

Chapter : 5. MATRICES

Exercise : 5A

Question: 1

If (i) Number of rows = 3

(ii) Number of columns = 4

(iii) Order of matrix = Number of rows x Number of columns = (3 x 4)

(iv) Number of entries = (Number of rows) x (Number of columns)

$$= 3 \times 4$$

$$= 12$$

(V) a_{ij} = element of i^{th} row and j^{th} column

$$a_{23} = 8$$

$$a_{31} = \sqrt{2}$$

$$a_{14} = 1$$

$$a_{33} = 4$$

$$a_{22} = 0$$

Question: 2

Solution:

$$i. A = \begin{bmatrix} 3 & 5 & 4 & -2 \\ 0 & \sqrt{3} & -1 & \frac{4}{9} \end{bmatrix}$$

Order of matrix = Number of rows x Number of columns

$$= (2 \times 4)$$

$$ii. B = \begin{bmatrix} 6 & -5 \\ 1 & 3 \\ 2 & 4 \\ -2 & -1 \end{bmatrix}$$

Order of matrix = Number of rows x Number of columns

$$= (4 \times 2)$$

$$iii. C = \begin{bmatrix} 7 - \sqrt{2} & 5 & 0 \end{bmatrix}$$

Order of matrix = Number of rows x Number of columns

$$= (1 \times 4)$$

$$iv. D = [8 - 3]$$

Order of matrix = Number of rows x Number of columns

$$= (1 \times 2)$$

$$v. E = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$$

Order of matrix = Number of rows x Number of columns

$$= (3 \times 1)$$

$$vi. F = [6]$$

Order of matrix = Number of rows x Number of columns

$$= \{1 \times 1\}$$

Question: 3

Solution:

$$\text{Number of entries} = (\text{Number of rows}) \times (\text{Number of columns}) = 18$$

If order is $\{a \times b\}$ then, Number of entries = $a \times b$

So now $a \times b = 18$ (in this case)

Possible cases are $\{1 \times 18\}, \{2 \times 9\}, \{3 \times 6\}, \{6 \times 3\}, \{9 \times 2\}, \{18 \times 1\}$

Conclusion: If a matrix has 18 elements, then possible orders are $\{1 \times 18\}, \{2 \times 9\}, \{3 \times 6\}, \{6 \times 3\}, \{9 \times 2\}, \{18 \times 1\}$

Question: 4

Solution:

$$\text{Number of entries} = (\text{Number of rows}) \times (\text{Number of columns}) = 7$$

If order is $\{a \times b\}$ then, Number of entries = $a \times b$

So now $a \times b = 7$ (in this case)

Possible cases are $\{1 \times 7\}, \{7 \times 1\}$

Conclusion: If a matrix has 7 elements, then possible orders are $\{1 \times 7\}, \{7 \times 1\}$

Question: 5

$$\text{Given: } a_{ij} = \{2i - j\}$$

$$\text{Now, } a_{11} = \{2 \times 1 - 1\} = 2 - 1 = 1$$

$$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$$

$$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$$

$$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$$

$$a_{31} = 2 \times 3 - 1 = 6 - 1 = 5$$

$$a_{32} = 2 \times 3 - 2 = 6 - 2 = 4$$

Therefore,

$$A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$$

Question: 6

Solution:

It is $\{4 \times 3\}$ matrix. So it has 4 rows and 3 columns

$$\text{Given } a_{ij} = \frac{i}{j}.$$

$$\text{So, } a_{11} = 1, a_{12} = \frac{1}{2}, a_{13} = \frac{1}{3},$$

$$a_{21} = 2, a_{22} = 1, a_{23} = \frac{2}{3},$$

$$a_{31} = 3, a_{32} = \frac{3}{2}, a_{33} = 1,$$

$$a_{41} = 4, a_{42} = 2, a_{43} = \frac{4}{3}$$

So, the matrix =
$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \\ 4 & 2 & \frac{4}{3} \end{bmatrix}$$

Conclusion: Therefore, Matrix is
$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \\ 4 & 2 & \frac{4}{3} \end{bmatrix}$$

Question: 7

Solution:

It is a (2 x 2) matrix. So, it has 2 rows and 2 columns.

Given $a_{ij} = \frac{(i+j)^2}{2}$

So, $a_{11} = \frac{9}{2}, a_{12} = \frac{25}{2},$

$a_{21} = 8, a_{22} = 18$

So, the matrix =
$$\begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

Conclusion: Therefore, Matrix is =
$$\begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

Question: 8

Solution:

It is a (2 x 3) matrix. So, it has 2 rows and 3 columns.

Given $a_{ij} = \frac{(i-j)^2}{2}$

So, $a_{11} = \frac{1}{2}, a_{12} = \frac{9}{2}, a_{13} = \frac{25}{2},$

$a_{21} = 0, a_{22} = 2, a_{23} = 8$

So, the matrix =
$$\begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{bmatrix}$$

Conclusion: Therefore, Matrix is
$$\begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{bmatrix}$$

Question: 9

Solution:

It is a (3 x 4) matrix. So, it has 3 rows and 4 columns.

Given $a_{ij} = \frac{|-3i+j|}{2}$

So, $a_{11} = 1, a_{12} = \frac{1}{2}, a_{13} = 0, a_{14} = \frac{1}{2},$

$a_{21} = \frac{5}{2}, a_{22} = 2, a_{23} = \frac{3}{2}, a_{24} = 1,$

$a_{31} = 4, a_{32} = \frac{7}{2}, a_{33} = 3, a_{34} = \frac{5}{2}$

So, the matrix =
$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

Conclusion: Therefore, Matrix is $\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

Exercise : 5B

Question: 1

If $A + B = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & -1 & 3 \\ 3 & -3 & 4 \end{bmatrix}$

$B + A = \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 5 & -1 & 3 \\ 3 & -3 & 4 \end{bmatrix} = B + A$

Therefore, $A + B = B + A$

This is true for any matrix

Conclusion: $A + B = B + A$

Question: 2

If $(A+B)+C = \left(\begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix} \right) + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix}$

$= \left(\begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 4 & 2 \end{bmatrix} \right) + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{bmatrix}$

$A+(B+C) = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \left(\begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix} \right)$

$= \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \left(\begin{bmatrix} -1 & -1 \\ 7 & -2 \\ -1 & 9 \end{bmatrix} \right)$

$= \begin{bmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{bmatrix}$

Therefore, $(A+B)+C = A+(B+C)$

It is true for any matrix

Conclusion: $(A+B)+C = A+(B+C)$

Question: 3

If $2A = 2 \left(\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -3 \end{bmatrix} \right)$

$= \begin{bmatrix} 6 & 2 & 4 \\ 2 & 4 & -6 \end{bmatrix}$

$(2A-B) = \begin{bmatrix} 6 & 2 & 4 \\ 2 & 4 & -6 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 8 & 2 & 0 \\ -3 & 7 & -8 \end{bmatrix}$

Conclusion: $(2A-B) = \begin{bmatrix} 8 & 2 & 0 \\ -3 & 7 & -8 \end{bmatrix}$

Question: 4

Let $A + 2B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + 2 \left(\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \right)$

$= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix}$

$$= \begin{bmatrix} 4 & 10 \\ -1 & 12 \end{bmatrix}$$

$$\text{Conclusion: } (A+2B) = \begin{bmatrix} 4 & 10 \\ -1 & 12 \end{bmatrix}$$

ii. $B - 4C$

$$B-4C = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - 4\left(\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$$

$$\text{Conclusion: } B-4C = \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$$

iii. $A - 2B + 3C$

$$A-2B+3C = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 2\left(\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\right) + 3\left(\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 13 \\ 16 & 4 \end{bmatrix}$$

$$\text{Conclusion: } A-2B+3C = \begin{bmatrix} -6 & 13 \\ 16 & 4 \end{bmatrix}$$

Question: 5

$$\text{Let } 5A-3B+4C = 5\left(\begin{bmatrix} 0 & 1 & -2 \\ 5 & -1 & -4 \end{bmatrix}\right) - 3\left(\begin{bmatrix} 1 & -3 & -1 \\ 0 & -2 & 5 \end{bmatrix}\right) + 4\left(\begin{bmatrix} 2 & -5 & 1 \\ -4 & 0 & 6 \end{bmatrix}\right)$$

$$= \left(\begin{bmatrix} 0 & 5 & -10 \\ 25 & -5 & -20 \end{bmatrix}\right) - \left(\begin{bmatrix} 3 & -9 & -3 \\ 0 & -6 & 15 \end{bmatrix}\right) + \left(\begin{bmatrix} 8 & -20 & 4 \\ -16 & 0 & 24 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -3 & 14 & -7 \\ 25 & 1 & -35 \end{bmatrix} + \begin{bmatrix} 8 & -20 & 4 \\ -16 & 0 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -6 & -3 \\ 9 & 1 & -11 \end{bmatrix}$$

$$\text{Conclusion: } 5A-3B+4C = \begin{bmatrix} 5 & -6 & -3 \\ 9 & 1 & -11 \end{bmatrix}$$

Question: 6

$$\text{If } 5A = \begin{bmatrix} 5 & 10 & -15 \\ 2 & 3 & 4 \\ 1 & 0 & -5 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{5}{5} & \frac{10}{5} & \frac{-15}{5} \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ \frac{1}{5} & \frac{0}{5} & \frac{-5}{5} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ \frac{1}{5} & 0 & -1 \end{bmatrix}$$

$$\text{Conclusion: } A = \begin{bmatrix} 1 & 2 & -3 \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ \frac{1}{5} & 0 & -1 \end{bmatrix}$$

Question: 7

Solution:

Add $(A+B)$ and $(A-B)$

$$\text{We get } (A+B)+(A-B) = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix} + \begin{bmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{bmatrix}$$

$$2A = \begin{bmatrix} -4 & -4 & 10 \\ 16 & 6 & -6 \\ 6 & 10 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & -2 & 5 \\ 8 & 3 & -3 \\ 3 & 5 & 6 \end{bmatrix}$$

Now Subtract (A-B) from (A+B)

$$(A+B) - (A-B) = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix} - \begin{bmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{bmatrix}$$

$$(2B) = \begin{bmatrix} 6 & 4 & -6 \\ -6 & 2 & -6 \\ 8 & -4 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 2 & -3 \\ -3 & 1 & -3 \\ 4 & -2 & 2 \end{bmatrix}$$

$$\text{Conclusion: } A = \begin{bmatrix} -2 & -2 & 5 \\ 8 & 3 & -3 \\ 3 & 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 & -3 \\ -3 & 1 & -3 \\ 4 & -2 & 2 \end{bmatrix}$$

Question: 8

Solution:

Add 2(2A-B) and (2B+A)

$$2(2A-B) + (2B+A) = 2 \left(\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \right) + \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$$

$$5A = \left(\begin{bmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{bmatrix} \right) + \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$$

$$5A = \begin{bmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

$$B = 2 \left(\begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix} \right) - \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -4 & 2 \\ -4 & 2 & -2 \end{bmatrix} - \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\text{Conclusion: } A = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

(GIVEN ANSWER IS WRONG for question 8)

Question: 9

Solution:

$$\text{Given } \begin{bmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{bmatrix} + x = \begin{bmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{bmatrix}$$

$$x = \begin{bmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{bmatrix} - \begin{bmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -3 & 12 \\ 5 & 4 & 13 \end{bmatrix}$$

$$\text{Conclusion : } x = \begin{bmatrix} 3 & -3 & 12 \\ 5 & 4 & 13 \end{bmatrix}$$

Question: 10

If Given $A + B - C = 0$

$$\begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix} - C = 0$$

$$C = \begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 5 \\ -3 & 8 \\ 7 & -2 \end{bmatrix}$$

$$\text{Conclusion: } C = \begin{bmatrix} 3 & 5 \\ -3 & 8 \\ 7 & -2 \end{bmatrix}$$

Question: 11

Solution:

$$\text{Given } 2A - B + X = 0$$

$$2\left(\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}\right) - \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix} + X = 0$$

$$X = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix} - 2\left(\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 6 & 2 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -1 \\ 0 & -1 \end{bmatrix}$$

$$\text{Conclusion: } X = \begin{bmatrix} -8 & -1 \\ 0 & -1 \end{bmatrix}$$

Question: 12

If Given $A+B+C$ is zero matrix i.e $A+B+C = 0$

$$\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix} + C = 0$$

$$C = -\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$$

$$\text{Conclusion: } C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$$

Question: 13

Solution:

If $Z = \text{diag}[a,b,c]$, then we can write it as

$$Z = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\text{So, } A+2B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} + 2\left(\begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} + \begin{bmatrix} -6 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 28 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 37 \end{bmatrix}$$

$$= \text{diag}[4,9,37]$$

$$\text{Conclusion: } A + 2B = \text{diag}[4,9,37]$$

[Given answer is wrong]

ii. $B + C - A$

If $Z = \text{diag}[a,b,c]$, then we can write it as

$$Z = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$B+C-A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$= \text{diag}[-1, 6, 8]$$

Conclusion: $B+C-A = \text{diag}[-1, 6, 8]$

iii. $2A + B - 5C$

If $Z = \text{diag}[a, b, c]$, then we can write it as

$$Z = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$2A+B-5C = 2 \left(\begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} \right) + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} - 5 \left(\begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 18 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} - \begin{bmatrix} 20 & 0 & 0 \\ 0 & -30 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} -19 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 17 \end{bmatrix}$$

$$= \text{diag}[-19, 27, 17]$$

Conclusion: $2A + B - 5C = \text{diag}[-19, 27, 17]$

(Given answer is wrong)

Question: 14

Solution:

$$\text{If } \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} e & f \\ g & h \end{bmatrix},$$

Then $a=e, b=f, c=g, d=h$

$$\text{Given } \begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

So, $x + y = 8$ and $x - y = 4$

Adding these two gives $2x = 12$

$$\Rightarrow x = 6$$

$$y = 2$$

Conclusion : $x = 6$ and $y = 2$

$$\text{ii. } \begin{bmatrix} 2x+5 & 7 \\ 0 & 3y-7 \end{bmatrix} = \begin{bmatrix} x-3 & 7 \\ 0 & -5 \end{bmatrix}$$

$$\text{Given, } \begin{bmatrix} 2x+5 & 7 \\ 0 & 3y-7 \end{bmatrix} = \begin{bmatrix} x-3 & 7 \\ 0 & -5 \end{bmatrix}$$

