Chapter: 31. PROBABILITY DISTRIBUTION

Exercise: 31

Question: 1

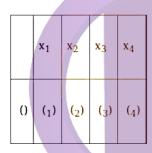
Solution:

(i) Given:

	0	1	2	3
0	$\frac{1}{6}$	$\frac{1}{2}$	3 10	$\frac{1}{30}$

To find: mean (s), variance (σ^2) and standard deviation (σ)

Formula used:



Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Standard deviation =
$$\sqrt{E(X^2) - E(X)^2}$$

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4)$$

Mean = E(X) =
$$O(\frac{1}{6}) + 1(\frac{1}{2}) + 2(\frac{3}{10}) + 3(\frac{1}{30}) = O(\frac{1}{2}) + \frac{6}{10} + \frac{3}{30} = \frac{15 + 18 + 3}{30} = \frac{36}{30} = \frac{6}{5}$$

Mean =
$$E(X) = 6 = 1.2$$

$$E(X)^2 = (1.2)^2 = 1.44$$

$$\mathrm{E}(\mathrm{X}^2) = \sum_{i=1}^{i=n} (x_i)^2. P(x_i) = (x_1)^2. P(x_1) + (x_2)^2. P(x_2) + (x_3)^2. P(x_3) + (x_4)^2. P(x_4)$$

$$E(X^{2}) = \underbrace{(0)^{2} \frac{(1)}{6} + (1)^{2} \frac{(1)}{2} + (2)^{2} \frac{(3)}{10} + (3)^{2} \frac{(1)}{30}}_{= 0} = \underbrace{0 + \frac{1}{2} + \frac{12}{10} + \frac{9}{30}}_{= 0} = \underbrace{15 + 36 + 9}_{= 0} = \underbrace{\frac{60}{30}}_{= 0}$$

$$E(X^2) = 2$$

Variance =
$$E(X^2)$$
 - = 2 - 1.44 = 0.56

Variance =
$$E(X^2) - \frac{1}{E(X)^2} = 0.56$$

Standard deviation =
$$= 0.74$$

 $E(X)^2$

$$\sqrt{E(X^2) - E(X)^2} \sqrt{0.56}$$

Variance = 0.56

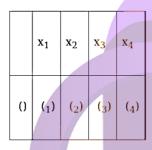
Standard deviation = 0.74

(ii) Given:

1	2	3	4
0.4	0.3	0.2	0.1

To find: mean (s), variance (σ^2) and standard deviation (σ)

Formula used:



Mean =
$$E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Standard deviation = $\sqrt{E(X^2) - E(X)^2}$

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4)$$

Mean =
$$E(X) = 1(0.4) + 2(0.3) + 3(0.2) + 4(0.1) = 0.4 + 0.6 + 0.6 + 0.4 = 2$$

$$Mean = E(X) = 2$$

$$E(X)^2 = (2)^2 = 4$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2} . P(x_{i}) = (x_{1})^{2} . P(x_{1}) + (x_{2})^{2} . P(x_{2}) + (x_{3})^{2} . P(x_{3}) + (x_{4})^{2} . P(x_{4})$$

$$E(X^2) = (1)^2(0.4) + (2)^2(0.3) + (3)^2(0.2) + (4)^2(0.1) = 0.4 + 1.2 + 1.8 + 1.6 = 5$$

$$E(X^2) = 5$$

Variance =
$$E(X^2) - E(X)^2 = 5 - 4 = 1$$

Variance =
$$E(X^2) - E(X)^2 = 1$$

Standard deviation =
$$\sqrt{E(X^2) - E(X)^2} = \sqrt{1} = 1$$

Mean = 2

Variance = 1

Standard deviation = 1

(iii) Given:

-3	-1	0	2
0.2	0.4	0.3	0.1

To find: mean (s), variance (σ^2) and standard deviation (σ)

Formula used:

	x ₁	x ₂	x ₃	x ₄
()	(1)	(₂)	(3)	(4)

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Standard deviation =
$$\sqrt{E(X^2) - E(X)^2}$$

$$\text{Mean} = \text{E}(\textbf{X}) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4)$$

Mean =
$$E(X) = -3(0.2) + (-1)(0.4) + o(0.3) + 2(0.1) = -0.6 - 0.4 + o + 0.2 = -0.8$$

$$Mean = E(X) = -0.8$$

$$E(X)^2 = (-0.8)^2 = 0.64$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2} P(x_{i}) = (x_{1})^{2} P(x_{1}) + (x_{2})^{2} P(x_{2}) + (x_{3})^{2} P(x_{3}) + (x_{4})^{2} P(x_{4})$$

$$E(X^2) = (-3)^2(0.2) + (-1)^2(0.4) + (0)^2(0.3) + (2)^2(0.1) = 1.8 + 0.4 + 0 + 0.4 = 2.6$$

$$E(X^2) = 2.6$$

Variance =
$$E(X^2) - E(X)^2 = 2.6 - 0.64 = 1.96$$

Variance =
$$E(X^2) - E(X)^2 = 1.96$$

Standard deviation =
$$\sqrt{E(X^2) - E(X)^2} = \sqrt{1.96} = 1.4$$

$$Mean = -0.8$$

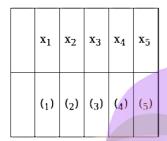
Standard deviation = 1.4

(iv) Given:

-2	-1	0	1	2
0.1	0.2	0.4	0.2	0.1

To find: mean (s), variance (σ^2) and standard deviation (σ)

Formula used:



Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Standard deviation =
$$\sqrt{E(X^2) - E(X)^2}$$

$$\text{Mean} = \text{E}(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4) + x_5 P(x_5)$$

Mean =
$$E(X) = -2(0.1) + (-1)(0.2) + o(0.4) + 1(0.2) + 2(0.1)$$

Mean =
$$E(X) = -0.2 - 0.2 + 0 + 0.2 + 0.2 = 0$$

$$Mean = E(X) = o$$

$$E(X)^2 = (0)^2 = 0$$

$$E(X^2) = \sum_{i=1}^{t=n} (x_i)^2 . P(x_i) = (x_1)^2 P(x_1) + (x_2)^2 . P(x_2) + (x_3)^2 . P(x_3) + (x_4)^2 . P(x_4) + (x_5)^2 . P(x_5)$$

$$E(X^{2}) = (-2)^{2}(0.1) + (-1)^{2}(0.2) + (0)^{2}(0.4) + (1)^{2}(0.2) + (2)^{2}(0.1)$$

$$E(X^2) = 0.4 + 0.2 + 0 + 0.2 + 0.4 = 1.2$$

$$E(X^2) = 1.2$$

Variance =
$$E(X^2) - E(X)^2 = 1.2 - 0 = 1.2$$

Variance =
$$E(X^2) - E(X)^2 = 1.2$$

Standard deviation =
$$\sqrt{E(X^2) - E(X)^2} = \sqrt{1.2} = 1.095$$

Mean = o

Variance = 1.2

Standard deviation = 1.095

Question: 2

Given: Two coins are tossed simultaneously

To find: mean (s), variance (σ²)

