Chapter : 29. PROBABILITY

Exercise : 29A

Question: 1

Solution:

Given - A and B be the events such that $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and

$P(A \cap B) = \frac{4}{13}$

To find – (i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(A \cup B)$ (iv) $P(\bar{B}/\bar{A})$

Formula to be used – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i)$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{4}{13} \div \frac{9}{13}$

$= \frac{4}{9}$

(ii)$P(B/A)$

$= \frac{P(A \cap B)}{P(A)}$

$= \frac{4}{13} \div \frac{7}{13}$

$= \frac{4}{7}$

(iii) $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{7}{13} + \frac{9}{13} - \frac{4}{13}$

$= \frac{12}{13}$

(iv) $P(\bar{B}/\bar{A}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})}$

Now, by De-Morgan's Law, $(A \cup B)^C = A^C \cap B^C$

$\therefore P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$\therefore \frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})}$

$= \frac{P(\overline{A \cup B})}{P(\bar{A})}$

$= \frac{1 - P(A \cup B)}{1 - P(A)}$

$$= \frac{1 - \frac{12}{13}}{1 - \frac{7}{13}}$$

$$= \frac{1}{6}$$

Question: 2

Solution:

Given - A and B be the events such that $P(A) = \frac{5}{11}$, $P(B) = \frac{6}{11}$ and

$$P(A \cup B) = \frac{7}{11}$$

To find – (i) $P(A \cap B)$ (ii) $P(A/B)$ (iii)$P(B/A)$ (iv) $P(\bar{A}/\bar{B})$

Formula to be used – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{5}{11} + \frac{6}{11} - \frac{7}{11}$$

$$= \frac{4}{11}$$

(ii) $P(A/B)$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{4}{11} \div \frac{6}{11}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

(iii)$P(B/A)$

$$= \frac{P(A \cap B)}{P(A)}$$

$$= \frac{4}{11} \div \frac{5}{11}$$

$$= \frac{4}{5}$$

(iv) $P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$

Now, by De-Morgan's Law, $(A \cup B)^C = A^C \cap B^C$

$\therefore P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$$\therefore \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$$

$$= \frac{P(\overline{A \cup B})}{P(\bar{A})}$$

CLASS24

$= \frac{1 - P(A \cup B)}{1 - P(B)}$

$= \frac{1 - \frac{7}{11}}{1 - \frac{6}{11}}$

$= \frac{4}{5}$

Question: 3

Solution:

Given - A and B be the events such that $P(A) = \frac{3}{10}$, $P(B) = \frac{1}{2}$ and

$P(B/A) = \frac{2}{5}$

To find – (i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A/B)$

Formula to be used – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i)$P(B/A) = \frac{P(A \cap B)}{P(A)}$

⇨ $P(A \cap B) = P(A)P(B/A)$

$= \frac{3}{10} \times \frac{2}{5}$

$= \frac{3}{25}$

(ii) $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{3}{10} + \frac{1}{2} - \frac{3}{25}$

$= \frac{15 + 25 - 6}{50}$

$= \frac{34}{50}$

$= \frac{17}{50}$

(iii) $P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{3}{25} \div \frac{1}{2}$

$= \frac{6}{25}$

Question: 4

Solution:

Given - A and B be the events such that $2P(A) = P(B) = \frac{5}{13}$ and

$P(A/B) = \frac{2}{5}$

To find – (i) $P(A \cap B)$ (ii) $P(A \cup B)$

Formula to be used – By conditional probability, $P(A/B) = \frac{P(A\cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i)$P(A/B) = \frac{P(A\cap B)}{P(B)}$

$\Rightarrow P(A \cap B) = P(B)P(A/B)$

$= \frac{5}{13} \times \frac{2}{5}$

$= \frac{2}{13}$

(ii) $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$

$= \frac{5 + 10 - 4}{26}$

$= \frac{11}{26}$

Question: 5

Solution:

A die has 6 faces and its sample space S={1,2,3,4,5,6}.

The total number of outcomes = 6.

Let P(A) be the probability of getting an even number.

The sample space of A = {2,4,6}

$\therefore P(A) = \frac{3}{6} = \frac{1}{2}$

Let P(B) be the probability of getting a number whose value is greater than 2.

The sample space of B = {3,4,5,6}

$\therefore (A \cap B) = \{4,6\}$

$\therefore P(A \cap B) = \frac{2}{6} = \frac{1}{3}$

Tip – By conditional probability, $P(A/B) = \frac{P(A\cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability of getting a number greater than 2 given that the outcome is even is given by:

$P(B/A)$

$= \frac{P(A \cap B)}{P(A)}$

$= \frac{1/3}{1/2}$

$= \frac{2}{3}$

Question: 6

Solution:

A coin has 2 sides and its sample space S={H,T}

The total number of outcomes = 2.

A coin is tossed twice.

