Chapter : 28. THE PLANE

CLASS24

Exercise : 28A

Question: 1

Solution:

(i) A(2, 2, -1), B(3, 4, 2) and C(7, 0, 6)

Given Points :

A = (2, 2, -1)

B = (3, 4, 2)

C = (7, 0, 6)

To Find : Equation of plane passing through points A, B & C

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Vector :

If A and B be two points with position vectors $\bar{a}$ & $\bar{b}$ respectively, where

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\overline{AB} = \bar{b} - \bar{a}$

$= (b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Equation of Plane :

If A = (a_1, a_2, a_3), B = (b_1, b_2, b_3), C = (c_1, c_2, c_3) are three non-collinear points,

Then, the vector equation of the plane passing through these points is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (2, 2, -1)

B = (3, 4, 2)

C = (7, 0, 6)

Position vectors are given by,

$\bar{a} = 2\hat{i} + 2\hat{j} - \hat{k}$

$\bar{b} = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$\bar{c} = 7\hat{i} + 6\hat{k}$

Now, vectors AB & AC are

$\overline{AB} = \bar{b} - \bar{a}$

$= (3-2)\hat{i} + (4-2)\hat{j} + (2+1)\hat{k}$

$\therefore \overline{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{AC} = \bar{c} - \bar{a}$

$= (7-2)\hat{i} + (0-2)\hat{j} + (6+1)\hat{k}$

$\therefore \overline{AC} = 5\hat{i} - 2\hat{j} + 7\hat{k}$

Therefore,

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix}$$

$= \hat{i}(2 \times 7 - (-2) \times 3) - \hat{j}(1 \times 7 - 5 \times 3) + \hat{k}(1 \times (-2) - 5 \times 2)$

$= 20\hat{i} + 8\hat{j} - 12\hat{k}$

Now,

$\bar{a}.(\overline{AB} \times \overline{AC}) = (2 \times 20) + (2 \times 8) + ((-1) \times (-12))$

$= 40 + 16 + 12$

$= 68$

$\therefore \bar{a}.(\overline{AB} \times \overline{AC}) = 68$ ....eq(1)

And

$\bar{r}.(\overline{AB} \times \overline{AC}) = (x \times 20) + (y \times 8) + (z \times (-12))$

$= 20x + 8y - 12z$

$\therefore \bar{r}.(\overline{AB} \times \overline{AC}) = 20x + 8y - 12z$ ....eq(2)

Vector equation of the plane passing through points A, B & C is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

From eq(1) and eq(2)

$20x + 8y - 12z = 68$

This is $5x + 2y - 3z = 17$ vector equation of required plane.

(ii) Given Points :

A = (0, -1, -1)

B = (4, 5, 1)

C = (3, 9, 4)

To Find : Equation of plane passing through points A, B & C

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

2) Vector :

If A and B be two points with position vectors $\bar{a}$ & $\bar{b}$ respectively, where

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\overline{AB} = \bar{b} - \bar{a}$

$= (b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & b are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Equation of Plane :

If A = (a_1, a_2, a_3), B = (b_1, b_2, b_3), C = (c_1, c_2, c_3) are three non-collinear points,

Then, vector equation of the plane passing through these points is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (0, -1, -1)

B = (4, 5, 1)

C = (3, 9, 4)

Position vectors are given by,

$\bar{a} = -\hat{\jmath} - \hat{k}$

$\bar{b} = 4\hat{\imath} + 5\hat{\jmath} + \hat{k}$

$\bar{c} = 3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$

Now, vectors $\overline{AB}$ & $\overline{AC}$ are

$\overline{AB} = \bar{b} - \bar{a}$

$= (4-0)\hat{\imath} + (5+1)\hat{\jmath} + (1+1)\hat{k}$

$\therefore \overline{AB} = 4\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$

$\overline{AC} = \bar{c} - \bar{a}$

$= (3-0)\hat{\imath} + (9+1)\hat{\jmath} + (4+1)\hat{k}$

$\therefore \overline{AC} = 3\hat{\imath} + 10\hat{\jmath} + 5\hat{k}$

Therefore,

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix}$$

$= \hat{\imath}(6 \times 5 - 10 \times 2) - \hat{\jmath}(4 \times 5 - 2 \times 3) + \hat{k}(4 \times 10 - 3 \times 6)$

$= 10\hat{\imath} - 14\hat{\jmath} + 22\hat{k}$

Now,

$\bar{a}.(\overline{AB} \times \overline{AC}) = (0 \times 10) + ((-1) \times (-14)) + ((-1) \times 22)$

= 0 + 14 – 22

= - 8

$\therefore \bar{a}.(\overline{AB} \times \overline{AC}) = -8$ ....eq(1)

And

$\bar{r}.(\overline{AB} \times \overline{AC}) = (x \times 10) + (y \times (-14)) + (z \times 22)$

= 10x - 14y + 22z

$\therefore \bar{r}.(\overline{AB} \times \overline{AC}) = 10x - 14y + 22z$ ....eq(2)

Vector equation of plane passing through points A, B & C is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

From eq(1) and eq(2)

10x - 14y + 22z = - 8

This is 5x - 7y + 11z = - 4 vector equation of required plane

(iii) Given Points :

A = (-2, 6, -6)

B = (-3, 10, 9)

C = (-5, 0, -6)

To Find : Equation of plane passing through points A, B & C

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Vector :

If A and B be two points with position vectors $\bar{a}$ & $\bar{b}$ respectively, where

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$b = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\overline{AB} = \bar{b} - \bar{a}$

$= (b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Equation of Plane :

If A = (a_1, a_2, a_3), B = (b_1, b_2, b_3), C = (c_1, c_2, c_3) are three non-collinear points,

Then, vector equation of the plane passing through these points is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (-2, 6, -6)

B = (-3, 10, 9)

C = (-5, 0, -6)

Position vectors are given by,

$\bar{a} = -2\hat{\imath} + 6\hat{\jmath} - 6\hat{k}$

$\bar{b} = -3\hat{\imath} + 10\hat{\jmath} + 9\hat{k}$

$\bar{c} = -5\hat{\imath} - 6\hat{k}$

Now, vectors $\overline{AB}$ & $\overline{AC}$ are

$\overline{AB} = \bar{b} - \bar{a}$

$= (-3+2)\hat{\imath} + (10-6)\hat{\jmath} + (9+6)\hat{k}$

$\therefore \overline{AB} = -\hat{\imath} + 4\hat{\jmath} + 15\hat{k}$

$\overline{AC} = \bar{c} - \bar{a}$

$= (-5+2)\hat{\imath} + (0-6)\hat{\jmath} + (-6+6)\hat{k}$

$\therefore \overline{AC} = -3\hat{\imath} - 6\hat{\jmath} + 0\hat{k}$

Therefore,

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 4 & 15 \\ -3 & -6 & 0 \end{vmatrix}$$

$= \hat{\imath}(4 \times 0 - (-6) \times 15) - \hat{\jmath}((-1) \times 0 - (-3) \times 15)$
$\quad + \hat{k}((-1) \times (-6) - (-3) \times 4)$

$= 90\hat{\imath} - 45\hat{\jmath} + 18\hat{k}$

Now,

$\bar{a}.(\overline{AB} \times \overline{AC}) = ((-2) \times 90) + (6 \times (-45)) + ((-6) \times 18)$

= - 180 - 270 - 108

= - 558

$\therefore \bar{a}.(\overline{AB} \times \overline{AC}) = -558$ ....eq(1)

And

$\bar{r}.(\overline{AB} \times \overline{AC}) = (x \times 90) + (y \times (-45)) + (z \times 18)$

= 90x - 45y + 18z

$\therefore \bar{r}.(\overline{AB} \times \overline{AC}) = 90x - 45y + 18z$ ....eq(2)

Vector equation of plane passing through points A, B & C is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

From eq(1) and eq(2)

90x - 45y + 18z = - 558

This is 10x - 5y + 2z = - 62 vector equation of required plane

Question: 2

Solution:

Given Points :

A = (3, 2, -5)

B = (-1, 4, -3)

C = (-3, 8, -5)

D = (-3, 2, 1)

To Prove : Points A, B, C & D are coplanar.

To Find : Equation of plane passing through points A, B, C & D.

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Equation of line

If A and B are two points having position vectors $\bar{a}$ & $\bar{b}$ then equation of line passing through two points is given by,

$\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} - a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Coplanarity of two lines :

If two lines $\overline{r_1} = \bar{a} + \lambda\bar{b}$ & $\overline{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$\bar{a}.(\bar{b} \times \bar{d}) = \bar{c}.(\bar{b} \times \bar{d})$

6) Equation of plane :

If two lines $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (3, 2, -5)

B = (-1, 4, -3)

C = (-3, 8, -5)

D = (-3, 2, 1)

Position vectors are given by,

$\bar{a} = 3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$

$\bar{b} = -1\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$

$\bar{c} = -3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}$

$\bar{d} = -3\hat{\imath} + 2\hat{\jmath} + \hat{k}$

Equation of line passing through points A & B is

$\overline{r_1} = \bar{a} + \lambda(\bar{b} - \bar{a})$

$\bar{b} - \bar{a} = (-1-3)\hat{\imath} + (4-2)\hat{\jmath} + (-3+5)\hat{k}$

$= -4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

$\therefore \overline{r_1} = (3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}) + \lambda(-4\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

Let, $\overline{r_1} = \overline{a_1} + \lambda b_1$

Where,

$\overline{a_1} = 3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ & $b_1 = -4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

And the equation of the line passing through points C & D is

$\overline{r_2} - \bar{c} + \mu(\bar{d} - \bar{c})$

$\bar{d} - \bar{c} = (-3+3)\hat{\imath} + (2-8)\hat{\jmath} + (1+5)\hat{k}$

$= -6\hat{\jmath} + 6\hat{k}$

$\therefore \overline{r_1} - (-3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}) + \lambda(-6\hat{\jmath} + 6\hat{k})$

Let, $\overline{r_2} = \overline{a_2} + \lambda b_2$

Where,

$\overline{a_2} = -3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}$ & $b_2 = -6\hat{\jmath} + 6\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4 & 2 & 2 \\ 0 & -6 & 6 \end{vmatrix}$$

$= \hat{\imath}(12 + 12) - \hat{\jmath}(-24 - 0) + \hat{k}(24 + 0)$

$\therefore (\overline{b_1} \times \overline{b_2}) = 24\hat{\imath} + 24\hat{\jmath} + 24\hat{k}$

Therefore,

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = (3 \times 24) + (2 \times 24) + ((-5) \times 24)$

= 72 + 48 – 120

= 0

$\therefore \overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 0$ ……… eq(1)

And

$\overline{a_2}.(\overline{b_1} \times \overline{b_2}) = ((-3) \times 24) + (8 \times 24) + ((-5) \times 24)$

= - 72 + 192 – 120

= 0

$\therefore \overline{a_2}.(\overline{b_1} \times \overline{b_2}) = 0$ ……… eq(2)

From eq(1) and eq(2)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = \overline{a_2}.(\overline{b_1} \times \overline{b_2})$

Hence lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar

Therefore, points A, B, C & D are also coplanar.

As lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar therefore equation of the plane passing through two lines containing four given points is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Now,

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = (x \times 24) + (y \times 24) + (z \times 24)$

= 24x + 24y + 24z

From eq(1)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 0$

Therefore, equation of required plane is

24x + 24y + 24z = 0

x + y + z = 0

Question: 3

Solution:

Given Points :

A = (0, -1, 0)

B = (2, 1, -1)

C = (1, 1, 1)

D = (3, 3, 0)

To Prove : Points A, B, C & D are coplanar.

To Find : Equation of plane passing through points A, B, C & D.

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Equation of line

If A and B are two points having position vectors $\bar{a}$ & $\bar{b}$ then equation of line passing through two points is given by,

$\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Coplanarity of two lines :

If two lines $\overline{r_1} = \bar{a} + \lambda\bar{b}$ & $\overline{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$\bar{a}.(\bar{b} \times \bar{d}) = \bar{c}.(\bar{b} \times \bar{d})$

6) Equation of plane :

If two lines $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (0, -1, 0)

B = (2, 1, -1)

C = (1, 1, 1)

D = (3, 3, 0)

Position vectors are given by,

$\bar{a} = -\hat{\jmath}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\bar{d} = 3\hat{\imath} + 3\hat{\jmath}$

Equation of line passing through points A & D is

$\overline{r_1} = \bar{a} + \lambda(\bar{d} - \bar{a})$

$\bar{d} - \bar{a} = (3 - 0)\hat{\imath} + (3 + 1)\hat{\jmath} + (0 - 0)\hat{k}$

$= 3\hat{\imath} + 4\hat{\jmath}$

$\therefore \overline{r_1} = (-\hat{\jmath}) + \lambda(3\hat{\imath} + 4\hat{\jmath})$

Let, $\overline{r_1} = \overline{a_1} + \lambda b_1$

Where,

$\overline{a_1} = -\hat{\jmath}$ & $b_1 = 3\hat{\imath} + 4\hat{\jmath}$

And equation of line passing through points B & C is

$\overline{r_2} = \bar{b} + \mu(\bar{c} - \bar{b})$

$\bar{c} - \bar{b} = (1 - 2)\hat{\imath} + (1 - 1)\hat{\jmath} + (1 + 1)\hat{k}$

$= -\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$

$\therefore \overline{r_1} = (2\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(-\hat{\imath} + 2\hat{k})$

Let, $\overline{r_2} = \overline{a_2} + \lambda b_2$

Where,

$\overline{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ & $b_2 = -\hat{\imath} + 2\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -1 & 0 & 2 \end{vmatrix}$$

$= \hat{i}(8-0) - \hat{j}(6-0) + \hat{k}(0+4)$

$\therefore (\overline{b_1} \times \overline{b_2}) = 8\hat{i} - 6\hat{j} + 4\hat{k}$

Therefore,

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = (0 \times 8) + ((-1) \times (-6)) + (0 \times 4)$

= 0 + 6 + 0

= 6

$\therefore \overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 6$ ……… eq(1)

And

$\overline{a_2}.(\overline{b_1} \times \overline{b_2}) = (2 \times 8) + (1 \times (-6)) + ((-1) \times 4)$

= 16 – 6 – 4

= 6

$\therefore \overline{a_2}.(\overline{b} \times \overline{d}) = 6$ ……… eq(2)

From eq(1) and eq(2)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = \overline{a_2}.(\overline{b_1} \times \overline{b_2})$

Hence lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar

Therefore, points A, B, C & D are also coplanar.

As lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar therefore equation of the plane passing through two lines containing four given points is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Now,

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = (x \times 8) + (y \times (-6)) + (z \times 4)$

= 8x - 6y + 4z

From eq(1)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 6$

Therefore, equation of required plane is

8x - 6y + 4z = 6

4x - 3y + 2z = 3

Question: 4

Solution:

Given :

X – intercept, a = 2

Y – intercept, b = - 4

Z – intercept, c = 5

To Find : Equation of plane

Formula :

If a, b & c are the intercepts made by plane on X, Y & Z axes respectively, then equation of the plane is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\therefore \frac{x}{2} + \frac{y}{-4} + \frac{z}{5} = 1$$

Multiplying above equation throughout by 40,

$$\therefore \frac{40x}{2} + \frac{40y}{-4} + \frac{40z}{5} = 40$$

20x – 10y + 8z = 40

10x – 5y + 4z = 20

This the equation of the required plane.

Question: 5

Solution:

Given :

Equation of plane : 4x – 3y + 2z = 12

To Find :

1) Equation of plane in intercept form

2) Intercepts made by the plane with the co-ordinate axes.

Formula :

If $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

is the equation of a plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Given the equation of plane:

4x – 3y + 2z = 12

Dividing the above equation throughout by 12

$\therefore \frac{4x}{12}+\frac{-3y}{12}+\frac{2z}{12}=1$

$\therefore \frac{x}{3}+\frac{y}{-4}+\frac{z}{6}=1$

This is the equation of a plane in intercept form.

Comparing the above equation with

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

We get,

a = 3

b = -4

c = 6

Therefore, intercepts made by plane with co-ordinate axes are

X-intercept = 3

Y-intercept = -4

Z-intercept = 6

Question: 6

Solution:

Equation of the plane making a, b & c intercepts with X, Y & Z axes respectively is

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

But, the plane makes equal intercepts on the co-ordinate axes

Therefore, a = b = c

Therefore the equation of the plane is

$\frac{x}{a}+\frac{y}{a}+\frac{z}{a}=1$

x + y + z = a

As plane passes through the point (2, -3, 7),

Substituting x = 2, y = -3 & z = 7

2 – 3 + 7 = a

Therefore, a = 6

Hence, required equation of plane is

x + y + z = 6

Question: 7

Solution:

Given :

X-intercept = A

Y-intercept = B

Z-intercept = C

Centroid of ΔABC = (1, -2, 3)

To Find : Equation of a plane

Formulae :

1) Centroid Formula :

For ΔABC if co-ordinates of A, B & C are

A = (x_1, x_2, x_3)

B = (y_1, y_2, y_3)

C = (z_1, z_2, z_3)

Then co-ordinates of the centroid of ΔABC are

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

2) Equation of plane :

Equation of the plane making a, b & c intercepts with X, Y & Z axes respectively is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

As the plane makes intercepts at points A, B & C on X, Y & Z axes respectively, let co-ordinates of A, B, C be

A = (a, 0, 0)

B = (0, b, 0)

C = (0, 0, c)

By centroid formula,

The centroid of ΔABC is given by

$$G = \left(\frac{a + 0 + 0}{3}, \frac{0 + b + 0}{3}, \frac{0 + 0 + c}{3}\right)$$

$$G = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

But, Centroid of ΔABC = (1, -2, 3) ...... given

$$\therefore \frac{a}{3} = 1, \frac{b}{3} = -2, \frac{c}{3} = 3$$

Therefore, a = 3, b = - 6, c = 9

Therefore,

X-intercept = a = 3

Y-intercept = b = - 6

Z-intercept = c = 9

Therefore, equation of required plane is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\therefore \frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$$

Question: 8

Solution:

Given :

A = (1, 2, 3)

Direction ratios of perpendicular vector = (2, 3, -4)

To Find : Equation of a plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If a plane is passing through point A, then the equation of a plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For point A = (1, 2, 3), position vector is

$\bar{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Vector perpendicular to the plane with direction ratios (2, 3, -4) is

$\bar{n} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

Now, $a.n = (1 \times 2) + (2 \times 3) + (3 \times (-4))$

= 2 + 6 – 12

= - 4

Equation of the plane passing through point A and perpendicular to vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}) = -4$

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}) = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k})$

= 2x + 3y – 4z

Therefore, equation of the plane is

2x + 3y – 4z = -4

Or

2x + 3y – 4z + 4 = 0

Question: 9

Solution:

Given :

P = (1, 2, -3)

O = (0, 0, 0)

$\bar{n} = \overline{OP}$

To Find : Equation of a plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Vector :

If A and B be two points with position vectors $\bar{a}$ & $\bar{b}$ respectively, where

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\overline{AD} - \bar{b} - \bar{a}$

$= (b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

3) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

4) Equation of plane :

If a plane is passing through point A, then the equation of a plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For points,

P = (1, 2, -3)

O = (0, 0, 0)

Position vectors are

$\bar{p} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{o} = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

Vector

$\overline{OP} = \bar{p} - \bar{o}$

$= (1 - 0)\hat{\imath} + (2 - 0)\hat{\jmath} + (3 - 0)\hat{k}$

$\therefore \overline{OP} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Now,

$\bar{p}.\overline{OP} = (1 \times 1) + (2 \times 2) + (3 \times 3)$

= 1 + 4 + 9

= 14

And

$\bar{r}.OP = (x \times 1) + (y \times 2) + (z \times 3)$

= x + 2y + 3z

Equation of the plane passing through point A and perpendicular to the vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

But,$\bar{n} = OP$

Therefore, the equation of the plane is

$\bar{r}.\overline{OP} = \bar{p}.\overline{OP}$

x + 2y + 3z = 14

x + 2y + 3z – 14 = 0

Exercise : 28B

Question: 1

Solution:

Given :

d = 5

$\hat{n} = \hat{k}$

To Find : Equation of a plane

Formulae :

1) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

2) Equation of plane :

Equation of plane which is at a distance of 5 units from the origin and having $\hat{n}$ as a unit vector normal to it is

$\bar{r}.\hat{n} = d$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given d = 5 and $\hat{n} = \hat{k}$,

Equation of plane is

$\bar{r}.\hat{n} = d$

$\therefore \bar{r}.\hat{k} = 5$

This is a vector equation of the plane

Now,

$\bar{r}.\hat{k} = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).\hat{k}$

$= (x \times 0) + (y \times 0) + (z \times 1)$

= z

$\therefore \bar{r}.\hat{k} = z$

Therefore, the equation of the plane is

This is - the Cartesian z = 5 equation of the plane.