Formula used:



	x ₁	x ₂	x ₃
0	(1)	(₂)	(3)

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Mean = E(X) =
$$\sum_{i=1}^{t=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

When two coins are tossed simultaneously,

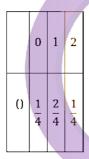
Total possible outcomes = TT, TH, HT, HH where H denotes head and T denotes tail.

$$P(0) = \frac{1}{4} (zero heads = 1 [TT])$$

$$P(1) = \frac{2}{4}$$
 (one heads = 2 [HT, TH])

$$P(2) = \frac{1}{4} \text{(two heads = 1 [HH])}$$

The probability distribution table is as follows,



Mean = E(X) =
$$O(\frac{1}{4}) + 1(\frac{2}{4}) + 2(\frac{1}{4}) = O(\frac{2}{4}) + \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$$

$$Mean = E(X) = 1$$

$$E(X)^2 = (1)^2 = 1$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2} . P(x_{i}) = (x_{1})^{2} . P(x_{1}) + P(x_{2}) + (x_{3})^{2} . P(x_{3})$$

$$E(X^2) = (0)^2 (\frac{1}{4}) + (1)^2 - (\frac{1}{4}) + (2)^2 (\frac{1}{4}) = 0 + (\frac{2}{4})^2 + (\frac{3}{4})^2 = 1.5$$

$$E(X^2) = 1.5$$

Variance =
$$E(X^2) - E(X)^2 = 1.5 - 1 = 0.5$$

Variance =
$$E(X^2) - E(X)^2 = 0.5$$

$$Mean = 1$$

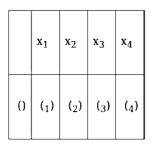
Question: 3

Solution:



To find: mean (s) and variance (σ^2)

Formula used:



Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Mean =
$$E(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

When three coins are tossed simultaneously,

 $Total\ possible\ outcomes = TTT\ , TTH\ , THT\ , HTT\ , THH\ , HHT\ , HHH\ where\ H\ denotes\ head\ and\ T\ denotes\ tail.$

$$P(0) = \frac{1}{8} (zero tails = 1 [HHH])$$

$$P(1) = \frac{3}{8} \text{ (one tail } = 3 \text{ [HTH, THH, HHT])}$$

$$P(2) = \frac{3}{8} \text{ (two tail} = 3 [HTT, THT, TTH])$$

$$P(3) = \frac{1}{8} \text{ (three tails = 1 [TTT])}$$

	0	1	2	3
0	1/8	3 8	$\frac{3}{8}$	1/8

Mean = E(X) =
$$\sum_{i=1}^{1=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4)$$

Mean = E(X) = $O(\frac{1}{8}) + I(\frac{3}{8}) + 2(\frac{3}{8}) + 3(\frac{1}{8}) = O(\frac{3}{8}) + \frac{6}{8} + \frac{3}{8} = \frac{3+6+3}{8} = \frac{12}{8} = \frac{3}{8}$

Mean =
$$E(X) = \frac{3}{2} = 1.5$$

$$E(X)^2 = (1.5)^2 = 2.25$$

$$E(X^2) = \sum_{i=1}^{i=n} (x_i)^2 . P(x_i) = (x_1)^2 . P(x_1) + P(x_2) + (x_3)^2 . P(x_3) + (x_4)^2 . P(x_4)^2 . P(x_4)^2$$

$$E(X^{2}) = (0)^{2} \frac{1}{8} + (1)^{2} \frac{3}{8} + (2)^{2} \frac{3}{8} + (3)^{2} \frac{1}{8} = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = \frac{3+12+9}{8} = \frac{24}{8} = 3$$

$$E(X^2) = 3$$

Variance =
$$E(X^2) - E(X)^2 = 3 - 2.25 = 0.75$$

Variance = $E(X^2) - E(X)^2 = 0.75$

Mean = 1.5

Variance = 0.75

Question: 4

Solution:

Given: A die is tossed twice and 'Getting an odd number on a toss' is considered a success.

To find : probability distribution of the number of successes and mean (s) and variance (σ^2) Formula used :



Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance = $E(X^2) - E(X)^2$

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

When a die is tossed twice,

Total possible outcomes =

$$\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\}$$

$$(2,1)$$
, $(2,2)$, $(2,3)$, $(2,4)$, $(2,5)$, $(2,6)$

$$(3,1)$$
, $(3,2)$, $(3,3)$, $(3,4)$, $(3,5)$, $(3,6)$

$$(6,1)$$
, $(6,2)$, $(6,3)$, $(6,4)$, $(6,5)$, $(6,6)$ }

'Getting an odd number on a toss' is considered a success.

$$P(0) = \frac{9}{36} = \frac{1}{4} (zero odd numbers = 9)$$

$$P(1) = \frac{18}{36} = \frac{1}{2}$$
 (one odd number = 18)

$$P(2) = \frac{9}{36} = \frac{1}{4}$$
 (two odd numbers = 9)

The probability distribution table is as follows,

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	0	1	2
0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$Mean = E(X) = o(\frac{1}{4}) + 1(\frac{1}{2}) + 2(\frac{1}{4}) = o + \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$$

$$Mean = E(X) = 1$$

$$E(X)^2 = (1)^2 = 1$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2} . P(x_{i}) = (x_{1})^{2} . P(x_{1}) + (x_{2})^{2} . P(x_{2}) + (x_{3})^{2} . P(x_{3})$$

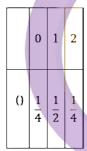
$$E(X^2) = (0)^2 (\frac{1}{4}) + (1)^2 (\frac{2}{4}) + (2)^2 (\frac{1}{4}) = 0 + \frac{2}{4} + \frac{4}{4} = \frac{6}{4} = \frac{3}{2} = 1.5$$

$$E(X^2) = 1.5$$

Variance =
$$E(X^2) - E(X)^2 = 1.5 - 1 = 0.5$$

Variance =
$$E(X^2) - E(X)^2 = 0.5$$

The probability distribution table is as follows,



Mean = 1

Variance = 0.5

Question: 5

Solution:

Given: A die is tossed twice and 'Getting a number greater than 4' is considered a success.

