^{x} \text{ ....} (\because \int 1\,dx = x)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(e^{x}) = \int (\cos x - \sin x).(e^{x})dx + c$$

Let, $f(x)=\cos x \Rightarrow f'(x) = -\sin x$

$$\therefore y.(e^{x}) = (e^{x}).\cos x + c$$

$$\text{.........}\left(\because \int e^{x}.\left(f(x) + f'(x)\right)dx = e^{x}.f(x)\right)$$

Dividing above equation by e^{x},

$$\therefore y = \cos x + \frac{c}{e^{x}}$$

Therefore general equation is

$$y = \cos x + ce^{-x}$$

Question: 26

Solution:

Given Differential Equation :

$$\sec x \frac{dy}{dx} - y = \sin x$$

Formula :

i) $\int \cos x\,dx = \sin x$

ii) $\int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx$

iii) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(kx) = k$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$\sec x \frac{dy}{dx} - y = \sin x$$

Dividing above equation by sec x,

$$\frac{dy}{dx} - \frac{1}{\sec x} y = \frac{\sin x}{\sec x}$$

$\therefore \frac{dy}{dx} - \cos x.y = \sin x.\cos x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -\cos x$ and $Q = \sin x.\cos x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int -\cos x\,dx}$$

$= e^{-\sin x}$ ....$(\because \int \cos x\,dx = \sin x)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.(e^{-\sin x}) = \int(\sin x.\cos x).(e^{-\sin x})dx + c$ ....eq(2)

Let,

$$I = \int(\sin x.\cos x).(e^{-\sin x})dx$$

Put sin x=t => cos x.dx=dt

$$\therefore I = \int e^{-t}.t\,dt$$

$$\therefore I = t.\int e^{-t}\,dt - \int\left(\frac{d}{dt}(t).\int e^{-t}\,dt\right)dt$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$$\therefore I = -t.e^{-t} - \int\left((1).(-e^{-t})\right)dt$$

$\text{.........}\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k} \ \&\ \frac{d}{dx}(kx) = k\right)$

$\therefore I = -t.e^{-t} + (-e^{-t})$ $\text{.........}\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$

$\therefore I = -\sin x \,.\, e^{-\sin x} - e^{-\sin x}$

Substituting I in eq(2),

$\therefore y.\left(e^{-\sin x}\right) = -\sin x\,.\,e^{-\sin x} - e^{-\sin x} + c$

$\therefore y.\left(e^{-\sin x}\right) = -e^{-\sin x}(\sin x + 1) + c$

$\therefore y.\left(e^{-\sin x}\right) = c - e^{-\sin x}(\sin x + 1)$

Dividing above equation by $e^{-\sin x}$,

$\therefore y = \frac{c}{e^{-\sin x}} - (\sin x + 1)$

Therefore general equation is

$y = ce^{-\sin x} - (\sin x + 1)$

Question: 27

Given Differential Equation :

$(1 + x^2)\frac{dy}{dx} + 2xy = \cot x$

Formula :

i) $\int \frac{f'(x)}{f(x)}dx = \log(f(x))$

ii) $a^{\log_a b} = b$

iii) $\int \cot x\, dx = \log|\sin x|$

iv) General solution :

For the differential equation in the form of

$\frac{dy}{dx} + Py = Q$

General solution is given by,

$y.(I.F.) = \int Q.(I.F.)dx + c$

Where, integrating factor,

$I.F. = e^{\int P\,dx}$

Answer :

Given differential equation is

$(1 + x^2)\frac{dy}{dx} + 2xy = \cot x$

Dividing above equation by $(1+x^2)$,

$\therefore \frac{dy}{dx} + \frac{2x}{(1+x^2)}y = \frac{\cot x}{(1+x^2)}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2x}{(1+x^2)}$ and $Q = \frac{\cot x}{(1+x^2)}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{2x}{(1+x^2)} dx}$$

Let, f(x) = (1+x²) => f ' (x) = 2x

$$= e^{\log(1+x^2)} \ldots\ldots\ldots \left(\because \int \frac{f'(x)}{f(x)} dx = \log(f(x))\right)$$

$$= (1 + x^2)\ldots\ldots\ldots \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(1 + x^2) = \int \frac{\cot x}{(1 + x^2)}.(1 + x^2)dx + c$$

$$\therefore y.(1 + x^2) = \int \cot x\,dx + c$$

$$\therefore y.(1 + x^2) = \log|\sin x| + c \ldots\ldots\ldots \left(\because \int \cot x\,dx = \log|\sin x|\right)$$

Therefore, general solution is

$$y.(1 + x^2) = \log|\sin x| + c$$

Question: 28

Solution:

Given Differential Equation :

$$\sin x \frac{dy}{dx} + (\cos x)y = \cos x.\sin^2 x$$

Formula :

v) $\int \cot x\,dx = \log(\sin x)$

vi) $a^{\log_a b} = b$

vii) $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

viii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\text{I. F.} = e^{\int P\,dx}$

Answer :

Given differential equation is

$$\sin x \frac{dy}{dx} + (\cos x)y = \cos x . \sin^2 x$$

Dividing above equation by sin x,

$$\therefore \frac{dy}{dx} - \frac{\cos x}{\sin x} y = \frac{\cos x . \sin^2 x}{\sin x}$$

$\therefore \frac{dy}{dx} + (\cot x)y = \cos x . \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = \sin x . \cos x$

Therefore, integrating factor is

$\text{I. F.} = e^{\int P\,dx}$

$= e^{\int \cot x\,dx}$

$= e^{\log(\sin x)}$ ....$(\because \int \cot x\,dx = \log(\sin x))$

$= \sin x$ ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.(\sin x) = \int (\sin x . \cos x).(\sin x)dx + c$$

$\therefore y.(\sin x) = \int (\sin^2 x . \cos x)dx + c$ ....eq(2)

Let,

$$I = \int (\sin^2 x . \cos x)dx$$

Put sin x=t => cos x.dx=dt

$$\therefore I = \int t^2 dt$$

$\therefore I = \frac{t^3}{3}$ ....$\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

$$\therefore I = \frac{\sin^3 x}{3}$$

Substituting I in eq(2),

$$\therefore y.(\sin x) = \frac{\sin^3 x}{3} + c$$

Therefore, general solution is

$y.(\sin x) = \frac{sin^3 x}{3} + c$

Question: 29

Solution:

Given Differential Equation :

$\frac{dy}{dx} + 2y(\cot x) = 3x^2 cosec^2 x$

Formula :

i) $\int \cot x \, dx = \log(\sin x)$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

v) General solution :

For the differential equation in the form of

$\frac{dy}{dx} + Py = Q$

General solution is given by,

$y.(I.F.) = \int Q.(I.F.)dx + c$

Where, integrating factor,

$I.F. = e^{\int P \, dx}$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y(\cot x) = 3x^2 cosec^2 x$ ....eq(1)

Equation (1) is of the form

$\frac{dy}{dx} + Py = Q$

Where, $P = 2\cot x$ and $Q = 3x^2 cosec^2 x$

Therefore, integrating factor is

$I.F. = e^{\int P \, dx}$

$= e^{\int 2\cot x \, dx}$

$= e^{2\log(\sin x)}$ ....$(\because \int \cot x \, dx = \log(\sin x))$

$= e^{\log(\sin x)^2}$ ....$(\because a\log b = \log b^a)$

$= \sin^2 x$ ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin^2 x) = \int (3x^2 \mathrm{cosec}^2 x).(\sin^2 x)dx + c$$

$$\therefore y.(\sin^2 x) = \int \left(3x^2 \frac{1}{\sin^2 x}\right).(\sin^2 x)dx + c$$

$$\therefore y.(\sin^2 x) = 3\int (x^2)dx + c$$

$$\therefore y.(\sin^2 x) = 3\frac{x^3}{3} + c \text{ ....} \left(\because \int x^n\, dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore y.(\sin^2 x) = x^3 + c$$

Therefore, general solution is

$$y.(\sin^2 x) = x^3 + c$$

Question: 30

Solution:

Given Differential Equation :

$$x\frac{dy}{dx} - y = 2x^2 \sec x$$

Formula :

vi) $\int \cot x\, dx = \log(\sin x)$

vii) $a \log b = \log b^a$

viii) $a^{\log_a b} = b$

ix) $\int x^n\, dx = \frac{x^{n+1}}{n+1}$

x) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = 2x^2 \sec x \text{ ....eq(1)}$$

Dividing above equation by x,

$$\therefore \frac{dy}{dx} - \frac{1}{x}y = 2x \sec x$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = 2x\sec x$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int \frac{-1}{x} dx}$$

$$= e^{-\log x} \text{ ....}\left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= e^{\log x^{-1}} \text{ ....}(\because a\log b = \log b^a)$$

$$= \frac{1}{x} \text{ ....}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int (2x\sec x).\left(\frac{1}{x}\right) dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = 2\int \sec x\, dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = 2\log|\sec x + \tan x| + c$$

$$\text{.........}\left(\because \int \sec x\, dx = \log|\sec x + \tan x|\right)$$

Multiplying above equation by x,

$$\therefore y = 2x\log|\sec x + \tan x| + cx$$

Therefore, general solution is

$$y = 2x\log|\sec x + \tan x| + cx$$

Question: 31

Solution:

Given Differential Equation :

$$\frac{dy}{dx} = y\tan x - 2\sin x$$

Formula :

i) $\int \tan x\, dx = \log|\sec x|$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $2\sin x.\cos x = \sin 2x$

v) $\int \sin x\, dx = -\cos x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$\frac{dy}{dx} = y\tan x - 2\sin x$$

$\frac{dy}{dx} - y\tan x = -2\sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -\tan x$ and $Q = -2\sin x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int -\tan x\,dx}$$

$= e^{-\log|\sec x|}$ ....$(\because \int \tan x\,dx = \log|\sec x|)$

$= e^{\log|\sec x|^{-1}}$ ....$(\because a\log b = \log b^a)$

$$= e^{\log\left(\frac{1}{\sec x}\right)}$$

$$= e^{\log(\cos x)}$$

$= \cos x$ .... $(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\cos x) = \int (-2\sin x).(\cos x)dx + c$$

$$\therefore y.(\cos x) = -\int (2\sin x).(\cos x)dx + c$$

$\therefore y.(\cos x) = -\int (\sin 2x)\,dx + c$ .... $(\because 2\sin x.\cos x = \sin 2x)$

$\therefore y.(\cos x) = \frac{\cos 2x}{2} + c$......... $(\because 2\int \sin x\,dx = -\cos x)$

Multiplying above equation by 2

$$\therefore 2y.(\cos x) = \cos 2x + 2c$$

$\therefore 2y.(\cos x) = \cos 2x + C$ where, C=2c

Therefore, general solution is

$2y.(\cos x) = \cos 2x + C$

Question: 32

Solution:

Given Differential Equation :

$$\frac{dy}{dx} + y\cot x = \sin 2x$$

Formula :

i) $\int \cot x\, dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v\, dx = u.\int v\, dx - \int\left(\frac{du}{dx}.\int v\, dx\right) dx$

iv) $\int \sin x\, dx = -\cos x$

v) $\frac{d}{dx}(\sin x) = \cos x$

vi) $2\sin x.\cos x = \sin 2x$

vii) $\cos 2x = (\cos^2 x - \sin^2 x)$

viii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$I.F. = e^{\int P\, dx}$

Answer :

Given differential equation is

$\frac{dy}{dx} + y\cot x = \sin 2x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = \sin 2x$

Therefore, integrating factor is

$I.F. = e^{\int P\, dx}$

$= e^{\int \cot x\, dx}$

$= e^{\log|\sin x|}$ ....($\because \int \cot x\, dx = \log|\sin x|$)

CLASS24

$= \sin x$ ....($\because a^{\log_a b} = b$)

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.(\sin x) = \int(\sin 2x).(\sin x)dx + c$ ....eq(2)

Let,

$$I = \int (\sin 2x).(\sin x)dx$$

Let, u=sin 2x & v=sin x

$$\therefore I = \sin 2x.\int \sin x\ dx - \int\left(\frac{d}{dt}(\sin 2x).\int \sin x\ dx\right) dx$$

.........$\left(\because \int u.v\, dx = u.\int v\, dx - \int\left(\frac{du}{dx}.\int v\, dx\right) dx\right)$

$$\therefore I = -\sin 2x.\cos x - \int\left((2\cos 2x).(-\cos x)\right) dx$$

.........$\left(\because \int \sin x\, dx = -\cos x\ \&\ \frac{d}{dx}(\sin x) = \cos x\right)$

$$\therefore I = -\sin 2x.\cos x + 2\int\left((\cos 2x).(\cos x)\right) dx$$

Again let, u=cos 2x & v=cos x

$$\therefore I = -\sin 2x.\cos x + 2\left\{\cos 2x.\int \cos x\ dx - \int\left(\frac{d}{dt}(\cos 2x).\int \cos x\ dx\right) dx\right\}$$

.........$\left(\because \int u.v\, dx = u.\int v\, dx - \int\left(\frac{du}{dx}.\int v\, dx\right) dx\right)$

$$\therefore I = -\sin 2x.\cos x + 2\left\{\cos 2x.\sin x - \int\left((-2\sin 2x).(\sin x)\right) dx\right\}$$

$$\therefore I = -\sin 2x.\cos x + 2\left\{\cos 2x.\sin x + 2\int\left((\sin 2x).(\sin x)\right) dx\right\}$$

$$\therefore I = -\sin 2x.\cos x + 2\{\cos 2x.\sin x + 2I\}$$

$$\therefore I = -\sin 2x.\cos x + 2\cos 2x.\sin x + 4I$$

$$\therefore I - 4I = -2\sin x\cos x.\cos x + 2(\cos^2 x - \sin^2 x).\sin x$$

.........$\left(\because \sin 2x = 2\sin x.\cos x\ \&\ \cos 2x = (\cos^2 x - \sin^2 x)\right)$

$$\therefore -3I = -2\sin x\cos^2 x + 2\sin x\cos^2 x - 2\sin^3 x$$

$$\therefore -3I = -2\sin^3 x$$

$$\therefore I = \frac{2}{3}\sin^3 x$$

Substituting I in eq(2),

$$\therefore y.(\sin x) = \frac{2}{3}\sin^3 x + c$$

Therefore, general solution is

$y.(\sin x) = \frac{2}{3} sin^3 x + c$

Question: 33

Given Differential Equation :

$$\frac{dy}{dx} + 2y \tan x = \sin x$$

Formula :

i) $\int \tan x \, dx = \log|\sec x|$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int \left(\frac{-1}{x^2}\right) dx = \frac{1}{x}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y \tan x = \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2 \tan x$ and $Q = \sin x$

Therefore, integrating factor is

$$I.F. = e^{\int P \, dx}$$

$$= e^{\int 2 \tan x \, dx}$$

$= e^{2 \log|\sec x|}$ ....($\because \int \tan x \, dx = \log|\sec x|$)

$= e^{\log|\sec x|^2}$ ....($\because a \log b = \log b^a$)

$= \sec^2 x$ ....($\because a^{\log_a b} = b$)

$$= \frac{1}{\cos^2 x}$$

General solution is

$y.(I.F.) = \int Q.(I.F.)dx + c$

$\therefore y.\left(\frac{1}{\cos^2 x}\right) = \int (\sin x).\left(\frac{1}{\cos^2 x}\right)dx + c$ ....eq(2)

Let,

$I = \int (\sin x).\left(\frac{1}{\cos^2 x}\right)dx$

Put, cos x=t => -sin x dx = dt

$\therefore I = \int \left(\frac{-1}{t^2}\right)dt$

$\therefore I = \frac{1}{t}$ ....$\left(\because \int \left(\frac{-1}{x^2}\right)dx = \frac{1}{x}\right)$

