Exercise : 4A

Question: 1

Solution:

NOTE:

Trigonometric Table

	0° (0)	30° $\left(\frac{\pi}{6}\right)$	45° $\left(\frac{\pi}{4}\right)$	60° $\left(\frac{\pi}{3}\right)$	90° $\left(\frac{\pi}{2}\right)$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
cosec	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Undefined
cot	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

(i) Let $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$\Rightarrow \frac{\sqrt{3}}{2} = \sin x$ [We know which value of x when placed in sin gives us this answer]

$\therefore x = \frac{\pi}{3}$

(ii) Let $\sin^{-1}\left(\frac{1}{2}\right) = x$

$\Rightarrow \frac{1}{2} = \sin x$ [We know which value of x when put in this expression will give us this result]

$\Rightarrow x = \frac{\pi}{6}$

(iii) Let $\cos^{-1}\left(\frac{1}{2}\right) = x$

$\Rightarrow \frac{1}{2} = \cos x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{3}$

(iv) Let $\tan^{-1}(1) = x$

$\Rightarrow 1 = \tan x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{4}$

(v) Let $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = x$

$\Rightarrow \frac{1}{\sqrt{3}} = \tan x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{6}$

(vi) Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = x$

$\Rightarrow \frac{2}{\sqrt{3}} = \sec x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{6}$

(vii) Let $\text{cosec}^{-1}\left(\sqrt{2}\right) = x$

$\Rightarrow \sqrt{2} = \text{cosec}\, x$

[We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{4}$

Question: 2

Solution:

(i) Let $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) = x$

$\Rightarrow -\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = x$ [Formula: $\sin^{-1}(-x) = -\sin^{-1} x$]

$\Rightarrow \frac{1}{\sqrt{2}} = -\sin x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = -\frac{\pi}{4}$

(ii) $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ [Formula: $\cos^{-1}(-x) = \pi - \cos^{-1} x$]

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$\Rightarrow \left(\frac{\sqrt{3}}{2}\right) = \cos x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{6}$

Putting this value back in the equation

$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

(iii) Let $\tan^{-1}(-\sqrt{3}) = x$

$\Rightarrow -\tan^{-1}(\sqrt{3}) = x$ [Formula: $\tan^{-1}(-x) = -\tan^{-1}(x)$]

$\Rightarrow \sqrt{3} = -\tan x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{-\pi}{3}$

(iv) $\sec^{-1}(-2) = \pi - \sec^{-1}(2)$ ...(i) [Formula: $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$]

Let $\sec^{-1}(2) = x$

$\Rightarrow 2 = \sec x$ [We know which value of x when put in this expression will give us this result]

$\therefore x = \frac{\pi}{3}$

Putting the value in (i)

$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

(v) Let $\text{cosec}^{-1}\left(-\sqrt{2}\right) = x$

$\Rightarrow -\text{cosec}^{-1}(\sqrt{2}) = x$ [Formula: $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}(x)$]

$\Rightarrow \sqrt{2} = -\text{cosec}\, x$

$\therefore x = -\frac{\pi}{4}$

(vi) $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \pi - \cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$ ... (i)

Let $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = x$

$\Rightarrow \frac{1}{\sqrt{3}} = \cot^{-1} x$ [We know which value of x when put in this expression will give us this result]

$\Rightarrow x = \frac{\pi}{3}$

Putting in (i)

$\pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

Question: 3

Solution:

$\cos\{\pi - \frac{\pi}{2} + \frac{\pi}{2}\}$ [Refer to question 2(ii)]

$= \cos\{\pi\}$

$= \cos\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$

= -1

Question: 4

Solution:

$\sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$

$= \sin\left(\frac{5\pi}{6}\right)$

$= \sin\left(\pi - \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{6}$

$= \frac{1}{2}$

Exercise : 4B

Question: 1

Solution:

$\sin^{-1}\left(\frac{-1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$ [Formula: $\sin^{-1}(-x) = \sin^{-1}(x)$]

$= -\frac{\pi}{6}$

Question: 2

Solution:

$\cos^{-1}\left(\frac{-1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$ [Formula: $\cos^{-1}(-x) = -\cos^{-1}(x)$]

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$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

Question: 3

Solution:

$\tan(-1) = -\tan(1)$ [Formula: $\tan^{-1}(-x) = -\tan^{-1}(x)$]

[We know that $\tan\frac{\pi}{4} = 1$, thus $\tan^{-1}\frac{\pi}{4} = 1$]

$= -\frac{\pi}{4}$

Question: 4

Solution:

$\sec^{-1}(-2) = \pi - \sec^{-1}(2)$

$= \pi - \frac{\pi}{3}$ [Formula: $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$]

$= \frac{2\pi}{3}$

Question: 5

Solution:

$\operatorname{cosec}^{-1}(-\sqrt{2}) = -\operatorname{cosec}^{-1}(\sqrt{2})$ [Formula: $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$]

$= -\frac{\pi}{4}$

This can also be solved as

$\operatorname{cosec}^{-1}(-\sqrt{2})$

Since cosec is negative in the third quadrant, the angle we are looking for will be in the third quadrant.

$= \pi + \frac{\pi}{4}$

$= \frac{5\pi}{4}$

Question: 6

Solution:

$\cot^{-1}(-1) = \pi - \cot^{-1}(1)$ [Formula: $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$]

$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

Question: 7

Solution:

$\tan^{-1}(-\sqrt{3}) = -\tan^{-1}(\sqrt{3})$ [Formula: $\tan^{-1}(-x) = -\tan^{-1}(x)$]

$= -\frac{\pi}{3}$

Question: 8

Solution:

$\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \pi - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ [Formula: $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$]

$= \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

Question: 9

Solution:

$\text{cosec}^{-1}(2)$

Putting the value directly

$= \frac{\pi}{6}$

Question: 10

Solution:

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$

[Formula: $\sin(\pi - x) = \sin x$]

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right)$

[Formula: $\sin^{-1}(\sin x) = x$]

$= \frac{\pi}{3}$

Question: 11

Solution:

$\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{4}\right)\right)$

[Formula: $\tan(\pi - x) = -\tan(x)$, as tan is negative in the second quadrant.]