To find : probability distribution of the number of successes and mean (s) and variance (σ^2) Formula used :

	x ₁	x ₂	x ₃
0	(1)	(₂)	(3)

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance = $E(X^2) - E(X)^2$

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

When a die is tossed twice,

Total possible outcomes =

$$\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\}$$

$$(2,1)$$
, $(2,2)$, $(2,3)$, $(2,4)$, $(2,5)$, $(2,6)$

$$(3,1)$$
, $(3,2)$, $(3,3)$, $(3,4)$, $(3,5)$, $(3,6)$

$$(6,1)$$
, $(6,2)$, $(6,3)$, $(6,4)$, $(6,5)$, $(6,6)$ }

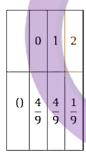
'Getting a number greater than 4' is considered a success.

$$P(o) = \frac{16}{36} = -\frac{4}{9} \text{ (zero numbers greater than } 4 = 16 \text{)}$$

$$P(1) = \frac{16}{36} = \frac{4}{9}$$
 (one number greater than 4= 16)

$$P(2) = \frac{4}{36} = -\frac{1}{9}$$
 (two numbers greater than 4= 4)

The probability distribution table is as follows,



Mean = E(X) =
$$o(\frac{4}{9}) + 1(\frac{4}{9}) + 2(\frac{1}{9}) = o + \frac{4}{9} + \frac{2}{9} = \frac{4+2}{9} = \frac{6}{9} = \frac{2}{9}$$

$$Mean = E(X) = \frac{2}{3}$$

$$E(X)^2 = (\frac{2}{3})^2 = \frac{4}{9}$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2}.P(x_{i}) = (x_{1})^{2}.P(x_{1}) + (x_{2})^{2}.P(x_{2}) + (x_{3})^{2}.P(x_{3})$$

$$E(X^{2}) = (0)^{2} \frac{4}{9} + (1)^{2} \frac{4}{9} + (2)^{2} \frac{1}{9} = 0 + \frac{4}{9} + \frac{4}{9} = \frac{8}{9}$$

$$E(X^2) = \frac{8}{9}$$

Variance =
$$E(X^2) - E(X)^2 = -\frac{8}{9} - \frac{4}{9} = -\frac{4}{9}$$

Variance =
$$E(X^2) - E(X)^2 = -\frac{4}{9}$$

	0	1	2
0	4 9	4 9	$\frac{1}{9}$

$$Mean = \frac{2}{3}$$

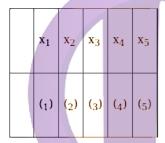
Variance =
$$\frac{4}{q}$$

Question: 6

Solution:

Given: A die is tossed twice and 'Getting a number greater than 4' is considered a success.

To find : probability distribution of the number of successes and mean (s) and variance (σ^2) Formula used :



Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

When a die is tossed 4 times,

Total possible outcomes = $6^2 = 36$

Getting a doublet is considered as a success

The possible doublets are (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)

Let p be the probability of success,

$$p = \frac{6}{26} = \frac{1}{6}$$

$$q = 1 - p = 1 - \frac{1}{6} = -\frac{5}{6}$$

$$q = \frac{5}{6}$$

since the die is thrown 4 times, n = 4

x can take the values of 1,2,3,4

$$P(x) = {^{n}C_{x}}p^{x}q^{n-x}$$

$$P(0) = 4C_0(\frac{1}{6})^0(\frac{5}{6})^4 = \frac{625}{1296}$$

$$P(1) = {}^{4}C_{1}(\frac{1}{6})^{1}(\frac{5}{6})^{3} = \frac{500}{1296} = \frac{125}{324}$$

$$P(2) = {}^{4}C_{2}(\frac{1}{6})^{2}(\frac{5}{6})^{2} = \frac{150}{1296} = \frac{25}{216}$$

$$P(3) = {}^{4}C_{3}(\frac{1}{6})^{3}(\frac{5}{6})^{1} = \frac{20}{1296} = \frac{5}{324}$$

$$P(4) = {}^{4}C_{4}(\frac{1}{6})^{4}(\frac{5}{6})^{0} = \frac{1}{1296}$$

The probability distribution table is as follows,

	0	1	2	3	4
	625 1296			5 324	1 1296

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4) + x_5 P(x_5)$$

Mean = E(X) =
$$0(\frac{625}{1296}) + 1(\frac{125}{324}) + 2(\frac{25}{216}) + 3(\frac{5}{324}) + 4(\frac{1296}{1296})$$

Mean = E(X) = 0 +
$$\frac{125}{224}$$
 + $\frac{50}{216}$ + $\frac{15}{221}$ + $\frac{4}{1296}$ = $\frac{500 + 300 + 60 + 4}{1296}$ = $\frac{864}{1296}$ = $\frac{2}{3}$

$$Mean = E(X) = \frac{2}{3}$$

$$E(X)^2 = (\frac{2}{3})^2 = -\frac{4}{9}$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2} P(x_{i}) = (x_{1})^{2} P(x_{1}) + (x_{2})^{2} P(x_{2}) + (x_{3})^{2} P(x_{3}) + (x_{4})^{2} P(x_{4}) + (x_{5})^{2} P(x_{5})$$

$$E(X^{2}) = (0)^{2} \left(\frac{625}{1296}\right) + (1)^{2} \left(\frac{125}{324}\right) + (2)^{2} \left(\frac{25}{216}\right) + (3)^{2} \left(\frac{5}{324}\right) + (4)^{2} \left(\frac{1}{1296}\right)$$

$$E(X^2) = O + \frac{125}{324} + \frac{100}{216} + \frac{45}{324} + \frac{16}{1296} = \frac{500 + 600 + 180 + 16}{1296} = \frac{1296}{1296}$$

$$E(X^2) = 1$$

Variance =
$$E(X^2) - E(X)^2 = 1 - \frac{4}{9} = \frac{5}{9}$$

Variance =
$$E(X^2) - E(X)^2 = \frac{5}{9}$$

	0	1	2	3	4
	625 1296	125 324	25 216	5 324	1 1296

Mean
$$\Rightarrow \frac{2}{3}$$

Variance =
$$-\frac{5}{9}$$

Question: 7

Given: A coin is tossed 4 times

To find: probability distribution of X and mean (s) and variance (σ^2)

Formula used:

	x ₁	x ₂	x ₃	x ₄	x ₅
	(1)	(₂)	(3)	(4)	(₅)

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

A coin is tossed 4 times,

Total possible outcomes = 24 = 16

X denotes the number of heads

Let p be the probability of getting a head,

$$\mathbf{p} = \frac{1}{2}$$

$$q = 1 - p = 1 - \frac{1}{2} = -\frac{1}{2}$$

$$q = \frac{1}{2}$$

since the coin is tossed 4 times, n = 4

X can take the values of 1,2,3,4

$$P(x) = {^{\mathrm{n}}}\mathrm{C}_x p^x q^{\mathrm{n}-x}$$

$$P(0) = {}^{4}C_{0}(\frac{1}{2})^{0}(\frac{1}{2})^{4} = \frac{1}{16}$$

$$P(1) = {}^{4}C_{1}(\frac{1}{2})^{1}(\frac{1}{2})^{3} = \frac{4}{16} = \frac{1}{4}$$

$$P(2) = {}^{4}C_{2}(\frac{1}{2})^{2}(\frac{1}{2})^{2} = \frac{6}{16} = \frac{3}{8}$$

$$P(3) = {}^{4}C_{3}(\frac{1}{2})^{3}(\frac{1}{2})^{1} = \frac{4}{16} = \frac{1}{4}$$

$$P(4) = {}^{4}C_{4}(\frac{1}{2})^{4}(\frac{1}{2})^{0} = \frac{1}{16}$$

0	1	2	3	4
$\frac{1}{16}$	$\frac{1}{4}$	3 - 8	$\frac{1}{4}$	$\frac{1}{16}$

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4) + x_5 P(x_5)$$

Mean = E(X) =
$$o(\frac{1}{16}) + 1(\frac{1}{4}) + 2(\frac{3}{8}) + 3(\frac{1}{4}) + 4(\frac{1}{16})$$