Let P(A) be the probability of getting at most 1 tail.

The sample space of A = {(H,H),(H,T),(T,H)}

Let P(B) be the probability of getting a head.

The sample space of B = {H}

$\therefore P(B) = \frac{1}{2}$

The probability of getting at most one tail and a head

$\text{i.e.} (A \cap B) = \{(H, H)\}$

$\therefore P(A \cap B) = \frac{1}{3}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that both head and tail have appeared:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/3}{1/2}$

$= \frac{2}{3}$

Question: 7

Solution:

When three coins are tossed simultaneously, the total number of outcomes = 2^3 = 8, and the sample space is given by S= {(H,H,H),(H,H,T),(H,T,T),(H,T,H),(T,H,T),(T,T,H),(T,H,H),(T,T,T)}

Let P(A) be the probability of getting 3 heads.

The sample space of A = {(H,H,H)}

$\therefore P(A) = \frac{1}{8}$

Let P(B) be the probability of getting at least head.

Probability of one head = 1 – probability of no heads = 1-1/8 = 7/8

$\therefore P(B) = \frac{7}{8}$

The probability that the throw is either all heads or at least one head i.e. $P(A \cup B) = \frac{}{8}$

Now,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B)$

$= P(A) + P(B) - P(A \cup B)$

$= \frac{1}{8} + \frac{7}{8} - \frac{7}{8}$

$= \frac{1}{8}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that all coins show heads if at least one of the coins

showed a head:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/8}{7/8}$

$= \frac{1}{7}$

Question: 8

Solution:

Two die having 6 faces each when tossed simultaneously will have a total outcome of 6^2=36

Let P(A) be the probability of getting a sum greater than 8.

Let P(B) be the probability of getting 4 on the first die.

The sample space of B = {(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)}

$\therefore P(B) = \frac{6}{36} = \frac{1}{6}$

Let $P(A \cap B)$ be the probability of getting 4 on the first die and the sum greater than or equal to 8

The sample space of $(A \cap B) = \{(4,4), (4,5), (4,6)\}$

$\therefore P(A \cap B) = \frac{3}{36} = \frac{1}{12}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that sum of the numbers is greater than or equal to 8 given that 4 was thrown first:

$P(A/B)$

CLASS24

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/12}{1/6}$$

$$= \frac{1}{2}$$

Question: 9

Solution:

A die thrown twice will have a total outcome of 6^2=36.

Let P(A) be the probability of getting the number 5 at least once.

Let P(B) be the probability of getting sum = 8.

The sample space of B = {(2,6),(3,5),(4,4),(5,3),(6,2)}

$$\therefore P(B) = \frac{5}{36}$$

Let $P(A \cap B)$ be the probability of getting the number 5 at least once and the sum equal to 8

The sample space of $(A \cap B) = \{(3,5), (5,3)\}$

$$\therefore P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that the number 5 have appeared at least once given that the sum = 8:

P(A/B)

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/18}{5/36}$$

$$= \frac{2}{5}$$

Question: 10

Solution:

Two die having 6 faces each when tossed simultaneously will have a total outcome of 6^2=36

Let P(A) be the probability of getting a sum equal to 5.

Let P(B) be the probability of getting 2 different numbers.

Probability of getting 2 different numbers

= 1 – probability of getting same numbers

= 1 – 1/6

=5/6

$$\therefore P(B) = \frac{5}{6}$$

Let $P(A \cap B)$ be the probability of getting a sum = 5 and two different numbers at the same time

The sample space of $(A \cap B) = \{(1,4),(2,3),(3,2),(4,1)\}$

$\therefore P(A \cap B) = \frac{4}{36} = \frac{1}{9}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that the sum = 5 given that two different numbers were thrown:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/9}{5/6}$

$= \frac{2}{15}$

Question: 11

Solution:

A coin is tossed and a die thrown.

A coin having two sides have a total outcome of 2 viz. {H,T}

A die has 6 faces and will have a total outcome of 6 i.e. {1, 2,3,4,5,6}

Let P(A) be the probability of getting the number 6.

$\therefore P(A) = \frac{1}{6}$

Let P(B) be the probability of getting a head.

The sample space of B ={H}

$\therefore P(B) = \frac{1}{2}$

Let $P(A \cap B)$ be the probability of getting the number 6 and a head.

$\therefore P(A \cap B) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that 6 came up given that head came up:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/12}{1/2}$

$= \frac{1}{6}$

Question: 12

CLASS24

Solution:

A couple has two children.

The sample space S = {(B,B),(B,G),(G,B),(G,G)}

Let P(A) be the probability of both being boys.

(i) Let P(B) be the probability of one of them being a boy.

The sample space of B ={(B,B),(B,G),(G,B)}

$\therefore P(B) = \frac{3}{4}$

Let $P(A \cap B)$ be the probability of one of them being a boy and both being boys.