$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$

[Formula: $\tan^{-1}(\tan x) = x$]

$= -\frac{\pi}{4}$

Question: 12

Solution:

$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{6}\right)\right)$

[Formula: $\cos(2\pi - x) = \cos(x)$, as cos has a positive vaule in the fourth quadrant.]

$= \cos^{-1}\left(\cos\frac{5\pi}{6}\right)$ [Formula: $\cos^{-1}(\cos x) = x$

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$= \frac{5\pi}{6}$

Question: 13

Solution:

$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi + \frac{\pi}{6}\right)\right)$

[Formula: $\cos(2\pi + x) = \cos x$, cos is positive in the first quadrant.]

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right)$ [Formula: $\cos^{-1}(\cos x) = x$]

$= \frac{\pi}{6}$

Question: 14

Solution:

$\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left(\tan\left(\pi + \frac{\pi}{6}\right)\right)$

[Formula: $\tan(\pi + x) = \tan x$, as tan is positive in the third quadrant.]

$=\tan^{-1}\left(\tan\frac{\pi}{6}\right)$ [Formula: $\tan^{-1}(\tan x) = x$]

$= \frac{\pi}{6}$

Question: 15

Solution:

$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

Putting the value of $\tan^{-1}\sqrt{3}$ and using the formula

$\cot^{-1}(-x) = \pi - \cot^{-1}x$

$= \frac{\pi}{3} - \left(\pi - \cot^{-1}(\sqrt{3})\right)$

Putting the value of $\cot^{-1}(\sqrt{3})$

$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right)$

$= \frac{\pi}{3} - \frac{5\pi}{6}$

$= -\frac{3\pi}{6}$

$= -\frac{\pi}{2}$

Question: 16

Solution:

[Formula: $\sin^{-1}(-x) = -\sin^{-1}x$]

$\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\}$

$=\sin\left\{\frac{\pi}{3}-\left(-\sin^{-1}\frac{1}{2}\right)\right\}$

$=\sin\left\{\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right\}$

Putting value of $\sin^{-1}\left(\frac{1}{2}\right)$

$=\sin\left\{\frac{\pi}{3}+\frac{\pi}{6}\right\}$

$=\sin\frac{3\pi}{6}$

$=\sin\frac{\pi}{2}$

= 1

Question: 17

Solution:

$\cot(\tan^{-1}x+\cot^{-1}x)=\cot\left(\frac{\pi}{2}\right)$ [Formula: $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$]

Putting value of $\cot\left(\frac{\pi}{2}\right)$

= 0

Question: 18

Solution:

$\operatorname{cosec}(\sin^{-1}x+\cos^{-1}x)=\operatorname{cosec}\frac{\pi}{2}$ [Formula: $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$]

Putting the value of $\operatorname{cosec}\frac{\pi}{2}$

= 1

Question: 19

Solution:

$\sin(\sec^{-1}x+\operatorname{cosec}^{-1}x)=\sin\left(\frac{\pi}{2}\right)$ [Formula: $\sec^{-1}x+\operatorname{cosec}^{-1}x=\frac{\pi}{2}$]

Putting the value of $\sin\left(\frac{\pi}{2}\right)$

=1

Question: 20

Solution:

Putting the values of the inverse trigonometric terms

$\frac{\pi}{3}+2\times\frac{\pi}{6}$

$=\frac{\pi}{3}+\frac{\pi}{3}$

$=\frac{2\pi}{3}$

Question: 21

Solution:

[Formula: $\cos^{-1}(-x)=\pi-\cos(x)$ and $\sin^{-1}(-x)=-\sin(x)$]

$\tan^{-1} 1 + \left(\pi - \cos^{-1}\left(\frac{1}{2}\right)\right) + \left(-\sin^{-1}\left(\frac{1}{2}\right)\right)$

Putting the values for each of the inverse trigonometric terms

$= \frac{\pi}{4} + \left(\pi - \frac{\pi}{3}\right) - \frac{\pi}{6}$

$= \frac{\pi}{12} + \frac{2\pi}{3}$

$= \frac{9\pi}{12}$

$= \frac{3\pi}{4}$

Question: 22

Solution:

$\sin^{-1}\left\{\sin\left(\frac{3\pi}{5}\right)\right\}$

$= \sin^{-1}\left\{\sin\left(\pi - \frac{2\pi}{5}\right)\right\}$

[Formula: $\sin(\pi - x) = \sin x$, as sin is positive in the second quadrant.]

$= \sin^{-1}\left\{\sin\frac{2\pi}{5}\right\}$ [Formula: $\sin^{-1}(\sin x) = x$]

$= \frac{2\pi}{5}$

Exercise : 4C

Question: 1 A

Solution:

To Prove: $\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \tan^{-1} x$

Formula Used: $\tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$

Proof:

$LHS = \tan^{-1}\left(\frac{1+x}{1-x}\right)$ ... (1)

Let $x = \tan A$ ... (2)

Substituting (2) in (1),

$LHS = \tan^{-1}\left(\frac{1+\tan A}{1-\tan A}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + A\right)\right)$

$= \frac{\pi}{4} + A$

From (2), $A = \tan^{-1} x$,

$\frac{\pi}{4} + A = \frac{\pi}{4} + \tan^{-1} x$

$= RHS$

Therefore, LHS = RHS

Hence proved.