$\therefore I = \frac{1}{\cos x}$

Substituting I in eq(2),

$\therefore y.\left(\frac{1}{\cos^2 x}\right) = \frac{1}{\cos x} + c$

Multiplying above equation by $\cos^2 x$,

$\therefore y = \cos x + c(\cos^2 x)$

Therefore, general solution is

$y = \cos x + c(cos^2 x)$

Question: 34

Solution:

Given Differential Equation :

$\frac{dy}{dx} + y \cot x = x^2 \cot x + 2x$

Formula :

i) $\int \cot x\, dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

iv) $\int \cos x\, dx = \sin x$

v) $\frac{d}{dx}(x^n) = nx^{n-1}$

vi) General solution :

For the differential equation in the form of

$\frac{dy}{dx} + Py = Q$

General solution is given by,

$y.(I.F.) = \int Q.(I.F.)dx + c$

Where, integrating factor,

$I.F. = e^{\int P\,dx}$

CLASS24

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + y\cot x = x^2 \cot x + 2x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = x^2 \cot x + 2x$

Therefore, integrating factor is

$I.F. = e^{\int P\,dx}$

$= e^{\int \cot x\,dx}$

$= e^{\log|\sin x|}$ ....($\because \int \cot x\,dx = \log|\sin x|$)

$= \sin x$ ....($\because a^{\log_a b} = b$)

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin x) = \int (x^2 \cot x + 2x).(\sin x)dx + c$$

$$\therefore y.(\sin x) = \int (x^2 \cot x.\sin x + 2x\sin x)\,dx + c$$

$$\therefore y.(\sin x) = \int \left(x^2 \frac{\cos x}{\sin x}.\sin x + 2x\sin x\right) dx + c$$

$$\therefore y.(\sin x) = \int (x^2 \cos x + 2x\sin x)\,dx + c$$

$\therefore y.(\sin x) = \int x^2 \cos x\,dx + \int 2x\sin x\,dx + c$ ....eq(2)

Let,

$$I = \int x^2 \cos x\,dx$$

Let, u=x² and v=cos x

$$\therefore I = x^2.\int \cos x\,dx - \int \left(\frac{d}{dt}(x^2).\int \cos x\,dx\right) dx$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx\right)$

$$\therefore I = x^2.\sin x - \int 2x.\sin x\,dx$$

.........$\left(\because \int \cos x\,dx = \sin x \ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$

Substituting I in eq(2),

$\therefore y.(\sin x) = x^2.\sin x - \int 2x.\sin x\,dx + \int 2x\sin x\,dx + c$

$\therefore y.(\sin x) = x^2.\sin x + c$

Dividing above equation by sin x,

$\therefore y = x^2 + \frac{c}{\sin x}$

Therefore, general solution is

$y = x^2 + c(cosec\ x)$

Question: 35

Solution:

Given Differential Equation :

$x\frac{dy}{dx} + y = x^3$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

iv) General solution :

For the differential equation in the form of

$\frac{dy}{dx} + Py = Q$

General solution is given by,

$y.(I.F.) = \int Q.(I.F.)dx + c$

Where, integrating factor,

$I.F. = e^{\int P\,dx}$

Answer :

Given differential equation is

$x\frac{dy}{dx} + y = x^3$

Dividing above equation by x,

$\therefore \frac{dy}{dx} + \frac{1}{x}.y = x^2$ ....eq(1)

Equation (1) is of the form

$\frac{dy}{dx} + Py = Q$

Where, $P = \frac{1}{x}$ and $Q = x^2$

Therefore, integrating factor is

$\text{I. F.} = e^{\int P\,dx}$

$= e^{\int \frac{1}{x} dx}$

$= e^{\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= x$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.(x) = \int x^2.(x)dx + c$$

$$\therefore xy = \int x^3 dx + c$$

$\therefore xy = \frac{x^4}{4} + c$ ....$\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

Dividing above equation by x,

$$\therefore y = \frac{x^3}{4} + \frac{c}{x}$$

Therefore general equation is

$$y = \frac{x^3}{4} + \frac{c}{x}$$

For particular solution put y=1 and x=2 in above equation,

$$\therefore 1 = \frac{2^3}{4} + \frac{c}{2}$$

$$\therefore 1 = \frac{8}{4} + \frac{c}{2}$$

$$\therefore 1 = 2 + \frac{c}{2}$$

$$\therefore \frac{c}{2} = -1$$

$$\therefore c = -2$$

Therefore, particular solution is

$$y = \frac{x^3}{4} - \frac{2}{x}$$

Question: 36

Solution:

Given Differential Equation :

$$\frac{dy}{dx} + y.\cot x = 4x \operatorname{cosec} x$$

Formula :

i) $\int \cot x\,dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int x^n\, dx = \frac{x^{n+1}}{n+1}$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y.\cot x = 4x \operatorname{cosec} x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = 4x \operatorname{cosec} x$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int \cot x\, dx}$$

$= e^{\log|\sin x|}$ ....$(\because \int \cot x\, dx = \log|\sin x|)$

$= \sin x$ ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin x) = \int (4x \operatorname{cosec} x).(\sin x)dx + c$$

$$\therefore y.(\sin x) = 4\int \left(x \frac{1}{\sin x}\right).(\sin x)dx + c$$

$$\therefore y.(\sin x) = 4\int (x)\, dx + c$$

$\therefore y.(\sin x) = 4\frac{x^2}{2} + c$ ....$\left(\because \int x^n\, dx = \frac{x^{n+1}}{n+1}\right)$

$$\therefore y.(\sin x) = 2x^2 + c$$

Therefore general equation is

$$y.(\sin x) = 2x^2 + c$$

For particular solution put y=0 and $x = \frac{\pi}{2}$ in above equation,

$\therefore 0 = 2\frac{\pi^2}{4} + c$

$\therefore 0 = \frac{\pi^2}{2} + c$

$\therefore c = -\frac{\pi^2}{2}$

Therefore, particular solution is

$$y.(\sin x) = 2x^2 - \frac{\pi^2}{2}$$

Question: 37

Solution:

Given Differential Equation :

$$\frac{dy}{dx} + 2xy = x$$

Formula :

i) $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

ii) $\int (e^{kx})dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2xy = x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2x$ and $Q = x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 2x\,dx}$$

$= e^{2\frac{x^2}{2}}$ ....$\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

$= e^{x^2}$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(e^{x^2}\right) = \int (x).\left(e^{x^2}\right)dx + c$$

$$\therefore y.\left(e^{x^2}\right) = \frac{1}{2}\int (2x).\left(e^{x^2}\right)dx + c \text{ ....eq(2)}$$

Let,

$$I = \int (2x).\left(e^{x^2}\right)dx$$

Put, $x^2=t \Rightarrow 2x\ dx = dt$

$$\therefore I = \int (e^t)dt$$

$$\therefore I = e^t \text{ ....}\left(\because \int \left(e^{kx}\right)dx = \frac{e^{kx}}{k}\right)$$

$$\therefore I = e^{x^2}$$

Substituting I in eq(2),

$$\therefore y.\left(e^{x^2}\right) = \frac{1}{2}.e^{x^2} + c$$

Therefore, general solution is

$$y.\left(e^{x^2}\right) = \frac{1}{2}.e^{x^2} + c$$

For particular solution put y=0 and x=0 in above equation,

$$\therefore 0 = \frac{1}{2}.e^0 + c$$

$$\therefore 0 = \frac{1}{2} + c$$

$$\therefore c = -\frac{1}{2}$$

Substituting c in general solution,

$$y.\left(e^{x^2}\right) = \frac{1}{2}.e^{x^2} - \frac{1}{2}$$

Multiplying above equation by $\frac{2}{e^{x^2}}$

$$\therefore 2y = 1 - e^{-x^2}$$

Therefore, particular solution is

$$2y = 1 - e^{-x^2}$$

Question: 38

Solution:

Given Differential Equation :

$\frac{dy}{dx} + 2y = e^{-2x}.\sin x$

Formula :

i) $\int 1\,dx = x$

ii) $\int(\sin x)dx = -\cos x$

iii) General solution :