So, $2x+5 = x-3$ and $3y-7 = -5$

$$\Rightarrow 3y = 2 \Rightarrow y = \frac{2}{3}$$

$$\Rightarrow 2x + 5 = x - 3 \Rightarrow x = -8$$

Conclusion : $x = -8$ and $y = \frac{2}{3}$

$$\text{iii. } 2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

$$2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

$$2x+3 = 7 \Rightarrow x = 2$$

$$2y-4 = 14 \Rightarrow y = 9$$

Conclusion : $x = 2$ and $y = 9$

(Given answer is wrong)

Question: 15

Solution:

Given

$$2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\text{So, } 2+y = 5 \text{ and } 2x+2 = 8$$

$$\text{i.e } y = 3 \text{ and } x = 3$$

$$\text{Therefore, } x+y=6$$

$$\text{Conclusion: Therefore } x+y = 6$$

Question: 16

$$\text{If } \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} e & f \\ g & h \end{bmatrix},$$

$$\text{Then } a=e, b=f, c=g, d=h$$

$$\text{Given, } \begin{bmatrix} x-y & 2y \\ 2y+z & x+y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix},$$

$$\text{So, } x-y = 1, x+y = 5, 2y = 4 \text{ and } 2y+z = 9$$

$$\text{Therefore, } x+y = 5$$

$$\text{Conclusion: } x+y = 5$$

(Given answer is wrong)

Exercise : 5C

Question: 1 A

Solution:

$$\text{Given : } A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$$

Matrix A is of order 3×2 , and Matrix B is of order 2×2

To find : matrix AB and BA

Formula used :

$$\begin{aligned} \text{Row } i & \rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & \boxed{b_{1j}} & \dots & b_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \dots & \boxed{b_{nj}} & \dots & b_{nn} \end{bmatrix} = \\ & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{i1} & c_{i2} & \dots & \boxed{c_{ij}} & \dots & c_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{array}{l} \text{entry on row } i \\ \text{column } j \end{array} \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

For matrix AB, $a = 3, b = c = 2, d = 2$, thus matrix AB is of order 3×2

$$\text{Matrix AB} = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \times \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2(-2) + (-1)(0) & 2(3) + (-1)(4) \\ 3(-2) + 0(0) & 3(3) + 0(4) \\ -1(-2) + 4(0) & -1(3) + 4(4) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -4 + 0 & 6 - 4 \\ -6 + 0 & 9 + 0 \\ 2 + 0 & -3 + 16 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$$

For matrix BA, $a = 3, b = c = 2, d = 2$, thus matrix BA exists, if and only if $d = a$

But $3 \neq 2$

Thus matrix BA does not exist

Question: 1 B

Solution:

$$\text{Given : } A = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix}$$

Matrix A is of order 3×2 , and Matrix B is of order 3×3

To find : matrix AB and BA

Formula used :

$$\begin{aligned} \text{Row } i & \rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & \boxed{b_{1j}} & \dots & b_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \dots & \boxed{b_{nj}} & \dots & b_{nn} \end{bmatrix} = \\ & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{i1} & c_{i2} & \dots & \boxed{c_{ij}} & \dots & c_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{array}{l} \text{entry on row } i \\ \text{column } j \end{array} \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of

order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

For matrix AB, $a = 3, b = 2, c = 3, d = 3$, thus matrix AB does not exist, as $d \neq a$

But $2 \neq 3$

Thus matrix AB does not exist

For matrix BA, $a = 3, b = 2, c = 3, d = 3$, thus matrix BA is of order 3×2

as $d = a = 3$

$$\text{Matrix BA} = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 3(-1) - 2(-2) + 1(-3) & 3(1) - 2(2) + 1(3) \\ 0(-1) + 1(-2) + 2(-3) & 0(1) + 1(2) + 2(3) \\ -3(-1) + 4(-2) - 5(-3) & -3(1) + 4(2) - 5(3) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -3 + 4 - 3 & 3 - 4 + 3 \\ 0 - 2 - 6 & 0 + 2 + 6 \\ 3 - 8 + 15 & -3 + 8 - 15 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$$

Question: 1 C

Solution:

Given :

$$A = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix}$$

Matrix A is of order 2×3 and Matrix B is of order 3×2

To find : matrices AB and BA

Formula used :

$$\begin{aligned} \text{Row } i & \rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \begin{bmatrix} b_{1j} \\ \vdots \\ b_{ij} \\ \vdots \\ b_{nj} \end{bmatrix} \end{matrix} = \\ & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix} \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

For matrix AB, $a = 2, b = 3, c = 3, d = 2$, matrix AB exists and is of order 2×2 , as

$b = c = 3$

$$\text{Matrix AB} = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0(1) + 1(-1) - 5(0) & 0(3) + 1(0) - 5(5) \\ 2(1) + 4(-1) + 0(0) & 2(3) + 4(0) + 0(5) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 0 - 1 - 0 & 0 + 0 - 25 \\ 2 - 4 + 0 & 6 + 0 + 0 \end{bmatrix} = \begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$$

For matrix BA, a = 2, b = 3, c = 3, d = 2, matrix BA exists and is of order 3 × 3, as

$$d = a = 2$$

$$\text{Matrix BA} = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1(0) + 3(2) & 1(1) + 3(4) & 1(-5) + 3(0) \\ -1(0) + 0(2) & -1(1) + 0(4) & -1(-5) + 0(0) \\ 0(0) + 5(2) & 0(1) + 5(4) & 0(-5) + 5(0) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 0 + 6 & 1 + 12 & -5 + 0 \\ 0 + 0 & -1 + 0 & 5 + 0 \\ 0 + 10 & 0 + 20 & 0 + 0 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 6 & 13 & -5 \\ 0 & -1 & 5 \\ 10 & 20 & 0 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 6 & 13 & -5 \\ 0 & -1 & 5 \\ 10 & 20 & 0 \end{bmatrix}$$

Question: 1 D

Solution:

Given : A = [1 2 3 4] and $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

Matrix A is of order 1 × 4 and Matrix B is of order 4 × 1

To find : matrices AB and BA

Formula used :

$$\begin{aligned} \text{Row } i \times \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{j1} & a_{j2} & a_{j3} & \dots & a_{jn} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} = \\ = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \end{aligned}$$

entry on row i column j

$$\text{Where } c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$$

If A is a matrix of order a × b and B is a matrix of order c × d, then matrix AB exists and is of order a × d, if and only if b = c

If A is a matrix of order a × b and B is a matrix of order c × d, then matrix BA exists and is of order c × b, if and only if d = a

For matrix AB, a = 1, b = 4, c = 4, d = 1, matrix AB exists and is of order 1 × 1, as

$$b = c = 4$$

$$\text{Matrix AB} = [1 \ 2 \ 3 \ 4] \times \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = [1(1) + 2(2) + 3(3) + 4(4)]$$

$$\text{Matrix } AB = [1 + 4 + 9 + 16] = [30]$$

$$\text{Matrix } AB = [30]$$

$$\text{Matrix } AB = [30]$$

For matrix BA, $a = 1, b = 4, c = 4, d = 1$, matrix BA exists and is of order 4×4 , as

$$d = a = 1$$

$$\text{Matrix } BA = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(1) & 1(2) & 1(3) & 1(4) \\ 2(1) & 2(2) & 2(3) & 2(4) \\ 3(1) & 3(2) & 3(3) & 3(4) \\ 4(1) & 4(2) & 4(3) & 4(4) \end{bmatrix}$$

$$\text{Matrix } BA = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$$

$$\text{Matrix } BA = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$$

Question: 1 E

Solution:

Given :

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

Matrix A is of order 3×2 and Matrix B is of order 2×3

To find : matrices AB and BA

Formula used :

$$\begin{matrix} \text{row } i & \leftarrow & \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \end{bmatrix} & \cdot & \begin{matrix} \text{column } j \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} & = & \end{matrix}$$

$$= \begin{bmatrix} c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

For matrix AB, $a = 3, b = 2, c = 2, d = 3$, matrix AB exists and is of order 3×3 , as

$$b = c = 2$$

$$\text{Matrix } AB = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(1) + 1(-1) & 2(0) + 1(2) & 2(1) + 1(1) \\ 3(1) + 2(-1) & 3(0) + 2(2) & 3(1) + 2(1) \\ -1(1) + 1(-1) & -1(0) + 1(2) & -1(1) + 1(1) \end{bmatrix}$$

$$\text{Matrix } AB = \begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

$$\text{Matrix } AB = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

For matrix BA, a = 3, b = 2, c = 2, d = 3, matrix BA exists and is of order 2 × 2, as

$$d = a = 3$$

$$\text{Matrix BA} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1(2) + 0(3) + 1(-1) & 1(1) + 0(2) + 1(1) \\ -1(2) + 2(3) + 1(-1) & -1(1) + 2(2) + 1(1) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 2 + 0 - 1 & 1 + 0 + 1 \\ -2 + 6 - 1 & -1 + 4 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Question: 2 A

Solution:

$$\text{Given : } A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

Matrix A is of order 2 × 2 and Matrix B is of order 2 × 2

To show : matrix AB ≠ BA

Formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d, then matrix AB exists and is of order a × d, if and only if b = c

If A is a matrix of order a × b and B is a matrix of order c × d, then matrix BA exists and is of order c × b, if and only if d = a

For matrix AB, a = 2, b = c = 2, d = 2, thus matrix AB is of order 2 × 2

$$\text{Matrix AB} = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5(2) - 1(3) & 5(1) - 1(4) \\ 6(2) + 7(3) & 6(1) + 7(4) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 10 - 3 & 5 - 4 \\ 12 + 21 & 6 + 28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

For matrix BA, a = 2, b = c = 2, d = 2, thus matrix BA is of order 2 × 2

$$\text{Matrix BA} = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 2(5) + 1(6) & 2(-1) + 1(7) \\ 3(5) + 4(6) & 3(-1) + 4(7) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 10 + 6 & -2 + 7 \\ 15 + 24 & -3 + 28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \text{ and Matrix AB} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

Question: 2 B**Solution:**

Given : $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

Matrix A is of order 3×3 , and Matrix B is of order 3×3 To show : matrix $AB \neq BA$

The formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

For matrix AB, $a = 3, b = c = 3, d = 3$, thus matrix AB is of order 3×3

Matrix AB =

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1) + 2(0) + 3(2) & 1(1) + 2(-1) + 3(3) & 1(0) + 2(1) + 3(4) \\ 0(-1) + 1(0) + 0(2) & 0(1) + 1(-1) + 0(3) & 0(0) + 1(1) + 0(4) \\ 1(-1) + 1(0) + 0(2) & 1(1) + 1(-1) + 0(3) & 1(0) + 1(1) + 0(4) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -1 + 0 + 6 & 1 - 2 + 9 & 0 + 2 + 12 \\ 0 + 0 + 0 & 0 - 1 + 0 & 0 + 1 + 0 \\ -1 + 0 + 0 & 1 - 1 + 0 & 0 + 1 + 0 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

For matrix BA, $a = 3, b = c = 3, d = 3$, thus matrix BA is of order 3×3

Matrix BA =

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1(1) + 1(0) + 0(1) & -1(2) + 1(1) + 0(1) & -1(3) + 1(0) + 0(0) \\ 0(1) - 1(0) + 1(1) & 0(2) - 1(1) + 1(1) & 0(3) - 1(0) + 1(0) \\ 2(1) + 3(0) + 4(1) & 2(2) + 3(1) + 4(1) & 2(3) + 3(0) + 4(0) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -1 + 0 + 0 & -2 + 1 + 0 & -3 + 0 + 0 \\ 0 - 1 + 1 & 0 - 1 + 1 & 0 + 0 + 0 \\ 2 + 0 + 4 & 4 + 3 + 4 & 6 + 0 + 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix} \text{ and Matrix AB} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Matrix $AB \neq BA$ **Question: 3 A****Solution:**

Given :

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \text{ and } B = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$

Matrix A is of order 2×2 and Matrix B is of order 2×2

To show : matrix $AB = BA$

Formula used :

$$\begin{aligned}
 \text{Row } i \rightarrow & \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} = \\
 & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

For matrix AB, $a = 2, b = c = 2, d = 2$, thus matrix AB is of order 2×2

Matrix AB =

$$\begin{aligned}
 & \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \\
 & = \begin{bmatrix} \cos\theta\cos\theta - \sin\theta\sin\theta & -\cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta + \cos\theta\sin\theta & -\sin\theta\sin\theta + \cos\theta\cos\theta \end{bmatrix}
 \end{aligned}$$

$$\text{Matrix AB} = \begin{bmatrix} \cos\theta\cos\theta - \sin\theta\sin\theta & -\cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta + \cos\theta\sin\theta & -\sin\theta\sin\theta + \cos\theta\cos\theta \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} \cos\theta\cos\theta - \sin\theta\sin\theta & -\cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta + \cos\theta\sin\theta & -\sin\theta\sin\theta + \cos\theta\cos\theta \end{bmatrix}$$

For matrix BA, $a = 2, b = c = 2, d = 2$, thus matrix BA is of order 2×2

Matrix BA =

$$\begin{aligned}
 & \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \\
 & = \begin{bmatrix} \cos\theta\cos\theta - \sin\theta\sin\theta & -\cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta + \cos\theta\sin\theta & -\sin\theta\sin\theta + \cos\theta\cos\theta \end{bmatrix}
 \end{aligned}$$

$$\text{Matrix BA} = \begin{bmatrix} \cos\theta\cos\theta - \sin\theta\sin\theta & -\cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta + \cos\theta\sin\theta & -\sin\theta\sin\theta + \cos\theta\cos\theta \end{bmatrix}$$

$$\text{Matrix BA} = \text{Matrix AB} = \begin{bmatrix} \cos\theta\cos\theta - \sin\theta\sin\theta & -\cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta + \cos\theta\sin\theta & -\sin\theta\sin\theta + \cos\theta\cos\theta \end{bmatrix}$$

