Chapter : 27. STRAIGHT LINE IN SPACE

Exercise : 27A

Question: 1

Solution:

Given: line passes through point (3, 4, 5) and is parallel to $2\hat{i}+2\hat{j}-3\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}-3\hat{k}$

Therefore,

Vector form:

$\vec{r}=3\hat{i}+4\hat{j}+5\hat{k}+\lambda(2\hat{i}+2\hat{j}-3\hat{k})$

Cartesian form:

$$\frac{x-3}{2}=\frac{y-4}{2}=\frac{z-5}{-3}$$

Question: 2

Solution:

Given: line passes through (2, 1, -3) and is parallel to $\hat{i}-2\hat{j}+3\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a}=2\hat{i}+\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$

Therefore,

Vector form:

$\vec{r}=2\hat{i}+\hat{j}-3\hat{k}+\lambda(\hat{i}-2\hat{j}+3\hat{k})$

Cartesian form:

$\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+3}{3}$

Question: 3

Solution:

Given: line passes through $2\hat{i}+\hat{j}-5\hat{k}$ and is parallel to $\hat{i}+3\hat{j}-\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a}=2\hat{i}+\hat{j}-5\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-\hat{k}$

Therefore,

Vector form:

$\vec{r}=2\hat{i}+\hat{j}-5\hat{k}+\lambda(\hat{i}+3\hat{j}-\hat{k})$

Cartesian form:

$\frac{x-2}{1}=\frac{y-1}{3}=\frac{z+5}{-1}$

Question: 4

Solution:

Given: line passes through $2\hat{i}-\hat{j}-4\hat{k}$ and is drawn in the direction of $\hat{i}+\hat{j}-2\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since line is drawn in the direction of $(\hat{i}+\hat{j}-\widehat{2k})$, it is parallel to $(\hat{i}+\hat{j}-2\hat{k})$

Here, $\vec{a}=2\hat{i}-\hat{j}-4\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-2\hat{k}$

Therefore,

Vector form:

$\vec{r}=2\hat{i}-\hat{j}-4\hat{k}+\lambda(\hat{i}+\hat{j}-2\hat{k})$

Cartesian form:

$\frac{x-2}{1}=\frac{y+1}{1}=\frac{z+4}{-2}$

Question: 5

Solution:

Given: Cartesian equation of line

$$\frac{x-3}{2} = \frac{y+2}{-5} = \frac{z-6}{4}$$

To find: equation of line in vector form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

From the Cartesian equation of the line, we can find $\vec{a}$ and $\vec{b}$

Here, $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$ and $\vec{b} = 2\hat{i} - 5\hat{j} + 4\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda(2\hat{i} - 5\hat{j} + 4\hat{k})$$

Question: 6

Solution:

Given: Cartesian equation of line are 3x + 1 = 6y – 2 = 1 - z

To find: fixed point through which the line passes through, its direction ratios and the vector equation.

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line and also its direction ratio.

Explanation:

The Cartesian form of the line can be rewritten as:

$$\frac{x+\frac{1}{3}}{\frac{1}{3}} = \frac{y-\frac{1}{3}}{\frac{1}{6}} = \frac{z-1}{-1} = \lambda$$

$$\Rightarrow \frac{x+\frac{1}{3}}{2} = \frac{y-\frac{1}{3}}{1} = \frac{z-1}{-6} = \lambda$$

Therefore, $\vec{a} = \frac{-1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 6\hat{k}$

So, the line passes through $\left(\frac{-1}{3}, \frac{1}{3}, 1\right)$ and direction ratios of the line are (2, 1, -6) and vector form is:

$\vec{r} = \frac{-1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k} + \lambda(2\hat{i} + \hat{j} - 6\hat{k})$

Question: 7

Solution:

Given: line passes through (1, 3, -2) and is parallel to the line

$$\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+3}{-6}$$

To find: equation of line in vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say L_1) is parallel to another line (say L_2), L_1 has the same direction ratios as that of L_2

Here, $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$

Since the equation of L_2 is

$$\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+3}{-6}$$

$\vec{b} = 3\hat{i} + 5\hat{j} - 6\hat{k}$

Therefore,

Vector form of the line is:

$\vec{r} = \hat{i} + 3\hat{j} - 2\hat{k} + \lambda(3\hat{i} + 5\hat{j} - 6\hat{k})$

Cartesian form of the line is:

$$\frac{x-1}{3} = \frac{y-3}{5} = \frac{z+2}{-6}$$

Question: 8

Solution:

Given: line passes through (1, -2, 3) and is parallel to the line

$$\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$$

To find: equation of line in vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say L_1) is parallel to another line (say L_2), L_1 has the same direction ratios as that of L_2

Here, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

Since the equation of L_2 is

$$\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$$

$$\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} - 4\hat{j} + 5\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{3} = \frac{y+2}{-4} = \frac{z-3}{5}$$

Question: 9

Solution:

Given: line passes through (1, 2, 3) and is parallel to the line

$$\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say L_1) is parallel to another line (say L_2), L_1 has the same direction ratios as that of L_2

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Equation of L_2 can be rewritten as:

$$\frac{x+2}{-1} = \frac{y+3}{7} = \frac{z-3}{\frac{3}{2}}$$

$$\Rightarrow \frac{x+2}{-2} = \frac{y+3}{14} = \frac{z-3}{3}$$

$$\vec{b} = -2\hat{i} + 14\hat{j} + 3\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-2\hat{i} + 14\hat{j} + 3\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{-2}=\frac{y-2}{14}=\frac{z-3}{3}$$

Question: 10

Solution:

Given: line passes through (-1, 3, -2) and is perpendicular to each of the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{\imath}+y_1\hat{\jmath}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ is a vector parallel to the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3=0$

Explanation:

Here, $\vec{a}=-\hat{\imath}+3\hat{\jmath}-2\hat{k}$

Let the direction ratios of the line be $b_1:b_2:b_3$

Direction ratios of the other two lines are 1 : 2 : 3 and -3 : 2 : 5

Since the other two line are perpendicular to the given line, we have

$b_1 + 2b_2 + 3b_3 = 0$

$-3b_1 + 2b_2 + 5b_3 = 0$

Solving,

$$\frac{b_1}{\begin{vmatrix}2 & 3\\ 2 & 5\end{vmatrix}}=\frac{-b_2}{\begin{vmatrix}1 & 3\\ -3 & 5\end{vmatrix}}=\frac{b_3}{\begin{vmatrix}1 & 2\\ -3 & 2\end{vmatrix}}$$

$$\Rightarrow\frac{b_1}{4}=\frac{b_2}{-14}=\frac{b_3}{8}$$

$$\Rightarrow\frac{b_1}{2}=\frac{b_2}{-7}=\frac{b_3}{4}$$

$\vec{b}=2\hat{\imath}-7\hat{\jmath}+4\hat{k}$

Therefore,

Vector form of the line is:

$$\vec{r}=-\hat{\imath}+3\hat{\jmath}-2\hat{k}+\lambda\left(2\hat{\imath}-7\hat{\jmath}+4\hat{k}\right)$$

Cartesian form of the line is:

$$\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$$

Question: 11

Solution:

Given: line passes through (1, 2, -4) and is perpendicular to each of the lines $\frac{x-8}{8}=\frac{y+19}{-16}=\frac{z-10}{7}$ and

$\frac{x-15}{3}=\frac{y+29}{8}=\frac{z-5}{-5}$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

If 2 lines of direction ratios a_1:a_2:a_3 and b_1:b_2:b_3 are perpendicular, then $a_1b_1+a_2b_2+a_3b_3=0$

Explanation:

Here, $\vec{a}=\hat{i}+2\hat{j}-4\hat{k}$

Let the direction ratios of the line be b_1:b_2:b_3

Direction ratios of other two lines are 8 : -16 : 7 and 3 : 8 : -5

Since the other two line are perpendicular to the given line, we have

$8b_1-16b_2+7b_3=0$

$3b_1+8b_2-5b_3=0$

Solving,

$$\frac{b_1}{\begin{vmatrix}-16 & 7\\ 8 & -5\end{vmatrix}}=\frac{-b_2}{\begin{vmatrix}8 & 7\\ 3 & -5\end{vmatrix}}=\frac{b_3}{\begin{vmatrix}8 & -16\\ 3 & 8\end{vmatrix}}$$

$$\Rightarrow \frac{b_1}{24}=\frac{b_2}{61}=\frac{b_3}{112}$$

$\vec{b}=24\hat{i}+61\hat{j}+112\hat{k}$

Therefore,

Vector form of the line is:

$\vec{r}=\ \hat{i}+2\hat{j}-4\hat{k}+\lambda(24\hat{i}+61\hat{j}+112\hat{k})$

Cartesian form of the line is:

$$\frac{x-1}{24}=\frac{y-2}{61}=\frac{z+4}{112}$$

Question: 12

Solution:

Given: The equations of the two lines are

$$\frac{x-4}{1}=\frac{y+3}{4}=\frac{z+1}{7} \text{ and } \frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$$

To Prove: The two lines intersect and to find their point of intersection.

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-4}{1}=\frac{y+3}{4}=\frac{z+1}{7}=\lambda_1$$

$$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}=\lambda_2$$

So a point on the first line is $(\lambda_1 + 4, 4\lambda_1 - 3, 7\lambda_1 - 1)$

A point on the second line is $(2\lambda_2 + 1, -3\lambda_2 - 1, 8\lambda_2 - 10)$

If they intersect they should have a common point.

$\lambda_1 + 4 = 2\lambda_2 + 1 \Rightarrow \lambda_1 - 2\lambda_2 = -3$ ... (1)

$4\lambda_1 - 3 = -3\lambda_2 - 1 \Rightarrow 4\lambda_1 + 3\lambda_2 = 2$ ... (2)

Solving (1) and (2),

$11\lambda_2 = 14$

$\lambda_2 = \frac{14}{11}$

Therefore, $\lambda_1 = \frac{-5}{11}$

Substituting for the z coordinate, we get

$7\lambda_1 - 1 = \frac{-46}{11}$ and $8\lambda_2 - 10 = \frac{2}{11}$

So, the lines do not intersect.

Question: 13

Solution:

Given: The equations of the two lines are

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-4}{5}=\frac{y-1}{2}=z$$

To Prove: The two lines intersect and to find their point of intersection.

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda_1$$

$$\frac{x-4}{5}=\frac{y-1}{2}=z=\lambda_2$$

So a point on the first line is $(2\lambda_1 + 1, 3\lambda_1 + 2, 4\lambda_1 + 3)$

A point on the second line is $(5\lambda_2 + 4, 2\lambda_2 + 1, \lambda_2)$

If they intersect they should have a common point.

$2\lambda_1 + 1 = 5\lambda_2 + 4 \Rightarrow 2\lambda_1 - 5\lambda_2 = 3$ ... (1)

$3\lambda_1 + 2 = 2\lambda_2 + 1 \Rightarrow 3\lambda_1 - 2\lambda_2 = -1$ ... (2)

Solving (1) and (2),

$-11\lambda_2 = 11$

$\lambda_2 = -1$

Therefore, $\lambda_1 = -1$

Substituting for the z coordinate, we get

$4\lambda_1 + 3 = -1$ and $\lambda_2 = -1$

So, the lines intersect and their point of intersection is (-1, -1, -1)

Question: 14

Solution:

Given: The equations of the two lines are

$$\frac{x-1}{2} = \frac{y+1}{3} = z \text{ and } \frac{x+1}{5} = \frac{y-2}{1}, z = 2$$

To Prove: the lines do not intersect each other.

