Chapter : 25. PRODUCT OF THREE VECTORS

Exercise : 25A

Question: 1

Solution:

i. $[\hat{\imath}\quad\hat{\jmath}\quad\hat{k}] = [\hat{\jmath}\quad\hat{k}\quad\hat{\imath}] = [\hat{k}\quad\hat{\imath}\quad\hat{\jmath}] = 1$

Let, $\hat{\imath}, \hat{\jmath}, \hat{k}$ be unit vectors in the direction of positive X-axis, Y-axis, Z-axis respectively.

Hence,

$Magnitude\ of\ \hat{\imath}\ is\ 1 \Rightarrow |\hat{\imath}| = 1$

$Magnitude\ of\ \hat{\jmath}\ is\ 1 \Rightarrow |\hat{\jmath}| = 1$

$Magnitude\ of\ \hat{k}\ is\ 1 \Rightarrow |\hat{k}| = 1$

To Prove :

$[\hat{\imath}\quad\hat{\jmath}\quad\hat{k}] = [\hat{\jmath}\quad\hat{k}\quad\hat{\imath}] = [\hat{k}\quad\hat{\imath}\quad\hat{\jmath}] = 1$

Formulae :

a) Dot Products :

i) $\hat{\imath}\,.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

ii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

b) Cross Products :

i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$

iii) $\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$

c) Scalar Triple Product :

$[\bar{a}\quad\bar{b}\quad\bar{c}] = \bar{a}\,.(\bar{b} \times \bar{c})$

Now,

(i) $[\hat{\imath}\quad\hat{\jmath}\quad\hat{k}] = \hat{\imath}\,.(\hat{\jmath} \times \hat{k})$

$= \hat{\imath}\,.\hat{\imath}$ .... $(\because \hat{\jmath} \times \hat{k} = \hat{\imath})$

= 1 .... $(\because \hat{\imath}\,.\hat{\imath} = 1)$

$\therefore\ [\hat{\imath}\quad\hat{\jmath}\quad\hat{k}] = 1$ .... eq(1)

(ii) $[\hat{\jmath}\quad\hat{k}\quad\hat{\imath}] = \hat{\jmath}\,.(\hat{k} \times \hat{\imath})$

$= \hat{\jmath}\,.\hat{\jmath}$ .... $(\because \hat{k} \times \hat{\imath} = \hat{\jmath})$

= 1 .... $(\because \hat{\jmath}\,.\hat{\jmath} = 1)$

$\therefore\ [\hat{\jmath}\quad\hat{k}\quad\hat{\imath}] = 1$ .... eq(2)

(iii) $[\hat{k}\quad\hat{\imath}\quad\hat{\jmath}] = \hat{k}\,.(\hat{\imath} \times \hat{\jmath})$

$= \hat{k}\,.\hat{k}$ .... $(\because \hat{\imath} \times \hat{\jmath} = \hat{k})$

= 1 .... $(\because \hat{k}\,.\hat{k} = 1)$

$\therefore\ [\hat{k}\quad \hat{\imath}\quad \hat{\jmath}] = 1$ ………… eq(3)

From eq(1), eq(2) and eq(3),

$[\hat{\imath}\quad \hat{\jmath}\quad \hat{k}] = [\hat{\jmath}\quad \hat{k}\quad \hat{\imath}] = [\hat{k}\quad \hat{\imath}\quad \hat{\jmath}] = 1$

Hence Proved.

Notes :

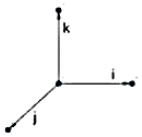
1. A cyclic change of vectors in a scalar triple product does not change its value i.e.

$[\bar{a}\quad \bar{b}\quad \bar{c}] = [\bar{b}\quad \bar{c}\quad \bar{a}] = [\bar{c}\quad \bar{a}\quad \bar{b}]$

2. Scalar triple product of unit vectors taken in a clockwise direction is 1, and that of unit vectors taken in anticlockwise direction is -1

$[\hat{\imath}\quad \hat{\jmath}\quad \hat{k}] = 1$

$[\hat{k}\quad \hat{\jmath}\quad \hat{\imath}] = -1$



ii. $[\hat{\imath}\quad \hat{k}\quad \hat{\jmath}] = [\hat{k}\quad \hat{\jmath}\quad \hat{\imath}] = [\hat{\jmath}\quad \hat{\imath}\quad \hat{k}] = -1$

Let, $\hat{\imath}, \hat{\jmath}, \hat{k}$ be unit vectors in the direction of positive X-axis, Y-axis, Z-axis respectively.

Hence,

$Magnitude\ of\ \hat{\imath}\ is\ 1 \Rightarrow |\hat{\imath}| = 1$

$Magnitude\ of\ \hat{\jmath}\ is\ 1 \Rightarrow |\hat{\jmath}| = 1$

$Magnitude\ of\ \hat{k}\ is\ 1 \Rightarrow |\hat{k}| = 1$

To Prove :

$[\hat{\imath}\quad \hat{k}\quad \hat{\jmath}] = [\hat{k}\quad \hat{\jmath}\quad \hat{\imath}] = [\hat{\jmath}\quad \hat{\imath}\quad \hat{k}] = -1$

Formulae :

a) Dot Products :

i) $\hat{\imath}\,.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

ii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

b) Cross Products :

i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$

iii) $\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$

c) Scalar Triple Product :

$[\bar{a}\quad \bar{b}\quad \bar{c}] = \bar{a}\,.(\bar{b} \times \bar{c})$

Answer :

(i) $[\hat{\imath}\quad \hat{k}\quad \hat{\jmath}] = \hat{\imath}\,.(\hat{k} \times \hat{\jmath})$

$= \hat{\imath}\,.(-\hat{\imath})$ ………… $(\because \hat{k} \times \hat{\jmath} = -\hat{\imath})$

$= -\hat{\imath}\,.\hat{\imath}$

= -1 ………… $(\because \hat{\imath}\,.\,\hat{\imath} = 1)$

$\therefore\ [\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = -1$ ………… eq(1)

(ii) $[\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = \hat{k}\,.\,(\hat{\jmath} \times \hat{\imath})$

$= \hat{k}\,.\,(-\hat{k})$ ………… $(\because \hat{\jmath} \times \hat{\imath} = -\hat{k})$

$= -\hat{k}\,.\,\hat{k}$

= -1 ………… $(\because \hat{k}\,.\,\hat{k} = 1)$

$\therefore\ [\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = -1$ ………… eq(2)

(iii) $[\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = \hat{\jmath}\,.\,(\hat{\imath} \times \hat{k})$

$= \hat{\jmath}\,.\,(-\hat{\jmath})$ ………… $(\because \hat{\imath} \times \hat{k} = -\hat{\jmath})$

$= -\hat{\jmath}\,.\,\hat{\jmath}$

= -1 ………… $(\because \hat{\jmath}\,.\,\hat{\jmath} = 1)$

$\therefore\ [\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = -1$ ………… eq(3)

From eq(1), eq(2) and eq(3),

$[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = [\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = [\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = -1$

Hence Proved.

Notes :

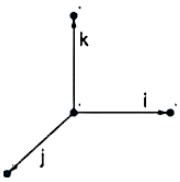
1. A cyclic change of vectors in a scalar triple product does not change its value i.e.

$[\bar{a} \quad \bar{b} \quad \bar{c}] = [\bar{b} \quad \bar{c} \quad \bar{a}] = [\bar{c} \quad \bar{a} \quad \bar{b}]$

2. Scalar triple product of unit vectors taken in a clockwise direction is 1, and that of unit vectors taken in anticlockwise direction is -1

$[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = 1$

$[\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = -1$



Question: 3

Find i. $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

2) $\bar{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

3) $\bar{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$\bar{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\bar{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$= 2(2 \times 2 - 1 \times 1) - 1((-1) \times 2 - 3 \times 1) + 3((-1) \times 1 - 3 \times 2)$

$= 2(3) - 1(-5) + 3(-7)$

= 6 + 5 − 21

= - 10

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -10$$

ii. $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

2) $\bar{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

3) $\bar{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\bar{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$= 2(2 \times 2 - (-1) \times (-1)) - (-3)(1 \times 2 - 3 \times (-1)) + 4(1 \times (-1) - 3 \times 2)$

$= 2(3) + 3(5) + 4(-7)$

= 6 + 15 – 28

= - 7

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -7$

iii. $\vec{a} = 2\hat{i} - 3\hat{j}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} - 3\hat{\jmath}$

2) $\bar{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$

3) $\bar{c} = 3\hat{\imath} - \hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 0\hat{k}$

$\bar{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = 3\hat{\imath} + 0\hat{\jmath} - \hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$$

$= 2(1 \times (-1) - (-1) \times 0) - (-3)(1 \times (-1) - 3 \times (-1)) + 0(1 \times 0 - 3 \times 1)$

$= 2(-1) + 3(2) + 0$

= - 2 + 6

= 4

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 4$$

Question: 3

Solution:

i. $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} - 2\hat{j} - \hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{c} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

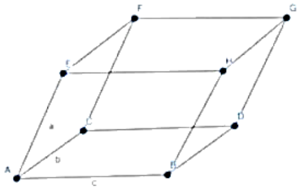
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{c} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$= 1((-1) \times (-1) - 2 \times 1) - 1(1 \times (-1) - 1 \times 1) + 1(1 \times 2 - 1 \times (-1))$

= 1(-1) -1(-2) + 1(3)

= -1+2+3

= 4

Therefore,

Volume of parallelepiped = 4 cubic unit

ii. $\bar{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \bar{b} = -5\hat{i} + 7\hat{j} - 3\hat{k},$
$\bar{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$

$\bar{b} = -5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$

$\bar{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

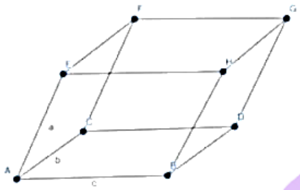
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$

$\bar{b} = -5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$

$\bar{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$$

$$= -3\big(7 \times (-3) - (-5) \times (-3)\big) - 7\big((-5) \times (-3) - 7 \times (-3)\big) + 5\big((-5) \times (-5) - 7 \times 7\big)$$

= -3(-36) -7(36) + 5(-24)

= 108 – 252 – 120

= -264

As volume is never negative

Therefore,

Volume of parallelepiped = 264 cubic unit

iii. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \hat{\jmath} + \hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

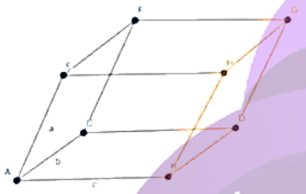
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = 0\hat{\imath} + \hat{\jmath} + \hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$= 1(1 \times 1 - 1 \times (-1)) - (-2)(2 \times 1 - 0 \times (-1)) + 3(2 \times 1 - 0 \times 1)$

= 1(2) +2(2) + 3(2)

= 2 + 4 + 6

= 12

Therefore,

Volume of parallelepiped = 12 cubic unit

iv. $\bar{a} = 6\hat{\imath}, \bar{b} = 2\hat{\jmath}, \bar{c} = 5\hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = 6\hat{\imath}$

$\bar{b} = 2\hat{\jmath}$

$\bar{c} = 5\hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

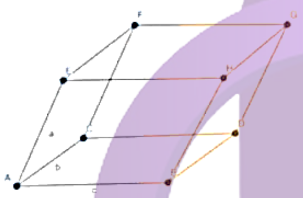
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = 6\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

$\bar{b} = 0\hat{\imath} + 2\hat{\jmath} + 0\hat{k}$

$\bar{c} = 0\hat{\imath} + 0\hat{\jmath} + 5\hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{vmatrix}$$

$= 6(2 \times 5 - 0 \times 0) - 0(0 \times 5 - 0 \times 0) + 0(0 \times 0 - 0 \times 2)$

$= 6(10) + 0 + 0$

$= 60$

Therefore,

Volume of parallelepiped = 60 cubic unit

Question: 4

Solution:

i. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

Given Vectors :

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = -2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

$\bar{c} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

CLASS24

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = -2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

$\bar{c} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= 1(3 \times 5 - (-3) \times (-4)) - (-2)((-2) \times 5 - 1 \times (-4)) + 3((-2) \times (-3) - 3 \times 1)$$

= 1(3) + 2(-6) + 3(3)

= 3 – 12 +9

= 0

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

ii. $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = 7\hat{j} + 3\hat{k}$

Given Vectors :

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Formulae :

1) Scalar Triple Product:

CLASS24

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix}$$

$= 1((-1) \times 3 - 7 \times (-1)) - 3(2 \times 3 - 0 \times (-1)) + 1(2 \times 7 - 0 \times (-1))$

$= 1(4) - 3(6) + 1(14)$

$= 4 - 18 + 14$

$= 0$

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

iii. $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$

Given Vectors :

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 2 \\ 1 & 2 & -3 \\ 3 & -4 & 7 \end{vmatrix}$$

$= 2(2 \times 7 - (-3) \times (-4)) - (-1)(1 \times 7 - 3 \times (-3)) + 2(1 \times (-4) - 3 \times 2)$

$= 2(2) + 1(16) + 2(-10)$

$= 4 + 16 - 20$

$= 0$

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Question: 5

Solution:

i. . $\bar{a} = \left(2\hat{i} - \hat{j} + \hat{k}\right), \bar{b} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$. and $\bar{c} = \left(3\hat{i} + \lambda\hat{j} + 5\hat{k}\right)$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{c} = 3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{c} = 3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & \lambda & 5 \end{vmatrix}$$

$= 2(2 \times 5 - 3 \times \lambda) - (-1)(1 \times 5 - 3 \times 3) + 1(1 \times \lambda - 3 \times 2)$

$= 2(10 - 3\lambda) - 4 + 1(\lambda - 6)$

$= 20 - 6\lambda - 4 + \lambda - 6$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 10 - 5\lambda$ ....eq(2)

From eq(1) and eq(2),

$10 - 5\lambda = 0$

$\therefore 5\lambda = 10$

$\therefore \lambda = 2$

ii. $\vec{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}, \vec{b} = -7\hat{i} - 5\hat{j}$ and $\vec{c} = \hat{i} - 4\hat{j} - 3\hat{k}$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$\bar{a} = \lambda\hat{\imath} - 10\hat{\jmath} - 5\hat{k}$

$\bar{b} = -7\hat{\imath} - 5\hat{\jmath}$

$\bar{c} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}$

$\bar{b} = -7\hat{i} - 5\hat{j} + 0\hat{k}$

$\bar{c} = \hat{i} - 4\hat{j} - 3\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} \lambda & -10 & -5 \\ -7 & -5 & 0 \\ 1 & -4 & -3 \end{vmatrix}$$

$$= \lambda((-5) \times (-3) - 0 \times (-4)) - (-10)((-7) \times (-3) - 0 \times 1) + (-5)((-7) \times (-4) - 1 \times (-5))$$

$= \lambda(15) + 10(21) - 5(33)$

$= 15\lambda + 45$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 15\lambda + 45$ ....eq(2)

From eq(1) and eq(2),

$15\lambda + 45 = 0$

$\therefore 15\lambda = 45$

$\therefore \lambda = -3$

iii. $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$\bar{a} = \hat{i} - \hat{j} + \hat{k}$

$\bar{b} = 2\hat{i} + \hat{j} - \hat{k}$

$\bar{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \lambda\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix}$$

$= 1(1 \times \lambda - (-1) \times (-1)) - (-1)(2 \times \lambda - (-1) \times \lambda) + 1(2 \times (-1) - \lambda \times 1)$

$= 1(\lambda - 1) + 1(3\lambda) + 1(-\lambda - 2)$

$= \lambda - 1 + 3\lambda - 2 - \lambda$

$= 3\lambda - 3$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 3\lambda - 3$ ....eq(2)

From eq(1) and eq(2),

$3\lambda - 3 = 0$

$\therefore 3\lambda = 3$

$\therefore \lambda = 1$

Question: 6

If Given Vectors :

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} - \hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} - \hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & -3 & -5 \\ 3 & -4 & -1 \end{vmatrix}$$

$$= 2((-3) \times (-1) - (-4) \times (-5)) - (-1)((-1) \times 1 - 3 \times (-5)) + 1((-4) \times 1 - 3 \times (-3))$$

$= 2(-17) + 1(14) + 1(5)$

= - 34 + 14 + 5

= - 15

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -15$$

Question: 7

Solution:

Given :

1) Coterminous edges of parallelepiped are

$\bar{a} = -12\hat{\imath} + \lambda\hat{k}$

$\bar{b} = 3\hat{\jmath} - \hat{k}$

$\bar{c} = 2\hat{\imath} + \hat{\jmath} - 15\hat{k}$

2) Volume of parallelepiped,

V = 546 cubic unit

To Find : value of λ

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

Given volume of parallelepiped,

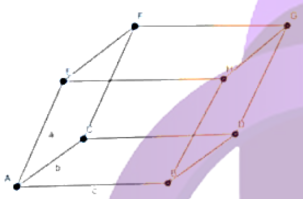
V = 546 cubic unit ....eq(1)

Volume of parallelopiped with coterminous edges

$\bar{a} = -12\hat{\imath} + \lambda\hat{k}$

$\bar{b} = 3\hat{\jmath} - \hat{k}$

$\bar{c} = 2\hat{\imath} + \hat{\jmath} - 15\hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} -12 & 0 & \lambda \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix}$$

$= -12(3 \times (-15) - 1 \times (-1)) - 0 + \lambda(0 \times 1 - 3 \times 2)$

$= 528 - 0 - 6\,\lambda$

$= 528 - 6\,\lambda$

$\therefore V = (528 - 6\,\lambda)$ cubic unit ....eq(2)

From eq(1) and eq(2)

$528 - 6\,\lambda = 546$

$\therefore -6\,\lambda = 18$

$\therefore \lambda = -3$

Question: 8

Solution:

Given Vectors :

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are parallel to same plane.

CLASS24

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

Vectors will be parallel to the same plane if they are coplanar.

