

Chapter : 7. ADJOINT AND INVERSE OF A MATRIX

Exercise : 7

Question: 1

Solution:

$$\text{Here, } A = \begin{bmatrix} 2 & 3 \\ 5 & 9 \end{bmatrix}$$

Now, we have to find adj A and for that we have to find co-factors:

$$a_{11} (\text{co - factor of } 2) = (-1)^{1+1}(9) = (-1)^2(9) = 9$$

$$a_{12} (\text{co - factor of } 3) = (-1)^{1+2}(5) = (-1)^3(5) = -5$$

$$a_{21} (\text{co - factor of } 5) = (-1)^{2+1}(3) = (-1)^3(3) = -3$$

$$a_{22} (\text{co - factor of } 9) = (-1)^{2+2}(2) = (-1)^4(2) = 2$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} 9 & -5 \\ -3 & 2 \end{bmatrix}$$

Now, adj A = Transpose of co-factor Matrix

$$\therefore \text{adj } A = \begin{bmatrix} 9 & -5 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$$

Calculating A (adj A)

$$\begin{aligned} A.(\text{adj } A) &= \begin{bmatrix} 2 & 3 \\ 5 & 9 \end{bmatrix} \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 2 \times 9 + 3 \times (-5) & 2 \times (-3) + 3 \times 2 \\ 5 \times 9 + 9 \times (-5) & 5 \times (-3) + 9 \times 2 \end{bmatrix} \\ &= \begin{bmatrix} 18 - 15 & -6 + 6 \\ 45 - 45 & -15 + 18 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \\ &= (3) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= 3I \end{aligned}$$

Calculating (adj A)A

$$\begin{aligned} (\text{adj } A).A &= \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 5 & 9 \end{bmatrix} \\ &= \begin{bmatrix} 9 \times 2 + (-3) \times 5 & 9 \times 3 + (-3) \times 9 \\ -5 \times 2 + 2 \times 5 & -5 \times 3 + 2 \times 9 \end{bmatrix} \\ &= \begin{bmatrix} 18 - 15 & 27 - 27 \\ -10 + 10 & -15 + 18 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \\ &= (3) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= 3I \end{aligned}$$

Calculating |A|.I

$$|A|.I = \begin{vmatrix} 2 & 3 \\ 5 & 9 \end{vmatrix} I$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A, is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= (2 \times 9 - 3 \times 5)I$$

$$= (18 - 15)I$$

$$= 3I$$

$$\text{Thus, } A(\text{adj } A) = (\text{adj } A)A = |A|I = 3I$$

$$\Rightarrow A(\text{adj } A) = (\text{adj } A)A = |A|I$$

Hence Proved

$$\text{Ans. } \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$$

Question: 2

Solution:

$$\text{Here, } A = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

Now, we have to find adj A and for that we have to find co-factors:

$$a_{11} \text{ (co - factor of 3)} = (-1)^{1+1}(2) = (-1)^2(2) = 2$$

$$a_{12} \text{ (co - factor of -5)} = (-1)^{1+2}(-1) = (-1)^3(-1) = 1$$

$$a_{21} \text{ (co - factor of -1)} = (-1)^{2+1}(-5) = (-1)^3(-5) = 5$$

$$a_{22} \text{ (co - factor of 2)} = (-1)^{2+2}(3) = (-1)^4(3) = 3$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

Now, adj A = Transpose of co-factor Matrix

$$\therefore \text{adj } A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Calculating A (adj A)

$$A.(\text{adj } A) = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 2 + (-5) \times 1 & 3 \times 5 + (-5) \times 3 \\ (-1) \times 2 + 2 \times 1 & (-1) \times 5 + 2 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 5 & 15 - 15 \\ -2 + 2 & -5 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Calculating (adj A)A

$$(\text{adj } A).A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 3 + 5 \times (-1) & 2 \times (-5) + 5 \times 2 \\ 1 \times 3 + 3 \times (-1) & 1 \times (-5) + 3 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 5 & -10 + 10 \\ 3 - 3 & -5 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Calculating $|A|.I$

$$|A|.I = \begin{vmatrix} 3 & -5 \\ -1 & 2 \end{vmatrix} I$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A , is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [3 \times 2 - (-1) \times (-5)]I$$

$$= [6 - (5)]I$$

$$= (1)I$$

$$= I$$

Thus, $A(\text{adj } A) = (\text{adj } A)A = |A|I = I$

$$\Rightarrow A(\text{adj } A) = (\text{adj } A)A = |A|I$$

Hence Proved

Ans. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Question: 3

Solution:

Here, $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

Now, we have to find $\text{adj } A$ and for that we have to find co-factors:

$$a_{11} \text{ (co - factor of } \cos \alpha) = (-1)^{1+1}(\cos \alpha) = (-1)^2(\cos \alpha) = \cos \alpha$$

$$a_{12} \text{ (co - factor of } \sin \alpha) = (-1)^{1+2}(\sin \alpha) = (-1)^3(\sin \alpha) = -\sin \alpha$$

$$a_{21} \text{ (co - factor of } \sin \alpha) = (-1)^{2+1}(\sin \alpha) = (-1)^3(\sin \alpha) = -\sin \alpha$$

$$a_{22} \text{ (co - factor of } \cos \alpha) = (-1)^{2+2}(\cos \alpha) = (-1)^4(\cos \alpha) = \cos \alpha$$

$$\therefore \text{ The co - factor matrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

Now, $\text{adj } A = \text{Transpose of co-factor Matrix}$

$$\therefore \text{adj } A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}^T = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

Calculating $A(\text{adj } A)$

$$A.(\text{adj } A) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\begin{aligned}
 &= \begin{bmatrix} \cos \alpha \times \cos \alpha + \sin \alpha \times (-\sin \alpha) & \cos \alpha \times (-\sin \alpha) + \sin \alpha \times \cos \alpha \\ \sin \alpha \times \cos \alpha + \cos \alpha \times (-\sin \alpha) & \sin \alpha \times (-\sin \alpha) + \cos \alpha \times \cos \alpha \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & -\cos \alpha \sin \alpha + \cos \alpha \sin \alpha \\ \sin \alpha \cos \alpha - \cos \alpha \sin \alpha & -\sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & 0 \\ 0 & \cos^2 \alpha - \sin^2 \alpha \end{bmatrix} \\
 &= (\cos^2 \alpha - \sin^2 \alpha) I
 \end{aligned}$$

Calculating $(\text{adj } A)A$

$$\begin{aligned}
 (\text{adj } A).A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\
 &= \begin{bmatrix} \cos \alpha \times \cos \alpha + (-\sin \alpha) \times \sin \alpha & \cos \alpha \times \sin \alpha + (-\sin \alpha) \times \cos \alpha \\ (-\sin \alpha) \times \cos \alpha + \cos \alpha \times \sin \alpha & (-\sin \alpha) \times \sin \alpha + \cos \alpha \times \cos \alpha \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & \cos \alpha \sin \alpha - \cos \alpha \sin \alpha \\ -\sin \alpha \cos \alpha + \cos \alpha \sin \alpha & -\sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & 0 \\ 0 & \cos^2 \alpha - \sin^2 \alpha \end{bmatrix} \\
 &= (\cos^2 \alpha - \sin^2 \alpha) I
 \end{aligned}$$

Calculating $|A|.I$

$$|A|.I = \begin{vmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{vmatrix} I$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A , is given by

$$\begin{aligned}
 |A| &= \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \\
 &= [\cos \alpha \times \cos \alpha - (\sin \alpha) \times (\sin \alpha)]I \\
 &= [\cos^2 \alpha - \sin^2 \alpha] I
 \end{aligned}$$

Thus, $A(\text{adj } A) = (\text{adj } A)A = |A|I = I$

$$\Rightarrow A(\text{adj } A) = (\text{adj } A)A = |A|I$$

Hence Proved

$$\text{Ans. } \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

Question: 4

Solution:

Here,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \end{bmatrix}$$

Now, we have to find $\text{adj } A$, and for that, we have to find co-factors:

$$a_{11} = \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} = 3 - (0) = 3$$

$$a_{12} = -\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -(9 - (-2)) = -(9 + 2) = -11$$

$$a_{13} = \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} = 0 - 1 = -1$$

$$a_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -(-3 - 0) = 3$$

$$a_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 3 - 2 = 1$$

$$a_{23} = -\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -(0 - (-1)) = -1$$

$$a_{31} = \begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} = 2 - 2 = 0$$

$$a_{32} = -\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -(-2 - 6) = 8$$

$$a_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = 1 - (-3) = 1 + 3 = 4$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} 3 & -11 & -1 \\ 3 & 1 & -1 \\ 0 & 8 & 4 \end{bmatrix}^T = \begin{bmatrix} 3 & 3 & 0 \\ -11 & 1 & 8 \\ -1 & -1 & 4 \end{bmatrix}$$

Calculating A (adj A)

$$A.(\text{adj } A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \\ -11 & 1 & 8 \\ -1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 + 11 - 2 & 3 - 1 - 2 & 0 - 8 + 8 \\ 9 - 11 + 2 & 9 + 1 + 2 & 0 + 8 - 8 \\ 3 - 0 - 3 & 3 + 0 - 3 & 0 + 0 + 12 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$$= 12 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= 12I$$

Calculating (adj A)A

$$(\text{adj } A).A = \begin{bmatrix} 3 & 3 & 0 \\ -11 & 1 & 8 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 + 9 + 0 & -3 + 3 + 0 & 6 - 6 + 0 \\ -11 + 3 + 8 & 11 + 1 + 0 & -22 - 2 + 24 \\ -1 - 3 + 4 & 1 - 1 + 0 & -2 + 2 + 12 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$$= 12 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= 12I$$

Calculating |A|.I

Expanding along C_1 , we get

$$|A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A|.I = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 1 & 0 & 3 \end{vmatrix} I$$

$$= [1(3 - 0) - (-1)\{9 - (-2)\} + 2(0 - 1)]I$$

$$= [3 + 1(11) + 2(-1)] I$$

$$= (3 + 11 - 2) I$$

$$= 12I$$

$$\text{Thus, } A(\text{adj } A) = (\text{adj } A)A = |A|I = 12I$$

$$\Rightarrow A(\text{adj } A) = (\text{adj } A)A = |A|I$$

Hence Proved

$$\text{Ans. } \begin{bmatrix} 3 & 3 & 0 \\ -11 & 1 & 8 \\ -1 & -1 & 4 \end{bmatrix}$$

Question: 5

Solution:

Here,

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Now, we have to find $\text{adj } A$, and for that, we have to find co-factors:

$$a_{11} = \begin{vmatrix} 6 & -5 \\ -2 & 2 \end{vmatrix} = 12 - (10) = 2$$

$$a_{12} = - \begin{vmatrix} -15 & -5 \\ 5 & 2 \end{vmatrix} = -(-30 - (-25)) = -(-30 + 25) = 5$$

$$a_{13} = \begin{vmatrix} -15 & 6 \\ 5 & -2 \end{vmatrix} = 30 - 30 = 0$$

$$a_{21} = - \begin{vmatrix} -1 & 1 \\ -2 & 2 \end{vmatrix} = -(-2 - (-2)) = 0$$

$$a_{22} = \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = 6 - 5 = 1$$

$$a_{23} = - \begin{vmatrix} 3 & -1 \\ 5 & -2 \end{vmatrix} = -(-6 - (-5)) = -(-6 + 5) = 1$$

$$a_{31} = \begin{vmatrix} -1 & 1 \\ 6 & -5 \end{vmatrix} = 5 - 6 = -1$$

$$a_{32} = - \begin{vmatrix} 3 & 1 \\ -15 & -5 \end{vmatrix} = -(-15 - (-15)) = -(-15 + 15) = 0$$

$$a_{33} = \begin{vmatrix} 3 & -1 \\ -15 & 6 \end{vmatrix} = 18 - 15 = 3$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Calculating $A(\text{adj } A)$

$$A.(\text{adj } A) = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 5 + 0 & 0 - 1 + 1 & -3 + 0 + 3 \\ -30 + 30 + 0 & 0 + 6 - 5 & 15 + 0 - 15 \\ 10 - 10 + 0 & 0 - 2 + 2 & -5 + 0 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