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-1}{2} = \frac{y+1}{3} = z = \lambda_1$$

$$\frac{x+1}{5} = \frac{y-2}{1} = \lambda_2, z = 2$$

So a point on the first line is $(2\lambda_1 + 1, 3\lambda_1 - 1, \lambda_1)$

A point on the second line is $(5\lambda_2 - 1, \lambda_2 + 1, 2)$

If they intersect they should have a common point.

$2\lambda_1 + 1 = 5\lambda_2 - 1 \Rightarrow 2\lambda_1 - 5\lambda_2 = -2$ ... (1)

$3\lambda_1 - 1 = \lambda_2 + 1 \Rightarrow 3\lambda_1 - \lambda_2 = 2$ ... (2)

Solving (1) and (2),

$-13\lambda_2 = -10$

$\lambda_2 = \frac{10}{13}$

Therefore, $\lambda_1 = \frac{33}{65}$

Substituting for the z coordinate, we get

$\lambda_1 = \frac{33}{65}$ and z = 2

So, the lines do not intersect.

Question: 15

Solution:

Given: Equation of line is $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.

To find: coordinates of foot of the perpendicular from (1, 2, 3) to the line. And find the length of the perpendicular.

Formula Used:

1. Equation of a line is

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

2. Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Explanation:

Let

$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda$

So the foot of the perpendicular is $(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$

Direction ratio of the line is 3 : 2 : -2

Direction ratio of the perpendicular is

$\Rightarrow (3\lambda + 6 - 1) : (2\lambda + 7 - 2) : (-2\lambda + 7 - 3)$

$\Rightarrow (3\lambda + 5) : (2\lambda + 5) : (-2\lambda + 4)$

Since this is perpendicular to the line,

$3(3\lambda + 5) + 2(2\lambda + 5) - 2(-2\lambda + 4) = 0$

$\Rightarrow 9\lambda + 15 + 4\lambda + 10 + 4\lambda - 8 = 0$

$\Rightarrow 17\lambda = -17$

$\Rightarrow \lambda = -1$

So the foot of the perpendicular is (3, 5, 9)

Distance $= \sqrt{(3-1)^2 + (5-2)^2 + (9-3)^2}$

$= \sqrt{4 + 9 + 36}$

= 7 units

Therefore, the foot of the perpendicular is (3, 5, 9) and length of perpendicular is 7 units.

Question: 16

Solution:

Given: Equation of line is $\frac{x-11}{10} = \frac{y+2}{4} = \frac{z+8}{11}$.

To find: coordinates of foot of the perpendicular from (2, -1, 5) to the line. And find the length of the perpendicular.

Formula Used:

1. Equation of a line is

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

2. Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Explanation:

Let

$$\frac{x - 11}{10} = \frac{y + 2}{-4} = \frac{z + 8}{-11} = \lambda$$

So the foot of the perpendicular is $(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$

Direction ratio of the line is 10 : -4 : -11

Direction ratio of the perpendicular is

$\Rightarrow (10\lambda + 11 - 2) : (-4\lambda - 2 + 1) : (-11\lambda - 8 - 5)$

$\Rightarrow (10\lambda + 9) : (-4\lambda - 1) : (-11\lambda - 13)$

Since this is perpendicular to the line,

$10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$

$\Rightarrow 100\lambda + 90 + 16\lambda + 4 + 121\lambda + 143 = 0$

$\Rightarrow 237\lambda = -237$

$\Rightarrow \lambda = -1$

So the foot of the perpendicular is (1, 2, 3)

Distance = $\sqrt{(1 - 2)^2 + (2 + 1)^2 + (3 - 5)^2}$

= $\sqrt{1 + 9 + 4}$

= $\sqrt{14}$ units

Therefore, the foot of the perpendicular is (1, 2, 3) and length of perpendicular is $\sqrt{14}$ units.

Question: 17

Solution:

Given: line passes through the points (3, 4, -6) and (5, -2, 7)

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = 3\hat{i} + 4\hat{j} - 6\hat{k}$

The direction ratios of the line are (3 - 5) : (4 + 2) : (-6 - 7)

$\Rightarrow$ -2 : 6 : -13

⇒ 2 : -6 : 13

So, $\vec{b} = 2\hat{i} - 6\hat{j} + 13\hat{k}$

Therefore,

Vector form:

$\vec{r} = 3\hat{i} + 4\hat{j} - 6\hat{k} + \lambda(2\hat{i} - 6\hat{j} + 13\hat{k})$

Cartesian form:

$$\frac{x-3}{2} = \frac{y-4}{-6} = \frac{z+6}{13}$$

Question: 18

Solution:

Given: line passes through the points (2, -3, 0) and (-2, 4, 3)

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = 2\hat{i} - 3\hat{j}$

The direction ratios of the line are (2 + 2) : (-3 - 4) : (0 - 3)

⇒ 4 : -7 : -3

⇒ -4 : 7 : 3

So, $\vec{b} = -4\hat{i} + 7\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$\vec{r} = 2\hat{i} - 3\hat{j} + \lambda(-4\hat{i} + 7\hat{j} + 3\hat{k})$

Cartesian form:

$$\frac{x-2}{-4} = \frac{y+3}{7} = \frac{z}{3}$$

Question: 19

Solution:

Given: line passes through the points whose position vectors are $(\hat{i} - 2\hat{j} + \hat{k})$ and $(\hat{i} + 3\hat{j} - 2\hat{k})$.

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$

The direction ratios of the line are (1 - 1) : (-2 - 3) : (1 + 2)

⇒ 0 : -5 : 3

⇒ 0 : 5 : -3

So, $\vec{b} = -5\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$$\vec{r} = \hat{i} - 2\hat{j} + \hat{k} + \lambda(5\hat{j} - 3\hat{k})$$

Cartesian form:

$$\frac{x-1}{0} = \frac{y+2}{5} = \frac{z-1}{-3}$$

Question: 20

Solution:

Given: line passes through the point (3, -2, 1) and is parallel to the line joining points B(-2, 4, 2) and C(2, 3, 3).

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$

The direction ratios of the line are (-2 - 2) : (4 - 3) : (2 - 3)

⇒ -4 : 1 : -1

⇒ 4 : -1 : 1

So, $\vec{b} = 4\hat{i} - \hat{j} + \hat{k}$

Therefore,

Vector form:

$$\vec{r} = 3\hat{i} - 2\hat{j} + \hat{k} + \lambda(4\hat{i} - \hat{j} + \hat{k})$$

Cartesian form:

$$\frac{x-3}{4} = \frac{y+2}{-1} = \frac{z-1}{1}$$

Question: 21

Solution:

Given: line passes through the point with position vector $\hat{i} + 2\hat{j} - 3\hat{k}$ and parallel to the line joining the points with position vectors $\hat{i} - \hat{j} + 5\hat{k}$ and $2\hat{i} + 3\hat{j} - 4\hat{k}$.

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

The direction ratios of the line are (1 - 2) : (-1 - 3) : (5 + 4)

⇒ -1 : -4 : 9

⇒ 1 : 4 : -9

So, $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$

Therefore,

Vector form:

$\vec{r} = \hat{i} + 2\hat{j} - 3\hat{k} + \lambda(\hat{i} + 4\hat{j} - 9\hat{k})$

Cartesian form:

$$\frac{x-1}{1} = \frac{y-2}{4} = \frac{z+3}{-9}$$

Question: 22

Solution:

Given: perpendicular drawn from point A (1, 2, 1) to line joining points B (1, 4, 6) and C (5, 4, 4)

To find: foot of perpendicular

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

B (1, 4, 6) is a point on the line.

Therefore, $\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

Also direction ratios of the line are (1 - 5) : (4 – 4) : (6 – 4)

⇒ -4 : 0 : 2

⇒ -2 : 0 : 1

So, equation of the line in Cartesian form is

$\frac{x-1}{-2} = \frac{y-4}{0} = \frac{z-6}{1} = \lambda$

Any point on the line will be of the form (-2λ + 1, 4, λ + 6)

So the foot of the perpendicular is of the form (-2λ + 1, 4, λ + 6)

The direction ratios of the perpendicular is

(-2λ + 1 – 1) : (4 - 2) : (λ + 6 - 1)

⇒ (-2λ) : 2 : (λ + 5)

From the direction ratio of the line and the direction ratio of its perpendicular, we have

-2(-2λ) + 0 + λ + 5 = 0

⇒ 4λ + λ = -5

⇒ λ = -1

So, the foot of the perpendicular is (3, 4, 5)

Question: 23

Solution:

Given: perpendicular drawn from point A (1, 8, 4) to line joining points B (0, -1, 3) and C (2, -3, -1)

To find: foot of perpendicular

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios a_1:a_2:a_3 and b_1:b_2:b_3 are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

B (0, -1, 3) is a point on the line.

Therefore, $\vec{a} = -\hat{j} + 3\hat{k}$

Also direction ratios of the line are (0 - 2) : (-1 + 3) : (3 + 1)

⇒ -2 : 2 : 4

⇒ -1 : 1 : 2

So, equation of the line in Cartesian form is

$\frac{x}{-1} = \frac{y+1}{1} = \frac{z-3}{2} = \lambda$

Any point on the line will be of the form (-λ, λ - 1, 2λ + 3)

So the foot of the perpendicular is of the form (-λ, λ - 1, 2λ + 3)

The direction ratios of the perpendicular is

(-λ - 1) : (λ – 1 - 8) : (2λ + 3 - 4)

⇒ (-λ - 1) : (λ – 9) : (2λ – 1)

From the direction ratio of the line and the direction ratio of its perpendicular, we have

-1(-λ - 1) + λ – 9 + 2(2λ – 1) = 0

$\Rightarrow \lambda + 1 + \lambda - 9 + 4\lambda - 2 = 0$

$\Rightarrow 6\lambda = 10$

$\Rightarrow \lambda = \frac{5}{3}$

So, the foot of the perpendicular is $\left(\frac{-5}{3}, \frac{2}{3}, \frac{19}{3}\right)$

Question: 24

Solution:

Given: Equation of line is $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

To find: image of point (0, 2, 3)

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1{:}a_2{:}a_3$ and $b_1{:}b_2{:}b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Mid-point of line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Explanation:

Let

$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$$

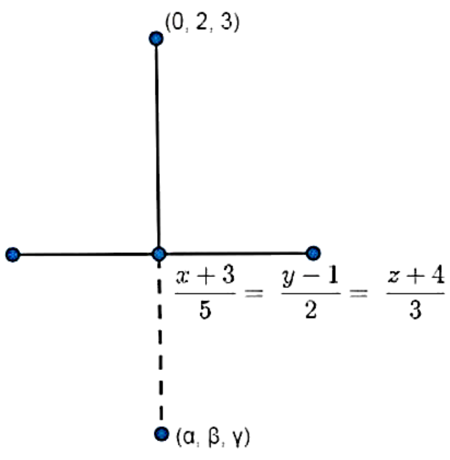
So the foot of the perpendicular is $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

The direction ratios of the perpendicular is

$(5\lambda - 3 - 0) : (2\lambda + 1 - 2) : (3\lambda - 4 - 3)$

$\Rightarrow (5\lambda - 3) : (2\lambda - 1) : (3\lambda - 7)$

Direction ratio of the line is 5 : 2 : 3



From the direction ratio of the line and the direction ratio of its perpendicular, we h

$5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$\Rightarrow 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$