Question: 1 B

Solution:

To Prove: $\tan^{-1} x + \cot^{-1}(x + 1) = \tan^{-1}(x^2 + x + 1)$

Formula Used:

1) $\cot^{-1} x = \tan^{-1}\frac{1}{x}$

2) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS = $\tan^{-1} x + \cot^{-1}(x + 1)$ ... (1)

$= \tan^{-1} x + \tan^{-1}\frac{1}{(x+1)}$

$= \tan^{-1}\left(\frac{x + \frac{1}{(x+1)}}{1 - \left(x \times \frac{1}{(x+1)}\right)}\right)$

$= \tan^{-1}\frac{x(x+1)+1}{x+1-x}$

$= \tan^{-1}(x^2 + x + 1)$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 2

Solution:

To Prove: $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1} x$

Formula Used: $\sin 2A = 2 \times \sin A \times \cos A$

Proof:

LHS $= \sin^{-1}(2x\sqrt{1-x^2})$ ... (1)

Let $x = \sin A$ ... (2)

Substituting (2) in (1),

LHS $= \sin^{-1}(2\sin A\sqrt{1-\sin^2 A})$

$= \sin^{-1}(2 \times \sin A \times \cos A)$

$= \sin^{-1}(\sin 2A)$

$= 2A$

From (2), $A = \sin^{-1} x$,

$2A = 2\sin^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 3 A

Solution:

To Prove: $\sin^{-1}(3x - 4x^3) = 3\sin^{-1}x$

Formula Used: $\sin 3A = 3\sin A - 4\sin^3 A$

Proof:

LHS = $\sin^{-1}(3x - 4x^3)$ ... (1)

Let $x = \sin A$ ... (2)

Substituting (2) in (1),

LHS = $\sin^{-1}(3\sin A - 4\sin^3 A)$

$= \sin^{-1}(\sin 3A)$

$= 3A$

From (2), $A = \sin^{-1}x$,

$3A = 3\sin^{-1}x$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 3 B

Solution:

To Prove: $\cos^{-1}(4x^3 - 3x) = 3\cos^{-1}x$

Formula Used: $\cos 3A = 4\cos^3 A - 3\cos A$

Proof:

LHS = $\cos^{-1}(4x^3 - 3x)$ ... (1)

Let $x = \cos A$ ... (2)

Substituting (2) in (1),

LHS = $\cos^{-1}(4\cos^3 A - 3\cos A)$

$= \cos^{-1}(\cos 3A)$

$= 3A$

From (2), $A = \cos^{-1}x$,

$3A = 3\cos^{-1}x$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 3 C

Solution:

To Prove: $tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = 3\,tan^{-1}x$

Formula Used: $tan\,3A = \frac{3\,tan\,A - tan^3 A}{1-3\,tan^2 A}$

Proof:

LHS $= tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ ... (1)

Let x = tan A ... (2)

Substituting (2) in (1),

LHS $= tan^{-1}\left(\frac{3\,tan\,A - tan^3 A}{1-3\,tan^2 A}\right)$

$= \tan^{-1}(\tan 3A)$

= 3A

From (2), $A = \tan^{-1} x$,

$3A = 3\tan^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 3 D

Solution:

To Prove: $tan^{-1}x + tan^{-1}\left(\frac{2x}{1-x^2}\right) = tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

Formula Used: $tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS $= tan^{-1}x + tan^{-1}\left(\frac{2x}{1-x^2}\right)$ ... (1)

$$= tan^{-1}\left(\frac{x+\left(\frac{2x}{1-x^2}\right)}{1-\left(x\times\left(\frac{2x}{1-x^2}\right)\right)}\right)$$

$$= tan^{-1}\left(\frac{x(1-x^2)+2x}{1-x^2-2x^2}\right)$$

$$= tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 4 A

Solution:

To Prove: $\cos^{-1}(1-2x^2) = 2\sin^{-1} x$

Formula Used: $\cos 2A = 1 - 2\sin^2 A$

Proof:

LHS = $\cos^{-1}(1 - 2x^2)$ ... (1)

Let $x = \sin A$ ... (2)

Substituting (2) in (1),

LHS = $\cos^{-1}(1 - 2\sin^2 A)$

$= \cos^{-1}(\cos 2A)$

$= 2A$

From (2), $A = \sin^{-1} x$,

$2A = 2\sin^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 4 B

Solution:

To Prove: $\cos^{-1}(2x^2 - 1) = 2\cos^{-1} x$

Formula Used: $\cos 2A = 2\cos^2 A - 1$

Proof:

LHS = $\cos^{-1}(2x^2 - 1)$ ... (1)

Let $x = \cos A$ ... (2)

Substituting (2) in (1),

LHS = $\cos^{-1}(2\cos^2 A - 1)$

$= \cos^{-1}(\cos 2A)$

$= 2A$

From (2), $A = \cos^{-1} x$,

$2A = 2\cos^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 4 C

Solution:

To Prove: $\sec^{-1}\left(\frac{1}{2x^2-1}\right) = 2\cos^{-1} x$

Formula Used:

1) $\cos 2A = 2\cos^2 A - 1$

2) $\cos^{-1} A = \sec^{-1}\left(\frac{1}{A}\right)$

Proof:

LHS = $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$

= $\cos^{-1}(2x^2 - 1)$... (1)

Let x = cos A ... (2)

Substituting (2) in (1),

LHS = $\cos^{-1}(2\cos^2 A - 1)$

= $\cos^{-1}(\cos 2A)$

= 2A

From (2), A = $\cos^{-1} x$,

2A = $2\cos^{-1} x$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 4 D

Solution:

To Prove: $\cot^{-1}\left(\sqrt{1+x^2} - x\right) = \frac{\pi}{2} - \frac{1}{2}\cot^{-1} x$

Formula Used:

1) $\tan\left(\frac{\pi}{4} + A\right) = \frac{1+\tan A}{1-\tan A}$

2) $\text{cosec}^2 A = 1 + \cot^2 A$

3) $1 - \cos A = 2\sin^2\left(\frac{A}{2}\right)$

4) $\sin A = 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$

Proof:

LHS = $\cot^{-1}\left(\sqrt{1+x^2} - x\right)$

Let x = cot A

LHS = $\cot^{-1}\left(\sqrt{1+\cot^2 A} - \cot A\right)$

= $\cot^{-1}(\text{cosec } A - \cot A)$

$= \cot^{-1}\left(\frac{1-\cos A}{\sin A}\right)$

$= \cot^{-1}\left(\frac{2\sin^2\left(\frac{A}{2}\right)}{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}\right)$