$\therefore (A \cap B) = \{(B, B)\}$

$\therefore P(A \cap B) = \frac{1}{4}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that both are boys given that one of them is a boy:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/4}{3/4}$

$= \frac{1}{3}$

(ii) Let P(B) be the probability of the elder being a boy.

The sample space of B ={(B,B),(B,G)}

$\therefore P(B) = \frac{1}{2}$

Let $P(A \cap B)$ be the probability of the elder being a boy and both being boys.

$\therefore (A \cap B) = \{(B, B)\}$

$\therefore P(A \cap B) = \frac{1}{4}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that both are boys given that the elder is a boy:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/4}{1/2}$

$= \frac{1}{2}$

CLASS24

Question: 13

Solution:

Let P(A) be the probability of students studying mathematics.

$\therefore P(A)=0.40$

Let P(B) be the probability of students studying biology.

$\therefore P(B)=0.25$

Let $P(A \cap B)$ be the probability of students studying both mathematics and biology.

$\therefore P(A\cap B)=0.15$

One student is selected at random.

Tip – By conditional probability, $P(A/B) = \frac{P(A\cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

(i) The probability that he studies mathematics given that he studies biology:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{0.15}{0.25}$

$= \frac{3}{5}$

(ii) The probability that he studies biology given that he studies mathematics:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{0.15}{0.40}$

$= \frac{3}{8}$

Question: 14

Solution:

One student is selected at random.

Let P(A) be the probability of students passing in English.

Let P(B) be the probability of students passing in Hindi.

$\therefore P(B) = \frac{4}{5}$

Let $P(A \cap B)$ be the probability of students passing in both English and Hindi.

$\therefore P(A \cap B) = \frac{1}{2}$

Tip – By conditional probability, $P(A/B) = \frac{P(A\cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that he will pass in English given that he passes in Hindi:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/2}{4/5}$

$= \frac{5}{8}$

Question: 15

Solution:

Let P(A) be the probability of a certain person buying a shirt.

$\therefore P(A)=0.2$

Let P(B) be the probability of him buying a coat.

$\therefore P(B)=0.3$

Let $P(A \cap B)$ be the probability that he buys both a shirt and a coat.

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

The probability that he will buy a shirt given that he buys a coat:

$P(A/B) = \frac{P(A \cap B)}{P(B)} = 0.4$

$\Rightarrow P(A \cap B)=P(B)P(A/B)$

$=0.3 \times 0.4$

$= 0.12$

Question: 16

Solution:

Let P(A) be the probability of students reading Hindi newspaper.

$\therefore P(A)=0.60$

Let P(B) be the probability of them reading English newspaper.

$\therefore P(B)=0.40$

Let $P(A \cap B)$ be the probability them reading both.

$\therefore P(A \cap B) = 0.20$

Let $P(A \cup B)$ be the probability them reading either one of them.

$\therefore P(A \cup B)$

$=P(A)+P(B)-P(A \cap B)$

$=0.60+0.40-0.20$

$=0.80$

(i) The probability that none of them reads either of them

$= 1 - 0.8$

$= 0.2$

$=1/5$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability o*

the event A given that B has already occurred.

(ii) The probability that he reads the English one given that he reads the Hindi one:

P(A/B)

$= \frac{P(A \cap B)}{P(A)}$

$= \frac{0.20}{0.60}$

$= \frac{1}{3}$

(iii) The probability that he reads the Hindi one given that he reads the English one:

P(A/B)

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{0.20}{0.40}$

$= \frac{1}{2}$

Question: 17

Solution:

Two integers are selected at random.

The first choice has 11 options from the 11 integers, and the second choice has 10 options from the remaining 10 integers.

Let P(A) be the probability of choosing both numbers odd.

Let P(B) be the probability of choosing the numbers to yield an even number.

Sample space of B = {(1,3),(1,5),(1,7),(1,9),(1,11),(3,5),(3,7),(3,9),(3,11),(5,7),(5,9),(5,11),(7,9),(7,11),(9,11),(2,4),(2,6),(2,8),(2,10),(4,6),(4,8),(4,10),(6,8),(6,10),(8,10)}

$\therefore P(B) = \frac{25}{11 \times 10} = \frac{25}{110}$

Let $P(A \cap B)$ be the probability of getting both odd numbers giving an even sum.

∴(A∩B)={(1,3),(1,5),(1,7),(1,9),(1,11),(3,5),(3,7),(3,9),(3,11),(5,7),

(5,9),(5,11),(7,9),(7,11),(9,11),}

$\therefore P(A \cap B) = \frac{15}{110}$

The probability of getting both numbers odd given that sum is even:

P(A/B)

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{15/_{110}}{25/_{110}}$

$= \frac{15}{25}$

$= \frac{3}{5}$

Exercise : 29B

Question: 1

Solution:

Given: A bag contains 17 tickets , numbered 1 to 17, and each trial is independent of the other.