$\Rightarrow 38\lambda = 38$

$\Rightarrow \lambda = 1$

So, the foot of the perpendicular is (2, 3, -1)

The foot of the perpendicular is the mid-point of the line joining (0, 2, 3) and (α, β, γ)

So, we have

$$\frac{\alpha + 0}{2} = 2 \Rightarrow \alpha = 4$$

$$\frac{\beta + 2}{2} = 3 \Rightarrow \beta = 4$$

$$\frac{\gamma + 3}{2} = -1 \Rightarrow \gamma = -5$$

So, the image is (4, 4, -5)

Question: 25

Solution:

Given: Equation of line is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

To find: image of point (5, 9, 3)

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Mid-point of line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Explanation:

Let

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

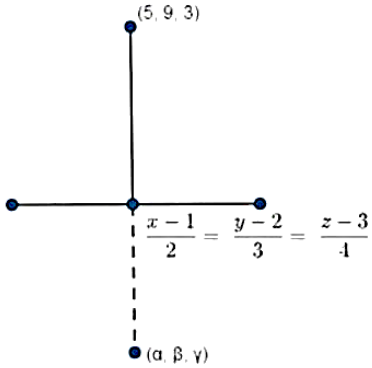
So the foot of the perpendicular is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

The direction ratios of the perpendicular is

$(2\lambda + 1 - 5) : (3\lambda + 2 - 9) : (4\lambda + 3 - 3)$

$\Rightarrow (2\lambda - 4) : (3\lambda - 7) : (4\lambda)$

Direction ratio of the line is 2 : 3 : 4



From the direction ratio of the line and the direction ratio of its perpendicular, we have

$2(2\lambda - 4) + 3(3\lambda - 7) + 4(4\lambda) = 0$

$\Rightarrow 4\lambda - 8 + 9\lambda - 21 + 16\lambda = 0$

$\Rightarrow 29\lambda = 29$

$\Rightarrow \lambda = 1$

So, the foot of the perpendicular is (3, 5, 7)

The foot of the perpendicular is the mid-point of the line joining (5, 9, 3) and (α, β, γ)

So, we have

$\frac{\alpha + 5}{2} = 3 \Rightarrow \alpha = 1$

$\frac{\beta + 9}{2} = 5 \Rightarrow \beta = 1$

$\frac{\gamma + 3}{2} = 7 \Rightarrow \gamma = 11$

So, the image is (1, 1, 11)

Question: 26

Solution:

Given: Point (2, -1, 5)

Equation of line $= \left(11\,\hat{i} - 2\,\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\,\hat{j} - 11\hat{k}\right)$

The equation of line can be re-arranged as $\frac{x-11}{10} = \frac{x+2}{-4} = \frac{x+8}{-11} = r$

The general point on this line is

$(10r + 11, -4r - 2, -11r - 8)$

Let N be the foot of the perpendicular drawn from the point P(2, 1, -5) on the given line.

Then, this point is N(10r + 11, -4r – 2, -11r – 8) for some fixed value of r.

D.r.'s of PN are (10r + 9, -4r - 3, -11r - 3)

D.r.'s of the given line is 10, -4, -11.

Since, PN is perpendicular to the given line, we have,

$10(10r + 9) - 4(-4r - 3) - 11(-11r - 3) = 0$

$100r + 90 + 16r + 12 + 121r + 33 = 0$

$237r = 135$

$r = \frac{135}{237}$

Then, the image of the point is

$$\frac{\alpha - 11}{-11} = 0, \frac{\beta + 2}{7} = 1, \frac{\gamma + 8}{9} = 1$$

Therefore, the image is (0, 5, 1).

Exercise : 27B

Question: 1

Solution:

Given -

A = (2,1,3)

B = (5,0,5)

C = (-4,3,-1)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a',b',c'),then the direction ratios of the line PQ is given by ((a'-a),(b'-b),(c'-c))

The direction ratios of the line AB can be given by

((5-2),(0-1),(5-3))

=(3,-1,-2)

Similarly, the direction ratios of the line BC can be given by

((-4-5),(3-0),(-1-5))

=(-9,3,-6)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(3,-1,-2)

=(-1/3)X(-9,3,-6)

=(-1/3)Xd.r. of BC

Hence, **A, B and C are collinear**

Question: 2

Solution:

Given -

A = (2,3,-4)

B = (1,-2,3)

C = (3,8,-11)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a',b',c'),then the direction ratios of the line PQ is given by ((a'-a),(b'-b),(c'-c))

The direction ratios of the line AB can be given by

((1-2),(-2-3),(3+4))

=(-1,-5,7)

Similarly, the direction ratios of the line BC can be given by

((3-1),(8+2),(-11-3))

=(2,10,-14)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(-1,-5,7)

=(-1/2)X(2,10,-14)

=(-1/2)Xd.r. of BC

Hence, **A, B and C are collinear**

Question: 3

Solution:

Given -

A = (2,5,1)

B = (1,2,-1)

C = (3,λ,3)

To find – The value of λ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a',b',c'),then the direction ratios of the line PQ is given by ((a'-a),(b'-b),(c'-c))

The direction ratios of the line AB can be given by

((1-2),(2-5),(-1-1))

=(-1,-3,-2)

Similarly, the direction ratios of the line BC can be given by

((3-1),(λ-2),(3+1))

=(2,λ-2,4)

Tip – If it is shown that direction ratios of AB=α times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(-1,-3,-2)

=(-1/2)X(2,λ-2,4)

=(-1/2)Xd.r. of BC

Since, A, B and C are collinear,

$\therefore -\frac{1}{2}(\lambda - 2) = -3$

$\Rightarrow \lambda - 2 = 6$

$\Rightarrow \boldsymbol{\lambda = 8}$

Question: 4

Solution:

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Given -

A = (3,2,-4)

B = (9,8,-10)

C = (λ,μ,-6)

To find – The value of λ and μ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a',b',c'),then the direction ratios of the line PQ is given by ((a'-a),(b'-b),(c'-c))

The direction ratios of the line AB can be given by

((9-3),(8-2),(-10+4))

=(6,6,-6)

Similarly, the direction ratios of the line BC can be given by

((λ-9),(μ-8),(-6+10))

=(λ-9,μ-8,4)

Tip – If it is shown that direction ratios of AB=α times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(6,6,-6)

=(-6/4)X(-4,-4,4)

=(-3/2)Xd.r. of BC

Since, A, B and C are collinear,

$\therefore -\frac{3}{2}(\lambda - 9) = 6$

⇨ $\lambda - 9 = -4$

⇨ $\boldsymbol{\lambda = 5}$

And,

$\therefore -\frac{3}{2}(\mu - 8) = 6$

⇨ $\mu - 8 = -4$

⇨ $\boldsymbol{\lambda = 4}$

Question: 5

Solution:

Given -

A = (-1,4,-2)

B = (λ,μ,1)

C = (0,2,-1)

To find – The value of λ and μ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a',b',c'),then the direction ratios of the line PQ is given by ((a'-a),(b'-b),(c'-c))

The direction ratios of the line AB can be given by ((λ+1),(μ-4),(1+2))

=(λ+1,μ-4,3)

Similarly, the direction ratios of the line BC can be given by

((0-λ),(2-μ),(-1-1))

=(-λ,2-μ,-2)

Tip – If it is shown that direction ratios of AB=α times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(λ+1,μ-4,3)

Say, α be an arbitrary constant such that d.r. of AB = α X d.r. of BC

So, 3 = α X (-2)

i.e. α = -3/2

Since, A, B and C are collinear,

$\therefore -\frac{3}{2}(-\lambda) = \lambda + 1$

$\Rightarrow 3\lambda = 2\lambda + 2$

$\Rightarrow \boldsymbol{\lambda = 2}$

And,

$\therefore -\frac{3}{2}(2 - \mu) = \mu - 4$

$\Rightarrow -6 + 3\mu = 2\mu - 8$

$\Rightarrow \boldsymbol{\mu = -2}$

Question: 6

Solution:

Given -

$\vec{A} = -4\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{B} = \hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{C} = -9\hat{i} + \hat{j} - 4\hat{k}$

It can thus be written as:

A = (-4,2,-3)

B = (1,3,-2)

C = (-9,1,-4)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a',b',c'),then the direction ratios of the line PQ is given by ((a'-a),(b'-b),(c'-c))

The direction ratios of the line AB can be given by

((1+4),(3-2),(-2+3))

=(5,1,1)

Similarly, the direction ratios of the line BC can be given by

((-9-1),(1-3),(-4+2))

=(-10,-2,-2)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(5,1,1)

=(-1/2)X(-10,-2,-2)

=(-1/2)Xd.r. of BC

Hence, **A, B and C are collinear**

Exercise : 27C

Question: 1

Solution:

Given – $\overrightarrow{L_1} = (3\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} - 2\hat{k})$

& $\overrightarrow{L_2} = (2\hat{i} - \hat{j} - 5\hat{k}) + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,-1,-2)

Direction ratios of L_2 = (3,-5,-4)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3 + (-1)\times(-5) + (-2)\times(-4)}{\sqrt{1^2+1^2+2^2}\sqrt{3^2+5^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{3+5+8}{\sqrt{6}\sqrt{50}}\right)$$

$$= \cos^{-1}\left(\frac{16}{5\sqrt{6}\sqrt{2}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)}$$

Question: 2

Solution:

Given – $\overrightarrow{L_1} = (3\hat{i} - 4\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 3\hat{k})$

& $\overrightarrow{L_2} = (5\hat{i}) + \mu(-\hat{i} + \hat{j} + \hat{k})$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,0,3)

Direction ratios of L_2 = (-1,1,1)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the sec

angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times(-1)+0\times 1+3\times 1}{\sqrt{1^2+0^2+3^2}\sqrt{1^2+1^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{-1+3}{\sqrt{10}\sqrt{3}}\right)$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{30}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{\sqrt{30}}{15}\right)}$$

Question: 3

Solution:

Given – $\overrightarrow{L_1} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$

& $\overrightarrow{L_2} = (3\hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (2,-2,1)

Direction ratios of L_2 = (1,2,-2)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2\times 1+(-2)\times 2+1\times(-2)}{\sqrt{2^2+2^2+1^2}\sqrt{1^2+2^2+2^2}}\right)$$

$$= \cos^{-1}\left(\frac{2-4-2}{3\times 3}\right)$$

$$= \mathbf{\cos^{-1}\left(-\frac{4}{9}\right)}$$

Question: 4

Solution:

Given – $\overrightarrow{L_1} = \frac{x-1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

& $\overrightarrow{L_2} = \frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,1,2)

Direction ratios of L_2 = (3,5,4)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3 + 1\times 5 + 2\times 4}{\sqrt{1^2+1^2+2^2}\sqrt{3^2+5^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{3+5+8}{\sqrt{6}\times\sqrt{50}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)}$$

Question: 5

Solution:

Given – $\overrightarrow{L_1} = \frac{x-4}{4} = \frac{y+1}{3} = \frac{z-6}{5}$

& $\overrightarrow{L_2} = \frac{x-5}{1} = \frac{y+5/2}{-1} = \frac{z-3}{1}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (4,3,5)

Direction ratios of L_2 = (1,-1,1)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{4\times 1 + 3\times(-1) + 5\times 1}{\sqrt{4^2+3^2+5^2}\sqrt{1^2+1^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{4-3+5}{5\sqrt{2}\times\sqrt{3}}\right)$$

$$= \cos^{-1}\left(\frac{6}{5\sqrt{6}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{2\sqrt{6}}{15}\right)}$$