Question: 2

Solution:

Given :

d = 7

$\bar{n} = 3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}$

To Find : Equation of plane

Formulae :

1) Unit Vector :

Let $\bar{a} - a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$

Where, $|\bar{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

Equation of plane which is at a distance of 5 units from the origin and having $\hat{n}$ as a unit vector normal to it is

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$\bar{r}.\hat{n} = d$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given normal vector

$\bar{n} = 3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}$

Unit vector normal to the plane is

$$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$$

$$\therefore \hat{n} = \frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{3^2 + 5^2 + (-6)^2}}$$

$$\therefore \hat{n} = \frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{9 + 25 + 36}}$$

$$\therefore \hat{n} = \frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{70}}$$

Equation of the plane is

$r.\bar{n} = d$

$$\therefore \bar{r}.\left(\frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{70}}\right) - 7$$

$\therefore \bar{r}.\left(3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}\right) = 7\sqrt{70}$

This is a vector equation of the plane.

Now,

$\bar{r}.\left(3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}\right) = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}\right)$

= (x × 3) + (y × 5) + (z × (-6))

= 3x + 5y – 6z

Therefore equation of the plane is

$3x + 5y - 6z = 7\sqrt{70}$

This is the Cartesian equation of the plane.

Question: 3

Solution:

Given :

$$d = \frac{6}{\sqrt{29}}$$

$\bar{n} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

To Find : Equation of a plane

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then the unit vector of $\bar{a}$ is

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

Equation of plane which is at a distance of 5 units from the origin and having $\hat{n}$ as a unit vector normal to it is

$\bar{r}.\hat{n} = d$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given normal vector

$\bar{n} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

Unit vector normal to the plane is

$$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$$

$$\therefore \hat{n} = \frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{2^2 + (-3)^2 + 4^2}}$$

$$\therefore \hat{n} = \frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{4 + 9 + 16}}$$

$$\therefore \hat{n} = \frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{29}}$$

Equation of the plane is

$r.\bar{n} = d$

$$\therefore \bar{r}.\left(\frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{29}}\right) - \frac{6}{\sqrt{29}}$$

$$\therefore \bar{r}.\left(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right) = 6$$

This is a vector equation of the plane.

Now,

$\bar{r}.\left(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right) = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right)$

= (x × 2) + (y × (-3)) + (z × 4)

= 2x - 3y + 4z

Therefore equation of the plane is

2x - 3y + 4z = 6

This is the Cartesian equation of the plane.

Question: 4

Solution:

Given :

d = 6

direction ratios of $\bar{n}$ are (2, -1, -2)

$\therefore \bar{n} = 2\hat{\imath} - \hat{\jmath} - 2\hat{k}$

To Find : Equation of plane

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then the unit vector of $\bar{a}$ is

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} - (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

Equation of plane which is at a distance of 5 units from the origin and having $\hat{n}$ as a unit vector normal to it is

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$\bar{r}.\hat{n} = d$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given normal vector

$\bar{n} = 2\hat{\imath} - \hat{\jmath} - 2\hat{k}$

Unit vector normal to the plane is

$$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$$

$$\therefore \hat{n} = \frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{\sqrt{2^2 + (-1)^2 + (-2)^2}}$$

$$\therefore \hat{n} = \frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{\sqrt{4+1+4}}$$

$$\therefore \hat{n} = \frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{\sqrt{9}}$$

$$\therefore \hat{n} = \frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{3}$$

Equation of the plane is

$\bar{r}.\hat{n} = d$

$$\therefore \bar{r}.\left(\frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{3}\right) = 6$$

$\therefore \bar{r}.\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right) = 18$

This is vector equation of the plane.

Now,

$\bar{r}.\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right) = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right)$

$= (x \times 2) + (y \times (-1)) + (z \times (-2))$

$= 2x - y - 2z$

Therefore equation of the plane is

$2x - y - 2z = 18$

This is Cartesian equation of the plane.

Question: 5

Solution:

Given :

A = (1, 4, 6)

$\bar{n} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

To Find : Equation of plane.

Formulae :

1) Position Vector :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

Equation of plane passing through point A and having $\bar{n}$ as a unit vector normal to it is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Position vector of point A = (1, 4, 6) is

$\bar{a} = \hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

Now,

$\bar{a}.\bar{n} = (\hat{\imath} + 4\hat{\jmath} + 6\hat{k}).(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

= (1×1) + (4×(-2)) + (6×1)

= 1 – 8 + 6

= - 1

Equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = -1$

This is vector equation of the plane.

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Therefore

$\bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

= (x × 1) + (y × (-2)) + (z × 1)

= x - 2y + z

Therefore equation of the plane is

$x - 2y + z = -1$

This is Cartesian equation of the plane.

Question: 6

Solution:

Given :

Equation of plane : $\bar{r}.(3\hat{\imath} - 12\hat{\jmath} - 4\hat{k}) + 39 = 0$

To Find :

i) Length of perpendicular = d

ii) Unit normal vector = $\hat{n}$

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$

Where, $|\bar{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

2) Length of perpendicular :

The length of the perpendicular from the origin to the plane

$\bar{r}.\bar{n} = p$ is given by,

$d = \frac{p}{|\bar{n}|}$

Given the equation of the plane is

$\bar{r}.(3\hat{\imath} \quad 12\hat{\jmath} \quad 4\hat{k})\ \text{ı}\ 39 = 0$

$\therefore \bar{r}.(3\hat{\imath} - 12\hat{\jmath} - 4\hat{k}) = -39$

$\therefore \bar{r}.(-3\hat{\imath} + 12\hat{\jmath} + 4\hat{k}) = 39$

Comparing the above equation with

$\bar{r}.\bar{n} = p$

We get,

$\bar{n} = -3\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$ & p = 39

Therefore,

$|\bar{n}| = \sqrt{(-3)^2 + 12^2 + 4^2}$

$= \sqrt{9 + 144 + 16}$

$= \sqrt{169}$

$= 13$

The length of the perpendicular from the origin to the given plane is

$d = \frac{p}{|\bar{n}|}$

$\therefore d = \frac{39}{13}$

$\therefore d = 3$

Vector normal to the plane is

$\bar{n} = -3\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$

Therefore, the unit vector normal to the plane is

$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$

$\therefore \hat{n} = \frac{-3\hat{\imath} + 12\hat{\jmath} + 4\hat{k}}{13}$

$\therefore \hat{n} = \frac{-3\hat{\imath}}{13} + \frac{12\hat{\jmath}}{13} + \frac{4\hat{k}}{13}$

Question: 7

Solution:

Given :

Vector equation of the plane is

$\bar{r}.(3\hat{\imath} + 5\hat{\jmath} - 9\hat{k}) = 8$

To Find : Cartesian equation of the given plane.

Formulae :

1) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$a.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

Given the equation of the plane is

$\bar{r}.(3\hat{\imath} + 5\hat{\jmath} - 9\hat{k}) = 8$

Here,

$\bar{r} - x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.(3\hat{\imath} + 5\hat{\jmath} - 9\hat{k}) = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(3\hat{\imath} + 5\hat{\jmath} - 9\hat{k})$

= (x × 3) + (y × 5) + (z × (-9))

= 3x + 5y – 9z

Therefore equation of the plane is

3x + 5y – 9z = 8

This is the Cartesian equation of the given plane.

Question: 8

Solution:

Given :

Cartesian equation of the plane is

5x – 7y + 2z + 4 = 0

To Find : Vector equation of the given plane.

Formulae :

1) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

Given the equation of the plane is

5x – 7y + 2z + 4 = 0

⇒ 5x - 7y + 2z = - 4

The term (5x - 7y + 2z) can be written as

$(5x - 7y + 2z) = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(5\hat{\imath} - 7\hat{\jmath} + 2\hat{k})$

But $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore (5x - 7y + 2z) = \bar{r}.(5\hat{\imath} - 7\hat{\jmath} + 2\hat{k})$

Therefore the equation of the plane is

$\bar{r}.(5\hat{\imath} - 7\hat{\jmath} + 2\hat{k}) = -4$

or

$\bar{r}.(-5\hat{\imath} + 7\hat{\jmath} - 2\hat{k}) = 4$

This is Vector equation of the given plane.

Question: 9

Solution:

Given :

Equation of plane : x – 2y + 2z = 6

To Find : unit normal vector = $\hat{n}$

Formula :

Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then the unit vector of $\bar{a}$ is

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

From the given equation of a plane

x – 2y + 2z = 6

direction ratios of vector normal to the plane are (1, -2, 2).

Therefore, the equation of normal vector is

$\bar{n} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

Therefore unit normal vector is given by

$$\hat{n} = \frac{n}{|\bar{n}|}$$

$$\therefore \hat{n} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{\sqrt{1^2 + (-2)^2 + 2^2}}$$

$$\therefore \hat{n} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{\sqrt{1 + 4 + 4}}$$

$$\therefore \hat{n} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{\sqrt{9}}$$

$$\therefore \hat{n} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3}$$

$\therefore \hat{n} = \frac{\hat{\imath}}{3} - \frac{2\hat{\jmath}}{3} + \frac{2\hat{k}}{3}$

Question: 10

Solution:

Given :

Equation of plane : 3x – 6y + 2z = 7

To Find : Direction cosines of the normal, i.e. l, m & n

Formula :

1) Direction cosines :

If a, b & c are direction ratios of the vector, then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

For the given equation of a plane

3x – 6y + 2z = 7

Direction ratios of normal vector are (3, -6, 2)

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{3^2 + (-6)^2 + 2^2}$$

$$= \sqrt{9 + 36 + 4}$$

$$= \sqrt{49}$$

$$= \pm 7$$

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \pm\frac{3}{7}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \mp\frac{6}{7}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \pm\frac{2}{7}$$

$$(l, m, n) = \pm\left(\frac{3}{7}, \frac{-6}{7}, \frac{2}{7}\right)$$

Question: 11

Solution:

(i) 2x + 3y – z = 5

Given :

Equation of plane : 2x + 3y – z = 5

To Find :

Direction cosines of the normal i.e. l, m & n

Distance of the plane from the origin = d

Formulae :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

2) The distance of the plane from the origin :

Distance of the plane from the origin is given by,

$$d = \frac{p}{|\bar{n}|}$$

For the given equation of plane

2x + 3y – z = 5

Direction ratios of normal vector are (2, 3, -1)

Therefore, equation of normal vector is

$$\bar{n} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{2^2 + 3^2 + (-1)^2}$$

$$= \sqrt{4 + 9 + 1}$$

$$= \sqrt{14}$$

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{2}{\sqrt{14}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{3}{\sqrt{14}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{-1}{\sqrt{14}}$$

$$(l, m, n) = \left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$$

Now, the distance of the plane from the origin is

$$d = \frac{p}{|\bar{n}|}$$

$$\therefore d = \frac{5}{\sqrt{14}}$$

(ii) Given :

Equation of plane : z = 3

To Find :

Direction cosines of the normal, i.e. l, m & n

The distance of the plane from the origin = d

Formulae :

3) Direction cosines :

If a, b & c are direction ratios of the vector, then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

4) The distance of the plane from the origin :

Distance of the plane from the origin is given by,

$$d = \frac{p}{|\bar{n}|}$$

For the given equation of a plane

z = 3

Direction ratios of normal vector are (0, 0, 1)

Therefore, equation of normal vector is

$$\bar{n} = \hat{k}$$

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{0^2 + 0^2 + 1^2}$$

$$= \sqrt{1}$$

$$= 1$$

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{0}{1} = 0$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{0}{1} = 0$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{1}{1} = 1$$

$(l, m, n) = (0,0,1)$

Now, the distance of the plane from the origin is

$$d = \frac{p}{|\bar{n}|}$$

$$\therefore d = \frac{3}{1}$$

$$\therefore d = 3$$

(iii) Given :

Equation of plane : 3y + 5 = 0

To Find :

Direction cosines of the normal, i.e. l, m & n

The distance of the plane from the origin = d

Formulae :

1) Direction cosines :

If a, b & c are direction ratios of the vector, then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

2) Distance of the plane from the origin :

Distance of the plane from the origin is given by,

$$d = \frac{p}{|\bar{n}|}$$

For the given equation of a plane

3y + 5 = 0

⇒-3y = 5

Direction ratios of normal vector are (0, -3, 0)

Therefore, equation of normal vector is

$$\bar{n} = -3j$$

$$\sqrt{a^2+b^2+c^2} = \sqrt{0^2+(-3)^2+0^2}$$

$$= \sqrt{9}$$

$$= 3$$

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{0}{3} = 0$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{-3}{3} = -1$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{0}{3} = 0$$

$(l, m, n) = (0,-1,0)$

Now, distance of the plane from the origin is

$$d = \frac{p}{|\bar{n}|}$$

$$\therefore d = \frac{5}{3}$$

Question: 12

Solution:

Given :

A = (2, -1, 1)

Direction ratios of perpendicular vector = (4, 2, -3)

To Find : Equation of a plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If a plane is passing through point A, then the equation of a plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For point A = (2, -1, 1), position vector is

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

Vector perpendicular to the plane with direction ratios (4, 2, -3) is

$\bar{n} = 4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

Now, $\bar{a}.\bar{n} = (2 \times 4) + ((-1) \times 2) + (1 \times (-3))$

= 8 – 2 – 3

= 3

Equation of the plane passing through point A and perpendicular to vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.(4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) = 3$

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.(4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(4\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$

= 4x + 2y – 3z

Therefore, the equation of the plane is

4x + 2y – 3z = 3

Or

4x + 2y – 3z - 3 = 0

Question: 13

Solution:

(i) 2x + 3y + 4z - 12 = 0

Given :

Equation of plane : 2x + 3y + 4z + 12 = 0

To Find :

coordinates of the foot of the perpendicular

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

From the given equation of the plane

2x + 3y + 4z − 12 = 0

⇒ 2x + 3y + 4z = 12

Direction ratios of the vector normal to the plane are (2, 3, 4)

Let, P = (x, y, z) be the foot of perpendicular perpendicular drawn from origin to the plane.

Therefore perpendicular drawn is OP.

$$\therefore \overline{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

Let direction ratios of $\overline{OP}$ are (x, y, z)

As normal vector and $\overline{OP}$ are parallel

$$\therefore \frac{x}{2} = \frac{y}{3} = \frac{z}{4} = k(say)$$

⇒x = 2k, y = 3k, z = 4k

As point P lies on the plane, we can write

2(2k) + 3(3k) + 4(4k) = 12

⇒ 4k + 9k + 16k = 12

⇒ 29k = 12

$$\therefore k = \frac{12}{29}$$

$$\therefore x = 2k = \frac{24}{29},$$

$$y = 3k = \frac{36}{29}$$

$$z = 4k = \frac{48}{29}$$

Therefore co-ordinates of the foot of perpendicular are

$$P(x, y, z) = \left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29}\right)$$

$$P = \left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29}\right)$$

(ii) Given :

Equation of plane : 5y + 8 = 0

To Find :

coordinates of the foot of the perpendicular

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

From the given equation of the plane

5y + 8 = 0

⇒ 5y = - 8

Direction ratios of the vector normal to the plane are (0, 5, 0)

Let, P = (x, y, z) be the foot of perpendicular perpendicular drawn from origin to the plane.