Hence the sample space is given by S = {1,2,3,……,17}

To find: the probability that both the tickets are drawn show even numbers.

Let , success : ticket drawn is even.i.e $\frac{8}{17}$

Now , the Probability of success in the first trial is

P_1(success) = $\frac{8}{17}$

Probability of success in the second trial without replacement of the first draw is given by

P_2(success) = $\frac{7}{16}$

Hence , the probability that both the tickets show even numbers with each trial being independent is given by

$P_1 \times P_2 = \frac{8}{17} \times \frac{7}{16} = \frac{7}{34}$

Question: 2

Solution:

Given: A box containing 3 black and 4 white marbles .Each trail is independent of the other trial.

Hence the sample space is given by S = {1B,2B,3B,1W,2W,3W,4W}

To find: the probability that both the marbles are drawn are black.

Let , success : marble drawn is black.i.e $\frac{3}{7}$

Now , the Probability of success in the first trial is

P_1(success) = $\frac{3}{7}$

Probability of success in the second trial without replacement of the first draw is given by

P_2(success) = $\frac{2}{6}$

Hence , the probability that both the marbles are drawn are black ,with each trial being independent is given by

$P_1 \times P_2 = \frac{3}{7} \times \frac{2}{6} = \frac{1}{7}$

Question: 3

Solution:

Given: a well shuffled deck of 52 cards. Each draw is independent of the other.

To find: the probability that the first card is drawn is a club and the second card is a spade.

Let , success for the first trail be getting a club.

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{13}{52}$

let , success for the second trail be getting a spade.

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{13}{51}$

Hence , the probability that the first card is drawn is a club and the second card is a spade ,with each trial being independent is given by

$P_1 \times P_2 = \frac{13}{52} \times \frac{13}{51} = \frac{13}{204}$

Question: 4

Solution:

Given: A box containing 30 bulbs of which 5 are defective.Each trail is independent of the other trial.

To find: the probability that both the bulbs are chosen are defective.

Let , success :bulb chosen is defective .i.e $\frac{5}{30}$

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{5}{30}$

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{4}{29}$

Hence , the probability that both the bulbs are chosen are defective,with each trial being independent is given by

$P_1 \times P_2 = \frac{5}{30} \times \frac{4}{29} = \frac{2}{87}$

Question: 5

Solution:

Given: A bag containing 10 white and 15 black balls .Each trial is independent of the other trial.

To find: the probability that the first ball is drawn is white and the second ball drawn is black.

Let , success in the first draw be getting a white ball.

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{10}{25}$

Let success in the second draw be getting a black ball.

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{15}{24}$

Hence , the probability that the first ball is drawn is white and the second ball drawn is black,with each trial being independent is given by

$P_1 \times P_2 = \frac{10}{25} \times \frac{15}{24} = \frac{1}{4}$

Question: 6

Solution:

Given: An urn containing 5 white and 8 black balls .Each trial is independent of the other trial.

To find: the probability that the first draws gives 3 white and the second draw gives 3 black balls.

Let , success in the first draw be getting 3 white balls.

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{{}^5c_3}{{}^{13}c_3} = \frac{10}{286} = \frac{5}{143}$

Let success in the second draw be getting 3 black balls.

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{{}^8c_3}{{}^{10}c_3} = \frac{56}{120} = \frac{7}{15}$

Hence , the probability that the first draws gives 3 white and the second draw gives 3 black balls,with each trial being independent is given by

$P_1 \times P_2 = \frac{5}{143} \times \frac{7}{15} = \frac{7}{429}$

Question: 7

Solution:

Given: E_1 and E_2 are two events such that $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{3}{5}$

To Find: i)$P(E_1 \cup E_2)$ when E_1 and E_2 are mutually exclusive.

We know that,

When two events are mutually exclusive $P(E_1 \cap E_2) = 0$

Hence, $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$= \frac{1}{3} + \frac{3}{5}$

$= \frac{14}{15}$

Therefore , $P(E_1 \cup E_2) = \frac{14}{15}$ when E_1 and E_2 are mutually exclusive.

ii) $P(E_1 \cap E_2)$ when E_1 and E_2 are independent.

We know that when E_1 and E_2 are independent ,

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= \frac{1}{3} \times \frac{3}{5}$

$= \frac{1}{5}$

Therefore, $P(E_1 \cap E_2) = \frac{1}{5}$ when E_1 and E_2 are independent.

Question: 8

Solution:

Given: E_1 and E_2 are two events such that $P(E_1) = \frac{1}{4}$ and $P(E_2) = \frac{1}{3}$ and

$P(E_1 \cup E_2) = \frac{1}{2}$

To show: E_1 and E_2 are independent events.

We know that,

Hence, $P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1 \cup E_2)$

$= \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$

$= \frac{1}{12}$ Equation 1

Since The condition for two events to be independent is

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{12}$ Equation 2

Since, Equation 1 = Equation 2

$\Rightarrow$ E_1 and E_2 are independent events.