Mean = E(X) = 0 +
$$\frac{1}{4}$$
 + $\frac{6}{6}$ + $\frac{3}{4}$ + $\frac{4}{16}$ = $\frac{4+12+12+4}{16}$ = $\frac{32}{16}$ = 2

$$Mean = E(X) = 2$$

$$E(X)^2 = (2)^2 = 4$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2} P(x_{i}) = (x_{1})^{2} P(x_{1}) + (x_{2})^{2} P(x_{2}) + (x_{3})^{2} P(x_{3}) + (x_{4})^{2} P(x_{4}) + (x_{5})^{2} P(x_{5})$$

$$E(X^{2}) = (0)^{2} (\frac{1}{16}) + (1)^{2} (\frac{1}{4}) + (2)^{2} (\frac{3}{6}) + (3)^{2} (\frac{1}{4}) + (4)^{2} (\frac{1}{16})$$

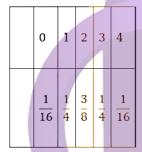
$$E(X^2) = O + \frac{1}{4} + \frac{12}{8} + \frac{9}{4} + \frac{16}{16} = \frac{0 + 4 + 24 + 36 + 16}{16} = \frac{80}{16} = 5$$

$$E(X^2) = 5$$

Variance =
$$E(X^2) - E(X)^2 = 5 - 4 = 1$$

Variance =
$$E(X^2) - E(X)^2 = 1$$

The probability distribution table is as follows,



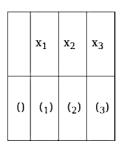
Mean = 2

Variance = 1

Question: 8

Given: Let X denote the number of times 'a total of 9' appears in two throws of a pair of dice

To find : probability distribution of X ,mean (s) and variance (σ^2) and standard deviation Formula used :



Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Standard deviation =
$$\sqrt{E(X^2) - E(X)^2}$$

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Total possible outcomes =

$$\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\}$$

$$(2,1)$$
, $(2,2)$, $(2,3)$, $(2,4)$, $(2,5)$, $(2,6)$

$$(3,1)$$
, $(3,2)$, $(3,3)$, $(3,4)$, $(3,5)$, $(3,6)$

Let X denote the number of times 'a total of 9' appears in two throws of a pair of dice

$$p = \frac{4}{36} = -\frac{1}{9}$$

$$= 1 - \frac{1}{9} = \frac{8}{9}$$

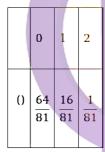
Two dice are tossed twice, hence n = 2

$$P(o) = {}^{2}C_{o}(\frac{1}{9})^{0}(\frac{8}{9})^{2} = \frac{64}{81}$$

$$P(1) = {}^{2}C_{1}(\frac{1}{9})^{1}(\frac{8}{9})^{1} = \frac{16}{81}$$

$$P(2) = {}^{2}C_{2}(\frac{1}{9})^{2}(\frac{9}{9})^{0} = \frac{1}{81}$$

The probability distribution table is as follows,



Mean = E(X) =
$$0(\frac{64}{81}) + 1(\frac{16}{81}) + 2(\frac{1}{81}) = 0 + \frac{16}{81} + \frac{2}{81} = \frac{16+2}{81} = \frac{18}{81} = \frac{2}{81}$$

$$Mean = E(X) = \frac{2}{9}$$

$$E(X)^2 = (\frac{2}{9})^2 = \frac{4}{81}$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2} . P(x_{i}) = (x_{1})^{2} . P(x_{1}) + (x_{2})^{2} . P(x_{2}) + (x_{3})^{2} . P(x_{3})$$

$$E(X^2) = (0)^2 \frac{64}{81} + \frac{16}{81} + (2)^2 (\frac{1}{81}) = 0 + \frac{16}{81} + \frac{4}{81} = \frac{20}{81}$$

$$E(X^2) = \frac{20}{91}$$

Variance =
$$E(X^2) - E(X)^2 = \frac{20}{81} - \frac{4}{81} = \frac{16}{81}$$

Variance =
$$E(X^2) - E(X)^2 = \frac{16}{81}$$

Standard deviation =
$$\sqrt{E(X^2) - E(X)^2} = \sqrt{\frac{16}{81}} = -\frac{4}{9}$$

	0	1	2
0	$\frac{64}{81}$	$\frac{16}{81}$	1 81

$$Mean = \frac{2}{9}$$

Variance =
$$\frac{16}{81}$$

Standard deviation =
$$\frac{4}{5}$$

Question: 9

Given: There are 5 cards, numbers 1 to 5, one number on each card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two cards drawn.

To find: mean (s) and variance (σ²) of X

Formula used:

x ₁	x ₂	x ₃	x ₄	x ₅	x ₆	x ₇	х ₈
(1)	(₂)	(₃)	(4)	(5)	(₆)	(₇)	(8)

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

There are 5 cards, numbers 1 to 5, one number on each card. Two cards are drawn at random without replacement.

X denote the sum of the numbers on two cards drawn

The minimum value of X will be 3 as the two cards drawn are 1 and 2

The maximum value of X will be 9 as the two cards drawn are 4 and 5

For X = 3 the two cards can be (1,2) and (2,1)

For X = 4 the two cards can be (1,3) and (3,1)

For X = 5 the two cards can be (1,4), (4,1), (2,3) and (3,2)

For X = 6 the two cards can be (1,5), (5,1), (2,4) and (4,2)

For X = 7 the two cards can be (3,4), (4,3), (2,5) and (5,2)

For X = 8 the two cards can be (5,3) and (3,5)

For X = 9 the two cards can be (4,5) and (4,5)

Total outcomes = 20

$$P(3) = \frac{2}{20} = \frac{1}{10}$$

$$P(4) = \frac{2}{20} = \frac{1}{10}$$

$$P(5) = \frac{4}{20} = \frac{1}{5}$$

$$P(6) = \frac{4}{20} = \frac{1}{5}$$

$$P(7) = \frac{4}{20} = \frac{1}{5}$$

$$P(8) = \frac{2}{20} = \frac{1}{10}$$

$$P(9) = \frac{2}{20} = \frac{1}{10}$$

The probability distribution table is as follows,

x _i	3	4	5	6	7	8	9
P _i	1 10	1 10	$\frac{1}{5}$	1 5	$\frac{1}{5}$	$\frac{1}{10}$	1 10

$$\text{Mean} = \text{E}(\textbf{X}) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4) + x_5 P(x_5) + x_6 P(x_6) + x_7 P(x_7) + x_8 P(x_8) + x_8 P$$