Therefore perpendicular drawn is $\overline{OP}$.

$$\therefore \overline{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

Let direction ratios of $\overline{OP}$ are (x, y, z)

As normal vector and $\overline{OP}$ are parallel

$$\therefore \frac{0}{x} = \frac{5}{y} = \frac{0}{z} = \frac{1}{k}(say)$$

⇒x = 0, y = 5k, z = 0

As point P lies on the plane, we can write

5(5k) = -8

⇒ 25k = -8

$\therefore k = \frac{-8}{25}$

$\therefore x = 0$

$y = 5k = 5 \times \frac{-8}{25} = \frac{-8}{5}$

$z = 0$

Therefore co-ordinates of the foot of perpendicular are

P(x, y, z) = $\left(0, \frac{-8}{5}, 0\right)$

P = $\left(0, \frac{-8}{5}, 0\right)$

Question: 14

Solution:

Given :

Equation of plane : 3x – y – z = 7

A = (2, 3, 7)

To Find :

i) Length of perpendicular = d

ii) coordinates of the foot of the perpendicular

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$\hat{a} - \frac{\bar{a}}{|\bar{a}|}$

Where, $|\bar{a}| - \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

2) Length of perpendicular :

The length of the perpendicular from point A with position vector $\bar{a}$ to the plane is given by,

$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

Given equation of the plane is

3x – y – z = 7 ....eq(1)

Therefore direction ratios of normal vector of the plane are

(3, -1, -1)

Therefore normal vector of the plane is

$\bar{n} = 3\hat{\imath} - \hat{\jmath} - \hat{k}$

$\therefore |\bar{n}| = \sqrt{3^2 + (-1)^2 + (-1)^2}$

$= \sqrt{9 + 1 + 1}$

$= \sqrt{11}$

From eq(1), p = 7

Given point A = (2, 3, 7)

Position vector of A is

$\bar{a} = 2\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$

Now,

$\bar{a}.\bar{n} = (2\hat{\imath} + 3\hat{\jmath} + 7\hat{k}).(3\hat{\imath} - \hat{\jmath} - \hat{k})$

= (2×3) + (3×(-1)) + (7×(-1))

= 6 – 3 – 7

= -4

Length of the perpendicular from point A to the plane is

$$d = \frac{|\bar{a}.\bar{n} - p|}{|n|}$$

$$\therefore d = \frac{|-4 - 7|}{\sqrt{11}}$$

$$\therefore d = \frac{11}{\sqrt{11}}$$

$$\therefore d = \sqrt{11}$$

Let P be the foot of perpendicular drawn from point A to the given plane,

Let P = (x, y, z)

$\overline{AP} = (x - 2)\hat{\imath} + (y - 3)\hat{\jmath} + (z - 7)\hat{k}$

As normal vector and $\overline{AP}$ are parallel

$$\therefore \frac{x - 2}{3} = \frac{y - 3}{-1} = \frac{z - 7}{-1} = k(say)$$

⇒x = 3k+2, y = - k+3, z = -k+7

As point P lies on the plane, we can write

3(3k+2) - (- k+3) - (-k+7) = 7

⇒ 9k + 6 + k – 3 + k – 7 = 7

⇒ 11k = 11

$\therefore k = 1$

$\therefore x = 3k + 2 = 5$

$y = -k + 3 = 2$

$z = -k + 7 = 6$

Therefore co-ordinates of the foot of perpendicular are

P(x, y, z) = (5, 2, 6)

P = (5, 2, 6)

Question: 15

Solution:

Given :

Equation of plane : $\bar{r}.(2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) + 5 = 0$

A = (1, 1, 2)

To Find :

i) Length of perpendicular = d

ii) coordinates of the foot of the perpendicular

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

2) Length of perpendicular :

The length of the perpendicular from point A with position vector $\bar{a}$ to the plane is given by,

$$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$$

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

Given equation of the plane is

$\bar{r}.\left(2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}\right) + 5 = 0$ ....eq(1)

$\therefore \bar{r}.\left(2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}\right) = -5$

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Therefore equation of plane is

2x – 2y + 4z = -5 .... eq(2)

From eq(1) normal vector of the plane is

$\bar{n} = 2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

$\therefore |\bar{n}| = \sqrt{2^2 + (-2)^2 + 4^2}$

$= \sqrt{4 + 4 + 16}$

$= \sqrt{24}$

From eq(1), p = -5

Given point A = (1, 1, 2)

Position vector of A is

$\bar{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$

Now,

$\bar{a}.\bar{n} = \left(\hat{\imath} + \hat{\jmath} + 2\hat{k}\right).\left(2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}\right)$

= (1×2) + (1×(-2)) + (2×4)

= 2 – 2 + 8

= 8

Length of the perpendicular from point A to the plane is

$$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$$

$$\therefore d = \frac{|8 + 5|}{\sqrt{24}}$$

$$\therefore d = \frac{13}{\sqrt{24}}$$

$$\therefore d = \frac{13\sqrt{6}}{\sqrt{24}.\sqrt{6}}$$

$$\therefore d = \frac{13\sqrt{6}}{\sqrt{144}}$$

$$\therefore d = \frac{13\sqrt{6}}{12}$$

Let P be the foot of perpendicular drawn from point A to the given plane,

Let P = (x, y, z)

$\overline{AP} = (x - 1)\hat{\imath} + (y - 1)\hat{\jmath} + (z - 2)\hat{k}$

As normal vector and $\overline{AP}$ are parallel

$$\therefore \frac{x - 1}{2} = \frac{y - 1}{-2} = \frac{z - 2}{4} = k(say)$$

⇒x = 2k+1, y = -2k+1, z = 4k+2

As point P lies on the plane, we can write

2(2k+1) – 2(-2k+1) + 4(4k+2) = -5

⇒ 4k + 2 + 4k – 2 + 16k + 8 = -5

⇒ 24k = -13

$\therefore k = \frac{-13}{24}$

$\therefore x = 2\left(\frac{13}{24}\right) + 1 = \frac{1}{12},$

$y = -2\left(\frac{-13}{24}\right) + 1 = \frac{25}{12}$

$z - 4\left(\frac{-13}{24}\right) + 2 - \frac{-1}{6}$

Therefore co-ordinates of the foot of perpendicular are

$P(x, y, z) = \left(\frac{-1}{12}, \frac{25}{12}, \frac{-1}{6}\right)$

$P \equiv \left(\frac{-1}{12}, \frac{25}{12}, \frac{-1}{6}\right)$

Question: 16

From the point P(

Solution:

Given :

Equation of plane : 2x + y – 2z + 3 = 0

P = (1, 2, 4)

To Find :

i) Equation of perpendicular

ii) Length of perpendicular = d

iii) coordinates of the foot of the perpendicular

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$

Where, $|\bar{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

2) Length of perpendicular :

The length of the perpendicular from point A with position vector $\bar{a}$ to the plane is given by,

$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

Given equation of the plane is

2x + y – 2z + 3 = 0

⇒2x + y – 2z = -3 ………… eq(1)

From eq(1) direction ratios of normal vector of the plane are

(2, 1, -2)

Therefore, equation of normal vector is

$\bar{n} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\therefore |\bar{n}| = \sqrt{2^2 + 1^2 + (-2)^2}$

$= \sqrt{4 + 1 + 4}$

$= \sqrt{9}$

= 3

From eq(1), p = -3

Given point P = (1, 2, 4)

Position vector of A is

$\bar{p} = \hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

Here, $\bar{a} = \bar{p}$

Now,

$\therefore \bar{a}.\bar{n} = (\hat{\imath} + 2\hat{\jmath} + 4\hat{k}).(2\hat{\imath} + \hat{\jmath} - 2\hat{k})$

= (1×2) + (2×1) + (4×(-2))

= 2 + 2 - 8

= -4

Length of the perpendicular from point A to the plane is

$$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$$

$$\therefore d = \frac{|-4 + 3|}{3}$$

$$\therefore d = \frac{1}{3}$$

Let Q be the foot of perpendicular drawn from point P to the given plane,

Let Q = (x, y, z)

$\overline{PQ} = (x - 1)\hat{\imath} + (y - 2)\hat{\jmath} + (z - 4)\hat{k}$

As normal vector and $\overline{PQ}$ are parallel, we can write,

$$\therefore \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2}$$

This is the equation of perpendicular.

$$\therefore \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2} = k(say)$$

⇒x = 2k+1, y = k+2, z = -2k+4

As point Q lies on the plane, we can write

2(2k+1) + (k+2) - 2(-2k+4) = -3

⇒ 4k + 2 + k + 2 + 4k - 8 = -3

⇒ 9k = 1

$$\therefore k = \frac{1}{9}$$

$$\therefore x = 2\left(\frac{1}{9}\right) + 1 = \frac{11}{9},$$

$$y = \frac{1}{9} + 2 = \frac{19}{9}$$

$$z = -2\left(\frac{1}{9}\right) + 4 = \frac{34}{9}$$

Therefore co-ordinates of the foot of perpendicular are

Q(x, y, z) = $\left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$

Q ≡ $\left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$

Question: 17

Solution:

Given :

Equation of plane : 2x - y + z + 1 = 0

P = (3, 2, 1)

To Find :

i) Length of perpendicular = d

ii) coordinates of the foot of the perpendicular

iii) Image of point P in the plane.

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

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Then unit vector of $\bar{a}$ is

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

2) Length of perpendicular :

The length of the perpendicular from point A with position vector $\bar{a}$ to the plane is given by,

$$d = \frac{|\bar{a}.\bar{n} - p|}{|n|}$$

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

Given equation of the plane is

2x - y + z + 1 = 0

⇒2x - y + z = -1 ....eq(1)

From eq(1) direction ratios of normal vector of the plane are

(2, -1, 1)

Therefore, equation of normal vector is

$$\bar{n} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$\therefore |\bar{n}| = \sqrt{2^2 + (-1)^2 + 1^2}$$

$$= \sqrt{4 + 1 + 1}$$

$$= \sqrt{6}$$

From eq(1), p = -1

Given point P = (3, 2, 1)

Position vector of A is

$$\bar{p} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$$

Here, $\bar{a} = \bar{p}$

Now,

$$\therefore \bar{a}.\bar{n} = (3\hat{\imath} + 2\hat{\jmath} + \hat{k}).(2\hat{\imath} - \hat{\jmath} + \hat{k})$$

= (3×2) + (2×(-1)) + (1×1)

= 6 - 2 + 1

= 5

Length of the perpendicular from point A to the plane is

$$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$$

$$\therefore d = \frac{|5 + 1|}{\sqrt{6}}$$

$$\therefore d = \frac{6}{\sqrt{6}}$$

$$\therefore d = \sqrt{6}$$

Let Q be the foot of perpendicular drawn from point P to the given plane,

Let Q = (x, y, z)

$$\overline{PQ} = (x - 3)\hat{\imath} + (y - 2)\hat{\jmath} + (z - 1)\hat{k}$$

As normal vector and $\overline{PA}$ are parallel, we can write,

$$\therefore \frac{x - 3}{2} = \frac{y - 2}{-1} = \frac{z - 1}{1} = k(say)$$

⇒x = 2k+3, y = -k+2, z = k+1

As point A lies on the plane, we can write

2(2k+3) - (-k+2) + (k+1) = -1

⇒ 4k + 6 + k – 2 + k + 1 = -1

⇒ 6k = -6

$\therefore k = -1$

$\therefore x = 2(-1) + 3 = 1$

$y = -(-1) + 2 = 3$

$z = (-1) + 1 = 0$

Therefore, co-ordinates of the foot of perpendicular are

Q(x, y, z) = (1,3,0)

Q ≡ (1,3,0)

Let, R(a, b, c) be image of point P in the given plane.

Therefore, the power of points P and R in the given plane will be equal and opposite.

2a – b + c + 1 = - (2(3) – 2 + 1 + 1)

⇒2a – b + c + 1 = - 6

⇒2a – b + c = - 7 ....eq(2)

Now, $\overline{PR} = (a-3)\hat{\imath} + (b-2)\hat{\jmath} + (c-1)\hat{k}$

As $\overline{PR}$ & $\bar{n}$ are parallel

$$\therefore \frac{a-3}{2} = \frac{b-2}{-1} = \frac{c-1}{1} = k(say)$$

⇒a = 2k+3, b = -k+2, c = k+1

substituting a, b, c in eq(2)

2(2k+3) - (-k+2) + (k+1) = -7

⇒ 4k + 6 + k – 2 + k + 1 = -7

⇒ 6k = -12

$\therefore k = -2$

$\therefore a = 2(-2) + 3 = -1$

$b = -(-2) + 2 = 4$

$c = (-2) + 1 = -1$

Therefore, co-ordinates of the image of P are

R(a, b, c) = (−1,4,−1)

R ≡ (−1,4,−1)

Question: 18

Solution:

Given :

Equation of plane : 2x - y + z + 3 = 0

P = (1, 3, 4)

To Find : Image of point P in the plane.

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

Given equation of the plane is

2x - y + z + 3 = 0

⇒2x - y + z = -3 ....eq(1)

From eq(1) direction ratios of normal vector of the plane are

(2, -1, 1)

Therefore, equation of normal vector is

$\bar{n} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

Given point is P = (1, 3, 4)

Let, R(a, b, c) be image of point P in the given plane.

Therefore, the power of points P and R in the given plane will be equal and opposite.

⇒2a – b + c + 3 = - (2(1) – 3 + 4 + 3)

⇒2a – b + c + 3 = - 6

⇒2a – b + c = - 9 ....eq(2)

Now, $\overline{PR} = (a-1)\hat{\imath} + (b-3)\hat{\jmath} + (c-4)\hat{k}$

As $\overline{PR}$ & $\bar{n}$ are parallel

$\therefore \frac{a-1}{2} = \frac{b-3}{-1} = \frac{c-4}{1} = k(say)$

⇒a = 2k+1, b = -k+3, c = k+4

substituting a, b, c in eq(2)

2(2k+1) - (-k+3) + (k+4) = -9

⇒ 4k + 2 + k – 3 + k + 4 = -9

⇒ 6k = -12

$\therefore k = -2$

$\therefore a = 2(-2) + 1 = -3,$

$b = -(-2) + 3 = 5$

$c = (-2) + 4 = 2$

Therefore, co-ordinates of the image of P are

R(a, b, c) = $(-3, 5, 2)$

Question: 19

Solution:

Given :

Equation of plane : 2x + 4y - z = 1

Equation of line :

$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$

To Find : Point of intersection of line and plane.

Let P(a, b, c) be point of intersection of plane and line.

As point P lies on the line, we can write,

$\frac{a-1}{2} = \frac{b-2}{-3} = \frac{c+3}{4} = k(say)$

⇒a = 2k+1, b = -3k+2, c = 4k-3 ....(1)

Also point P lies on the plane

2a + 4b – c = 1

⇒2(2k+1) + 4(-3k+2) – (4k-3) = 1 .....from (1)

⇒4k + 2 – 12k + 8 – 4k + 3 = 1

⇒-12k = -12

⇒k = 1

$\therefore a = 2(1) + 1 = 3,$

$b = -3(1) + 2 = -1$

$c = 4(1) - 3 = 1$

Therefore, co-ordinates of point of intersection of given line and plane are

P ≡ $(3, -1, 1)$

Question: 20

Solution:

Given :

Equation of plane : 2x + y + z = 7

Points :

A = (3, -4, -5)

B = (2, -3, 1)

To Find : Point of intersection of line and plane.

Formula :

Equation of line passing through A = (x_1, y_1, z_1) &

B = (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$$

Equation of line passing through A = (3, -4, -5) & B = (2, -3, 1) is

$$\frac{x-3}{3-2}=\frac{y+4}{-4+3}=\frac{z+5}{-5-1}$$

$$\therefore \frac{x-3}{1}=\frac{y+4}{-1}=\frac{z+5}{-6}$$

Let P(a, b, c) be point of intersection of plane and line.

As point P lies on the line, we can write,

$$\frac{a-3}{1}=\frac{b+4}{-1}=\frac{c+5}{-6}=k(say)$$

⇒a = k+3, b = -k - 4, c = -6k-5 ....(1)

Also point P lies on the plane

2a + b + c = 7

⇒2(k+3) + (-k-4) + (-6k-5) = 7 .....from (1)

⇒2k + 6 – k - 4 – 6k - 5 = 7

⇒-5k = 10

⇒k = -2

$\therefore a=(-2)+3=1,$

$b=-(-2)-4=-2$

$c=-6(-2)-5=7$

Therefore, co-ordinates of point of intersection of given line and plane are

P ≡ (1,−2,7)

Question: 21

Solution:

Given :

Equation of plane : 3x + 2y + 2z + 5 = 0

Equation of line :

$$\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$$

Point : P = (2, 3, 4)

To Find : Distance of point P from the given plane parallel to the given line.

Formula :

1) Equation of line :

Equation of line passing through A = (x_1, y_1, z_1) & having direction ratios (a, b, c) is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

2) Distance formula :

The distance between two points A = (a_1, a_2, a_3) & B = (b_1, b_2, b_3) is

$$d=\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$$

For the given line,

$$\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$$

Direction ratios are (a, b, c) = (3, 6, 2)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (2, 3, 4) and with direction ratios (3, 6, 2) is

$$\frac{x-2}{3}=\frac{y-3}{6}=\frac{z-4}{2}$$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$$\frac{u-2}{3}=\frac{v-3}{6}=\frac{w-4}{2}=k(say)$$

⇒u = 3k+2, v = 6k+3, w = 2k+4 ....(1)

Also point Q lies on the plane

3u + 2v + 2w = -5

⇒3(3k+2) + 2(6k+3) + 2(2k+4) = -5 .....from (1)

⇒9k + 6 + 12k + 6 + 4k + 8 = -5

⇒25k = -25

⇒k = -1

$\therefore u=3(-1)+2=-1,$

$v=6(-1)+3=-3$

$w=2(-1)+4=2$

Therefore, co-ordinates of point Q are

Q = $(-1,-3,2)$

Now distance between points P and Q by distance formula is

$d=\sqrt{(2+1)^2+(3+3)^2+(4-2)^2}$

$=\sqrt{(3)^2+(6)^2+(2)^2}$

$=\sqrt{9+36+4}$

$=\sqrt{49}$

= 7

Therefore distance of point P from the given plane measured parallel to the given line is

d = 7 units

Question: 22

Solution:

Given :

Equation of plane : x + 2y - z = 1

Equation of line :

$$\frac{x+1}{3}=\frac{y+1}{2}=\frac{z}{3}$$

Point : P = (0, -3, 2)

To Find : Distance of point P from the given plane parallel to the given line.

Formula :

1) Equation of line :

Equation of line passing through A = (x_1, y_1, z_1) & having direction ratios (a, b, c) is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

2) Distance formula :

The distance between two points A = (a_1, a_2, a_3) & B = (b_1, b_2, b_3) is

$$d=\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$$

For the given line,

$$\frac{x+1}{3}=\frac{y+1}{2}=\frac{z}{3}$$

Direction ratios are (a, b, c) = (3, 2, 3)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (0, -3, 2) and with direction ratios (3, 2, 3) is

$$\frac{x-0}{3}=\frac{y+3}{2}=\frac{z-2}{3}$$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$$\frac{u}{3}=\frac{v+3}{2}=\frac{w-2}{3}=k(say)$$

⇒u = 3k, v = 2k-3, w = 3k+2 ....(1)

Also point Q lies on the plane

u + 2v - w = 1

⇒(3k) + 2(2k-3) - (3k+2) = 1 .....from (1)

⇒3k + 4k - 6 - 3k - 2 = 1

⇒4k = 9

$\Rightarrow k=\frac{9}{4}$

$\therefore u=3\left(\frac{9}{4}\right)=\frac{27}{4},$

$$v=2\left(\frac{9}{4}\right)-3=\frac{6}{4}$$

$$w-3\left(\frac{9}{4}\right)+2-\frac{35}{4}$$

Therefore, co-ordinates of point Q are

$Q\equiv\left(\frac{27}{4},\frac{6}{4},\frac{35}{4}\right)$

Now distance between points P and Q by distance formula is

$$d=\sqrt{\left(0-\frac{27}{4}\right)^2+\left(-3-\frac{6}{4}\right)^2+\left(2-\frac{35}{4}\right)^2}$$

$$=\sqrt{\left(\frac{-27}{4}\right)^2+\left(\frac{-18}{4}\right)^2+\left(\frac{-27}{4}\right)^2}$$

$$=\sqrt{45.5625+20.25+45.5625}$$

$$=\sqrt{111.375}$$

= 10.55

Therefore distance of point P from the given plane measured parallel to the given line is

d = 10.55 units

Question: 23

Solution:

Given :

Equation of plane : x + y - z = 8

Equation of line :

$$\frac{x-1}{3}=\frac{y}{2}=\frac{z+1}{7}$$

Point : P = (4, 6, 2)

To Find : Equation of line.

Formula :

Equation of line passing through A = (x_1, y_1, z_1) &

B = (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$$

let Q (a, b, c) be point of intersection of plane and line.

As point Q lies on the line, we can write,

$\frac{a-1}{3}=\frac{b}{2}=\frac{c+1}{7}=k(say)$

⇒a =3k+1, b= 2k, c= 7k-1

Also point Q lies on the plane,

a + b – c = 8

⇒(3k+1) + (2k) – (7k-1) = 8

⇒3k + 1 + 2k – 7k + 1 = 8

⇒-2k = 6

⇒k = -3

$\therefore a-3(-3)+1--8,$

$b=-2(-3)=-6$

$c=7(-3)-1=-22$

Therefore, co-ordinates of point of intersection of given line and plane are Q = (-8, -6, -22)

Now, equation of line passing through P(4,6,2) and

Q(-8, -6, -22) is

$\frac{x-4}{4+8}=\frac{y-6}{6+6}=\frac{z-2}{2+22}$

$\therefore \frac{x-4}{12}-\frac{y-6}{12}-\frac{z-2}{24}$

$\therefore \frac{x-4}{1}=\frac{y-6}{1}=\frac{z-2}{2}$

This is the equation of required line

Question: 24

Solution:

Given :

Equation of plane : x - y + z = 5

Equation of line :

$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$

Point : P = (-1, -5, -10)

To Prove : Distance of point P from the given plane parallel to the given line is 13 units.