For vectors $\bar{a}, \bar{b}, \bar{c}$ to be coplanar, $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Now, for given vectors,

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix}$$

$= 1(3 \times (-1) - 7 \times (-1)) - 3(2 \times 3 - 0 \times (-1)) + 1(2 \times 7 - 0 \times (-1))$

= 1(4) – 3(6) + 1(14)

= 4 – 18 + 14

= 0

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Hence, given vectors are parallel to the same plane.

Question: 9

Solution:

Given : vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar. Where,

$\bar{a} = a\hat{\imath} + a\hat{\jmath} + c\hat{k}$

$\bar{b} = \hat{\imath} + \hat{k}$

$\bar{c} = c\hat{\imath} + c\hat{\jmath} + b\hat{k}$

To Prove : $c^2 = ab$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = a\hat{\imath} + a\hat{\jmath} + c\hat{k}$

$\bar{b} = \hat{\imath} + \hat{k}$

$\bar{c} = c\hat{\imath} + c\hat{\jmath} + b\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix}$$

$= a(0 \times b - c \times 1) - a(1 \times b - 1 \times c) + c(1 \times c - 0 \times c)$

= a.(- c) – a.(b - c) + c(c)

= – ac – ab + ac + c²

= - ab + c²

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -ab + c^2$ ....eq(2)

From eq(1) and eq(2),

- ab + c² = 0

Therefore,

$c^2 = ab$

Hence proved.

Note : Three vectors $\bar{a}, \bar{b}$ & $\bar{c}$ are coplanar if and only if

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Question: 10

Solution:

Given :

Let A, B, C & D be four points with position vectors $\bar{a}, \bar{b}, \bar{c}$ & $\bar{d}$:

Therefore,

$\bar{a} = 4\hat{\imath} + 8\hat{\jmath} + 12\hat{k}$

$\bar{b} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

$\bar{c} = 3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

$\bar{d} = 5\hat{\imath} + 8\hat{\jmath} + 5\hat{k}$

To Prove : Points A, B, C & D are coplanar.

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = 4\hat{\imath} + 8\hat{\jmath} + 12\hat{k}$

$\bar{b} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

$\bar{c} = 3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

$\bar{d} = 5\hat{\imath} + 8\hat{\jmath} + 5\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (4 - 2)\hat{\imath} + (8 - 4)\hat{\jmath} + (12 - 6)\hat{k}$

$\therefore\ \overline{BA} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (4-3)\hat{\imath} + (8-5)\hat{\jmath} + (12-4)\hat{k}$

$\therefore\ \overline{CA} = \hat{\imath} + 3\hat{\jmath} + 8\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (4-5)\hat{\imath} + (8-8)\hat{\jmath} + (12-5)\hat{k}$

$\therefore\ \overline{DA} = -\hat{\imath} + 0\hat{\jmath} + 7\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

$\overline{CA} = \hat{\imath} + 3\hat{\jmath} + 8\hat{k}$

$\overline{DA} = -\hat{\imath} + 0\hat{\jmath} + 7\hat{k}$

$$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 3 & 8 \\ -1 & 0 & 7 \end{vmatrix}$$

$= 2(3\times 7 - 0\times 8) - 4(1\times 7 - (-1)\times 8) + 6(1\times 0 - (-1)\times 3)$

= 2(21) – 4(15) + 6(3)

= 42 – 60 + 18

= 0

$\therefore [\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$

Question: 11

Solution:

Given :

Let A, B, C & D be four points with position vectors $\bar{a}, \bar{b}, \bar{c}$ & $\bar{d}$.

Therefore,

$\bar{a} = 6\hat{\imath} - 7\hat{\jmath}$

$\bar{b} = 16\hat{\imath} - 19\hat{\jmath} - 4\hat{k}$

$\bar{c} = 3\hat{\jmath} - 6\hat{k}$

$\bar{d} = 2\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$

To Prove : Points A, B, C & D are coplanar.

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = 6\hat{\imath} - 7\hat{\jmath}$

$\bar{b} = 16\hat{\imath} - 19\hat{\jmath} - 4\hat{k}$

$\bar{c} = 3\hat{\jmath} - 6\hat{k}$

$\bar{d} = 2\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (6 - 16)\hat{\imath} + (-7 + 19)\hat{\jmath} + (0 + 4)\hat{k}$

$\therefore \ \overline{BA} = -10\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (6 - 0)\hat{\imath} + (-7 - 3)\hat{\jmath} + (0 + 6)\hat{k}$

$\therefore \ \overline{CA} = 6\hat{\imath} - 10\hat{\jmath} + 6\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (6 - 2)\hat{\imath} + (-7 + 5)\hat{\jmath} + (0 - 10)\hat{k}$

$\therefore \ \overline{DA} = 4\hat{\imath} - 2\hat{\jmath} - 10\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -10\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$

$\overline{CA} = 6\hat{\imath} - 10\hat{\jmath} + 6\hat{k}$

$\overline{DA} = 4\hat{\imath} - 2\hat{\jmath} - 10\hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} -10 & 12 & 4 \\ 6 & -10 & 6 \\ 4 & -2 & -10 \end{vmatrix}$$

$$= -10\big((-10) \times (-10) - (-2) \times 6\big) - 12(6 \times (-10) - 4 \times 6) + 4(6 \times (-2) - (-10) \times 4)$$

= -10(112) – 12(-84) + 4(28)

= -1120 + 1008 + 112

= 0

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Question: 12

Find the value of

Solution:

Given :

Let, A, B, C & D be four points with given position vectors

$\bar{a} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{c} = -2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$\bar{d} = 6\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

To Find : value of λ

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{c} = -2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$\bar{d} = 6\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (1-3)\hat{\imath} + (2+1)\hat{\jmath} + (3-2)\hat{k}$

$\therefore\ \overline{BA} = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (1+2)\hat{\imath} + (2-\lambda)\hat{\jmath} + (3-1)\hat{k}$

$\therefore\ \overline{CA} = 3\hat{\imath} + (2-\lambda)\hat{\jmath} + 2\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (1-6)\hat{\imath} + (2+4)\hat{\jmath} + (3-2)\hat{k}$

$\therefore\ \overline{DA} = -5\hat{\imath} + 6\hat{\jmath} + \hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\overline{CA} = 3\hat{\imath} + (2-\lambda)\hat{\jmath} + 2\hat{k}$

$\overline{DA} = -5\hat{\imath} + 6\hat{\jmath} + \hat{k}$

$$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = \begin{vmatrix} -2 & 3 & 1 \\ 3 & (2-\lambda) & 2 \\ -5 & 6 & 1 \end{vmatrix}$$

$$= -2\big((2-\lambda)\times 1 - 2\times 6\big) - 3(3\times 1 - 2\times(-5)) + 1(6\times 3 - (2-\lambda)\times(-5))$$

= -2(- λ – 10) – 3(13) + 1(28 – 5λ)

= 2λ + 20 – 39 + 28 – 5λ

= 9 – 3λ

$\therefore [\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 9 - 3\lambda$ ....eq(4)

Four points A, B, C & D are coplanar if and only if

$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$ ....eq(5)

From eq(4) and eq(5)

9 – 3λ = 0

3λ = 9

λ = 3

Question: 13

Solution:

Given :

Let, A, B, C & D be four points with given position vectors

$\bar{a} = -\hat{j} + \hat{k}$

$\bar{b} = 2\hat{i} - \hat{j} - \hat{k}$

$\bar{c} = \hat{i} + \lambda\hat{j} + \hat{k}$

$\bar{d} = 3\hat{j} + 3\hat{k}$

To Find : value of λ

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = -\hat{j} + \hat{k}$

$\bar{b} = 2\hat{i} - \hat{j} - \hat{k}$

$\bar{c} = \hat{i} + \lambda\hat{j} + \hat{k}$

$\bar{d} = 3\hat{j} + 3\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (0 - 2)\hat{i} + (-1 + 1)\hat{j} + (1 + 1)\hat{k}$

$\therefore \overline{BA} = -2\hat{i} + 0\hat{j} + 2\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (0-1)\hat{\imath} + (-1-\lambda)\hat{\jmath} + (1-1)\hat{k}$

$\therefore\ \overline{CA} = -\hat{\imath} + (-1-\lambda)\hat{\jmath} + 0\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (0-0)\hat{\imath} + (-1-3)\hat{\jmath} + (1-3)\hat{k}$

$\therefore\ \overline{DA} = 0\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -2\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$

$\overline{CA} = -\hat{\imath} + (-1-\lambda)\hat{\jmath} + 0\hat{k}$

$\overline{DA} = 0\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$

$$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = \begin{vmatrix} -2 & 0 & 2 \\ -1 & (-1-\lambda) & 0 \\ 0 & -4 & -2 \end{vmatrix}$$

$$= -2\big((-1-\lambda)\times(-2) - (-4)\times 0\big) - 0\big((-1)\times(-2) - 0\times 0\big) + 2\big((-1)\times(-4) - (-1-\lambda)\times 0\big)$$

$= -2(2 + 2\lambda) - 0 + 2(4)$

$= -4 - 4\lambda + 8$

$= 4 - 4\lambda$

$\therefore [\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 4 - 4\lambda$ ....eq(4)

Four points A, B, C & D are coplanar if and only if

$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$ ....eq(5)

From eq(4) and eq(5)

$4 - 4\lambda = 0$

$4\lambda = 4$

$\boxed{\lambda = 1}$

Question: 14

Solution:

Given Points :

A ≡ (4, 5, 1)

B ≡ (0, -1, -1)

C ≡ (3, 9, 4)

D ≡ (-4, 4, 4)

To Prove : Points A, B, C & D are coplanar.

Formulae :

4) Position Vectors :

If A is a point with co-ordinates (a_1, a_2, a_3)

then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

5) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$

Where,

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$

6) Scalar Triple Product:

If

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

7) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given points,

A ≡ (4, 5, 1)

B ≡ (0, -1, -1)

C ≡ (3, 9, 4)

D ≡ (-4, 4, 4)

Position vectors of above points are,

$\bar{a} = 4\hat{i} + 5\hat{j} + \hat{k}$

$\bar{b} = 0\hat{i} - \hat{j} - \hat{k}$

$\bar{c} = 3\hat{i} + 9\hat{j} + 4\hat{k}$

$\bar{d} = -4\hat{i} + 4\hat{j} + 4\hat{k}$

Vectors $\overline{BA}$, $\overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (4 - 0)\hat{i} + (5 + 1)\hat{j} + (1 + 1)\hat{k}$

$\therefore\ \overline{BA} = 4\hat{i} + 6\hat{j} + 2\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (4 - 3)\hat{i} + (5 - 9)\hat{j} + (1 - 4)\hat{k}$

$\therefore\ \overline{CA} = \hat{i} - 4\hat{j} - 3\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (4+4)\hat{\imath} + (5-4)\hat{\jmath} + (1-4)\hat{k}$

$\therefore\ \overline{DA} = 8\hat{\imath} + 1\hat{\jmath} - 3\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = 4\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$

$\overline{CA} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

$\overline{DA} = 8\hat{\imath} + 1\hat{\jmath} - 3\hat{k}$

$$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = \begin{vmatrix} 4 & 6 & 2 \\ 1 & -4 & -3 \\ 8 & 1 & -3 \end{vmatrix}$$

$$= 4((-4)\times(-3) - 1\times(-3)) - 6(1\times(-3) - (-3)\times 8) + 2(1\times 1 - (-4)\times 8)$$

= 4(15) – 6(21) + 2(33)

= 60 – 126 + 66

= 0

$\therefore [\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$

Question: 15

Solution:

Given :

Points A, B, C & D are coplanar where,

A ≡ (3, 2, 1)

B ≡ (4, λ, 5)

C ≡ (4, 2, -2)

D ≡ (6, 5, -1)

To Find : value of λ

Formulae :

1) Position Vectors :

If A is a point with co-ordinates (a_1, a_2, a_3)

then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

3) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

4) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given points,

A ≡ (3, 2, 1)

B ≡ (4, λ, 5)

C ≡ (4, 2, -2)

D ≡ (6, 5, -1)

Position vectors of above points are,

$\bar{a} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\bar{b} = 4\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$

$\bar{c} = 4\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

$\bar{d} = 6\hat{\imath} + 5\hat{\jmath} - \hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (3-4)\hat{\imath} + (2-\lambda)\hat{\jmath} + (1-5)\hat{k}$

$\therefore\ \overline{BA} = -\hat{\imath} + (2-\lambda)\hat{\jmath} - 4\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (3-4)\hat{\imath} + (2-2)\hat{\jmath} + (1+2)\hat{k}$

$\therefore\ \overline{CA} = -\hat{\imath} + 0\hat{\jmath} + 3\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (3-6)\hat{\imath} + (2-5)\hat{\jmath} + (1+1)\hat{k}$

$\therefore\ \overline{DA} = -3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -\hat{\imath} + (2-\lambda)\hat{\jmath} - 4\hat{k}$

$\overline{CA} = -\hat{\imath} + 0\hat{\jmath} + 3\hat{k}$

$\overline{DA} = -3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} -1 & (2-\lambda) & -4 \\ -1 & 0 & 3 \\ -3 & -3 & 2 \end{vmatrix}$$

$$= -1\big(0 \times 2 - 3 \times (-3)\big) - (2-\lambda)(2 \times (-1) - (-3) \times 3) - 4((-1) \times (-3) - (-3) \times 0)$$

= - 1(9) – (2 - λ).(7) – 4(3)

= - 9 – 14 + 7λ – 12

= 7λ – 35

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 7\lambda - 35$ ………… eq(4)

But points A, B, C & D are coplanar if and only if

$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$ ………… eq(5)

From eq(4) and eq(5)

7λ – 35 = 0

$\therefore 7\lambda = 35$

Exercise : 25B

Question: 1

If $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}, \vec{b} = 2\vec{i} + \vec{j} - \vec{k}, \vec{c} = \vec{j} + \vec{k}$

$\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$

$\vec{b} = 3\hat{\imath} - y\hat{\jmath} + \hat{k}$

Since, these two vectors are equal, therefore comparing these two vectors we get,

x = 3 , - y = 2 , - z = 1

⇒x = 3,y = - 2,z = - 1

∴x + y + z = 3 + (- 2) + (- 1) = 3 - 2 - 1 = 0

Ans:x + y + z = 0

Question: 2

Solution:

Let $\vec{s}$ be the sum of the vectors $\vec{a}$ and $\vec{b}$

$\Rightarrow \vec{s} = \vec{a} + \vec{b}$

$\Rightarrow \vec{s} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k} + 2\hat{\imath} + \hat{\jmath} - 7\hat{k}$

$\Rightarrow \vec{s} = 4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}$

$|\vec{s}| = (4^2 + 3^2 + (-12)^2)^{1/2}$

$\Rightarrow |\vec{s}| = (16 + 9 + 144)^{1/2} = (169)^{1/2} = 13$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}}{13}$$

Ans: $\hat{s} = \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{13}$

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Question: 3

Solution:

$\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$

$\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

Since these two vectors are perpendicular the dot product of these two vectors is zero.

i.e.: $\vec{a}.\vec{b} = 0$

$\Rightarrow (2\hat{i} + \lambda\hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + 3\hat{k}) = 0$

$\Rightarrow 2 + \lambda \times (-2) + 3 = 0$

$\Rightarrow 5 = 2\lambda$

$\Rightarrow \lambda = 5/2$

Ans: $\lambda = 5/2$

Question: 4

Find the value of

Solution:

$\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$

$\vec{b} = \hat{i} - 2p\hat{j} + 3\hat{k}$

Since these two vectors are parallel

$\therefore \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$

$\Rightarrow \frac{3}{1} = \frac{1}{-p}$

$\Rightarrow p = \frac{-1}{3}$

Ans: $p = \frac{-1}{3}$

Question: 5

Solution:

$\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$

$\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (2^2 + 6^2 + 3^2)^{1/2}$

$\Rightarrow |\vec{b}| = (4 + 36 + 9)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{b}{|\vec{b}|} = \frac{2\hat{i} + 6\hat{j} + 3\hat{k}}{7}$$

Now it is given that: $\vec{a}.\hat{b} = 4$

$\Rightarrow (\lambda\hat{i} + \hat{j} + 4\hat{k}).\left(\frac{2\hat{i} + 6\hat{j} + 3\hat{k}}{7}\right) = 4$

$\Rightarrow 2\lambda + 6 + (3\times4) = 28$

$\Rightarrow \lambda = (28 - 12 - 6)/2$

$\Rightarrow \lambda = 10/2 = 5$

Ans: $\lambda = 5$

Question: 6

If $\Rightarrow \theta = \frac{\pi}{2}$

Now,

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 13^2 = 5^2 + |\vec{b}|^2 + 0$ ...($\cos\theta = \cos\frac{\pi}{2} = 0$)

$\Rightarrow |\vec{b}|^2 = 169 - 25 = 144$

$\Rightarrow |\vec{b}| = 12$

Ans: $|\vec{b}| = 12$

Question: 7

If $(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 15$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 15$

$\Rightarrow |\vec{x}|^2 = |\vec{a}|^2 + 15$

Now, a is a unit vector,

$\Rightarrow |\vec{a}| = 1$

$\Rightarrow |\vec{x}|^2 = 1^2 + 15$

$\Rightarrow |\vec{x}|^2 = 16$

$\Rightarrow |\vec{x}| = 4$

Ans: $|\vec{x}| = 4$

Question: 8

Solution:

$\vec{a} = \hat{i} - 3\hat{k}$

$\vec{b} = 2\hat{j} - \hat{k}$

$\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$

Now,

$\vec{a} + \vec{b} + \vec{c} = \hat{i} - 3\hat{j} + 2\hat{j} - \hat{k} + 2\hat{i} - 3\hat{j} + 2\hat{k}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$

Ans: $\vec{a} + \vec{b} + \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$

Question: 9

Solution:

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$\vec{a} = \hat{\imath} - 2\hat{\jmath}$

$\vec{b} = 2\hat{\imath} - 3\hat{\jmath}$

$\vec{c} = 2\hat{\imath} + 3\hat{k}$

Now ,

$\vec{a} + \vec{b} + \vec{c} = \hat{\imath} - 2\hat{\jmath} + 2\hat{\imath} - 3\hat{\jmath} + 2\hat{\imath} + 3\hat{k}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 5\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$

Ans: $\vec{a} + \vec{b} + \vec{c} = 5\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$

Question: 10

Solution:

projection of a on b is given by:

∴ the projection of the vector $\vec{a}.\hat{b}$ $(\hat{i} + \hat{j} + \hat{k})$ along the vector $\hat{j}$. is :

$(\hat{\imath} + \hat{\jmath} + \hat{k}).\hat{\jmath} = 0 + 1 + 0 = 1$

Ans: the projection of the vector $(\hat{i} + \hat{j} + \hat{k})$ along the vector $\hat{j}$. is:1

Question: 11

Write the project

Solution:

$\vec{a} = 7\hat{\imath} + \hat{\jmath} - 4\hat{k}$

$\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (2^2 + 6^2 + 3^2)^{1/2}$

$\Rightarrow |\vec{b}| = (4 + 36 + 9)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}$$

$$\vec{a}.\hat{b} = (7\hat{\imath} + \hat{\jmath} - 4\hat{k}).\left(\frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}\right) = \frac{(7 \times 2) + (1 \times 6) - (4 \times 3)}{7}$$

$$= \frac{14 + 6 - 12}{7} = \frac{8}{7}$$

Ans: the projection of the vector $(7\hat{i} + \hat{j} - 4\hat{k})$ on the vector $(2\hat{i} + 6\hat{j} + 3\hat{k})$.