Thus Matrix $AB = BA$

Question: 3 B

Solution:

$$\text{Given : } A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

Matrix A is of order 3×3 and Matrix B is of order 3×3

To show : matrix $AB \neq BA$

$$\begin{aligned}
 & \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \\
 \text{Formula used :} & \\
 & = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}
 \end{aligned}$$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

For matrix AB, $a = 3, b = c = 3, d = 3$, thus matrix AB is of order 3×3

$$\text{Matrix AB} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1(10) + 2(-11) + 1(-9) & 1(-4) + 2(5) + 1(-5) & 1(-1) + 2(0) + 1(1) \\ 3(10) + 4(-11) + 2(-9) & 3(-4) + 4(5) + 2(-5) & 3(-1) + 4(0) + 2(1) \\ 1(10) + 3(-11) + 2(-9) & 1(-4) + 3(5) + 2(-5) & 1(-1) + 3(0) + 2(1) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 10 - 22 - 9 & -4 + 10 - 5 & -1 + 0 + 1 \\ 30 - 44 - 18 & -12 + 20 - 10 & -3 + 0 + 2 \\ 10 - 33 - 18 & -4 + 15 - 10 & -1 + 0 + 2 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ -32 & -2 & -1 \\ -41 & 1 & 1 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -3 & 1 & 0 \\ -32 & -2 & -1 \\ -41 & 1 & 1 \end{bmatrix}$$

For matrix BA, $a = 3, b = c = 3, d = 3$, thus matrix AB is of order 3×3

Matrix BA =

$$\begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 10(1) - 4(3) - 1(1) & 10(2) - 4(4) - 1(3) & 10(1) - 4(2) - 1(2) \\ -11(1) + 5(3) + 0(1) & -11(2) + 5(4) + 0(3) & -11(1) + 5(2) + 0(2) \\ 9(1) - 5(3) + 1(1) & 9(2) - 5(4) + 1(3) & 9(1) - 5(2) + 1(2) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 10 - 12 - 1 & 20 - 16 - 3 & 10 - 8 - 2 \\ -11 + 15 + 0 & -22 + 20 + 0 & -11 + 10 + 0 \\ 9 - 15 + 1 & 18 - 20 + 3 & 9 - 10 + 2 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ -4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix}$$

Matrix AB $\neq$ BA

Question: 3 C

Solution:

Given :

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

Matrix A is of order 3×3 and Matrix B is of order 3×3

To show : matrix AB = BA

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Formula used :

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

For matrix AB, $a = 3, b = c = 3, d = 3$, thus matrix AB is of order 3×3

$$\text{Matrix AB} = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} =$$

$$\begin{bmatrix} 1(-2) + 3(-1) - 1(-6) & 1(3) + 3(2) - 1(9) & 1(-1) + 3(-1) - 1(-4) \\ 2(-2) + 2(-1) - 1(-6) & 2(3) + 2(2) - 1(9) & 2(-1) + 2(-1) - 1(-4) \\ 3(-2) + 0(-1) - 1(-6) & 3(3) + 0(2) - 1(9) & 3(-1) + 0(-1) - 1(-4) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -2+3+6 & 3+6-9 & -1-3+4 \\ -4-2+6 & 6+4-9 & -2-2+4 \\ -6+0+6 & 9+0-9 & -3+0+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For matrix BA, $a = 3, b = c = 3, d = 3$, thus matrix AB is of order 3×3

$$\text{Matrix BA} = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -2(1)+3(2)-1(3) & -2(3)+3(2)-1(0) & -2(-1)+3(-1)-1(-1) \\ -1(1)+2(2)-1(3) & -1(3)+2(2)-1(0) & -1(-1)+2(-1)-1(-1) \\ -6(1)+9(2)-4(3) & -6(3)+9(2)-4(0) & -6(-1)+9(-1)-4(-1) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -2+6-3 & -6+6+0 & 2-3+1 \\ -1+2-3 & -3+4+0 & 1-2+1 \\ -6+18-12 & -18+18+0 & 6-9+4 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Matrix AB} = \text{Matrix BA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question: 4

$$\text{If Given : } A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Matrix A is of order 3×3 and Matrix B is of order 3×3

To show : matrix $AB = A, BA = B$

$$\begin{aligned} \text{Formula used : } & \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \\ & = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix} \end{aligned}$$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

For matrix AB, $a = 3, b = c = 3, d = 3$, thus matrix AB is of order 3×3

$$\text{Matrix AB} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \times \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} =$$

$$\begin{bmatrix} 2(2)-3(-1)-5(1) & 2(-2)-3(3)-5(-2) & 2(-4)-3(4)-5(-3) \\ -1(2)+4(-1)+5(1) & -1(-2)+4(3)+5(-2) & -1(-4)+4(4)+5(-3) \\ 1(2)-3(-1)-4(1) & 1(-2)-3(3)-4(-2) & 1(-4)-3(4)-4(-3) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & +2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+8 & -4-12+12 \end{bmatrix} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} = \text{Matrix A}$$

Matrix AB = Matrix A

For matrix BA, $a = 3, b = c = 3, d = 3$, thus matrix AB is of order 3×3

$$\text{Matrix BA} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 2(2) - 2(-1) - 4(1) & 2(-3) - 2(4) - 4(-3) & 2(-5) - 2(5) - 4(-4) \\ -1(2) + 3(-1) + 4(1) & -1(-3) + 3(4) + 4(-3) & -1(-5) + 3(5) + 4(-4) \\ 1(2) - 2(-1) - 3(1) & 1(-3) - 2(4) - 3(-3) & 1(-5) - 2(5) - 3(-4) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 4 + 2 - 4 & -6 - 8 + 12 & -10 - 10 + 16 \\ -2 - 3 + 4 & +3 + 12 - 12 & +5 + 15 - 16 \\ 2 + 2 - 3 & -3 - 8 + 9 & -5 - 10 + 12 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \text{Matrix B}$$

$$\text{Matrix BA} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \text{Matrix B}$$

MATRIX AB = A and MATRIX BA = B

Question: 5

If

Solution:

$$\text{Given : } A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

Matrix A is of order 3×3 , matrix B is of order 3×3 and matrix C is of order 3×3

To show : AB is a zero matrix

Formula used :

$$\begin{aligned} \text{Row } i \times \text{column } j &= \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} = \\ &= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \end{aligned}$$

entry on row i
column j

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

AB

$$\begin{aligned} &= \begin{bmatrix} 0 \times a^2 + c \times ab - b \times ac & 0 \times ab + c \times b^2 - b \times bc & 0 \times ac + c \times bc - b \times c^2 \\ -c \times a^2 + 0 \times ab + a \times ac & -c \times ab + 0 \times b^2 + a \times bc & -c \times ac + 0 \times bc + a \times c^2 \\ b \times a^2 - a \times ab + a \times ac & b \times ab - a \times b^2 + a \times bc & b \times ac - a \times bc + a \times c^2 \end{bmatrix} \\ &= \begin{bmatrix} abc - abc & b^2c - b^2c & bc^2 - bc^2 \\ -a^2c + a^2c & -abc + abc & -ac^2 + ac^2 \\ a^2b - a^2b & ab^2 - ab^2 & abc - abc \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

= 0 matrix

Hence, Proved

Question: 16 A

For the following

$$\text{Given : } A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$$

Matrix A is of order 2×3 , matrix B is of order 3×3 and matrix C is of order 3×1

To show : matrix $A(BC) = (AB)C$

Formula used :

$$\begin{aligned}
 \text{Row } i \times \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} & \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & \boxed{b_{1j}} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & \boxed{b_{ij}} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & \boxed{b_{nj}} & \dots & b_{nn} \end{bmatrix} = \\
 & = \begin{bmatrix} c_{i1} & c_{i2} & \dots & \boxed{c_{ij}} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & \boxed{c_{ij}} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & \boxed{c_{nj}} & \dots & c_{nn} \end{bmatrix} \quad \begin{array}{l} \text{entry on row } i \\ \text{column } j \end{array}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

For matrix BC, $a = 3, b = c = 3, d = 1$, thus matrix BC is of order 3×1

$$\text{Matrix BC} = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2(1) + 3(4) + 0(5) \\ 1(1) + 0(4) + 4(5) \\ 1(1) - 1(4) + 2(5) \end{bmatrix} = \begin{bmatrix} 2 + 12 + 0 \\ 1 + 0 + 20 \\ 1 - 4 + 10 \end{bmatrix}$$

$$\text{Matrix BC} = \begin{bmatrix} 14 \\ 21 \\ 7 \end{bmatrix}$$

For matrix A(BC), $a = 2, b = c = 3, d = 1$, thus matrix A(BC) is of order 2×1

$$\text{Matrix A(BC)} = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 14 \\ 21 \\ 7 \end{bmatrix} = \begin{bmatrix} 1(14) + 2(21) + 5(7) \\ 0(14) + 1(21) + 3(7) \end{bmatrix} = \begin{bmatrix} 14 + 42 + 35 \\ 0 + 21 + 21 \end{bmatrix}$$

$$\text{Matrix A(BC)} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

$$\text{Matrix A(BC)} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

For matrix AB, $a = 2, b = c = 3, d = 3$, thus matrix BC is of order 2×3

$$\text{Matrix AB} = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 1(2) + 2(1) + 5(1) & 1(3) + 2(0) + 5(-1) & 1(0) + 2(4) + 5(2) \\ 0(2) + 1(1) + 3(1) & 0(3) + 1(0) + 3(-1) & 0(0) + 1(4) + 3(2) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 2 + 2 + 5 & 3 + 0 - 5 & 0 + 8 + 10 \\ 0 + 1 + 3 & 0 + 0 - 3 & 0 + 4 + 6 \end{bmatrix} = \begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix}$$

For matrix (AB)C, $a = 2, b = c = 3, d = 1$, thus matrix (AB)C is of order 2×1

$$\text{Matrix (AB)C} = \begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 9(1) - 2(4) + 18(5) \\ 4(1) - 3(4) + 10(5) \end{bmatrix}$$

$$\text{Matrix (AB)C} = \begin{bmatrix} 9 - 8 + 90 \\ 4 - 12 + 50 \end{bmatrix} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

$$\text{Matrix } (AB)C = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

$$\text{Matrix } A(BC) = (AB)C = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$$

Question: 6 B

For the following

Solution:

$$\text{Given : } A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \text{ and } C = [1 \ -2]$$

Matrix A is of order 2×3 , matrix B is of order 3×1 and matrix C is of order 1×2

To show : matrix $A(BC) = (AB)C$

Formula used :

$$\begin{aligned} \text{Row } i \times \text{column } j &= \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} = \\ &= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \end{aligned}$$

column j

column j

column j

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

For matrix BC, $a = 3, b = c = 1, d = 2$, thus matrix BC is of order 3×2

$$\text{Matrix } BC = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \times [1 \ -2] = \begin{bmatrix} 1(1) & 1(-2) \\ 1(1) & 1(-2) \\ 2(1) & 2(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}$$

$$\text{Matrix } BC = \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}$$

For matrix A(BC), $a = 2, b = c = 3, d = 2$, thus matrix A(BC) is of order 2×2

$$\text{Matrix } A(BC) = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 2(1) + 3(1) - 1(2) & 2(-2) + 3(-2) - 1(-4) \\ 3(1) + 0(1) + 2(2) & 3(-2) + 0(-2) + 2(-4) \end{bmatrix}$$

$$\text{Matrix } A(BC) = \begin{bmatrix} 2 + 3 - 2 & -4 - 6 + 4 \\ 3 + 0 + 4 & -6 + 0 - 8 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

$$\text{Matrix } A(BC) = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

$$\text{Matrix } A(BC) = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

For matrix AB, $a = 2, b = c = 3, d = 1$, thus matrix AB is of order 2×1

$$\text{Matrix } AB = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2(1) + 3(1) - 1(2) \\ 3(1) + 0(1) + 2(2) \end{bmatrix}$$

$$\text{Matrix } AB = \begin{bmatrix} 2 + 3 - 2 \\ 3 + 0 + 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\text{Matrix } AB = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

For matrix $(AB)C$, $a = 2, b = c = 1, d = 2$, thus matrix $(AB)C$ is of order 2×2

$$\text{Matrix } (AB)C = \begin{bmatrix} 3 \\ 7 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(-2) \\ 7(1) & 7(-2) \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

$$\text{Matrix } (AB)C = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

$$\text{Matrix } A(BC) = (AB)C = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$$

Question: 7 A

Verify that $A(B + C)$

Solution:

$$\text{Given : } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

Matrix A is of order 2×2 , matrix B is of order 2×2 and matrix C is of order 2×2

To verify : $A(B + C) = (AB + AC)$

Formula used :

$$\begin{aligned} \text{row } i \times \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{j1} & a_{j2} & a_{j3} & \dots & a_{jn} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} & \times \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} = \\ & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \end{aligned}$$

entry on row i
column j

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

$$B + C = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+1 & 0-1 \\ 1+0 & -3+1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

Matrix $A(B + C)$ is of order 2×2

$$A(B + C) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1(3) + 2(1) & 1(-1) + 2(-2) \\ 3(3) + 4(1) & 3(-1) + 4(-2) \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 3+2 & -1-4 \\ 9+4 & -3-8 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

For matrix AB, $a = b = c = d = 2$, matrix AB is of order 2×2

$$\text{Matrix } AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(1) & 1(0) + 2(-3) \\ 3(2) + 4(1) & 3(0) + 4(-3) \end{bmatrix}$$

$$\text{Matrix } AB = \begin{bmatrix} 2+2 & 0-6 \\ 6+4 & 0-12 \end{bmatrix} = \begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix}$$

$$\text{Matrix } AB = \begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix}$$