Formula :

1) Equation of line :

Equation of line passing through A = (x_1, y_1, z_1) & having direction ratios (a, b, c) is

$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

2) Distance formula :

The distance between two points A = (a_1, a_2, a_3) & B = (b_1, b_2, b_3) is

$d-\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$

For the given line,

$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$

Direction ratios are (a, b, c) = (3, 4, 12)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (-1, -5, -10) and with direction ratios (3, 4, 12) is

$\frac{x+1}{3}=\frac{y+5}{4}=\frac{z+10}{12}$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$\frac{u+1}{3}=\frac{v+5}{4}=\frac{w+10}{12}=k(say)$

$\Rightarrow$u = 3k-1, v = 4k-5, w = 12k-10 ....(1)

Also point Q lies on the plane

u - v + w = 5

$\Rightarrow$(3k-1) - (4k-5) + (12k-10) = 5 .....from (1)

$\Rightarrow$3k - 1 - 4k + 5 + 12k - 10 = 5

$\Rightarrow$11k = 11

$\Rightarrow k = 1$

$\therefore u = 3(1) - 1 = 2,$

$v = 4(1) - 5 = -1$

$w = 12(1) - 10 = 2$

Therefore, co-ordinates of point Q are

$Q \equiv (2, -1, 2)$

Now distance between points P and Q by distance formula is

$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2}$

$= \sqrt{(-3)^2 + (-4)^2 + (-12)^2}$

$= \sqrt{9 + 16 + 144}$

$= \sqrt{169}$

= 13

Therefore distance of point P from the given plane measured parallel to the given line is

d = 13 units

Hence proved.

Question: 25

Solution:

Given :

Equation of plane : $\bar{r}.(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$

Equation of line :

$\bar{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$

Point : P = (-1, -5, -10)

To Find : Distance of point P from the given plane parallel to the given line.

Formula :

1) Equation of line :

Equation of line passing through A = (x_1, y_1, z_1) & having direction ratios (a, b, c) is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

2) Distance formula :

The distance between two points A = (a_1, a_2, a_3) & B = (b_1, b_2, b_3) is

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

for the given plane,

$\bar{r}.(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$

Here, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$(x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$

$\Rightarrow$x – y + z = 5 ....eq(1)

For the given line,

$\bar{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$

Here, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore (3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})\lambda = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}) - (2\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$\therefore 3\lambda\hat{\imath} + 4\lambda\hat{\jmath} + 2\lambda\hat{k} = (x-2)\hat{\imath} + (y+1)\hat{\jmath} + (z-2)\hat{k}$

Comparing coefficients of $\hat{\imath}, \hat{\jmath}$ & $\hat{k}$

$\Rightarrow 3\lambda = (x-2), 4\lambda = (y+1)$ & $2\lambda = (z-2)$

$\Rightarrow \lambda = \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ ………eq(2)

Direction ratios for above line are (a, b, c) = (3, 4, 2)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (-1, -5, -10) and with direction ratios (3, 4, 2) is

$$\frac{x+1}{3} = \frac{y+5}{4} = \frac{z+10}{2}$$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$$\frac{u+1}{3} = \frac{v+5}{4} = \frac{w+10}{2} = k(say)$$

⇒u = 3k-1, v = 4k-5, w = 2k-10 ………(3)

Also point Q lies on the given plane

Therefore from eq(1), we can write,

u - v + w = 5

⇒(3k-1) - (4k-5) + (2k-10) = 5 …..from (3)

⇒3k - 1 - 4k + 5 + 2k - 10 = 5

⇒k = 11

$\Rightarrow k = 11$

$\therefore u = 3(11) - 1 = 32,$

$v = 4(11) - 5 = 39$

$w = 2(11) - 10 = 12$

Therefore, co-ordinates of point Q are

$Q \equiv (32, 39, 12)$

Now the distance between points P and Q by distance formula is

$d = \sqrt{(-1-32)^2 + (-5-39)^2 + (-10-12)^2}$

$= \sqrt{(-33)^2 + (-44)^2 + (-22)^2}$

$= \sqrt{1089 + 1936 + 484}$

$= \sqrt{3509}$

= 59.24

Therefore distance of point P from the given plane measured parallel to the given line is

d = 59.24 units

Question: 26

Solution:

Given :

Equations of plane are :

4x + 11y + 2z + 3 = 0

3x – 2y + 5z = 8

To Prove : $\overline{n_1}$ & $\overline{n_2}$ are perpendicular.

Formula :

1) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

Note :

Direction ratios of the plane given by

ax + by + cz = d

are (a, b, c).

For plane

4x + 11y + 2z + 3 = 0

direction ratios of normal vector are (4, 11, 2)

therefore, equation of normal vector is

$\overline{n_1} = 4\hat{\imath} + 11\hat{\jmath} + 2\hat{k}$

And for plane

3x – 2y + 5z = 8

direction ratios of the normal vector are (3, -2, 5)

therefore, the equation of normal vector is

$\overline{n_2} = 3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$

Now,

$\overline{n_1}.\overline{n_2} = (4\hat{\imath} + 11\hat{\jmath} + 2\hat{k}).(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k})$

$= (4 \times 3) + (11 \times (-2)) + (2 \times 5)$

= 12 – 22 + 10

= 0

$\therefore \overline{n_1}.\overline{n_2} = 0$

Therefore, normals to the given planes are perpendicular.

Question: 27

Solution:

Given :

Equation of plane : : $\bar{r}.(\hat{\imath} + 5\hat{\jmath} + \hat{k}) = 7$

Equation of a line :

$\bar{r} = (2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + 4\hat{k})$

To Prove : Given line is parallel to the given plane.

Comparing given plane i.e.

$\bar{r}.(\hat{\imath} + 5\hat{\jmath} + \hat{k}) = 7$

with $\bar{r}.\bar{n} = \bar{a}.\bar{n}$, we get,

$\bar{n} = \hat{\imath} + 5\hat{\jmath} + \hat{k}$

This is the vector perpendicular to the given plane.

Now, comparing the given the equation of line i.e.

$\bar{r} = (2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + 4\hat{k})$

with $\bar{r} - \bar{a} + \lambda\bar{b}$, we get,

$\bar{b} = \hat{\imath} - \hat{\jmath} + 4\hat{k}$

Now,

$\bar{n}.\bar{b} = (\hat{\imath} + 5\hat{\jmath} + \hat{k}).(\hat{\imath} - \hat{\jmath} + 4\hat{k})$

$= (1 \times 1) + (5 \times (-1)) + (1 \times 4)$

= 1 – 5 + 4

= 0

$\therefore \bar{n}.\bar{b} = 0$

Therefore, a vector normal to the plane is perpendicular to the vector parallel to the line.

Hence, the given line is parallel to the given plane.

Question: 28

Solution:

Given :

$d = 3\sqrt{3}$

$\alpha = \beta = \gamma$

To Find : Equation of plane

Formulae :

1) Distance of plane from the origin :

If $\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is the vector normal to the plane, then distance of the plane from the origin is

$$d = \frac{p}{|\bar{n}|}$$

Where, $|\bar{n}| = \sqrt{a^2 + b^2 + c^2}$

2) $l^2 + m^2 + n^2 = 1$

Where $l = \cos\alpha\,, m = \cos\beta\,, n = \cos\gamma$

3) Equation of plane :

If $\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is the vector normal to the plane, then equation of the plane is

$$\bar{r}.\bar{n} = p$$

As $\alpha = \beta = \gamma$

$\therefore \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow$ l = m = n

$l^2 + m^2 + n^2 = 1$

$\therefore 3l^2 = 1$

$\therefore l = \frac{1}{\sqrt{3}}$

Therefore equation of normal vector of the plane having direction cosines l, m, n is

$\bar{n} = l\hat{\imath} + m\hat{\jmath} + n\hat{k}$

$\therefore \bar{n} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$

$\therefore |\bar{n}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}$

$= \sqrt{1}$

$= 1$

Now,

distance of the plane from the origin is

$d = \frac{p}{|\bar{n}|}$

$\therefore 3\sqrt{3} = \frac{p}{1}$

$\therefore p = 3\sqrt{3}$

Therefore equation of required plane is

$\bar{r}.\bar{n} = p$

$\therefore (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).\left(\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}\right) = 3\sqrt{3}$

$\therefore \frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 3\sqrt{3}$

$\therefore x + y + z = 3\sqrt{3}.\sqrt{3}$

$\therefore x + y + z - 9$

This is the required equation of the plane.

Question: 29

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Solution:

Given :

$|\bar{n}| = 8$

$\alpha = 45°$

$\beta = 60°$

P = (√2, -1, 1)

To Find : Equation of plane

Formulae :

1) $l^2 + m^2 + n^2 = 1$

Where $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$

2) *Equation of plane :*

If $\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ *is the vector normal to the plane, then equation of the plane is*

$\bar{r}.\bar{n} = p$

As $\alpha = 45°$ & $\beta = 60°$

$\therefore l = \cos\alpha = \cos 45° = \frac{1}{\sqrt{2}}$ *and*

$m = \cos\beta = \cos 60° = \frac{1}{2}$

But, $l^2 + m^2 + n^2 = 1$

$\therefore \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + n^2 = 1$

$\therefore \frac{1}{2} + \frac{1}{4} + n^2 = 1$

$\therefore n^2 = 1 - \frac{3}{4}$

$\therefore n^2 = \frac{1}{4}$

$\therefore n = \frac{1}{2}$

Therefore direction cosines of the normal vector of the plane are (l, m, n)

Hence direction ratios are (kl, km, kn)

Therefore the equation of normal vector is

$\bar{n} = kl\hat{\imath} + km\hat{\jmath} + kn\hat{k}$

$\therefore |\bar{n}| = \sqrt{(kl)^2 + (km)^2 + (kn)^2}$

$\therefore |\bar{n}| = \sqrt{\left(\frac{k}{\sqrt{2}}\right)^2 + \left(\frac{k}{2}\right)^2 + \left(\frac{k}{2}\right)^2}$

$\therefore 8 = \sqrt{\frac{k^2}{2} + \frac{k^2}{4} + \frac{k^2}{4}}$

$\therefore 8 = \sqrt{k^2}$

$\therefore k = 8$

$\bar{n} = \left(\frac{8}{\sqrt{2}}\right)\hat{\imath} + \left(\frac{8}{2}\right)\hat{\jmath} + \left(\frac{8}{2}\right)\hat{k}$

$\therefore \bar{n} = 4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$

Now, equation of the plane is

$\bar{r}.\bar{n} - p$

$\therefore \bar{r}.(4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}) = p$ ………*eq(1)*

But $\bar{r} = (x\hat{\imath} + y\hat{\jmath} + z\hat{k})$

$\therefore (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}) = p$

$\Rightarrow 4\sqrt{2}x + 4y + 4z = p$

As point P (√2, -1, 1) lies on the plane by substituting it in above equation,

$4\sqrt{2}(\sqrt{2}) + 4(-1) + 4(1) = p$

$\Rightarrow 8 - 4 + 4 = p$

$\Rightarrow P = 8$

From eq(1)

$\therefore \bar{r}.\left(4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}\right) = 8$

Dividing throughout by 4

$\therefore \bar{r}.\left(\sqrt{2}\hat{\imath} + \hat{\jmath} + \hat{k}\right) = 2$

This is the equation of required plane.

Question: 30

Find the vector e

Solution:

Given :

$\bar{a} = 2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

Equation of plane : $\bar{r}.\left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right) = -2$

To Find :

Equation of line

Point of intersection

Formula :

Equation of line passing through point A with position vector $\bar{a}$ *and parallel to vector* $\bar{b}$ *is*

$\bar{r} = \bar{a} + \lambda\bar{b}$

Where, $\bar{r} = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right)$

From the given equation of the plane

$\bar{r}.\left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right) = -2$ ....*eq(1)*

The normal vector of the plane is

$\bar{n} = 6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

As the given line is perpendicular to the plane therefore $\bar{n}$ *will be parallel to the line.*

$\therefore \bar{n} = b$

Now, the equation of the line passing through $\bar{a} = \left(2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}\right)$ *and parallel to* $\bar{b} = \left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right)$ *is*

$\bar{r} = \bar{a} + \lambda\bar{b}$

$\therefore \bar{r} = \left(2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}\right) + \lambda\left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right)$

.........*eq(2)*

This is the required equation line.

Substituting $\bar{r} = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right)$ *in eq(1)*

$\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right) = -2$

$\Rightarrow 6x - 3y + 5z = -2$ ....*eq(3)*

Also substituting $\bar{r} = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right)$ *in eq(2)*

$\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right) = \left(2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}\right) + \lambda\left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right)$

$\therefore \left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right)\lambda = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right) - \left(2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}\right)$

$\therefore 6\lambda\hat{\imath} - 3\lambda\hat{\jmath} + 5\lambda\hat{k} = (x-2)\hat{\imath} + (y+3)\hat{\jmath} + (z+5)\hat{k}$

Comparing coefficients of $\hat{\imath}, \hat{\jmath}$ & $\hat{k}$

$\Rightarrow 6\lambda = (x-2), -3\lambda = (y+3)$ & $5\lambda = (z+5)$

$\lambda = \frac{x-2}{6} = \frac{y+3}{-3} = \frac{z+5}{5}$ ....*eq(4)*

Let Q(a, b, c) be the point of intersection of given line and plane

As point Q lies on the given line.

Therefore from eq(4)

$$\frac{a-2}{6}=\frac{b+3}{-3}=\frac{c+5}{5}=k(say)$$

⇒$a = 6k+2,\ b = -3k-3,\ c = 5k-5$

Also point Q lies on the plane.

Therefore from eq(3)

$6a - 3b + 5c = -2$

⇒$6(6k+2) - 3(-3k-3) + 5(5k-5) = -2$

⇒$36k + 12 + 9k + 9 + 25k - 25 = -2$

⇒$70k = 2$

⇒ $k=\frac{1}{35}$

$$\therefore a=6\left(\frac{1}{35}\right)+2=\frac{76}{35}$$

$$b=-3\left(\frac{1}{35}\right)-3=\frac{-108}{35}$$

$$c=5\left(\frac{1}{35}\right)-5=\frac{-170}{35}=\frac{-34}{7}$$

Therefore co-ordinates of the point of intersection of line and plane are

$$Q\equiv\left(\frac{76}{35},\frac{-108}{35},\frac{-34}{7}\right)$$

Exercise : 28C

Question: 1

Solution:

Formula : Distance $=\frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$

where (x_1,y_1,z_1) *is point from which distance is to be calculated*

Therefore ,

Plane $r.(3i-4j+12k)=9$ *can be written in cartesian form as*

$3x - 4y + 12z = 9$

$3x - 4y + 12z - 9 = 0$

Point = (2i - j - 4k)

Which can be also written as

Point = (2 , - 1 , - 4)

Distance $=\frac{|(2\times3)+(-1\times-4)+(-4\times12)+(-9)|}{\sqrt{(3)^2+(-4)^2+12^2}}$

$$=\frac{|6+4-48-9|}{\sqrt{9+16+144}}$$

$$=\frac{|-47|}{\sqrt{169}}$$

$$=\frac{47}{13}\ units$$

Question: 2

Solution:

Formula : Distance $=\frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$

where (x_1,y_1,z_1) *is point from which distance is to be calculated*

Therefore ,

Plane $r.(i+j+k)+17=0$ *can be written in cartesian form as*

$x + y + z + 17 = 0$

Point = (i + 2j + 5k)

Which can be also written as

Point = (1 , 2 , 5)

$Distance = \frac{|(1\times1)+(2\times1)+(5\times1)+(17)|}{\sqrt{(1)^2+(1)^2+1^2}}$

$= \frac{|1 + 2 + 5 + 17|}{\sqrt{1 + 1 + 1}}$

$= \frac{|25|}{\sqrt{3}}$

$= \frac{25\sqrt{3}}{3}$ *units*

Question: 3

Solution:

Formula : $Distance = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) *is point from which distance is to be calculated*

Therefore ,

Plane $r.(2i - 5j + 3k) = 13$ *can be written in cartesian form as*

2x - 5y + 3z = 13

2x - 5y + 3z - 13 = 0

Point = (3 , 4 , 5)

$Distance = \frac{|(3\times2)+(4\times-5)+(5\times3)-(13)|}{\sqrt{(2)^2+(-5)^2+3^2}}$

$= \frac{|6 - 20 + 15 - 13|}{\sqrt{4 + 25 + 9}}$

$= \frac{|-12|}{\sqrt{38}}$

$= \frac{12\sqrt{38}}{38} = \frac{6\sqrt{38}}{19}$ *units*

Question: 4

Solution:

Formula : $Distance = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) *is point from which distance is to be calculated*

Therefore ,

Plane $r.(2i - 2j + 4k) + 5 = 0$ *can be written in cartesian form as*

2x - 2y + 4z + 5 = 0

Point = (1 , 1 , 2)

$Distance = \frac{|(1\times2)+(1\times-2)+(2\times4)+(5)|}{\sqrt{(2)^2+(-2)^2+(4)^2}}$

$= \frac{|2 - 2 + 8 + 5|}{\sqrt{4 + 4 + 16}}$

$= \frac{|13|}{\sqrt{24}}$

$= \frac{13}{2\sqrt{6}} = \frac{13\sqrt{6}}{12}$ *units*

Question: 5

Solution:

Formula : $Distance = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) *is point from which distance is to be calculated*

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Therefore ,

2x + y + 2z + 5 = 0

Point = (2 , 1 , 0)

Distance $= \frac{|(2\times2)+(1\times1)+(0\times2)+(5)|}{\sqrt{(2)^2+(1)^2+(2)^2}}$

$= \frac{|4 + 1 + 0 + 5|}{\sqrt{4 + 1 + 4}}$

$= \frac{|10|}{\sqrt{9}}$

$= \frac{10}{3}$ units

Question: 6

Solution:

Formula : Distance $= \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2+B^2+C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

x - 2y + 4z = 9

x - 2y + 4z – 9 = 0

Point = (2 , 1 , - 1)

Distance $= \frac{|(2\times1)+(1\times-2)+(-1\times4)-(9)|}{\sqrt{(1)^2+(-2)^2+(4)^2}}$

$= \frac{|2-2-4-9|}{\sqrt{1 + 4 + 16}}$

$= \frac{|-13|}{\sqrt{21}}$

$= \frac{13}{\sqrt{21}} = \frac{13\sqrt{21}}{21}$ units

Question: 7

Solution:

Formula : Distance $= \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2+B^2+C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

First Plane $r.(i + 2j - 2k) = 5$ can be written in cartesian form as

x + 2y - 2z = 5

x + 2y - 2z – 5 = 0

Point = (1 , 2 , 1)

Distance for first plane $= \frac{|(1\times1)+(2\times2)+(1\times-2)-(5)|}{\sqrt{(1)^2+(2)^2+(-2)^2}}$

$= \frac{|1 + 4 - 2 - 5|}{\sqrt{1 + 4 + 4}}$

$= \frac{|-2|}{\sqrt{9}}$

$= \frac{2}{3}$ units

Second Plane $r.(2i - 2j + k) + 3 = 0$ can be written in cartesian form as

2x - 2y + z + 3 = 0

Point = (1 , 2 , 1)

Distance for second plane $= \frac{|(1\times2)+(2\times-2)+(1\times1)+(3)|}{\sqrt{(2)^2+(-2)^2+(1)^2}}$

$= \frac{|2 - 4 + 1 + 3|}{\sqrt{4 + 4 + 1}}$

$= \frac{|2|}{\sqrt{9}}$

$= \frac{2}{3}$ *units*

Hence proved.

