

Chapter : 24. CROSS, OR VECTOR, PRODUCT OF VECTORS

Exercise : 24

Question: 1 A

Here,

We

have

$$\vec{a} = i - j + 2k \text{ and } \vec{b} = 2i + 3j - 4k$$

$$\Rightarrow a_1 = 1, a_2 = -1, a_3 = 2 \text{ and } b_1 = 2, b_2 = 3, b_3 = -4$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = ((-1 \times -4) - 3 \times 2)i + (2 \times 2 - (-4) \times 1)j + (1 \times 3 - 2 \times (-1))k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-2)^2 + 8^2 + 5^2}$$

$$\vec{a} \times \vec{b} = (-2\hat{i} + 8\hat{j} + 5\hat{k}) \text{ and } |\vec{a} \times \vec{b}| = \sqrt{93}$$

Question: 1 B

Solution:

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \vec{a} = 2i - j + 3k \text{ and } \vec{b} = 3i + 5j - 2k$$

$$\Rightarrow a_1 = 2, a_2 = -1, a_3 = 3 \text{ and } b_1 = 3, b_2 = 5, b_3 = -2$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = ((-1 \times -2) - 5 \times 3)i + (3 \times 3 - (-2) \times 2)j + (2 \times 5 - 3 \times (-1))k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-17)^2 + 13^2 + 7^2} = 13\sqrt{3}$$

$$\Rightarrow \vec{a} \times \vec{b} = (-17)i + (13)j + (7)k$$

Question: 1 C

Solution:

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \vec{a} = i - 7j + 7k \text{ and } \vec{b} = 3i - 2j + 2k$$

$$\Rightarrow a_1 = 1, a_2 = -7, a_3 = 7 \text{ and } b_1 = 3, b_2 = -2, b_3 = 2$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = ((-7 \times 2) - (-2 \times 7))\mathbf{i} + (7 \times 3 - 1 \times 2)\mathbf{j} + ((-2) \times 1 - 3 \times (-7))\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(0)^2 + 19^2 + 19^2} = 19\sqrt{2}$$

$$\Rightarrow \vec{a} \times \vec{b} = (0)\mathbf{i} + (19)\mathbf{j} + (19)\mathbf{k}$$

Question: 1 D

Solution:

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 4\mathbf{i} + \mathbf{j} - 2\mathbf{k} \text{ and } \vec{b} = 3\mathbf{i} + 0\mathbf{j} + \mathbf{k}$$

$$\Rightarrow a_1 = 4, a_2 = 1, a_3 = -2 \text{ and } b_1 = 3, b_2 = 0, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (1 \times 1 - (0) \times (-2))\mathbf{i} + (-2 \times 3 - 1 \times 4)\mathbf{j} + (4 \times 0 - 3 \times 1)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{1^2 + (-10)^2 + (-3)^2} = \sqrt{110}$$

$$\Rightarrow \vec{a} \times \vec{b} = \mathbf{i} - 10\mathbf{j} - 3\mathbf{k}$$

Question: 1 E

Solution:

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 3\mathbf{i} + 4\mathbf{j} + 0\mathbf{k} \text{ and } \vec{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$$

$$\Rightarrow a_1 = 3, a_2 = 4, a_3 = 0 \text{ and } b_1 = 1, b_2 = 1, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (4 \times 1 - 1 \times 0)\mathbf{i} + (0 \times 1 - 1 \times 3)\mathbf{j} + (3 \times 1 - 1 \times 4)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{4^2 + (-3)^2 + (-1)^2} = \sqrt{26}$$

$$\Rightarrow \vec{a} \times \vec{b} = 4\mathbf{i} - 3\mathbf{j} - \mathbf{k}$$

Question: 2

λ Solution:

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

have $\vec{a} = 2i + 6j + 14k$ and $\vec{b} = i - \lambda j + 7k$

$$\Rightarrow a_1 = 2, a_2 = 6, a_3 = 14 \text{ and } b_1 = 1, b_2 = \lambda, b_3 = 7$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (6 \times 7 - (-\lambda) \times 14)i + (14 \times 1 - 2 \times 7)j + (2 \times (-\lambda) - 1 \times 6)k$$

$$\Rightarrow \vec{a} \times \vec{b} = 0i + 0j + 0k$$

$$\Rightarrow 42 + 14\lambda = 0,$$

$$\Rightarrow \lambda = -3$$

Question: 3

Solution:

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \vec{a} = -3i + 4j - 7k \text{ and } \vec{b} = 6i + 2j - 3k$$

$$\Rightarrow a_1 = -3, a_2 = 4, a_3 = -7 \text{ and } b_1 = 6, b_2 = 2, b_3 = -3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (4 \times (-3) - 2 \times (-7))i + ((-7) \times 6 - (-3) \times (-3))j + ((-3) \times 2 - 6 \times 4)k$$

$$\Rightarrow \vec{a} \times \vec{b} = 2i - 51j - 30k$$

If \vec{a} and $\vec{a} \times \vec{b}$ are perpendicular to each other then,

$$\Rightarrow \vec{a} \cdot (\vec{a} \times \vec{b}) = 0$$

i.e.,

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = (-6) - (204) + (210) = 0$$

And in the similar way, we have,

$$\vec{b} \cdot (\vec{a} \times \vec{b}) = (12) - (102) + (90) = 0$$

Hence proved.

Question: 4

Solution:

i.

The value of $(i \times j) \cdot k + i \cdot j$ is, ... As $i \times j = k$ and $i \cdot j = 0$

$$\Rightarrow (k) \cdot k + 0 = 1$$

ii.

The value of $(j \times k) \cdot i + j \cdot k$ is, ... As $j \times k = i$ and $j \cdot k = 0$

$$\Rightarrow (i) \cdot i + 0 = 1$$

iii.

The value of $i \times (j + k) + j \times (k + i) + k \times (i + j)$ is,
As $i \times k = -j, i \times j = k, j \times k = i, j \times i = -k, k \times i = j, k \times j = -i$

$$\Rightarrow k - j + i - k + j - i = 0$$

Question: 5 A

Solution:

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k.(\vec{a} \times \vec{b}) \text{ ...where } k \text{ is a scalar}$$

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \vec{a} = 3i + j - 2k \text{ and } \vec{b} = 2i + 3j - k$$

$$\Rightarrow a_1 = 3, a_2 = 1, a_3 = -2 \text{ and } b_1 = 2, b_2 = 3, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (1 \times -1 - 3 \times -2)i + (-2 \times 2 - (-1) \times 3)j + (3 \times 3 - 2 \times 1)k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(5)^2 + (-1)^2 + (7)^2} = 5\sqrt{3}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{5i - 1j + 7k}{5\sqrt{3}}$$

$$\Rightarrow \vec{r} = \pm \frac{5i - 1j + 7k}{5\sqrt{3}}$$

Question: 5 B

Solution:

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k.(\vec{a} \times \vec{b}) \text{ ...where } k \text{ is a scalar}$$

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \vec{a} = i - 2j + 3k \text{ and } \vec{b} = i + 2j - k$$

$$\Rightarrow a_1 = 1, a_2 = -2, a_3 = 3 \text{ and } b_1 = 1, b_2 = 2, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-2 \times -1 - 2 \times 3)\mathbf{i} + (3 \times 1 - (-1) \times 1)\mathbf{j} + (1 \times 2 - (-2) \times 1)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(-4)^2 + (4)^2 + (4)^2} = 4\sqrt{3}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{-4\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}}{4\sqrt{3}}$$

$$\Rightarrow \vec{r} = \pm \frac{-\mathbf{i} + \mathbf{j} + \mathbf{k}}{\sqrt{3}}$$

Question: 5 C

Solution:

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k.(\vec{a} \times \vec{b}) \text{ ...where } k \text{ is a scalar}$$

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} \text{ and } \vec{b} = -\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}$$

$$\Rightarrow a_1 = 1, a_2 = 3, a_3 = -2 \text{ and } b_1 = -1, b_2 = 0, b_3 = 3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (9 - 0)\mathbf{i} + (2 - 3)\mathbf{j} + (0 - (-3))\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(9)^2 + (-1)^2 + (3)^2} = \sqrt{91}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{9\mathbf{i} - \mathbf{j} + 3\mathbf{k}}{\sqrt{91}}$$

$$\Rightarrow \vec{r} = \pm \frac{9\mathbf{i} - \mathbf{j} + 3\mathbf{k}}{\sqrt{91}}$$