Hence proved.

Question: 9

Solution:

Given: E_1 and E_2 are two independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$

To Find: i)$P(E_1 \cap E_2)$

We know that,

when E_1 and E_2 are independent ,

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= 0.3 \times 0.4$

$= 0.12$

Therefore, $P(E_1 \cap E_2) = 0.12$ when E_1 and E_2 are independent.

ii) $P(E_1 \cup E_2)$ when E_1 and E_2 are independent.

We know that,

Hence, $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= 0.3 + 0.4 - (0.3 \times 0.4)$

$= 0.58$

Therefore , $P(E_1 \cup E_2) = 0.58$ when E_1 and E_2 are Independent.

iii) $P(\overline{E_1} \cap \overline{E_2}) = P(\overline{E_1}) \times P(\overline{E_2})$

since , $P(E_1) = 0.3$ and $P(E_2) = 0.4$

⇒ $P(\overline{E_1}) = 1 - P(E_1) = 0.7$ and $P(\overline{E_2}) = 1 - P(E_2) = 0.6$

Since, E_1 and E_2 are two independent events

⇒ $\overline{E_1}$ and $\overline{E_2}$ are also independent events.

Therefore, $P(\overline{E_1} \cap \overline{E_2}) = 0.7 \times 0.6 = 0.42$

iv) $P(\overline{E_1} \cap E_2) = P(\overline{E_1}) \times P(E_2)$

$= 0.7 \times 0.4$

$= 0.28$

Therefore , $P(\overline{E_1} \cap E_2) = 0.28$

Question: 10

Solution:

Given: A and B are the events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{7}{12}$ and

P(not A or not B) = $\frac{1}{4}$

To Find: i)If A and B are mutually exclusive

Since P(not A or not B) = $\frac{1}{4}$ i.e., $P(\bar{A} \cup \bar{B}) = \frac{1}{4}$

we know that , $P(\bar{A} \cup \bar{B}) = P(A \cap B)' = 1 - P(A \cap B) = \frac{1}{4}$

⇒ $P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}$ Equation 1

Since for two mutually exclusive events $P(A \cap B) = 0$

But here $P(A \cap B) \neq 0$

Therefore , A and B are not mutually exclusive.

ii)If A and B are independent

The condition for two events to be independent is given by

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= \frac{1}{2} \times \frac{7}{12}$

$= \frac{7}{24}$ Equation 2

Since Equation 1 ≠ Equation 2

⇒ A and B are not independent

Question: 11

Solution:

Given: let A denote the event 'kamal is selected' and let B denote the event 'vimal is selected'.

Therefore , $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$

Also, A and B are independent .A and not B are independent, not A and B are independent.

To Find:The probability that only one of them will be selected.

Now ,

P(only one of them is selected) = P(A and not B or B and not A)

= P(A and not B) + (B and not A)

= P($A \cap \bar{B}$) + P($B \cap \bar{A}$)

= P(A) ×P($\bar{B}$) + P(B) × P($\bar{A}$)

= P(A) ×[1-P(B)]+ P(B) × [1-P(A)]

= $\frac{1}{3}[1-\frac{1}{5}] + \frac{1}{5}[1-\frac{1}{3}]$

= $\frac{4}{15} + \frac{2}{15}$

= $\frac{2}{5}$

Therefore , The probability that only one of them will be selected is $\frac{2}{5}$

Question: 12

Solution:

Given : let A denote the event 'Arun is selected' and let B denote the event 'ved is selected'.

Therefore , P(A) = $\frac{1}{4}$ and P($\bar{B}$) = $\frac{2}{3}$ ⇒P(B) = $\frac{1}{3}$ and P($\bar{A}$) = $\frac{3}{4}$

Also, A and B are independent .A and not B are independent, not A and B are independent.

To Find: The probability that atleast one of them will get selected.

Now,

P(atleast one of them getting selected) = P(selecting only Arun) + P(selecting only ved) + P(selecting both)

= P(A and not B) +P (B and not A) +P (A and B)

= P($A \cap \bar{B}$) + P($B \cap \bar{A}$) + P($A \cap B$)

= P(A) ×P($\bar{B}$) + P(B) × P($\bar{A}$) + P(A) ×P(B)

= $(\frac{1}{4} \times \frac{2}{3}) + (\frac{1}{3} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{1}{3})$

= $\frac{2}{12} + \frac{3}{12} + \frac{1}{12}$

= $\frac{1}{2}$

Therefore , The probability that atleast one of them will get selected is $\frac{1}{2}$

Question: 13

Solution:

Given : A and B appear for an interview ,then P(A) = $\frac{1}{6}$ and P(B) = $\frac{1}{4}$ ⇒ P($\bar{A}$) = $\frac{5}{6}$ and P($\bar{B}$) = $\frac{3}{4}$

Also, A and B are independent .A and not B are independent, not A and B are independent.