Question: 6

Solution:

Given – $\overrightarrow{L_1} = \frac{x-3}{2} = \frac{y+5}{1} = \frac{z-1}{-3}$

& $\overrightarrow{L_2} = \frac{x}{3} = \frac{y-1}{2} = \frac{z+2}{-1}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (2,1,-3)

Direction ratios of L_2 = (3,2,-1)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$=\cos^{-1}\left(\frac{2\times 3+1\times 2+(-3)\times(-1)}{\sqrt{2^2+1^2+3^2}\sqrt{3^2+2^2+1^2}}\right)$$

$$=\cos^{-1}\left(\frac{6+2+3}{\sqrt{14}\times\sqrt{14}}\right)$$

$$=\cos^{-1}\left(\frac{11}{14}\right)$$

Question: 7

Solution:

Given – $\overrightarrow{L_1} = \frac{x}{1} = \frac{y}{0} = \frac{z}{1}$

& $\overrightarrow{L_2} = \frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,0,-1)

Direction ratios of L_2 = (3,4,5)

Tip – If (a,b,c) be the direction ratios of the first line and (a’,b’,c’) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$=\cos^{-1}\left(\frac{1\times 3+0\times 4+(-1)\times 5}{\sqrt{1^2+0^2+1^2}\sqrt{3^2+4^2+5^2}}\right)$$

$$=\cos^{-1}\left(\frac{3-5}{5\sqrt{2}\times\sqrt{2}}\right)$$

$$=\cos^{-1}\left(\frac{1}{5}\right)$$

Question: 8

Solution:

Given – $\overrightarrow{L_1} = \frac{x-5}{-3} = \frac{y+3}{-2} = \frac{z-5}{0}$

& $\overrightarrow{L_2} = \frac{x-1}{1} = \frac{y-1}{-3} = \frac{z-5}{2}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (-3,-2,0)

Direction ratios of L_2 = (1,-3,2)

Tip – If (a,b,c) be the direction ratios of the first line and (a’,b’,c’) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{(-3)\times 1+(-2)\times(-3)+0\times 2}{\sqrt{3^2+2^2+0^2}\sqrt{1^2+3^2+2^2}}\right)$

$= \cos^{-1}\left(\frac{-3+6}{\sqrt{13}\times\sqrt{14}}\right)$

$= \cos^{-1}\left(\frac{3}{\sqrt{182}}\right)$

Question: 9

Solution:

Given – $\overrightarrow{L_1} = \frac{x-3}{2} = \frac{y+1}{-3} = \frac{z-2}{4}$

& $\overrightarrow{L_2} = \frac{x+2}{2} = \frac{y-4}{4} = \frac{z+5}{2}$

To prove – The lines are perpendicular to each other

Direction ratios of L_1 = (2,-3,4)

Direction ratios of L_2 = (2,4,2)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{2\times 2+(-3)\times 4+4\times 2}{\sqrt{2^2+3^2+4^2}\sqrt{2^2+4^2+2^2}}\right)$

$= \cos^{-1}\left(\frac{4-12+8}{\sqrt{29}\times\sqrt{24}}\right)$

$= \cos^{-1}(0)$

$= \frac{\pi}{2}$

Hence, **the lines are perpendicular to each other.**

Question: 10

Solution:

Given – $\overrightarrow{L_1} = \frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$

& $\overrightarrow{L_2} = \frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{z-6}{-5}$

To find – The value of λ

Direction ratios of L_1 = (-3,2λ,2)

Direction ratios of L_2 = (3λ,1,-5)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

Since the lines are perpendicular to each other,

The angle between the lines

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$\Rightarrow \cos^{-1}\left(\frac{(-3)\times 3\lambda + 2\lambda \times 1 + 2\times(-5)}{\sqrt{3^2+(2\lambda)^2+2^2}\sqrt{(3\lambda)^2+1^2+5^2}}\right) = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\left(\frac{-9\lambda + 2\lambda - 10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right) = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\left(\frac{-7\lambda - 10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right) = \frac{\pi}{2}$

$\Rightarrow \left(\frac{-7\lambda - 10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right) = \cos\frac{\pi}{2} = 0$

$\Rightarrow -7\lambda - 10 = 0$

$\Rightarrow \boldsymbol{\lambda = -\frac{10}{7}}$

Question: 11

Solution:

Given – $\overrightarrow{L_1} = \frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$

& $\overrightarrow{L_2} = \frac{x+2}{2} = \frac{y-1/2}{1} = \frac{z-1}{-2}$

To prove – The lines are perpendicular to each other

Direction ratios of L_1 = (2,-2,1)

Direction ratios of L_2 = (2,1,-2)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{2\times 2+(-2)\times 1+1\times(-2)}{\sqrt{2^2+2^2+1^2}\sqrt{1^2+1^2+2^2}}\right)$

$= \cos^{-1}\left(\frac{4-2-2}{\sqrt{29}\times\sqrt{24}}\right)$

$= \cos^{-1}(0)$

$= \boldsymbol{\frac{\pi}{2}}$

Hence, **the lines are perpendicular to each other.**

Question: 12

Solution:

(i) : Given – Direction ratios of L_1 = (2,1,2) & Direction ratios of L_2 = (4,8,1)

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

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$= \cos^{-1}\left(\frac{2\times4+1\times8+2\times1}{\sqrt{2^2+1^2+2^2}\sqrt{4^2+8^2+1^2}}\right)$

$= \cos^{-1}\left(\frac{8+8+2}{3\times9}\right)$

$= \cos^{-1}\left(\frac{18}{27}\right)$

$= \boldsymbol{\cos^{-1}\left(\frac{2}{3}\right)}$

(ii) : Given – Direction ratios of L_1 = (5,-12,13) & Direction ratios of L_2 = (-3,4,5)

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{5\times(-3)+(-12)\times4+13\times5}{\sqrt{5^2+12^2+13^2}\sqrt{3^2+4^2+5^2}}\right)$

$= \cos^{-1}\left(\frac{-15-48+65}{13\sqrt{2}\times5\sqrt{2}}\right)$

$= \cos^{-1}\left(\frac{2}{130}\right)$

$= \boldsymbol{\cos^{-1}\left(\frac{1}{65}\right)}$

(iii) Given – Direction ratios of L_1 = (1,1,2) & Direction ratios of L_2 = (√3-1,-√3-1,4)

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{1\times(\sqrt{3}-1)+1\times(-\sqrt{3}-1)+2\times4}{\sqrt{1^2+1^2+2^2}\sqrt{(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2+4^2}}\right)$

$= \cos^{-1}\left(\frac{\sqrt{3}-1-\sqrt{3}-1+8}{\sqrt{6}\sqrt{24}}\right)$

$= \cos^{-1}\left(\frac{1}{2}\right)$

$= \boldsymbol{\frac{\pi}{3}}$

(iv) Given – Direction ratios of L_1 = (a,b,c) & Direction ratios of L_2 = ((b-c),(c-a),(a-b))

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{a \times (b-c) + b \times (c-a) + c \times (a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right)$$

$$= \cos^{-1}\left(\frac{0}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right)$$

$$= \cos^{-1}(0)$$

$$= \frac{\pi}{2}$$

Question: 13

Solution:

Given -

A = (1,2,3)

B = (4,5,7)

C = (-4,3,-6)

D = (2,9,2)

Formula to be used – If P = (a,b,c) and Q = (a’,b’,c’),then the direction ratios of the line PQ is given by ((a’-a),(b’-b),(c’-c))

The direction ratios of the line AB can be given by

((4-1),(5-2),(7-3))

=(3,3,4)

Similarly, the direction ratios of the line CD can be given by

((2+4),(9-3),(2+6))

=(6,6,8)

To find – Angle between the two pair of lines AB and CD

Tip – If (a,b,c) be the direction ratios of the first line and (a’,b’,c’) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2+b^2+c^2} \times \sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{3 \times 6 + 3 \times 6 + 4 \times 8}{\sqrt{3^2+3^2+4^2}\sqrt{6^2+6^2+8^2}}\right)$$

$$= \cos^{-1}\left(\frac{18+18+32}{\sqrt{34} \times 2\sqrt{34}}\right)$$

$$= \cos^{-1}\left(\frac{68}{2 \times 34}\right)$$

$$= \cos^{-1} 1$$

$$= \mathbf{0}$$

Exercise : 27D

Question: 1

Solution:

Given equations :

$\bar{r} = (\hat{\imath} + \hat{\jmath}) + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$

$\bar{r} = (2\hat{\imath} + \hat{\jmath} - \hat{k}) + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

Answer :

For given lines,

$\bar{r} = (\hat{\imath} + \hat{\jmath}) + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$

$\bar{r} = (2\hat{\imath} + \hat{\jmath} - \hat{k}) + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$

Here,

$\overline{a_1} = \hat{\imath} + \hat{\jmath}$

$\overline{b_1} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\overline{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\overline{b_2} = 3\hat{\imath} - 5\hat{\jmath} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

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$= \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$

$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{i} - \hat{j} - 7\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{3^2 + (-1)^2 + (-7)^2}$

$= \sqrt{9 + 1 + 49}$

$= \sqrt{59}$

$\overline{a_2} - \overline{a_1} = (2-1)\hat{i} + (1-1)\hat{j} + (-1-0)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} + 0\hat{j} - \hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (3\hat{i} - \hat{j} - 7\hat{k}).(\hat{i} + 0\hat{j} - \hat{k})$

$= (3 \times 1) + ((-1) \times 0) + ((-7) \times (-1))$

$= 3 + 0 + 7$

$= 10$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{10}{\sqrt{59}}\right|$$

Question: 2

Solution:

Given equations :

$\bar{r} = (-4\hat{i} + 4\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$

$\bar{r} = (-3\hat{i} - 8\hat{j} - 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 3\hat{k})$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

Answer :

For given lines,

$\bar{r} = (-4\hat{i} + 4\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$

$\bar{r} = (-3\hat{i} - 8\hat{j} - 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 3\hat{k})$

Here,

$\overline{a_1} = -4\hat{i} + 4\hat{j} + \hat{k}$

$\overline{b_1} = \hat{i} + \hat{j} - \hat{k}$

$\overline{a_2} = -3\hat{i} - 8\hat{j} - 3\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 3\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 3 & 3 \end{vmatrix}$$

$= \hat{i}(3 + 3) - \hat{j}(3 + 2) + \hat{k}(3 - 2)$

$\therefore \overline{b_1} \times \overline{b_2} = 6\hat{i} - 5\hat{j} + \hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{6^2 + (-5)^2 + 1^2}$

$= \sqrt{36 + 25 + 1}$

$= \sqrt{62}$

$\overline{a_2} - \overline{a_1} = (-3 + 4)\hat{i} + (-8 - 4)\hat{j} + (-3 - 1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} - 12\hat{j} - 4\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (6\hat{i} - 5\hat{j} + \hat{k}).(\hat{i} - 12\hat{j} - 4\hat{k})$

$= (6 \times 1) + ((-5) \times (-12)) + (1 \times (-4))$

$= 6 + 60 - 4$

$= 62$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{62}{\sqrt{62}} \right|$$

$d = \sqrt{62}\ units$

Question: 3

Solution:

Given equations :

$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$

$\bar{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and $\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

Answer :

For given lines,

$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$

$\bar{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$

$\overline{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + \hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3+6)$

$\therefore \overline{b_1} \times \overline{b_2} = -9\hat{i} + 3\hat{j} + 9\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-9)^2 + 3^2 + 9^2}$