Question: 8

Solution:

Formula : Distance $= \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) *is point from which distance is to be calculated*

Therefore ,

Plane = 3x + 4y - 12z + 13 = 0

First Point = (- 3 , 0 , 1)

Distance for first point $= \frac{|(-3\times3) + (0\times4) + (1\times-12) + (13)|}{\sqrt{(3)^2 + (4)^2 + (-12)^2}}$

$= \frac{|-9 + 0 - 12 + 13|}{\sqrt{9 + 16 + 144}}$

$= \frac{|-8|}{\sqrt{169}}$

$= \frac{8}{13}$ *units*

Plane = 3x + 4y - 12z + 13 = 0

Second Point = (1 , 1 , 1)

Distance for first point $= \frac{|(1\times3) + (1\times4) + (1\times-12) + (13)|}{\sqrt{(3)^2 + (4)^2 + (-12)^2}}$

$= \frac{|3 + 4 - 12 + 13|}{\sqrt{9 + 16 + 144}}$

$= \frac{|8|}{\sqrt{169}}$

$= \frac{8}{13}$ *units*

Hence proved.

Question: 9

Solution:

Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

Distance $= \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$

where (d_1, d_2) *are constants of the planes*

Therefore ,

First Plane 2x + 3y + 4 = 4

2x + 3y + 4 – 4 = 0 …… (1)

Second plane 4x + 6y + 8z = 12

4x + 6y + 8z - 12 = 0

2(2x + 3y + 4z – 6) = 0

2x + 3y + 4z – 6 = 0 …… (2)

Using equation (1) and (2)

Distance between both planes $= \frac{|-6-(-4)|}{\sqrt{(2)^2+(3)^2+(4)^2}}$

$= \frac{|-6 + 4|}{\sqrt{4 + 9 + 16}}$

$- \frac{|-2|}{\sqrt{29}}$

$= \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$ units

Question: 10

Solution:

Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Palne 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

where (d_1, d_2) are costants of the planes

Therefore ,

First Plane x + 2y - 2z + 4 = 0 ...... (1)

Second plane x + 2y – 2z – 8 = 0 ...... (2)

Using equation (1) and (2)

Distance between both planes $= \frac{|-8-(4)|}{\sqrt{(1)^2+(2)^2+(2)^2}}$

$= \frac{|-12|}{\sqrt{1 + 4 + 4}}$

$= \frac{12}{\sqrt{9}}$

$= \frac{12}{3} = 4$ units

Question: 11

Solution:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane x – 2y + 2z – 3 = 0

Normal vector = (i - 2j + 2k)

∴ Normal vector of required plane = (i - 2j + 2k)

Equation of required planes r . (i - 2j + 2k) = d

In cartesian form x – 2y + 2y = d

It should be at unit distance from point (1,1,1)

Distance $= \frac{|(1\times1)+(1\times-2)+(1\times2)-(d)|}{\sqrt{(1)^2+(-2)^2+(2)^2}}$

$= \frac{|1 - 2 + 2 - d|}{\sqrt{1 + 4 + 4}}$

$= \frac{|1 - d|}{\sqrt{9}}$

$1 = \frac{\pm(1 - d)}{3}$

$3 = \pm(1-d)$

For + sign = > 3 = 1 - d = > d = - 2

For - sign = > 3 = - 1 + d = > d = 4

Therefore equations of planes are : -

For d = - 2 For d = 4

x – 2y + 2y = d x – 2y + 2y = d

x – 2y + 2y = - 2 x – 2y + 2y = 4

x – 2y + 2y + 2 = 0 x – 2y + 2y – 4 = 0

Required planes = x − 2y + 2y + 2 = 0

x − 2y + 2y − 4 = 0

Question: 12

Solution:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Palne 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane 2x – 3y + 5z + 7 = 0

Normal vector = (2i - 3j + 5k)

∴ Normal vector of required plane = (2i - 3j + 5k)

Equation of required plane r . (2i - 3j + 5k) = d

In cartesian form 2x – 3y + 5y = d

Plane passes through point (3,4, - 1) therefore it will satisfy it.

2(3) – 3(4) + 5(- 1) = d

6 – 12 - 5 = d

d = - 11

Equation of required plane 2x − 3y + 5z = - 11

2x – 3y + 5z + 11 = 0

Therefore ,

First Plane 2x – 3y + 5z + 7 = 0 …… (1)

Second plane 2x – 3y + 5z + 11 = 0 …… (2)

Using equation (1) and (2)

$$\text{Distance between both planes} = \frac{|11-(7)|}{\sqrt{(2)^2 + (-3)^2 + (5)^2}}$$

$$= \frac{|4|}{\sqrt{4 + 9 + 25}}$$

$$= \frac{4}{\sqrt{38}}$$

$$= \frac{4\sqrt{38}}{38} = \frac{2\sqrt{38}}{19} \text{ units}$$

Question: 13

Solution:

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Formula : The equation of mid parallel plane is , say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

$$\text{Mid parallel plane} = ax + by + cy + \frac{(d_1 + d_2)}{2} = 0$$

where (d_1, d_2) are constants of the planes

Therefore ,

First Plane 2x – 3y + 6z + 21 = 0 ...... (1)

Second plane 2x – 3y + 6z – 14 = 0 ...... (2)

Using equation (1) and (2)

$$\text{Mid parallel plane} = 2x - 3y + 6z + \frac{21-14}{2} = 0$$

4x – 6y + 12z + 7 = 0

Exercise : 28D

Question: 1

Solution:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are either same or proportional to each other

Therefore ,

Plane 1 : - 2x – y + 6z = 5

Normal vector (Plane 1) = (2i – j + 6k) .....(1)

Plane 2 : - 5x – 2.5y + 15z = 12

Normal vector (Plane 2) = (5i – 2.5j + 15k) .....(2)

Multiply equation(1) by 5 and equation(2) by 2

Normal vector (Plane 1) = 5(2i – j + 6k)

= 10i – 5j + 30k

Normal vector (Plane 2) = 2(5i – 2.5j + 15k)

= 10i – 5j + 30k

Since, both normal vectors are same .Therefore both planes are parallel

Question: 2

Solution:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (2i – 3j + 5k) + 5 = 0

Normal vector = (2i - 3j + 5k)

∴ Normal vector of required plane = (2i - 3j + 5k)

Equation of required plane r . (2i - 3j + 5k) = d

In cartesian form 2x – 3y + 5z = d

Plane passes through point (3,4, - 1) therefore it will satisfy it.

2(3) – 3(4) + 5(- 1) = d

$6 – 12 - 5 = d$

$d = - 11$

Equation of required plane r . (2i - 3j + 5k) = - 11

r . (2i - 3j + 5k) + 11 = 0

Question: 3

Solution:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (i + j + k) = 2

Normal vector = (i + j + k)

∴ Normal vector of required plane = (i + j + k)

Equation of required plane r . (i + j + k) = d

In cartesian form $x + y + z = d$

Plane passes through point (a,b,c) therefore it will satisfy it.

$(a) + (b) + (c) = d$

$d = a + b + c$

Equation of required plane r . (i + j + k) = a + b + c

Question: 4

Solution:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (2i - j + 2k) = 5

Normal vector = (2i - j + 2k)

∴ Normal vector of required plane = (2i - j + 2k)

Equation of required plane r . (2i - j + 2k) = d

In cartesian form $2x - y + 2z = d$

Plane passes through point (1,1,1) therefore it will satisfy it.

$2(1) - (1) + 2(1) = d$

$d = 2 – 1 + 2 = 3$

Equation of required plane r . (2i - j + 2k) = 3

Question: 5

Solution:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane $2x – y + 3z + 7 = 0$

Normal vector = (2i - j + 3k)

∴ Normal vector of required plane = (2i - j + 3k)

Equation of required plane r . (2i - j + 3k) = d

In cartesian form 2x - y + 3z = d

Plane passes through point (1,4, - 2) therefore it will satisfy it.

2(1) - (4) + 3(- 2) = d

d = 2 – 4 – 6 = - 8

Equation of required plane 2x - y + 3z = - 8

2x - y + 3z + 8 = 0

Question: 6

Solution:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane 2x – 3y + 7z + 13 = 0

Normal vector = (2i - 3j + 7k)

∴ Normal vector of required plane = (2i - 3j + 7k)

Equation of required plane r . (2i - 3j + 7k) = d

In cartesian form 2x - 3y + 7z = d

Plane passes through point (0,0,0) therefore it will satisfy it.

2(0) - (0) + 3(0) = d

d = 0

Equation of required plane 2x - 3y + 7z = 0

Question: 7

Find the eq

Solution:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane 3x – 5y + 4z = 11

Normal vector = (3i - 5j + 4k)

∴ Normal vector of required plane = (3i - 5j + 4k)

Equation of required plane r . (3i - 5j + 4k) = d

In cartesian form 3x - 5y + 4z = d

Plane passes through point (- 1,0,7) therefore it will satisfy it.

3(- 1) - 5(0) + 4(7) = d

d = - 3 + 28 = 25

Equation of required plane 3x - 5y + 4z = 25

Question: 8

Solution:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane x – 2y + 2z – 3 = 0

Normal vector = (i - 2j + 2k)

∴ Normal vector of required plane = (i - 2j + 2k)

Equation of required planes r . (i - 2j + 2k) = d

In cartesian form x – 2y + 2y = d

It should be at unit distance from point (1,2,3)

Distance $= \frac{|(1\times1)+(2\times-2)+(3\times2)-(d)|}{\sqrt{(1)^2+(-2)^2+(2)^2}}$

$= \frac{|1-4+\ 6-d|}{\sqrt{1\ +\ 4\ +\ 4}}$

$= \frac{|3-d|}{\sqrt{9}}$

$1\ =\ \frac{\pm(3-d)}{3}$

$3\ =\ \pm(3-d)$

For + sign = > 3 = 3 - d = > d = 0

For - sign = > 3 = - 3 + d = > d = 6

Therefore equations of planes are : -

For d = 0 For d = 6

x – 2y + 2y = d x – 2y + 2y = d

x – 2y + 2y = 0 x – 2y + 2y = 6

Required planes = x − 2y + 2y = 0

x – 2y + 2y − 6 = 0

Question: 9

Solution:

Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

$\text{Distance}\ =\ \frac{|d_2-d_1|}{\sqrt{a^2\ +\ b^2\ +\ c^2}}$

where (d_1, d_2) *are constants of the planes*

Therefore ,

First Plane x + 2y + 3z + 7 = 0

2(x + 2y + 3z + 7) = 0

2x + 4y + 6z + 14 = 0 …… (1)

Second plane 2x + 4y + 6z + 7 = 0 …… (2)

Using equation (1) and (2)

Distance between both planes $= \frac{|7-(14)|}{\sqrt{(2)^2+(4)^2+(6)^2}}$

$= \frac{|-7|}{\sqrt{4\ +\ 16\ +\ 36}}$

$= \frac{|-7|}{\sqrt{56}}$

$= \frac{7}{\sqrt{56}}$ *units*

Exercise : 28E

CLASS24

Question: 1

Solution:

Equation of plane through the line of intersection of planes in Cartesian form is

$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0$ *(1)*

For the standard equation of planes,

$A_1x+B_1y+C_1z+D_1$ and $A_2x+B_2y+C_2z+D_2$

So, putting in equation (1), we have

$x + y + z-6 + \lambda(2x + 2y + 4z + 5)=0$

$(1 + 2\lambda)x + (1 + 2\lambda)y + (1 + 4\lambda)z-6 + 5\lambda=0$ (2)

Now plane passes through (1,1,1) then it must satisfy the plane equation,

$(1 + 2\lambda).1 + (1 + 2\lambda).1 + (1 + 4\lambda).1-6 + 5\lambda=0$

$1 + 2\lambda + 1 + 2\lambda + 1 + 4\lambda-6 + 5\lambda=0$

$3 + 8\lambda-6 + 5\lambda=0$

$13\lambda=3$

$$\lambda=\frac{3}{13}$$

Putting in equation (2)

$$\left(1+2.\frac{3}{13}\right)x+\left(1+2.\frac{3}{13}\right)y+\left(1+4.\frac{3}{13}\right)z-6+5.\frac{3}{13}=0$$

$$\left(\frac{13+6}{13}\right)x+\left(\frac{13+6}{13}\right)y+\left(\frac{13-12}{13}\right)z+\frac{-78+15}{13}=0$$

$19x + 19y + 25z-63=0$

So, the required equation of plane is $19x + 19y + 25z=63$.

Question: 2

Solution:

Equation of plane through the line of intersection of planes in Cartesian form is

$A_1x+B_1y+C_1z+D_1+\lambda(A_2x-B_2y+C_2z+D_2)=0$ *(1)*

For the standard equation of planes,

$A_1x+B_1y+C_1z+D_1$ and $A_2x+B_2y+C_2z+D_2$

So, putting in equation (1), we have

$x-3y + z + 6 + \lambda(x + 2y + 3z + 5)=0$

$(1 + \lambda)x + (-3 + 2\lambda)y + (1 + 3\lambda)z + 6 + 5\lambda=0$ (2)

Now plane passes through (0,0,0) then it must satisfy the plane equation,

$(1 + \lambda).0 + (-3 + 2\lambda).0 + (1 + 3\lambda).0 + 6 + 5\lambda=0$

$5\lambda=-6$

$$\lambda=\frac{-6}{5}$$

Putting in equation (2)

$$\left(1+\frac{-6}{5}\right)x+\left(-3+2.\frac{-6}{5}\right)y+\left(1+3.\frac{-6}{5}\right)z+6+5.\frac{-6}{5}=0$$

$$\left(\frac{5+(-6)}{5}\right)x+\left(\frac{-15-12}{5}\right)y+\left(\frac{5+(-18)}{5}\right)z+\frac{30+(-30)}{5}=0$$

$-x-27y-13z=0$

$x + 27y + 13z=0$

So, required equation of plane is $x + 27y + 13z=0$.

Question: 3

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0 \quad (1)$$

For the standard equation of planes,

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

2x + 3y−z + 1 + λ(x + y−2z + 3)=0

(2 + λ)x + (3 + λ)y + (-1-2λ)z + 1 + 3λ=0 (2)

Now as the plane 3x-y-2z-4=0 is perpendicular the given plane,

For θ=90°, cos90°=0

$$A_1A_2+B_1B_2+C_1C_2=0 \quad (3)$$

On comparing with standard equations in Cartesian form,

$$A_1=2+\lambda, B_1=3+\lambda, C_1=-1-2\lambda \text{ and } A_2=3, B_2=-1, C_2=-2$$

Putting values in equation (3), we have

(2 + λ).3 + (3 + λ).(-1) + (-1-2λ).(-2)=0

6 + 3λ-3-λ + 2 + 4λ=0

5 + 6λ=0

$$\lambda=\frac{-5}{6}$$

Putting in equation(2)

$$\left(2+\frac{-5}{6}\right)x+\left(3+\frac{-5}{6}\right)y+\left(-1-2.\frac{-5}{6}\right)z+1+3.\frac{-5}{6}=0$$

$$\left(\frac{12-5}{6}\right)x+\left(\frac{18-5}{6}\right)y+\left(\frac{-6+10}{6}\right)z+\frac{6-15}{6}=0$$

7x + 13y + 4z-9=0

7x + 13y + 4z=9

So, required equation of plane is 7x + 13y + 4z=9.

Question: 4

Solution:

Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0 \quad (1)$$

For the standard equation of planes,

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

2x-y + λ(3z-y)=0

2x + (-1-λ)y + 3λz=0 (2)

Now as the plane is perpendicular the given plane,

For θ=90°, cos90°=0

$$A_1A_2+B_1B_2+C_1C_2=0 \quad (3)$$

On comparing with standard equations in Cartesian form,

$$A_1=2, B_1=-1-\lambda, C_1=3\lambda \text{ and } A_2=4, B_2=5, C_2=-3$$

Putting values in equation(3),

2.4 + (-1-λ).5 + 3λ.-3=0

8-5-5λ-9λ=0

-14λ=-3

$$\lambda=\frac{3}{14}$$

Putting in equation(2)

$$2x+\left(-1-\frac{3}{14}\right)y+3\left(\frac{3}{14}\right)z=0$$

$$2x+\left(\frac{-14-3}{14}\right)y+\frac{9}{14}.z=0$$

28x-17y + 9z=0

So, required equation of plane is 28x-17y + 9z=0.

Question: 5

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0 \quad (1)$$

For the standard equation of planes,

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

x-2y + z-1 + λ(2x + y + z-8)=0

(1 + 2λ)x + (-2 + λ)y + (1 + λ)z-1-8λ=0 (2)

For plane the normal is perpendicular to line given parallel to this i.e.

$$A_1A_2+B_1B_2+C_1C_2=0$$

Where A_1, B_1, C_1 are direction ratios of plane and A_2, B_2, C_2 are of line.

(1 + 2λ).1 + (-2 + λ).2 + (1 + λ).1=0

1 + 2λ-4 + 2λ + 1 + λ=0

-2 + 5λ=0

$$\lambda=\frac{2}{5}$$

Putting the value of λ in equation (2)

$$\left(1+2.\left(\frac{2}{5}\right)\right).x+\left(-2+\frac{2}{5}\right).y+\left(1+\frac{2}{5}\right).z-1-8.\left(\frac{2}{5}\right)=0$$

$$\left(\frac{5+4}{5}\right)x+\left(\frac{-10+2}{5}\right)y+\left(\frac{5+2}{5}\right)z+\frac{-5-16}{5}=0$$

9x-8y + 7z-21=0

9x-8y + 7z=21

For the equation of plane Ax + By + Cz=D and point (x1,y1,z1), a distance of a point from a plane can be calculated as

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

$$\left|\frac{9.1-8.1+7.1-21}{\sqrt{(9)^2+(-8)^2+(7)^2)}}\right| \Rightarrow \left|\frac{9-8+7-21}{\sqrt{81+64+49}}\right|=\left|\frac{13}{\sqrt{194}}\right|$$

So, the required equation of the plane is 9x-8y + 7z=21, and distance of the plane from (1,1,1) is

$$d-\frac{13}{\sqrt{194}}$$

Question: 6

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0 \quad (1)$$

For the standard equation of planes in Cartesian form

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation 1 we have

x + 2y + 3 z-5 + λ(3x -2y -z + 1)=0

(1 + 3λ)x + (2-2λ)y + (3-λ)z-5 + λ=0

Now equation of plane in intercept form

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

As given equal intercept means a=c

First, we transform equation of a plane in intercept form

$$\frac{x}{\frac{1}{(1+3\lambda)}}+\frac{y}{\frac{1}{(2-2\lambda)}}+\frac{z}{\frac{1}{(3-\lambda)}}=5-\lambda$$

$$\frac{x}{\frac{5-\lambda}{(1+3\lambda)}}+\frac{y}{\frac{5-\lambda}{(2-2\lambda)}}+\frac{z}{\frac{5-\lambda}{(3-\lambda)}}=1$$

On comparing with the standard equation of a plane in intercept form

$$a=\frac{5-\lambda}{(1+3\lambda)},c=\frac{5-\lambda}{(3-\lambda)}$$

Now as a=b=c

$$\frac{5-\lambda}{(1+3\lambda)}=\frac{5-\lambda}{(3-\lambda)}\Rightarrow 3-\lambda=1+3\lambda$$

$$4\lambda=2\Rightarrow\lambda=\frac{1}{2}$$

Putting in equation (2), we have

$$\left(1+3.\frac{1}{2}\right)x+\left(2-2.\frac{1}{2}\right)y+\left(3-\frac{1}{2}\right)z-5+\frac{1}{2}=0$$

$$\left(\frac{2+3}{2}\right)x+\left(\frac{4-2}{2}\right)y+\left(\frac{6-1}{2}\right)z+\frac{-10+1}{2}=0$$

5x + 2y + 5z-9=0

5x + 2y + 5z=9

So, required equation of plane is 5x + 2y + 5z=9.