$= \cot^{-1}\left(\tan\left(\frac{A}{2}\right)\right)$

$= \frac{\pi}{2} - \tan^{-1}\left(\tan\left(\frac{A}{2}\right)\right)$

$= \frac{\pi}{2} - \frac{A}{2}$

From (2), $A = \cot^{-1} x$,

$$\frac{\pi}{2} - \frac{A}{2} = \frac{\pi}{2} - \frac{1}{2}\cot^{-1} x$$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 5 A

Solution:

To Prove: $\tan^{-1}\left(\frac{\sqrt{x}+\sqrt{y}}{1-\sqrt{xy}}\right) = \tan^{-1}\sqrt{x} + \tan^{-1}\sqrt{y}$

We know that, $\tan A + \tan B = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Also, $\tan^{-1}\left(\frac{A+B}{1-AB}\right) = \tan^{-1} A + \tan^{-1} B$

Taking $A = \sqrt{x}$ and $B = \sqrt{y}$

We get,

$$\tan^{-1}\left(\frac{\sqrt{x}+\sqrt{y}}{1-\sqrt{xy}}\right) = \tan^{-1}\sqrt{x} + \tan^{-1}\sqrt{y}$$

Hence, Proved.

Question: 5 B

Solution:

We know that,

$$\tan^{-1}\left(\frac{A+B}{1-AB}\right) = \tan^{-1} A + \tan^{-1} B$$

Now, taking $A = x$ and $B = \sqrt{x}$

We get,

$$\tan^{-1} x + \tan^{-1}\sqrt{x} = \tan^{-1}\left(\frac{x+\sqrt{x}}{1-x^{3/2}}\right)$$

As, $x.x^{1/2} = x^{3/2}$

Hence, Proved.

Question: 5 C

Solution:

To Prove: $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \frac{x}{2}$

Formula Used:

1) $\sin A = 2 \times \sin\frac{A}{2} \times \cos\frac{A}{2}$

2) $1 + \cos A = 2\cos^2\frac{A}{2}$

Proof:

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LHS $= \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

$= \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$= \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{x}{2}\right)$

$= \frac{x}{2}$

= RHS

Therefore LHS = RHS

Hence proved.

Question: 6 A

Solution:

To Prove: $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\frac{3}{4}$

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS $= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11}$

$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{11}}{1-\left(\frac{1}{2}\times\frac{2}{11}\right)}\right)$

$= \tan^{-1}\left(\frac{11+4}{22-2}\right)$

$= \tan^{-1}\frac{15}{20}$

$= \tan^{-1}\frac{3}{4}$

= RHS

Therefore LHS = RHS

Hence proved.

Question: 6 B

Solution:

To Prove: $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS $= \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$= \tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\left(\frac{2}{11}\times\frac{7}{24}\right)}\right)$

$= \tan^{-1}\left(\frac{48+77}{264-14}\right)$

$= \tan^{-1}\frac{125}{250}$

$= \tan^{-1}\frac{1}{2}$

= RHS

Therefore LHS = RHS

Hence proved.

Question: 6 C

Solution:

To Prove: $\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{2}$

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS = $\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$

$= \tan^{-1}1 + \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\times\frac{1}{3}\right)}\right)$

$= \tan^{-1}1 + \tan^{-1}\left(\frac{5}{6-1}\right)$

$= \tan^{-1}1 + \tan^{-1}1$

$= \frac{\pi}{4} + \frac{\pi}{4}$

$= \frac{\pi}{2}$

= RHS

Therefore LHS = RHS

Hence proved.

Question: 6 D

Solution:

To Prove: $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Proof:

LHS = $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{3}}{1-\left(\frac{1}{3}\times\frac{1}{3}\right)}\right)+\tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left(\frac{6}{9-1}\right)+\tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\left(\frac{3}{4}\times\frac{1}{7}\right)}\right)$

$= \tan^{-1}\left(\frac{21+4}{28-3}\right)$

$= \tan^{-1}\frac{25}{25}$

$= \tan^{-1} 1$

$= \frac{\pi}{4}$

= RHS

Therefore LHS = RHS

Hence proved.

Question: 6 E

Solution:

To Prove: $\tan^{-1} 2 - \tan^{-1} 1 = \tan^{-1}\frac{1}{3}$

Formula Used: $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ where $xy > -1$

Proof:

LHS = $\tan^{-1} 2 - \tan^{-1} 1$

$= \tan^{-1}\left(\frac{2-1}{1+2}\right)$

$= \tan^{-1}\left(\frac{1}{3}\right)$

= RHS

Therefore LHS = RHS

Hence proved.

Question: 6 F

Solution:

To Prove: $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

Formula Used: $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy > 1$

Proof:

LHS = $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$

$= \frac{\pi}{4} + \pi + \tan^{-1}\left(\frac{2+3}{1-(2\times3)}\right)$ {since 2 × 3 = 6 > 1}

$= \frac{5\pi}{4} + \tan^{-1}\left(\frac{5}{-5}\right)$

$= \frac{5\pi}{4} + \tan^{-1}(-1)$

$= \frac{5\pi}{4} - \frac{\pi}{4}$

$= \pi$

= RHS

Therefore LHS = RHS

Hence proved.

Question: 6 G

Solution:

To Prove:

$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy < 1$

Proof:

LHS = $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}$

$= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\left(\frac{1}{5}\times\frac{1}{8}\right)}\right)$

$= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{8+5}{40-1}\right)$

$= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{13}{39}\right)$

$= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$

$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\times\frac{1}{3}\right)}\right)$

$= \tan^{-1}\left(\frac{3+2}{6-1}\right)$

$= \tan^{-1} 1$

$= \frac{\pi}{4}$

= RHS

Therefore LHS = RHS

Hence proved.