$= \sqrt{81 + 9 + 81}$

$= \sqrt{171}$

$\overline{a_2} - \overline{a_1} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-9\hat{i} + 3\hat{j} + 9\hat{k}).(3\hat{i} + 3\hat{j} + 3\hat{k})$

$= ((-9) \times 3) + (3 \times 3) + (9 \times 3)$

$= - 27 + 9 + 27$

$= 9$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{9}{\sqrt{171}}\right|$$

$$\therefore d = \frac{9}{\sqrt{19}.\sqrt{9}}$$

$$\therefore d = \frac{3}{\sqrt{19}}$$

$$\therefore d = \frac{3\sqrt{19}}{19}$$

Question: 4

Solution:

Given equations :

$\bar{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

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2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

Answer :

For given lines,

$\bar{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} + \hat{k}$

$\overline{b_1} = \hat{i} - \hat{j} + \hat{k}$

$\overline{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

$\overline{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$= \hat{i}(-2 - 1) - \hat{j}(2 - 2) + \hat{k}(1 + 2)$

$\therefore \overline{b_1} \times \overline{b_2} = -3\hat{i} + 0\hat{j} + 3\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-3)^2 + 0^2 + 3^2}$

$= \sqrt{9 + 0 + 9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{i} + (-1 - 2)\hat{j} + (-1 - 1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} - 3\hat{j} - 2\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-3\hat{i} + 0\hat{j} + 3\hat{k}).(\hat{i} - 3\hat{j} - 2\hat{k})$

$= ((-3) \times 1) + (0 \times (-3)) + (3 \times (-2))$

$= -3 + 0 - 6$

$= -9$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{-9}{3\sqrt{2}}\right|$$

$$\therefore d = \frac{3}{\sqrt{2}}$$

$$\therefore d = \frac{3\sqrt{2}}{2}$$

Question: 5

Solution:

Given equations :

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and $\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

Answer :

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\overline{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\overline{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\overline{b_2} = -2\hat{i} + 3\hat{j} + 8\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$$

$= \hat{i}(24 - 18) - \hat{j}(16 + 12) + \hat{k}(6 - 6)$

$\therefore \overline{b_1} \times \overline{b_2} = 6\hat{i} - 28\hat{j} + 0\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{6^2 + (-28)^2 + 0^2}$

$= \sqrt{36 + 784 + 9}$

$= \sqrt{820}$

$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (6\hat{i} - 28\hat{j} + 0\hat{k}).(2\hat{i} + \hat{j} - \hat{k})$

$= (6 \times 2) + ((-28) \times 1) + (0 \times (-1))$

= 12 – 28 + 0

= - 16

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{-16}{\sqrt{820}} \right|$$

$$d = \frac{16}{\sqrt{820}} \ units$$

Question: 6

Solution:

Given equations :

$\bar{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$

$\bar{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

Answer :

For given lines,

$\bar{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$

$\bar{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$

Here,

$\overline{a_1} = 6\hat{i} + 3\hat{k}$

$\overline{b_1} = 2\hat{i} - \hat{j} + 4\hat{k}$

$\overline{a_2} = -9\hat{i} + \hat{j} - 10\hat{k}$

$\overline{b_2} = 4\hat{i} + \hat{j} + 6\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 4 & 1 & 6 \end{vmatrix}$$

$= \hat{i}(-6 - 4) - \hat{j}(12 - 16) + \hat{k}(2 + 4)$

$\therefore \overline{b_1} \times \overline{b_2} = -10\hat{i} + 4\hat{j} + 6\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-10)^2 + 4^2 + 6^2}$

$= \sqrt{100 + 16 + 36}$

$= \sqrt{152}$

$\overline{a_2} - \overline{a_1} = (-9 - 6)\hat{i} + (1 - 0)\hat{j} + (6 - 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -15\hat{i} + \hat{j} + 3\hat{k}$

Now,

$(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1}) = (-10\hat{i} + 4\hat{j} + 6\hat{k}).(-15\hat{i} + \hat{j} + 3\hat{k})$

$= ((-10) \times (-15)) + (4 \times 1) + (6 \times 3)$

$= 150 + 4 + 18$

$= 172$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

$$\therefore d = \left|\frac{172}{\sqrt{152}}\right|$$

$$\therefore d = \frac{172}{2\sqrt{38}}$$

$$\therefore d = \frac{86}{\sqrt{38}}$$

$$d = \frac{86}{\sqrt{38}} \text{ units}$$

Question: 7

Solution:

Given equations :

$\bar{r} = (3 - t)\hat{i} + (4 + 2t)\hat{j} + (t - 2)\hat{k}$

$\bar{r} = (1 + s)\hat{i} + (3s - 7)\hat{j} + (2s - 2)\hat{k}$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

<u>Answer</u>:

Given lines,

$\bar{r} = (3 - t)\hat{i} + (4 + 2t)\hat{j} + (t - 2)\hat{k}$

$\bar{r} = (1 + s)\hat{i} + (3s - 7)\hat{j} + (2s - 2)\hat{k}$

Above equations can be written as

$\bar{r} = (3\hat{i} + 4\hat{j} - 2\hat{k}) + t(-\hat{i} + 2\hat{j} + \hat{k})$

$\bar{r} = (\hat{i} - 7\hat{j} - 2\hat{k}) + s(\hat{i} + 3\hat{j} + 2\hat{k})$

Here,

$\overline{a_1} = 3\hat{i} + 4\hat{j} - 2\hat{k}$

$\overline{b_1} = -\hat{i} + 2\hat{j} + \hat{k}$

$\overline{a_2} = \hat{i} - 7\hat{j} - 2\hat{k}$

$\overline{b_2} = \hat{i} + 3\hat{j} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$= \hat{i}(4 - 3) - \hat{j}(-2 - 1) + \hat{k}(-3 - 2)$

$\therefore \overline{b_1} \times \overline{b_2} = \hat{i} + 3\hat{j} - 5\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{1^2 + 3^2 + (-5)^2}$

$= \sqrt{1 + 9 + 25}$

$= \sqrt{35}$

$\overline{a_2} - \overline{a_1} = (1 - 3)\hat{i} + (-7 - 4)\hat{j} + (-2 + 2)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -2\hat{i} - 11\hat{j} + 0\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (\hat{i} + 3\hat{j} - 5\hat{k}).(-2\hat{i} - 11\hat{j} + 0\hat{k})$

$= (1 \times (-2)) + (3 \times (-11)) + ((-5) \times 0)$

$= -2 - 33 + 0$

$= -35$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$\therefore d = \left|\frac{-35}{\sqrt{35}}\right|$

$\therefore d = \sqrt{35}$

$d = \sqrt{35}\ units$

Question: 8

Solution:

Given equations :

$\bar{r} = (\lambda - 1)\hat{\imath} + (\lambda + 1)\hat{\jmath} - (\lambda + 1)\hat{k}$

$\bar{r} = (1 - \mu)\hat{\imath} + (2\mu - 1)\hat{\jmath} + (\mu + 2)\hat{k}$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b_2}$ is given by,

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

Answer :

Given lines,

$\bar{r} = (\lambda - 1)\hat{\imath} + (\lambda + 1)\hat{\jmath} - (\lambda + 1)\hat{k}$

$\bar{r} = (1 - \mu)\hat{\imath} + (2\mu - 1)\hat{\jmath} + (\mu + 2)\hat{k}$

Above equations can be written as

$\bar{r} = (-\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(\hat{\imath} + \hat{\jmath} - \hat{k})$

$\bar{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + s(-\hat{\imath} + 2\hat{\jmath} + \hat{k})$

Here,

$\bar{a_1} = -\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{b_1} = \hat{\imath} + \hat{\jmath} - \hat{k}$

$\overline{a_2} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\overline{b_2} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$= \hat{\imath}(1 + 2) - \hat{\jmath}(1 - 1) + \hat{k}(2 + 1)$

$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{\imath} - 0\hat{\jmath} + 3\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{3^2 + 0^2 + 3^2}$

$= \sqrt{9 + 0 + 9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$\overline{a_2} - \overline{a_1} = (1 + 1)\hat{\imath} + (-1 - 1)\hat{\jmath} + (2 + 1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (3\hat{\imath} - 0\hat{\jmath} + 3\hat{k}).(2\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$

$= (3 \times 2) + (0 \times (-2)) + (3 \times 3)$

$= 6 + 0 + 9$

$= 15$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{15}{3\sqrt{2}}\right|$$

$$\therefore d = \frac{5}{\sqrt{2}}$$

$$\therefore d = \frac{5\sqrt{2}}{2}$$

$$d = \frac{5\sqrt{2}}{2}\ units$$

Question: 9

Solution:

<u>Given equations</u> :

$\bar{r} = (\hat{\imath} - \hat{\jmath}) + \lambda(2\hat{\imath} - \hat{k})$

$\bar{r} = (2\hat{\imath} - \hat{\jmath}) + \mu(\hat{\imath} - \hat{\jmath} - \hat{k})$

<u>To Find</u> : d

<u>Formula</u> :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and $\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

<u>Answer</u>:

For given lines,

$\bar{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} - \hat{k})$

$\bar{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} - \hat{k})$

Here,

$\overline{a_1} = \hat{i} - \hat{j}$

$\overline{b_1} = 2\hat{i} - \hat{k}$

$\overline{a_2} = 2\hat{i} - \hat{j}$

$\overline{b_2} = \hat{i} - \hat{j} - \hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 1 & -1 & -1 \end{vmatrix}$$

$= \hat{i}(0 - 1) - \hat{j}(-2 + 1) + \hat{k}(-2 - 0)$

$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + \hat{j} - 2\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-1)^2 + 1^2 + (-2)^2}$

$= \sqrt{1 + 1 + 4}$

$= \sqrt{6}$

$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{i} + (-1 + 1)\hat{j} + (0 - 0)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} + 0\hat{j} + 0\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-\hat{i} + \hat{j} - 2\hat{k}).(\hat{i} + 0\hat{j} + 0\hat{k})$

$= ((-1) \times 1) + (1 \times 0) + ((-2) \times 0)$

= - 1 + 0 + 0

= - 1

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{-1}{\sqrt{6}}\right|$$

$$\therefore d = \frac{1}{\sqrt{6}}$$

$$\therefore d = \frac{\sqrt{6}}{6}$$

$$d = \frac{\sqrt{6}}{6} \text{ units}$$

As $d \neq 0$

Hence, the given lines do not intersect.

Question: 10

Solution:

Given equations :

$\bar{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$

$\bar{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

Answer :

For given lines,

$\bar{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$

$\bar{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$

Here,

$\overline{a_1} = 3\hat{i} - 15\hat{j} + 9\hat{k}$

$\overline{b_1} = 2\hat{i} - 7\hat{j} + 5\hat{k}$

$\overline{a_2} = -\hat{i} + \hat{j} + 9\hat{k}$

$\overline{b_2} = 2\hat{i} + \hat{j} - 3\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$= \hat{i}(21 - 5) - \hat{j}(-6 - 10) + \hat{k}(2 + 14)$

$\therefore \overline{b_1} \times \overline{b_2} = 17\hat{i} + 16\hat{j} + 16\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{17^2 + 16^2 + 17^2}$

$= \sqrt{289 + 256 + 289}$

$= \sqrt{834}$

$\overline{a_2} - \overline{a_1} = (-1 - 3)\hat{i} + (1 + 15)\hat{j} + (9 - 9)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -4\hat{i} + 16\hat{j} + 0\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (17\hat{i} + 16\hat{j} + 16\hat{k}).(-4\hat{i} + 16\hat{j} + 0\hat{k})$

$= (17 \times (-4)) + (16 \times 16) + (16 \times 0)$

$= -68 + 256 + 0$

$= 188$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{188}{\sqrt{834}}\right|$$

$$\therefore d = \frac{188}{\sqrt{834}} \text{units}$$

As $d \neq 0$

Hence, the given lines do not intersect.