Mean = E(X) =
$$3\frac{(1)}{10}$$
 + $4(\frac{)}{10}$ + $5(\frac{)}{5}$ + $6(\frac{)}{5}$ + $7(\frac{)}{5}$ + $8(\frac{)}{10}$ + $9(\frac{1}{10})$

Mean = E(X) =
$$\frac{3}{10} + \frac{4}{10} + \frac{5}{5} + \frac{6}{5} + \frac{7}{5} + \frac{8}{10} + \frac{9}{10} = \frac{3+4+10+12+14+8+9}{10} = \frac{60}{10} = \frac{60}{10}$$

$$Mean = E(X) = 6$$

$$E(X)^2 = (6)^2 = 36$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2}.P(x_{i}) = (x_{1})^{2}P(x_{1}) + (x_{2})^{2}.P(x_{2}) + (x_{3})^{2}.P(x_{3}) + (x_{4})^{2}.P(x_{4}) + (x_{5})^{2}.P(x_{5}) + (x_{6})^{2}.P(x_{6}) + (x_{7})^{2}.P(x_{7})$$

$$E(X^{2}) = (3)^{2}(\frac{1}{10} + (4)^{2}(\frac{1}{10}) + (5)^{2}(\frac{1}{5} + (6)^{2}(\frac{1}{5} + (7)^{2}(\frac{1}{5}) + (8)^{2}(\frac{1}{10}) + (9)^{2}(\frac{1}{10})$$

$$E(X^2) = \frac{9}{10} + \frac{16}{10} + \frac{25}{5} + \frac{36}{5} + \frac{49}{5} + \frac{64}{10} + \frac{81}{10} = \frac{9 + 16 + 50 + 72 + 98 + 64 + 81}{10} = \frac{390}{10} = 39$$

$$E(X^2) = 39$$

Variance =
$$E(X^2) - E(X)^2 = 39 - 36 = 3$$

Variance =
$$E(X^2) - E(X)^2 = 3$$

$$Mean = 6$$

Variance = 3

Question: 10

Solution:

Given: Two cards are drawn from a well-shuffled pack of 52 cards.

To find: probability distribution of the number of kings and variance (σ^2)

Formula used:



	x ₁	x ₂	x ₃
0	(1)	(₂)	(3)

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Mean = E(X) =
$$\sum_{i=1}^{t=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Two cards are drawn from a well-shuffled pack of 52 cards.

Let X denote the number of kings in the two cards

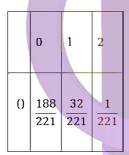
There are 4 king cards present in a pack of well-shuffled pack of 52 cards.

$$P(o) = \frac{{}^{48}_{2}C}{{}^{52}_{2}C} = \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$$

$$P(1) = \frac{{}^{48}C \times {}^{4}C}{{}^{52}C} = \frac{48 \times 4 \times 2}{52 \times 51} = \frac{32}{221}$$

$$P(2) = \frac{\frac{4}{2}C}{\frac{5}{2}C} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$

The probability distribution table is as follows,



Mean = E(X) =
$$O(\frac{189}{221}) + I(\frac{32}{221}) + 2(\frac{1}{221}) = O + \frac{32}{221} + \frac{2}{221} = \frac{32+2}{221} = \frac{34}{221}$$

$$Mean = E(X) = \frac{34}{221}$$

$$E(X)^2 = (\frac{34}{221})^2 = \frac{1156}{48941}$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2}.P(x_{i}) = (x_{1})^{2}.P(x_{1}) + (x_{2})^{2}.P(x_{2}) + (x_{3})^{2}.P(x_{3})$$

$$E(X^2) = (0)^2 (\frac{188}{221}) + (1)^2 (\frac{32}{221}) + (2)^2 (\frac{1}{221}) = o + \frac{32}{221} + \frac{4}{221} = \frac{36}{221}$$

$$E(X^2) = \frac{36}{221}$$

Variance =
$$E(X^2)$$
 - $E(X)^2 = \frac{36}{221} \cdot \frac{1156}{48841} = \frac{7956 - 1156}{48841} = \frac{6800}{48841} = \frac{400}{2873}$

Variance =
$$E(X^2) - E(X)^2 = \frac{400}{2873}$$

	0	1	2
0	$\frac{188}{221}$	$\frac{32}{221}$	1 221

Variance =
$$\frac{400}{2873}$$

Question: 11

Given : A box contains 16 bulbs, out of which 4 bulbs are defective. Three bulbs are drawn at random

To find: mean (s) and variance (σ²)

Formula used:

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Mean = E(X) =
$$\sum_{i=1}^{l=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

A box contains 16 bulbs, out of which 4 bulbs are defective. Three bulbs are drawn at random

Let X denote the number of defective bulbs drawn

There are 4 defective bulbs present in 16 bulbs

$$P(0) = \frac{{}^{12}C}{{}^{12}C} = \frac{12 \times 11 \times 10}{16 \times 15 \times 14} = \frac{11}{28}$$

$$P(1) = \frac{{}^{12}_{2}C \times {}^{4}_{1}C}{{}^{10}_{0}C} = \frac{12 \times 11 \times 4 \times 3 \times 2}{16 \times 15 \times 14 \times 2} = \frac{33}{70}$$

$$P(2) = \frac{{}^{12}_{1}\text{C} \times {}^{4}_{2}\text{C}}{{}^{16}_{2}\text{C}} = \frac{12 \times 4 \times 3 \times 3 \times 2}{16 \times 15 \times 14 \times 2} = \frac{9}{70}$$

$$P(3) = \frac{\frac{4}{3}C}{\frac{16}{3}C} = \frac{4 \times 3 \times 2}{16 \times 15 \times 14} = \frac{1}{140}$$

	0	1	2	3
0	$\frac{11}{28}$	33 70	9 70	1/140

$$Mean = E(X) = O(\frac{11}{28}) + I(\frac{33}{70}) + 2(\frac{9}{70}) + 3(\frac{1}{140}) = O(\frac{33}{70}) + \frac{18}{70} + \frac{3}{140} = \frac{66 + 36 + 3}{140}$$

Mean = E(X) =
$$\frac{105}{140}$$
 = $-\frac{3}{4}$

$$E(X)^2 = (\frac{3}{4})^2 = \frac{9}{16}$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2}.P(x_{i}) = (x_{1})^{2}.P(x_{1}) + (x_{2})^{2}.P(x_{2}) + (x_{3})^{2}.P(x_{3})^{2}$$

$$E(X^{2}) = (0)^{2} \left(\frac{11}{28}\right) + (1)^{2} \left(\frac{33}{70}\right) + (2)^{2} \left(\frac{9}{70}\right) + (3)^{2} \left(\frac{1}{140}\right) = O + \frac{33}{70} + \frac{36}{70} + \frac{9}{140} = \frac{66 + 72 + 9}{140}$$

$$E(X^2) = \frac{147}{140}$$

Variance =
$$E(X^2)$$
 - $E(X)^2 = \frac{147}{140} - \frac{9}{16} = \frac{588 - 315}{560} = \frac{273}{560} = \frac{39}{80}$

Variance =
$$E(X^2) - E(X)^2 = \frac{39}{80}$$

$$Mean = E(X) = 3$$

Variance =
$$\frac{39}{80}$$

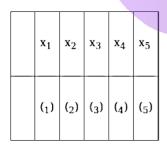
Question: 12

Given: 20% of the bulbs produced by a machine are defective.