For matrix AC, $a = b = c = d = 2$, matrix AC is of order 2×2

$$\text{Matrix } AC = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(1) + 2(0) & 1(-1) + 2(1) \\ 3(1) + 4(0) & 3(-1) + 4(1) \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 1+0 & -1+2 \\ 3+0 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$$

$$\text{Matrix AB} + \text{AC} = \begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 4+1 & -6+1 \\ 10+3 & -12+1 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

$$\text{Matrix AB} + \text{AC} = \text{A(B + C)} = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$$

$$\text{A(B + C)} = (\text{AB} + \text{AC})$$

Question: 7 B

Verify that $\text{A(B + C)} = (\text{AB} + \text{AC})$

Solution:

$$\text{Given : } A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}.$$

Matrix A is of order 3×2 , matrix B is of order 2×2 and matrix C is of order 2×2

To verify : $\text{A(B + C)} = (\text{AB} + \text{AC})$

Formula used :

$$\begin{aligned} \text{row } i \rightarrow & \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} = \\ & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \end{aligned}$$

entry on row i
column j

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

$$B + C = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5-1 & -3+2 \\ 2+3 & 1+4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix}$$

For Matrix A(B + C) , $a = 3, b = c = d = 2$, thus matrix A(B + C) is of order 3×2

$$\text{A(B + C)} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 2(4) + 3(5) & 2(-1) + 3(5) \\ -1(4) + 4(5) & -1(-1) + 4(5) \\ 0(4) + 1(5) & 0(-1) + 1(5) \end{bmatrix}$$

$$\text{A(B + C)} = \begin{bmatrix} 8 + 15 & -2 + 15 \\ -4 + 20 & 1 + 20 \\ 0 + 5 & 0 + 5 \end{bmatrix} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

$$\text{A(B + C)} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

For matrix AB, $a = 3, b = c = d = 2$, matrix AB is of order 3×2

$$\text{Matrix AB} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(5) + 3(2) & 2(-3) + 3(1) \\ -1(5) + 4(2) & -1(-3) + 4(1) \\ 0(5) + 1(2) & 0(-3) + 1(1) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 10 + 6 & -6 + 3 \\ -5 + 8 & 3 + 4 \\ 0 + 2 & 0 + 1 \end{bmatrix} = \begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix}$$

For matrix AC, $a = 3, b = c = d = 2$, matrix AC is of order 3×2

$$\text{Matrix AC} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2(-1) + 3(3) & 2(2) + 3(4) \\ -1(-1) + 4(3) & -1(2) + 4(4) \\ 0(-1) + 1(3) & 0(2) + 1(4) \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} -2 + 9 & 4 + 12 \\ 1 + 12 & -2 + 16 \\ 0 + 3 & 0 + 4 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix}$$

$$\text{Matrix AB} + \text{AC} = \begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 16 + 7 & 16 - 3 \\ 3 + 13 & 7 + 21 \\ 2 + 3 & 1 + 4 \end{bmatrix} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

$$\text{Matrix AB} + \text{AC} = \text{A(B + C)} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

$$\text{A(B + C)} = (\text{AB} + \text{AC})$$

Question: 8

$$\text{If Given : } A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} :$$

Matrix A is of order 3×3 ; matrix B is of order 3×3 and matrix C is of order 3×3

To verify : $\text{A(B - C)} = (\text{AB} - \text{AC})$.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

The formula used :

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix BA exists and is of order $c \times b$, if and only if $d = a$

$$B - C = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 - 1 & 5 - 5 & -4 - 2 \\ -2 + 1 & 1 - 1 & 3 - 0 \\ 1 - 0 & 0 + 1 & 2 - 1 \end{bmatrix}$$

$$B - C = \begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

For Matrix A(B - C) , $a = 3, b = c = d = 3$, thus matrix A(B - C) is of order 3×3

$$\text{A(B - C)} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\text{A(B - C)} = \begin{bmatrix} 1(-1) + 0(-1) - 2(1) & 1(0) + 0(0) - 2(1) & 1(-6) + 0(3) - 2(1) \\ 3(-1) - 1(-1) + 0(1) & 3(0) - 1(0) + 0(1) & 3(-6) - 1(3) + 0(1) \\ -2(-1) + 1(-1) + 1(1) & -2(0) + 1(0) + 1(1) & -2(-6) + 1(3) + 1(1) \end{bmatrix}$$

$$\text{A(B - C)} = \begin{bmatrix} -1 + 0 - 2 & 0 + 0 - 2 & -6 + 0 - 2 \\ -3 + 1 + 0 & 0 + 0 + 0 & -18 - 3 + 0 \\ 2 - 1 + 1 & 0 + 0 + 1 & 12 + 3 + 1 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$$

$$\text{A(B - C)} = \begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$$

For matrix AB, $a = 3, b = c = d = 3$, matrix AB is of order 3×3

$$\text{Matrix AB} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 1(0) + 0(-2) - 2(1) & 1(5) + 0(1) - 2(0) & 1(-4) + 0(3) - 2(2) \\ 3(0) - 1(-2) + 0(1) & 3(5) - 1(1) + 0(0) & 3(-4) - 1(3) + 0(2) \\ -2(0) + 1(-2) + 1(1) & -2(5) + 1(1) + 1(0) & -2(-4) + 1(3) + 1(2) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 0 + 0 - 2 & 5 + 0 + 0 & -4 + 0 - 4 \\ 0 + 2 + 0 & 15 - 1 + 0 & -12 - 3 + 0 \\ 0 - 2 + 1 & -10 + 1 + 0 & 8 + 3 + 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix}$$

For matrix AC, a = 3, b = c = d = 3, matrix AC is of order 3 x 3

$$\text{Matrix AC} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 1(1) + 0(-1) - 2(0) & 1(5) + 0(1) - 2(-1) & 1(2) + 0(0) - 2(1) \\ 3(1) - 1(-1) + 0(0) & 3(5) - 1(1) + 0(-1) & 3(2) - 1(0) + 0(1) \\ -2(1) + 1(-1) + 1(0) & -2(5) + 1(1) + 1(-1) & -2(2) + 1(0) + 1(1) \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 1 + 0 + 0 & 5 + 0 + 2 & 2 + 0 - 2 \\ 3 + 1 + 0 & 15 + 1 + 0 & 6 + 0 + 0 \\ -2 - 1 + 0 & -10 + 1 - 1 & -4 + 0 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$\text{Matrix AB} - \text{AC} = \begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix} = \begin{bmatrix} -2 - 1 & 5 - 7 & -8 - 0 \\ 2 - 4 & 14 - 16 & -15 - 6 \\ -1 + 3 & -9 + 10 & 13 + 3 \end{bmatrix}$$

$$\text{Matrix AB} - \text{AC} = \begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$$

$$A(B - C) = (AB - AC) = \begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$$

Question: 9

$$\text{If Given : } A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

Matrix A is of order 2 x 2

To show : $A^2 = O$

Formula used :

$$C_{ij} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix}$$

entry on row i
column j

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \times \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} = \begin{bmatrix} ab(ab) + b^2(-a^2) & ab(b^2) + b^2(-ab) \\ -a^2(ab) - ab(-a^2) & -a^2(b^2) - ab(-ab) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = O$$

Question: 10

$$\text{If Given : } A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}.$$

Matrix A is of order 3×3

To show : $A^2 = A$

$$\begin{aligned} \text{Formula used : } & \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \\ &= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix} \end{aligned}$$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$\begin{aligned} A^2 &= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \\ A^2 &= \begin{bmatrix} 2(2) - 2(-1) - 4(1) & 2(-2) - 2(3) - 4(-2) & 2(-4) - 2(4) - 4(-3) \\ -1(2) + 3(-1) + 4(1) & -1(-2) + 3(3) + 4(-2) & -1(-4) + 3(4) + 4(-3) \\ 1(2) - 2(-1) - 3(1) & 1(-2) - 2(3) - 3(-2) & 1(-4) - 2(4) - 3(-3) \end{bmatrix} \\ A^2 &= \begin{bmatrix} 4 + 2 - 4 & -4 - 6 + 8 & -8 - 8 + 12 \\ -2 - 3 + 4 & 2 + 9 - 8 & 4 + 12 - 12 \\ 2 + 2 - 3 & -2 - 6 + 6 & -4 - 8 + 9 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \\ A^2 &= A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \end{aligned}$$

Question: 11

$$\text{If Given : } A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}.$$

Matrix A is of order 3×3

To show : $A^2 = I$

$$\begin{aligned} \text{Formula used : } & \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \\ &= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix} \end{aligned}$$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$\begin{aligned} A^2 &= \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \times \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \\ A^2 &= \begin{bmatrix} 4(4) - 1(3) - 4(3) & 4(-1) - 1(0) - 4(-1) & 4(-4) - 1(-4) - 4(-3) \\ 3(4) + 0(3) - 4(3) & 3(-1) + 0(0) - 4(-1) & 3(-4) + 0(-4) - 4(-3) \\ 3(4) - 1(3) - 3(3) & 3(-1) - 1(0) - 3(-1) & 3(-4) - 1(-4) - 3(-3) \end{bmatrix} \\ A^2 &= \begin{bmatrix} 16 - 3 - 12 & -4 + 0 + 4 & -16 + 4 + 12 \\ 12 + 0 - 12 & -3 + 0 + 4 & -12 + 0 + 12 \\ 12 - 3 - 9 & -3 + 0 + 3 & -12 + 4 + 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$A^2 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question: 12

If Given : $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$.

Matrix A is of order 2×2 , Matrix B is of order 2×2

To find : $3A^2 - 2B + I$

Formula used :

$$\text{row } i \times \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2(2) - 1(3) & 2(-1) - 1(2) \\ 3(2) + 2(3) & 3(-1) + 2(2) \end{bmatrix} = \begin{bmatrix} 4 - 3 & -2 - 2 \\ 6 + 6 & -3 + 4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$$

$$3A^2 = 3 \times \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix}$$

$$3A^2 = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix}$$

$$2B = 2 \times \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$$

$$2B = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$3A^2 - 2B + I = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 - 0 + 1 & -12 - 8 + 0 \\ 36 + 2 + 0 & 3 - 14 + 1 \end{bmatrix}$$

$$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

$$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

Question: 13

If Given : $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$

Matrix A is of order 2×2 .

To find : $-A^2 + 6A$

Formula used :

$$\text{Row } i \leftarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$A^2 = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 2(2) - 2(-3) & 2(-2) - 2(4) \\ -3(2) + 4(-3) & -3(-2) + 4(4) \end{bmatrix} = \begin{bmatrix} 4 + 6 & -4 - 8 \\ -6 - 12 & 6 + 16 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 10 & -12 \\ -18 & 22 \end{bmatrix}$$

$$-A^2 = -\begin{bmatrix} 10 & -12 \\ -18 & 22 \end{bmatrix} = \begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix}$$

$$6A = 6 \times \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix}$$

$$6A = \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix}$$

$$-A^2 + 6A = \begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix} + \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix} = \begin{bmatrix} -10 + 12 & 12 - 12 \\ 18 - 18 & -22 + 24 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$-A^2 + 6A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Question: 14

If Given : $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

Matrix A is of order 2×2 .

To show : $A^2 - 5A + 7I = 0$

Formula used :

$$\text{Row } i \leftarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3(3) + 1(-1) & 3(1) + 1(2) \\ -1(3) + 2(-1) & -1(1) + 2(2) \end{bmatrix} = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5 \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$7I = 7 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 - 5A + 7I = 0$$

Question: 15

Show that the mat

Solution:

$$\text{Given : } A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Matrix A is of order 2×2 .

$$\text{To show : } A^3 - 4A^2 + A = 0$$

Formula used :

$$\begin{aligned} \text{row } i \rightarrow & \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} = \\ & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \end{aligned}$$

column j

entry of row i column j

$$\text{Where } c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$$

A^2 and A^3 are matrices of order 2×2 .

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2(2) + 3(1) & 2(3) + 3(2) \\ 1(2) + 2(1) & 1(3) + 2(2) \end{bmatrix} = \begin{bmatrix} 4 + 3 & 6 + 6 \\ 2 + 2 & 3 + 4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7(2) + 12(1) & 7(3) + 12(2) \\ 4(2) + 7(1) & 4(3) + 7(2) \end{bmatrix} = \begin{bmatrix} 14 + 12 & 21 + 24 \\ 8 + 7 & 12 + 14 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

$$4A^2 = 4 \times \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix}$$

$$4A^2 = \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix}$$

$$A^3 - 4A^2 + A = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^3 - 4A^2 + A = 0$$

Question: 16

$$\text{If Given : } A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}, A^2 = kA - 2I.$$

Matrix A is of order 2×2 .

To find : k

$$\begin{aligned}
 \text{row } i &\rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} = \\
 &= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

A^2 is a matrix of order 2×2 .

$$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3(3) - 2(4) & 3(-2) - 2(-2) \\ 4(3) - 2(4) & 4(-2) - 2(-2) \end{bmatrix} = \begin{bmatrix} 9 - 8 & -6 + 4 \\ 12 - 8 & -8 + 4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

$$kA = k \times \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$$

$$kA - 2I = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - 2 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3k - 2 & -2k \\ 4k & -2k - 2 \end{bmatrix}$$

It is the given that $A^2 = kA - 2I$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k - 2 & -2k \\ 4k & -2k - 2 \end{bmatrix}$$

Equating like terms,

$$3k - 2 = 1$$

$$3k = 1 + 2 = 3$$

$$3k = 3$$

$$k = \frac{3}{3} = 1$$

$$k = 1$$

Question: 17

$$\text{If Given : } A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}, \text{ and } f(x) = x^2 - 2x + 3.$$

Matrix A is of order 2×2 .

To find : $f(A)$

Formula used :

$$\begin{aligned}
 \text{row } i &\rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} = \\
 &= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

A^2 is a matrix of order 2×2 .

$$f(x) = x^2 - 2x + 3$$

$$f(A) = A^2 - 2A + 3I$$

$$A^2 = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -1(-1) + 2(3) & -1(2) + 2(1) \\ 3(-1) + 1(3) & 3(2) + 1(1) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+6 & -2+2 \\ -3+3 & 6+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$2A = 2 \times \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix}$$

$$2A = \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix}$$

$$3I = 3 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$3I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$f(A) = A^2 - 2A + 3I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7+2+3 & -4+0 \\ 0-6+0 & 7-2+3 \end{bmatrix}$$

$$f(A) = A^2 - 2A + 3I = \begin{bmatrix} 12 & -4 \\ -6 & 8 \end{bmatrix}$$

$$f(A) = A^2 - 2A + 3I = \begin{bmatrix} 12 & -4 \\ -6 & 8 \end{bmatrix}$$

Question: 18

If Given : $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and $f(x) = 2x^3 + 4x + 5$

Matrix A is of order 2×2 .