Question: 7

Solution:

Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x-B_2y+C_2z+D_2)=0 \quad (1)$$

For the standard equation of planes in Cartesian form

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

3x – 4y + 5z -10 + λ(2x + 2y - 3z - 4)=0

(3 + 2λ)x + (-4 + 2λ)y + (5-3λ)z-10-4λ=0

Given line is parallel to plane then the normal of plane is perpendicular to line,

$$A_1A_2+B_1B_2+C_1C_2=0$$

Where A_1, B_1, C_1 are direction ratios of plane and A_2, B_2, C_2 are of line.

(3 + 2λ).6 + (-4 + 2λ).3 + (5-3λ).2=0

18 + 12λ-12 + 6λ + 10-6λ=0

16 + 12λ=0

$$\lambda=\frac{-16}{12}\Rightarrow\frac{-4}{3}$$

Putting the value of λ in equation (2)

$$\left(3+2.\left(\frac{-4}{3}\right)\right)x+\left(-4+2.\left(\frac{-4}{3}\right)\right)y+\left(5-3\left(\frac{-4}{3}\right)\right)z-10-4.\left(\frac{-4}{3}\right)=0$$

$$\left(\frac{9-8}{3}\right)x+\left(\frac{-12-8}{3}\right)y+\left(\frac{15+12}{3}\right)z+\frac{-30+16}{3}=0$$

x-20y + 27z-14=0

So, required equation of plane is x-20y + 27z-14=0.

Question: 8

Solution:

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ *(1)*

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation(1)

$\vec{r}(\hat{i}+3\hat{j}-\hat{k}+\lambda(\hat{j}+2\hat{k})=0+\lambda.0$

$\vec{r}\left(\hat{i}+(3+\lambda)\hat{j}+(-1+2\lambda)\hat{k}\right)=0$ *(2)*

Now as the plane passes through (2,1,-1)

$\vec{r}=2\hat{i}+\hat{j}-\hat{k}$

Putting in equation (2)

$\left(2\hat{i}+\hat{j}-\hat{k}\right)(\left(\hat{i}+(3+\lambda)\hat{j}+(-1+2\lambda)\hat{k}\right)=0$

2.1 + 1.(3 + λ) + (-1)(-1 + 2λ)=0

2 + 3 + λ + 1-2λ=0

λ=6

Putting the value of λ in equation (2)

$\vec{r}\left(\hat{i}+(3+6)\hat{j}+(-1+2(6))\hat{k}\right)=0$

$\vec{r}\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0$

So, required equation of plane is $\vec{r}\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0.$

Question: 9

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ *(1)*

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation(1)

$\vec{r}(\hat{i}-\hat{j}+3\hat{k}+\lambda(2\hat{i}+\hat{j}-\hat{k})=-1+\lambda.5$

$\vec{r}\left((1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(3-\lambda)\hat{k}\right)=-1+5\lambda$ *(2)*

Now as the plane passes through (1,1,1)

$\vec{r}=\hat{i}+\hat{j}+\hat{k}$

Putting in equation (2)

$\left(\hat{i}+\hat{j}+\hat{k}\right)(\left((1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(3-\lambda)\hat{k}\right)=-1+5\lambda$

1.(1 + 2λ) + 1.(-1 + λ) + 1.(3-λ)=-1 + 5λ

1 + 2λ-1 + λ + 3- λ + 1-5λ=0

-3λ + 4=0

$\lambda=\frac{4}{3}$

Putting the value of λ in equation (2)

$\vec{r}\left(\left(1+2.\frac{4}{3}\right)\hat{i}+\left(-1+\frac{4}{3}\right)\hat{j}+\left(3-\frac{4}{3}\right)\hat{k}\right)=-1+5.\frac{4}{3}$

$$\vec{r}\left(\left(\frac{3+8}{3}\right)\hat{i}+\left(\frac{-3+4}{3}\right)\hat{j}+\left(\frac{9-4}{3}\right)\hat{k}\right)=\frac{-3+20}{3}$$

$$\vec{r}\left(11\hat{i}+\hat{j}+5\hat{k}\right)=17$$

So, required equation of plane is $\vec{r}\left(11\hat{i}+\hat{j}+5\hat{k}\right)=17$.

Question: 10

Solution:

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ *(1)*

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation(1)

$$\vec{r}(2\hat{i}-7\hat{j}+4\hat{k}+\lambda\left(3\hat{i}-5\hat{j}+4\hat{k}\right)=3-\lambda.11$$

$\vec{r}\left((2+3\lambda)\hat{i}+(-7-5\lambda)\hat{j}+(4+4\lambda)\hat{k}\right)=3-11\lambda$ *(2)*

Now as the plane passes through (-2,1,3)

$$\vec{r}--2\hat{i}+\hat{j}+3\hat{k}$$

Putting in equation (2)

$$\left(-2\hat{i}+\hat{j}+3\hat{k}\right)(\left((2+3\lambda)\hat{i}-(-7-5\lambda)\hat{j}+(4+4\lambda)\hat{k}\right)=3-11\lambda$$

-2.(2 + 3λ) + 1.(-7-5λ) + 3.(4 + 4λ)=3-11λ

-4-6λ-7-5λ + 12 + 12λ-3 + 11λ=0

-14 + 12 + 12λ=0

$$\lambda=\frac{1}{6}$$

Putting the value of λ in equation (2)

$$\vec{r}\left(\left(2+3.\frac{1}{6}\right)\hat{i}+\left(-7-5.\frac{1}{6}\right)\hat{j}+\left(4+4\frac{1}{6}\right)\hat{k}\right)=3-11.\frac{1}{6}$$

$$\vec{r}\left(\left(\frac{12+3}{6}\right)\hat{i}+\left(\frac{-42-5}{6}\right)\hat{j}+\left(\frac{24+4}{6}\right)\hat{k}\right)=\frac{18-11}{6}$$

$$\vec{r}\left(15\hat{i}-47\hat{j}+28\hat{k}\right)=7$$

So, required equation of plane is $\vec{r}\left(15\hat{i}-47\hat{j}+28\hat{k}\right)=7$.

Question: 11

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ *(1)*

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation (1), we have

$$\vec{r}(2\hat{i}-3\hat{j}+4\hat{k}+\lambda\left(\hat{i}-\hat{j}\right)=1-\lambda.4$$

$\vec{r}\left((2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}\right)=1-4\lambda$ *(2)*

Given a plane perpendicular to this plane, So if n1 and n2 are normal

vectors of planes

$$\vec{n}_1.\vec{n}_2=0$$

$(2\hat{i}-\hat{j}+\hat{k}).\left((2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}\right)=0$

2.(2 + λ) + (-1).(-3-λ) + 1.4=0

4 + 2λ + 3 + λ + 4=0

11 + 3λ=0

$\lambda=\frac{-11}{3}$

Putting the value of λ in equation (2)

$\vec{r}\left(\left(2+\frac{-11}{3}\right)\hat{i}+\left(-3-\frac{-11}{3}\right)\hat{j}+4\hat{k}\right)=1-4.\frac{-11}{3}$

$\vec{r}\left(\left(\frac{6-11}{3}\right)\hat{i}+\left(\frac{-9+11}{3}\right)\hat{j}+4\hat{k}\right)=\frac{3+44}{3}$

$\vec{r}\left(-5\hat{i}-2\hat{j}+12\hat{k}\right)=47$

So required equation of plane is $\vec{r}\left(-5\hat{i}-2\hat{j}+12\hat{k}\right)=47.$

Question: 12

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ *(1)*

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation (1)

$\vec{r}(\hat{i}-\hat{j}+\lambda(3\hat{i}+3\hat{j}-4\hat{k})=6+\lambda.0$

$\vec{r}\left((1+3\lambda)\hat{i}+(-1+3\lambda)\hat{j}+(-4\lambda)\hat{k}\right)=6$ *(2)*

For the equation of plane Ax + By + Cz=D and point (x1,y1,z1), a distance of a point from a plane can be calculated as

$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$

$\left|\frac{(1+3\lambda)0+(-1+3\lambda).0+(-4\lambda).0-6}{\sqrt{(1+3\lambda)^2+(-1+3\lambda)^2+(-4\lambda)^2)}}\right|=1$

$\left|\frac{-6}{\sqrt{1+9\lambda^2+6\lambda+1+9\lambda^2-6\lambda+16\lambda^2}}\right|=1$

$\sqrt{2+34\lambda^2}=-6$

$2+34\lambda^2=(-6)^2$

$34\lambda^2=36-2$

$34\lambda^2=34$

$\lambda^2=1\Rightarrow\lambda=1,-1$

Putting value of λ in equation (2)

λ=1

$\vec{r}\left((1+3.1)\hat{i}+(-1+3.1)\hat{j}+(-4.1)\hat{k}\right)=6$

$\vec{r}\left(4\hat{i}+2\hat{j}-4\hat{k}\right)=6\Rightarrow\vec{r}.\left(2\hat{i}+\hat{j}-2\hat{k}\right)=3$

λ=-1

$\vec{r}\left((1+3.(-1))\hat{i}+(-1+3(-1))\hat{j}+(-4(-1))\hat{k}\right)=6$

$\vec{r}\left(-2\hat{i}-4\hat{j}+4\hat{k}\right)=6\Rightarrow\vec{r}.\left(\hat{i}+2\hat{j}-2\hat{k}\right)=-3$

For equations in Cartesian form put

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

For λ=1

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(2\hat{i} + \hat{j} - 2\hat{k} - 3\right) = 0$$

x.2 + y.1 + z.(-2)-3=0

2x + y-2z-3=0

For λ=-1

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(\hat{i} + 2\hat{j} - 2\hat{k} + 3\right) = 0$$

x.1 + y.2 + z.(-2) + 3=0

x + 2y-2z + 3=0

So.required equation of plane

in vector form are $\vec{r}.\left(2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$ for $\lambda = 1$

$$\vec{r}.\left(\hat{i} + 2\hat{j} - 2\hat{k}\right) = -3 \text{ for } \lambda = -1$$

In Cartesian form are 2x + y-2z-3=0 & x + 2y-2z + 3=0

Exercise : 28F

Question: 1

Solution:

To find the angle between two planes, we simply find the angle between the normal vectors of planes. So if n1 and n2 are normal vectors and θ is the angle between both then,

$$\cos\theta = \left|\frac{\vec{n}_1 \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|$$

(i) On comparing with the standard equation of planes in vector form

$$\vec{r}.\vec{n}_1 = d_1 \text{ and } \vec{r}.\vec{n}_2 = d_2$$

$$\vec{n}_1 = \hat{i} + \hat{j} - 2\hat{k} \text{ and } \vec{n}_2 = 2\hat{i} - 2\hat{j} - \hat{k}$$

Then

$$\cos\theta = \left|\frac{\left(\hat{i} + \hat{j} - 2\hat{k}\right).\left(2\hat{i} + 2\hat{j} - \hat{k}\right)}{\left|\hat{i} + \hat{j} - 2\hat{k}\right|\left|2\hat{i} + 2\hat{j} - \hat{k}\right|}\right| \Rightarrow \left|\frac{1.2 + 1.2 + (-2).(-1)}{\left(\sqrt{1^2 + 1^2 + (-2)^2}\right).\left(\sqrt{2^2 + 2^2 + (-1)^2}\right)}\right| = \left|\frac{2 + 2 + 2}{\sqrt{1 + 1 + 4}\sqrt{4 + 4 + 1}}\right|$$

$$\Rightarrow \left|\frac{6}{\sqrt{6}.\sqrt{9}}\right| = \left|\frac{\sqrt{6}}{3}\right|$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{6}}{3}\right)$$

(ii) On comparing with the standard equation of planes in vector form

$$\vec{r}.\vec{n}_1 = d_1 \text{ and } \vec{r}.\vec{n}_2 = d_2$$

$$\vec{n}_1 = \hat{i} + 2\hat{j} - \hat{k} \text{ and } \vec{n}_2 = 2\hat{i} - \hat{j} - \hat{k}$$

Then

$$\cos\theta = \left|\frac{\left(\hat{i} + 2\hat{j} - \hat{k}\right).\left(2\hat{i} - \hat{j} - \hat{k}\right)}{\left|\hat{i} + 2\hat{j} - \hat{k}\right|\left|2\hat{i} - \hat{j} - \hat{k}\right|}\right| \Rightarrow \left|\frac{1.2 + 2.(-1) + (-1).(-1)}{\left(\sqrt{1^2 + 2^2 + (-1)^2}\right).\left(\sqrt{2^2 + (-1)^2 + (-1)^2}\right)}\right| = \left|\frac{2 - 2 + 1}{\sqrt{1 + 4 + 1}\sqrt{4 + 1 + 1}}\right|$$

$$\Rightarrow \left|\frac{1}{\sqrt{6}.\sqrt{6}}\right| = \left|\frac{1}{6}\right|$$

$\theta = \cos^{-1}\left(\frac{1}{6}\right)$

(iii) On comparing with the standard equation of planes in vector form

$\vec{r}.\vec{n}_1 = d_1 \text{ and } \vec{r}.\vec{n}_2 = d_2$

$\vec{n}_1 = 2\hat{i} - 3\hat{j} + 4\hat{k} \text{ and } \vec{n}_2 = -\hat{i} + \hat{j}$

Then

$$\cos\theta = \left|\frac{(2\hat{i} - 3\hat{j} + 4\hat{k}).(-\hat{i} + \hat{j})}{|2\hat{i} - 3\hat{j} + 4\hat{k}||-\hat{i} + \hat{j}|}\right| \Rightarrow \left|\frac{2.(-1) + (-3).1 + 4.0}{\left(\sqrt{2^2 + (-3)^2 + 4^2}\right).\left(\sqrt{(-1)^2 + 1^2}\right)}\right| = \left|\frac{-2 + (-3)}{(\sqrt{4 + 9 + 16})(\sqrt{1 + 1})}\right|$$

$$\Rightarrow \left|\frac{-5}{\sqrt{29}\sqrt{2}}\right| = \left|\frac{-5}{\sqrt{58}}\right|$$

$\theta = \cos^{-1}\left(\frac{5}{\sqrt{58}}\right)$

(iv) On comparing with the standard equation of planes in vector for

$\vec{r}.\vec{n}_1 = d_1 \text{ and } \vec{r}.\vec{n}_2 = d_2$

$\vec{n}_1 = 2\hat{i} - 3\hat{j} + 6\hat{k} \text{ and } \vec{n}_2 = 3\hat{i} + 4\hat{j} - 12\hat{k}$

Then

$$\cos\theta = \left|\frac{(2\hat{i} - 3\hat{j} + 6\hat{k}).(3\hat{i} + 4\hat{j} - 12\hat{k})}{|2\hat{i} - 3\hat{j} + 6\hat{k}||3\hat{i} + 4\hat{j} - 12\hat{k}|}\right| \Rightarrow \left|\frac{2.3 + (-3).4 + 6.(-12)}{\left(\sqrt{2^2 + (-3)^2 + 6^2}\right).\left(\sqrt{3^2 + 4^2 + (-12)^2}\right)}\right|$$

$$= \left|\frac{6 + (-12) + (-72)}{(\sqrt{4 + 9 + 36})(\sqrt{9 + 16 + 144})}\right|$$

$$\Rightarrow \left|\frac{-78}{\sqrt{49}\sqrt{169}}\right| = \left|\frac{-78}{7.13}\right|$$

$\theta = \cos^{-1}\left(\frac{6}{7}\right)$

Question: 2

To show the right angle between two planes, we simply find the angle between the normal vectors of planes. So if n1 and n2 are normal vectors and θ is the angle between both then

$\cos\theta = \left|\frac{\vec{n}_1\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|$ *,for right angle θ=90°*

Cos90°=0

$\vec{n}_1.\vec{n}_2 = 0$ *(1)*

(i) On comparing with standard equation

$\vec{n}_1 = 4\hat{i} - 7\hat{j} - 8\hat{k} \text{ and } \vec{n}_2 = 3\hat{i} - 4\hat{j} + 5\hat{k}$

$\text{LHS} = \vec{n}_1.\vec{n}_2 \Rightarrow (4\hat{i} - 7\hat{j} - 8\hat{k}).(3\hat{i} - 4\hat{j} + 5\hat{k}) = 4.3 + (-7).(-4) + (-8).5$

$\Rightarrow 12 + 28 - 40 = 40 - 40 \Rightarrow 0 = \text{RHS}$

Hence proved planes at right angles.

(ii) On comparing with the standard equation of a plane

$\vec{n}_1 = 2\hat{i} + 6\hat{j} + 6\hat{k} \text{ and } \vec{n}_2 = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$\text{LHS} = \vec{n}_1.\vec{n}_2 \rightarrow (2\hat{i} + 6\hat{j} + 6\hat{k}).(3\hat{i} + 4\hat{j} - 5\hat{k}) = 2.3 + 6.4 + 6.(-5)$

$\Rightarrow 6 + 24 - 30 = 30 - 30 \Rightarrow 0 = \text{RHS}$

Hence proved planes at right angles.