Question: 12

Find $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\vec{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$\vec{b}\times\vec{c} = (-\hat{\imath} + 2\hat{\jmath} + \hat{k})\times(3\hat{\imath} + \hat{\jmath} + 2\hat{k}) = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \hat{\imath}(4-1) - \hat{\jmath}(-2-3) + \hat{k}(-1-6) = 3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$

$\therefore \vec{b}\times\vec{c} = 3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$

$\therefore \vec{a}.(\vec{b}\times\vec{c}) = (2\hat{\imath} + \hat{\jmath} + 3\hat{k}).(3\hat{\imath} + 5\hat{\jmath} - 7\hat{k})$ = (2×3) + (1×5) + (3× - 7)

= 6 + 5 - 21 = - 10

Ans: - 10

Question: 13

Solution:

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$

$|\vec{a}| = (2^2 + (-3)^2 + 6^2)^{1/2}$

$\Rightarrow |\vec{a}| = (4 + 9 + 36)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}}{7}$

a vector in the direction of $(2\hat{i} - 3\hat{j} + 6\hat{k})$ which has magnitude 21 units.

$= 21\hat{a} = 21\times\frac{2\hat{i}-3\hat{j}+6\hat{k}}{7} = 3(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}) = 6\hat{\imath} - 9\hat{\jmath} + 18\hat{k}$

Ans: $6\hat{\imath} - 9\hat{\jmath} + 18\hat{k}$

Question: 14

If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\vec{c} = 3\hat{\imath} + \hat{\jmath}$

$\vec{a} + \lambda\vec{b} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(-\hat{\imath} + 2\hat{\jmath} + \hat{k})$

$\Rightarrow \vec{a} + \lambda\vec{b} = (2-\lambda)\hat{\imath} + (2 + 2\lambda)\hat{\jmath} + (3 + \lambda)\hat{k}$

Since $\vec{a} + \lambda\vec{b}$ *is perpendicular to* $\vec{c}$

$\Rightarrow \overrightarrow{(a} + \lambda\vec{b}).\vec{c} = 0$

$\Rightarrow \left((2-\lambda)\hat{\imath} + (2 + 2\lambda)\hat{\jmath} + (3 + \lambda)\hat{k}\right).(3\hat{\imath} + \hat{\jmath}) = 0$

$\Rightarrow (2 - \lambda)\times 3 + (2 + 2\lambda)\times 1 = 0$

$\Rightarrow 6 + 2 - 3\lambda + 2\lambda = 0$

$\Rightarrow \lambda = 8$

Ans: $\lambda = 8$

Question: 15

Solution:

$\vec{a} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$| = (1^2 + (-2)^2 + 2^2)^{1/2}$

$\Rightarrow|\vec{a}| = (1 + 4 + 4)^{1/2} = (9)^{1/2} = 3$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3}$$

a vector in the direction of $(\hat{i} - 2\hat{j} + 2\hat{k})$. which has magnitude 15 units.

$$= 15\hat{a} = 15 \times \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3} = 5(\hat{\imath} - 2\hat{\jmath} + 2\hat{k}) = 5\hat{\imath} - 10\hat{\jmath} + 10\hat{k}.$$

Ans: $5\hat{\imath} - 10\hat{\jmath} + 10\hat{k}$.

Question: 16

If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\therefore (2\vec{a} - \vec{b} + 3\vec{c}) = 2(\hat{\imath} + \hat{\jmath} + \hat{k}) - (4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + 3(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$\Rightarrow (2\vec{a} - \vec{b} + 3\vec{c}) = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

LET, $(2\vec{a} - \vec{b} + 3\vec{c}) = \vec{s}$

$\vec{s} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$|\vec{s}| = (1^2 + (-2)^2 + 2^2)^{1/2}$

$\Rightarrow |\vec{s}| = (1 + 4 + 4)^{1/2} = (9)^{1/2} = 3$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3}$$

a vector of magnitude 6 units which is parallel to the vector $(2\vec{a} - \vec{b} + 3\vec{c})$. is:

$$6\hat{s} = 6 \times \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3} = 2(\hat{\imath} - 2\hat{\jmath} + 2\hat{k}) = 2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}.$$

Ans: $2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$

Question: 17

Solution:

$\vec{a} = \hat{\imath} - \hat{\jmath}$

$\vec{b} = \hat{\imath} + \hat{\jmath}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (1^2 + 1^2 + 0^2)^{1/2}$

$\Rightarrow |\vec{b}| = (1 + 1)^{1/2} = (2)^{1/2}$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}$$

$$\vec{a}.\hat{b} = (\hat{\imath} - \hat{\jmath}).\left(\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}\right) = \frac{(1 \times 1) + (-1 \times 1)}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0$$

Ans: the projection of the vector $\left(7\hat{i} + \hat{j} - 4\hat{k}\right)$ on the vector $\left(2\hat{i} + 6\hat{j} + 3\hat{k}\right)$.

Question: 18

Solution:

$|\vec{a}| = \sqrt{3}$

$|\vec{b}| = 2$

Since, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|cos\theta$

Substituting the given values we get:

$\Rightarrow \sqrt{6} = \sqrt{3} \times 2 \times cos\theta$

$\Rightarrow cos\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \cos^{-1}\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 45^{\circ} = \frac{\pi}{4}$

Ans: $\theta = 45^{\circ} = \frac{\pi}{4}$

Question: 19

If $\vec{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k}$

$\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$$\vec{a} \times \vec{b} = (\hat{\imath} - 7\hat{\jmath} + 7\hat{k}) \times (3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}) = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix} = \hat{\imath}(-14 - (-14)) - \hat{\jmath}(2 - 21) + \hat{k}(-2 - (-21))$$
$$= 0\hat{\imath} + 19\hat{\jmath} + 19\hat{k}$$

$\therefore \vec{a} \times \vec{b} = 0\hat{\imath} + 19\hat{\jmath} + 19\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = (0^2 + 19^2 + 19^2)^{1/2} = (2 \times 19^2)^{1/2} = 19\sqrt{2}$

Ans: $\therefore |\vec{a} \times \vec{b}| = 19\sqrt{2}$

Question: 20

Solution:

| = 1

$|\vec{b}| = 2$

Since, $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|sin\theta$

Substituting the given values we get:

$\Rightarrow\sqrt{3} = 1 \times 2 \times sin\theta$

$\Rightarrow sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow\theta = \sin^{-1}\frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 60^{\circ} = \frac{\pi}{3}$

Ans: $\theta = 60^{\circ} = \frac{\pi}{3}$

Question: 21

Solution:

It is given that:

$\vec{a} \times \vec{b} = \vec{0}$ *and* $\vec{a}.\vec{b} = \vec{0}$

$\Rightarrow|\vec{a}||\vec{b}|sin\theta = |\vec{a}||\vec{b}|cos\theta = \vec{0}$

Since sinθ and cosθ cannot be 0 simultaneously $\therefore|\vec{a}| = |\vec{b}| = 0$

Conclusion: when $\vec{a} \times \vec{b} = \vec{0}$ *and* $\vec{a}.\vec{b} = \vec{0}$

Then $|\vec{a}| = |\vec{b}| = 0$

Question: 22

Solution:

$\vec{a} = \hat{\imath} + \lambda\hat{\jmath} + 3\hat{k}$

$\vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$

It is given that $\vec{a} \parallel \vec{b}$

$\Rightarrow\frac{1}{3} = \frac{\lambda}{2} = \frac{3}{9}$

$\Rightarrow\frac{1}{3} = \frac{\lambda}{2}$

$\Rightarrow \lambda = 2 \times \frac{1}{3} = \frac{2}{3}$

Ans: λ = 2/3

Question: 23

Solution:

We know that:

$\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath},$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$

$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

$\therefore \hat{\imath}.(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{\imath} \times \hat{k}) + \hat{k}.(\hat{\imath} \times \hat{\jmath}) = \hat{\imath}.\hat{\imath} + \hat{\jmath}.(-\hat{\jmath}) + \hat{k}.\hat{k} = 1 - 1 + 1 = 1$

Ans: $\hat{\imath}.(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{\imath} \times \hat{k}) + \hat{k}.(\hat{\imath} \times \hat{\jmath}) = 1$

Question: 24

Solution:

Scalar triple product geometrically represents the volume of the parallelepiped whose three coterminous edges are represented by $\vec{a}, \vec{b}, \vec{c}$.i.e. V = $[\vec{a}\vec{b}\vec{c}]$

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\vec{c} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$\therefore$ V = $[\vec{a}\vec{b}\vec{c}]$ = $\begin{bmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -2 & 2 \end{bmatrix}$ = 2(4 - 2) - (- 3)(2 - (- 3)) + 4(- 2 - 6) = 4 + 15 - 32 = | - 13| =

13 cubic units.

Ans:13 cubic units.

Question: 25

If $\vec{a} = -2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

$\vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

$\vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

If $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $[\vec{a}\vec{b}\vec{c}] = 0$

L.H.S = $\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & -2 \\ 4 & -2 & -2 \end{bmatrix}$ = - 2(- 8 - 4) + 2(4 + 8) + 4(4 - 16) = 24 + 24 - 48 = 0 = R.H.S

$\therefore$L.H.S = R.H.S

Hence proved that the vectors $\vec{a} = -2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

$\vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

$\vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

Are coplanar.

Question: 26

If $\vec{a} = 2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}$

$\vec{b} = \hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k}$

It is given that $\vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow (2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}) \times (\hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k}) = 0$

$\Rightarrow \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{bmatrix} = 0 = \hat{\imath}(6\mu - 27\lambda) - \hat{\jmath}(2\mu - 27) + \hat{k}(2\lambda - 6)$

$\Rightarrow 2\lambda - 6 = 0$

$\Rightarrow \lambda = 6/2 = 3$

$\Rightarrow 2\mu - 27 = 0$

$\Rightarrow \mu = 27/2$

Ans: $\lambda = 3$, $\mu = 27/2$

Question: 27

Solution:

It is given that:

$|\vec{a} \times \vec{b}| = |\vec{a}.\vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}| sin\theta = |\vec{a}||\vec{b}| cos\theta$

$\Rightarrow$sinθ = cosθ

$\Rightarrow$tanθ = 1

$\Rightarrow \theta = \tan^{-1} 1 = \frac{\pi}{4}$

Ans: $\theta = \frac{\pi}{4}$

Question: 28

Solution:

When the two vectors are parallel or collinear, they can be added in a scalar way because the angle between them is zero degrees,they are I the same or opposite direction.

Therefore when two vectors $\vec{a}$ *and* $\vec{b}$ are either parallel or collinear then

$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$

Question: 29

Find the directio

Solution:

Let the inclination with:

x - axis =-α

y - axis =-β

z - axis =-γ

The vector is equally inclined to the three axes.

$\Rightarrow \alpha = \beta = \gamma$

Direction cosines: $cos\alpha, cos\beta, cos\gamma$

We know that:$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$ ...($\alpha = \beta = \gamma$)

$\Rightarrow 3\cos^2\alpha = 1$

$cos\alpha = \frac{1}{\sqrt{3}}$

$\Rightarrow cos\alpha = \frac{1}{\sqrt{3}}$

$cos\beta = \frac{1}{\sqrt{3}}$

$cos\gamma = \frac{1}{\sqrt{3}}$

Ans: $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Question: 30

CLASS24

Solution:

Let P(x_1,y_1,z_1) and Q(x_2,y_2,z_2) be the two points then Direction ratios of line joining P and Q i.e. PQ are x_2 - x_1,y_2 - y_1,z_2 - z

Here, P is(1, 5, 4) and Q is (4, 1, - 2)

Direction ratios of PQ are:(4 - 1),(1 - 5),(- 2 - 4) = 3, - 4, - 6

Ans: the direction ratios of $\overrightarrow{PQ}$. are: 3, - 4, - 6

Question: 31

Find the directio

Solution:

$\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Let the inclination with:

x - axis =α

y - axis =β

z - axis =γ

Direction cosines: $cos\alpha, cos\beta, cos\gamma = l, m, n$

For a vector $\vec{a} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, l = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$\therefore l = \frac{1}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{1}{\sqrt{1 + 4 + 9}} = \frac{1}{\sqrt{14}}.$$

$$\therefore m = \frac{2}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{2}{\sqrt{1 + 4 + 9}} = \frac{2}{\sqrt{14}}$$

$$\therefore n = \frac{3}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{3}{\sqrt{1 + 4 + 9}} = \frac{3}{\sqrt{14}}$$

Ans: $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

Question: 32

If It is given that $\hat{a}$ *and* $\hat{b}$ are unit vectors ,as well as $(\hat{a} + \hat{b})$ is also a unit vector

$\Rightarrow |\hat{a}| = |\hat{b}| = |\hat{a} + \hat{b}| = 1$

Since the modulus of a unit vector is unity.

Now,

$|\hat{a} + \hat{b}|^2 = |\hat{a}|^2 + |\hat{b}|^2 + 2|\hat{a}||\hat{b}|\cos\theta$

$\Rightarrow 1^2 = 1^2 + 1^2 + 2\times1\times1\times \cos\theta$

$\Rightarrow \cos\theta = (1 - 1 - 1)/2$

$\Rightarrow \cos\theta = \frac{-1}{2}$

$\Rightarrow \theta = \cos^{-1}\frac{-1}{2} = \frac{2\pi}{3}$

Ans: $\frac{2\pi}{3}$

Question: 1

If Thus, the given vectors $\vec{a}$ and $\vec{b}$ are equal if and only if

x = 3, - y = 2, - z = 1

x = 3, y = - 2, z = - 1

x + y + z = 3 + (- 2) + (- 1) = 3 - 3 = 0

Question: 2

The sum of vectors is

$$\vec{a} + \vec{b} = 2\hat{i} + 2\hat{j} - 5\hat{k} + 2\hat{i} + \hat{j} - 7\hat{k}$$

$$= 4\hat{i} + 3\hat{j} - 12\hat{k}$$

Let the unit vector in the direction of $\vec{a} + \vec{b}$ be $\hat{c}$, then

$$\hat{c} = \frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} \Rightarrow \frac{(4\hat{i} + 3\hat{j} - 12\hat{k})}{|4\hat{i} + 3\hat{j} - 12\hat{k}|} = \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{\sqrt{(4^2 + 3^2 + (-12)^2)}}$$

$$\Rightarrow \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{\sqrt{169}} = \frac{1}{13}(4\hat{i} + 3\hat{j} - 12\hat{k})$$

Question: 3

Solution:

If the scalar product (dot product) is zero, two non - zero vectors are perpendicular.

$$\vec{a}.\vec{b} \Rightarrow (2\hat{i} + \lambda\hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + 3\hat{k}) = 0$$

$$\Rightarrow 2.1 + \lambda.(-2) + 1.3 = 0\,(\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1)$$

2 - 2λ + 3 = 0

- 2λ = 5

$$\lambda = \frac{5}{2}$$

Question: 4

Solution:

Two nonzero vectors are parallel if their vector product (cross product) is zero,

$$\vec{a} \times \vec{b} \Rightarrow \begin{vmatrix} i & j & k \\ 3 & 2 & 9 \\ 1 & -2p & 3 \end{vmatrix} = 0$$

$$\Rightarrow (2.3 - (-2p).9)\hat{i} - (3.3 - 9.1)\hat{j} + (3.(-2p) - 1.2)\hat{k} = 0$$

$\Rightarrow (6+18p)\hat{i} - (9-9)\hat{j} + (-6p-2)\hat{k} = 0$

$\Rightarrow (6+18p)\hat{i} - 0.\hat{j} + (-6p-2)\hat{k} = 0 \Rightarrow 0.\hat{i} - 0.\hat{j} + 0.\hat{k}$

On comparing with right hand side,

6 + 18p = 0

$p = \frac{-6}{18} \Rightarrow -\frac{1}{3}$ (You can solve using - 6p - 2)

Question: 5

Solution:

Projection of vector $\vec{a}$ on vector

$$\vec{b} = \frac{1}{|\vec{b}|}(\vec{a}.\vec{b})$$

So we first calculate the magnitude of vector b and the scalar product of a vector $\vec{a}$ and $\vec{b}$:

$|\vec{b}| = \sqrt{(2^2 + 6^2 + 3^2)} \Rightarrow \sqrt{(4+36+9)} = \sqrt{49} \Rightarrow 7$

$\vec{a}.\vec{b} = (\lambda\hat{i} + \hat{j} + 4\hat{k}).(2\hat{i} + 6\hat{j} + 3\hat{k}) \Rightarrow \lambda.2 + 1.6 + 4.3 = 2\lambda + 6 + 12 = 2\lambda + 18$

Projection of vector $\vec{a}$ on vector $\vec{b} = \frac{1}{|\vec{b}|}(\vec{a}.\vec{b}) = 4$ (1)

Putting the values from above in equation (1),

$\frac{2\lambda + 18}{7} = 4 \Rightarrow 2\lambda = 28 - 18$

$2\lambda = 10$

$\lambda = 5$

Question: 6

If $\vec{a}$ and $\vec{b}$ is perpendicular, $\vec{a}.\vec{b} = 0$. So, using $(\vec{a} + \vec{b})^2$

$(\vec{a} + \vec{b})^2 \Rightarrow (\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = |\vec{a} + \vec{b}|^2$ $\left(\text{using } \vec{a}.\vec{a} = |\vec{a}|^2\right)$

$\Rightarrow \vec{a}.\vec{a} + \vec{b}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{b} = 13^2$

$\Rightarrow |\vec{a}|^2 + 2.\vec{a}.\vec{b} + |\vec{b}|^2 = 169$

$\Rightarrow 5^2 + 2.0 + |\vec{b}|^2 = 169$

$\Rightarrow |\vec{b}|^2 = 169 - 25$

$\Rightarrow |\vec{b}|^2 = 144$

$|\vec{b}| = \sqrt{144} \Rightarrow 12$ (Negative value not considered as magnitude is positive).