To find : $f(A)$

Formula used :

$$\text{Row } i \rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{1j} & b_{12} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{ij} & b_{i2} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots \\ b_{nj} & b_{n2} & \dots & b_{nn} \end{bmatrix} \end{matrix} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix}$$

entry on row i
column j

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

A^3 is a matrix of order 2×2 .

$$f(x) = 2x^3 + 4x + 5$$

$$f(A) = 2A^3 + 4A + 5I$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1(1) + 2(4) & 1(2) + 2(-3) \\ 4(1) - 3(4) & 4(2) - 3(-3) \end{bmatrix} = \begin{bmatrix} 1+8 & 2-6 \\ 4-12 & 8+9 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 9(1) - 4(4) & 9(2) - 4(-3) \\ -8(1) + 17(4) & -8(2) + 17(-3) \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 9-16 & 18+12 \\ -8+68 & -16-51 \end{bmatrix} = \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix}$$

$$2A^3 = 2 \times \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix} = \begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix}$$

$$2A^3 = \begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix}$$

$$4A = 4 \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix}$$

$$4A = \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix}$$

$$5I = 5 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$5I = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$2A^3 + 4A + 5I = \begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -14 + 4 + 5 & 60 + 8 + 0 \\ 120 + 16 + 0 & -134 - 12 + 5 \end{bmatrix}$$

$$f(A) = 2A^3 + 4A + 5I = \begin{bmatrix} -5 & 68 \\ 136 & -141 \end{bmatrix}$$

$$f(A) = 2A^3 + 4A + 5I = \begin{bmatrix} -5 & 68 \\ 136 & -141 \end{bmatrix}$$

Question: 19

Solution:

Given :

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

To find x and y

Formula used :

$$\text{Row } i \rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \end{bmatrix} \cdot \begin{bmatrix} \text{column } j \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{bmatrix} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix}$$

entry of row i column j

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

The resulting matrix obtained on multiplying $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$ is of order 2×1

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Equating similar terms,

$$2x - 3y = 1 \text{ equation 1}$$

$$x + y = 3 \text{ equation 2}$$

equation 1 + 3(equation 2) and solving the above equations,

$$\begin{array}{r} 2x - 3y = 1 \\ + \\ 3x + 3y = 9 \\ \hline 5x = 10 \end{array}$$

$$5x = 10$$

$$x = \frac{10}{5} = 2$$

$x = 2$, substituting $x = 2$ in equation 2,

$$2 + y = 3$$

$$y = 3 - 2 = 1$$

$$x = 2 \text{ and } y = 1$$

Question: 20

Solution:

Given :

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$$

To find x and y

Formula used :

$$C_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

The resulting matrix obtained on multiplying $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$ is of order 2×1

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x - 4y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 3x - 4y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$$

Equating similar terms,

$$3x - 4y = 3 \text{ equation 1}$$

$$x + 2y = 11 \text{ equation 2}$$

equation 1 + 2(equation 2) and solving the above equations,

$$\begin{array}{r} 3x - 4y = 3 \\ + \\ 2x + 4y = 22 \\ \hline \end{array}$$

$$5x = 3 + 22 = 25$$

$$5x = 25$$

$$x = \frac{25}{5} = 5$$

$x = 5$, substituting $x = 2$ in equation 2,

$$5 + 2y = 11$$

$$2y = 11 - 5 = 6$$

$$2y = 6$$

$$y = \frac{6}{2} = 3$$

$$x = 5 \text{ and } y = 3$$

Question: 21

$$\text{If Given : } A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}, A^2 + xI = yA.$$

A is a matrix of order 2×2

To find : x and y

Formula used :

$$\begin{aligned}
 \text{row } i & \rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \begin{bmatrix} b_{1j} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} = \\
 & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

A^2 is a matrix of order 2×2

$$A^2 = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 3(3) + 1(7) & 3(1) + 1(5) \\ 7(3) + 5(7) & 7(1) + 5(5) \end{bmatrix} = \begin{bmatrix} 9 + 7 & 3 + 5 \\ 21 + 35 & 7 + 25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 + 7 & 3 + 5 \\ 21 + 35 & 7 + 25 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$$

$$xI = x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$$

$$xI = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$$

$$A^2 + xI = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 16 + x & 8 + 0 \\ 56 + 0 & 32 + x \end{bmatrix} = \begin{bmatrix} 16 + x & 8 \\ 56 & 32 + x \end{bmatrix}$$

$$A^2 + xI = \begin{bmatrix} 16 + x & 8 \\ 56 & 32 + x \end{bmatrix}$$

$$yA = y \times \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$$yA = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

It is given that $A^2 + xI = yA$,

$$\begin{bmatrix} 16 + x & 8 \\ 56 & 32 + x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

Equating similar terms in the given matrices,

$$16 + x = 3y \text{ and } 8 = y,$$

hence $y = 8$

Substituting $y = 8$ in equation $16 + x = 3y$

$$16 + x = 3 \times 8 = 24$$

$$16 + x = 24$$

$$x = 24 - 16 = 8$$

$$x = 8$$

$$x = 8, y = 8$$

Question: 22

$$\text{If Given : } A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}, A^2 + aA + bI = O$$

A is a matrix of order 2×2

To find : a and b

Formula used :

$$\begin{aligned}
 \text{Row } i & \leftarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \begin{bmatrix} b_{1j} & b_{12} & \dots & \boxed{b_{1j}} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & \boxed{b_{nj}} & \dots & b_{nn} \end{bmatrix} \end{matrix} = \\
 & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & \boxed{c_{ij}} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

A^2 is a matrix of order 2×2

$$A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3(3) + 2(1) & 3(2) + 2(1) \\ 1(3) + 1(1) & 1(2) + 1(1) \end{bmatrix} = \begin{bmatrix} 9 + 2 & 6 + 2 \\ 3 + 1 & 2 + 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$

$$aA = a \times \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3a & 2a \\ 1a & 1a \end{bmatrix}$$

$$bI = b \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$$

$$bI = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$$

$$A^2 + aA + bI = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ 1a & 1a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 11 + 3a + b & 8 + 2a + 0 \\ 4 + a + 0 & 3 + a + b \end{bmatrix}$$

$$A^2 + aA + bI = \begin{bmatrix} 11 + 3a + b & 8 + 2a \\ 4 + a & 3 + a + b \end{bmatrix}$$

It is given that $A^2 + aA + bI = 0$

$$\begin{bmatrix} 11 + 3a + b & 8 + 2a \\ 4 + a & 3 + a + b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Equating similar terms in the matrices, we get

$$4 + a = 0 \text{ and } 3 + a + b = 0$$

$$a = 0 - 4 = -4$$

$$a = -4$$

substituting $a = -4$ in $3 + a + b = 0$

$$3 - 4 + b = 0$$

$$-1 + b = 0$$

$$b = 0 + 1 = 1$$

$$b = 1$$

$$a = -4 \text{ and } b = 1$$

Question: 23

Solution:

Given :

$$\text{To find matrix } A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}, A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}.$$

Formula used :

$$\begin{aligned}
 \text{Row } i & \rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{1j} & b_{12} & \dots & \boxed{b_{1j}} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} = \\
 & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & \boxed{c_{ij}} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

IF $XA = B$, then $A = X^{-1}B$

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \cdot A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$\text{To find } \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1}$$

$$\text{Determinant of given matrix} = \begin{vmatrix} 5 & -7 \\ -2 & 3 \end{vmatrix} = 5(3) - (-7)(-2) = 15 - 14 = 1$$

$$\text{Adjoint of matrix } \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} = \frac{1}{1} \times \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3(-16) + 7(7) & 3(-6) + 7(2) \\ 2(-16) + 5(7) & 2(-6) + 5(2) \end{bmatrix} = \begin{bmatrix} -48 + 49 & -18 + 14 \\ -32 + 35 & -12 + 10 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$$

Question: 24

Solution:

Given : A.

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$$

To find : matrix A

Formula used :

$$\begin{aligned}
 \text{Row } i & \rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{1j} & b_{12} & \dots & \boxed{b_{1j}} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} = \\
 & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & \boxed{c_{ij}} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

IF $AX = B$, then $A = BX^{-1}$

$$A \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1}$$

To find $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1}$

$$\text{Determinant of given matrix} = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 5(2) - (4)(3) = 10 - 12 = -2$$

$$\text{Adjoint of matrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{1}{-2} \times \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} = \frac{1}{-2} \cdot \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{1}{-2} \cdot \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \frac{1}{-2} \cdot \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$A = \frac{1}{-2} \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} = \frac{1}{-2} \cdot \begin{bmatrix} 0(5) - 4(-4) & 0(-3) - 4(2) \\ 10(5) + 3(-4) & 10(-3) + 3(2) \end{bmatrix}$$

$$A = \frac{1}{-2} \cdot \begin{bmatrix} 0 + 16 & 0 - 8 \\ 50 - 12 & -30 + 6 \end{bmatrix} = \frac{1}{-2} \cdot \begin{bmatrix} 16 & -8 \\ 38 & -24 \end{bmatrix} = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$$

Question: 25

$$\text{If Given : } A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix}$$

$$(A + B)^2 = (A^2 + B^2)$$

To find : a and b

Formula used :

$$\begin{aligned} \text{Row } i \times \text{column } j & \rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{j1} & a_{j2} & a_{j3} & \dots & a_{jn} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} = \\ & = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{array}{l} \text{entry on row } i \\ \text{column } j \end{array} \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} 1+a & -1-1 \\ 2+b & -1-1 \end{bmatrix} = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix} \times \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix} = \begin{bmatrix} (1+a)(1+a) - 2(2+b) & (1+a)(-2) - 2(-2) \\ (2+b)(1+a) - 2(2+b) & (2+b)(-2) - 2(-2) \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} 1 + a^2 + 2a - 4 - 2b & -2 - 2a + 4 \\ 2 + 2a + b + ab - 4 - 2b & -4 - 2b + 4 \end{bmatrix} = \begin{bmatrix} a^2 + 2a - 2b - 3 & 2 - 2a \\ 2a - b + ab - 2 & -2b \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} a^2 + 2a - 2b - 3 & 2 - 2a \\ 2a - b + ab - 2 & -2b \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1(1) - 1(2) & 1(-1) - 1(-1) \\ 2(1) - 1(2) & 2(-1) - 1(-1) \end{bmatrix} = \begin{bmatrix} 1-2 & -1+1 \\ 2-2 & -2+2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} \times \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a(a) - 1(b) & a(-1) - 1(-1) \\ b(a) - 1(b) & b(-1) - 1(-1) \end{bmatrix} = \begin{bmatrix} a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix}$$

$$(A^2 + B^2) = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix} = \begin{bmatrix} -1 + a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix}$$

$$(A^2 + B^2) = \begin{bmatrix} -1 + a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix}$$

It is given that $(A + B)^2 = (A^2 + B^2)$

$$\begin{bmatrix} a^2 + 2a - 2b - 3 & 2 - 2a \\ 2a - b + ab - 2 & -2b \end{bmatrix} = \begin{bmatrix} -1 + a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix}$$

Equating similar terms in the given matrices we get,

$$2 - 2a = -a + 1 \text{ and } -2b = -b + 1$$

$$2 - 1 = -a + 2a \text{ and } -2b + b = 1$$

$$1 = a \text{ and } -b = 1$$

$$a = 1 \text{ and } b = -1$$

Question: 26

If Given : $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

To show : $F(x) \cdot F(y) = F(x + y)$.

Formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x + y) = \begin{bmatrix} \cos(x + y) & -\sin(x + y) & 0 \\ \sin(x + y) & \cos(x + y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x) \cdot F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x(\cos y) - \sin x(\sin y) + 0(0) & \cos x(-\sin y) - \sin x(\cos y) + 0(0) & \cos x(0) - \sin x(0) + 0(1) \\ \sin x(\cos y) + \cos x(\sin y) + 0(0) & \sin x(-\sin y) + \cos x(\cos y) + 0(0) & \sin x(0) + \cos x(0) + 0(1) \\ 0(\cos y) + 0(\sin y) + 1(0) & 0(-\sin y) + 0(\cos y) + 1(0) & 0(0) + 0(0) + 1(1) \end{bmatrix}$$

$$F(x) \cdot F(y) = \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We know that,

$$\cos x(\cos y) - \sin x(\sin y) = \cos(x + y) \text{ and } -\cos x(\sin y) - \sin x(\cos y) = -\sin(x + y)$$

$$F(x) \cdot F(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x+y) = F(x) \cdot F(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x+y) = F(x) \cdot F(y)$$

Question: 27

If Given : $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$,

To show : $A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

Formula used :

$$\text{Row } i \times \begin{matrix} \text{column } j \\ \uparrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} = \begin{matrix} \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \\ \text{entry of row } i \text{ column } j \end{matrix}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \times \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos \alpha (\cos \alpha) + \sin \alpha (-\sin \alpha) & \cos \alpha (\sin \alpha) + \sin \alpha (\cos \alpha) \\ -\sin \alpha (\cos \alpha) + \cos \alpha (-\sin \alpha) & -\sin \alpha (\sin \alpha) + \cos \alpha (\cos \alpha) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & -2\sin \alpha \cos \alpha \\ -2\sin \alpha \cos \alpha & -\sin^2 \alpha + \cos^2 \alpha \end{bmatrix}$$

We know that $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ and $\sin 2\alpha = 2\sin \alpha \cos \alpha$

$$A^2 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

Question: 28

If Given : $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = O$.