Question: 3

Solution:

For planes perpendicular Cos90°=0

$\vec{n}_1.\vec{n}_2 = 0$ *(1)*

(i) On comparing with the standard equation of a plane

$\vec{n}_1 = 2\hat{i} - \hat{j} - \lambda\hat{k} \text{ and } \vec{n}_2 = 3\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{n}_1.\vec{n}_2 - \left(2\hat{i} - \hat{j} - \lambda\hat{k}\right).\left(3\hat{i} + 2\hat{j} + 2\hat{k}\right) - 0$

2.3 + (-1).2 + (-λ).2=0

6-2-2λ=0

2λ=4

λ=2

(ii) On comparing with the standard equation of a plane

$\vec{n}_1 = \lambda\hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{n}_2 = \hat{i} + 2\hat{j} - 7\hat{k}$

$\vec{n}_1.\vec{n}_2 = \left(\lambda\hat{i} + 2\hat{j} + 3\hat{k}\right).\left(\hat{i} + 2\hat{j} - 7\hat{k}\right) = 0$

λ.1 + 2.2 + 3.(-7)=0 λ + 4-21=0 λ=17

Question: 4

$A_1x + B_1y + C_1z + D_1 - 0 \text{ and } A_1x + B_2y + C_2z + D_2 - 0$

$$\cos\theta = \left|\frac{A_1A_2 - B_1B_2 + C_1C_2}{\sqrt{\left(A_1^2 + B_1^2 + C_1^2\right)}\sqrt{\left(A_2^2 + B_2^2 + C_2^2\right)}}\right|$$

(i) On comparing with the standard equation of planes

$A_1 = 2, B_1 = -1, C_1 = 1 \text{ and } A_2 = 1, B_2 = 1, C_2 = 2$

$$\cos\theta = \left|\frac{2.1 + (-1).1 + 1.2}{\sqrt{2^2 + (-1)^2 + 1^2}\sqrt{1^2 + 1^2 + 2^2}}\right| \Rightarrow \left|\frac{2 + (-1) + 2}{\sqrt{4 + 1 + 1}\sqrt{1 + 1 + 4}}\right| = \left|\frac{3}{\sqrt{6}\sqrt{6}}\right|$$

$$= \frac{3}{6} \Rightarrow \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \frac{\pi}{3}$$

(ii) On comparing with the standard equation of planes

$A_1 = 1, B_1 = 2, C_1 = 2 \text{ and } A_2 = 2, B_2 = -3, C_2 = 6$

$$\cos\theta = \left|\frac{1.2 + 2.(-3) + 2.6}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{2^2 + (-3)^2 + 6^2}}\right| \Rightarrow \left|\frac{2 + (-6) + 12}{\sqrt{1 + 4 + 4}\sqrt{4 + 9 + 36}}\right| = \left|\frac{8}{\sqrt{9}\sqrt{49}}\right|$$

$$= \frac{8}{3.7} \Rightarrow \frac{8}{21}$$

$$\theta = \cos^{-1}\left(\frac{8}{21}\right)$$

(iii) On comparing with standard equation of planes

$A_1 = 1, B_1 = 1, C_1 = -1 \text{ and } A_2 = 1, B_2 = 2, C_2 = 1$

$$\cos\theta = \left|\frac{1.1 + 1.2 + (-1).1}{\sqrt{1^2 + 1^2 + (-1)^2}\sqrt{1^2 + 2^2 + 1^2}}\right| \Rightarrow \left|\frac{1 + 2 + (-1)}{\sqrt{1 + 1 + 1}\sqrt{1 + 4 + 1}}\right| = \left|\frac{2}{\sqrt{3}\sqrt{6}}\right|$$

$$= \frac{\sqrt{2}}{3}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

(iv) On comparing with the standard equation of planes

$$A_1 - 1, B_1 - 1, C_1 - -2 \text{ and } A_2 - 2, B_2 - -2, C_2 - 1$$

$$\cos\theta = \left|\frac{1.2 + 1.(-2) + (-2).1}{\sqrt{1^2 + 1^2 + (-2)^2}\sqrt{2^2 + (-2)^2 + 1^2}}\right| \Rightarrow \left|\frac{2 + (-2) + (-2)}{\sqrt{1+1+4}\sqrt{4+4+1}}\right| = \left|\frac{-2}{\sqrt{6}\sqrt{9}}\right|$$

$$= \frac{2}{\sqrt{6}.3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)$$

Question: 5

$$A_1x + B_1y + C_1z + D_1 = 0 \text{ and } A_1x + B_2y + C_2z + D_2 = 0 \;\; \cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{\left(A_1^2 + B_1^2 + C_1^2\right)}\sqrt{\left(A_2^2 + B_2^2 + C_2^2\right)}}\right|$$

For θ=90°, cos90°=0

$$A_1A_2 + B_1B_2 + C_1C_2 - 0$$

(i) On comparing with the standard equation of a plane

$$A_1 = 3, B_1 = 4, C_1 = -5 \text{ and } A_2 = 2, B_2 = 6, C_2 = 6$$

$$LHS = A_1A_2 + B_1B_2 + C_1C_2 \Rightarrow 3.2 + 4.6 + (-5).6 = 6 + 24 - 30$$

=0=RHS

Hence proved that the angle between planes is 90°.

(ii) On comparing with the standard equation of a plane

$$A_1 = 1, B_1 = -2, C_1 = 4 \text{ and } A_2 = 18, B_2 = 17, C_2 = 4$$

$$LHS = A_1A_2 + B_1B_2 + C_1C_2 \Rightarrow 1.18 + (-2).17 + 4.4 = 18 - (-34) + 16$$

=0=RHS

Hence proved that angle between planes is 90°.

Question: 6

$$A_1A_2 + B_1B_2 + C_1C_2 = 0$$

Where A_1, B_1, C_1 are direction ratios of plane and A_2, B_2, C_2 are of other plane.

2.1 + 2.2 + 4.2=2 + 4 + 8=14≠0

Hence, planes are not perpendicular.

Similarly for the other plane

2.5 + 2.6 + 2.7=10 + 12 + 14=36≠0

Hence, planes are not perpendicular.

Question: 7

Solution:

To show that planes are parallel

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

On comparing with the standard equation of a plane

$$A_1 - 2, B_1 - -2, C_1 - 4 \text{ and } A_2 - 3, B_2 - -3, C_2 - 6$$

$$\frac{A_1}{A_2} = \frac{2}{3}, \frac{B_1}{B_2} = \frac{-2}{-3} \Rightarrow \frac{2}{3}, \frac{C_1}{C_2} = \frac{4}{6} \Rightarrow \frac{2}{3}$$

So,

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\frac{2}{3}$$

Hence proved that planes are parallel.

Question: 8

Solution:

To find an angle in Cartesian form, for the standard equation of planes

$$A_1x+B_1y+C_1z+D_1=0 \text{ and } A_1x+B_2y+C_2z+D_2=0$$

$$\cos\theta=\left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{\left(A_1^2+B_1^2+C_1^2\right)}\sqrt{\left(A_2^2+B_2^2+C_2^2\right)}}\right|$$

For θ=90°, cos90°=0

$$A_1A_2+B_1B_2+C_1C_2=0$$

On comparing with the standard equation of the plane,

$$A_1=1, B_1=-4, C_1=\lambda \text{ and } A_2=2, B_2=2, C_2=3$$

$$A_1A_2+B_1B_2+C_1C_2 \Rightarrow 1.2+(-4).2+\lambda.3=0$$

2 + (-8) + 3λ=0

-6 + 3λ=0

λ=2

Question: 9

$$A_1x+B_1y+C_1z+D_1=0$$

Direction ratios of parallel planes are related to each other as

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=k\left(\text{constant}\right)$$

Putting the values from the equation of a given parallel plane,

$$\frac{A_1}{5}=\frac{B_1}{-3}=\frac{C_1}{7}=k$$

$$A_1=5k, B_1=-3k, C_1=7k$$

Putting in equation plane

$$5kx-3ky+7kz+D_1=0$$

As the plane is passing through (0,0,0), it must satisfy the plane,

$$5k.0-3k.0+7k.0+D_1=0$$

$$D_1=0$$

5kx-3ky + 7kz=0

5x-3y + 7z=0

So, required equation of plane is 5x-3y + 7z=0.

Question: 10

Solution:

Let the equation of a plane

$$\vec{r}.\left(x_1\hat{i}+y_1\hat{j}+z_1\hat{k}\right)=d \quad (1)$$

Direction ratios of parallel planes are related to each other as

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\lambda\left(\text{constant}\right)$$

Putting the values from the equation of a given parallel plane,

$$\frac{x_1}{1}=\frac{x_1}{1}=\frac{z_1}{1}=\lambda$$

$$x_1=y_1=z_1=\lambda$$

Putting values in equation (1), we have

$$\vec{r}.\left(\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k}\right)=d \quad (2)$$

A plane passes through (a,b,c) then it must satisfy the equation of a plane

$$\left(a\hat{i}+b\hat{j}+c\hat{k}\right).\left(\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k}\right)=d$$

$$\lambda\left(a\hat{i}+b\hat{j}+c\hat{k}\right)\left(\hat{i}+\hat{j}+\hat{k}\right)=d$$

λ(a.1 + b.1 + c.1)=d

λ(a + b + c)=d

Putting value in equation (2)

$$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)\lambda=\lambda(a+b+c)$$

$$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=a+b+c$$

So, required equtaion of plane is $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=a+b+c$.

Question: 11

$$A_1x+B_1y+C_1z+D_1=0$$

Direction ratios of parallel planes are related to each other as

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=k\,(\text{constant})$$

Putting the values from the equation of a given parallel plane,

$$\frac{A_1}{5}=\frac{B_1}{4}=\frac{C_1}{-11}=k$$

$$A_1=5k, B_1=4k, C_1=-11k$$

Putting in the equation of a plane

$$5kx+4ky-11kz+D_1=0$$

As the plane is passing through (1,-2,7), it must satisfy the plane,

$$5k.1+4k.(-2)-11k.7+D_1=0 \quad (1)$$

$$5k \quad 8k \quad 77k + D_1=0$$

$$D_1=80k$$

Putting value in equation (1), we have

$$5kx+4ky-11kz+80k=0$$

5x + 4y-11z + 80=0

So, the required equation of the plane is 5x + 4y-11z + 80=0.

Question: 12

$$AA_1+BB_1+CC_1=0$$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of another plane.

$$3.A_1+2B_1-3C_1=0 \quad (1)$$

$$5.A_1-4B_1+C_1=0 \quad (2)$$

And plane passes through (-1,-1,2),

A(x + 1) + B(y + 1) + C(z-2)=0 (3)

On solving equation (1) and (2)

$A = \frac{5B}{9} \text{ and } C = \frac{11B}{9}$

Putting values in equation (3)

$\frac{5B}{9}.(x+1)+B(y+1)+\frac{11B}{9}.(z-2) = 0$

B(5x + 5 + 9y + 9 + 11z-22)=0

5x + 9y + 11z-8=0

So, required equation of plane is 5x + 9y + 11z=8.

Question: 13

Solution:

Applying condition of perpendicularity between planes,

$AA_1 + BB_1 + CC_1 = 0$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of other plane.

$1.A + 2.B - 1.C = 0$

$A + 2B - C = 0$ *(1)*

$3.A - 4.B + C = 0$

$3A - 4B + C = 0$ *(2)*

And plane passes through (0, 0, 0),

A(x-0) + B(y-0) + C(z-0)=0

Ax + By + Cz=0 (3)

On solving equation (1) and (2)

$A = \frac{B}{2} \text{ and } C = \frac{5B}{2}$

Putting values in equation(3)

$\frac{B}{2}.x + By + \frac{5B}{2}.z = 0$

B(x + 2y + 5z)=0

x + 2y + 5z=0

So, required equation of plane is x + 2y + 5z=0.

Question: 14

$AA_1 + BB_1 + CC_1 = 0$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of other plane.

$3.A + 3.B - 2.C = 0$

$3A + 3B - 2C = 0$ *(1)*

$1.A + 2.B - 3C = 0$

$A + 2B - 3C = 0$ *(2)*

And plane contains the point (1,-1,2),

A(x-1) + B(y + 1) + C(z-2)=0 (3)

On solving equation (1) and (2)

$A = \frac{-5B}{7} \text{ and } C = \frac{3B}{7}$

Putting values in equation (3)

$\frac{-5B}{7}.(x-1)+B(y+1)+\frac{3B}{7}.(z-2) = 0$

$B(-5(x-1)+7(y+1)+3(z-2)) = 0$

-5x + 5 + 7y + 7 + 3z-6=0

-5x + 7y + 3z + 6=0

5x-7y-3z-6=0

For equation of plane Ax + By + Cz=D and point (x1,y1,z1), distance of a point from a plane can be calculated as

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

$$\left|\frac{5.(-2)-7.5-3.5-6}{\sqrt{(5)^2+(-7)^2+(-3)^2)}}\right| \Rightarrow \left|\frac{-10-35-15-6}{\sqrt{25}+49+9}\right| = \left|\frac{-66}{\sqrt{83}}\right| \Rightarrow \frac{66}{\sqrt{83}}$$

Question: 15

A(x-1) + B(y-1) + C(z-2)=0 (1)

A(x-2) + B(y + 2) + C(z-2)=0 (2)

Subtracting (1) from (2),

A(x-2-x + 1) + B(y + 2-y-1)=0

A-3B=0 (3)

Now plane is perpendicular to 6x-2y + 2z=9

6A-2B + 2C=0 (4)

Using (3) in (4)

18A-2B + 2C=0

16B + 2C=0

C=-8B

Putting values in equation (1)

3B(x-1) + B(y + 2)-8B(z-2)=0

B(3x-3 + y + 2-8z + 16)=0

3x + y-8z + 15=0

Question: 16

A(x + 1) + B(y-1) + C(z-1)=0 (1)

A(x-1) + B(y + 1) + C(z-1)=0 (2)

Subtracting (1) from (2),

A(x-1-x-1) + B(y + 1-y + 1)=0

-2A + 2B=0

A=B (3)

Now plane is perpendicular to x + 2y + 2z=5

A + 2B + 2C=0 (4)

Using (3) in (4)

B + 2B + 2C=0

3B + 2C=0

$$C = \frac{-3}{2}B$$

Putting values in equation (1)

$$B(x+1)+B(y-1)+\frac{-3}{2}B(z-1)=0$$

B(2(x + 1) + 2(y-1)-3(z-1)=0

2x + 2y-3z + 2-2-3=0

2x + 2y-3z-3=0

Question: 17

Find the equation

Solution:

Plane passes through (3,4,2) and (7,0,6), *A(x-3) + B(y-4) + C(z-2)=0 (1)*

A(x-7) + B(y-0) + C(z-6)=0 (2)

Subtracting (1) from (2),

A(x-7-x + 3) + B(y-y + 4) + C(z-6-z + 2)=0

-4A + 4B-4C=0

A-B + C=0

B=A + C (3)

Now plane is perpendicular to 2x-5y=15

2A-5B=0 (4)

Using (3) in (4)

2A-5(A + C)=0

2A-5A-5C=0

-3A-5C=0

$$C = \frac{-3}{5}A$$

$$B - A + \frac{-3}{5}A \Rightarrow \frac{2}{5}A$$

Putting values in equation (1)

$$A(x-3) + \frac{2}{5}A(y-4) + \frac{-3}{5}A(z-2) = 0$$

A(5(x-3) + 2(y-4)-3(z-2)=0

5x + 2y-3z-15-8 + 6=0

5x + 2y-3z-17=0

So, required equation of plane is 5x + 2y-3z-17=0.

Question: 18

Plane passes through (2,1,-1) and (-1,3,4),

A(x-2) + B(y-1) + C(z + 1)=0 (1)

A(x + 1) + B(y-3) + C(z-4)=0 (2)

Subtracting (1) from (2),

A(x + 1-x + 2) + B(y-3-y + 1) + C(z-4-z-1)=0

3A-2B-5C=0 (3)

Now plane is perpendicular to x-2y + 4z=10

A-2B + 4C=0 (4)

Using (3) in (4)

2A-9C=0

$$C - \frac{2}{9}A$$

$$2B = A + 4.\frac{2}{9}A \Rightarrow \left(\frac{9+8}{9}\right)A = \frac{17}{9}A$$

$$B = \frac{17}{18}A$$

Putting values in equation (1)

$$A(x-2) + \frac{17}{18}A(y-1) + \frac{2}{9}A(z+1) = 0$$

A(18(x-2) + 17(y-1) + 4(z + 1)=0

18x + 17y + 4z-36-17 + 4=0

18x + 17y + 4z-49=0

So, the required equation of plane is 18x + 17y + 4z-49=0

If plane contains $\bar{r} = -\hat{i} + 3\hat{j} + 4\hat{k} + \left(3\hat{i} - 2\hat{j} - 5\hat{k}\right)$ then $(-1, 3, 4)$ satisfies plane and normal vector of plane is perpendicular

LHS=18(-1) + 17.3 + 4.4-49

=-18 + 51 + 16-49

=-2 + 2=0=RHS

In vector form normal of plane

$$\vec{n} = 18\hat{i} + 17\hat{j} + 4\hat{k}$$

LHS=18.3 + 17(-2) + 4.(-5)=54-34-20=0=RHS

Hence line is contained in plane.