To find probability distribution of a number of defective bulbs in a sample of 4 bulbs chosen at random.

Formula used:

The probability distribution table is given by,



Where
$$P(x) = {}^{n}C_{x}p^{x}q^{n-x}$$

Here p is the probability of getting a defective bulb.

$$q = 1 - p$$

Let the total number of bulbs produced by a machine be x

20% of the bulbs produced by a machine are defective.

Number of defective bulbs produced by a machine = $\frac{20}{100} \times (\chi) = \frac{x}{5}$

Let p be the probability of getting a defective bulb,

$$p = -\frac{x}{5} = -\frac{1}{5}$$

$$p = -\frac{1}{5}$$

$$q = 1 - p = 1 - \frac{1}{5} = -\frac{4}{5}$$

$$q = \frac{4}{5}$$

since 4 bulbs are chosen at random, n = 4

X can take the values of 0,1,2,3,4

$$P(x) = {}^{n}C_{x}p^{x}q^{n-x}$$

$$P(0) = 4C_0(\frac{1}{5})^0(\frac{4}{5})^4 = \frac{256}{625}$$

$$P(1) = 4C_1(\frac{1}{5})^1(\frac{4}{5})^3 = \frac{256}{625}$$

$$P(2) = 4C_2(\frac{1}{5})^2(\frac{4}{5})^2 = \frac{96}{625}$$

$$P(3) = {}^{4}C_{3}(\frac{1}{5})^{3}(\frac{4}{5})^{1} = \frac{16}{625}$$

$$P(4) = {}^{4}C_{4}(\frac{1}{2})^{4}(\frac{4}{5})^{0} = \frac{1}{625}$$

The probability distribution table is as follows,

0	1	2	3	4
256	256	96	16	1
625	625	625	625	625

Question: 13

Solution:

Given: Four bad eggs are mixed with 10 good ones. Three eggs are drawn one by one without replacement.

To find: mean (s) and variance (σ^2)

Formula used:

	x ₁	x ₂	x ₃
0	(1)	(₂)	(3)

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance = $E(X^2) - E(X)^2$

$$\text{Mean} = \text{E}(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Four bad eggs are mixed with 10 good ones. Three eggs are drawn one by one without replacement.

Let X denote the number of bad eggs drawn

There are 4 bad eggs present in 14 eggs

$$P(0) = \frac{{}^{10}C}{{}^{14}C} = \frac{10 \times 9 \times 8}{14 \times 13 \times 12} = \frac{30}{91}$$

$$P(1) = \frac{{}^{1}{}^{0}_{2}C \times {}^{4}_{1}C}{{}^{1}_{3}C} = \frac{10 \times 9 \times 4 \times 3 \times 2}{14 \times 13 \times 12 \times 2} = \frac{45}{91}$$

$$P(2) = \frac{\frac{{}^{10}\text{C}}{{}^{12}\text{C}}}{\frac{{}^{12}\text{C}}{{}^{12}\text{C}}} = \frac{10 \times 4 \times 3 \times 3 \times 2}{14 \times 13 \times 12 \times 2} = \frac{15}{91}$$

$$P(3) = \frac{{}_{3}^{4}C}{{}_{1}^{4}C} = \frac{4 \times 3 \times 2}{14 \times 13 \times 12} = \frac{1}{91}$$

The probability distribution table is as follows,



$$\underline{\text{Mean} = E(X) = o(\frac{30}{91}) + 1(\frac{45}{91} + 2(\frac{15}{91} + 3(\frac{1}{91}) = o + \frac{45}{91} + \frac{30}{91} - \frac{3}{91} = \frac{45 + 30 + 3}{91}}$$

Mean =
$$E(X) = \frac{78}{91} = \frac{6}{7}$$

$$E(X)^2 = (\frac{6}{7})^2 = \frac{36}{49}$$

$$E(X_2) = \sum_{i=1}^{i=n} (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_2) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^{2}) = (0)^{2} \left(\frac{30}{91}\right) + (1)^{2} \left(\frac{45}{91}\right) + (2)^{2} \left(\frac{15}{91}\right) + (3)^{2} \left(\frac{1}{91}\right) = 0 + \frac{45}{91} + \frac{60}{91} + \frac{9}{91} = \frac{45 + 60 + 9}{91}$$

$$E(X^2) = \frac{114}{91}$$

Variance =
$$E(X^2) - E(X)^2 = \frac{114}{91} - \frac{36}{49} = \frac{798 - 468}{637} = \frac{330}{637}$$

Variance =
$$E(X^2) - E(X)^2 = \frac{330}{637}$$

$$Mean = E(X) = 6$$

Variance =
$$\frac{330}{637}$$

Question: 14

Given: Four rotten oranges are mixed with 16 good ones. Three oranges are drawn one by one without replacement.

To find: mean (5) and variance (σ²)



	x ₁	x ₂	x ₃
0	(1)	(₂)	(3)

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance = $E(X^2) - E(X)^2$

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Four rotten oranges are mixed with 16 good ones. Three oranges are drawn one by one without replacement.

Let X denote the number of rotten oranges drawn

There are 4 rotten oranges present in 20 oranges

$$P(o) = \frac{{}^{16}C}{{}^{20}C} = \frac{{}^{16} \times 15 \times 14}{{}^{20} \times 19 \times 18} = \frac{{}^{28}}{57}$$

$$\mathbf{P(1)} = \frac{{}^{1}{}^{0}\mathbf{C} \times {}^{4}\mathbf{C}}{{}^{2}{}^{0}\mathbf{C}} = \frac{16 \times 15 \times 4 \times 3 \times 2}{20 \times 19 \times 18 \times 2} = \frac{8}{19}$$

$$P(2) = \frac{{}^{1}_{1}C \times {}^{4}_{2}C}{{}^{2}_{2}C} = \frac{{}^{1}_{0} \times 4 \times 3 \times 3 \times 2}{{}^{2}_{0} \times 19 \times 18 \times 2} = \frac{8}{95}$$

$$P(3) = \frac{{}_{3}^{4}C}{{}_{3}^{2}C} = \frac{4 \times 3 \times 2}{20 \times 19 \times 18} = \frac{1}{285}$$