To find : x

Formula used :

$$\begin{aligned}
 (C \cdot A)_{ij} &= \sum_{k=1}^n a_{ik} b_{kj} \\
 &= \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} = \\
 &= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{array}{l} \text{entry on row } i \\ \text{column } j \end{array}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$[1 \ x \ 1] \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$$

$$[1 \ x \ 1] \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} = [1(1) + x(4) + 1(3) \quad 1(2) + x(5) + 1(2) \quad 1(3) + x(6) + 1(5)]$$

$$[1 \ x \ 1] \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} = [1 + 4x + 3 \quad 2 + 5x + 2 \quad 6x + 8]$$

$$[1 \ x \ 1] \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} = [4x + 4 \quad 5x + 4 \quad 6x + 8]$$

$$[1 \ x \ 1] \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = [4x + 4 \quad 5x + 4 \quad 6x + 8] \times \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$$[4x + 4 \quad 5x + 4 \quad 6x + 8] \times \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = [(4x + 4)(1) + (5x + 4)(-2) + (6x + 8)(3)]$$

$$[4x + 4 \quad 5x + 4 \quad 6x + 8] \times \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = [4x + 4 - 10x - 8 + 18x + 24] = [12x + 20]$$

$$[1 \ x \ 1] \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = [12x + 20] = 0$$

$$12x + 20 = 0$$

$$12x = -20$$

$$x = \frac{-20}{12} = \frac{-5}{3}$$

$$x = \frac{-5}{3}$$

Question: 29

$$\text{If Given : } \begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0.$$

To find : x

Formula used :

$$\begin{aligned}
 \text{Row } i &\rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \begin{bmatrix} b_{1j} \\ \vdots \\ b_{ij} \\ \vdots \\ b_{nj} \end{bmatrix} \dots \begin{bmatrix} b_{1n} \\ \vdots \\ b_{in} \\ \vdots \\ b_{nn} \end{bmatrix} \end{matrix} = \\
 &= \begin{bmatrix} c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} = [x(2) + 4(1) + 1(0) \quad x(1) + 4(0) + 1(2) \quad x(2) + 4(2) + 1(-4)]$$

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} = [2x + 4 \quad x + 2 \quad 2x + 4]$$

$$[2x + 4 \quad x + 2 \quad 2x + 4] \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = [(2x + 4)(x) + 4(x + 2) + (2x + 4)(-1)]$$

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = [2x^2 + 4x + 4x + 8 - 2x - 4] = [2x^2 + 6x + 4] = 0$$

$$2x^2 + 6x + 4 = 0$$

$$x^2 + 3x + 2 = 0$$

$$(x + 1)(x + 2) = 0$$

$$x + 1 = 0 \text{ or } x + 2 = 0$$

$$x = -1 \text{ or } x = -2$$

$$x = -1 \text{ or } x = -2$$

Question: 30

Find the values of

Solution:

$$\text{Given: } \begin{bmatrix} a & b \\ -a & 2b \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

To find: a and b

Formula used:

$$\begin{aligned}
 \text{Row } i &\rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \begin{bmatrix} b_{1j} \\ \vdots \\ b_{ij} \\ \vdots \\ b_{nj} \end{bmatrix} \dots \begin{bmatrix} b_{1n} \\ \vdots \\ b_{in} \\ \vdots \\ b_{nn} \end{bmatrix} \end{matrix} = \\
 &= \begin{bmatrix} c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$, then matrix AB exists and is of order $a \times d$, if and only if $b = c$

$$\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} a(2) + b(-1) \\ -a(2) + 2b(-1) \end{bmatrix} = \begin{bmatrix} 2a - b \\ -2a - 2b \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 2a - b \\ -2a - 2b \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Equating similar terms,

$$2a - b = 5$$

$$-2a - 2b = 4$$

Adding the above two equations, we get

$$-3b = 9$$

$$b = \frac{9}{-3} = -3$$

$$b = -3$$

substituting $b = -3$ in $2a - b = 5$, we get

$$2a + 3 = 5$$

$$2a = 5 - 3 = 2$$

$$a = 1$$

$$a = 1 \text{ and } b = -3$$

Question: 31

If Given : $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, and $f(x) = x^2 - 5x + 7$.

Matrix A is of order 2×2 .

To find : $f(A)$

Formula used :

$$f(A) = A^2 - 5A + 7I$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix}$$

entry on row i column j

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

A^2 is a matrix of order 2×2 .

$$f(x) = x^2 - 5x + 7$$

$$f(A) = A^2 - 5A + 7I$$

$$A^2 = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 3(3) + 4(-4) & 3(4) + 4(-3) \\ -4(3) - 3(-4) & -4(4) - 3(-3) \end{bmatrix} = \begin{bmatrix} 9 - 16 & 12 - 12 \\ -12 + 12 & -16 + 9 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$$

$$5A = 5 \times \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}$$

$$5A = \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}$$

$$7I = 7 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$f(A) = A^2 - 5A + 7I = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} - \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -7-15+7 & 0-20+0 \\ 0+20+0 & -7+15+7 \end{bmatrix}$$

$$f(A) = A^2 - 5A + 7I = \begin{bmatrix} -15 & -20 \\ 20 & 15 \end{bmatrix}$$

$$f(A) = A^2 - 5A + 7I = \begin{bmatrix} -15 & -20 \\ 20 & 15 \end{bmatrix}$$

Question: 32

If Given : $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

Matrix A is of order 2×2 .

To prove : $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

Proof :

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Let us assume that the result holds for A^{n-1}

$$A^{n-1} = \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix}$$

We need to prove that the result holds for A^n by mathematical induction .

$$A^n = A^{n-1} \times A = \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(1) + (n-1)(0) & 1(1) + (n-1)(1) \\ 0(1) + 1(0) & 0(1) + 1(1) \end{bmatrix}$$

$$A^n = \begin{bmatrix} 1+0 & 1+n-1 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$$

Question: 33

Solution:

Given : $A \neq 0, B \neq 0, AB = 0, BA \neq 0$

To Find : matrix A and B

Formula used :

$$C_{ij} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix}$$

entry on row i
column j

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$A \neq 0, B \neq 0$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1(0) + 0(1) & 1(0) + 0(0) \\ 0(0) + 0(1) & 0(0) + 0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$BA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0(1) + 0(0) & 0(0) + 0(0) \\ 1(1) + 0(0) & 1(0) + 0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Question: 34

Solution:

Given : $AB = AC$ and $B \neq C$.

To Find : matrix A and B

Formula used :

$$\text{Row } i \rightarrow \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \cdot \begin{matrix} \text{column } j \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \quad \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

$$\text{Let } A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$B \neq C$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1(0) + 0(1) & 1(0) + 0(0) \\ 0(0) + 0(1) & 0(0) + 0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(0) + 0(0) & 1(0) + 0(1) \\ 0(0) + 0(0) & 0(0) + 0(1) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$AB = AC = 0$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Question: 35

$$\text{If Given : } A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}.$$

Matrices A and B are of order 2×2 .

To find : $(3A^2 - 2B + I)$.

Formula used :

$$\begin{aligned}
 \text{row } i \times \begin{matrix} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{1j} & b_{12} & \dots & \boxed{b_{1j}} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn} \end{bmatrix} \end{matrix} &= \\
 &= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & \boxed{c_{ij}} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

A^2 is a matrix of order 2×2 .

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1(1) + 0(-1) & 1(0) + 0(7) \\ -1(1) + 7(-1) & -1(0) + 7(7) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$$

$$3A^2 = 3 \times \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix}$$

$$3A^2 = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix}$$

$$2B = 2 \times \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$$

$$2B = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$3A^2 - 2B + I = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3-0+1 & 0-8+0 \\ -24+2+0 & 147-14+1 \end{bmatrix}$$

$$3A^2 - 2B + I = \begin{bmatrix} 4 & -8 \\ -22 & 134 \end{bmatrix}$$

Question: 36

$$\text{If Given : } \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

To find : x

Formula used :

$$\begin{aligned}
 \text{row } i \times \begin{matrix} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ \boxed{b_{ij}} & b_{i2} & \dots & b_{in} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix} \end{matrix} &= \\
 &= \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \dots & \boxed{c_{ij}} & \dots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn} \end{bmatrix} \begin{matrix} \text{entry on row } i \\ \text{column } j \end{matrix}
 \end{aligned}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{in}b_{nj}$

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 2(1) + 3(-2) & 2(-3) + 3(4) \\ 5(1) + 7(-2) & 5(-3) + 7(4) \end{bmatrix} = \begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

Equating similar terms in the two matrices, we get

$$x = 13$$

$$x = 13$$

Exercise : 5D

Question: 1

Hence transpose of matrix A is,

$$A' = \begin{bmatrix} 2 & 0 \\ -3 & 7 \\ 5 & -4 \end{bmatrix}$$

$$(A')' = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{bmatrix} \quad (A')' = A \text{ Hence, Proved.}$$

Question: 2

$$A = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix}$$

To Prove: $(2A)' = 2A'$

Proof: Let us consider, $B = 2A$

$$\text{Now, } B = 2 \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & 10 \\ -4 & 0 \\ 8 & -12 \end{bmatrix}$$

$$\text{LHS} \Rightarrow B' = \begin{bmatrix} 6 & -4 & 8 \\ 10 & 0 & -12 \end{bmatrix}$$

Again to find RHS, we will find the transpose of matrix A

$$A' = \begin{bmatrix} 3 & -2 & 4 \\ 5 & 0 & -6 \end{bmatrix}$$

$$\text{RHS} = 2A'$$

$$\Rightarrow 2 \begin{bmatrix} 3 & -2 & 4 \\ 5 & 0 & -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & -4 & 8 \\ 10 & 0 & -12 \end{bmatrix}$$

$$\text{LHS} = \text{RHS}$$

Hence proved.

Question: 3

$$\text{Given } A = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$$

To Prove: $(A + B)' = A' + B'$

Proof: Let us consider $C = A + B$

$$C = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix} + \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -3 & -3 \\ -2 & 1 & 2 \end{bmatrix}$$

Now $LHS = C'$

$$\Rightarrow \begin{bmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{bmatrix}$$

To find RHS, we will find transpose of matrix A and B

$$A' = \begin{bmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{bmatrix} \text{ And } B' = \begin{bmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{bmatrix}$$

$RHS = A' + B'$

$$\Rightarrow \begin{bmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{bmatrix} + \begin{bmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{bmatrix}$$

$LHS = RHS$

Hence proved.

Question: 4

$$\text{Given } P = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix} \text{ and } Q = \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$$

To Prove: $(P + Q)' = P' + Q'$

Proof: Let us consider $R = P + Q$.

$$R = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & -1 \\ -2 & -1 \\ 2 & 11 \end{bmatrix}$$

$LHS = R \Rightarrow (P + Q)'$

$$LHS = \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix}$$

To find RHS, we will first find the transpose of matrix P and Q

$$P' = \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix} \text{ And } Q' = \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix}$$

$RHS = P' + Q'$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix}$$

LHS = RHS

Hence proved.

Question: 5

$$A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$$

To Prove: $A + A'$ is symmetric. (Note: A matrix P is symmetric if $P' = P$)

Proof: We will find A' ,

$$A' = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$$

Now let us take $P = A + A'$

$$P = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$$

$$\text{Now } P' = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$$

$$\Rightarrow P' = P$$

Hence $A + A'$ is a symmetric matrix.

Question: 6

$$\text{Given } A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

To prove: $A - A'$ is a skew-symmetric matrix. (Note: A matrix P is skew-symmetric if $P' = -P$)

Proof: First we will find the transpose of matrix A

$$A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

Let us take $P = A - A'$

$$P = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

$$\Rightarrow P' = P$$

Hence $A - A'$ is a skew symmetric matrix.

Question: 7

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

To Prove: A is a skew symmetric matrix.

Proof: As for a matrix to be skew symmetric $A' = -A$

We will find A' .

$$A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$= - \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$\Rightarrow A' = -A$$

So A is A skew symmetric matrix.

Question: 8

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, \text{ As for a symmetric matrix } A' = A \text{ hence}$$

$$A + A' = 2A$$

$$A = \frac{1}{2}(A + A') \Rightarrow P \text{ (Symmetric Matrix)}$$

Similarly for a skew symmetric matrix since $A' = -A$ hence

$$A - A' = 2A$$

$$A = \frac{1}{2}(A - A') \Rightarrow Q \text{ (Skew Symmetric Matrix)}$$

So a matrix can be represented as a sum of a symmetric matrix P and skew symmetric matrix Q.

First, we will find the transpose of matrix A,

$$A' = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

Now using the above formulas,

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2} \left(\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \right)$$

$$\Rightarrow \frac{1}{2} \begin{bmatrix} 4 & 2 \\ 2 & 8 \end{bmatrix}$$

$$P = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$$

$$Q = \frac{1}{2} \left(\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \right)$$

$$\Rightarrow \frac{1}{2} \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

Hence $A = P + Q$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \text{ [Matrix A as the sum of P and Q]}$$

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

Question: 9

Given $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, to express as the sum of symmetric matrix P and skew symmetric matrix Q.

$A = P + Q$

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$, we will find transpose of matrix A

$$A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

Now using the above formulas

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2} \left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} \right)$$

$$\Rightarrow \frac{1}{2} \begin{bmatrix} 6 & -3 \\ -3 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2} \left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} \right)$$

$$\Rightarrow \frac{1}{2} \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

Hence $A = P + Q$

$$\Rightarrow \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} \text{ [Matrix A as the sum of P and Q]}$$

$$\Rightarrow \begin{bmatrix} 3 & -\frac{8}{2} \\ \frac{2}{2} & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

Question: 10

$A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$, to express as sum of symmetric matrix P and skew symmetric matrix Q.

$$A = P + Q$$

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$.