Exercise : 28G

Question: 1

Solution:

Given - $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ *and* $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) = 4$

To find *– The angle between the line and the plane*

Direction ratios of the line = (1, - 1, 1)

Direction ratios of the normal of the plane = (2, - 1, 1)

Formula to be used *– If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by*

$$\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{1 \times 2 + (-1) \times (-1) + 1 \times 1}{\sqrt{1^2 + 1^2 + 1^2}\sqrt{2^2 + 1^2 + 1^2}}\right)$$

$$= \sin^{-1}\left(\frac{2 + 1 + 1}{\sqrt{3}\sqrt{6}}\right)$$

$$= \sin^{-1}\left(\frac{4}{3\sqrt{2}}\right)$$

$$= \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

Question: 2

Solution:

Given - $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} - \hat{j} - 2\hat{k})$ *and* $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 3$

To find *– The angle between the line and the plane*

Direction ratios of the line = (3, - 1, 2)

Direction ratios of the normal of the plane = (1, 1, 1)

Formula to be used *– If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by*

$$\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{3 \times 1 + (-1) \times 1 + 2 \times 1}{\sqrt{3^2 + 1^2 + 2^2}\sqrt{1^2 + 1^2 + 1^2}}\right)$$

$$= \sin^{-1}\left(\frac{3 - 1 + 2}{\sqrt{14}\sqrt{3}}\right)$$

$$= \sin^{-1}\left(\frac{4}{\sqrt{42}}\right)$$

Question: 3

Solution:

Given - $\vec{r} = (3\hat{i} + \hat{k}) + \lambda(\hat{i} + \hat{k})$ *and* $\vec{r}.(2\hat{i} - \hat{j} + 2\hat{k}) = 1$

To find *– The angle between the line and the plane*

Direction ratios of the line = (0, 1, 1)

Direction ratios of the normal of the plane = (2, - 1, 2)

Formula to be used – *If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by*

$$\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{0\times 2 + 1\times(-1) + 1\times 2}{\sqrt{0^2 + 1^2 + 1^2}\sqrt{2^2 + 1^2 + 2^2}}\right)$$

$$= \sin^{-1}\left(\frac{-1 + 2}{3\sqrt{2}}\right)$$

$$= \mathbf{\sin^{-1}\left(\frac{1}{3\sqrt{2}}\right)}$$

Question: 4

Solution:

Given - $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ *and* $3x + 4y + z + 5 = 0$

To find – *The angle between the line and the plane*

Direction ratios of the line = (3, - 1, 2)

Direction ratios of the normal of the plane = (3, 4, 1)

Formula to be used – *If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by*

$$\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{3\times 3 + (-1)\times 4 + 2\times 1}{\sqrt{3^2 + 1^2 + 2^2}\sqrt{3^2 + 4^2 + 1^2}}\right)$$

$$= \sin^{-1}\left(\frac{9-4+2}{\sqrt{14}\sqrt{26}}\right)$$

$$= \sin^{-1}\left(\frac{7}{\sqrt{2}\sqrt{7}\times\sqrt{2}\times\sqrt{13}}\right)$$

$$= \mathbf{\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)}$$

Question: 5

Solution:

Given - $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ *and* $10x + 2y - 11z = 3$

To find – *The angle between the line and the plane*

Direction ratios of the line = (2, 3, 6)

Direction ratios of the normal of the plane = (10, 2, - 11)

Formula to be used – *If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by*

$$\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{2\times 10 + 3\times 2 + 6\times(-11)}{\sqrt{2^2 + 3^2 + 6^2}\sqrt{10^2 + 2^2 + 11^2}}\right)$$

$$= \sin^{-1}\left(\frac{20 + 6 - 66}{7\times 15}\right)$$

$$= \sin^{-1}\left(\frac{-40}{7\times 15}\right)$$

$$= \mathbf{\sin^{-1}\left(-\frac{8}{21}\right)}$$

Question: 6

Solution:

***Given** - A = (3, - 4, - 2) , B = (12, 2, 0) and 3x - y + z = 1*

***To find** – The angle between the line joining the points A and B and the plane*

***Tip** – If P = (a, b, c) and Q = (a', b', c'), then the direction ratios of the line PQ is given by ((a' - a), (b' - b), (c' - c))*

The direction ratios of the line AB can be given by

((12 - 3), (2 + 4), (0 + 2))

= (9, 6, 2)

Direction ratios of the normal of the plane = (3, - 1, 1)

***Formula to be used** – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by*

$$\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{9\times 3 + 6\times(-1) + 2\times 1}{\sqrt{9^2+6^2+2^2}\sqrt{3^2+1^2+1^2}}\right)$$

$$= \sin^{-1}\left(\frac{27-6+2}{11\times\sqrt{11}}\right)$$

$$= \mathbf{\sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)}$$

Question: 7

Solution:

***Given** -* $y - z - 0$ *and* $2x - 3y - 6z - 13$

***To find** – The angle between the line and the plane*

Direction ratios of the line = (1, 0, 0)

Direction ratios of the normal of the plane = (2, - 3, - 6)

***Formula to be used** – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by*

$$\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{1\times 2 + 0\times(-3) + 0\times(-6)}{\sqrt{1^2+0^2+0^2}\sqrt{2^2+3^2+9^2}}\right)$$

$$= \mathbf{\sin^{-1}\left(\frac{2}{7}\right)}$$

Question: 8

Solution:

***Given** -* $\vec{r} = (2\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda(\hat{i} + 3\hat{j} + 4\hat{k})$ *and* $\vec{r}.(\hat{i} + \hat{j} - \hat{k}) = 7$

***To prove** – The line and the plane are parallel &*

***To find** – The distance between them*

Direction ratios of the line = (1, 3, 4)

Direction ratios of the normal of the plane = (1, 1, - 1)

***Formula to be used** – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by*

$$\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{1\times 1 + 3\times 1 + 4\times(-1)}{\sqrt{1^2+3^2+4^2}\sqrt{1^2+1^2+1^2}}\right)$$

$= \sin^{-1}\left(\frac{1+3-4}{\sqrt{26}\sqrt{3}}\right)$

$= \sin^{-1}(0)$

$= 0$

Hence, the line and the plane are parallel.

Now, the equation of the plane may be written as x + y - z = 7.

Tip – *If ax + by + c + d = 0 be a plane and* $\vec{r} = (a'\hat{i} + b'\hat{j} + c'\hat{k}) + \lambda(a''\hat{i} + b''\hat{j} + c''\hat{k})$ *be a line vector, then the distance between them is given by* $\left|\frac{a \times a' + b \times b' + c \times c' + d}{\sqrt{a^2 + b^2 + c^2}}\right|$

The distance between the plane and the line

$= \left|\frac{1 \times 2 + 1 \times 5 - 1 \times 7 - 7}{\sqrt{1^2 + 1^2 + 1^2}}\right|$

$= \left|\frac{2 + 5 - 7 - 7}{\sqrt{3}}\right|$

$= \frac{7}{\sqrt{3}}$ ***units***

Question: 9

Solution:

Given - $\vec{r} = (\hat{i} + 2\hat{k}) + \lambda(2\hat{i} - m\hat{j} - 3\hat{k})$ *and* $\vec{r}.(m\hat{i} + 3\hat{j} + \hat{k}) = 4$ *and they are parallel*

To find – *The value of m*

Direction ratios of the line = (2, - m, - 3)

Direction ratios of the normal of the plane = (m, 3, 1)

Formula to be used – *If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by*

$\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$

$\therefore \sin^{-1}\left(\frac{2 \times m + (-m) \times 3 + (-3) \times 1}{\sqrt{2^2 + m^2 + 3^2}\sqrt{m^2 + 3^2 + 1^2}}\right) = 0$

$\Rightarrow \sin^{-1}\left(\frac{2m - 3m - 3}{\sqrt{13 + m^2}\sqrt{10 + m^2}}\right) = 0$

$\Rightarrow \frac{-m - 3}{\sqrt{13 + m^2}\sqrt{10 + m^2}} = 0$

$\Rightarrow \mathbf{m = -3}$

Question: 10

Solution:

Given - $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) = 3$

To find – *The vector equation of the line passing through the origin and perpendicular to the given plane*

Tip – *The equation of a plane can be given by* $\vec{r}.\hat{n} = d$ *where* $\hat{n}$ *is the normal of the plane*

A line parallel to the given plane will be in the direction of the normal and will have the direction ratios same as that of the normal.

Formula to be used – *If a line passes through the point (a, b, c) and has the direction ratios as (a', b', c'), then its vector equation is given by* $\vec{r} = (a\hat{i} + b\hat{j} + c\hat{k}) + \lambda(a'\hat{i} + b'\hat{j} + c'\hat{k})$ *where λ is any scalar constant*

The required equation will be $\vec{r} = (0.\hat{i} + 0.\hat{j} + 0.\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$= \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ ***for some scalar λ***

Question: 11

Solution:

Given - $\vec{r}.(2\hat{i} - 3\hat{j} - \hat{k}) = 0$ *and the vector has position vector* $(\hat{i} - 2\hat{j} + 5\hat{k})$

To find – *The vector equation of the line passing through (1, - 2, 5) and perpendicular to the given plane*

Tip – *The equation of a plane can be given by* $\vec{r}.\hat{n} = d$ *where* $\hat{n}$ *is the normal of the plane*

A line parallel to the given plane will be in the direction of the normal and will have the dir ratios same as that of the normal.

Formula to be used *– If a line passes through the point (a, b, c) and has the direction ratios as (a', b', c'), then its vector equation is given by* $\vec{r} = (a\hat{i} + b\hat{j} + c\hat{k}) + \lambda(a'\hat{i} + b'\hat{j} + c'\hat{k})$ *where λ is any scalar constant*

The required equation will be $\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} - 3\hat{j} - \hat{k})$ ***for some scalar λ***

Question: 12

Solution:

Given – *The equation of the plane is given by ax + by + d = 0*

To prove – *The plane is parallel to z - axis*

Tip – *If ax + by + cz + d is the equation of the plane then its angle with the z - axis will be given by* $\sin^{-1}\left(\frac{c}{\sqrt{a^2 + b^2 + c^2}}\right)$

Considering the equation, the direction ratios of its normal is given by (a, b, 0)

The angle the plane makes with the z - axis = $\sin^{-1}[0/\sqrt{(a^2 + b^2)}] = 0$

Hence, the plane is parallel to the z - axis

To find – *Equation of the plane parallel to z - axis and passing through points A = (2, - 3, 1) and B = (- 4, 7, 6)*

The given equation ax + by + d = 0 passes through (2, - 3, 1) & (- 4, 7, 6)

$\therefore 2a - 3b + d = 0$ ………*(i)*

$\therefore -4a + 7b + d = 0$ ……*(ii)*

Solving (i) and (ii),

$\therefore \frac{a}{\begin{vmatrix} -3 & 1 \\ 7 & 1 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 1 & 2 \\ 1 & -4 \end{vmatrix}} = \alpha$ *[α → arbitrary constant]*

$\therefore a = -10\alpha$

$\therefore b = -6\alpha$

Substituting the values of a and b in eqn (i), we get,

- 2X10α + 3X6α + d = 0 i.e. d = - 2α

Putting the value of a, b and d in the equation ax + by + d = 0,

(- 10α)x + (- 6α)y + (- 2α) = 0

i.e. ***5x + 3y + 1 = 0***

Question: 13

Solution:

Given – *A plane passes through points (1, 2, 3) and (0, - 1, 0) and is parallel to the line* $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$

To find – *Equation of the plane*

Tip – *If a plane passes through points (a', b', c'), then its equation may be given as a(x - a') + b(y - b') + c(z - c') = 0*

Taking points (1, 2, 3):

a(x - 1) + b(y - 2) + c(z - 3) = 0…………..*(i)*

The plane passes through (0, - 1, 0):

a(0 - 1) + b(- 1 - 2) + c(0 - 3) = 0

i.e. a + 3b + 3c = 0…………………*(ii)*

The plane is parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{3}$

Tip – *The normal of the plane will be normal to the given line since both the line and plane are parallel.*

Direction ratios of the line is (2, 3, - 3)

Direction ratios of the normal of the plane is (a, b, c)

So, 2a + 3b - 3c = 0…………………………*(iii)*

Solving equations (ii) and (iii),

$\therefore \frac{a}{\begin{vmatrix} 3 & 3 \\ 3 & -3 \end{vmatrix}} = -\frac{b}{\begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix}} = \alpha$ *[α → arbitrary constant]*

$\therefore a = -18\alpha$

$\therefore b = 9\alpha$

$\therefore c = -3\alpha$

Putting these values in equation (i) we get,

$-18\alpha(x-1) + 9\alpha(y-2) - 3\alpha(z-3) = 0$

$\Rightarrow 18(x-1) - 9(y-2) + 3(z-3) = 0$

$\Rightarrow 6(x-1) - 3(y-2) + (z-3) = 0$

$\Rightarrow 6x - 3y + z - 3 = 0$

$\Rightarrow \mathbf{6x - 3y + z = 3}$

Question: 14

Solution:

Given – *A plane passes through (2, - 1, 5), perpendicular to the plane x + 2y - 3z = 7 and parallel to the line* $\frac{x+5}{3} = \frac{y+1}{-1} = \frac{z-2}{1}$

To find – *The equation of the plane*

Let the equation of the required plane be ax + by + cz + d = 0……(a)

The plane passes through (2, - 1, 5)

So, 2a - b + 5c + d = 0………………… (i)

The direction ratios of the normal of the plane is given by (a, b, c)

Now, this plane is perpendicular to the plane x + 2y - 3z = 7 having direction ratios (1, 2, - 3)

So, a + 2b - 3c = 0……………(ii)

This plane is also parallel to the line having direction ratios (3, - 1, 1)

So, the direction of the normal of the required plane is also at right angles to the given line.

So, 3a - b + c = 0…………………(iii)

Solving equations (ii) and (iii),

$\therefore \frac{a}{\begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}} = \alpha$ *[α → arbitrary constant]*

$\therefore a = -\alpha$

$\therefore b = -10\alpha$

$\therefore c = -7\alpha$

Putting these values in equation (i) we get,

2X(- α) - (- 10α) + 5(- 7α) + d = 0 i.e. d = 27α

Substituting all the values of a, b, c and d in equation (a) we get,

$-\alpha x - 10\alpha y - 7\alpha z + 27\alpha = 0$

$\Rightarrow \mathbf{x + 10y + 7z + 27 = 0}$

Question: 15

Solution:

Given – *A plane passes through the intersection of 5x - y + z = 10 and x + y - z = 4 and parallel to the line with direction ratios (2, 1, 1)*

To find – *Equation of the plane*

Tip – *If ax + by + cz + d = 0 and a’x + b’y + c’z + d’ = 0 be two planes, then the equation of the plane passing through their intersection will be given by*

(ax + by + cz + d) + λ(a’x + b’y + c’z + d’) = 0, where λ is any scalar constant

So, the equation of the plane maybe written as

(5x - y + z - 10) + λ(x + y - z - 4) = 0

⇒ (5 + λ)x + (- 1 + λ)y + (1 - λ)z + (– 10 - 4λ) = 0

This is plane parallel to a line with direction ratios (2, 1, 1)

So, the normal of this line with direction ratios ((5 + λ), (- 1 + λ), (1 - λ)) will be perpendicular to *the given line.*

Hence,

2(5 + λ) + (- 1 + λ) + (1 - λ) = 0

$\Rightarrow \lambda = -5$

The equation of the plane will be

(5 + (- 5))x + (- 1 + (- 5))y + (1 - X(- 5))z + (- 10 - 4X(- 5)) = 0

$\Rightarrow -6y + 6z + 10 = 0$

$\Rightarrow$ ***3y - 3z = 5***

To find – *Perpendicular distance of point (1, 1, 1) from the plane*

Formula to be used - *If ax + by + c + d = 0 be a plane and (a', b', c') be the point, then the distance between them is given by* $\left|\frac{a \times a' + b \times b' + c \times c' + d}{\sqrt{a^2 + b^2 + c^2}}\right|$

The distance between the plane and the line

$= \left|\frac{0 \times 2 + 3 \times 1 - 3 \times 1 - 5}{\sqrt{0^2 + 3^2 + 3^2}}\right|$

$= \left|\frac{3 - 3 - 5}{2\sqrt{3}}\right|$

$= \frac{5}{2\sqrt{3}}$ ***units***

Exercise : 28H

Question: 1

Solution:

Given - $\vec{r} = \hat{i} + \hat{j} - \hat{k}$ & $\vec{r'} = 3\hat{i} - \hat{k}$ *are two lines to which a plane is parallel and it passes through the origin.*

To find – *The equation of the plane*

Tip – *A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product*

$\therefore \vec{r} \times \vec{r'}$

$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$

$= \hat{i}(-1 - 0) + \hat{j}(-3 + 1) + \hat{k}(0 - 3)$

$= -\hat{i} - 2\hat{j} - 3\hat{k}$

The plane passes through origin (0, 0, 0).

Formula to be used – *If a line passes through the point (a, b, c) and has the direction ratios as (a', b', c'), then its vector equation is given by* $\vec{r} = (a\hat{i} + b\hat{j} + c\hat{k}) + \lambda(a'\hat{i} + b'\hat{j} + c'\hat{k})$ *where λ is any scalar constant*

The required plane will be

$\vec{r} = (0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}) + \lambda'(-\hat{i} - 2\hat{j} - 3\hat{k})$

$\Rightarrow \vec{r} = \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

The vector equation : $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) = 0$

The Cartesian equation : ***x + 2y + 3z = 0***

Question: 3

Solution:

Given - $\vec{r} = (-\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 5\hat{j} - \hat{k})$ & $\vec{r} = (\hat{i} - 3\hat{j} + \hat{k}) + \mu(-5\hat{i} + 4\hat{j})$. *A plane is parallel to both these lines and passes through (3, - 1, 2).*

To find – *The equation of the plane*

Tip – *A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product*

$\vec{R} = 2\hat{i} - 5\hat{j} - \hat{k}$ & $\vec{R'} = -5\hat{i} + 4\hat{j}$, *where the two vectors represent the directions*

$\therefore \vec{R} \times \vec{R'}$

$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & -1 \\ -5 & 4 & 0 \end{bmatrix}$

$= \hat{i}(0 + 4) + \hat{j}(5 - 0) + \hat{k}(8 - 25)$

$- 4\hat{i} + 5\hat{j} - 17\hat{k}$

The equation of the plane maybe represented as 4x + 5y - 17z + d = 0

Now, this plane passes through the point (3, - 1, 2)

Hence,

4 × 3 + 5 × (- 1) – 17 × 2 + d = 0

□ d = 27

The Cartesian equation of the plane : ***4x + 5y - 17z + 27 = 0***

The vector equation : $\vec{\mathbf{r}}.(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 17\hat{\mathbf{k}}) + 27 = 0$

Question: 3

Solution:

Given – *The lines have direction ratios of (1, - 1, - 2) and (- 1, 0, 2). The plane parallel to these lines passes through (1, 2, 3)*

To find – *The vector equation of the plane*

Tip – *A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product*

$\vec{R} = \hat{i} - \hat{j} - 2\hat{k}$ & $\vec{R'} = -\hat{i} + 2\hat{k}$, *where the two vectors represent the directions*

$\therefore \vec{R} \times \vec{R'}$

$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{bmatrix}$

$= \hat{i}(-2 - 0) + \hat{j}(2 - 2) + \hat{k}(0 - 1)$

$= -2\hat{i} - \hat{k}$

The equation of the plane maybe represented as - 2x - z + d = 0

Now, this plane passes through the point (1, 2, 3)

Hence,

(- 2) × 1 - 3 + d = 0

□ d = 5

The Cartesian equation of the plane : - 2x - z + 5 = 0 i.e. 2x + z = 5

The vector equation : $\vec{\mathbf{r}}.(2\hat{\mathbf{i}} + \hat{\mathbf{k}}) = 5$

Question: 4

Find the Cartesia

Solution:

Given - $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ & $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z}{-1}$. *A plane is parallel to both these lines and passes through (1, 2, - 4).*

To find – *The equation of the plane*

Tip – *A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product*

The direction ratios of the given lines are (2, 3, 6) and (1, 1, - 1)

$\therefore \vec{R} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ & $\vec{R'} = \hat{i} + \hat{j} - \hat{k}$

$\therefore \vec{R} \times \vec{R'}$

$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{bmatrix}$

$= \hat{i}(-3 - 6) + \hat{j}(6 + 2) + \hat{k}(2 - 3)$

$= -9\hat{i} + 8\hat{j} - \hat{k}$

The equation of the plane maybe represented as - 9x + 8y - z + d = 0

Now, this plane passes through the point (1, 2, - 4)

Hence,

(- 9) × 1 + 8 × 2 - (- 4) + d = 0

 d = - 11

The Cartesian equation of the plane : - 9x + 8y - z - 11 = 0 i.e. ***9x - 8y + z + 11 = 0***

The vector equation : $\vec{r}.\left(9\hat{i} - 8\hat{j} + \hat{k}\right) + 11 = 0$

Question: 5

Find

Solution:

Given - $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k}$ *&* $\vec{r'} = \hat{i} - \hat{j} + \hat{k}$ *are two lines to which a plane is parallel and it passes through the point* $3\hat{i} + 4\hat{j} + 2\hat{k}$

To find – *The equation of the plane*

Tip – *A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product*

$\therefore \vec{r} \times \vec{r'}$

$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$

$= \hat{i}(2 + 3) + \hat{j}(3 - 1) + \hat{k}(-1 - 2)$

$= 5\hat{i} + 2\hat{j} - 3\hat{k}$

The equation of the plane maybe represented as 5x + 2y - 3z + d = 0

Now, this plane passes through the point (3, 4, 2)

Hence,

5 × 3 + 2 × 4 – 3 × 2 + d = 0

 d = - 17

The Cartesian equation of the plane : 5x + 2y - 3z - 17 = 0 i.e. 5x + 2y - 3z = 17

The vector equation $\vec{r}.\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 17$