For the differential equation in the form of

$\frac{dy}{dx} + Py = Q$

General solution is given by,

$y.(I.F.) = \int Q.(I.F.)dx + c$

Where, integrating factor,

$I.F. = e^{\int P\,dx}$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y = e^{-2x}.\sin x$ ....eq(1)

Equation (1) is of the form

$\frac{dy}{dx} + Py = Q$

Where, $P = 2$ and $Q = e^{-2x}.\sin x$

Therefore, integrating factor is

$I.F. = e^{\int P\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$ .... $(\because \int 1\,dx = x)$

General solution is

$y.(I.F.) = \int Q.(I.F.)dx + c$

$\therefore y.(e^{2x}) = \int (e^{-2x}.\sin x).(e^{2x})dx + c$

$\therefore y.(e^{2x}) = \int \left(\frac{1}{e^{2x}}.\sin x\right).(e^{2x})dx + c$

$\therefore y.(e^{2x}) = \int (\sin x)dx + c$

$\therefore y.(e^{2x}) = -\cos x + c$ .... $(\because \int(\sin x)dx = -\cos x)$

Therefore, general solution is

$y.(e^{2x}) = -\cos x + c$

For particular solution put y=0 and x=0 in above equation,

$\therefore 0 = -\cos 0 + c$

$\therefore 0 = -1 + c$

$\therefore c = 1$

Substituting c in general solution,

$y.(e^{2x}) = -\cos x + 1$

Therefore, particular solution is

$y.(e^{2x}) = -\cos x + 1$

Question: 39

Solution:

Given Differential Equation :

$$(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$$

Formula :

i) $\int \frac{f(x)}{f'(x)} dx = \log f(x)$

ii) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$I.F. = e^{\int P\, dx}$

Answer :

Given differential equation is

$$(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$$

Dividing above equation by $(1+x^2)$,

$\therefore \frac{dy}{dx} + \frac{2x}{(1+x^2)} y = \frac{4x^2}{(1+x^2)}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2x}{(1+x^2)}$ and $Q = \frac{4x^2}{(1+x^2)}$

Therefore, integrating factor is

$$\text{I. F.} = e^{\int P\,dx}$$

$$= e^{\int \frac{2x}{(1+x^2)} dx}$$

Let, $f(x) = (1 + x^2)$ $\therefore f'(x) = 2x$

$$\therefore \text{I. F.} = e^{\log(1+x^2)} \ldots\ldots\ldots \left(\because \int \frac{f(x)}{f'(x)} dx = \log f(x)\right)$$

$$= (1 + x^2)$$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.(1 + x^2) = \int \left(\frac{4x^2}{(1 + x^2)}\right).(1 + x^2)dx + c$$

$$\therefore y.(1 + x^2) = 4\int x^2 dx + c$$

$$\therefore y.(1 + x^2) = 4\frac{x^3}{3} + c \ldots\ldots\ldots \left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

Therefore, general solution is

$$y.(1 + x^2) = 4\frac{x^3}{3} + c$$

For particular solution put y=0 and x=0 in above equation,

$$\therefore 0 = 0 + c$$

$$\therefore c = 0$$

Substituting c in general solution,

$$\therefore y.(1 + x^2) = 4\frac{x^3}{3}$$

Dividing above equation by $(1+x^2)$,

$$\therefore y = \frac{4x^3}{3(1 + x^2)}$$

Therefore, particular solution is

$$y = \frac{4x^3}{3(1 + x^2)}$$

Question: 40

Solution:

Given Differential Equation :

$$x\frac{dy}{dx} - y = \log x$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx$

v) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

vi) $\frac{d}{dx}(kx) = k$

vii) $\log 1 = 0$

viii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = \log x$$

Dividing above equation by x,

$\therefore \frac{dy}{dx} - \frac{1}{x}y = \frac{\log x}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{\log x}{x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}dx}$$

$= e^{-\log(x)}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= e^{\log x^{-1}}$ ....$(\because a\log b = \log b^a)$

$$= e^{\log\left(\frac{1}{x}\right)}$$

$= \frac{1}{x}$ ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.\left(\frac{1}{x}\right) = \int\left(\frac{\log x}{x}\right).\left(\frac{1}{x}\right)dx + c$ ....eq(2)

Let,

$I = \int\left(\frac{\log x}{x}\right).\left(\frac{1}{x}\right)dx$

Put, log x =t => x=e^t

Therefore, (1/x) dx = dt

$\therefore I = \int\left(\frac{t}{e^t}\right) dt$

$\therefore I = \int t.e^{-t}\, dt$

Let, u=t and v=e^{-t}

$\therefore I = t.\int e^{-t}\,dt - \int\left(\frac{d}{dt}(t).\int e^{-t}\,dt\right)dt$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$\therefore I = -t.e^{-t} - \int\left((1).(-e^{-t})\right)dt$

.........$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(kx) = k\right)$

$\therefore I = -t.e^{-t} - e^{-t}$ ....$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$

$\therefore I = -\frac{\log x}{x} - \frac{1}{x}$

Substituting I in eq(2),

$\therefore y.\left(\frac{1}{x}\right) = -\frac{\log x}{x} - \frac{1}{x} + c$

Multiplying above equation by x,

$\therefore y = -\log x - 1 + cx$

Therefore, general solution is

$y = -\log x - 1 + cx$

For particular solution put y=0 and x=1 in above equation,

$\therefore 0 = -\log 1 - 1 + c$

$\therefore c = 1$ ....$(\because \log 1 = 0)$

Substituting c in general solution,

$\therefore y = -\log x - 1 + x$

$\therefore y = x - \log x - 1$

Therefore, particular solution is

$y = x - \log x - 1$

Question: 41

Find a particular

Solution:

Given Differential Equation :

$$\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$$

Formula :

i) $\int \tan x \, dx = \log|\sec x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v \, dx = u.\int v \, dx - \int \left(\frac{du}{dx}.\int v \, dx\right) dx$

iv) $\int \sec x . \tan x \, dx = \sec x$

v) $\frac{d}{dx}(x^n) = nx^{n-1}$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \tan x$ and $Q = 2x + x^2 \tan x$

Therefore, integrating factor is

$$I.F. = e^{\int P \, dx}$$

$$= e^{\int \tan x \, dx}$$

$= e^{\log|\sec x|}$ .... $(\because \int \tan x \, dx = \log|\sec x|)$

$= \sec x$......... $(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sec x) = \int (2x + x^2 \tan x).(\sec x)dx + c$$

$\therefore y.(\sec x) = \int (x^2 \tan x.\sec x + 2x \sec x)\, dx + c$

$\therefore y.(\sec x) = \int x^2 \tan x.\sec x\ dx + \int 2x \sec x\, dx + c$ ....eq(2)

Let,

$I = \int x^2 \tan x.\sec x\ dx$

Let, u=x² and v= tan x. sec x

$\therefore I = x^2.\int \sec x.\tan x\ dx - \int \left(\frac{d}{dt}(x^2).\int \sec x.\tan x\ dx\right) dx$

.........$\left(\because\ \int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx\right)$

$\therefore I = x^2.\sec x - \int 2x.\sec x\ dx$

.........$\left(\because\ \int \sec x.\tan x\, dx = \sec x\ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$

Substituting I in eq(2),

$\therefore y.(\sec x) = x^2.\sec x - \int 2x.\sec x\ dx + \int 2x \sec x\, dx + c$

$\therefore y.(\sec x) = x^2.\sec x + c$

$\therefore y.\left(\frac{1}{\cos x}\right) = x^2.\left(\frac{1}{\cos x}\right) + c$

Multiplying above equation by cos x,

$\therefore y = x^2 + c.(\cos x)$

Therefore, general solution is

$y = x^2 + c.(\cos x)$

For particular solution put y=1 and x=0 in above equation,

$\therefore 1 = 0 + c$

$\therefore c = 1$

Substituting c in general solution,

$\therefore y = x^2 + \cos x$

Therefore, particular solution is

$y = x^2 + \cos x$

Question: 42

A curve pas

Solution:

Formula :

i) $\int 1\, dx = x$

ii) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

iii) $\int e^{kx}\, dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

The slope of the tangent to the curve $= \frac{dy}{dx}$

The slope of the tangent to the curve is equal to the sum of the coordinates of the point.

$$\therefore \frac{dy}{dx} = x + y$$

Therefore differential equation is

$$\therefore \frac{dy}{dx} = x + y$$

$\therefore \frac{dy}{dx} - y = x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -1$ and $Q = x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int -1\,dx}$$

$= e^{-x}$ ....($\because \int 1\,dx = x$)

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.(e^{-x}) = \int (x).(e^{-x})dx + c$ ....eq(2)

Let,

$$I = \int (x).(e^{-x})dx$$

Let, u=x and v= e^{-x}

$$\therefore I = x.\int e^{-x}\,dx - \int \left(\frac{d}{dx}(x).\int e^{-x}\,dx\right) dx$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$\therefore I = -x.e^{-x} - \int (1).(-e^{-x})\,dx$

.........$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$

$\therefore I = -x.e^{-x} - e^{-x}$ .... $\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$

Substituting I in eq(2),

$\therefore y.(e^{-x}) = -x.e^{-x} - e^{-x} + c$

Dividing above equation by e^{-x},

$\therefore y = -x - 1 + c.e^{x}$

Therefore, general solution is

$y + x + 1 = c.e^{x}$

The curve passes through origin , therefore the above equation satisfies for x=0 and y=0,

$\therefore 0 + 0 + 1 = c.e^{0}$

$\therefore c = 1$

Substituting c in general solution,

$\therefore y + x + 1 = e^{x}$

Therefore, equation of the curve is

$y + x + 1 = e^{x}$

Question: 43

A curve pas

Solution:

Formula :

i) $\int 1\,dx = x$

ii) $\int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx$

iii) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$\frac{dy}{dx} + Py = Q$

General solution is given by,

$y.(I.F.) = \int Q.(I.F.)dx + c$

Where, integrating factor,

$I.F. = e^{\int P\,dx}$

Answer :

The slope of the tangent to the curve $= \frac{dy}{dx}$

The sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at the given point by 5.

$$\therefore 5 + \frac{dy}{dx} = x + y$$

Therefore differential equation is

$$\therefore 5 + \frac{dy}{dx} = x + y$$

$\therefore \frac{dy}{dx} - y = x - 5$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -1$ and $Q = x - 5$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int -1\, dx}$$

$= e^{-x}$ ....$(\because \int 1\, dx = x)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.(e^{-x}) = \int (x-5).(e^{-x})dx + c$ ....eq(2)

Let,

$$I = \int (x-5).(e^{-x})dx$$

Let, u=x-5 and v= e^{-x}

$$\therefore I = (x-5).\int e^{-x}\, dx - \int \left(\frac{d}{dt}(x-5).\int e^{-x}\, dx\right) dx$$

.........$\left(\because \int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx\right)$

$$\therefore I = -(x-5).e^{-x} - \int (1).(-e^{-x})\, dx$$

.........$\left(\because \int e^{kx}\, dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$

$\therefore I = -(x-5).e^{-x} - e^{-x}$ .... $\left(\because \int e^{kx}\, dx = \frac{e^{kx}}{k}\right)$

Substituting I in eq(2),

$$\therefore y.(e^{-x}) = -(x-5).e^{-x} - e^{-x} + c$$

Dividing above equation by e^{-x},

$$\therefore y = -(x-5) - 1 + c.e^{x}$$

$\therefore y = -x + 5 - 1 + c.e^x$

$\therefore y = -x + 4 + c.e^x$

Therefore, general solution is

$y = -x + 4 + c.e^x$

The curve passes through point (0,2) , therefore the above equation satisfies for x=0 and y=2,

$\therefore 2 = -0 + 4 + c.e^0$

$\therefore c = -2$

Substituting c in general solution,

$\therefore y = -x + 4 - 2e^x$

Therefore, equation of the curve is

$y = 4 - x - 2e^x$

Question: 44

Solution:

Given Differential Equation :

$ydx - (x + 2y^2)dy = 0$

Formula :

i) $\int \frac{1}{x}dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$I.F. = e^{\int P\,dy}$

Answer :

Given differential equation is

$ydx - (x + 2y^2)dy = 0$

$\therefore ydx = (x + 2y^2)dy$

$$\therefore \frac{dx}{dy} = \frac{(x + 2y^2)}{y}$$

$\therefore \frac{dx}{dy} = \frac{x}{y} + 2y$

$\therefore \frac{dx}{dy} - \frac{1}{y}.x = 2y$ ....eq(1)

Equation (1) is of the form

$\frac{dx}{dy} + Px = Q$

Where, $P = \frac{-1}{y}$ and $Q = 2y$

Therefore, integrating factor is

$I.F. = e^{\int P\,dy}$

$= e^{\int \frac{-1}{y} dy}$

$= e^{-\log y}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log \frac{1}{y}}$ ....$(\because a \log b = \log b^a)$

$= \frac{1}{y}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$x.(I.F.) = \int Q.(I.F.)dy + c$

$\therefore x.\left(\frac{1}{y}\right) = \int (2y).\left(\frac{1}{y}\right)dy + c$

$\therefore \frac{x}{y} = \int (2)dy + c$

$\therefore \frac{x}{y} = 2y + c$ ....$(\because \int 1dx = x)$

Multiplying above equation by y,

$\therefore x = 2y^2 + cy$

Therefore, general solution is

$\therefore x = 2y^2 + cy$

Question: 45

Solution:

Given Differential Equation :

$ydx + (x - y^2)dy = 0$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int 1dx = x$

iv) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

Answer :

Given differential equation is

$$ydx + (x - y^2)dy = 0$$

$$\therefore ydx = -(x - y^2)dy$$

$$\therefore ydx = (y^2 - x)dy$$

$$\therefore \frac{dx}{dy} = \frac{(y^2 - x)}{y}$$

$$\therefore \frac{dx}{dy} = -\frac{x}{y} + y$$

$$\therefore \frac{dx}{dy} + \frac{1}{y}.x = y \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{1}{y}$ and $Q = y$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int \frac{1}{y} dy}$$

$$= e^{\log y} \text{ ....}\left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= y \text{ ....}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$$\therefore x.(y) = \int (y).(y)dy + c$$

$$\therefore xy = \int y^2\,dy + c$$

$$\therefore xy = \frac{y^3}{3} + c \text{ ....}(\because \int 1dx = x)$$

Dividing above equation by y,

$\therefore x = \frac{1}{3}y^2 + \frac{c}{y}$

Therefore, general solution is

$$x = \frac{1}{3}y^2 + \frac{c}{y}$$

Question: 46

Solution:

Given Differential Equation :

$ydx + (x - y^2)dy = 0$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int 1 dx = x$

iv) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$I.F. = e^{\int P\, dy}$

Answer :

Given differential equation is

$ydx + (x - y^2)dy = 0$

$\therefore ydx = -(x - y^2)dy$

$\therefore ydx = (y^2 - x)dy$

$\therefore \frac{dx}{dy} = \frac{(y^2 - x)}{y}$

$\therefore \frac{dx}{dy} = -\frac{x}{y} + y$

$\therefore \frac{dx}{dy} + \frac{1}{y}.x = y$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{1}{y}$ and $Q = y$

Therefore, integrating factor is

$I.F. = e^{\int P\,dy}$

$= e^{\int \frac{1}{y} dy}$

$= e^{\log y}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= y$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$x.(I.F.) = \int Q.(I.F.)dy + c$

$\therefore x.(y) = \int (y).(y)dy + c$

$\therefore xy = \int y^2\,dy + c$

$\therefore xy = \frac{y^3}{3} + c$ ....$(\because \int 1dx = x)$

Dividing above equation by y,

$\therefore x = \frac{1}{3}y^2 + \frac{c}{y}$

Therefore, general solution is

$x = \frac{1}{3}y^2 + \frac{c}{y}$

Question: 47

Solution:

Given Differential Equation :

$(x + 3y^3)\frac{dy}{dx} = y$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int x^n dx = \frac{x^{n+1}}{n+1}$

v) General solution :