First, we find A'

$$A' = \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix}$$

Now using the above mentioned formulas

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2} \left(\begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix} \right)$$

$$\Rightarrow \frac{1}{2} \begin{bmatrix} -2 & 7 & 8 \\ 7 & 6 & 4 \\ 8 & 4 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 9 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2} \left(\begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix} \right)$$

$$\Rightarrow \frac{1}{2} \begin{bmatrix} 0 & 3 & -6 \\ -3 & 0 & 4 \\ 6 & -4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{3}{2} & -3 \\ \frac{-3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$$

Now $A = P + Q$

$$\Rightarrow \begin{bmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 9 \end{bmatrix} + \begin{bmatrix} 0 & \frac{3}{2} & -3 \\ \frac{-3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix} \quad [\text{Matrix A as sum of P and Q}]$$

$$\Rightarrow \begin{bmatrix} -1 & \frac{10}{2} & 1 \\ \frac{4}{2} & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$$

Question: 11

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}, \text{ to express as sum of symmetric matrix P and skew symmetric matrix Q}$$

$$A = P + Q$$

$$\text{Where } P = \frac{1}{2}(A + A') \text{ and } Q = \frac{1}{2}(A - A')$$

First we will find A' ,

$$A' = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}$$

Now using above mentioned formulas,

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2} \left(\begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix} \right)$$

$$\Rightarrow \frac{1}{2} \begin{bmatrix} 6 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2} \left(\begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix} \right)$$

$$\Rightarrow \frac{1}{2} \begin{bmatrix} 0 & -3 & -1 \\ 3 & 0 & 4 \\ 1 & -4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{bmatrix}$$

Now $A = P + Q$

$$\Rightarrow \begin{bmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{bmatrix} \quad [\text{Matrix A as sum of P and Q}]$$

$$\Rightarrow \begin{bmatrix} 3 & \frac{-2}{2} & 0 \\ \frac{4}{2} & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$$

Question: 12

Given $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$, to express as sum of symmetric matrix P and skew symmetric matrix Q.

$$A = P + Q$$

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$.

First we will find A'

$$A' = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}$$

Now using above mentioned formulas

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2} \left(\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} \right)$$

$$\Rightarrow \frac{1}{2} \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2} \left(\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} \right)$$

$$\Rightarrow \frac{1}{2} \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & \frac{-3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 0 \end{bmatrix}$$

Now $A = P + Q$

$$\Rightarrow \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix} + \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & \frac{-3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$$

Question: 13 A

$$C = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+6 & 4+15 \\ 2+8 & 8+20 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}$$

$$\text{LHS} \Rightarrow C' = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$$A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ And } B' = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$$

$$\text{RHS} = B'A'$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+6 & 2+8 \\ 3+16 & 8+20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

$$\text{LHS} = \text{RHS}$$

Hence proved.

Question: 13 B

$$C = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+(-2) & -9+1 \\ 2+(-4) & -6+2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -8 \\ -2 & -4 \end{bmatrix}$$

$$\text{LHS} \Rightarrow C' = \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$$B' = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix} \text{ And } A' = \begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\text{RHS} = B'A'$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+(-2) & 2+(-4) \\ -9+1 & -6+2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$$

$$\text{LHS} = \text{RHS}$$

Hence proved.

Question: 13 C

Let us take $C = AB$

$$C = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{bmatrix}$$

$$\text{LHS} = C'$$

$$\Rightarrow \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$$A' = \begin{bmatrix} -1 & 2 & 3 \end{bmatrix} \text{ And } B' = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix}$$

$$\text{RHS} = B'A'$$

$$\Rightarrow \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$$

$$\text{LHS} = \text{RHS}$$

Hence proved.

Question: 13 D

$$C = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3 + 4 + 3 & 4 + 2 + 0 \\ 12 + (-10) + (-6) & -16 + (-5) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 6 \\ -4 & -21 \end{bmatrix}$$

$$\text{LHS} = C'$$

$$\Rightarrow \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$$A' = \begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix} \text{ And } B' = \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix}$$

$$\text{RHS} = B'A'$$

$$\Rightarrow \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3 + 4 + 3 & 12 + (-10) + (-6) \\ 4 + 2 & -16 + (-5) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

LHS = RHS

Hence proved.

Question: 14

Given $A = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix}$, We will find A'

$$A' = \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$$

LHS = $A'A$

$$\Rightarrow \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix} \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos^2 a + \sin^2 a & \cos a \sin a + (-\sin a \cos a) \\ \sin a \cos a + (-\cos a \sin a) & \sin^2 a + \cos^2 a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ [Using } \cos^2 a + \sin^2 a = 1 \text{ and commutative law } a.b = b.a \text{ i.e.}$$

$$\sin a \cos a = \cos a \sin a]$$

$$\text{RHS} = 1 \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

LHS = RHS

Hence proved.

Question: 15

Given $A = [1 \ 2 \ 3]$

We will find A' to calculate AA' ,

$$A' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Now

$$AA' = [1 \ 2 \ 3] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow [1 + 4 + 9]$$

$$\Rightarrow [14]$$

Exercise : 5E

Question: 1

Solution:

$$\text{Let, } A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array} \right], \text{ where } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array} \right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array} \right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{cc|cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array} \right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 2

Solution:

Let, A =

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array} \right], \text{ where } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array} \right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -5 & -2 & 1 \end{array} \right] \xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array} \right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{5} & \frac{2}{5} \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array} \right]$$

Here, the matrix A is converted into the Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 3

Solution:

Let, A =

$$\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|cc} 2 & 5 & 1 & 0 & 0 \\ -3 & 1 & 0 & 1 & 1 \end{array} \right], \text{ where } I = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right]$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\begin{aligned} \left[\begin{array}{ccc|cc} 2 & 5 & 1 & 0 & 0 \\ -3 & 1 & 0 & 1 & 1 \end{array} \right] &\xrightarrow{R_2+R_1} \left[\begin{array}{ccc|cc} 2 & 5 & 1 & 0 & 0 \\ -1 & 6 & 1 & 1 & 1 \end{array} \right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|cc} 1 & 11 & 2 & 1 & 1 \\ -1 & 6 & 1 & 1 & 1 \end{array} \right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|cc} 1 & 11 & 2 & 1 & 1 \\ 0 & 17 & 3 & 2 & 2 \end{array} \right] \\ &\xrightarrow{\frac{1}{17}R_2} \left[\begin{array}{ccc|cc} 1 & 11 & 2 & 1 & 1 \\ 0 & 1 & \frac{3}{17} & \frac{2}{17} & \frac{2}{17} \end{array} \right] \xrightarrow{R_1-11R_2} \left[\begin{array}{ccc|cc} 1 & 0 & \frac{1}{17} & -\frac{5}{17} & -\frac{5}{17} \\ 0 & 1 & \frac{3}{17} & \frac{2}{17} & \frac{2}{17} \end{array} \right] \end{aligned}$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \left[\begin{array}{cc} \frac{1}{17} & -\frac{5}{17} \\ \frac{3}{17} & \frac{2}{17} \end{array} \right] = \frac{1}{17} \left[\begin{array}{cc} 1 & -5 \\ 3 & 2 \end{array} \right] \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 4

Solution:

Let, $A =$

$$\left[\begin{array}{cc} 2 & -3 \\ 4 & 5 \end{array} \right]$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array} \right], \text{ where } I = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right]$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\begin{aligned} \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array} \right] &\xrightarrow{R_2-2R_1} \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 0 & 11 & -2 & 1 \end{array} \right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 11 & -2 & 1 \end{array} \right] \xrightarrow{\frac{1}{11}R_2} \left[\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{11} & \frac{1}{11} \end{array} \right] \\ &\xrightarrow{R_1+\frac{3}{2}R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{5}{22} & \frac{3}{22} \\ 0 & 1 & -\frac{2}{11} & \frac{1}{11} \end{array} \right] \end{aligned}$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \left[\begin{array}{cc} \frac{5}{22} & \frac{3}{22} \\ -\frac{2}{11} & \frac{1}{11} \end{array} \right] \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 5

Solution:

$$\text{Let, } A = \begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{cc|cc} 4 & 0 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array} \right], \text{ where } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\begin{aligned} \left[\begin{array}{cc|cc} 4 & 0 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array} \right] &\xrightarrow{R_1 - 2R_2} \left[\begin{array}{cc|cc} 0 & -10 & 1 & -2 \\ 2 & 5 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|cc} 2 & 5 & 0 & 1 \\ 0 & -10 & 1 & -2 \end{array} \right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|cc} 1 & \frac{5}{2} & 0 & \frac{1}{2} \\ 0 & -10 & 1 & -2 \end{array} \right] \\ &\xrightarrow{-\frac{1}{10}R_2} \left[\begin{array}{cc|cc} 1 & \frac{5}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{10} & \frac{1}{5} \end{array} \right] \xrightarrow{R_1 - \frac{5}{2}R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{4} & 0 \\ 0 & 1 & -\frac{1}{10} & \frac{1}{5} \end{array} \right] \end{aligned}$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ -\frac{1}{10} & \frac{1}{5} \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 6

Solution:

$$\text{Let, } A = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{cc|cc} 6 & 7 & 1 & 0 \\ 8 & 9 & 0 & 1 \end{array} \right], \text{ where } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\begin{aligned} \left[\begin{array}{cc|cc} 6 & 7 & 1 & 0 \\ 8 & 9 & 0 & 1 \end{array} \right] &\xrightarrow{R_2 - R_1} \left[\begin{array}{cc|cc} 6 & 7 & 1 & 0 \\ 2 & 2 & -1 & 1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|cc} 2 & 2 & -1 & 1 \\ 6 & 7 & 1 & 0 \end{array} \right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|cc} 2 & 2 & -1 & 1 \\ 0 & 1 & 4 & -3 \end{array} \right] \\ &\xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|cc} 1 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 4 & -3 \end{array} \right] \xrightarrow{R_1 - R_2} \left[\begin{array}{cc|cc} 1 & 0 & -\frac{9}{2} & \frac{7}{2} \\ 0 & 1 & 4 & -3 \end{array} \right] \end{aligned}$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} -\frac{9}{2} & \frac{7}{2} \\ 4 & -3 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 7

Using elementary

Solution:

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$$\text{Let, } A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array} \right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\begin{aligned} & \left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 - 3R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array} \right] \\ & \xrightarrow{R_3 + 4R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 4 & -3 & 1 \end{array} \right] \xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array} \right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array} \right] \\ & \xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 3 & 1 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array} \right] \xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 3 & 1 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array} \right] \xrightarrow{R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -4 & 3 & 1 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array} \right] \end{aligned}$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} \quad [\text{Answer}]$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 8

Solution:

$$\text{Let, } A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 3 & -2 & 2 & 0 & 0 & 1 \end{array} \right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 3 & -2 & 2 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2-R_1} \left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 3 & -2 & 2 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3-R_1} \left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_1-R_3} \left[\begin{array}{ccc|ccc} 1 & -4 & 4 & 2 & 0 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 \end{array} \right] \xrightarrow{R_3-R_1} \left[\begin{array}{ccc|ccc} 1 & -4 & 4 & 2 & 0 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 0 & 5 & -5 & -3 & 0 & 2 \end{array} \right] \xrightarrow{R_3-R_2} \left[\begin{array}{ccc|ccc} 1 & -4 & 4 & 2 & 0 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 0 & 0 & -5 & -2 & -1 & 2 \end{array} \right]$$

$$\xrightarrow{R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 1 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 0 & 0 & -5 & -2 & -1 & 2 \end{array} \right] \xrightarrow{\frac{1}{5}R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 1 & -1 \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & -5 & -2 & -1 & 2 \end{array} \right] \xrightarrow{-\frac{1}{5}R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 1 & -1 \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{array} \right]$$

$$\xrightarrow{R_1-R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & \frac{6}{5} & \frac{4}{5} & -1 \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{array} \right] \xrightarrow{R_1-4R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{2}{5} & 0 & \frac{3}{5} \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{array} \right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \left[\begin{array}{ccc} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{array} \right] = -\frac{1}{5} \left[\begin{array}{ccc} 2 & 0 & -3 \\ 1 & -1 & 0 \\ -2 & -1 & 2 \end{array} \right] \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 9

Solution:

$$\text{Let, } A = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 1 & 5 & 9 & 0 & 1 & 0 \\ 6 & 4 & 7 & 0 & 0 & 1 \end{array} \right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\left[\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 1 & 5 & 9 & 0 & 1 & 0 \\ 6 & 4 & 7 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 3 & 0 & 2 & 1 & 0 & 0 \\ 6 & 4 & 7 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3-2R_2} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 3 & 0 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_2-3R_1} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 0 & -15 & -25 & 1 & -3 & 0 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array} \right] \xrightarrow{R_2+4R_3} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array} \right] \xrightarrow{R_1-R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -3 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_3 - 4R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 0 & 55 & 26 & 12 & -15 \end{array} \right] \xrightarrow{\frac{1}{55}R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array} \right]$$

$$\xrightarrow{R_2 + 13R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & 0 & -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array} \right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{157}{55} & \frac{64}{55} & -\frac{80}{55} \\ 0 & 1 & 0 & -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array} \right]$$

$$\xrightarrow{R_1 - 6R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{55} & -\frac{8}{55} & \frac{10}{55} \\ 0 & 1 & 0 & -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array} \right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \left[\begin{array}{ccc} \frac{1}{55} & -\frac{8}{55} & \frac{10}{55} \\ -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array} \right] = -\frac{1}{55} \left[\begin{array}{ccc} -1 & 8 & -10 \\ 47 & 9 & -25 \\ -26 & -12 & 15 \end{array} \right] \quad [\text{Answer}]$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 10

Solution:

$$\text{Let, } A = \left[\begin{array}{ccc} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{array} \right]$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 3 & -3 & -4 & 0 & 0 & 1 \end{array} \right], \text{ where } I = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 3 & -3 & -4 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 1 & -6 & -6 & 0 & -1 & 1 \end{array} \right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & 1 & 5 & -1 & 1 & 0 \\ 1 & -6 & -6 & 0 & -1 & 1 \end{array} \right]$$

$$\xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & 1 & 5 & -1 & 1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array} \right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & -1 & 8 & -2 & 1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array} \right] \xrightarrow{-R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{R_3+7R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & 0 & -67 & 15 & -9 & 1 \end{array} \right] \xrightarrow{-\frac{1}{67}R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array} \right]$$

$$\xrightarrow{R_2+8R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array} \right] \xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{39}{67} & -\frac{10}{67} & \frac{16}{67} \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array} \right]$$

$$\xrightarrow{R_1+3R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array} \right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \left[\begin{array}{ccc|ccc} -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array} \right] = \frac{1}{67} \left[\begin{array}{ccc} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{array} \right] \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 11