Question: 6 H

Solution:

To Prove: $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\tan^{-1}\frac{4}{3} \Rightarrow 2\left(\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}\right) = \tan^{-1}\frac{4}{3}$

CLASS24

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy < 1$

Proof:

$LHS = 2(\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9})$

$$= 2\left(\tan^{-1}\left(\frac{\frac{1}{4} + \frac{2}{9}}{1 - \left(\frac{1}{4} \times \frac{2}{9}\right)}\right)\right)$$

$$= 2\ \tan^{-1}\left(\frac{9+8}{36-2}\right)$$

$$= 2\tan^{-1}\frac{17}{34}$$

$$= 2\tan^{-1}\frac{1}{2}$$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{2}$$

$$= \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{2}}{1 - \left(\frac{1}{2} \times \frac{1}{2}\right)}\right)$$

$$= \tan^{-1}\left(\frac{1}{\frac{4-1}{4}}\right)$$

$$= \tan^{-1}\frac{4}{3}$$

= RHS

Therefore LHS = RHS

Hence proved.

Question: 7 A

Solution:

To Prove: $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

Formula Used: $\cos^{-1}x + \cos^{-1}y = \cos^{-1}(xy - \sqrt{1-x^2} \times \sqrt{1-y^2})$

Proof:

$LHS = \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$

$$= \cos^{-1}\left(\frac{4}{5} \times \frac{12}{13} - \sqrt{1-\left(\frac{4}{5}\right)^2} \times \sqrt{1-\left(\frac{12}{13}\right)^2}\right)$$

$$= \cos^{-1}\left(\frac{48}{65} - \sqrt{1-\frac{16}{25}} \times \sqrt{1-\frac{144}{169}}\right)$$

$$= \cos^{-1}\left(\frac{48}{65} - \left(\sqrt{\frac{25-16}{25}} \times \sqrt{\frac{169-144}{169}}\right)\right)$$

$= \cos^{-1}\left(\frac{48}{65} - \left(\sqrt{\frac{9}{25}} \times \sqrt{\frac{25}{169}}\right)\right)$

$= \cos^{-1}\left(\frac{48}{65} - \frac{3}{13}\right)$

$= \cos^{-1}\left(\frac{48-15}{65}\right)$

$= \cos^{-1}\frac{33}{65}$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 7 B

Solution:

To Prove: $\sin^{-1}\frac{1}{\sqrt{5}} + \sin^{-1}\frac{2}{\sqrt{5}} = \frac{\pi}{2}$

Formula Used: $\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x \times \sqrt{1-y^2} + y \times \sqrt{1-x^2})$

Proof:

LHS $= \sin^{-1}\frac{1}{\sqrt{5}} + \sin^{-1}\frac{2}{\sqrt{5}}$

$= \sin^{-1}\left(\frac{1}{\sqrt{5}} \times \sqrt{1-\left(\frac{2}{\sqrt{5}}\right)^2} + \frac{2}{\sqrt{5}} \times \sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2}\right)$

$= \sin^{-1}\left(\frac{1}{\sqrt{5}} \times \sqrt{1-\frac{4}{5}} + \frac{2}{\sqrt{5}} \times \sqrt{1-\frac{1}{5}}\right)$

$= \sin^{-1}\left(\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{5}} + \frac{2}{\sqrt{5}} \times \frac{2}{\sqrt{5}}\right)$

$= \sin^{-1}\left(\frac{1}{5} + \frac{4}{5}\right)$

$= \sin^{-1}\frac{5}{5}$

$= \sin^{-1} 1$

$= \frac{\pi}{2}$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 7 C

Solution:

To Prove:

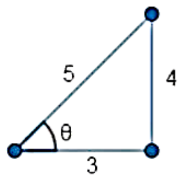
$$\cos^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13} = \sin^{-1}\frac{56}{65}$$

Formula Used: $\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x \times \sqrt{1-y^2} + y \times \sqrt{1-x^2})$

Proof:

LHS $= \cos^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13}$ ... (1)

Let $\cos\theta = \frac{3}{5}$



Therefore $\theta = \cos^{-1}\frac{3}{5}$ ... (2)

From the figure, $\sin\theta = \frac{4}{5}$

$\Rightarrow \theta = \sin^{-1}\frac{4}{5}$ ... (3)

From (2) and (3),

$$\cos^{-1}\frac{3}{5} = \sin^{-1}\frac{4}{5}$$

Substituting in (1), we get

LHS $= \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{12}{13}$

$$= \sin^{-1}\left(\frac{4}{5} \times \sqrt{1-\left(\frac{12}{13}\right)^2} + \frac{12}{13} \times \sqrt{1-\left(\frac{4}{5}\right)^2}\right)$$

$$= \sin^{-1}\left(\frac{4}{5} \times \sqrt{1-\frac{144}{169}} + \frac{12}{13} \times \sqrt{1-\frac{16}{25}}\right)$$

$$= \sin^{-1}\left(\frac{4}{5} \times \sqrt{\frac{25}{169}} + \frac{12}{13} \times \sqrt{\frac{9}{25}}\right)$$

$$= \sin^{-1}\left(\frac{4}{5} \times \frac{5}{13} + \frac{12}{13} \times \frac{3}{5}\right)$$

$$= \sin^{-1}\left(\frac{20}{65} + \frac{36}{65}\right)$$

$$= \sin^{-1}\frac{56}{65}$$

$=$ RHS

Therefore, LHS = RHS

Hence proved.

Question: 7 D

Solution:

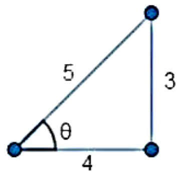
To Prove: $\cos^{-1}\frac{4}{5} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{27}{11}$

Formula Used: $\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x \times \sqrt{1-y^2} + y \times \sqrt{1-x^2})$

Proof:

LHS $= \cos^{-1}\frac{4}{5} + \sin^{-1}\frac{3}{5}$ ... (1)

Let $\cos\theta = \frac{4}{5}$



Therefore $\theta = \cos^{-1}\frac{4}{5}$ ... (2)

From the figure, $\sin\theta = \frac{3}{5}$

$\Rightarrow \theta = \sin^{-1}\frac{3}{5}$ ... (3)

From (2) and (3),

$\cos^{-1}\frac{4}{5} = \sin^{-1}\frac{3}{5}$

Substituting in (1), we get

LHS $= \sin^{-1}\frac{3}{5} + \sin^{-1}\frac{3}{5}$

$= \sin^{-1}\left(2 \times \frac{3}{5} \times \sqrt{1-\left(\frac{3}{5}\right)^2}\right)$