Question: 7

If $\vec{x}-\vec{a}).(\vec{x}+\vec{a}) \Rightarrow \vec{x}.\vec{x}-\vec{a}.\vec{x}+\vec{a}.\vec{x}-\vec{a}.\vec{a}=15$

$\Rightarrow |\vec{x}|^2-|\vec{a}|^2=15$ (Using $\vec{x}.\vec{a}=\vec{a}.\vec{x}$, commutative law)

$\Rightarrow |\vec{x}|^2-1^2=15$ (As magnitude of unit vector is 1)

$\Rightarrow |\vec{x}|^2=15+1$

$\Rightarrow |\vec{x}|=\sqrt{16} \Rightarrow 4$

Question: 8

$\vec{a}+\vec{b}+\vec{c}=\hat{i}-3\hat{k}+2\hat{j}-\hat{k}+2\hat{i}-3\hat{j}+2\hat{k}$

$=(1+2)\hat{i}+(2-3)\hat{j}+(-3-1+2)\hat{k}$

$=3\hat{i}-\hat{j}-2\hat{k}$

Question: 9

$\vec{a}+\vec{b}+\vec{c}=\hat{i}-2\hat{j}+2\hat{i}-3\hat{j}-2\hat{i}-3\hat{k}$

$=(1+2+2)\hat{i}+(-2-3)\hat{j}+3\hat{k}$

$=5\hat{i}-5\hat{j}+3\hat{k}$

Question: 10

Solution:

Projection of vector $\vec{a}$ on vector $\vec{b}=\frac{1}{|\vec{b}|}(\vec{a}.\vec{b})$;

$=\frac{1}{1}\left((\hat{i}+\hat{j}+\hat{k}).\hat{j}\right)$

$=0+1+0\left(\hat{i}.\hat{j}=\hat{j}.\hat{k}=\hat{k}.\hat{i}=0\right)=1$

Question: 11

$\vec{a}$ on vector $\vec{b}=\frac{1}{|\vec{b}|}(\vec{a}.\vec{b})$

$=\frac{1}{\sqrt{(2^2+6^2+3^2)}}\left((7\hat{i}+\hat{j}-4\hat{k}).(2\hat{i}+6\hat{j}+3\hat{k}\right)$

$=\frac{1}{\sqrt{(4+36+9)}}(7.2+1.6+(-4).3)$

$=\frac{1}{\sqrt{49}}(14+6-12)$

$= \frac{1}{7}(20-12)$

$= \frac{8}{7}$

Question: 12

$\vec{b}$ and $\vec{c}$ then scalar product of that with $\vec{a}$:

$$\vec{b}\times\vec{c} \Rightarrow \begin{vmatrix} i & j & k \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = (2.2-1.1)\hat{i} - ((-1).2-3.1)\hat{j} + ((-1).1-3.2)\hat{k}$$

$= (4-1)\hat{i} - (-2-3)\hat{j} + (-1-6)\hat{k}$

$= 3\hat{i} + 5\hat{j} - 7\hat{k}$

$\vec{a}.(\vec{b}\times\vec{c}) = (2\hat{i} + \hat{j} + 3\hat{k}).(3\hat{i} + 5\hat{j} - 7\hat{k})$

= 2.3 + 1.(5) + 3.(- 7)

= 6 + 5 - 21

= - 10

Question: 13

$\hat{a} \Rightarrow \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{(2^2+(-3)^2+6^2)}}$

$\Rightarrow \frac{2\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{(4+9+36)}} = \frac{2\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{49}}$

$= \frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}$

Now vector in the direction of the given vector and with magnitude 21 is

$= 21.\frac{2\hat{i}-3\hat{j}+6\hat{k}}{7} \Rightarrow 3(2\hat{i}-3\hat{j}+6\hat{k})$

$= 6\hat{i} - 9\hat{j} + 18\hat{k}$

Question: 14

$(\vec{a} + \lambda\vec{b}).\vec{c} = 0$

$(2\hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-\hat{i} + 2\hat{j} + \hat{k})).(3\hat{i} + \hat{j}) = 0$

$((2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}).(3\hat{i} + \hat{j}) = 0$

(2 - λ).3 + (2 + 2λ).1 + (3 + λ).0 = 0

6 - 3λ + 2 + 2λ = 0

8 - λ = 0

$\lambda = 8$

Question: 15

$$\hat{a} \Rightarrow \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{\left(1^2 + (-2)^2 + 2^2\right)}}$$

$$\Rightarrow \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{(1 + 4 + 4)}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{9}}$$

$$= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

Now vector in the direction of the given vector and with magnitude 15 is

$$= 15.\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} \Rightarrow 5\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

$$= 5\hat{i} - 10\hat{j} + 10\hat{k}$$

Question: 16

$$2\vec{a} - \vec{b} + 3\vec{c} = 2\left(\hat{i} + \hat{j} + \hat{k}\right) - \left(4\hat{i} - 2\hat{j} + 3\hat{k}\right) + 3\left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= (2 - 4 + 3)\hat{i} + (2 + 2 - 6)\hat{j} + (2 - 3 + 3)\hat{k}$$

$$= \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\left|2\vec{a} - \vec{b} + 3\vec{c}\right| = \left|\hat{i} - 2\hat{j} + 2\hat{k}\right| \Rightarrow \sqrt{\left(1^2 + (-2)^2 + 2^2\right)} = \sqrt{(1 + 4 + 4)}$$

$$\Rightarrow \sqrt{9} = 3$$

$$\hat{a} \Rightarrow \frac{\vec{a}}{|\vec{a}|} = \frac{2\vec{a} - \vec{b} + 3\vec{c}}{\left|2\vec{a} - \vec{b} + 3\vec{c}\right|} \Rightarrow \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

Vector with magnitude 6 in the direction of the vector is

$$\vec{m} = 6.\left(\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}\right) \Rightarrow 2\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) = 2\hat{i} - 4\hat{j} + 4\hat{k}$$

Question: 17

Projection of vector $\vec{a}$ on vector $\vec{b} = \frac{1}{|\vec{b}|}\left(\vec{a}.\vec{b}\right)$

$$= \frac{1}{\left|\hat{i} + \hat{j}\right|}\left(\left(\hat{i} - \hat{j}\right).(\hat{i} + \hat{j}\right)_{)}$$

$$= \frac{1}{\sqrt{\left(1^2 + 1^2\right)}}\left(\left(\hat{i} - \hat{j}\right).(\hat{i} + \hat{j}\right)$$

$$= \frac{1}{\sqrt{(1 + 1)}}(1.1 + (-1.1))$$

$= \frac{1}{\sqrt{2}}(1-1)$

= 0

So, projection of vector on other is 0.

Question: 18

$$\cos\theta = \frac{|\vec{a}.\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{\sqrt{6}}{\sqrt{3.2}} = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

Question: 19

$$\vec{a}\times\vec{b} \Rightarrow \begin{vmatrix} i & j & k \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix} = ((-7).2-(-2).7)\hat{i} + (1.2-3.7)\hat{j} + (1.(-2)-3.(-7))\hat{k}$$

$$= (-14-(-14))\hat{i} + (2-21)\hat{j} + (-2-(-21))\hat{k}$$

$$= 0.\hat{i} - 19\hat{j} + 19\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{(19^2+19^2)} = \sqrt{2.19^2} = 19\sqrt{2}$$

Question: 20

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{\sqrt{3}}{1.2} = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}$$

Question: 21

$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$ and $\cos\theta = \frac{|\vec{a}.\vec{b}|}{|\vec{a}||\vec{b}|}$, using scalar product and vector product.

Now $\vec{a}\times\vec{b} = 0$ and $\vec{a}.\vec{b} = 0$ also.

As cosθ and sinθ cannot be 0 simultaneously So, then either vector a is o or b is 0.

Question: 22

Find the value of

Solution:

If the vector product is zero, two vectors are parallel.

$$\vec{a}\times\vec{b} \Rightarrow \begin{vmatrix} i & j & k \\ 1 & \lambda & 3 \\ 3 & 2 & 9 \end{vmatrix} = 0$$

$$(9\lambda - 2.3)\hat{i} - (1.9-3.3)\hat{j} + (1.2-3.\lambda) = 0$$

$$(9\lambda - 6)\hat{i} - 0.\hat{j} + (2 - 3\lambda)\hat{k} = 0$$

On comparing with the right hand side, we have

9λ - 6 = 0

$$\lambda = \frac{6}{9} = \frac{2}{3}$$

Question: 23

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

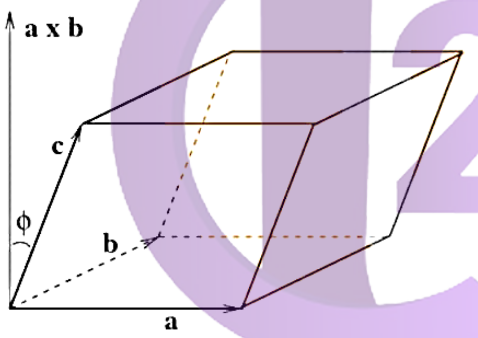
Then putting values in the equation

$$\hat{i}.\left(\hat{j} \times \hat{k}\right) + \hat{j}\left(\hat{i} \times \hat{k}\right) + \hat{k}\left(\hat{i} \times \hat{j}\right) = \hat{i}.\hat{i} + \hat{j}.\left(-\hat{j}\right) + \hat{k}.\hat{k}$$

= 1 - 1 + 1 = 1

Question: 24

$$= \left|\left(\vec{a} \times \vec{b}\right).\vec{c}\right|$$



$$\left(\vec{a} \times \vec{b}\right).\vec{c} = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} 3 & -2 & 2 \\ 2 & -3 & 4 \\ 1 & 2 & -1 \end{vmatrix}$$

= (- 3).(- 1) - 2.4).3 - (2.(- 1) - 1.4).(- 2) + (2.2 - 1.(- 3)).

= (3 - 8).3 + (- 2 - 4).2 + (4 - (- 3)).2

= - 15 - 12 + 14

= - 27 + 14

= - 13

Volume of parallelepiped $= \left|\left(\vec{a} \times \vec{b}\right).\vec{c}\right| = |-13| = 13$ cubic unit

Question: 25

The volume of parallelepiped $= \left|\left(\vec{a} \times \vec{b}\right).\vec{c}\right| = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$$= \begin{vmatrix} 4 & -2 & -2 \\ -2 & -2 & 4 \\ -2 & 4 & -2 \end{vmatrix}$$

= (- 2. - 2 - 4.4)4 - (- 2. - 2 - 4. - 2) - 2 + (4. - 2 - (- 2). - 2) - 2

= (4 - 16)4 + (4 + 8)2 - (- 8 - 4)2

= - 48 + 24 - (- 24)

= - 48 + 48 = 0

So, planes are coplanar.

Question: 26

$$\vec{a} \times \vec{b} \Rightarrow \begin{vmatrix} i & j & k \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = \vec{0}$$

$$= (6.\mu - 27.\lambda)\hat{i} + (2.\mu - 27.1)\hat{j} + (2.\lambda - 1.6)\hat{k} = \vec{0}$$

On comparing with the right hand side, we have

$6\mu - 27\lambda = 0$

$2\mu - 27 = 0$

$$\mu = \frac{27}{2}$$

$2\lambda - 6 = 0$

$$\lambda = \frac{6}{2} \Rightarrow 3$$

Question: 27

$$|\vec{a}.\vec{b}| = |\vec{a}|.|\vec{b}|.\cos\theta$$

$$|\vec{a} \times \vec{b}| = |\vec{a}|.|\vec{b}|.\sin\theta$$

Equating both

$$|\vec{a} \times \vec{b}| = |\vec{a}.\vec{b}| \; |\vec{a}|.|\vec{b}|.\sin\theta = |\vec{a}|.|\vec{b}|.\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1$$

$$\theta = \frac{\pi}{4}$$

Question: 28

$$(\vec{a} + \vec{b})^2 = \vec{a}^2 + \vec{b}^2 + 2\vec{a}.\vec{b}$$

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2.|\vec{a}|.|\vec{b}|.\cos\theta \left(\text{using } \vec{a}^2 = |\vec{a}|^2\right)$$

$$(|\vec{a}| + |\vec{b}|)^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2.|\vec{a}|.|\vec{b}|.\cos\theta$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2.|\vec{a}|.|\vec{b}| = |\vec{a}|^2 + |\vec{b}|^2 + 2.|\vec{a}|.|\vec{b}|.\cos\theta$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2.|\vec{a}|.|\vec{b}| - |\vec{a}|^2 - |\vec{b}|^2 - 2.|\vec{a}|.|\vec{b}|.\cos\theta = 0$$

$$2.|\vec{a}|.|\vec{b}|(1 - \cos\theta) = 0$$

As, magnitude of a vector cannot be zero (leaving zero vector)

$1 - \cos\theta = 0$

$\text{Cos}\theta = 1$

$\theta = 0°$

So, vectors are either parallel or collinear.

Question: 29

$$l^2 + m^2 + n^2 = 1$$

Now given that equally inclined to three axes with an angle of θ. Then direction cosines l, m, n are

$l = m = n = \cos\theta$

Putting values of direction cosines in equation,

$\text{Cos}^2\theta + \text{Cos}^2\theta + \text{Cos}^2\theta = 1$

$3\text{Cos}^2\theta = 1$

$$\cos^2\theta = \frac{1}{3} \Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$$

$$l = m = n = \cos\theta = \frac{1}{\sqrt{3}}$$

Question: 30

$$\overrightarrow{PQ} \Rightarrow \vec{P} - \vec{Q} = (4-1)\hat{i} + (1-5)\hat{j} + (-2-4)\hat{k}$$

$$= 3\hat{i} - 4\hat{j} - 6\hat{k}$$

So direction ratios are 3, - 4, - 6.

Question: 31

$l = \frac{a}{r}, m = \frac{b}{r}, n = \frac{c}{r}$, where a, b, c are direction ratios and r is magnitude.

Now direction ratios are 1, 2, 3 respectively and magnitude of vector is

$$r = \sqrt{(1^2 + 2^2 + 3^2)}\sqrt{(1+4+9)} = \sqrt{14}$$

Putting the values

$$l = \frac{1}{\sqrt{14}}, m = \frac{2}{\sqrt{14}}, n = \frac{3}{\sqrt{14}}$$

Question: 32

$$(\hat{a} + \hat{b})^2 = \hat{a}^2 + \hat{b}^2 + 2\hat{a}.\hat{b}$$

$$\left|\hat{a}+\hat{b}\right|^2 = |\hat{a}|^2 + \left|\hat{b}\right|^2 + 2.|\hat{a}|.\left|\hat{b}\right|.\cos\theta$$

$$1^2 = 1^2 + 1^2 + 2.1.1.\cos\theta$$

1 - 1 - 1 = 2cosθ

- 1 = 2cosθ

$$\cos\theta = -\frac{1}{2} \Rightarrow \theta = \pi - \frac{\pi}{3} \Rightarrow \frac{2\pi}{3}$$

Exercise : OBJECTIVE QUESTIONS

Question: 1

Solution:

Tip – A vector in the direction of another vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\lambda(a\hat{i} + b\hat{j} + c\hat{k})$ and the unit vector is given by $\frac{\lambda(ai+bj+c\hat{k})}{\sqrt{(a\lambda)^2+(b\lambda)^2+(c\lambda)^2}}$

So, a vector parallel to $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is given by $\lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$ where λ is an arbitrary constant.

Now, $|\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = 7$

Hence, the required unit vector

$$= \frac{\lambda(2\hat{i} - 3\hat{j} + 6\hat{k})}{\sqrt{(2\lambda)^2 + (3\lambda)^2 + (6\lambda)^2}}$$

$$= \frac{\lambda(2\hat{i} - 3\hat{j} + 6\hat{k})}{\lambda\sqrt{2^2 + 3^2 + 6^2}}$$

$$= \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

Question: 1

Mark (√) against

Solution:

Tip – A vector in the direction of another vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\lambda(a\hat{i} + b\hat{j} + c\hat{k})$ and the unit vector is given by $\frac{\lambda(ai+bj+c\hat{k})}{\sqrt{(a\lambda)^2+(b\lambda)^2+(c\lambda)^2}}$

So, a vector parallel to $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is given by $\lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$ where λ is an arbitrary constant.