	0	1	2	3
()	28 57	8 19	8 95	$\frac{1}{285}$

Mean = E(X) =
$$0(\frac{28}{57}) + 1(\frac{8}{19}) + 2(\frac{8}{95}) + 3(\frac{1}{285}) = 0 + \frac{8}{19} + \frac{16}{95} + \frac{3}{285} = \frac{120 + 48 + 3}{285}$$

Mean =
$$E(X) = \frac{171}{285} = \frac{3}{5}$$

$$E(X)^2 = (\frac{3}{5})^2 = \frac{9}{25}$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2} . P(x_{i}) = (x_{1})^{2} . P(x_{1}) + (x_{2})^{2} . P(x_{2}) + (x_{3})^{2} . P(x_{3})$$

$$E(X^{2}) = (0)^{2} \left(\frac{28}{57}\right) + (1)^{2} \cdot \left(\frac{8}{15}\right) + (2)^{2} \left(\frac{8}{95}\right) + (3)^{2} \left(\frac{1}{285}\right) = 0 + \frac{8}{19} + \frac{32}{95} + \frac{9}{285} = \frac{120 + 96 + 9}{285}$$

$$E(X^2) = \frac{225}{285} = \frac{15}{19}$$

Variance =
$$E(X^2)$$
 - $E(X)^2 = \frac{15}{19} - \frac{9}{25} = \frac{375 - 171}{475} = \frac{204}{475}$

Variance =
$$E(X^2) - E(X)^2 = \frac{204}{475}$$

$$Mean = E(X) = \frac{3}{5}$$

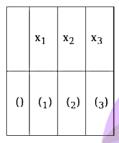
Variance =
$$\frac{204}{475}$$

Question: 15

Given: Three balls are drawn simultaneously from a bag containing 5 white and 4 red balls.

To find: mean (s) and variance (σ^2) of X

Formula used:



Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Three balls are drawn simultaneously from a bag containing 5 white and 4 red balls.

Let X be the number of red balls drawn.

$$P(0) = \frac{{}_{3}^{5}C}{{}_{2}^{9}C} = \frac{5 \times 4}{{}_{9 \times 9 \times 7}} = \frac{5}{{}_{126}}$$

$$P(1) = \frac{{\frac{5}{2}C \times {\frac{4}{1}C}}}{{\frac{9}{2}C}} = \frac{5 \times 4 \times 4 \times 3 \times 2}{9 \times 8 \times 7 \times 2} = \frac{10}{21}$$

$$P(2) = \frac{{}_{1}^{5}C \times {}_{2}^{4}C}{{}_{3}^{9}C} = \frac{5 \times 4 \times 3 \times 3 \times 2}{9 \times 8 \times 7 \times 2} = \frac{5}{14}$$

$$P(3) = \frac{{}_{2}^{4}C}{{}_{9}^{6}C} = \frac{4 \times 3 \times 2}{9 \times 8 \times 7} = \frac{1}{21}$$

	0	1	2	3
0	5 126	10 21	5 14	1/21

Mean = E(X) =
$$O(\frac{5}{126}) + 1(\frac{10}{21}) + 2(\frac{5}{14}) + 3(\frac{1}{21}) = O(\frac{10}{21}) + \frac{10}{14} + \frac{3}{21} = \frac{20 + 30 + 6}{42}$$

Mean = E(X) =
$$\frac{56}{42}$$
 = $\frac{4}{3}$

$$E(X)^2 = (\frac{4}{3})^2 = \frac{16}{9}$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{1})^{2} . P(x_{i}) = (x_{1})^{2} . P(x_{1}) + (x_{2})^{2} . P(x_{2}) + (x_{3})^{2} . P(x_{3})$$

$$E(X^2) = (0)^2 \left(\frac{5}{126}\right) + (1)^2 \left(\frac{10}{21}\right) + (2)^2 \left(\frac{5}{14}\right) + (3)^2 \left(\frac{1}{21}\right) = O + \frac{10}{21} + \frac{20}{14} + \frac{9}{21} = \frac{20 + 60 + 18}{42}$$

$$E(X^2) = \frac{98}{42} = -\frac{7}{3}$$

Variance =
$$E(X^2) - E(X)^2 = -\frac{7}{3} - \frac{16}{9} = \frac{21 - 16}{9} = -\frac{5}{9}$$

Variance =
$$E(X^2) - E(X)^2 = \frac{5}{6}$$

$$Mean = E(X) = 4$$

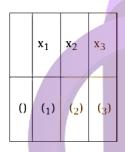
Variance =
$$-\frac{5}{9}$$

Question: 16

Given: Two cards are drawn without replacement from a well-shuffled deck of 52 cards.

To find: mean (s) and variance (σ^2) of X

Formula used:



Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Two cards are drawn without replacement from a well-shuffled deck of 52 cards.

Let X denote the number of face cards drawn

There are 12 face cards present in 52 cards

$$P(0) = \frac{{}^{40}_{2}C}{{}^{52}_{2}C} = \frac{40 \times 39}{52 \times 51} = \frac{10}{17}$$

$$P(1) = \frac{{}^{40}_{1}\text{C} \times {}^{12}_{1}\text{C}}{{}^{52}_{2}\text{C}} = \frac{40 \times 12 \times 2}{52 \times 51} = \frac{80}{221}$$

$$P(2) = \frac{{}^{12}_{2}C}{{}^{52}_{2}C} = \frac{12 \times 11}{52 \times 51} = \frac{11}{221}$$

	0	1	2
0	10 17	80 221	11 221

$$Mean = E(X) = O(\frac{10}{17}) + 1(\frac{80}{221}) + 2(\frac{11}{221}) = O(\frac{80}{221}) + \frac{22}{221} = \frac{80 + 22}{221} = \frac{102}{221} = \frac{6}{13}$$

$$Mean = E(X) = \frac{6}{13}$$

$$E(X)^2 = (\frac{6}{13})^2 = \frac{36}{169}$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2}.P(x_{i}) = (x_{1})^{2}.P(x_{1}) + (x_{2})^{2}.P(x_{2}) + (x_{3})^{2}.P(x_{3})$$

$$E(X^{2}) = (0)^{2} (\frac{10}{17}) + (1)^{2} (\frac{80}{221}) + (2)^{2} (\frac{11}{221}) = 0 + \frac{80}{221} + \frac{44}{221} = \frac{80 + 44}{221}$$

$$E(X^2) = \frac{124}{221}$$

Variance =
$$E(X^2)$$
 - $E(X)^2 = \frac{124}{221} - \frac{36}{169} = \frac{1612 - 612}{2873} = \frac{1000}{2873}$

Variance =
$$E(X^2) - E(X)^2 = \frac{1000}{2873}$$

$$Mean = E(X) = \frac{6}{12}$$

Variance =
$$\frac{1000}{2873}$$

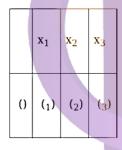
Question: 17

Solution:

Given : Two cards are drawn with replacement from a well-shuffled deck of 52 cards.