$= \sin^{-1}\left(2 \times \frac{3}{5} \times \sqrt{1-\frac{9}{25}}\right)$

$= \sin^{-1}\left(2 \times \frac{3}{5} \times \sqrt{\frac{16}{25}}\right)$

$= \sin^{-1}\left(2 \times \frac{3}{5} \times \frac{4}{5}\right)$

$= \sin^{-1}\frac{24}{25}$

Question: 7 E

Solution:

To Prove: $\tan^{-1}\frac{1}{3} + \sec^{-1}\frac{\sqrt{5}}{2} = \frac{\pi}{4}$

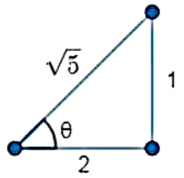
Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy < 1$

Proof:

LHS $= \tan^{-1}\frac{1}{3} + \sec^{-1}\frac{\sqrt{5}}{2}$ ... (1)

Let $\sec\theta = \frac{\sqrt{5}}{2}$

Therefore $\theta = \sec^{-1}\frac{\sqrt{5}}{2}$ ... (2)



From the figure, $\tan\theta = \frac{1}{2}$

$\Rightarrow \theta = \tan^{-1}\frac{1}{2} \dots (3)$

From (2) and (3),

$$\sec^{-1}\frac{\sqrt{5}}{2} = \tan^{-1}\frac{1}{2}$$

Substituting in (1), we get

$\text{LHS} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$

$$= \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\left(\frac{1}{3}\times\frac{1}{2}\right)}\right)$$

$$= \tan^{-1}\left(\frac{2+3}{6-1}\right)$$

$$= \tan^{-1}\frac{5}{5}$$

$= \tan^{-1} 1$

$$= \frac{\pi}{4}$$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 7 F

Solution:

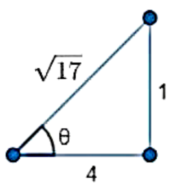
To Prove: $\sin^{-1}\frac{1}{\sqrt{17}} + \cos^{-1}\frac{9}{\sqrt{85}} = \tan^{-1}\frac{1}{2}$

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy < 1$

Proof:

$\text{LHS} = \sin^{-1}\frac{1}{\sqrt{17}} + \cos^{-1}\frac{9}{\sqrt{85}} \dots (1)$

Let $\sin\theta = \frac{1}{\sqrt{17}}$



Therefore $\theta = \sin^{-1}\frac{1}{\sqrt{17}} \dots (2)$

From the figure, $\tan\theta = \frac{1}{4}$

$\Rightarrow \theta = \tan^{-1}\frac{1}{4}$ ... (3)

From (2) and (3),

$\sin^{-1}\frac{1}{\sqrt{17}} = \tan^{-1}\frac{1}{4}$ ... (3)

Now, let $\cos\theta = \frac{9}{\sqrt{85}}$

Therefore $\theta = \cos^{-1}\frac{9}{\sqrt{85}}$ ... (4)

From the figure, $\tan\theta = \frac{2}{9}$

$\Rightarrow \theta = \tan^{-1}\frac{2}{9}$ ... (5)

From (4) and (5),

$\cos^{-1}\frac{9}{\sqrt{85}} = \tan^{-1}\frac{2}{9}$ ... (6)

Substituting (3) and (6) in (1), we get

LHS $= \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}$

$= \tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\left(\frac{1}{4}\times\frac{2}{9}\right)}\right)$

$= \tan^{-1}\left(\frac{9+8}{36-2}\right)$

$= \tan^{-1}\frac{17}{34}$

$= \tan^{-1}\frac{1}{2}$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 7 G

Solution:

To Prove: $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$

Formula Used:

1) $2\sin^{-1}x = \sin^{-1}(2x \times \sqrt{1-x^2})$

2) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy < 1$

Proof:

LHS $= 2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$ ... (1)

$2\sin^{-1}\frac{3}{5} = \sin^{-1}\left(2\times\frac{3}{5}\times\sqrt{1-\left(\frac{3}{5}\right)^2}\right)$

$= \sin^{-1}\left(\frac{6}{5} \times \frac{4}{5}\right)$

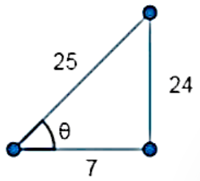
$= \sin^{-1}\frac{24}{25}$ ... (2)

Substituting (2) in (1), we get

LHS $= \sin^{-1}\frac{24}{25} - \tan^{-1}\frac{17}{31}$ ... (3)

Let $\sin\theta = \frac{24}{25}$

Therefore $\theta = \sin^{-1}\frac{24}{25}$ ... (4)



From the figure, $\tan\theta = \frac{24}{7}$

$\Rightarrow \theta = \tan^{-1}\frac{24}{7}$ ... (5)

From (4) and (5),

$\sin^{-1}\frac{24}{25} = \tan^{-1}\frac{24}{7}$ ... (6)

Substituting (6) in (3), we get

LHS $= \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$

$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \left(\frac{24}{7} \times \frac{17}{31}\right)}\right)$

$= \tan^{-1}\left(\frac{744 - 119}{217 + 408}\right)$

$= \tan^{-1}\frac{625}{625}$

$= \tan^{-1} 1$

$= \frac{\pi}{4}$

= RHS

Therefore, LHS = RHS

Hence proved.