Now, $|\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = 7$

Hence, the required unit vector

$$= \frac{\lambda(2\hat{i} - 3\hat{j} + 6\hat{k})}{\sqrt{(2\lambda)^2 + (3\lambda)^2 + (6\lambda)^2}}$$

$$= \frac{\lambda(2\hat{i} - 3\hat{j} + 6\hat{k})}{\lambda\sqrt{2^2 + 3^2 + 6^2}}$$

$$= \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

Question: 2

Solution:

Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{i} + \hat{j} - 5\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2 + 1^2 + 5^2}}, \frac{1}{\sqrt{2^2 + 1^2 + 5^2}}, \frac{-5}{\sqrt{2^2 + 1^2 + 5^2}}\right)$$

$$= \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$$

Question: 2

Solution:

Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{i} + \hat{j} - 5\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2 + 1^2 + 5^2}}, \frac{1}{\sqrt{2^2 + 1^2 + 5^2}}, \frac{-5}{\sqrt{2^2 + 1^2 + 5^2}}\right)$$

$$= \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$$

Question: 3

Solution:

Given - A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$

Tip – If P(a_1,b_1,c_1) and Q(a_2,b_2,c_2) be two points then the vector $\overrightarrow{PQ}$ is represented by $(a_2 - a_1)\hat{i} + (b_2 - b_1)\hat{j} + (c_2 - c_1)\hat{k}$

Hence, $\overrightarrow{AB} = (-1 - 1)\hat{i} + (-2 - 2)\hat{j} + (1 + 3)\hat{k} = -2\hat{i} - 4\hat{j} + 4\hat{k}$

Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{i} - 4\hat{j} + 4\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2 + 4^2 + 4^2}}, \frac{-4}{\sqrt{2^2 + 4^2 + 4^2}}, \frac{4}{\sqrt{2^2 + 4^2 + 4^2}}\right)$$

$$= \left(\frac{-2}{6}, \frac{-4}{6}, \frac{4}{6}\right)$$

$$= \frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$$

Question: 3

Solution:

Given - A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$

Tip – If P(a_1,b_1,c_1) and Q(a_2,b_2,c_2) be two points then the vector $\overrightarrow{PQ}$ is represented by $(a_2 - a_1)\hat{i} + (b_2 - b_1)\hat{j} + (c_2 - c_1)\hat{k}$

Hence, $\overrightarrow{AB} = (-1 - 1)\hat{i} + (-2 - 2)\hat{j} + (1 + 3)\hat{k} = -2\hat{i} - 4\hat{j} + 4\hat{k}$

Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{i} - 4\hat{j} + 4\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2 + 4^2 + 4^2}}, \frac{-4}{\sqrt{2^2 + 4^2 + 4^2}}, \frac{4}{\sqrt{2^2 + 4^2 + 4^2}}\right)$$

$$= \left(\frac{-2}{6}, \frac{-4}{6}, \frac{4}{6}\right)$$

$$= \frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$$

Question: 4

Solution:

Given - A vector makes angle α, β and γ with the x-axis, y-axis and z-axis respectively.

To Find - $(\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$

Formula to be used - $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Hence,

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$=(1-\cos^2\alpha) + (1-\cos^2\beta) + (1-\cos^2\gamma)$

$= 3-(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$=3-1$

$=2$

Question: 4

Solution:

Given - A vector makes angle α, β and γ with the x-axis, y-axis and z-axis respectively.

To Find - $(\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$

Formula to be used - $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Hence,

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$=(1-\cos^2\alpha) + (1-\cos^2\beta) + (1-\cos^2\gamma)$

$= 3-(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$=3-1$

$=2$

Question: 5

Solution:

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

A unit vector is a vector whose magnitude = 1.

Formula to be used - $\sin^2\theta + \cos^2\theta = 1$

Hence, magnitude of $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\sin\beta)\hat{j} + (\sin\alpha)\hat{k}$ will be given by $\sqrt{(\cos\alpha\cos\beta)^2 + (\cos\alpha\sin\beta)^2 + (\sin\alpha)^2}$

$= \sqrt{\cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha}$

$= \sqrt{\cos^2\alpha + \sin^2\alpha}$

= 1 i.e a unit vector

Question: 5

Solution:

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by

A unit vector is a vector whose magnitude = 1. $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Formula to be used - $\sin^2\theta + \cos^2\theta = 1$

Hence, magnitude of $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\sin\beta)\hat{j} + (\sin\alpha)\hat{k}$ will be given by $\sqrt{(\cos\alpha\cos\beta)^2 + (\cos\alpha\sin\beta)^2 + (\sin\alpha)^2}$

$= \sqrt{\cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha}$

$= \sqrt{\cos^2\alpha + \sin^2\alpha}$

= 1 i.e a unit vector

Question: 6

Solution:

Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ is given by

$$\left(\frac{1}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}, \frac{1}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}, \frac{\sqrt{2}}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}\right)$$

$$= \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}$$

$$= \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$$

-θ

The direction cosine of z-axis = $\frac{1}{\sqrt{2}}$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ where is the angle the vector makes with the z-axis.

$$\therefore \theta = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

Question: 6

Solution:

Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ is given by

$$\left(\frac{1}{\sqrt{1^2+1^2+(\sqrt{2})^2}}, \frac{1}{\sqrt{1^2+1^2+(\sqrt{2})^2}}, \frac{\sqrt{2}}{\sqrt{1^2+1^2+(\sqrt{2})^2}}\right)$$

$$= \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}$$

$$= \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$$

The direction cosine of z-axis = $\frac{1}{\sqrt{2}}$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ where θ is the angle the vector makes with the z-axis.

$$\therefore \theta = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

Question: 7

Solution:

Given – $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

To find – Angle between $\vec{a}$ and $\vec{b}$:

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $\sqrt{6} = 2\sqrt{3}\cos\theta$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ $\quad \therefore \theta = \frac{\pi}{4}$

Question: 7

Solution:

Given – $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

To find – Angle between $\vec{a}$ and $\vec{b}$:

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $\sqrt{6} = 2\sqrt{3}\cos\theta$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ $\quad \therefore \theta = \frac{\pi}{4}$

Question: 8

Solution:

Given – $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a}.\vec{b} = -1$

To find – Angle between $\vec{a}$ and $\vec{b}$:

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $-1 = \sqrt{2}\sqrt{2}\cos\theta$ i.e. $\cos\theta = \frac{1}{2}$ $\quad \therefore \theta = \frac{\pi}{3}$

Question: 8

Solution:

Given – $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a}.\vec{b} = -1$

To find – Angle between $\vec{a}$ and $\vec{b}$:

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $-1 = \sqrt{2}\sqrt{2}\cos\theta$ i.e. $\cos\theta = \frac{1}{2}$ $\quad \therefore \theta = \frac{\pi}{3}$

Question: 9

Solution:

Given - $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\imath} + \hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Tip – Magnitude of a vector $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a}.\vec{b} = (\hat{\imath} - 2\hat{\jmath} + 3\hat{k}).(3\hat{\imath} - 2\hat{\jmath} + \hat{k})$=3+4+3=10

$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

Hence, $10 = \sqrt{14}\sqrt{14}\cos\theta$ i.e. $\cos\theta = \frac{10}{14} = \frac{5}{7}$

$\therefore \theta = \cos^{-1}\frac{5}{7}$

Question: 9

Solution:

Given - $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\imath} + \hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Tip – Magnitude of a vector $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a}.\vec{b} = (\hat{\imath} - 2\hat{\jmath} + 3\hat{k}).(3\hat{\imath} - 2\hat{\jmath} + \hat{k})$=3+4+3=10

$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

Hence, $10 = \sqrt{14}\sqrt{14}\cos\theta$ i.e. $\cos\theta = \frac{10}{14} = \frac{5}{7}$

$\therefore \theta = \cos^{-1}\frac{5}{7}$

Question: 10

Solution:

Given - $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - \hat{\imath} + 2\hat{k}$

To find – Angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 4\hat{i} + \hat{j} - \hat{k}$

and $\vec{a} - \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = -2\hat{i} + 3\hat{j} - 5\hat{k}$

$\therefore (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = (4\hat{i} + \hat{j} - \hat{k}).(-2\hat{i} + 3\hat{j} - 5\hat{k}) = -8 + 3 + 5 = 0$

$|\vec{a} + \vec{b}| = \sqrt{4^2 + 1^2 + 1^2} = \sqrt{18}$

$|\vec{a} - \vec{b}| = \sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$

Hence, $0 = \sqrt{18}\sqrt{38}\cos\theta$ i.e. $\cos\theta = 0$

$\therefore \theta = \frac{\pi}{2}$

Question: 10

Solution:

Given - $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{i} + 2\hat{k}$

To find – Angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 4\hat{i} + \hat{j} - \hat{k}$

and $\vec{a} - \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = -2\hat{i} + 3\hat{j} - 5\hat{k}$

$\therefore (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = (4\hat{i} + \hat{j} - \hat{k}).(-2\hat{i} + 3\hat{j} - 5\hat{k}) = -8 + 3 + 5 = 0$

$|\vec{a} + \vec{b}| = \sqrt{4^2 + 1^2 + 1^2} = \sqrt{18}$

$|\vec{a} - \vec{b}| = \sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$

Hence, $0 = \sqrt{18}\sqrt{38}\cos\theta$ i.e. $\cos\theta = 0$

$\therefore \theta = \frac{\pi}{2}$

Question: 11

Solution:

Given - $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{i} + 2\hat{k}$

To find – Angle between $2\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $2\vec{a} + \vec{b} = 2(\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 5\hat{i} + 3\hat{j} - 4\hat{k}$

and
$\vec{a} + 2\vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + 2(3\hat{i} - \hat{j} + 2\hat{k}) = 7\hat{i} + \hat{k}$

$\therefore (2\vec{a} + \vec{b}).(\vec{a} - 2\vec{b}) = (5\hat{i} + 3\hat{j} - 4\hat{k}).(7\hat{i} + \hat{k}) = 35 - 4 = 31$

$|2\vec{a} + \vec{b}| = \sqrt{5^2 + 3^2 + 4^2} = \sqrt{50}$

$|\vec{a} - 2\vec{b}| = \sqrt{7^2 + 1^2} = \sqrt{50}$

Hence, $31 = \sqrt{50}\sqrt{50}\cos\theta$ i.e. $\cos\theta = \frac{31}{50}$

$\therefore \theta = \cos^{-1}\frac{31}{50}$

Question: 11

Solution:

Given - $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{i} + 2\hat{k}$

To find – Angle between $2\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$∶

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $2\vec{a} + \vec{b} = 2(\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 5\hat{i} + 3\hat{j} - 4\hat{k}$

and $\vec{a} + 2\vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + 2(3\hat{i} - \hat{j} + 2\hat{k}) = 7\hat{i} + \hat{k}$

$\therefore (2\vec{a} + \vec{b}).(\vec{a} - 2\vec{b}) = (5\hat{i} + 3\hat{j} - 4\hat{k}).(7\hat{i} + \hat{k}) = 35 - 4 = 31$

$|2\vec{a} + \vec{b}| = \sqrt{5^2 + 3^2 + 4^2} = \sqrt{50}$

$|\vec{a} - 2\vec{b}| = \sqrt{7^2 + 1^2} = \sqrt{50}$

Hence, $31 = \sqrt{50}\sqrt{50}\cos\theta$ i.e. $\cos\theta = \frac{31}{50}$

$\therefore \theta = \cos^{-1}\frac{31}{50}$

Question: 12

Solution:

Given - and

To find – Value of λ

$\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$ $\vec{a} \perp \vec{b}$

Formula to be used - where and are two vectors

Tip – For perpendicular vectors, i.e. i.e. the dot product=0

$\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ $\vec{p}$

Hence, $\theta = \frac{\pi}{2}$ $\cos\theta = 0$

$\therefore (2\hat{i} + 4\hat{j} - \hat{k}).(3\hat{i} - 2\hat{j} + \lambda\hat{k}) = 0$

$\vec{a}.\vec{b} = 0$

$\Rightarrow 6 - 8 - \lambda = 0$

i.e.

Question: 12

$\lambda = -2$

Solution:

Given - and

To find – Value of λ

$\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$ $\vec{a} \perp \vec{b}$

Formula to be used - where and are two vectors

Tip – For perpendicular vectors, i.e. i.e. the dot product=0

$\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ $\vec{p}$

$\theta = \frac{\pi}{2}$ $\cos\theta = 0$

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Hence, $\vec{a}.\vec{b} = 0$

$\therefore (2\hat{i} + 4\hat{j} - \hat{k}).(3\hat{i} - 2\hat{j} + \lambda\hat{k}) = 0$

$\Rightarrow 6 - 8 - \lambda = 0$

i.e. $\lambda = -2$

Question: 13

Solution:

Given - $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

To find – Projection of $\vec{a}$ on $\vec{b}$ i.e. $\vec{a}\cos\theta$

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – If $\vec{p}$ and $\vec{q}$ are two vectors, then the projection of $\vec{p}$ on $\vec{q}$ is defined as $\vec{p}\cos\theta$

Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow (2\hat{i} - \hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + \hat{k}) = \sqrt{1^2 + 2^2 + 1^2}|\vec{a}|\cos\theta$

$\Rightarrow |\vec{a}|\cos\theta = \frac{2 + 2 + 1}{\sqrt{6}}$

$\Rightarrow |\vec{a}|\cos\theta = \frac{5}{\sqrt{6}}$

Question: 13

Mark (√) against

Solution:

Given - $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

To find – Projection of $\vec{a}$ on $\vec{b}$ i.e. $\vec{a}\cos\theta$

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – If $\vec{p}$ and $\vec{q}$ are two vectors, then the projection of $\vec{p}$ on $\vec{q}$ is defined as $\vec{p}\cos\theta$

Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow (2\hat{i} - \hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + \hat{k}) = \sqrt{1^2 + 2^2 + 1^2}|\vec{a}|\cos\theta$

$\Rightarrow |\vec{a}|\cos\theta = \frac{2 + 2 + 1}{\sqrt{6}}$

$\Rightarrow |\vec{a}|\cos\theta = \frac{5}{\sqrt{6}}$

Question: 14

Solution:

Given - $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

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Tip – If $\vec{a}$ and $\vec{b}$ are two vectors then $|\vec{a} \pm \vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

Hence,

$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

$\Rightarrow \sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

$\Rightarrow a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta$

$\Rightarrow 4ab\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

i.e. $\theta = \frac{\pi}{2}$

So, $\vec{a} \perp \vec{b}$

Question: 14

Solution:

Given - $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

Tip – If $\vec{a}$ and $\vec{b}$ are two vectors then $|\vec{a} \pm \vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

Hence,

$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

$\Rightarrow \sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

$\Rightarrow a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta$

$\Rightarrow 4ab\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

i.e. $\theta = \frac{\pi}{2}$

So, $\vec{a} \perp \vec{b}$

Question: 15

Solution:

Given - $\vec{a}$ and $\vec{b}$ are two mutually perpendicular unit vectors i.e. $|\vec{a}| = |\vec{b}| = 1$

To Find – $(3\vec{a}+2\vec{b}).(5\vec{a}-6\vec{b})$

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip - $\vec{a} \perp \vec{b}$

$\therefore |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\cos\frac{\pi}{2} = 0$

$\therefore \vec{a}.\vec{b} = \vec{b}.\vec{a} = 0$

Hence,

$(3\vec{a}+2\vec{b}).(5\vec{a}-6\vec{b})$

$= 15|\vec{a}|^2 + 10\vec{b}.\vec{a} - 18\vec{a}.\vec{b} - 12|\vec{b}|^2$

$= 15 - 12$

$= 3$

Question: 15

Solution:

Given –$\vec{a}$ and-$\vec{b}$ are two mutually perpendicular unit vectors i.e. $|\vec{a}| = |\vec{b}| = 1$

To Find – $(3\vec{a} + 2\vec{b}).(5\vec{a} - 6\vec{b})$

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where-$\vec{p}$ and-$\vec{q}$ are two vectors

Tip - $\vec{a} \perp \vec{b}$

$\therefore |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\cos\frac{\pi}{2} = 0$

$\therefore \vec{a}.\vec{b} = \vec{b}.\vec{a} = 0$

Hence,

$(3\vec{a} + 2\vec{b}).(5\vec{a} - 6\vec{b})$

$= 15|\vec{a}|^2 + 10\vec{b}.\vec{a} - 18\vec{a}.\vec{b} - 12|\vec{b}|^2$

$= 15 - 12$

$= 3$

Question: 16

Mark (√) against

Solution:

Given - $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{a} \perp \vec{b}$

To find – Value of-λ

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where and -$\vec{p}$ are two vectors

Tip – For perpendicular vectors, $\theta = \frac{\pi}{2}$ i.e. $\cos\theta = 0$ i.e. the dot product=0

Hence, $\vec{a}.\vec{b} = 0$

$\therefore (3\hat{i} + \hat{j} - 2\hat{k}).(\hat{i} + \lambda\hat{j} - 3\hat{k}) = 0$

$\Rightarrow 3 + \lambda + 6 = 0$

i.e. $\lambda = -9$

Question: 16

Mark (√) against

Solution:

Given - $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{a} \perp \vec{b}$

To find – Value of-λ

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where and -$\vec{p}$ are two vectors

Tip – For perpendicular vectors, $\theta = \frac{\pi}{2}$ i.e. $\cos\theta = 0$ i.e. the dot product=0

Hence, $\vec{a}.\vec{b} = 0$

$\therefore (3\hat{i} + \hat{j} - 2\hat{k}).(\hat{i} + \lambda\hat{j} - 3\hat{k}) = 0$

$\Rightarrow 3 + \lambda + 6 = 0$

i.e. $\lambda = -9$

Question: 17

Solution:

Given - $\hat{a}$ and-$\hat{b}$ are two unit vectors with an angle-θ between them

To find - $\frac{1}{2}|\hat{a} - \hat{b}|$

Formula used - If $\vec{a}$ and-$\vec{b}$ are two vectors then $|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

$\cos 2\theta = 1 - 2\sin^2\theta$

Tip - $|\hat{a}|^2 = |\hat{b}|^2 = 1$ & $\hat{a}.\hat{b} = 1$

Hence,

$$\frac{1}{2}|\hat{a} - \hat{b}|$$

$$= \frac{1}{2}\sqrt{|\hat{a}|^2 + |\hat{b}|^2 + 2ab\cos\theta}$$

$$= \frac{1}{2}\sqrt{2 + 2\cos\theta}$$

$$= \frac{1}{\sqrt{2}}\sqrt{1 + \cos\theta}$$

$$= \frac{1}{\sqrt{2}} \times \sqrt{2\sin^2\frac{\theta}{2}}$$

$$= \sin\frac{\theta}{2}$$

Question: 17

Solution:

Given - $\hat{a}$ and-$\hat{b}$ are two unit vectors with an angle-θ between them

To find - $\frac{1}{2}|\hat{a} - \hat{b}|$

Formula used - If $\vec{a}$ and-$\vec{b}$ are two vectors then $|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

$\cos 2\theta = 1 - 2\sin^2\theta$

Tip - $|\hat{a}|^2 = |\hat{b}|^2 = 1$ & $\hat{a}.\hat{b} = 1$

Hence,

$$\frac{1}{2}|\hat{a} - \hat{b}|$$

$$= \frac{1}{2}\sqrt{|\hat{a}|^2 + |\hat{b}|^2 + 2ab\cos\theta}$$

$$= \frac{1}{2}\sqrt{2 + 2\cos\theta}$$

$= \frac{1}{\sqrt{2}}\sqrt{1 + \cos\theta}$

$= \frac{1}{\sqrt{2}} \times \sqrt{2\sin^2\frac{\theta}{2}}$

$= \sin\frac{\theta}{2}$

Question: 18

Solution:

Given - $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ are two vectors.