To Find: i) The probability that both of them are selected.

We know that, P(both of them are selected) = P($A \cap B$) = P(A) × P(B)

$= \frac{1}{6} \times \frac{1}{4}$

$= \frac{1}{24}$

Therefore , The probability that both of them are selected is $\frac{1}{24}$

ii) P(only one of them is selected) = P(A and not B or B and not A)

= P(A and not B) + (B and not A)

$= P(A \cap \bar{B}) + P(B \cap \bar{A})$

$= P(A) \times P(\bar{B}) + P(B) \times P(\bar{A})$

$= (\frac{1}{6} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{5}{6})$

$= \frac{3}{24} + \frac{5}{24}$

$= \frac{1}{3}$

Therefore, the probability that only one of them Is selected is $\frac{1}{3}$

iii)none is selected

we know that P(none is selected) = $P(\bar{A} \cap \bar{B})$

$= P(\bar{A}) \times P(\bar{B})$

$= \frac{5}{6} \times \frac{3}{4}$

$= \frac{5}{8}$

Therefore , the probability that none is selected is $\frac{5}{8}$

iv) atleast one of them is selected

Now, P(atleast one of them is selected) = P(selecting only A) + P(selecting only B) + P(selecting both)

= P(A and not B) +P (B and not A) +P (A and B)

$= P(A \cap \bar{B}) + P(B \cap \bar{A}) + P(A \cap B)$

$= P(A) \times P(\bar{B}) + P(B) \times P(\bar{A}) + P(A) \times P(B)$

$= (\frac{1}{6} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{5}{6}) + (\frac{1}{6} \times \frac{1}{4})$

$= \frac{3}{24} + \frac{5}{24} + \frac{1}{24}$

$= \frac{3}{8}$

Therefore, the probability that atleast one of them is selected is $\frac{3}{8}$

Question: 14

Solution:

Given : Here probability of A and B that can solve the same problem is given , i.e., $P(A) = \frac{2}{3}$ and $P(B) = \frac{3}{5} \Rightarrow P(\bar{A}) = \frac{1}{3}$ and $P(\bar{B}) = \frac{2}{5}$

Also, A and B are independent . not A and not B are independent.

To Find: i) atleast one of A and B will solve the problem

Now , P(atleast one of them will solve the problem) = 1 – P(both are unable to solve)

$= 1 - P(\bar{A} \cap \bar{B})$

$= 1 - P(\bar{A}) \times P(\bar{B})$

$= 1 - (\frac{1}{3} \times \frac{2}{5})$

$= \frac{13}{15}$

Therefore , atleast one of A and B will solve the problem is $\frac{13}{15}$

ii) none of the two will solve the problem

Now, P(none of the two will solve the problem) = $P(\bar{A} \cap \bar{B})$

$= P(\bar{A}) \times P(\bar{B})$

$= \frac{1}{3} \times \frac{2}{5}$

$= \frac{2}{15}$

Therefore , none of the two will solve the problem is $\frac{2}{15}$

Question: 15

Solution:

Given : let A , B and C be three students whose chances of solving a problem is given i.e , $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{5}$ and $P(C) = \frac{1}{6}$.

$\Rightarrow P(\bar{A}) = \frac{3}{4}$, $P(\bar{B}) = \frac{4}{5}$ and $P(\bar{C}) = \frac{5}{6}$

To Find: The probability that the problem is solved .

Here , P(the problem is solved) = 1 - P(the problem is not solved)

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - [P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})]$

$= 1 - [\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}]$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

Therefore , The probability that the problem is solved is $\frac{1}{2}$.

Question: 16

Solution:

Given : let A , B and C be three students whose chances of solving a problem is given i.e , $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{6}$.

$\Rightarrow P(\bar{A}) = \frac{2}{3}$, $P(\bar{B}) = \frac{3}{4}$ and $P(\bar{C}) = \frac{5}{6}$

To Find: The probability that excatly one of them will solve it .

Now, P(excatly one of them will solve it) = P(A and not B and not c) +P (B and not A and not C) +P (C and not A and not B)

$= P(A \cap \bar{B} \cap \bar{C}) + P(B \cap \bar{A} \cap \bar{C}) + P(C \cap \bar{A} \cap \bar{B})$

$= P(A) \times P(\bar{B}) \times P(\bar{C}) + P(B) \times P(\bar{A}) \times P(\bar{C}) + P(C) \times P(\bar{B}) \times P(\bar{A})$

$= [\frac{1}{3} \times \frac{3}{4} \times \frac{5}{6}] + [\frac{1}{4} \times \frac{2}{3} \times \frac{5}{6}] + [\frac{1}{6} \times \frac{3}{4} \times \frac{2}{3}]$

$= \frac{15}{72} + \frac{10}{72} + \frac{6}{72}$

$= \frac{31}{72}$

Therefore , The probability that excatly one of them will solve the problem is $\frac{31}{72}$

Question: 17

Solution:

Given : let A , B and C chances of hitting a target is given i.e , $P(A) = \frac{4}{5}$, $P(B) = \frac{3}{4}$ and $P(C) = \frac{2}{3}$.