Question: 8 A

Solution:

To find: value of x

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy < 1$

Given: $\tan^{-1}(x + 1) + \tan^{-1}(x - 1) = \tan^{-1}\frac{8}{31}$

CLASS24

$LHS = \tan^{-1}\left(\frac{x+1+x-1}{1-\{(x+1)\times(x-1)\}}\right)$

$= \tan^{-1}\frac{2x}{1-(x^2-x+x-1)}$

$= \tan^{-1}\frac{2x}{2-x^2}$

Therefore, $\tan^{-1}\frac{2x}{2-x^2} = \tan^{-1}\frac{8}{31}$

Taking tangent on both sides, we get

$\frac{2x}{2-x^2} = \frac{8}{31}$

$\Rightarrow 62x = 16 - 8x^2$

$\Rightarrow 8x^2 + 62x - 16 = 0$

$\Rightarrow 4x^2 + 31x - 8 = 0$

$\Rightarrow 4x^2 + 32x - x - 8 = 0$

$\Rightarrow 4x \times (x + 8) - 1 \times (x + 8) = 0$

$\Rightarrow (4x - 1) \times (x + 8) = 0$

$\Rightarrow x = \frac{1}{4}$ or $x = -8$

Therefore, $x = \frac{1}{4}$ or $x = -8$ are the required values of x.

Question: 8 B

Solution:

To find: value of x

Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy < 1$

Given: $\tan^{-1}(2+x) + \tan^{-1}(2-x) = \tan^{-1}\frac{2}{3}$

$LHS = \tan^{-1}\left(\frac{2+x+2-x}{1-\{(2+x)\times(2-x)\}}\right)$

$= \tan^{-1}\frac{4}{1-(4-2x+2x-x^2)}$

$= \tan^{-1}\frac{4}{x^2-3}$

Therefore, $\tan^{-1}\frac{4}{x^2-3} = \tan^{-1}\frac{2}{3}$

Taking tangent on both sides, we get

$\frac{4}{x^2-3} = \frac{2}{3}$

$\Rightarrow 12 = 2x^2 - 6$

$\Rightarrow 2x^2 = 18$

$\Rightarrow x^2 = 9$

$\Rightarrow x = 3$ or $x = -3$

Therefore, $x = \pm 3$ are the required values of x.

Question: 8 C

Solution:

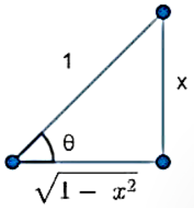
To find: value of x

Given: $\cos(\sin^{-1} x) = \frac{1}{9}$

LHS = $\cos(\sin^{-1} x)$ ... (1)

Let $\sin \theta = x$

Therefore $\theta = \sin^{-1} x$ ... (2)



From the figure, $\cos\theta = \sqrt{1 - x^2}$

$\Rightarrow \theta = \cos^{-1}\sqrt{1 - x^2}$ ... (3)

From (2) and (3),

$\sin^{-1} x = \cos^{-1}\sqrt{1 - x^2}$ ... (4)

Substituting (4) in (1), we get

LHS $= \cos(\cos^{-1}\sqrt{1 - x^2})$

$= \sqrt{1 - x^2}$

Therefore, $\sqrt{1 - x^2} = \frac{1}{9}$

Squaring and simplifying,

$\Rightarrow 81 - 81x^2 = 1$

$\Rightarrow 81x^2 = 80$

$\Rightarrow x^2 = \frac{80}{81}$

$\Rightarrow x = \pm\frac{4\sqrt{5}}{9}$

Therefore, $x = \pm\frac{4\sqrt{5}}{9}$ are the required values of x.

Question: 8 D

Solution:

To find: value of x

Formula Used: $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1 - x^2})$

Given: $\cos(2\sin^{-1} x) = \frac{1}{9}$

LHS = $\cos(2\sin^{-1} x)$

Let $\theta = \sin^{-1} x$

So, $x = \sin\theta$ ... (1)

LHS = $\cos(2\theta)$

$= 1 - 2\sin^2\theta$

Substituting in the given equation,

$$1 - 2\sin^2\theta = \frac{1}{9}$$

$$2\sin^2\theta = \frac{8}{9}$$

$$\sin^2\theta = \frac{4}{9}$$

Substituting in (1),

$$x^2 = \frac{4}{9}$$

$$x = \pm\frac{2}{3}$$

Therefore, $x = \pm\frac{2}{3}$ are the required values of x.

Question: 8 E

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Solution:

To find: value of x

Given: $\sin^{-1}\frac{8}{x} + \sin^{-1}\frac{15}{x} = \frac{\pi}{2}$

We know $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

Let $\sin^{-1}\frac{8}{x} = P$

$$\Rightarrow \sin P = \frac{8}{x}$$

Therefore, $\cos P = \frac{\sqrt{x^2-64}}{x}$

$$P = \cos^{-1}\frac{\sqrt{x^2-64}}{x}$$

$$\cos^{-1}\frac{\sqrt{x^2-64}}{x} + \sin^{-1}\frac{15}{x} = \frac{\pi}{2}$$

Therefore, $\frac{\sqrt{x^2-64}}{x} = \frac{15}{x}$

$$\Rightarrow \sqrt{x^2-64} = 15$$

Squaring both sides,

$\Rightarrow x^2 - 64 = 225$

$\Rightarrow x^2 = 289$

$\Rightarrow x = \pm 17$

Therefore, $x = \pm 17$ are the required values of x.

Question: 9 A

Solution:

To find: value of x

Given: $\cos(\sin^{-1}x) = \frac{1}{2}$

LHS $= \cos(\sin^{-1}x)$

$= \cos(\cos^{-1}(\sqrt{1-x^2}))$

$= \sqrt{1-x^2}$

Therefore, $\sqrt{1-x^2} = \frac{1}{2}$

Squaring both sides,

$$1 - x^2 = \frac{1}{4}$$

$$x^2 = 1 - \frac{1}{4}$$

$$x^2 = \frac{3}{4}$$

$$x = \pm\frac{\sqrt{3}}{2}$$

Therefore, $x = \pm\frac{\sqrt{3}}{2}$ are the required values of x.

Question: 9 B

Solution:

To find: value of x

Given: $\tan^{-1}x = \sin^{-1}\frac{1}{\sqrt{2}}$

We know that $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Therefore, $\frac{\pi}{4} = \sin^{-1}\frac{1}{\sqrt{2}}$

Substituting in the given equation,

$$\tan^{-1}x = \frac{\pi}{4}$$

$$x = \tan\frac{\pi}{4}$$

$\Rightarrow x = 1$

Therefore, x = 1 is the required value of x.