Question: 11

Solution:

Given equations :

$\bar{r} = (2\hat{i} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$\bar{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

Answer :

For given lines,

$\bar{r} = (2\hat{i} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$\bar{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$

Here,

$\overline{a_1} = 2\hat{i} - 3\hat{k}$

$\overline{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Therefore,

$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$

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$= \hat{i}(12 - 9) - \hat{j}(4 - 6) + \hat{k}(3 - 4)$

$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{i} + 2\hat{j} - \hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{3^2 + 2^2 + (-1)^2}$

$= \sqrt{9 + 4 + 1}$

$= \sqrt{14}$

$\overline{a_2} - \overline{a_1} = (2 - 2)\hat{i} + (6 - 0)\hat{j} + (3 + 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 0\hat{i} + 6\hat{j} + 6\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1}) = (3\hat{i} + 2\hat{j} - \hat{k}) . (0\hat{i} + 6\hat{j} + 6\hat{k})$

$= (3 \times 0) + (2 \times 6) + ((-1) \times 6)$

$= 0 + 12 - 6$

$= 6$

Therefore, the shortest distance between the given lines is

$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$

$\therefore d = \left| \frac{6}{\sqrt{14}} \right|$

$\therefore d = \frac{6}{\sqrt{14}}$ units

As $d \neq 0$

Hence, the given lines do not intersect.

Question: 12

Solution:

Given equations :

$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\bar{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

Answer :

For given lines,

$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\bar{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overline{a_2} = 4\hat{i} + \hat{j}$

$\overline{b_2} = 5\hat{i} + 2\hat{j} + \hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(3 - 8) - \hat{j}(2 - 20) + \hat{k}(4 - 15)$

$\therefore \overline{b_1} \times \overline{b_2} = -5\hat{i} + 18\hat{j} - 11\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$

$= \sqrt{25 + 324 + 121}$

$= \sqrt{470}$

$\overline{a_2} - \overline{a_1} = (4 - 1)\hat{i} + (1 - 2)\hat{j} + (0 - 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 3\hat{i} - \hat{j} - 3\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-5\hat{i} + 18\hat{j} - 11\hat{k}).(3\hat{i} - \hat{j} - 3\hat{k})$

$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$

$= - 15 - 18 + 33$

= 0

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{0}{\sqrt{470}}\right|$$

$\therefore d = 0$ units

As d = 0

Hence, the given lines not intersect each other.

Now, to find point of intersection, let us convert given vector equations into Cartesian equations.

For that substituting $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$$\therefore L1 : x\hat{i} + y\hat{j} + z\hat{k} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\therefore L2 : x\hat{i} + y\hat{j} + z\hat{k} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

$$\therefore L1 : (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$$

$$\therefore L2 : (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$$

$$\therefore L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$$\therefore L2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$$

General point on L1 is

$x_1 = 2\lambda+1$, $y_1 = 3\lambda+2$, $z_1 = 4\lambda+3$

let, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Therefore, point P satisfies equation of line L2.

$$\therefore \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = \frac{4\lambda + 3 - 0}{1}$$

$$\therefore \frac{2\lambda - 3}{5} = \frac{3\lambda + 1}{2}$$

$\Rightarrow 4\lambda - 6 = 15\lambda + 5$

$\Rightarrow 11\lambda = -11$

$\Rightarrow \lambda = -1$

Therefore, $x_1 = 2(-1)+1$, $y_1 = 3(-1)+2$, $z_1 = 4(-1)+3$

$\Rightarrow x_1 = -1$, $y_1 = -1$, $z_1 = -1$

Hence point of intersection of given lines is (-1, -1, -1).

Question: 13

Solution:

Given equations :

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

To Find : d

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Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two parallel lines :

The shortest distance between the parallel lines $\bar{r} = \bar{a_1} + \lambda\bar{b}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{|(\bar{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|}\right|$$

Answer :

For given lines,

$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Here,

$\bar{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\bar{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\bar{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\bar{b_2} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

As $\bar{b_1} = \bar{b_2} = \bar{b}$ (say) , given lines are parallel to each other.

Therefore,

$\bar{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\therefore |\bar{b}| = \sqrt{2^2 + 3^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

$\bar{a_2} - \bar{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

$$(\overline{a_2} - \overline{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$= \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(6-2)$

$\therefore (\overline{a_2} - \overline{a_1}) \times \bar{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

$\therefore \left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right| = \sqrt{9^2 + (-14)^2 + 4^2}$

$= \sqrt{81 + 196 + 16}$

$= \sqrt{293}$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right|}{|\bar{b}|}\right|$$

$$\therefore d = \left|\frac{\sqrt{293}}{7}\right|$$

$$d = \frac{\sqrt{293}}{7} \; units$$

Question: 14

Solution:

Given equations :

$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two parallel lines :

The shortest distance between the parallel lines $\bar{r} = \bar{a_1} + \lambda\bar{b}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{|(\bar{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|}\right|$$

Answer:

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$$

Here,

$$\bar{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\bar{a_2} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\bar{b} = \hat{i} - \hat{j} + \hat{k}$$

$$\therefore |\bar{b}| = \sqrt{1^2 + (-1)^2 + 1^2}$$

$$= \sqrt{1 + 1 + 1}$$

$$= \sqrt{3}$$

$$\bar{a_2} - \bar{a_1} = (2 - 1)\hat{i} + (-1 - 2)\hat{j} + (-1 - 3)\hat{k}$$

$$\therefore \bar{a_2} - \bar{a_1} = \hat{i} - 3\hat{j} - 4\hat{k}$$

$$(\bar{a_2} - \bar{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(-3 - 4) - \hat{j}(1 + 4) + \hat{k}(-1 + 3)$$

$$\therefore (\bar{a_2} - \bar{a_1}) \times \bar{b} = -7\hat{i} - 5\hat{j} + 2\hat{k}$$

$$\therefore |(\bar{a_2} - \bar{a_1}) \times \bar{b}| = \sqrt{(-7)^2 + (-5)^2 + 2^2}$$

$$= \sqrt{49 + 25 + 4}$$

$$= \sqrt{78}$$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{|(\bar{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|}\right|$$

$$\therefore d = \left|\frac{\sqrt{78}}{\sqrt{3}}\right|$$

$$\therefore d = \sqrt{26}$$

$$d = \sqrt{26}\ units$$

Question: 15

Solution:

Given : point $A \equiv (2, 3, 2)$

Equation of line : $\bar{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$

To Find : i) equation of line

ii) distance d

Formulae :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and parallel to vector $\bar{b} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by

$\bar{r} = \bar{a} + \lambda\bar{b}$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

4. Shortest distance between two parallel lines :

The shortest distance between the parallel lines $\bar{r} = \overline{a_1} + \lambda\bar{b}$ and

$\bar{r} = \overline{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{\left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right|}{|\bar{b}|}\right|$$

Answer :

As the required line is parallel to the line

$\bar{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$

Therefore, the vector parallel to the required line is

$\bar{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Given point A ≡ (2, 3, 2)

$\therefore \bar{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

Therefore, equation of line passing through A and parallel to $\bar{b}$ is

$\bar{r} = \bar{a} + \mu\bar{b}$

$\therefore \bar{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$

Now, to calculate distance between above line and given line,

$\bar{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$

$\bar{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$

Here,

$\bar{a_1} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$\bar{a_2} = -2\hat{i} + 3\hat{j}$

$\bar{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\therefore |\bar{b}| = \sqrt{2^2 + (-3)^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

$= 7$

$\bar{a_2} - \bar{a_1} = (-2 - 2)\hat{i} + (3 - 3)\hat{j} + (0 - 2)\hat{k}$

$\therefore \bar{a_2} - \bar{a_1} = -4\hat{i} + 0\hat{j} - 2\hat{k}$

$$(\bar{a_2} - \bar{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 0 & -2 \\ 2 & -3 & 6 \end{vmatrix}$$

$= \hat{i}(0 - 6) - \hat{j}(-24 + 4) + \hat{k}(12 - 0)$

$\therefore (\bar{a_2} - \bar{a_1}) \times \bar{b} = -6\hat{i} + 20\hat{j} + 12\hat{k}$

$\therefore |(\bar{a_2} - \bar{a_1}) \times \bar{b}| = \sqrt{(-6)^2 + 20^2 + 12^2}$

$= \sqrt{36 + 400 + 144}$

$= \sqrt{580}$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{|(\bar{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|} \right|$$

$$\therefore d = \left| \frac{\sqrt{580}}{7} \right|$$

$$\therefore d = \frac{\sqrt{580}}{7}$$

$$d = \frac{\sqrt{580}}{7} \text{ units}$$

Question: 16

Solution:

Given : Cartesian equations of lines

$$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

$$L2 : \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

To Find : i) vector equations of given lines

ii) distance d

Formulae :

CLASS24

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$\bar{r} = \bar{a} + \lambda\bar{b}$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

4. Shortest distance between two parallel lines :

The shortest distance between the parallel lines $\bar{r} = \bar{a_1} + \lambda\bar{b}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{|(\bar{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|}\right|$$

Answer :

Given Cartesian equations of lines

$$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

Line L1 is passing through point (1, 2, -4) and has direction ratios (2, 3, 6)

Therefore, vector equation of line L1 is

$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

And

$$L2 : \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

Line L2 is passing through point (3, 3, -5) and has direction ratios (4, 6, 12)

Therefore, vector equation of line L2 is

$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$

$\therefore \bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + 2\mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Now, to calculate distance between the lines,

$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + 2\mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\overline{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\overline{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

As $\overline{b_1} = \overline{b_2} = \bar{b}$ (say) , given lines are parallel to each other.