To find: mean (s) and variance (o2) of X

Formula used:



Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Two cards are drawn with replacement from a well-shuffled deck of 52 cards.

Let X denote the number of ace cards drawn

There are 4 face cards present in 52 cards

X can take the value of 0,1,2.

$$P(0) = \frac{48}{52} \times \frac{48}{52} = \frac{144}{169}$$

$$P(1) = {}_{1}^{2}C \times {}_{52}^{4} \times {}_{52}^{48} = {}_{52 \times 52}^{2 \times 4 \times 48} = {}_{169}^{24}$$

$$P(2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

	0	1	2
0	144 169	24 169	1/169

Mean = E(X) =
$$O(\frac{144}{169}) + 1(\frac{24}{169}) + 2(\frac{1}{169}) = O + \frac{24}{169} + \frac{2}{169} = \frac{24+2}{169} = \frac{26}{169} = \frac{2}{13}$$

$$Mean = E(X) = \frac{2}{13}$$

$$E(X)^2 = (\frac{2}{13})^2 = \frac{4}{169}$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2} . P(x_{i}) = (x_{1})^{2} . P(x_{1}) + (x_{2})^{2} . P(x_{2}) + (x_{3})^{2} . P(x_{3})$$

$$E(X^2) = (0)^2 (\frac{144}{169}) + (1)^2 (\frac{24}{169}) + (2)^2 (\frac{1}{169}) = o + \frac{24}{169} + \frac{4}{169} = \frac{28}{169}$$

$$E(X^2) = \frac{28}{169}$$

Variance =
$$E(X^2) - E(X)^2 = \frac{28}{169} - \frac{4}{169} = \frac{24}{169}$$

Variance =
$$E(X^2) - E(X)^2 = \frac{24}{169}$$

$$Mean = E(X) = \frac{2}{13}$$

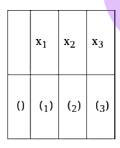
Variance =
$$\frac{24}{169}$$

Question: 18

Given : Three cards are drawn successively with replacement from a well – shuffled deck of 52 cards.

To find: mean (s) and variance (σ^2) of X

Formula used:



Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i)$$

Variance =
$$E(X^2) - E(X)^2$$

Mean = E(X) =
$$\sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Three cards are drawn successively with replacement from a well - shuffled deck of 52 cards.

Let X be the number of hearts drawn.

Number of hearts in 52 cards is 13

$$P(o) = \frac{39}{52} \times \frac{39}{52} \times \frac{39}{52} = \frac{27}{64}$$

$$P(1) = {}^{3}_{1}C \times {}^{13}_{52} \times {}^{39}_{52} \times {}^{39}_{52} = {}^{27}_{64}$$

$$P(2) = {}^{3}_{2}C \times {}^{13}_{52} \times {}^{13}_{52} \times {}^{39}_{52} = {}^{9}_{64}$$

$$P(3) = \frac{13}{52} \times \frac{13}{52} \times \frac{13}{52} = \frac{1}{64}$$

The probability distribution table is as follows,

		0	1	2	3
0	ľ	27 64	27 64	9 64	1 64

Mean = E(X) =
$$o(\frac{27}{64}) + 1(\frac{27}{64}) + 2(\frac{9}{64}) + 3(\frac{1}{64}) = o + \frac{27}{64} + \frac{18}{64} + \frac{3}{64} = \frac{48}{64} = \frac{3}{4}$$

$$Mean = E(X) = \frac{3}{4}$$

$$E(X)^2 = (\frac{3}{4})^2 = \frac{9}{16}$$

$$E(X^{2}) = \sum_{i=1}^{i=n} (x_{i})^{2} P(x_{i}) = (x_{1})^{2} P(x_{1}) + (x_{2})^{2} P(x_{2}) + (x_{3})^{2} P(x_{3})$$

$$E(X^{2}) = (0)^{2} {\binom{27}{64}} + (1)^{2} {\binom{27}{64}} + (2)^{2} {\binom{9}{64}} + (3)^{2} {\binom{1}{64}} = 0 + \frac{27}{64} + \frac{36}{64} + \frac{9}{64} = \frac{72}{64} = \frac{9}{8}$$

$$E(X^2) = -\frac{9}{8}$$

Variance =
$$E(X^2) - E(X)^2 = \frac{9}{8} - \frac{9}{16} = \frac{18 - 9}{16} = \frac{9}{16}$$

Variance =
$$E(X^2) - E(X)^2 = \frac{9}{16}$$

$$Mean = E(X) = 3$$

Variance =
$$\frac{9}{16}$$

Question: 19

Given: Five defective bulbs are accidently mixed with 20 good ones.

To find: probability distribution from this lot

Formula used:

	x ₁	x ₂	x ₃	x ₄	x ₅
	(1)	(2)	(3)	(4)	(₅)

Five defective bulbs are accidently mixed with 20 good ones.

Total number of bulbs = 25

X denote the number of defective bulbs drawn

X can draw the value o , 1 , 2 , 3 , 4.

since the number of bulbs drawn is 4, n = 4

P(o) = P(getting a no defective bulb) =
$$\frac{{}^{20}\text{C}}{{}^{25}\text{C}} = \frac{20 \times 19 \times 18 \times 17}{25 \times 24 \times 23 \times 22} = \frac{969}{2530}$$

P(1) = P(getting 1 defective bulb and 3 good ones) =
$$\frac{{}_{1}^{5}C \times {}_{3}^{2}C}{{}_{2}^{5}C} = \frac{5 \times 20 \times 19 \times 18 \times 4}{25 \times 24 \times 23 \times 22}$$

$$P(1) = \frac{1140}{2530} = \frac{114}{253}$$

$$P(2) = P(\text{getting 2 defective bulbs and 2 good one}) = \frac{\frac{5}{2}C \times \frac{20}{2}C}{\frac{25}{2}C}$$

$$P(2) = \frac{5 \times 4 \times 20 \times 19 \times 4 \times 3 \times 2}{25 \times 24 \times 23 \times 22 \times 2 \times 2} = \frac{380}{2530} = \frac{38}{253}$$

P(3) = P(getting 3 defective bulbs and 1 good one) =
$$\frac{\frac{5}{2}C \times \frac{20}{1}C}{\frac{25}{5}C} = \frac{5 \times 4 \times 20 \times 4 \times 3 \times 2}{25 \times 24 \times 23 \times 22 \times 2}$$

$$P(3) = \frac{40}{2530} = \frac{4}{253}$$

P(4) = P(getting all defective bulbs) =
$$\frac{{}_{4}^{5}C}{{}_{4}^{25}C} = \frac{5 \times 4 \times 3 \times 2}{25 \times 24 \times 23 \times 22} = \frac{1}{2530}$$

$$P(4) = \frac{1}{2530}$$

0	1	2	3	4
969	114	38	4	1
2530	253	253	253	2530