Question: 9 C

Solution:

Given: $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

We know that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

So, $\sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x$

CLASS24

Substituting in the given equation,

$$\frac{\pi}{2} - \cos^{-1} x - \cos^{-1} x = \frac{\pi}{6}$$

Rearranging,

$$2\cos^{-1} x = \frac{\pi}{2} - \frac{\pi}{6}$$

$$2\cos^{-1} x = \frac{\pi}{3}$$

$$\cos^{-1} x = \frac{\pi}{6}$$

$$x = \frac{\sqrt{3}}{2}$$

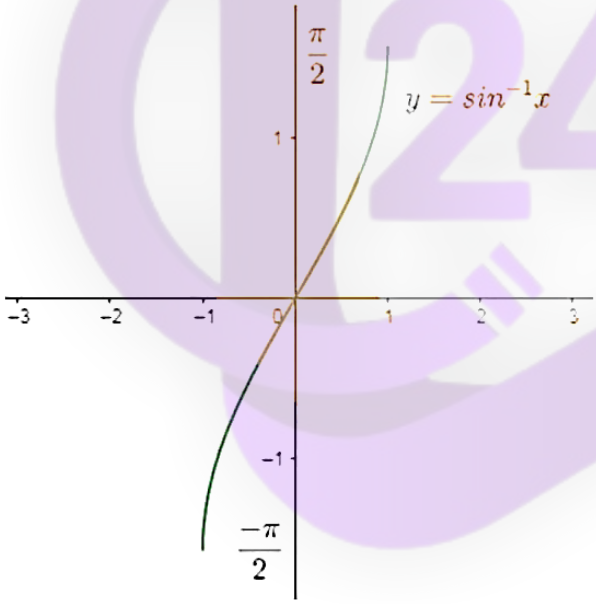
Therefore, $x = \frac{\sqrt{3}}{2}$ is the required value of x.

Exercise : 4D

Question: 1

Solution:

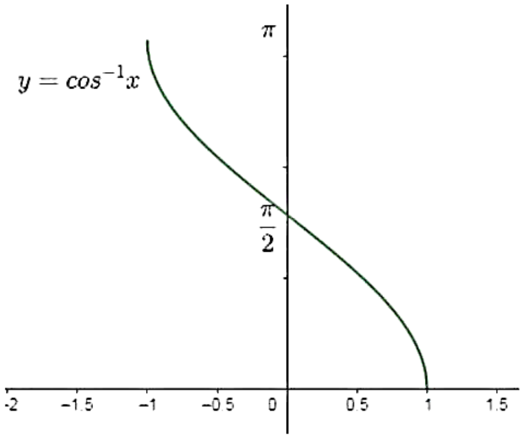
Principal value branch of $\sin^{-1} x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$



Question: 2

Solution:

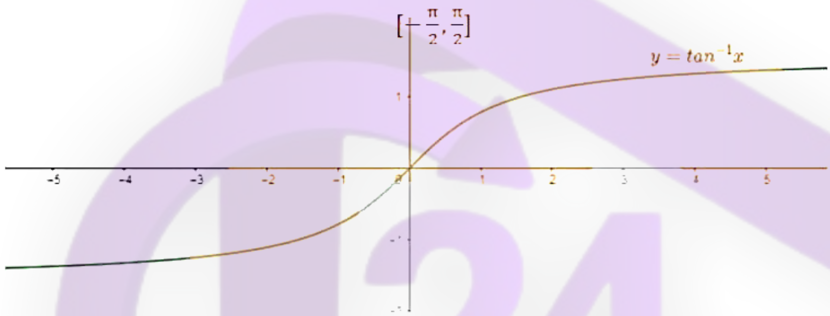
Principal value branch of $\cos^{-1} x$ is $[0, \pi]$



Question: 3

Solution:

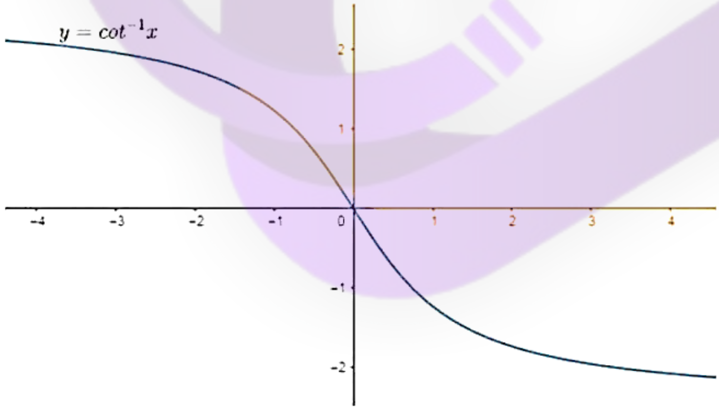
Principal value branch of $\tan^{-1}$ x is



Question: 4

Solution:

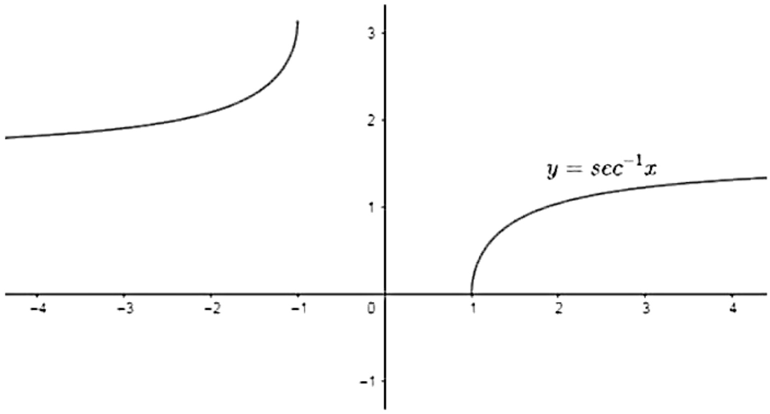
Principal value branch of $\cot^{-1}$ x is (0, π)



Question: 5

Solution:

Principal value branch of $\sec^{-1}$ x is $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



Question: 6

Solution:

Principal value branch of cosec^{-1} x is