Question: 5 D

Solution:

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k.(\vec{a} \times \vec{b}) \text{ ...where } k \text{ is a scalar}$$

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k} \text{ and } \vec{b} = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$$

$$\Rightarrow a_1 = 4, a_2 = 2, a_3 = -1 \text{ and } b_1 = 1, b_2 = 4, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (2 \times -1 - (-1) \times 4)\mathbf{i} + (-1 \times 1 - (-1) \times 4)\mathbf{j} + (4 \times 4 - 1 \times 2)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(2)^2 + (3)^2 + (14)^2} = \sqrt{209}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{2\mathbf{i} + 3\mathbf{j} + 14\mathbf{k}}{\sqrt{209}}$$

$$\Rightarrow \vec{r} = \pm \frac{2\mathbf{i} + 3\mathbf{j} + 14\mathbf{k}}{\sqrt{209}}$$

Question: 6

Solution:

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k(\vec{a} \times \vec{b}) \text{ ...where } k \text{ is a scalar}$$

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 2\mathbf{i} - 6\mathbf{j} - 3\mathbf{k} \text{ and } \vec{b} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$$

$$\Rightarrow a_1 = 2, a_2 = -6, a_3 = -3 \text{ and } b_1 = 4, b_2 = 3, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-6 \times (-1) - 3 \times (-3))\mathbf{i} + (-3 \times 4 - (-1) \times 2)\mathbf{j} + (2 \times 3 - 4 \times (-6))\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(15)^2 + (-10)^2 + (30)^2} = \sqrt{1225}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}}{7}$$

$$\vec{r} = \pm \frac{3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}}{7}$$

Question: 7

Solution:

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k(\vec{a} \times \vec{b}) \text{ ...where } k \text{ is a scalar}$$

Thus, we have r is vector of magnitude 6,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

have $\vec{a} = 4i - j + 3k$ and $\vec{b} = -2i + j - 2k$

$$\Rightarrow a_1 = 4, a_2 = -1, a_3 = 3 \text{ and } b_1 = -2, b_2 = 1, b_3 = -2$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-1 \times (-2) - 1 \times (3))i + (3 \times (-2) - (-2) \times 4)j + (4 \times 1 - (-2) \times (-1))k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-1)^2 + (2)^2 + (2)^2} = 3$$

$$\Rightarrow \hat{a} \times \hat{b} = \frac{-i+2j+2k}{3}$$

$$\vec{r} = \pm k \cdot \frac{-i + 2j + 2k}{3}$$

Here, as r is of magnitude 6 thus,

$$k = 6,$$

$$\text{Thus, } \vec{r} = \pm 2(-i + 2j + 2k)$$

Question: 8

Solution:

$$\vec{a} + \vec{b} = 2i + 3j + 4k = \vec{l}$$

$$\vec{a} - \vec{b} = 0i - i - 2k = \vec{m}$$

Let \vec{r} be the vector which is perpendicular to \vec{l} & \vec{m} then we have,

$$\vec{r} = k(\vec{l} \times \vec{m}) \dots \text{where } k \text{ is a scalar}$$

Thus, we have r is vector of magnitude 5,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \vec{l} = 2i + 3j + 4k \text{ and } \vec{m} = 0i - j - 2k$$

$$\Rightarrow a_1 = 2, a_2 = 3, a_3 = 4 \text{ and } b_1 = 0, b_2 = -1, b_3 = -2$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{l} \times \vec{m} = (-2)i + (4)j + (-2)k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-2)^2 + (4)^2 + (-2)^2} = \sqrt{24}$$

$$\Rightarrow \hat{a} \times \hat{b} = \frac{-i+2j-k}{\sqrt{6}}$$

$$\vec{r} = \pm k \cdot \frac{-i + 2j - k}{\sqrt{6}}$$

Here, as r is of magnitude 5 thus,

$$k = 5,$$

$$\text{Thus, } \vec{r} = \pm 5 \left(\frac{-i + 2j - k}{\sqrt{6}} \right)$$

Solution:

We are given that $|\vec{a}| = 1$ and $|\vec{b}| = 2$.