$\Rightarrow P(\bar{A}) = \frac{1}{5}$, $P(\bar{B}) = \frac{1}{4}$ and $P(\bar{C}) = \frac{1}{3}$

To Find: i)The probability that A , B and C all hit the target.

Now ,P(all hitting the target) = $P(A \cap B \cap C)$

$= P(A) \times P(B) \times P(C)$

$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$

$= \frac{2}{5}$

Hence , The probability that A , B and C all hit the target is $\frac{2}{5}$

ii) B and C hit and A does not hit the target

Here, P(B and C hit and not A) = $P(B \cap C \cap \bar{A})$

$= P(B) \times P(C) \times P(\bar{A})$

$= \frac{3}{4} \times \frac{2}{3} \times \frac{1}{5}$

$= \frac{1}{10}$

Hence , the probability that B and C hit and A does not hit the target is $\frac{1}{10}$

Question: 18

Solution:

Given : let A , B and C represent the subjects physics,chemistry and mathematics respectively ,the probability of neelam getting A grade in these three subjects is given i.e , P(A) = 0.2 , P(B) = 0.3 and P(C) =0.9

$\Rightarrow P(\bar{A}) = 0.8$, $P(\bar{B}) = 0.7$ and $P(\bar{C}) = 0.1$

To Find: i)The probability that neelam gets all A grades

Here, P(getting all A grades) = P($A \cap B \cap C$)

= P(A) ×P(B) × P(C)

=0.2 × 0.3 × 0.9

= 0.054

Therefore, The probability that neelam gets all A grades is 0.054.

ii)no A grade

Here , P(getting no A grade) = P($\bar{A} \cap \bar{B} \cap \bar{C}$)

= P($\bar{A}$) × P($\bar{B}$) × P($\bar{C}$)

= 0.8 × 0.7 ×0.1

= 0.056

Therefore, The probability that neelam gets no A grade is 0.056.

iii)excatly 2 a grades

P(getting excatly 2 A grades) = P(A and B and not c) +P (B and C and not A) +P (C and A and not B)

= P($A \cap B \cap \bar{C}$) + P($B \cap C \cap \bar{A}$) + P($C \cap A \cap \bar{B}$)

= P(A) ×P(B) × P($\bar{C}$) + P(B) × P(C) × P($\bar{A}$) + P(C) ×P(A) × P($\bar{B}$)

= [0.2 × 0.3 ×0.1] + [0.3 × 0.9 × 0.8] + [0.9 × 0.2 ×0.7]

= 0.006 + 0.216 +0.126

= 0.348

Therefore, The probability that neelam gets excatly 2 A grades is 0.348.

Question: 19

Solution:

Given: X and Y are the two parts of a company that manufactures an article.

Here the probability of the parts being defective is given i.e, P(X) = $\frac{8}{100}$ and P(Y) = $\frac{5}{100}$ $\Rightarrow$ P($\bar{X}$) = $\frac{92}{100}$ and P($\bar{Y}$) = $\frac{95}{100}$

To Find: the probability that the assembled product will not be defective.

Here,

P(product assembled will not be defective) = 1 – P(product assembled to be defective)

=1 – [P(X and not Y) + P(Y and not X) +P(both)]

= 1- [P($X \cap \bar{Y}$) + P($Y \cap \bar{X}$) + P($X \cap Y$)]

= 1- [P(X) ×P($\bar{Y}$) + P(Y) × P($\bar{X}$) + P(X) ×P(Y)]

= 1-[($\frac{8}{100} \times \frac{95}{100}$) + ($\frac{5}{100} \times \frac{92}{100}$)+ ($\frac{8}{100} \times \frac{5}{100}$)]

=1- [$\frac{760}{10000} + \frac{460}{10000} + \frac{40}{10000}$]

= $\frac{437}{500}$

Therefore, The probability that the assembled product will not be defective is $\frac{437}{500}$.