To find - $|\vec{a} \times \vec{b}|$

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 3 & -4 \end{vmatrix}$

$= \hat{i}(4 - 6) + \hat{j}\,(4 + 4) + \hat{k}(3 + 2)$

$= -2\hat{i} + 8\hat{j} + 5\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{2^2 + 8^2 + 5^2} = \sqrt{93}$

Question: 18

Solution:

Given - $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ are two vectors.

To find - $|\vec{a} \times \vec{b}|$

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 3 & -4 \end{vmatrix}$

$= \hat{i}(4 - 6) + \hat{j}\,(4 + 4) + \hat{k}(3 + 2)$

$= -2\hat{i} + 8\hat{j} + 5\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{2^2 + 8^2 + 5^2} = \sqrt{93}$

Question: 19

Mark (√) against

Solution:

Given - $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{k}$ are two vectors.

To find - $|\vec{b} \times 2\vec{a}|$

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{b} \times 2\vec{a}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -3 \\ 2 & -4 & 6 \end{vmatrix}$

$= \hat{i}(12) + \hat{j}\,(-6 - 6) + \hat{k}(-4)$

$= 12\hat{i} - 12\hat{j} - 4\hat{k}$

$\therefore |\vec{b} \times 2\vec{a}| = \sqrt{12^2 + 12^2 + 4^2} = \sqrt{304} = 4\sqrt{19}$

Question: 19

Mark (√) against

Solution:

Given - $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{k}$ are two vectors.

To find - $|\vec{b} \times 2\vec{a}|$

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{b} \times 2\vec{a}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -3 \\ 2 & -4 & 6 \end{vmatrix}$

$= \hat{i}(12) + \hat{j}\,(-6 - 6) + \hat{k}(-4)$

$= 12\hat{i} - 12\hat{j} - 4\hat{k}$

$\therefore |\vec{b} \times 2\vec{a}| = \sqrt{12^2 + 12^2 + 4^2} = \sqrt{304} = 4\sqrt{19}$

Question: 20

Solution:

Given - $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$

~~Tip~~ – $|\vec{p} \times \vec{q}| = \left||\vec{p}||\vec{q}|\sin\theta\hat{n}\right| = |\vec{p}||\vec{q}|\sin\theta$ & magnitude of a vector $\vec{p} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$: $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence, $|\vec{a} \times \vec{b}| = |3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}| = \sqrt{3^2 + 2^2 + 6^2} = 7$

$\therefore 7 = 2 \times 7\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

Question: 20

Solution:

Given - $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$

~~Tip~~ – $|\vec{p} \times \vec{q}| = \left||\vec{p}||\vec{q}|\sin\theta\hat{n}\right| = |\vec{p}||\vec{q}|\sin\theta$ & magnitude of a vector $\vec{p} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence, $|\vec{a} \times \vec{b}| = |3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}| = \sqrt{3^2 + 2^2 + 6^2} = 7$

$\therefore 7 = 2 \times 7\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

Question: 21

Solution:

Given – $|\vec{a}| = \sqrt{26}$, $|\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$

To find - $\vec{a}.\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$ & $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ & $\vec{q}$ are any two vectors

Tip – $|\vec{p} \times \vec{q}| = \left||\vec{p}||\vec{q}|\sin\theta\hat{n}\right| = |\vec{p}||\vec{q}|\sin\theta$

So,

$|\vec{a} \times \vec{b}| = 35$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = 35$

$\Rightarrow \sin\theta = \frac{35}{7\sqrt{26}} = \frac{5}{\sqrt{26}}$

$\therefore \cos\theta = \sqrt{1 - \left(\frac{5}{\sqrt{26}}\right)^2} = \frac{1}{\sqrt{26}}$

$\therefore \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = \sqrt{26} \times 7 \times \frac{1}{\sqrt{26}} = 7$

Question: 21

Solution:

Given – $|\vec{a}| = \sqrt{26}$, $|\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$

To find - $\vec{a}.\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$ & $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ & $\vec{q}$ are any two vectors

Tip – $|\vec{p} \times \vec{q}| = \left||\vec{p}||\vec{q}|\sin\theta\hat{n}\right| = |\vec{p}||\vec{q}|\sin\theta$

So,

$|\vec{a} \times \vec{b}| = 35$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = 35$

$\Rightarrow \sin\theta = \frac{35}{7\sqrt{26}} = \frac{5}{\sqrt{26}}$

$\therefore \cos\theta = \sqrt{1 - \left(\frac{5}{\sqrt{26}}\right)^2} = \frac{1}{\sqrt{26}}$

$\therefore \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = \sqrt{26} \times 7 \times \frac{1}{\sqrt{26}} = 7$

Question: 22

Solution:

Given - Two adjacent sides of a || gm are represented by the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

To find – Area of the parallelogram

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

Tip – Area of ||gm = $|\vec{a} \times \vec{b}|$ and magnitude of a vector $\vec{p} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$

$= \hat{\imath}(-4 - 1) + \hat{\jmath}(4 - 3) + \hat{k}(-3 - 1)$

$= -5\hat{\imath} + \hat{\jmath} - 4\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{5^2 + 1^2 + 4^2} = \sqrt{42}$

i.e. the area of the parallelogram = $\sqrt{42}$ sq. units

Question: 23

Solution:

Given - Two diagonals of a || gm are represented by the vectors $\overrightarrow{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$; $\overrightarrow{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$

To find – Area of the parallelogram

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

~~Tip – Area of ||gm~~ $= \frac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right|$ and magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\overrightarrow{d_1} \times \overrightarrow{d_2}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$

$= \hat{i}(4 - 6) + \hat{j}(-2 - 12) + \hat{k}(-9 - 1)$

$= -2\hat{i} - 14\hat{j} - 10\hat{k}$

$\therefore \left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right| = \sqrt{2^2 + 14^2 + 10^2} = \sqrt{300}$

i.e. the area of the parallelogram $= \frac{1}{2} \times \sqrt{300} = 5\sqrt{3}$ sq. units

Question: 1

Solution:

Given - Two adjacent sides of a triangle are represented by the vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$

To find – Area of the triangle

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Area of triangle $= \frac{1}{2}\left|\vec{a} \times \vec{b}\right|$ and magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix}$

$= \hat{k}(21 + 20)$

$= 41\hat{k}$

$\therefore \left|\vec{a} \times \vec{b}\right| = \sqrt{41^2} = 41$

i.e. the area of the parallelogram $= \frac{41}{2}$ sq. units

Question: 1

Solution:

Given - Two adjacent sides of a triangle are represented by the vectors $\vec{a} = 3\hat{i} +$ ·
$\vec{b} = -5\hat{i} + 7\hat{j}$

To find – Area of the triangle

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Area of triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$ and magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by

$|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix}$

$= \hat{k}(21 + 20)$

$= 41\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{41^2} = 41$

i.e. the area of the parallelogram $= \frac{41}{2}$ sq. units

Question: 25

Solution:

Given - $\vec{a} = \hat{i} - \hat{j} - \hat{k}$ & $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

To find – A unit vector perpendicular to the two given vectors.

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – A vector perpendicular to two given vectors is their cross product.

The unit vector of any vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{(a\hat{i}+b\hat{j}+c\hat{k})}{\sqrt{a^2+b^2+c^2}}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$

$= -2\hat{j} + 2\hat{k}$, which the vector perpendicular to the two given vectors.

The required unit vector $= \frac{-2\hat{j}+2\hat{k}}{\sqrt{2^2+2^2}} = \frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

Question: 25

Solution:

Given - $\vec{a} = \hat{i} - \hat{j} - \hat{k}$ & $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

To find – A unit vector perpendicular to the two given vectors.

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath}$

Tip – A vector perpendicular to two given vectors is their cross product.

The unit vector of any vector $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is given by $\frac{(ai+bj+c\hat{k})}{\sqrt{a^2+b^2+c^2}}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$

$= -2\hat{\jmath} + 2\hat{k}$, which the vector perpendicular to the two given vectors.

The required unit vector $= \frac{-2j+2\hat{k}}{\sqrt{2^2+2^2}} = \frac{1}{\sqrt{2}}(-\hat{\jmath} + \hat{k})$

Question: 26

Solution:

Given - $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors and $(\vec{a} + \vec{b} + \vec{c}) = 0$

To find - $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

Tip – $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

So,

$(\vec{a} + \vec{b} + \vec{c})^2 = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\Rightarrow 3 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\Rightarrow (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = \frac{-3}{2}$

Question: 26

Solution:

Given - $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors and $(\vec{a} + \vec{b} + \vec{c}) = 0$

To find - $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

Tip – $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

So,

$(\vec{a} + \vec{b} + \vec{c})^2 = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\Rightarrow 3 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\Rightarrow (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = \frac{-3}{2}$

Question: 27

Mark (√) against

Solution:

Given - $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors

To find - $[\vec{a} + \vec{b} + \vec{c}]$

Tip – $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ & $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

So,

$(\vec{a} + \vec{b} + \vec{c})^2$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

$=3$

$\therefore [\vec{a} + \vec{b} + \vec{c}] = \sqrt{3}$

Question: 27

Solution:

Given - $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors

To find - $[\vec{a} + \vec{b} + \vec{c}]$

Tip – $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ & $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

So,

$(\vec{a} + \vec{b} + \vec{c})^2$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

$=3$

$\therefore [\vec{a} + \vec{b} + \vec{c}] = \sqrt{3}$

Question: 28

Solution:

To find - $[\hat{i} \;\; \hat{j} \;\; \hat{k}]$

Formula to be used - $[\hat{a} \;\; \hat{b} \;\; \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$\therefore [\hat{i} \;\; \hat{j} \;\; \hat{k}]$

$= \hat{i}\,.(\hat{j} \times \hat{k})$

$= \hat{i}.\hat{i}$

$= |\hat{i}|^2$

$= 1$

Question: 28

Solution:

To find - $[\hat{i} \;\; \hat{j} \;\; \hat{k}]$

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b}\times\hat{c})$

$\therefore [\hat{i}\ \ \hat{j}\ \ \hat{k}]$

$= \hat{i}\,.(\hat{j}\times\hat{k})$

$= \hat{i}.\hat{i}$

$= |\hat{i}|^2$

$= 1$

Question: 29

Solution:

Given – The three coterminous edges of a parallelepiped are $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$,

$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ & $\vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$

To find – The volume of the parallelepiped

~~Formula to be used -~~ $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b}\times\hat{c})$

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ ~~where~~ $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - The volume of the parallelepiped $= \left|[\hat{a}\ \ \hat{b}\ \ \hat{c}]\right|$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}]$

$= \hat{a}\,.(\hat{b}\times\hat{c})$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).\{(\hat{i} + 2\hat{j} - \hat{k})\times(3\hat{i} - \hat{j} - 2\hat{k})\}$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).(-5\hat{i} - \hat{j} - 7\hat{k})$

$= -10 + 3 - 28$

$= -35$

The volume = 35 sq units

Question: 29

Solution:

Given – The three coterminous edges of a parallelepiped are $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$,

$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ & $\vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$

To find – The volume of the parallelepiped

~~Formula to be used -~~ $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b}\times\hat{c})$

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ ~~where~~ $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - The volume of the parallelepiped $= \left|[\hat{a}\ \ \hat{b}\ \ \hat{c}]\right|$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}]$

$= \hat{a}.(\hat{b} \times \hat{c})$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).\{(\hat{i} + 2\hat{j} - \hat{k}) \times (3\hat{i} - \hat{j} - 2\hat{k})\}$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).(-5\hat{i} - \hat{j} - 7\hat{k})$

$= -10 + 3 - 28$

$= -35$

The volume = 35 sq units

Question: 30

Solution:

Given – The three coterminous edges of a parallelepiped are $\vec{a} = 5\hat{i} - 4\hat{j} + \hat{k}$,

$\vec{b} = 4\hat{i} + 3\hat{j} + \lambda\hat{k}$ & $\vec{c} = \hat{i} - 2\hat{j} + 7\hat{k}$

To find – The value of λ

~~Formula to be used –~~ $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ ~~where~~ $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - The volume of the parallelepiped $= |[\hat{a}\ \ \hat{b}\ \ \hat{c}]|$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}]$

$= \hat{a}.(\hat{b} \times \hat{c})$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\{(4\hat{i} + 3\hat{j} + \lambda\hat{k}) \times (\hat{i} - 2\hat{j} + 7\hat{k})\}$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & \lambda \\ 1 & -2 & 7 \end{vmatrix}$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\left((21 + 2\lambda)\hat{i} + (\lambda - 28)\hat{j} - 11\hat{k}\right)$

$=5(21+2\lambda)-4(\lambda-28)-11$

$=206+6\lambda$

The volume $=206+6\lambda$

But, the volume = 216 sq units

So, $206+6\lambda=216 \Rightarrow \lambda=\frac{10}{6} = \frac{5}{3}$

Question: 30

Solution:

Given – The three coterminous edges of a parallelepiped are

$\vec{a} = 5\hat{i} - 4\hat{j} + \hat{k}$,

$\vec{b} = 4\hat{i} + 3\hat{j} + \lambda\hat{k}$ & $\vec{c} = \hat{i} - 2\hat{j} + 7\hat{k}$

To find – The value of λ

————————— $[\hat{a}\ \hat{b}\ \hat{c}] = \hat{a}.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ ——— $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - The volume of the parallelepiped $= \left| [\hat{a}\ \hat{b}\ \hat{c}] \right|$

Hence,

$[\hat{a}\ \hat{b}\ \hat{c}]$

$= \hat{a}.(\hat{b} \times \hat{c})$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\{(4\hat{i} + 3\hat{j} + \lambda\hat{k}) \times (\hat{i} - 2\hat{j} + 7\hat{k})\}$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & \lambda \\ 1 & -2 & 7 \end{vmatrix}$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\left((21 + 2\lambda)\hat{i} + (\lambda - 28)\hat{j} - 11\hat{k}\right)$

$=5(21+2\lambda)-4(\lambda-28)-11$

$=206+6\lambda$

The volume $=206+6\lambda$

But, the volume = 216 sq units

So, $206+6\lambda=216 \Rightarrow \lambda=\frac{10}{6}=\frac{5}{3}$

Question: 31

Solution:

Given – The vectors $\vec{a} = 2\hat{i} - 2\hat{k}, \vec{b} = \hat{i} + (\lambda + 1)\hat{j}$ & $\vec{c} = 4\hat{i} + 2\hat{k}$ are coplanar

To find – The value of λ

~~Formula to be used –~~ $[\hat{a}\ \hat{b}\ \hat{c}] = \hat{a}.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ ~~where~~ $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – For vectors to be coplanar, $[\hat{a}\ \hat{b}\ \hat{c}] = 0$

Hence,

$[\hat{a}\ \hat{b}\ \hat{c}] = 0$

$\Rightarrow \hat{a}.(\hat{b} \times \hat{c}) = 0$

$\Rightarrow (2\hat{i} - 2\hat{k}).\{(\hat{i} + (\lambda + 1)\hat{j}) \times (4\hat{i} + 2\hat{k})\} = 0$

$\Rightarrow (2\hat{i} - 2\hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \lambda + 1 & 0 \\ 4 & 0 & 2 \end{vmatrix} = 0$

$\Rightarrow (2\hat{i} - 2\hat{k}).(2(\lambda + 1)\hat{i} - 2\hat{j} - 4(\lambda + 1)\hat{k}) = 0$

$\Rightarrow 4(\lambda-1)+8(\lambda-1)=0$

⇨ 12(λ-1)=0 i.e. λ= 1

Question: 31

Solution:

Given – The vectors $\vec{a} = 2\hat{i} - 2\hat{k}, \vec{b} = \hat{i} + (\lambda + 1)\hat{j}\ \&\ \vec{c} = 4\hat{i} + 2\hat{k}$ are coplanar

To find – The value of λ

Formula used – $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – For vectors to be coplanar, $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = 0$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}] = 0$

$\Rightarrow \hat{a}\,.(\hat{b} \times \hat{c}) = 0$

$\Rightarrow (2\hat{i} - 2\hat{k}).\{(\hat{i} + (\lambda + 1)\hat{j}) \times (4\hat{i} + 2\hat{k})\} = 0$

$$\Rightarrow (2\hat{i} - 2\hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \lambda + 1 & 0 \\ 4 & 0 & 2 \end{vmatrix} = 0$$

$\Rightarrow (2\hat{i} - 2\hat{k}).(2(\lambda + 1)\hat{i} - 2\hat{j} - 4(\lambda + 1)\hat{k}) = 0$

⇨ 4(λ-1)+8(λ-1)=0

⇨ 12(λ-1)=0 i.e. λ= 1

Question: 32

Solution:

Tip - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c}) = \hat{b}\,.(\hat{c} \times \hat{a}) = \hat{c}\,.(\hat{a} \times \hat{b}) = (\hat{a} \times \hat{b}).\,\hat{c}$ since, dot product is commutative

Hence, $\hat{a} \times (\hat{b}.\hat{c})$ is meaningless.