$$\text{And } |\vec{a} \times \vec{b}| = \sqrt{3}.$$

So we have,

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin \theta = \sqrt{3}$$

$$\Rightarrow |\vec{a}| \cdot |\vec{b}| \sin \theta = 1 \times 2 \times \sin \theta$$

$$\Rightarrow 2 \sin \theta = \sqrt{3}$$

$$\Rightarrow \theta = \sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

Question: 10**Solution:**

Given that

Let \vec{d} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{d} = k. (\vec{a} \times \vec{b}) \text{ ...where } k \text{ is a scalar}$$

We have,

$$\vec{a} \times \vec{b} = (a_2 b_3 - b_2 a_3) \hat{i} + (a_3 b_1 - b_3 a_1) \hat{j} + (a_1 b_2 - b_1 a_2) \hat{k}$$

Here,

We

$$\text{have } \vec{a} = \hat{i} - \hat{j} \text{ and } \vec{b} = 0\hat{i} + 3\hat{j} - \hat{k}$$

$$\Rightarrow a_1 = 1, a_2 = -1, a_3 = 0 \text{ and } b_1 = 0, b_2 = 3, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (1)\hat{i} + (1)\hat{j} + (3)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(1)^2 + (1)^2 + (3)^2} = \sqrt{11}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{11}}$$

$$\vec{d} = \pm k. \frac{\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{11}}$$

$$\text{Given that } \vec{c} \cdot \vec{d} = 1$$

$$\vec{c} = 7\hat{i} - \hat{k}$$

$$\Rightarrow \vec{c} \cdot \vec{d} = \frac{7k - 3k}{\sqrt{11}} = 1,$$

$$\Rightarrow k = \frac{\sqrt{11}}{4}$$

$$\Rightarrow \vec{d} = \frac{\hat{i} + \hat{j} + 3\hat{k}}{4}$$

Question: 11

Solution:

Given that

Let \vec{d} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{d} = k(\hat{a} \times \hat{b}) \dots \text{where } k \text{ is a scalar}$$

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 4\mathbf{i} + 5\mathbf{j} - \mathbf{k} \text{ and } \vec{b} = \mathbf{i} - 4\mathbf{j} + \mathbf{k}$$

$$\Rightarrow a_1 = 4, a_2 = 5, a_3 = -1 \text{ and } b_1 = 1, b_2 = -4, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (1)\mathbf{i} + (-5)\mathbf{j} + (-21)\mathbf{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(1)^2 + (-5)^2 + (-21)^2} = \sqrt{467}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{\mathbf{i} - 5\mathbf{j} - 21\mathbf{k}}{\sqrt{467}}$$

$$\vec{d} = \pm k \cdot \frac{\mathbf{i} - 5\mathbf{j} - 21\mathbf{k}}{\sqrt{467}}$$

$$\text{Given that } \vec{c} \cdot \vec{d} = 21$$

$$\vec{c} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$$

$$\Rightarrow \vec{c} \cdot \vec{d} = \frac{19k}{\sqrt{467}} = 21,$$

$$\Rightarrow k = \frac{\sqrt{467}}{19 \times 21}$$

$$\vec{d} = \frac{\mathbf{i} - 5\mathbf{j} - 21\mathbf{k}}{319} \times \sqrt{467}$$

Question: 12

Solution:

$$\text{We know that } |\vec{a} \cdot \vec{b}| = |\vec{a}||\vec{b}|\cos\theta|$$

$$\text{And } |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta|$$

So,

$$\tan\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a} \cdot \vec{b}|}$$

Hence, proved.

Question: 13

Solution:

As the vectors are parallel vectors so, $\vec{a} \times \vec{b} = 0$

Thus,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \vec{a} = 3i + 2j + 9k \text{ and } \vec{b} = i + pj + 3k$$

$$\Rightarrow a_1 = 3, a_2 = 2, a_3 = 9 \text{ and } b_1 = 1, b_2 = p, b_3 = 3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (6 - 9p)i + (0)j + (3p - 2)k = 0$$

$$\Rightarrow 6 - 9p = 0$$

$$\Rightarrow \text{Thus, } p = \frac{2}{3}.$$

Question: 14 A

Solution:

$$\text{To verify } \vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})$$

We need to prove L.H.S = R.H.S

L.H.S we have,

$$\text{Given, } \vec{a} = \hat{i} - \hat{j} - 3\hat{k} \quad \vec{b} = 4\hat{i} - 3\hat{j} + \hat{k} \quad \vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = (i - j - 3k) \times (6i - 4j + 3k)$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \vec{a} = i - j - 3k \text{ and } \vec{b} + \vec{c} = 6i - 4j + 3k$$

$$\Rightarrow a_1 = 1, a_2 = -1, a_3 = -3 \text{ and } b_1 = 6, b_2 = -4, b_3 = 3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times (\vec{b} + \vec{c}) = (-3 - 12)i + (3 + 18)j + (-4 + 6)k$$

$$\Rightarrow (-15)i + (21)j + (2)k$$

RHS is

$$(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-10i + 13j + k) + (-5i + 8j + k)$$

$$\Rightarrow (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-15)i + (21)j + (2)k$$

Thus, LHS = RHS.

Question: 14 B

Solution:

To verify $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})$

We need to prove L.H.S = R.H.S

L.H.S we have,

$$\text{Given, } \vec{a} = 4\hat{i} - \hat{j} + \hat{k} \quad \vec{b} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{c} = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = (4\hat{i} - \hat{j} + \hat{k}) \times (2\hat{i} + 0\hat{j} + 2\hat{k})$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = 4\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{b} + \vec{c} = 2\hat{i} + 0\hat{j} + 2\hat{k}$$

$$\Rightarrow a_1 = 4, a_2 = -1, a_3 = 1 \text{ and } b_1 = 2, b_2 = 0, b_3 = 2$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times (\vec{b} + \vec{c}) = (-2)\hat{i} + (-2)\hat{j} + (2)\hat{k}$$

$$\Rightarrow (-2)\hat{i} + (-2)\hat{j} + (2)\hat{k}$$

RHS is

$$(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-2\hat{i} - 3\hat{j} + 5\hat{k}) + (0\hat{i} + \hat{j} - 3\hat{k})$$

$$\Rightarrow (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-2)\hat{i} + (-2)\hat{j} + (2)\hat{k}$$

Thus, LHS = RHS.

Question: 15 A

Solution:

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where \vec{a} and \vec{b} are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{b} = -3\hat{i} - 2\hat{j} + \hat{k}$$

$$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3 \text{ and } b_1 = -3, b_2 = -2, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (8)\hat{i} + (-10)\hat{j} + (4)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(8)^2 + (-10)^2 + (4)^2} = \sqrt{180}$$

$$\Rightarrow \text{area} = 6\sqrt{5} \text{ sq units}$$

Question: 15 B

Solution:

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k} \text{ and } \vec{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$$

$$\Rightarrow a_1 = 3, a_2 = 1, a_3 = 4 \text{ and } b_1 = 1, b_2 = -1, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (5)\mathbf{i} + (-1)\mathbf{j} + (-4)\mathbf{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(5)^2 + (-1)^2 + (-4)^2} = \sqrt{42}$$

$$\Rightarrow \text{area} = \sqrt{42} \text{ sq units}$$

Question: 15 C

Solution:

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} \text{ and } \vec{b} = \mathbf{i} - \mathbf{j} + 0\mathbf{k}$$

$$\Rightarrow a_1 = 2, a_2 = 1, a_3 = 3 \text{ and } b_1 = 1, b_2 = -1, b_3 = 0$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (3)\mathbf{i} + (3)\mathbf{j} + (-3)\mathbf{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(3)^2 + (3)^2 + (-3)^2} = 3\sqrt{3}$$

$$\Rightarrow \text{area} = 3\sqrt{3} \text{ sq units}$$

Question: 15 D

Solution:

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 2\mathbf{i} + 0\mathbf{j} + 0\mathbf{k} \text{ and } \vec{b} = 0\mathbf{i} + 3\mathbf{j} + 0\mathbf{k}$$

$$\Rightarrow a_1 = 2, a_2 = 0, a_3 = 0 \text{ and } b_1 = 0, b_2 = 3, b_3 = 0$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (6)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = 6$$

$$\Rightarrow \text{area} = 6 \text{ sq units}$$

Question: 16 A

Solution:

The diagonals are $\vec{a} + \vec{b} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ & $\vec{a} - \vec{b} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

Thus, $\vec{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\vec{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where \mathbf{a} and \mathbf{b} are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

have $\vec{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\vec{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$

$$\Rightarrow a_1 = 2, a_2 = -1, a_3 = 1 \text{ and } b_1 = 1, b_2 = 2, b_3 = -3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (3 - 2)\mathbf{i} + 7\mathbf{j} + (5)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(1)^2 + (7)^2 + (5)^2} = 5\sqrt{3}$$

\Rightarrow

$$\Rightarrow \text{area} = 5\sqrt{3} \text{ sq units}$$

Question: 16 B

Solution:

The diagonals are $\vec{a} + \vec{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ & $\vec{a} - \vec{b} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$

Thus, $\vec{a} = \frac{5}{2}\mathbf{i} + \frac{3}{2}\mathbf{j}$, $\vec{b} = -\frac{1}{2}\mathbf{i} - \frac{5}{2}\mathbf{j} + \mathbf{k}$

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where \mathbf{a} and \mathbf{b} are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

have, $\vec{a} = \frac{5}{2}\mathbf{i} + \frac{3}{2}\mathbf{j}$, $\vec{b} = -\frac{1}{2}\mathbf{i} - \frac{5}{2}\mathbf{j} + \mathbf{k}$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = \left(\frac{3}{2}\right)\mathbf{i} - \frac{5}{2}\mathbf{j} + \left(-\frac{11}{2}\right)\mathbf{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{5}{2}\right)^2 + \left(-\frac{11}{2}\right)^2} = \frac{1}{2}\sqrt{155}$$

\Rightarrow

$$\Rightarrow \text{area} = \frac{1}{2}\sqrt{155} \text{ sq units}$$

Question: 16 C

Solution:

The diagonals are $\vec{a} + \vec{b} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ & $\vec{a} - \vec{b} = -\mathbf{i} + 2\mathbf{j} + 0\mathbf{k}$

Thus, $\vec{a} = 0\mathbf{i} - \frac{1}{2}\mathbf{j} + \mathbf{k}$, $\vec{b} = \mathbf{i} - \frac{5}{2}\mathbf{j} + \mathbf{k}$

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 0\mathbf{i} - \frac{1}{2}\mathbf{j} + \mathbf{k} \text{ and } \vec{b} = \mathbf{i} - \frac{5}{2}\mathbf{j} + \mathbf{k}$$

$$\Rightarrow a_1 = 0, a_2 = -\frac{1}{2}, a_3 = 1 \text{ and } b_1 = 1, b_2 = -\frac{5}{2}, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (2)\mathbf{i} + 1\mathbf{j} + \left(\frac{1}{2}\right)\mathbf{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(2)^2 + (1)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2}\sqrt{21}$$

\Rightarrow

$$\Rightarrow \text{area} = \frac{\sqrt{21}}{2} \text{ sq units}$$

Question: 17 A

Solution:

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = -2\mathbf{i} + 0\mathbf{j} - 5\mathbf{k} \text{ and } \vec{b} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$$

$$\Rightarrow a_1 = -2, a_2 = 0, a_3 = -5 \text{ and } b_1 = 1, b_2 = -2, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (8)\mathbf{i} + (-10)\mathbf{j} + (4)\mathbf{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$$

$$\Rightarrow \text{area} = \frac{\sqrt{165}}{2} \text{ sq units}$$

Question: 17 B

Solution:

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 3\mathbf{i} + 4\mathbf{j} + 0\mathbf{k} \text{ and } \vec{b} = -5\mathbf{i} + 7\mathbf{j} + 0\mathbf{k}$$

$$\Rightarrow a_1 = 3, a_2 = 4, a_3 = 0 \text{ and } b_1 = -5, b_2 = 7, b_3 = 0$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (41)\mathbf{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = 41$$

$$\Rightarrow \text{area} = \frac{41}{2} \text{ sq units}$$

Question: 18 A

Solution:

Through the vertices we get the adjacent vectors as,

$$\vec{AB} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \text{ and } \vec{AC} = 4\mathbf{j} + 3\mathbf{k}$$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{AB} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \text{ and } \vec{AC} = 4\mathbf{j} + 3\mathbf{k}$$

$$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3 \text{ and } b_1 = 0, b_2 = 4, b_3 = 3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-6)\mathbf{i} + (-3)\mathbf{j} + (4)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(-6)^2 + (-3)^2 + (4)^2} = \sqrt{61}$$

$$\Rightarrow \text{area} = \frac{\sqrt{61}}{2} \text{ sq units}$$

Question: 18 B

Solution:

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = \mathbf{i} - 3\mathbf{j} + 1\mathbf{k} \text{ and } \overrightarrow{AC} = 3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \overrightarrow{AB} = \mathbf{i} - 3\mathbf{j} + \mathbf{k} \text{ and } \overrightarrow{AC} = 3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$$

$$\Rightarrow a_1 = 1, a_2 = -3, a_3 = 1 \text{ and } b_1 = 3, b_2 = 3, b_3 = -2$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (3)\mathbf{i} + (5)\mathbf{j} + (12)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(3)^2 + (5)^2 + (12)^2} = \sqrt{178}$$

$$\Rightarrow \text{area} = \frac{\sqrt{178}}{2} \text{ sq units}$$

Question: 18 C

Solution:

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = -2\mathbf{i} + 0\mathbf{j} - 5\mathbf{k} \text{ and } \overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \overrightarrow{AB} = -2\mathbf{i} - 5\mathbf{k} \text{ and } \overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$$

$$\Rightarrow a_1 = -2, a_2 = 0, a_3 = -5 \text{ and } b_1 = 1, b_2 = -2, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-10)\mathbf{i} + (-7)\mathbf{j} + (4)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$$

$$\Rightarrow \text{area} = \frac{\sqrt{165}}{2} \text{ sq units}$$

Question: 18 D

Solution:

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \text{ and } \overrightarrow{AC} = 2\mathbf{i}$$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \overrightarrow{AB} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \text{ and } \overrightarrow{AC} = 2\mathbf{i}$$

$$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3 \text{ and } b_1 = 0, b_2 = 4, b_3 = 3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-6) + (-4)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(-6)^2 + (-4)^2} = \sqrt{52}$$

$$\Rightarrow \text{area} = \frac{\sqrt{52}}{2} \text{ sq units}$$

Question: 19 A

Solution:

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = -4\mathbf{i} + 5\mathbf{j} + 7\mathbf{k} \text{ and } \overrightarrow{AC} = 4\mathbf{i} - 5\mathbf{j} - 7\mathbf{k}$$

To prove that A, B, C are collinear we need to prove that

$$\vec{a} \times \vec{b} = 0.$$

So,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \overrightarrow{AB} = -4\mathbf{i} + 5\mathbf{j} + 7\mathbf{k} \text{ and } \overrightarrow{AC} = 4\mathbf{i} - 5\mathbf{j} - 7\mathbf{k}$$

$$\Rightarrow a_1 = -4, a_2 = 5, a_3 = 7 \text{ and } b_1 = 4, b_2 = -5, b_3 = -7$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (0)i + (0)j + (0)k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = 0$$

Question: 19 B

Solution:

Through the vertices we get the adjacent vectors as,

$$\vec{AB} = -4i + 4j + 2k \text{ and } \vec{AC} = -2i + 2j + k$$

To prove that A, B, C are collinear we need to prove that

$$\vec{a} \times \vec{b} = 0.$$

So,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \vec{AB} = -4i + 4j + 2k \text{ and } \vec{AC} = -2i + 2j + k$$

$$\Rightarrow a_1 = -4, a_2 = 4, a_3 = 2 \text{ and } b_1 = -2, b_2 = 2, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (0)i + (0)j + (0)k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = 0$$

Thus, A, B and C are collinear.

Question: 20

Solution:

Through the vertices we get the adjacent vectors as,

$$\vec{AB} = -2i + 3j - 3k \text{ and } \vec{AC} = -4i + 6j - 6k$$

To prove that A, B, C are collinear we need to prove that

$$\vec{a} \times \vec{b} = 0.$$

So,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \vec{AB} = -2i + 3j - 3k \text{ and } \vec{AC} = -4i + 6j - 6k$$

$$\Rightarrow a_1 = -2, a_2 = 3, a_3 = -3 \text{ and } b_1 = -4, b_2 = 6, b_3 = -6$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (0)i + (0)j + (0)k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = 0$$

Thus, A, B and C are collinear.