Solution:

$$\text{Let, } A = \left[\begin{array}{ccc} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{array} \right]$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 3 & -5 & 0 & 0 & 0 & 1 \end{array} \right], \text{ where } I = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 3 & -5 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3-R_1} \left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array} \right] \xrightarrow{R_1-R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array} \right] \xrightarrow{R_3+2R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & -5 & 6 & 1 \end{array} \right] \xrightarrow{\frac{1}{4}R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array} \right]$$

$$\begin{aligned}
 &\xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 0 & -\frac{3}{4} & \frac{6}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array} \right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array} \right] \\
 &\xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & \frac{5}{8} & -\frac{2}{8} & -\frac{1}{8} \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array} \right] \xrightarrow{R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{5}{8} & \frac{10}{8} & \frac{1}{8} \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array} \right]
 \end{aligned}$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \left[\begin{array}{ccc|ccc} -\frac{5}{8} & \frac{10}{8} & \frac{1}{8} \\ -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array} \right] = -\frac{1}{8} \left[\begin{array}{ccc|ccc} 5 & -10 & -1 \\ 3 & -6 & 1 \\ 10 & -12 & -2 \end{array} \right] \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 12

Solution:

$$\text{Let, } A = \left[\begin{array}{ccc} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{array} \right]$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array} \right], \text{ where } I = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array} \right] \xrightarrow{R_2 + 3R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 9 & -7 & 3 & 1 & 0 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array} \right] \xrightarrow{-R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & 5 & -4 & 2 & 0 & -1 \end{array} \right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -1 & -2 \end{array} \right]$$

$$\xrightarrow{R_2 - 4R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 5 & 9 \\ 0 & 1 & -1 & 1 & -1 & -2 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 & -2 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array} \right]$$

$$\xrightarrow{R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array} \right] \xrightarrow{R_1 + 2R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -5 & 10 & 18 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array} \right] \xrightarrow{R_1 - 3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -3 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array} \right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 13

Solution:

$$\text{Let, } A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 7 & 0 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array} \right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\begin{aligned} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 7 & 0 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array} \right] &\xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array} \right] &\xrightarrow{R_3+2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right] \\ &\xrightarrow{R_2-R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right] &\xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 9 & -2 & 2 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right] &\xrightarrow{R_1-3R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -1 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right] \end{aligned}$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 14

Solution:

$$\text{Let, } A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 3 & 0 & -1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array} \right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\left[\begin{array}{ccc|ccc} 3 & 0 & -1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 9 & 2 & -2 & 3 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array} \right]$$

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$$\xrightarrow{R_2 - 2R_3} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 - 4R_2} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array} \right] \xrightarrow{R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & -3 & 0 & 9 & -13 & 9 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array} \right]$$

$$\xrightarrow{R_1 + 3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -4 & 3 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array} \right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question: 15

Solution:

$$\text{Let, } A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array} \right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1} .

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 0 & 3 & 5 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 - 3R_2} \left[\begin{array}{ccc|ccc} 0 & 3 & 5 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array} \right]$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 3 & 5 & 1 & 1 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array} \right] \xrightarrow{R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & -2 & -3 & 1 & -2 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array} \right] \xrightarrow{R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array} \right]$$

$$\xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 0 & 2 & 3 & -1 & 2 & -1 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array} \right] \xrightarrow{R_3 + 3R_1} \left[\begin{array}{ccc|ccc} 0 & 2 & 3 & -1 & 2 & -1 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -3 & 3 & -2 \end{array} \right] \xrightarrow{R_1 - 2R_3} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & 3 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -3 & 3 & -2 \end{array} \right]$$

$$\xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & 3 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -8 & 7 & -5 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 5 & -4 & 3 \\ 0 & 1 & 0 & -8 & 7 & -5 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -8 & 7 & -5 \\ 0 & 0 & 1 & 5 & -4 & 3 \end{array} \right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Exercise : 5F

Question: 1

Solution:

Here, i is the subscript for a row, and j is the subscript for column

And the given matrix is 3×2 , so $1 \leq i \leq 3$ and $1 \leq j \leq 2$

$$\text{Hence for } i=1, j=1, a_{11} = \frac{1}{2}(1 - (2 \times 1))^2 = \frac{1}{2}$$

$$\text{For } i=1, j=2, a_{12} = \frac{1}{2}(1 - (2 \times 2))^2 = \frac{9}{2}$$

$$\text{For } i=2, j=1, a_{21} = \frac{1}{2}(2 - (2 \times 1))^2 = 0$$

$$\text{For } i=2, j=2, a_{22} = \frac{1}{2}(2 - (2 \times 2))^2 = 2$$

$$\text{For } i=3, j=1, a_{31} = \frac{1}{2}(3 - (2 \times 1))^2 = \frac{1}{2}$$

$$\text{For } i=3, j=2, a_{32} = \frac{1}{2}(3 - (2 \times 2))^2 = \frac{1}{2}$$

$$\text{Hence the required matrix is :- } \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Question: 2

Solution:

$$\text{The elements of the matrix are given by, } a_{ij} = \frac{1}{2}|-3j + j|$$

Matrix is 2×3 hence, $1 \leq i \leq 2, 1 \leq j \leq 3$

Here, i is the subscript for a row, and j is the subscript for column

$$\text{For } i=1, j=1, a_{11} = \frac{1}{2}|-3(1) + 1| = 1$$

$$\text{For } i=1, j=2, a_{12} = \frac{1}{2}|-3(1) + 2| = \frac{1}{2}$$

$$\text{For } i=1, j=3, a_{13} = \frac{1}{2}|-3(1) + 3| = 0$$

$$\text{For } i=2, j=1, a_{21} = \frac{1}{2}|-3(2) + 1| = \frac{5}{2}$$

$$\text{For } i=2, j=2, a_{22} = \frac{1}{2}|-3(2) + 2| = 2$$

$$\text{For } i=2, j=3, a_{23} = \frac{1}{2}|-3(2) + 3| = \frac{3}{2}$$

Hence the required matrix is :-

Question: 3

Solution:

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 5 & 2 & 3 \\ 2 & 2 & \frac{3}{2} \end{bmatrix}$$

On comparing L.H.S. and R. H.S we get,

$$\begin{bmatrix} x + 2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$$

On comparing each term we get,

$$x + 2y = -4 \dots(i)$$

$$-y = 3 \dots(ii)$$

$$3x = 6 \dots(iii)$$

From (i), (ii) and (iii), we get,

$$y = -3 \text{ and } x = 2$$

Question: 4

Solution:

Given,

$$2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Using the property of matrix multiplication such that h is scalar, $h \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ah & bh \\ ch & dh \end{bmatrix}$

Using the matrix property of matrix addition, when two matrices are of the same order then, each element gets added to the corresponding element,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix}$$

$$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Comparing each element we get,

$$2+y=5, \Rightarrow y=3$$

$$2x+2=8, \Rightarrow x=3$$

Question: 5

Solution:

$$\text{Given, } x. \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y. \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix}$$

And we have,

$$\begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Solving the linear equations, we get,

$$x = 3, y = -4$$

Question: 6

Solution:

Given,

$$\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$$

On comparing each element of the two matrices we get,

$$x=3,$$

$$3x-y=2$$

$$y=7$$

$$2x+z=4,$$

$$z=-2,$$

$$3y-w=7,$$

$$w=14$$

Question: 7

Solution:

Given,

$$\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = 3 \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

First applying matrix addition then, comparing each element of the matrix with the corresponding element we get,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix}$$

$$\begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix} = \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix}$$

We now have, $x+4=3x$,(i)

$$x=2$$

$$2w+3=3w$$
,(ii)

$$w=3$$

$6+x+y=3y$, substituting x from (i) we get,

$$y=4,$$

And $-1+z+w=3z$, substituting w from (ii), we get,

$$z=1$$

Question: 8

Solution:

We are given two diagonal matrices A and B,

On adding the two diagonal matrices of order (3×3) we get an diagonal matrix of order (3×3)

Each of the elements get added to the corresponding element hence, we get after adding,

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, we get $A+B = \text{diag}(4 \ 1 \ 1)$

Question: 9

Solution:

We have to show that

$$\cos \theta \cdot \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \cdot \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiplying the scalars with we get,

$$\begin{bmatrix} \cos \theta \times \cos \theta & \cos \theta \times \sin \theta \\ \cos \theta \times (-\sin \theta) & \cos \theta \times \cos \theta \end{bmatrix} + \begin{bmatrix} \sin \theta \times \sin \theta & \sin \theta \times (-\cos \theta) \\ \sin \theta \times \cos \theta & \sin \theta \times \sin \theta \end{bmatrix}$$

$$\begin{bmatrix} \cos^2 \theta + \sin^2 \theta & 0 \\ 0 & \cos^2 \theta + \sin^2 \theta \end{bmatrix}$$

And we know that $\cos^2 \theta + \sin^2 \theta = 1$

$$\begin{bmatrix} \cos^2 \theta + \sin^2 \theta & 0 \\ 0 & \cos^2 \theta + \sin^2 \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, proved.

Question: 10

Solution:

Given, $A+B+C =$ zero matrix

We know that zero matrix is a matrix whose all elements are zero, so we have,

$$A = \begin{bmatrix} 1 & -5 \\ -3 & 2 \\ 4 & -2 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ -2 & 3 \end{bmatrix}$$

WE have $A+B+C=0$,

So $C = -A+B$,

$$-C = \begin{bmatrix} 1 & -5 \\ -3 & 2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ -2 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} -4 & 4 \\ 1 & -1 \\ -2 & -1 \end{bmatrix}$$

Question: 11

Solution:

$$\text{Given, } A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

Here, A' i.e. A transpose is $\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

We are given that $A+A'=I$

$$\text{So, } \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

After doing addition of matrices, we get,

$$\begin{bmatrix} \cos \alpha + \cos \alpha & \sin \alpha - \sin \alpha \\ \sin \alpha - \sin \alpha & \cos \alpha + \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \cos \alpha & 0 \\ 0 & 2 \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing the elements we get,

$$2 \cos \alpha = 1$$

$$\text{This implies, } \cos \alpha = \frac{1}{2}$$

$$\text{For } \alpha \text{ belongs } 0 \text{ to } \pi, \alpha = \frac{\pi}{3}$$

Question: 12

Solution:

Given,

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Applying matrix multiplication we get,

$$\begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

On comparing the elements we get, $2x - 3y = 1$,

$$x + y = 3,$$

On solving the equations we get, $x = 2, y = 1$

Question: 13

Solution:

Given,

$$\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Applying matrix multiplication we have, $\begin{bmatrix} x + 2y \\ 3y + 2x \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

On comparing the elements with each other we get,

The linear equations, $x + 2y = 3, 3y + 2x = 5$

On solving these equations we get $x = 1, y = 1$

Question: 14

If <

Solution:

$$\text{Given, } A = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} \text{ and } A' = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$$

$$\text{Then, } (A + A') \text{ will be, } \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$$

The matrix $\begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$ is a symmetrical matrix.

Question: 15

Solution:

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \text{ and}$$

$$A' = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$

$$(A - A') = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

The matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is skew-symmetric.

Question: 16

Solution:

$$\text{Given, } A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$$

We need to a matrix X such that, $A + 2B + X = 0$

We have, $X = -(A + 2B)$,

$$X = - \left[\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} + 2 \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} \right]$$

$$X = - \left[\begin{bmatrix} 2 + (-2) & -3 + (2 \times 2) \\ 4 + 0 & 5 + (2 \times 3) \end{bmatrix} \right]$$

$$X = \begin{bmatrix} 0 & -1 \\ -4 & -11 \end{bmatrix}$$

Question: 17

Solution:

Given, $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

We have $3A - 2B + X = 0$

So $X = -(3A - 2B)$

Thus,

$$X = -3 \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} - 2 \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$X = - \begin{bmatrix} 3 \times 4 + 2 \times 2 & 3 \times 2 - 2 \times 1 \\ 3 \times 1 - 2 \times 3 & 3 \times 3 - 2 \times 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -16 & -4 \\ 3 & -5 \end{bmatrix}$$

Question: 18

Solution:

Given, $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

$$A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

Then, $AA' = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

Applying matrix multiplication we get,

$$AA' =$$

$$\begin{bmatrix} \cos \alpha \times \cos \alpha + \sin \alpha \times \sin \alpha & \cos \alpha \times (-\sin \alpha) + \sin \alpha \times \cos \alpha \\ (-\sin \alpha) \times \cos \alpha + \cos \alpha \times \sin \alpha & (-\sin \alpha) \times (-\sin \alpha) + \cos \alpha \times \cos \alpha \end{bmatrix}$$

$$AA' = \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & 0 \\ 0 & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix}$$

Hence, $AA' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

As we know that $\cos^2 \alpha + \sin^2 \alpha = 1$

Question: 19

Solution:

We are given that A and B are symmetric matrices of the same order then, we need to show that $(AB - BA)$ is a skew symmetric matrix.

Let us consider P is a matrix of the same order as A and B

And let $P = (AB - BA)$,

we have $A = A'$ and $B = B'$

then, $P' = (AB - BA)'$

$P' = [(AB)' - (BA)']$ using reversal law we have $(CD)' = D'C'$

$P' = (B'A' - A'B')$

$P' = (BA - AB)$

$P' = -P$

Hence, P is a skew symmetric matrix.

Question: 20

Solution:

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$f(x) = x^2 - 4x + 1,$

$f(A) = A^2 - 4A + I,$

$f(A) = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$f(A) = \begin{bmatrix} 4 + 3 - 8 + 1 & 6 + 6 - 12 + 0 \\ 2 + 2 - 4 + 0 & 3 + 4 - 8 + 1 \end{bmatrix}$

$f(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Question: 21

Solution:

Given that matrix A is both symmetric and skew symmetric, then,

We have $A = A'$ (i)

And $A = -A'$ (ii)

From (i) and (ii) we get,

$A' = -A',$

$2A' = 0$

$A' = 0$

Then, $A = 0$

Hence proved.