Question: 32

Solution:

Tip - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c}) = \hat{b}\,.(\hat{c} \times \hat{a}) = \hat{c}\,.(\hat{a} \times \hat{b}) = (\hat{a} \times \hat{b}).\,\hat{c}$ since, dot product is commutative

Hence, $\hat{a} \times (\hat{b}.\hat{c})$ is meaningless.

Question: 33

Solution:

Tip – The cross product of two vectors is the vector perpendicular to both the vectors.

$\therefore \vec{a} \times \vec{b}$ gives a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

Now,

$\vec{a}.(\vec{a} \times \vec{b})$

$= |\vec{a}||\vec{b}|\cos\theta$

$= |\vec{a}||\vec{b}|\cos\frac{\pi}{2}$

$= 0$

Question: 33

Solution:

Tip – The cross product of two vectors is the vector perpendicular to both the vectors.

$\therefore \vec{a} \times \vec{b}$ gives a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

Now,

$\vec{a}.(\vec{a} \times \vec{b})$

$= |\vec{a}||\vec{b}|\cos\theta$

$= |\vec{a}||\vec{b}|\cos\frac{\pi}{2}$

$= 0$

Question: 34

Solution:

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}.(\hat{b} \times \hat{c}) = \hat{b}.(\hat{c} \times \hat{a})$ for any three arbitrary vectors

$\therefore [\hat{a} - \hat{b}\ \ \hat{b} - \hat{c}\ \ \hat{c} - \hat{a}]$

$= (\hat{a} - \hat{b}).\{(\hat{b} - \hat{c}) \times (\hat{c} - \hat{a})\}$

$= (\hat{a} - \hat{b}).(\hat{b} \times \hat{c} - \hat{c} \times \hat{c} - \hat{b} \times \hat{a} + \hat{c} \times \hat{a})$

$= (\hat{a} - \hat{b}).(\hat{b} \times \hat{c} - \hat{b} \times \hat{a} + \hat{c} \times \hat{a})$

$= [\hat{a}.(\hat{b} \times \hat{c}) - \hat{b}(\hat{b} \times \hat{c}) - \hat{a}.(\hat{b} \times \hat{a}) + \hat{b}(\hat{b} \times \hat{a}) + \hat{a}.(\hat{c} \times \hat{a}) - \hat{b}.(\hat{c} \times \hat{a})]$

$= [\hat{a}\ \hat{b}\ \hat{c}] - [\hat{a}\ \hat{b}\ \hat{c}] = 0$

Question: 34

Solution:

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}.(\hat{b} \times \hat{c}) = \hat{b}.(\hat{c} \times \hat{a})$ for any three arbitrary vectors

$\therefore [\hat{a} - \hat{b}\ \ \hat{b} - \hat{c}\ \ \hat{c} - \hat{a}]$

$= (\hat{a} - \hat{b}).\{(\hat{b} - \hat{c}) \times (\hat{c} - \hat{a})\}$

$= (\hat{a} - \hat{b}).(\hat{b} \times \hat{c} - \hat{c} \times \hat{c} - \hat{b} \times \hat{a} + \hat{c} \times \hat{a})$

$= (\hat{a} - \hat{b}).(\hat{b} \times \hat{c} - \hat{b} \times \hat{a} + \hat{c} \times \hat{a})$

$= [\hat{a}.(\hat{b} \times \hat{c}) - \hat{b}(\hat{b} \times \hat{c}) - \hat{a}.(\hat{b} \times \hat{a}) + \hat{b}(\hat{b} \times \hat{a}) + \hat{a}.(\hat{c} \times \hat{a}) - \hat{b}.(\hat{c} \times \hat{a})]$

$= [\hat{a}\ \hat{b}\ \hat{c}] - [\hat{a}\ \hat{b}\ \hat{c}] = 0$

Question: 1

Mark (√) against

Solution:

Given vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Property : The unit vector corresponding to the vector $a\hat{i} + b\hat{j} + c\hat{k} = \frac{ai+bj+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$

Therefore the unit vector corresponding to the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

is

$$\hat{a} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{2^2 + (-3)^2 + 6^2}}$$

$$\hat{a} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{4 + 9 + 16}}$$

$$\hat{a} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{49}}$$

$$\hat{a} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$$

$$\hat{a} = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

Question: 2

Solution:

Given vector $\vec{r} = -2\hat{i} + 1\hat{j} - 5\hat{k}$

Property: for the vector $\hat{i} + b\hat{j} + c\hat{k}$

1) Direction ratios dr's are a,b,c

2) Direction cosines dc's are $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$

Therefore the dc's of the vector $-2\hat{i} + 1\hat{j} - 5\hat{k} = \frac{-2}{\sqrt{(-2)^2+1^2+(-5)^2}}, \frac{1}{\sqrt{(-2)^2+1^2+(-5)^2}}, \frac{-5}{\sqrt{(-2)^2+1^2+(-5)^2}}$

$= \frac{-2}{\sqrt{4+1+25}}, \frac{1}{\sqrt{4+1+25}}, \frac{-5}{\sqrt{4+1+25}}$

$= \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$

Question: 3

Solution:

Given A(1,2,-3) and B(-1,-2,1)

Property: The position vector of the of the vector joining two points $(x_1,y_1,z_1)$and (x_2,y_2,z_2) is $(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

So, the position vector of the line joining A and B is

$\overrightarrow{AB} = (-1 - 1)\hat{i} + (-2 - 2)\hat{j} + [1 - (-1)\}\hat{k}$

$\overrightarrow{AB} = -2\hat{i} - 4\hat{j} + 4\hat{k}$

Property: for the vector$\hat{i} + b\hat{j} + c\hat{k}$, Direction cosines dc's are $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{}{\sqrt{a^2+b^2+c^2}}$

Therefore the Dc's of the vector $\overrightarrow{AB} = \frac{-2}{\sqrt{(-2)^2+(-4)^2+4^2}}, \frac{-4}{\sqrt{(-2)^2+(-4)^2+4^2}}, \frac{4}{\sqrt{(-2)^2+(-4)^2+4^2}}$

$= \frac{-2}{\sqrt{4+16+16}}, \frac{-4}{\sqrt{4+16+16}}, \frac{4}{\sqrt{4+16+16}}$

$= \frac{-2}{\sqrt{36}}, \frac{-4}{\sqrt{36}}, \frac{4}{\sqrt{36}}$

$= -\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}$

$= -\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

Question: 4

Solution:

Given α , β and γ are the angles made by the vector with X,Y and z axes respectively

⇒ $\cos\alpha, \cos\beta, \cos\gamma$ are the direction cosines .

As we know that if $\cos\alpha, \cos\beta, \cos\gamma$ are the direction cosines , then the relation between them is $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

We also know that $\cos^2\theta = 1 - \sin^2\theta$

So we can write $(1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$

$\Rightarrow 3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Question: 5

Solution:

Given vector

$\cos\alpha\cos\beta\,\hat{i} + \cos\alpha\sin\beta\,\hat{j} + \sin\alpha\,\hat{k}$

UNIT VECTOR: the vector with magnitude as 1.

Property: The magnitude of the vector $a\hat{i} + b\hat{j} + c\hat{k} = \sqrt{a^2 + b^2 + c^2}$

The magnitude of the given vector is $\sqrt{(\cos\alpha\cos\beta)^2 + (\cos\alpha\sin\beta)^2 + \sin^2\alpha}$

$= \sqrt{\cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha}$

$= \sqrt{\cos^2\alpha + \sin^2\alpha}$

$= 1$

As the magnitude of the given vector is 1, it is a UNIT VECTOR.

Question: 6

Solution:

Given vector is

$1\hat{i} + 1\hat{j} + \sqrt{2}\hat{k}$

Property: for the vector $\hat{\imath} + b\hat{\jmath} + c\hat{k}$, Direction cosines dc's are $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}$ CLASS24

Therefore the dc's of the given vector is $\frac{1}{\sqrt{1^2+1^2+\sqrt{2}^2}}, \frac{1}{\sqrt{1^2+1^2+\sqrt{2}^2}}, \frac{\sqrt{2}}{\sqrt{1^2+1^2+\sqrt{2}^2}}$

$= \frac{1}{\sqrt{1+1+2}}, \frac{1}{\sqrt{1+1+2}}, \frac{\sqrt{2}}{\sqrt{1+1+2}}$

$= \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{4}}, \frac{\sqrt{2}}{\sqrt{4}}$

$= \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$

Let the angle made by the vector with the Z axis be γ.

we got that the cosine of the angle γ is $\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\gamma = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos\gamma = \cos\left(\frac{\pi}{4}\right)$

$\Rightarrow \gamma = \frac{\pi}{4}$

Question: 7

Solution:

Given $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$

And $\vec{a}.\vec{b} = \sqrt{6}$

Let angle between the vectors-$\vec{a}$ and -$\vec{b}$ be θ

Using the dot product property of the vectors,

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Substituting the given values in the equation,

$\sqrt{6} = \sqrt{3} \times 2 \times \cos\theta$

$\Rightarrow \cos\theta = \frac{\sqrt{6}}{\sqrt{3}\times 2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}$

Question: 8

Solution:

Given

Given $|\vec{a}| = \sqrt{2}, |\vec{b}| = \sqrt{2}$

And $\vec{a}.\vec{b} = -1$

Let angle between the vectors-$\vec{a}$ and -$\vec{b}$ be θ

Using the dot product property of the vectors,

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Substituting the given values in the equation,

$-1 = \sqrt{2} \times \sqrt{2} \times \cos\theta$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow -\cos\theta = \frac{1}{2}$

$\Rightarrow \cos(\pi - \theta) = \cos\frac{\pi}{3}$

$\Rightarrow \pi - \theta = \frac{\pi}{3}$

$\Rightarrow \theta = \frac{2\pi}{3}$

Question: 9

Solution:

Given vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and

Magnitude $|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

Magnitude of $|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$

Property:

$\vec{r_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{r_2} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\vec{r_1}.\vec{r_2} = (x_1.x_2)\hat{i} + (y_1.y_2)\hat{j} + (z_1.z_2)\hat{k}$

Then

$\vec{a}.\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k}).(3\hat{i} - 2\hat{j} + \hat{k})$

= (1 x 3)+(-2 x -2)+(3 x 1)

=3+4+3

= 10

Let angle between the vectors-$\vec{a}$ and -$\vec{b}$ be θ

Using the dot product property of the vectors,

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Substituting the given values in the equation,

$10 = \sqrt{14} \times \sqrt{14} \times \cos\theta$

$\Rightarrow \cos\theta = \frac{10}{14}$

$\Rightarrow \cos\theta = \frac{5}{7}$

$\Rightarrow \theta = \cos^{-1}\frac{5}{7}$

Question: 10

Mark (√) against

Solution:

Given vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - 1\hat{j} + 2\hat{k}$

$\vec{a} + \vec{b} = 4\hat{i} + \hat{j} - \hat{k}$

$\vec{a} - \vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$

$(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = (4\hat{i} + \hat{j} - \hat{k}).(-2\hat{i} + 3\hat{j} - 5\hat{k})$

= -8+3+5

=0

As $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$, then the cosine of angle between the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is 0.

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$.

Question: 11

Solution:

Given vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - 1\hat{j} + 2\hat{k}$

$2\vec{a} = 2\hat{i} + 4\hat{j} - 6\hat{k}$

$2\vec{b} = 6\hat{i} - 2\hat{j} + 4\hat{k}$

Let the vector $2\vec{a} + \vec{b}$ be $\vec{U}$

$\vec{U} = 2\vec{a} + \vec{b} = 2\hat{i} + 4\hat{j} - 6\hat{k} + 3\hat{i} - 1\hat{j} + 2\hat{k}$

$\vec{U} = 2\vec{a} + \vec{b} = 5\hat{i} + 3\hat{j} - 4\hat{k}$

$|\vec{U}| = \sqrt{5^2 + 3^2 + (-4)^2} = \sqrt{25 + 9 + 16} = \sqrt{50}$

Let the vector $2\vec{b} + \vec{a}$ be $\vec{V}$

$\vec{V} = \vec{a} + 2\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k} + 6\hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{V} = \vec{a} + 2\vec{b} = 7\hat{i} + 0\hat{j} + \hat{k}$

$|\vec{V}| = \sqrt{7^2 + 0^2 + 1^2} = \sqrt{49 + 1} = \sqrt{50}$

$\vec{U}.\vec{V} = (5\hat{i} + 3\hat{j} - 4\hat{k}).(7\hat{i} + 0\hat{j} + \hat{k})$

= (5 × 7) + 0 -(4 × 1)

=35-4

=31

Let angle between the vectors $\vec{U}$ and $\vec{V}$ be θ

Using the dot product property of the vectors,

$\vec{U}.\vec{V} = |\vec{U}||\vec{V}|\cos\theta$

Substituting the given values in the equation,

$31 = \sqrt{50} \times \sqrt{50} \times \cos\theta$

$\Rightarrow \cos\theta = \frac{31}{50}$

$\Rightarrow \theta = \cos^{-1}\frac{31}{50}$

Question: 12

Solution:

Given vectors $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$

Also given that $\vec{a} \perp \vec{b}$

Let the angle between the vectors $\vec{a}$ and $\vec{b}$ be θ.

$\Rightarrow \theta = \frac{\pi}{2}$

$= \cos\theta = 0$

$\Rightarrow \vec{a}.\vec{b} = 0$

So, $(2\hat{i} + 4\hat{j} - \hat{k}).(3\hat{i} - 2\hat{j} + \lambda\hat{k}) = 0$

$\Rightarrow (2 \times 3) + (4 \quad -2) + (-1\times \lambda) = 0$

$\Rightarrow 6 - 8 - \lambda = 0$

$\Rightarrow \lambda = -2$

Question: 13

Mark (√) against

Solution:

Given vectors $\vec{a} = 2\hat{i} - 1\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 1\hat{k}$

Property:

Projection of the vector $\vec{a}$ on $-\vec{b}$ is $\vec{a}.\frac{\vec{b}}{|\vec{b}|} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

Therefore the projection of $\vec{a}$ on $-\vec{b}$ is $\frac{(2\hat{i}-1\hat{j}+\hat{k}).(\hat{i}-2\hat{j}+1\hat{k})}{\sqrt{1^2+(-2)^2+1^2}}$

$= \frac{(2x1)+(-1x-2)+(1x1)}{\sqrt{1+4+1}}$

$= \frac{2+2+1}{\sqrt{6}}$

$= \frac{5}{\sqrt{6}}$

Question: 14

Solution:

Given $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

Squaring on both the sides,

$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a}.\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a}.\vec{b})$

$\Rightarrow 4.\vec{a}.\vec{b}=0$

$\Rightarrow \vec{a}.\vec{b}=0$

$\Rightarrow \vec{a} \perp \vec{b}$

Question: 15

Solution:

Given $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors

$\Rightarrow |\vec{a}| = |\vec{b}| = 1$

And angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$ and $\vec{a}.\vec{b}=0$

Asking to find $(3\vec{a} + 2\vec{b}).(5\vec{a} - 6\vec{b})$

Multiplying ,

$=(3\times5)\ |\vec{a}|^2 - (3\times6)\ (\vec{a}.\vec{b}) + (2\times5)(\vec{b}.\vec{a}) - (2\times6)\ |\vec{b}|^2$

$= 15|\vec{a}|^2 - 18(\vec{a}.\vec{b}) + 10(\vec{a}.\vec{b}) - 12|\vec{b}|^2$ [reason: dot product is commutative i.e, $\vec{a}.\vec{b} = \vec{b}.\vec{a}$]

$=15-8(\vec{a}.\vec{b})-12$

$=15-12$ [reason: $\vec{a}.\vec{b}=0$]

$= 3$

Question: 16

Solution:

Given vectors $\vec{a} = 3\hat{i} + 1\hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$

Also given $\vec{a} \perp \vec{b}$

As they are perpendicular, $\vec{a}.\vec{b}=0$

$\Rightarrow (3\hat{i} + 1\hat{j} - 2\hat{k}).(\hat{i} + \lambda\hat{j} - 3\hat{k}) = 0$

$\Rightarrow (3\times 1) + (1\times \lambda) + (-2 \times -3) =0$

$\Rightarrow 3+\lambda+6=0$

$\Rightarrow \lambda = -9$

Question: 17

Solution:

Given $\hat{a}$ and $\hat{b}$ are unit vectors

Let θ be the angle between them.