Therefore,

$\bar{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\therefore |\bar{b}| = \sqrt{2^2 + 3^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

$= 7$

$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

$(\overline{a_2} - \overline{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$

$= \hat{i}(6 + 3) - \hat{j}(12 + 2) + \hat{k}(6 - 2)$

$\therefore (\overline{a_2} - \overline{a_1}) \times \bar{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

$\therefore |(\overline{a_2} - \overline{a_1}) \times \bar{b}| = \sqrt{9^2 + (-14)^2 + 4^2}$

$= \sqrt{81 + 196 + 16}$

$= \sqrt{293}$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{|(\overline{a_2} - \overline{a_1}) \times \bar{b}|}{|\bar{b}|} \right|$$

$$\therefore d = \left| \frac{\sqrt{293}}{7} \right|$$

$$d = \frac{\sqrt{293}}{7} \text{ units}$$

Question: 17

Solution:

Given : Cartesian equations of lines

$$L1 : \frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4}$$

$L2 : \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$

To Find : i) vector equations of given lines

ii) distance d

Formulae :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$\bar{r} = \bar{a} + \lambda\bar{b}$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

4. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

Answer :

Given Cartesian equations of lines

$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

Line L1 is passing through point (1, 2, 3) and has direction ratios (2, 3, 4)

Therefore, vector equation of line L1 is

$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

And

$L2 : \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$

Line L2 is passing through point (2, 3, 5) and has direction ratios (3, 4, 5)

Therefore, vector equation of line L2 is

$\bar{r} = (3\hat{i} + 3\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$

Now, to calculate distance between the lines,

$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\bar{r} = (3\hat{i} + 3\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overline{a_2} = 3\hat{i} + 3\hat{j} + 5\hat{k}$

$\overline{b_2} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9)$

$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-1)^2 + 2^2 + (-1)^2}$

$= \sqrt{1 + 4 + 1}$

$= \sqrt{6}$

$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (5 - 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} + 2\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-\hat{i} + 2\hat{j} - \hat{k}).(2\hat{i} + \hat{j} + 2\hat{k})$

$= ((-1) \times 2) + (2 \times 1) + ((-1) \times 2)$

= - 2 + 2 - 2

= -2

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{-2}{\sqrt{6}} \right|$$

$$\therefore d = \frac{2}{\sqrt{3}.\sqrt{2}}$$

$$\therefore d = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\therefore d = \sqrt{\frac{2}{3}}$$

$d = \sqrt{\frac{2}{3}}\ units$

Question: 18

Solution:

Given : Cartesian equations of lines

$L1 : \frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$

$L2 : \frac{x-1}{2} = \frac{y+1}{2} = \frac{z+1}{-2}$

To Find : distance d

Formulae :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$\bar{r} = \bar{a} + \lambda\bar{b}$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

4. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

Answer :

Given Cartesian equations of lines

$L1 : \frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$

Line L1 is passing through point (1, -2, 3) and has direction ratios (-1, 1, -2)

Therefore, vector equation of line L1 is

$\bar{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$

And

$L2 : \frac{x-1}{2} = \frac{y+1}{2} = \frac{z+1}{-2}$

Line L2 is passing through point (1, -1, -1) and has direction ratios (2, 2, -2)

Therefore, vector equation of line L2 is

$\bar{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + 2\hat{j} - 2\hat{k})$

Now, to calculate distance between the lines,

$\bar{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$

$\bar{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + 2\hat{j} - 2\hat{k})$

Here,

$\overline{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\overline{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$

$\overline{a_2} = \hat{i} - \hat{j} - \hat{k}$

$\overline{b_2} = 2\hat{i} + 2\hat{j} - 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 2 & 2 & -2 \end{vmatrix}$$

$= \hat{i}(-2 + 4) - \hat{j}(2 + 4) + \hat{k}(-2 - 2)$

$\therefore \overline{b_1} \times \overline{b_2} = 2\hat{i} - 6\hat{j} - 4\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{2^2 + (-6)^2 + (-4)^2}$

$= \sqrt{4 + 36 + 16}$

$= \sqrt{56}$

$\overline{a_2} - \overline{a_1} = (1 - 1)\hat{i} + (-1 + 2)\hat{j} + (-1 - 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 0\hat{i} + \hat{j} - 4\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (2\hat{i} - 6\hat{j} - 4\hat{k}).(0\hat{i} + \hat{j} - 4\hat{k})$

$= (2 \times 0) + ((-6) \times 1) + ((-4) \times (-4))$

= 0 - 6 + 16

= 10

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$\therefore d = \left|\frac{10}{\sqrt{56}}\right|$

$\therefore d = \frac{10}{\sqrt{56}}$

$d = \frac{10}{\sqrt{56}}\ units$

Question: 19

Solution:

Given : Cartesian equations of lines

L1 : $\frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2}$

L2 : $\frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-23}{3}$

To Find : distance d

Formulae :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$\bar{r} = \bar{a} + \lambda\bar{b}$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

4. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b_2}$ is given by,

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

Answer:

Given Cartesian equations of lines

$$L1 : \frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2}$$

Line L1 is passing through point (12, 1, 5) and has direction ratios (-9, 4, 2)

Therefore, vector equation of line L1 is

$$\bar{r} = (12\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-9\hat{i} + 4\hat{j} + 2\hat{k})$$

And

$$L2 : \frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-23}{3}$$

Line L2 is passing through point (23, 10, 23) and has direction ratios (-6, -4, 3)

Therefore, vector equation of line L2 is

$$\bar{r} = (23\hat{i} + 10\hat{j} + 23\hat{k}) + \mu(-6\hat{i} - 4\hat{j} + 3\hat{k})$$

Now, to calculate distance between the lines,

$$\bar{r} = (12\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-9\hat{i} + 4\hat{j} + 2\hat{k})$$

$$\bar{r} = (23\hat{i} + 10\hat{j} + 23\hat{k}) + \mu(-6\hat{i} - 4\hat{j} + 3\hat{k})$$

Here,

$$\overline{a_1} = 12\hat{i} + \hat{j} + 5\hat{k}$$

$$\overline{b_1} = -9\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\overline{a_2} = 23\hat{i} + 10\hat{j} + 23\hat{k}$$

$$\overline{b_2} = -6\hat{i} - 4\hat{j} + 3\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -9 & 4 & 2 \\ -6 & -4 & 3 \end{vmatrix}$$

$$= \hat{i}(12+8) - \hat{j}(-27+12) + \hat{k}(36+24)$$

$$\therefore \overline{b_1} \times \overline{b_2} = 20\hat{i} + 15\hat{j} + 60\hat{k}$$

$$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{20^2 + 15^2 + 60^2}$$

$$= \sqrt{400 + 225 + 3600}$$

$$= \sqrt{4225}$$

$$= 65$$

$$\overline{a_2} - \overline{a_1} = (23-12)\hat{i} + (10-1)\hat{j} + (23-5)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 11\hat{i} + 9\hat{j} + 18\hat{k}$$

Now,

$$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (20\hat{i} + 15\hat{j} + 60\hat{k}).(11\hat{i} + 9\hat{j} + 18\hat{k})$$

$$= (20 \times 11) + (15 \times 9) + (60 \times 18)$$

$$= 220 + 135 + 1080$$

$$= 1435$$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{1435}{65}\right|$$

$$\therefore d = \frac{287}{13}$$

$$d = \frac{287}{13}\ units$$

Exercise : 27E

Question: 1

Solution:

Given : Cartesian equations of lines

$$L1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

$$L2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

Formulae :

1. Condition for perpendicularity :

If line L1 has direction ratios (a_1, a_2, a_3) and that of line L2 are (b_1, b_2, b_3) then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

2. Distance formula :

Distance between two points $A\equiv(a_1, a_2, a_3)$ and $B\equiv(b_1, b_2, b_3)$ is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

3. Equation of line :

Equation of line passing through points $A\equiv(x_1, y_1, z_1)$ and $B\equiv(x_2, y_2, z_2)$ is given by,

$$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2}$$

Answer :

Given equations of lines

$$L1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

$$L2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

Direction ratios of L1 and L2 are (3, -1, 1) and (-3, 2, 4) respectively.

Let, general point on line L1 is $P\equiv(x_1, y_1, z_1)$

$x_1 = 3s+3$, $y_1 = -s+8$, $z_1 = s+3$

and let, general point on line L2 is $Q\equiv(x_2, y_2, z_2)$

$x_2 = -3t - 3$, $y_2 = 2t - 7$, $z_2 = 4t + 6$

$\therefore \overline{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= (-3t - 3 - 3s - 3)\hat{i} + (2t - 7 + s - 8)\hat{j} + (4t + 6 - s - 3)\hat{k}$

$\therefore \overline{PQ} = (-3t - 3s - 6)\hat{i} + (2t + s - 15)\hat{j} + (4t - s + 3)\hat{k}$

Direction ratios of $\overline{PQ}$ are ((-3t – 3s - 6), (2t + s - 15), (4t – s + 3))

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

3(-3t – 3s - 6) – 1(2t + s - 15) + 1(4t – s + 3) = 0 and

-3(-3t – 3s - 6) + 2(2t + s - 15) + 4(4t – s + 3) = 0

⇒ -9t – 9s – 18 – 2t – s + 15 + 4t – s + 3 = 0 and

9t + 9s + 18 + 4t + 2s – 30 + 16t – 4s + 12 = 0

⇒ -7t – 11s = 0 and

29t + 7s = 0

Solving above two equations, we get,

t = 0 and s = 0

therefore,

P ≡ (3, 8, 3) and Q ≡ (-3, -7, 6)

Now, distance between points P and Q is

$d = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$

$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$

$= \sqrt{36 + 225 + 9}$

$= \sqrt{270}$

$= 3\sqrt{30}$

Therefore, the shortest distance between two given lines is

$d = 3\sqrt{30}\ units$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x-3}{3+3} = \frac{y-8}{8+7} = \frac{z-3}{3-6}$$

$$\therefore \frac{x-3}{6} = \frac{y-8}{15} = \frac{z-3}{-3}$$

$$\therefore \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

Question: 2

Solution:

Given : Cartesian equations of lines

L1 : $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$

L2 : $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$

Formulae :

1. Condition for perpendicularity :

If line L1 has direction ratios (a_1, a_2, a_3) and that of line L2 are (b_1, b_2, b_3) then lines L1 and L2 will be perpendicular to each other if

$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$

2. Distance formula :

Distance between two points $A \equiv (a_1, a_2, a_3)$ and $B \equiv (b_1, b_2, b_3)$ is given by,

$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$

3. Equation of line :

Equation of line passing through points $A \equiv (x_1, y_1, z_1)$ and $B \equiv (x_2, y_2, z_2)$ is given by,

$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$

Answer :

Given equations of lines

L1 : $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$

L2 : $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$

Direction ratios of L1 and L2 are (-1, 2, 1) and (1, 3, 2) respectively.

Let, general point on line L1 is $P \equiv (x_1, y_1, z_1)$

$x_1 = -s+3$, $y_1 = 2s+4$, $z_1 = s-2$

and let, general point on line L2 is $Q \equiv (x_2, y_2, z_2)$

$x_2 = t+1$, $y_2 = 3t - 7$, $z_2 = 2t - 2$

$\therefore \overline{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= (t + 1 + s - 3)\hat{i} + (3t - 7 - 2s - 4)\hat{j} + (2t - 2 - s + 2)\hat{k}$

$\therefore \overline{PQ} = (t + s - 2)\hat{i} + (3t - 2s - 11)\hat{j} + (2t - s)\hat{k}$

Direction ratios of $\overline{PQ}$ are $((t + s - 2), (3t - 2s - 11), (2t - s))$

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

$-1(t + s - 2) + 2(3t - 2s - 11) + 1(2t - s) = 0$ and

$1(t + s - 2) + 3(3t - 2s - 11) + 2(2t - s) = 0$

$\Rightarrow - t - s + 2 + 6t - 4s - 22 + 2t - s = 0$ and

$t + s - 2 + 9t - 6s - 33 + 4t - 2s = 0$

$\Rightarrow 7t - 6s = 20$ and

$14t - 7s = 35$

Solving above two equations, we get,

t = 2 and s = -1

therefore,

P ≡ (4, 2, -3) and Q ≡ (3, -1, 2)

Now, distance between points P and Q is

$$d = \sqrt{(4-3)^2 + (2+1)^2 + (-3-2)^2}$$

$$= \sqrt{(1)^2 + (3)^2 + (-5)^2}$$

$$= \sqrt{1 + 9 + 25}$$

$$= \sqrt{35}$$

Therefore, the shortest distance between two given lines is

$$d = \sqrt{35}\ units$$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x-4}{4-3} = \frac{y-2}{2+1} = \frac{z+3}{-3-2}$$

$$\therefore \frac{x-4}{1} = \frac{y-2}{3} = \frac{z+3}{-5}$$

$$\therefore \frac{x-4}{-1} = \frac{y-2}{-3} = \frac{z+3}{5}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x-4}{-1} = \frac{y-2}{-3} = \frac{z+3}{5}$$

Question: 3

Solution:

Given : Cartesian equations of lines

$$L1 : \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$$

$$L2 : \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$$

Formulae :

1. Condition for perpendicularity :

If line L1 has direction ratios (a_1, a_2, a_3) and that of line L2 are (b_1, b_2, b_3) then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

2. Distance formula :

Distance between two points $A \equiv (a_1, a_2, a_3)$ and $B \equiv (b_1, b_2, b_3)$ is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

3. Equation of line :

Equation of line passing through points $A \equiv (x_1, y_1, z_1)$ and $B \equiv (x_2, y_2, z_2)$ is given by,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

Answer:

Given equations of lines

$$L1 : \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$$

$$L2 : \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$$

Direction ratios of L1 and L2 are (2, 1, -3) and (2, -7, 5) respectively.