For the differential equation in the form of

$\frac{dx}{dy} + Px = Q$

General solution is given by,

$x.(I.F.) = \int Q.(I.F.)dy + c$

Where, integrating factor,

$I.F. = e^{\int P\,dy}$

<u>Answer</u> :

Given differential equation is

$$(x+3y^3)\frac{dy}{dx} = y$$

$$\therefore \frac{dx}{dy} = \frac{(x+3y^3)}{y}$$

$$\therefore \frac{dx}{dy} = \frac{x}{y} + 3y^2$$

$\therefore \frac{dx}{dy} - \frac{1}{y}.x = 3y^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{-1}{y}$ and $Q = 3y^2$

Therefore, integrating factor is

$I.F. = e^{\int P\,dy}$

$= e^{\int \frac{-1}{y}dy}$

$= e^{-\log y}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= e^{\log \frac{1}{y}}$ ....$(\because a\log b = \log b^a)$

$= \frac{1}{y}$ ....$(\because a^{\log_a b} = b)$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$$\therefore x.\left(\frac{1}{y}\right) = \int (3y^2).\left(\frac{1}{y}\right)dy + c$$

$$\therefore \frac{x}{y} = 3\int (y)dy + c$$

$\therefore \frac{x}{y} = \frac{3y^2}{2} + c$ ....$\left(\because \int x^n dx = \frac{x^{n+1}}{n+1}\right)$

Multiplying above equation by y,

$$\therefore x = \frac{3}{2}y^3 + cy$$

Therefore, general solution is

$$x = \frac{3}{2}y^3 + cy$$

Question: 48

Find the general

Solution:

Given Differential Equation :

$$(x+y)\frac{dy}{dx}=1$$

Formula :

i) $\int 1\,dx = x$

ii) $\int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx$

iii) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy}+Px=Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

Answer :

Given differential equation is

$$(x+y)\frac{dy}{dx}=1$$

$$\therefore \frac{dx}{dy} = x+y$$

$$\therefore \frac{dx}{dy} - x = y \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dx}{dy}+Px=Q$$

Where, $P=-1$ and $Q=y$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int -1\,dy}$$

$$= e^{-y} \text{ ....} (\because \int 1dx = x)$$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$\therefore x.(e^{-y}) = \int (y).(e^{-y})dy + c$ ....eq(2)

Let,

$$I = \int (y).(e^{-y})dy$$

Let, u=y and v= e^{-y}

$$\therefore I = y.\int e^{-y}\,dy - \int \left(\frac{d}{dy}(y).\int e^{-y}\,dy\right) dy$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx\right)$

$$\therefore I = -y.e^{-y} - \int (1).(-e^{-y})\,dy$$

.........$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k} \ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$

$\therefore I = -y.e^{-y} - e^{-y}$ .... $\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$

Substituting I in eq(2),

$\therefore x.(e^{-y}) = -y.e^{-y} - e^{-y} + c$

$\therefore x.(e^{-y}) + y.e^{-y} + e^{-y} = c$

$\therefore e^{-y}(x + y + 1) = c$

Therefore, general solution is

$e^{-y}(x + y + 1) = c$

Question: 49

Solution:

Given Differential Equation :

$$(x + y + 1)\frac{dy}{dx} = 1$$

Formula :

i) $\int 1dx = x$

ii) $\int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx$

iii) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$I.F. = e^{\int P\,dy}$

Answer :

Given differential equation is

$$(x + y + 1)\frac{dy}{dx} = 1$$

$$\therefore \frac{dx}{dy} = x + y + 1$$

$\therefore \frac{dx}{dy} - x = y + 1$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = -1$ and $Q = y + 1$

Therefore, integrating factor is

$I.F. = e^{\int P\,dy}$

$= e^{\int -1\,dy}$

$= e^{-y}$ ....$(\because \int 1 dx = x)$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$\therefore x.(e^{-y}) = \int (y + 1).(e^{-y})dy + c$ ....eq(2)

Let,

$$I = \int (y + 1).(e^{-y})dy$$

Let, u=y+1 and v= e^{-y}

$$\therefore I = (y + 1).\int e^{-y}\,dy - \int\left(\frac{d}{dy}(y + 1).\int e^{-y}\,dy\right)dy$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$$\therefore I = -(y + 1).e^{-y} - \int (1).(-e^{-y})\,dy$$

.........$\left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$

$\therefore I = -(y + 1).e^{-y} - e^{-y}$ ....$\left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$

Substituting I in eq(2),

$$\therefore x.(e^{-y}) = -(y + 1).e^{-y} - e^{-y} + c$$

$$\therefore x.(e^{-y}) = -e^{-y}(y + 1 + 1) + c$$

$$\therefore x.(e^{-y}) = -e^{-y}(y + 2) + c$$

$\therefore x.(e^{-y}) = c - e^{-y}(y+2)$

Dividing above equation by e^{-y}

$\therefore x = ce^{y} - (y+2)$

Therefore, general solution is

$x = ce^{y} - (y+2)$

Question: 50

Solve

Solution:

Given Equation: $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$

Re-arranging, we get,

$$\frac{1}{2e^{-y}-1}dy = \frac{dx}{(x+1)}$$

$$\frac{e^{y}}{2-e^{y}}dy = \frac{dx}{(x+1)}$$

Let $2 - e^{y} = t$

$-e^{y}dy = dt$

Therefore,

$$\frac{dt}{t} = \frac{dx}{x+1}$$

Integrating both sides, we get,

$\log t = \log(x+1) + C$

$\log(2 - e^{y}) = \log(x+1) + C$

At $x = 0, y = 0$.

Therefore,

$\log(2) = \log(1) + C$

Therefore,

$C = \log 2$

Now, we have,

$\log(2 - e^{y}) - \log(x+1) - \log 2 = 0$

$$y = \log\left|\frac{2x+1}{x+1}\right|$$

Question: 51

Solve

Solution:

Given Differential Equation :

$(1+y^{2})dx + \left(x - e^{-\tan^{-1}y}\right)dy = 0$

Formula :

i) $\int \frac{1}{(1+x^2)} dx = \tan^{-1}x$

ii) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

Answer :

Given differential equation is

$$(1+y^2)dx + \left(x - e^{-\tan^{-1}y}\right)dy = 0$$

$$\therefore (1+y^2)dx = -\left(x - e^{-\tan^{-1}y}\right)dy$$

$$\therefore (1+y^2)dx = \left(e^{-\tan^{-1}y} - x\right)dy$$

$$\therefore \frac{dx}{dy} = \frac{\left(e^{-\tan^{-1}y} - x\right)}{(1+y^2)}$$

$$\therefore \frac{dx}{dy} = \frac{e^{-\tan^{-1}y}}{(1+y^2)} - \frac{x}{(1+y^2)}$$

$$\therefore \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{-\tan^{-1}y}}{(1+y^2)} \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{1}{(1+y^2)}$ and $Q = \frac{e^{-\tan^{-1}y}}{(1+y^2)}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int \frac{1}{(1+y^2)} dy}$$

$$= e^{\tan^{-1}y} \text{ ....} \left(\because \int \frac{1}{(1+x^2)} dx = \tan^{-1}x\right)$$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right) = \int \left(\frac{e^{-\tan^{-1}y}}{(1+y^2)}\right).\left(e^{\tan^{-1}y}\right)dy + c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right) = \int \left(\frac{1}{e^{\tan^{-1}y}.(1+y^2)}\right).\left(e^{\tan^{-1}y}\right)dy + c$$

$\therefore x.\left(e^{\tan^{-1}y}\right) = \int \frac{1}{(1+y^2)} dy + c$

$\therefore x.\left(e^{\tan^{-1}y}\right) = \tan^{-1}y + c \;.........\left(\because \int \frac{1}{(1+x^2)} dx = \tan^{-1}x\right)$

Putting x=0 and y=0

$\therefore 0 = 0 + c$

$\therefore c = 0$

Therefore, general solution is

$x.\left(e^{tan^{-1}y}\right) = tan^{-1}y$

Exercise : OBJECTIVE QUESTIONS

Question: 1

Solution:

Given, $\frac{dy}{dx} = e^{x+y}$

$\frac{dy}{dx} = e^x e^y$

$e^{-y}dy = e^x dx$

On integrating on both sides, we get

$-e^{-y} + c_1 = e^x + c_2$

$e^{-y} + e^x = c$

Conclusion: Therefore, $e^{-y} + e^x = c$ is the solution of $\frac{dy}{dx} = e^{x+y}$

Question: 2

Solution:

Given, $\frac{dy}{dx} = 2^{x+y}$

$\frac{dy}{dx} = 2^x 2^y$

$2^{-y}dy = 2^x dx$

On integrating on both sides, we get

$-\frac{2^{-y}}{\log 2} + c_2 = \frac{2^x}{\log 2} + c_2$

$2^x + 2^{-y} = c_3 \log 2$

$2^x + 2^{-y} = c$

Conclusion: Therefore, $2^x + 2^{-y} = c$ is the solution of $\frac{dy}{dx} = 2^{x+y}$

Question: 3

Solution:

$(e^x + 1)y\,dy = (y + 1)e^x dx$

$$\frac{y\,dy}{y+1} = \frac{e^x\,dx}{(e^x+1)}$$

Let, $e^x + 1 = t$

On differentiating on both sides we get $e^x dx = dt$

Now we can write this equation as $\frac{y\,dy}{y+1} = \frac{e^x\,dx}{(e^x+1)}$

$$\frac{((y+1)-1)\,dy}{y+1} = \frac{e^x\,dx}{(e^x+1)}$$

$$\left(1 - \frac{1}{y+1}\right)dy = \frac{e^x\,dx}{(e^x+1)}$$

$$\left(1 - \frac{1}{y+1}\right)dy = \frac{dt}{t}$$

On integrating on both sides, we get

$y - \log(y + 1) = \log(e^x + 1) + \log c$

$y = \log(y + 1) + \log(e^x + 1) + \log c$

$y = \log(y + 1)\,(e^x + 1)c$

$e^y = c(y + 1)(e^x + 1)$

Conclusion: Therefore, $e^y = c(y + 1)(e^x + 1)$ is the solution of $(e^x + 1)y\,dy = (y + 1)e^x dx$

Question: 4

Solution:

Given xdy + ydx = 0

xdy = -ydx

$$-\frac{dy}{y} = \frac{dx}{x}$$

On integrating on both sides we get,

-log y = log x + c

log x + log y = c

log xy = c

xy = C

Conclusion: Therefore xy = c is the solution of xdy + ydx = 0

Question: 5

Solution:

Given: $x\frac{dy}{dx} = \cot y$

Separating the variables, we get,

$$\frac{dy}{\cot y} = \frac{dx}{x}$$

$\tan y\, dy = \frac{dx}{x}$

Integrating both sides, we get,

$\int \tan y\, dy = \int \frac{dx}{x}$

log sec y = log x + log c

xcosy = c

Hence, A is the correct answer.

Question: 6

Solution:

Given $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

On integrating on both sides, we get

$\tan^{-1} y = \tan^{-1} x + c$

$\tan^{-1} y - \tan^{-1} x = c$

$\frac{y-x}{1+yx} = c$ (since $\tan^{-1} y - \tan^{-1} x = \frac{y-x}{1+yx}$)

y-x = C(1+yx)

Conclusion: Therefore, y-x = C(1+yx) is the solution of $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Question: 7

Solution:

$\frac{dy}{dx} = 1 - x + y - xy$

$\frac{dy}{dx} = 1 - x + y(1 - x)$

$\frac{dy}{dx} = (1 + y)(1 - x)$

$\frac{dy}{1+y} = (1 - x)dx$

On integrating on both sides, we get

$\log(1 + y) = x - \frac{x^2}{2} + c$

Conclusion: Therefore, $\log(1 + y) = x - \frac{x^2}{2} + c$ is the solution of $\frac{dy}{dx} = 1 - x + y - xy$

Question: 8

Solution:

Given $\frac{dy}{dx} = e^{x+y} + x^2e^y$

$(e^{-y})dy = (e^x + x^2)dx$

On integrating on both sides, we get

$$-e^{-y} = e^x + \frac{x^3}{3} + C$$

$$e^{-y} + e^x + \frac{x^3}{3} = C$$

Conclusion: Therefore, $e^{-y} + e^x + \frac{x^3}{3} = C$ is the

solution of $\frac{dy}{dx} = e^{x+y} + x^2e^y$

Question: 9

Solution:

Given $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

$$-\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

On integrating on both sides, we get

$-\sin^{-1} y = \sin^{-1} x + C$ (As $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$)

$\sin^{-1} y + \sin^{-1} x = C$

Conclusion: Therefore, $\sin^{-1} y + \sin^{-1} x = C$ is the

solution of $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

Question: 10

Solution:

Given $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

$$\frac{dy}{dx} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$\frac{dy}{dx} = \tan^2\frac{x}{2}$$

$$dy = dx(\tan^2\frac{x}{2})$$

On integrating on both sides, we get

$$y = 2\tan\frac{x}{2} - x + C$$

Conclusion: Therefore, $y = 2\tan\frac{x}{2} - x + C$ is the solution

of $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

Question: 11

Mark (√) against

Solution:

Given $\frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$

$$\frac{dy}{y} = \frac{-2xdx}{(x^2+1)}$$

Let $x^2 + 1 = t$

On differentiating on both sides we get 2xdx = dt

$$\frac{dy}{y} = \frac{-dt}{t}$$

On integrating on both sides, we get

$\log y = -\log t + C$

$\log y + \log t = C$

$\log yt = C$

yt = C

As $t = x^2 + 1$

$y(x^2 + 1) = C$

Conclusion: Therefore, $y(x^2 + 1) = C$ is the solution of $\frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$

Question: 12

Solution:

Given cos x (1+cos y) dx – sin y (1+sin x) dy = 0

Let 1+cos y = t and 1+sin x = u

On differentiating both equations, we get

-sin y dy = dt and cos x dx = du

Substitute this in the first equation

t du + u dt = 0

$$-\frac{du}{u} = \frac{dt}{t}$$

-log u = log t + C

log u + log t = C

log ut = C

ut = C

(1+sin x)(1+cos y) = C

Conclusion: Therefore, (1+sin x)(1+cos y) = C is the solution of cos x (1+cos y) dx – sin y (1+sin x) dy = 0

Question: 13

Solution:

Given $x\cos y\ dy = (xe^x \log x + e^x)dx$

$$\cos y\ dy = \frac{(xe^x \log x + e^x)}{x}dx$$

On integrating on both sides we get

$$\sin y = \log x \int e^x dx - \int \frac{1}{x}\left(\int e^x\right)dx + \int \frac{e^x}{x}dx$$

$$\sin y = \log x\,(e^x) - \int \frac{e^x}{x}dx + \int \frac{e^x}{x}dx + C$$

$$\sin y = e^x \log x + C$$

Conclusion: Therefore, $\sin y = e^x \log x + C$ the solution of

$x\cos y\ dy = (xe^x \log x + e^x)dx$

Question: 14

Solution:

Given $\frac{dy}{dx} + y\log y \cot x = 0$

$$\frac{dy}{y\log y} = -\cot x\ dx$$

Let log y = t

On differentiating we get

$$\frac{1}{y}\ dy = dt$$

$$\frac{dt}{t} = -\cot x\ dx$$

log t = -log (sin x) + C

log t + log(sin x) = C

log(tsin x) = C

tsin x = C

(log y)(sin x) = C

Conclusion: Therefore, (log y)(sin x) = C is the solution of $\frac{dy}{dx} + y\log y \cot x = 0$

Question: 15

Solution:

Given $(1 + x^2)dy - xy\,dx = 0$

$$\frac{dy}{y} = \frac{x}{1 + x^2}dx$$

Let $1 + x^2 = t$

2x dx = dt

$$\frac{dy}{y} = \frac{dt}{2t}$$

On integrating on both sides we get

$\log y = \frac{\log t}{2} + C$

$2 \log y = \log t + C$

$\log y^2 = \log t + C$

$y^2 = (1 + x^2)c$

Conclusion: Therefore, $y^2 = (1 + x^2)c$ is the solution of

$(1 + x^2)dy - xy\,dx = 0$

Question: 16

Solution:

Given $x\sqrt{1 + y^2}dx + y\sqrt{1 + x^2}\,dy = 0$

$\frac{ydy}{\sqrt{1 + y^2}} = -\frac{xdx}{\sqrt{1 + x^2}}$

Let $1 + y^2 = t$ and $1 + x^2 = u$

$2y\,dy = dt$ and $2x\,dx = du$

$\frac{dt}{\sqrt{t}} = -\frac{du}{\sqrt{u}}$

On integrating on both sides we get

$\sqrt{t} = -\sqrt{u} + C$

$\sqrt{1 + y^2} + \sqrt{1 + x^2} = C$

Conclusion: Therefore, $\sqrt{1 + y^2} + \sqrt{1 + x^2} = C$ is the solution of $x\sqrt{1 + y^2}dx + y\sqrt{1 + x^2}\,dy = 0$ **Question: 17**

Mark (√) against

Solution:

Given $\log(\frac{dy}{dx}) = (ax + by)$

$\frac{dy}{dx} = e^{ax+by}$

$\frac{dy}{e^{by}} = e^{ax}dx$

On integrating on both sides we get

$-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + C$

Conclusion: Therefore, $-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + C$ is the solution of

$\log(\frac{dy}{dx}) = (ax + by)$

Question: 18

Mark (√) against

Solution:

Given $\frac{dy}{dx} = (\sqrt{1-x^2})(\sqrt{1-y^2})$

$$\frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}\,dx$$

Let x = sin t

dx = cos t dt

We know $\cos t = \sqrt{1-x^2}$

On integrating on both sides we get

$$\sin^{-1} y = \frac{t}{2} + \frac{\sin 2t}{4}$$

Sin 2t = 2 sin t cost

$= 2x\sqrt{1-x^2}$

$$\sin^{-1} y = \frac{\sin^{-1}x}{2} + \frac{x\sqrt{1-x^2}}{2} + C$$

$$2\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$$

Conclusion: Therefore, $2\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$ is the solution of $\frac{dy}{dx} = (\sqrt{1-x^2})(\sqrt{1-y^2})$

Question: 19

Solution:

Given $\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{x^2v^2 - x^2}{2vx^2} = v + x\frac{dv}{dx}$$

$$\frac{v^2-1}{2v} - v = x\frac{dv}{dx}$$

$$\frac{-v^2-1}{2v} = x\frac{dv}{dx}$$

$$\frac{dx}{x} + \frac{2vdv}{v^2+1} = 0$$

On integrating on both sides, we get

$$\log x + \log(v^2+1) = c$$

$$\log(x(v^2+1)) = c$$

$$x\left(\frac{y^2}{x^2} + 1\right) = C$$

$$y^2 + x^2 = Cx$$

Conclusion: Therefore, $y^2 + x^2 = Cx$ is the solution of

$$\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$$

Question: 20

Solution:

Given $x^2\frac{dy}{dx}=x^2+xy+y^2$

$$\frac{dy}{dx}=1+\frac{y}{x}+\frac{y^2}{x^2}$$

Let y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$1+v+v^2=v+x\frac{dv}{dx}$$

$$1+v^2=x\frac{dv}{dx}$$

$$\frac{dx}{x}=\frac{dv}{v^2+1}$$

On integrating on both sides, we get

$\log x=\tan^{-1}v+C$

$\tan^{-1}\frac{y}{x}=\log x+C$

Conclusion: Therefore, $\tan^{-1}\frac{y}{x}=\log x+C$ is the solution of

$$x^2\frac{dy}{dx}=x^2+xy+y^2$$

Question: 21

Mark (√) ag

Solution: Given DE: $x\frac{dy}{dx}=y+x\tan\frac{y}{x}$ Now,Dividing both sides by x, we get, $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$ Let y = vxDifferentiating both sides,dy/dx = v + xdv/dxNow, our differential equation becomes, $v+x\frac{dv}{dx}=v+\tan v$ On separating the variables, we get, $\frac{dv}{\tan v}=\frac{dx}{x}$ Integrating both sides, we get,sinv = CxPutting the value of v we get, $\sin\left(\frac{y}{x}\right)=Cx$ Hence, B is the correct answer.

Question: 22

Solution:

Given $2xy\,dy+(x^2-y^2)dx=0$

$$\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$$

Let y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\frac{x^2v^2 - x^2}{2vx^2} = v + x\frac{dv}{dx}$$

$$\frac{v^2 - 1}{2v} - v = x\frac{dv}{dx}$$

$$\frac{-v^2 - 1}{2v} = x\frac{dv}{dx}$$

$$\frac{dx}{x} + \frac{2vdv}{v^2 + 1} = 0$$

On integrating on both sides, we get

$$\log x + \log(v^2 + 1) = c$$

$$\log(x(v^2 + 1)) = c$$

$$x\left(\frac{y^2}{x^2} + 1\right) = C$$

$$y^2 + x^2 = Cx$$

Conclusion: Therefore, $y^2 + x^2 = Cx$ is the solution of

$2xy\,dy + (x^2 - y^2)dx = 0$

Question: 23

Solution:

Given (x-y)dy + (x+y) dx = o

$$\frac{dy}{dx} = \frac{x + y}{y - x}$$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx + x}{vx - x}$$

$$v + x\frac{dv}{dx} = \frac{v + 1}{v - 1}$$

$$x\frac{dv}{dx} = \frac{v + 1 - v^2 + v}{v - 1}$$

$$x\frac{dv}{dx} = \frac{2v + 1 - v^2}{v - 1}$$

Question is wrong. I think subtraction should be there instead of addition in LHS(left hand side)

Question: 24

Solution:

Given $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = v + \sin v$$

$$x\frac{dv}{dx} = \sin v$$

$$\frac{dv}{\sin v} = \frac{dx}{x}$$

$$\log\tan\frac{v}{2} = \log x + C$$

$$\tan\frac{v}{2} = Cx$$

$$\tan\frac{y}{2x} = Cx$$

Conclusion: Therefore, $\tan\frac{y}{2x} = Cx$ is the solution of $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Question: 25

Solution:

Given $\frac{dy}{dx} + y\tan x = \sec x$

It is in the form $\frac{dy}{dx} + py = Qx$

Integrating factor $= e^{\int \tan x dx} = e^{\log \sec x} = \sec x$

General solution $y\sec x = \int(\sec x)(\sec x)\,dx + C$

$$y\sec x = \int \sec^2 x\,dx + C$$

y sec x = tan x + C

y = sin x + C cos x

Conclusion: Therefore, y = sin x + C cos x is the solution of $\frac{dy}{dx} + y\tan x = \sec x$

Question: 26

Solution:

Given $\frac{dy}{dx} + y\cot x = 2\cos x$

It is in the form $\frac{dy}{dx} + py = Qx$

Integrating factor $= e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

General solution is $y\sin x = \int 2\cos x \sin x\,dx + C$

$$y\sin x = \int \sin 2x\,dx + C$$

$$y\sin x = -\frac{\cos 2x}{2} + C$$

$$y\sin x = \sin^2 x + C$$

(y-sin x)sin x = C

Conclusion: Therefore, (y-sin x)sin x = C is the solution of $\frac{dy}{dx} + y\cot x = 2\cos x$

Question: 27

Solution:

Given $\frac{dy}{dx}+\frac{y}{x}=x^2$

It is in the form $\frac{dy}{dx}+py=Qx$

Integrating factor $=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

General solution is $yx=\int x^2.xdx+C$

$$yx=\frac{x^4}{4}+C$$

Conclusion: Therefore, $yx=\frac{x^4}{4}+C$ is the solution of $\frac{dy}{dx}+\frac{y}{x}=x^2$