Asking us to find the value of $\frac{1}{2}|\hat{a} - \hat{b}|$

Let this value de T

$\Rightarrow T=\frac{1}{2}|\hat{a}-\hat{b}|$

Squaring on both the sides

$T^2=\frac{1}{4}|(\hat{a})^2+(\hat{b})^2-2.(\hat{a}.\hat{b})|$

$T^2=\frac{1}{4}|1+1-2.(\hat{a}.\hat{b})|$

$T^2=\frac{1}{4}|2-2.(\hat{a}.\hat{b})|$

$T^2=\frac{2}{4}|1-(\hat{a}.\hat{b})|$

$T^2=\frac{1}{2}|1-(\hat{a}.\hat{b})|$

$T^2=\frac{1}{2}|1-(|\hat{a}||\hat{b}|)\cos\theta)|$

$T^2=\frac{1}{2}.|1-(1.\cos\theta)|$

$(1-\cos\theta)$ can be written as $2.\sin^2\frac{\theta}{2}$

$\Rightarrow T^2=\frac{1}{2}.|1-(1.\cos\theta)|$

$= T^2=\frac{1}{2}.|2.\sin^2\frac{\theta}{2}|$

$T^2=\sin^2\frac{\theta}{2}$

$\Rightarrow T=\sin\frac{\theta}{2}$

Question: 18

Solution:

Given vectors $\vec{a}=\hat{i}-1\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}-4\hat{k}$

$\vec{a} \times \vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ 2 & 3 & -4\end{vmatrix}$

$=\hat{i}[(-1 \times -4)-(2 \times 3)] - \hat{j}[(1 \times -4)-(2 \times 2)] + \hat{k}[(1 \times 3)-(2 \times -1)]$

$=\hat{i}[4-6] - \hat{j}[-4-4] + \hat{k}[3+2]$

$=-2\hat{i}+8\hat{j}+5\hat{k}$

$|\vec{a} \times \vec{b}|=\sqrt{(-2)^2+8^2+5^2}=\sqrt{4+64+25}=\sqrt{93}$

Question: 19

Solution:

Given vectors $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=\hat{i}-3\hat{k}$

Asking us to find, $|\vec{b} \times 2\vec{a}|$

$2\vec{a}=2\hat{i}-4\hat{j}+6\hat{k}$

$\vec{b} \times 2\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -3 \\ 2 & -4 & 6 \end{vmatrix}$

$= -\hat{i}[0-(-4 \times -3)] - \hat{j}[(1 \times 6)-(2 \times -3)] + -\hat{k}[(1 \times -4)-0]$

$= \hat{i}(-12) - \hat{j}(6+6) + \hat{k}(-4)$

$= -12\hat{i} + 12\hat{j} - 4\hat{k}$

$|\vec{b} \times 2\vec{a}| = \sqrt{(-12)^2 + 12^2 + (-4)^2} = \sqrt{414 + 144 + 16} = \sqrt{304}$

$= \sqrt{16.19}$

$= 4.\sqrt{19}$

Question: 20

Solution:

Given

$|\vec{a}| = 2$

And $|\vec{b}| = 7$

$\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{3^2 + 2^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$

Let the angle between the vector be θ

As we know that,

$|\vec{a} \times \vec{b}| = |\vec{a}|.|\vec{b}| \sin\theta$

Substituting the values,

$7 = 2 \times 7 \times \sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

Question: 21

Solution:

Given

$|\vec{a}| = \sqrt{26}$

And $|\vec{b}| = 7$

$|\vec{a} \times \vec{b}| = 35$ and $|\vec{a}.\vec{b}| = ?$

As we know that, $|\vec{a}.\vec{b}| = |\vec{a}||\vec{b}|\cos\theta$ and $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

Adding and subtracting the above equations,

$|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2\cos^2\theta + |\vec{a}|^2|\vec{b}|^2\sin^2\theta$

$|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2(\cos^2\theta + \sin^2\theta)$

$|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2(1)$

Substituting the given values, we get

$|\vec{a}.\vec{b}|^2+35^2 = \sqrt{26}^2 7^2$

$|\vec{a}.\vec{b}|^2+1225=26.49$

$|\vec{a}.\vec{b}|^2+1225=1274$

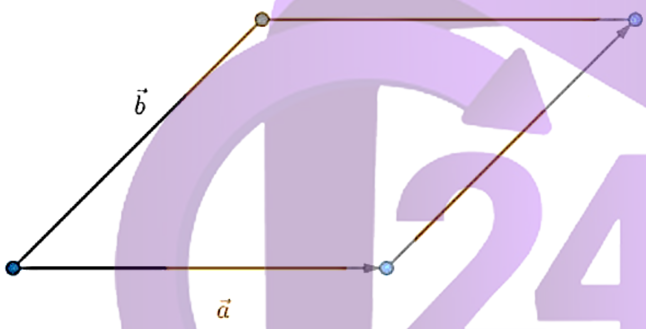
$|\vec{a}.\vec{b}|^2=1274-1225$

$|\vec{a}.\vec{b}|^2=49$

$|\vec{a}.\vec{b}| = 7$

Question: 22

Solution:



Given the adjacent sides of the parallelogram

$\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

Property: The area of the parallelogram with the adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$

Therefore the area of the parallelogram is

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

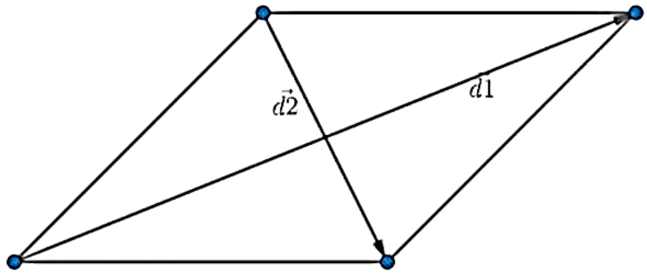
$= \hat{i}[1 - (-4)] - \hat{j}[3 - 4] + \hat{k}[-3 - 1]$

$= 5\hat{i} + \hat{j} - 4\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{5^2 + 1^2 + (-4)^2} = \sqrt{25 + 1 + 16} = \sqrt{42}$ sq.units

Question: 23

Solution:



Given diagonals of the parallelogram $\overrightarrow{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$

Area of the parallelogram as $\overrightarrow{d_1}$ and $\overrightarrow{d_2}$ as diagonals is $\frac{1}{2}|\overrightarrow{d_1} x \overrightarrow{d_2}|$

$\overrightarrow{d_1} x \overrightarrow{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$

$= \hat{i}[4-6] - \hat{j}[12-(-2)] + \hat{k}[-9-1]$

$= -2\hat{i} - 14\hat{j} - 10\hat{k}$

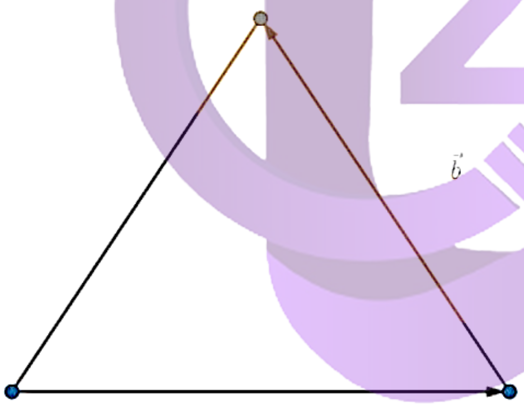
$|\overrightarrow{d_1} x \overrightarrow{d_2}| = \sqrt{(-2)^2 + (-14)^2 + (-10)^2} = \sqrt{4 + 196 + 100} = \sqrt{300} = 10 \times \sqrt{3}$

Therefore the area of the parallelogram is $\frac{1}{2}|\overrightarrow{d_1} x \overrightarrow{d_2}| = \frac{1}{2} \times 10 \times \sqrt{3}$

$=.5\sqrt{3}$ sq units

Question: 24

Solution:



Given the adjacent sides of the triangle are $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$

Property: The area of the triangle with the sides $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a} x \vec{b}|$

$\vec{a} x \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix}$

$= \hat{k}[21 - (-20)]$

$= 41\hat{k}$

$|\vec{a} x \vec{b}| = 41$

Therefore area of the triangle $=\frac{1}{2}\times 41=\frac{41}{2}$ sq. units

Question: 25

Solution:

Given the plane is passing through $\vec{a}=\hat{i}-\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}+\hat{k}$

Property: The normal to the plane passing through-$\vec{a}$ and-$\vec{b}$ is $\vec{a}x\vec{b}$

Here ,

$\vec{a}x\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -1\\ 1 & 1 & 1\end{vmatrix}$

$=\hat{i}[-1-(-1)]-\hat{j}[1-(-1)]+\hat{k}[1-(-1)]$

$=-2\hat{j}+2\hat{k}$

As it is a unit normal vector,

$\Rightarrow \vec{a}x\vec{b}$ is divided by its magnitude.

Therefore the unit normal vector is $\frac{-2\hat{j}+2\hat{k}}{\sqrt{(-2)^2+2^2}}$

$=\frac{-2\hat{j}+2\hat{k}}{\sqrt{4+4}}$

$=\frac{-2\hat{j}+2\hat{k}}{\sqrt{8}}$

$=\frac{-2\hat{j}+2\hat{k}}{2\sqrt{2}}$

$=\frac{-\hat{j}+\hat{k}}{\sqrt{2}}$

Question: 26

Solution:

Given $\vec{a},\vec{b},\vec{c}$ are unit vectors and $\vec{a}+\vec{b}+\vec{c}=0$

$|\vec{a}|=1, |\vec{b}|=1, |\vec{c}|=1$

Let the angle between $\vec{a}$ and $\vec{b}$ be θ

We can write the given relation as $\vec{a}+\vec{b}=-\vec{c}$

Squaring on both the sides

$(\vec{a}+\vec{b})^2=\vec{c}^2$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2(\vec{a}.\vec{b})=|\vec{c}|^2$

$\Rightarrow$ **1+1+2$(\vec{a}.\vec{b})$=1**

$\Rightarrow 2(\vec{a}.\vec{b})$=-1

$\Rightarrow \vec{a}.\vec{b}=-\frac{1}{2}$

Similarly we can prove that $\vec{b}.\vec{c} = 0$ and $\vec{c}.\vec{a} = 0$

Asking us to find the value of $(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

$= -\frac{1}{2} - \frac{1}{2} - \frac{1}{2}$

$= -\frac{3}{2}$

Question: 27

Solution:

Given $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors

$|\vec{a}| = 1, |\vec{b}| = 1, |\vec{c}| = 1$

And $\vec{a}.\vec{b} = 0; \vec{b}.\vec{c} = 0; \vec{c}.\vec{a} = 0$

Let the value of $\vec{a} + \vec{b} + \vec{c} = T$

Squaring on both the sides,

$(\vec{a} + \vec{b} + \vec{c})^2 = T^2$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = T^2$

$\Rightarrow |\vec{a}|^2 + (\vec{a}.\vec{b}) + (\vec{a}.\vec{c}) + |\vec{b}|^2 + (\vec{b}.\vec{a}) + (\vec{b}.\vec{c}) + |\vec{c}|^2 + (\vec{c}.\vec{a}) + (\vec{c}.\vec{b}) = T^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = T^2$

$\Rightarrow$1+1+1= T^2

$\Rightarrow T^2 = 3$

$\Rightarrow T = \sqrt{3}$

Question: 28

Solution:

Asking us to find the value of

$[\hat{i}\,\hat{j}\,\hat{k}] = \hat{i}.(\hat{j} \times \hat{k})$ or $(\hat{i} \times \hat{j}).\hat{k}$ $[\hat{i}\,\hat{j}\,\hat{k}]$

The value of $\hat{j} \times \hat{k} = \hat{i}$ and $\hat{i} \times \hat{j} = \hat{k}$

$\Rightarrow \hat{i}.(\hat{j} \times \hat{k}) = \hat{i}.(\hat{i})$ or $(\hat{i} \times \hat{j}).\hat{k} = \hat{k}.\hat{k}$

=1 =1

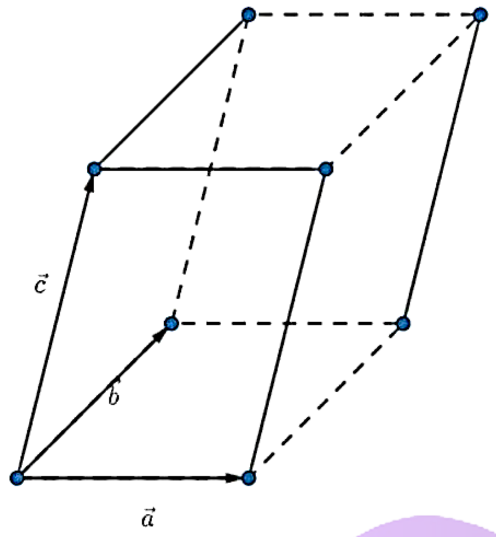
Question: 29

Solution:

Given $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

And $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

$\vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$ are the coterminous edges of the parallelepiped.



Property:

If $\vec{a}, \vec{b}, \vec{c}$ are the coterminous edges of the parallelepiped, the the volume of the parallelepiped is [$\vec{a}\,\vec{b}\,\vec{c}$]

[$\vec{a}\,\vec{b}\,\vec{c}$] is the scalar triple product.

$[\vec{a}\,\vec{b}\,\vec{c}] = |\vec{a}.(\vec{b} \times \vec{c})|$

$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$

$=\hat{i}[-4-1]-\hat{j}[-2-(-3)]+\hat{k}[-1-6]$

$=-5\hat{i}-\hat{j}-7\hat{k}$

$\vec{a}.(\vec{b} \times \vec{c})=(2\hat{i}-3\hat{j}+4\hat{k}).(-5\hat{i}-\hat{j}-7\hat{k})$

$= -10+3-28$

$= -35$

$|\vec{a}.(\vec{b} \times \vec{c})|=35$ cubic units

OR

$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$

$=2\,[-4-1]-(-3)[-2-(-3)]+4[-1-6]$

$= -35$

Therefore the volume of the parallelepiped with the given coterminous edges is 35 cubic units

Question: 30

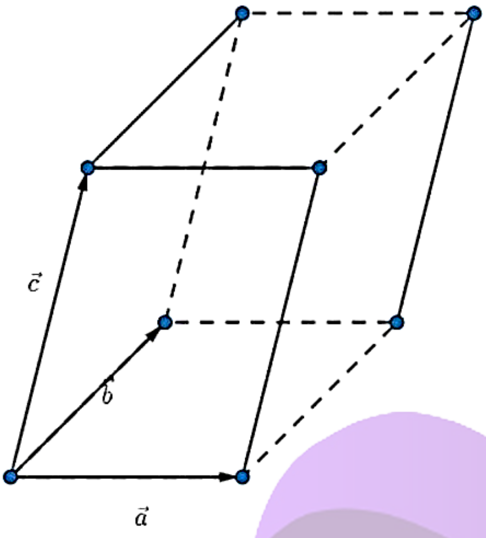
Solution:

Given volume of the parallelepiped is 216 cubic units

Given $\vec{a} = 5\hat{i} - 4\hat{j} + \hat{k}$

And $\vec{b} = 4\hat{i} + 3\hat{j} - \lambda\hat{k}$

$\vec{c} = \hat{i} - 2\hat{j} + 7\hat{k}$ are the coterminous edges of the parallelepiped.



$[\vec{a}\ \vec{b}\ \vec{c}] = 216$

$\Rightarrow 216 = \begin{vmatrix} 5 & -4 & 1 \\ 4 & 3 & \lambda \\ 1 & -2 & 7 \end{vmatrix}$

$\Rightarrow\ 216 = 5[21-(-2\lambda)]-(-4)[28-\lambda]+1[-8-3]$

$\Rightarrow\ 216 = 5[21+2\lambda]+4[28-\lambda]+1[-11]$

$\Rightarrow 216 = 105 + 10\lambda + 112 - 4\lambda - 11$

$\Rightarrow 216-105-112+11 = 6\lambda$

$\Rightarrow 6\lambda = 10$

$\Rightarrow \lambda = \frac{10}{6}$

$\Rightarrow \lambda = \frac{5}{3}$

Question: 31

Solution:

Given $\vec{a} = 2\hat{i} - 2\hat{k}$

And $\vec{b} = 1\hat{i} + (1+\lambda)\hat{j}$

$\vec{c} = 4\hat{i} + 2\hat{k}$ are the coplanar.

If three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 2 & 0 & -2 \\ 1 & 1+\lambda & 0 \\ 4 & 0 & 2 \end{vmatrix} = 0$

$\Rightarrow 2[2(1+\lambda)]-2[-4(1+\lambda)=0$

$\Rightarrow 4(1+\lambda)+8(1+\lambda)=0$

$\Rightarrow 12(1+\lambda)=0$

$\Rightarrow \lambda=-1$

Question: 32

Solution:

Option B is meaningless

Reason:

The term $(\vec{b}.\vec{c})$ is a scalar term and $\vec{a}$ is a vector. Cross product can only be applied in between the vectors . It is meaning less if used in between scalars or between scalar and vector.

Question: 33

Solution:

Asking us to find $\vec{a}.(\vec{a}\times\vec{b})$

By the definition of the scalar triple product,

$\vec{a}.(\vec{a}\times\vec{b}) = (\vec{a}.\vec{b}).\vec{a} - (\vec{a}.\vec{a})\vec{b}$

Also $(\vec{a}.\vec{b}).\vec{a} = (\vec{a}.\vec{a})\vec{b}$ [reason : dot product is associative]

$\Rightarrow \vec{a}.(\vec{a}\times\vec{b}) = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{a})\vec{b}$

=0

Question: 34

Solution:

Asking us to find the value of $[\vec{a}-\vec{b} \quad \vec{b}-\vec{c} \quad \vec{c}-\vec{a}]$

$$[\vec{a}-\vec{b} \quad \vec{b}-\vec{c} \quad \vec{c}-\vec{a}] = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} . [\vec{a} \quad \vec{b} \quad \vec{c}]$$

Coefficients Of $\vec{a}$ | coefficients of $\vec{b}$ | coefficients of $\vec{c}$

$$= \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} . [\vec{a}\,\vec{b}\,\vec{c}]$$

=1[1]-(-1)[-1]

=1-1

=0