Let, general point on line L1 is $P \equiv (x_1, y_1, z_1)$

$x_1 = 2s-1$, $y_1 = s+1$, $z_1 = -3s+9$

and let, general point on line L2 is $Q \equiv (x_2, y_2, z_2)$

$x_2 = 2t+3$, $y_2 = -7t - 15$, $z_2 = 5t + 9$

$\therefore \overline{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= (5t + 9 - 2s + 1)\hat{i} + (-7t - 15 - s - 1)\hat{j} + (5t + 9 + 3s - 9)\hat{k}$

$\therefore \overline{PQ} = (5t - 2s + 10)\hat{i} + (-7t - s - 16)\hat{j} + (5t + 3s)\hat{k}$

Direction ratios of $\overline{PQ}$ are ((5t - 2s + 10), (-7t – s – 16), (5t + 3s))

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

2(5t - 2s + 10) + 1(-7t – s – 16) - 3(5t + 3s) = 0 and

2(5t - 2s + 10) – 7(-7t – s – 16) + 5(5t + 3s) = 0

⇒ 10t – 4s + 20 - 7t – s - 16 - 15t – 9s = 0 and

10t - 4s + 20 + 49t + 7s + 112 + 25t + 15s = 0

⇒ -12t – 14s = -4 and

84t + 18s = -132

Solving above two equations, we get,

t = -2 and s = 2

therefore,

P ≡ (3, 3, 3) and Q ≡ (-1, -1, -1)

Now, distance between points P and Q is

$d = \sqrt{(3+1)^2 + (3+1)^2 + (3+1)^2}$

$= \sqrt{(4)^2 + (4)^2 + (4)^2}$

$= \sqrt{16 + 16 + 16}$

$= \sqrt{48}$

$= 4\sqrt{3}$

Therefore, the shortest distance between two given lines is

$d = 4\sqrt{3}\ units$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x - 3}{3 + 1} = \frac{y - 3}{3 + 1} = \frac{z - 3}{3 + 1}$$

$$\therefore \frac{x - 3}{4} = \frac{y - 3}{4} = \frac{z - 3}{4}$$

$$\therefore x - 3 = y - 3 = z - 3$$

$$\Rightarrow x = y = z$$

Therefore, equation of line of shortest distance between two given lines is

$x = y = z$

Question: 4

Solution:

Given : Cartesian equations of lines

$$L1 : \frac{x - 6}{3} = \frac{y - 7}{-1} = \frac{z - 4}{1}$$

$$L2 : \frac{x}{-3} = \frac{y + 9}{2} = \frac{z - 2}{4}$$

Formulae :

1. Condition for perpendicularity :

If line L1 has direction ratios (a_1, a_2, a_3) and that of line L2 are (b_1, b_2, b_3) then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

2. Distance formula :

Distance between two points $A \equiv (a_1, a_2, a_3)$ and $B \equiv (b_1, b_2, b_3)$ is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

3. Equation of line :

Equation of line passing through points $A \equiv (x_1, y_1, z_1)$ and $B \equiv (x_2, y_2, z_2)$ is given by,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

Answer :

Given equations of lines

$$L1 : \frac{x - 6}{3} = \frac{y - 7}{-1} = \frac{z - 4}{1}$$

$$L2 : \frac{x}{-3} = \frac{y + 9}{2} = \frac{z - 2}{4}$$

Direction ratios of L1 and L2 are (3, -1, 1) and (-3, 2, 4) respectively.

Let, general point on line L1 is $P \equiv (x_1, y_1, z_1)$

$x_1 = 3s+6$, $y_1 = -s+7$, $z_1 = s+4$

and let, general point on line L2 is $Q \equiv (x_2, y_2, z_2)$

$x_2 = -3t$, $y_2 = 2t - 9$, $z_2 = 4t + 2$

$\therefore \overline{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= (-3t - 3s - 6)\hat{i} + (2t - 9 + s - 7)\hat{j} + (4t + 2 - s - 4)\hat{k}$

$\therefore \overline{PQ} = (-3t - 3s - 6)\hat{i} + (2t + s - 16)\hat{j} + (4t - s - 2)\hat{k}$

Direction ratios of $\overline{PQ}$ are ((-3t - 3s - 6), (2t + s – 16), (4t – s – 2))

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

3(-3t - 3s - 6) - 1(2t + s – 16) + 1(4t – s – 2) = 0 and

-3(-3t - 3s - 6) + 2(2t + s – 16) + 4(4t – s – 2) = 0

⇒ -9t - 9s - 18 - 2t – s + 16 + 4t – s – 2 = 0 and

9t + 9s + 18 + 4t + 2s – 32 + 16t – 4s – 8 = 0

⇒ -7t – 11s = 4 and

29t + 7s = -22

Solving above two equations, we get,

t = 1 and s = -1

therefore,

P ≡ (3, 8, 3) and Q ≡ (-3, -7, 6)

Now, distance between points P and Q is

$d = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$

$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$

$= \sqrt{36 + 225 + 9}$

$= \sqrt{270}$

$= 3\sqrt{30}$

Therefore, the shortest distance between two given lines is

$d = 3\sqrt{30}\ units$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x-3}{3+3} = \frac{y-8}{8+7} = \frac{z-3}{3-6}$$

$$\therefore \frac{x-3}{6} = \frac{y-8}{15} = \frac{z-3}{-3}$$

$$\therefore \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

Question: 5

Solution:

Given : Cartesian equations of lines

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$$L1 : \frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$$

$$L2 : \frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$$

To Find : distance d

Formulae :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$\bar{r} = \bar{a} + \lambda\bar{b}$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

4. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

Answer :

Given Cartesian equations of lines

$$L1 : \frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$$

Line L1 is passing through point (0, 2, -3) and has direction ratios (1, 2, 3)

Therefore, vector equation of line L1 is

$\bar{r} = (0\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

And

$L2 : \frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$

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Line L2 is passing through point (2, 6, 3) and has direction ratios (2, 3, 4)

Therefore, vector equation of line L2 is

$\bar{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$

Now, to calculate distance between the lines,

$\bar{r} = (0\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$\bar{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$

Here,

$\overline{a_1} = 0\hat{i} + 2\hat{j} - 3\hat{k}$

$\overline{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$= \hat{i}(8-9) - \hat{j}(4-6) + \hat{k}(3-4)$

$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-1)^2 + 2^2 + (-1)^2}$

$= \sqrt{1+4+1}$

$= \sqrt{6}$

$\overline{a_2} - \overline{a_1} = (2-0)\hat{i} + (6-2)\hat{j} + (3+3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + 4\hat{j} + 6\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-\hat{i} + 2\hat{j} - \hat{k}).(2\hat{i} + 4\hat{j} + 6\hat{k})$

$= ((-1) \times 2) + (2 \times 4) + ((-1) \times 6)$

$= -2 + 8 - 6$

$= 0$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{0}{\sqrt{14}} \right|$$

$\therefore d = 0$ units

As d = 0

Hence, given lines intersect each other.

Now, general point on L1 is

$x_1 = \lambda$, $y_1 = 2\lambda+2$, $z_1 = 3\lambda-3$

let, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Therefore, point P satisfies equation of line L2.

$$\therefore \frac{\lambda - 2}{2} = \frac{2\lambda + 2 - 6}{3} = \frac{3\lambda - 3 - 3}{4}$$

$$\therefore \frac{\lambda - 2}{2} = \frac{2\lambda - 4}{3}$$

$\Rightarrow 3\lambda - 6 = 4\lambda - 8$

$\Rightarrow \lambda = 2$

Therefore, $x_1 = 2$, $y_1 = 2(2)+2$, $z_1 = 3(2)-3$

$\Rightarrow x_1 = 2$, $y_1 = 6$, $z_1 = 3$

Hence point of intersection of given lines is (2, 6, 3).

Question: 6

Solution:

Given : Cartesian equations of lines

$$L1 : \frac{x - 1}{3} = \frac{y + 1}{2} = \frac{z - 1}{5}$$

$$L2 : \frac{x - 2}{2} = \frac{y - 1}{3} = \frac{z + 1}{-2}$$

To Find : distance d

Formulae :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$$\bar{r} = \bar{a} + \lambda\bar{b}$$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

4. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b_2}$ is given by,

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

Answer :

Given Cartesian equations of lines

$$L1 : \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$$

Line L1 is passing through point (1, -1, 1) and has direction ratios (3, 2, 5)

Therefore, vector equation of line L1 is

$$\bar{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$$

And

$$L2 : \frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$$

Line L2 is passing through point (2, 1, -1) and has direction ratios (2, 3, -2)

Therefore, vector equation of line L2 is

$$\bar{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} - 2\hat{k})$$

Now, to calculate distance between the lines,

$$\bar{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$$

$$\bar{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} - 2\hat{k})$$

Here,

$$\bar{a_1} = \hat{i} - \hat{j} + \hat{k}$$

$$\bar{b_1} = 3\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\bar{a_2} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\bar{b_2} = 2\hat{i} + 3\hat{j} - 2\hat{k}$$

Therefore,

$$\bar{b_1} \times \bar{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 5 \\ 2 & 3 & -2 \end{vmatrix}$$

$$= \hat{i}(-4 - 15) - \hat{j}(-6 - 10) + \hat{k}(9 - 4)$$

$$\therefore \bar{b_1} \times \bar{b_2} = -19\hat{i} + 16\hat{j} + 5\hat{k}$$

$$\therefore |\bar{b_1} \times \bar{b_2}| = \sqrt{(-19)^2 + 16^2 + 5^2}$$

$$= \sqrt{361 + 256 + 25}$$

$$= \sqrt{642}$$

$$\bar{a_2} - \bar{a_1} = (2 - 1)\hat{i} + (1 + 1)\hat{j} + (-1 - 1)\hat{k}$$

$$\therefore \bar{a_2} - \bar{a_1} = \hat{i} + 2\hat{j} - 2\hat{k}$$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(-19\hat{\imath} + 16\hat{\jmath} + 5\hat{k}\right).\left(\hat{\imath} + 2\hat{\jmath} - 2\hat{k}\right)$

$= ((-19) \times 1) + (16 \times 2) + (5 \times (-2))$

= - 19 + 32 – 10

= 3

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{3}{\sqrt{642}}\right|$$

$$\therefore d = \frac{3}{\sqrt{642}} \text{ units}$$

As d ≠ 0

Hence, given lines do not intersect each other.