Question: 21

Solution:

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = \vec{b} - \vec{a} \text{ and } \overrightarrow{AC} = \vec{c} - \vec{a} = 2\vec{a} + 2\vec{b}$$

To prove that A, B, C are collinear we need to prove that

$$\overrightarrow{AB} \times \overrightarrow{AC} = 0.$$

So,

Here,

We

$$\text{have } \overrightarrow{AB} = \vec{b} - \vec{a} \text{ and } \overrightarrow{AC} = 2\vec{a} + 2\vec{b}$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = (\vec{b} - \vec{a}) \times (2\vec{a} + 2\vec{b})$$

$$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = \vec{b} \times 2\vec{a} + 0 - 0 - \vec{a} \times 2\vec{b} = 0$$

Thus, A, B and C are collinear.

Question: 22

Solution:

$$\text{We have, } A = -2\vec{a} + 3\vec{b} + 5\vec{c}, B = \vec{a} + 2\vec{b} + 3\vec{c}, C = 7\vec{a} - \vec{c}$$

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = 3\vec{a} - \vec{b} - 2\vec{c} \text{ and } \overrightarrow{AC} = 9\vec{a} - 3\vec{b} - 6\vec{c}$$

To prove that A, B, C are collinear we need to prove that

$$\overrightarrow{AB} \times \overrightarrow{AC} = 0.$$

So,

Here,

We

have

$$\overrightarrow{AB} = 3\vec{a} - \vec{b} - 2\vec{c} \text{ and } \overrightarrow{AC} = 9\vec{a} - 3\vec{b} - 6\vec{c}$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = (3\vec{a} - \vec{b} - 2\vec{c}) \times (9\vec{a} - 3\vec{b} - 6\vec{c})$$

$$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = 0$$

Thus, A, B and C are collinear.

Question: 23

Solution:

A unit vector perpendicular to the plane ABC will be,

$$\pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = -2\mathbf{i} + 0\mathbf{j} - 5\mathbf{k} \text{ and } \overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \overrightarrow{AB} = -2\mathbf{i} + 0\mathbf{j} - 5\mathbf{k} \text{ and } \overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$$

$$\Rightarrow a_1 = -2, a_2 = 0, a_3 = -5 \text{ and } b_1 = 1, b_2 = -2, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-10)\mathbf{i} + (-7)\mathbf{j} + (4)\mathbf{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$$

$$\Rightarrow \text{unit vector} = \frac{-10\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}}{\sqrt{165}}$$

Question: 24

Solution:

$$\vec{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \text{ and } \vec{b} = \mathbf{i} - 3\mathbf{k}$$

$$\text{Then, } |\vec{b} \times \vec{a}|,$$

$$\text{We have, } \vec{b} \times \vec{a} = (-2a_2b_3 + 2b_2a_3)\mathbf{i} - (a_3b_1 - 2b_3a_1)\mathbf{j} - (a_1b_2 - 2b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \text{ and } \vec{b} = \mathbf{i} - 3\mathbf{k}$$

$$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3 \text{ and } b_1 = 1, b_2 = 0, b_3 = -3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-12)\mathbf{i} + (12)\mathbf{j} + (-4)\mathbf{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-12)^2 + (12)^2 + (-4)^2} = 4\sqrt{19}$$

Question: 25

Solution:

$$\text{We have, } |\vec{a}|^2 |\vec{b}|^2 = |\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2$$

$$\text{So, } |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - |\vec{a} \times \vec{b}|^2$$

$$\Rightarrow |\vec{a} \cdot \vec{b}|^2 = 10^2 - 8^2 = 6^2$$

$$\Rightarrow |\vec{a} \cdot \vec{b}| = 6$$

Solution:

We have, $|\vec{a}|^2 |\vec{b}|^2 = |\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2$

$$\Rightarrow \vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{3^2 + 2^2 + 6^2} = 7$$

$$\Rightarrow 7 = 7 \times 2 \sin \theta$$

$$\Rightarrow \sin \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \sin^{-1} \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