CLASS24

Question: 20

Solution:

Given: Let A and B be two fire extinguishing engines . The probability of availability of each of the two fire extinguishing engines is given i.e., P(A) = 0.95 and P(B) = 0.95 ⇒ P($\bar{A}$) = 0.05 and P($\bar{B}$) = 0.05

To Find: i) The probability that neither of them is available when needed

Here, P(not A and not B) = P($\bar{A} \cap \bar{B}$)

= P($\bar{A}$) ×P($\bar{B}$)

=0.05 × 0.05

= 0.0025 = $\frac{1}{400}$

Therefore, The probability that neither of them is available when needed is $\frac{1}{400}$

ii)an engine is available when needed

Here, P(A and not B or B and not A) = P(A ∩ $\bar{B}$) + P(B ∩ $\bar{A}$)

= P(A) ×P($\bar{B}$) + P(B) × P($\bar{A}$)

= (0.95 ×0.05) + (0.95 × 0.05)

= 0.0475 + 0.0475

= 0.095

= $\frac{19}{200}$

Therefore, The probability that an engine is available when needed is $\frac{19}{200}$

Question: 21

Solution:

Given: let A ,B and C be the three components of a machine which works only if all its three compenents function.the probabilities of the failures of A,B

and C respectively is given i.e, P(A) = 0.14 ,P(B) = 0.10 and P(C) = 0.05

⇒ P($\bar{A}$) = 0.86 and P($\bar{B}$) = 0.90 and P($\bar{C}$) = 0.95

To Find: The probability that the machine will fail.

Here, P(the machine will fail) = 1 – P(the machine will function)

= 1 – P(all three components are working)

= 1- P($\bar{A} \cap \bar{B} \cap \bar{C}$)

= 1 – [P($\bar{A}$) × P($\bar{B}$) × P($\bar{C}$)]

= 1- [0.86× 0.90 × 0.95]

= 1 – 0.7353

= 0.2647

Therefore, The probability that the machine will fail is 0.2647.

Question: 22

Solution:

Given:Let A ,B , C and Dbe first second third and fourth shots whose probability of hitting the plane is given i.e, P(A) = 0.4 ,P(B) = 0.3 , P(C) = 0.2 and P(D) = 0.1 respectively

$\Rightarrow$ P($\bar{A}$) = 0.6 and P($\bar{B}$) = 0.7 and P($\bar{C}$) = 0.8 and P($\bar{D}$) = 0.9

To Find: The probability that atleast one shot hits the plane .

Here , P(atleast one shot hits the plane) = 1 – P(none of the shots hit the plane)

= 1- P($\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$)

= 1 – [P($\bar{A}$) × P($\bar{B}$) × P($\bar{C}$) × P($\bar{D}$)]

= 1- [0.6× 0.7 × 0.8× 0.9]

= 1 – 0.3024

= 0.6976

Therefore, The probability that atleast one shot hits the plane is 0.6976.

Question: 23

Solution:

Given: S_1 and S_2 are two swiches whose probabilities of working be given by

$P(S_1) = \frac{4}{5}$ and $P(S_2) = \frac{9}{10}$

To Find: the probability that the current flows from terminal A to terminal B when

S_1 and S_2 are connected in series.

Now, since the current in series flows from end to end

$\Rightarrow$ the flow of current from terminal A to terminal B is given by

$P(S_1 \cap S_2) = P(S_1) \times P(S_2)$

$= \frac{4}{5} \times \frac{9}{10}$

$= \frac{18}{25}$

Therefore, The probability that the current flows from terminal A to terminal B when S_1 and S_2 are connected in series is $\frac{18}{25}$

Question: 24

Solution:

Given: S_1 and S_2 are two swiches whose probabilities of working be given by

$P(S_1) = \frac{2}{3}$ and $P(S_2) = \frac{3}{4}$

To Find: the probability that the current flows from terminal A to terminal B when

S_1 and S_2 are connected in parallel.

Now, since current in parallel flows in two or more paths and hence the sum of currents through each path is equal to total current that flows from the source.

$\Rightarrow$ the flow of current from terminal A to terminal B in a parallel circuit is given by

$P(S_1 \cup S_2) = P(S_1) + P(S_2) - P(S_1 \cap S_2)$

$= P(S_1) + P(S_2) - [P(S_1) \times P(S_2)]$

$= \frac{2}{3} + \frac{3}{4} - \frac{1}{2}$

$= \frac{11}{12}$

Therefore, The probability that the current flows from terminal A to terminal B when S_1 and S_2 are connected in parallel is $\frac{11}{12}$

Question: 25
Solution:

Given : let H be head, and T be tails where as 1,2,3,4,5,6 be the numbers on the dice which are thrown when a head comes up or else coin is tossed again if its tail.

According to the question ,sample space S = {(TH),(TT) ,(H1),(H2),(H3),(H4),(H5),(H6)}

To Find: i)the probability of obtaining two tails

From sample space, it is clear that the probability of obtaining two tails is $\frac{1}{8}$

i.e., {TT} with total no of elements in sample space as 8.

ii) the probability of obtaining a head and the number 6

From sample space, it is clear that the probability of obtaining a head and the number 6 is $\frac{1}{8}$

i.e., {H6} with total no of elements in sample space as 8.

iii) the probability of obtaining a head and an even number

From sample space, it is clear that the probability of obtaining a head and an even number is $\frac{3}{8}$

i.e, {H2,H4,H6} with total no of elements in sample space as 8.