Calculating $(\text{adj } A)A$

$$(\text{adj } A).A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6+0-5 & -2+0+2 & 2+0-2 \\ 15-15+0 & -5+6+0 & 5-5+0 \\ 0-15+15 & 0+6-6 & 0-5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

Calculating $|A|.I$

Expanding along C_1 , we get

$$|A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A|.I = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{vmatrix} I$$

$$= [3(12-10) - (-15)\{-2-(-2)\} + 5(5-6)]I$$

$$= [3(2) + 15(0) + 5(-1)]I$$

$$= (6-5)I$$

$$= I$$

Thus, $A(\text{adj } A) = (\text{adj } A)A = |A|.I = I$

$$\Rightarrow A(\text{adj } A) = (\text{adj } A)A = |A|.I$$

Hence Proved

$$\text{Ans. } \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Question: 6

Solution:

Here,

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Now, we have to find $\text{adj } A$, and for that, we have to find co-factors:

$$a_{11} = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 2-3 = -1$$

$$a_{12} = -\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = -(1-9) = 8$$

$$a_{13} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1-6 = -5$$

$$a_{21} = -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -(1-2) = 1$$

$$a_{22} = \begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix} = 0 - 6 = -6$$

$$a_{23} = -\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} = -(0 - 3) = 3$$

$$a_{31} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1$$

$$a_{32} = -\begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix} = -(0 - 2) = 2$$

$$a_{33} = \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = 0 - 1 = -1$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

Calculating A (adj A)

$$\begin{aligned} A.(\text{adj } A) &= \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 0 + 8 - 10 & 0 - 6 + 6 & 0 + 2 - 2 \\ -1 + 16 - 15 & 1 - 12 + 9 & -1 + 4 - 3 \\ -3 + 8 - 5 & 3 - 6 + 3 & -3 + 2 - 1 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \\ &= -2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= -2I \end{aligned}$$

Calculating (adj A)A

$$\begin{aligned} (\text{adj } A).A &= \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 + 1 - 3 & -1 + 2 - 1 & -2 + 3 - 1 \\ 0 - 6 + 6 & 8 - 12 + 2 & 16 - 18 + 2 \\ 0 + 3 - 3 & -5 + 6 - 1 & -10 + 9 - 1 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \\ &= -2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= -2I \end{aligned}$$

Calculating |A|.I

Expanding along C_1 , we get

$$\begin{aligned} |A| &= a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} \\ &\quad + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ |A|.I &= \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} I \\ &= [0(2 - 3) - (1)(1 - 2) + 3(3 - 4)]I \end{aligned}$$

$$= [0 - 1(-1) + 3(-1)] I$$

$$= (1 - 3)I$$

$$= -2I$$

$$\text{Thus, } A(\text{adj } A) = (\text{adj } A)A = |A|I = -2I$$

$$\Rightarrow A(\text{adj } A) = (\text{adj } A)A = |A|I$$

Hence Proved

$$\text{Ans. } \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

Question: 7

Solution:

Here,

$$A = \begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix}$$

Now, we have to find $\text{adj } A$ and for that we have to find co-factors:

$$a_{11} = \begin{vmatrix} -1 & 4 \\ 8 & 2 \end{vmatrix} = -2 - 32 = -34$$

$$a_{12} = - \begin{vmatrix} 5 & 4 \\ 6 & 2 \end{vmatrix} = -(10 - 24) = -(-14) = 14$$

$$a_{13} = \begin{vmatrix} 5 & -1 \\ 6 & 8 \end{vmatrix} = 40 - (-6) = 40 + 6 = 46$$

$$a_{21} = - \begin{vmatrix} 7 & 3 \\ 8 & 2 \end{vmatrix} = -(14 - 24) = 10$$

$$a_{22} = \begin{vmatrix} 9 & 3 \\ 6 & 2 \end{vmatrix} = 18 - 18 = 0$$

$$a_{23} = - \begin{vmatrix} 9 & 7 \\ 6 & 8 \end{vmatrix} = -(72 - 42) = -30$$

$$a_{31} = \begin{vmatrix} 7 & 3 \\ -1 & 4 \end{vmatrix} = 28 - (-3) = 31$$

$$a_{32} = - \begin{vmatrix} 9 & 3 \\ 5 & 4 \end{vmatrix} = -(36 - 15) = -21$$

$$a_{33} = \begin{vmatrix} 9 & 7 \\ 5 & -1 \end{vmatrix} = -9 - 35 = -44$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} -34 & 14 & 46 \\ 10 & 0 & -30 \\ 31 & -21 & -44 \end{bmatrix}^T = \begin{bmatrix} -34 & 10 & 31 \\ 14 & 0 & -21 \\ 46 & -30 & -44 \end{bmatrix}$$

Calculating $A(\text{adj } A)$

$$\begin{aligned} A.(\text{adj } A) &= \begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix} \begin{bmatrix} -34 & 10 & 31 \\ 14 & 0 & -21 \\ 46 & -30 & -44 \end{bmatrix} \\ &= \begin{bmatrix} -306 + 98 + 138 & 90 + 0 - 90 & 279 - 147 - 132 \\ -170 - 14 + 184 & 50 + 0 - 120 & 155 + 21 - 176 \\ -204 + 112 + 92 & 60 + 0 - 60 & 186 - 168 - 88 \end{bmatrix} \\ &= \begin{bmatrix} -70 & 0 & 0 \\ 0 & -70 & 0 \\ 0 & 0 & -70 \end{bmatrix} \end{aligned}$$

$$= -70 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= -70 I$$

Calculating $(\text{adj } A)A$

$$(\text{adj } A).A = \begin{bmatrix} -34 & 10 & 31 \\ 14 & 0 & -21 \\ 46 & -30 & -44 \end{bmatrix} \begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -306 + 50 + 186 & -238 - 10 + 248 & -102 + 40 + 62 \\ 126 + 0 - 126 & 98 + 0 - 168 & 42 + 0 - 42 \\ 414 - 150 - 264 & 322 + 30 - 352 & 138 - 120 - 88 \end{bmatrix}$$

$$= \begin{bmatrix} -70 & 0 & 0 \\ 0 & -70 & 0 \\ 0 & 0 & -70 \end{bmatrix}$$

$$= -70 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= -70 I$$

Calculating $|A|.I$

Expanding along C_1 , we get

$$|A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A|.I = \begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix} I$$

$$= [9(-2 - 32) - (5)(14 - 24) + 6(28 - (-3))]I$$

$$= [9(-34) - 5(-10) + 6(31)]I$$

$$= (-306 + 50 + 186)I$$

$$= -70 I$$

$$\text{Thus, } A(\text{adj } A) = (\text{adj } A)A = |A|I = -70 I$$

$$\Rightarrow A(\text{adj } A) = (\text{adj } A)A = |A|I$$

Hence Proved

$$\text{Ans. } \begin{bmatrix} -34 & 10 & 31 \\ 14 & 0 & -21 \\ 46 & -30 & -44 \end{bmatrix}$$

Question: 8

Solution:

Here,

$$A = \begin{bmatrix} 4 & 5 & 3 \\ 1 & 0 & 6 \end{bmatrix}$$

Now, we have to find $\text{adj } A$ and for that we have to find co-factors:

$$a_{11} = \begin{vmatrix} 0 & 6 \\ 7 & 9 \end{vmatrix} = 0 - 42 = -42$$

$$a_{12} = - \begin{vmatrix} 1 & 6 \\ 2 & 9 \end{vmatrix} = -(9 - 12) = 3$$

$$a_{13} = \begin{vmatrix} 1 & 0 \\ 2 & 7 \end{vmatrix} = 7 - 0 = 7$$

$$a_{21} = - \begin{vmatrix} 5 & 3 \\ 7 & 9 \end{vmatrix} = -(45 - 21) = -24$$

$$a_{22} = \begin{vmatrix} 4 & 3 \\ 2 & 9 \end{vmatrix} = 36 - 6 = 30$$

$$a_{23} = - \begin{vmatrix} 4 & 5 \\ 2 & 7 \end{vmatrix} = -(28 - 10) = -18$$

$$a_{31} = \begin{vmatrix} 5 & 3 \\ 0 & 6 \end{vmatrix} = 30 - 0 = 30$$

$$a_{32} = - \begin{vmatrix} 4 & 3 \\ 1 & 6 \end{vmatrix} = -(24 - 3) = -(21) = -21$$

$$a_{33} = \begin{vmatrix} 4 & 5 \\ 1 & 0 \end{vmatrix} = 0 - 5 = -5$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} -42 & -3 & 7 \\ -24 & 30 & -18 \\ 30 & -21 & -5 \end{bmatrix}^T = \begin{bmatrix} -42 & -24 & 30 \\ -3 & 30 & -21 \\ 7 & -18 & -5 \end{bmatrix}$$

Calculating A (adj A)

$$\begin{aligned} A.(\text{adj } A) &= \begin{bmatrix} 4 & 5 & 3 \\ 1 & 0 & 6 \\ 2 & 7 & 9 \end{bmatrix} \begin{bmatrix} -42 & -24 & 30 \\ -3 & 30 & -21 \\ 7 & -18 & -5 \end{bmatrix} \\ &= \begin{bmatrix} -168 + 15 + 21 & -96 + 150 - 54 & 120 - 105 - 15 \\ -42 + 0 + 42 & -24 + 0 - 108 & 30 + 0 - 30 \\ -84 + 21 + 63 & -48 + 210 - 162 & 60 - 147 - 45 \end{bmatrix} \\ &= \begin{bmatrix} -132 & 0 & 0 \\ 0 & -132 & 0 \\ 0 & 0 & -132 \end{bmatrix} \\ &= -132 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= -132I \end{aligned}$$

Calculating (adj A)A

$$\begin{aligned} (\text{adj } A).A &= \begin{bmatrix} -42 & -24 & 30 \\ 3 & 30 & -21 \\ 7 & -18 & -5 \end{bmatrix} \begin{bmatrix} 4 & 5 & 3 \\ 1 & 0 & 6 \\ 2 & 7 & 9 \end{bmatrix} \\ &= \begin{bmatrix} -168 - 24 + 60 & -210 + 0 + 210 & -126 - 144 + 270 \\ 12 + 30 - 42 & 15 + 0 - 147 & 9 + 180 - 189 \\ 28 - 18 - 10 & 35 + 0 - 35 & 21 - 108 - 45 \end{bmatrix} \\ &= \begin{bmatrix} -132 & 0 & 0 \\ 0 & -132 & 0 \\ 0 & 0 & -132 \end{bmatrix} \\ &= -132 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= -132I \end{aligned}$$

Calculating |A|.I

Expanding along C_1 , we get

$$|A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A|.I = \begin{vmatrix} 4 & 5 & 3 \\ 1 & 0 & 6 \\ 2 & 7 & 9 \end{vmatrix} I$$

$$= [4(0 - 42) - (1)\{45 - 21\} + 2(30 - 0)]I$$

$$= [4(-42) - 1(24) + 2(30)]I$$

$$= [-168 - 24 + 60]I$$

$$= -132I$$

$$\text{Thus, } A(\text{adj } A) = (\text{adj } A)A = |A|I = -132I$$

$$\Rightarrow A(\text{adj } A) = (\text{adj } A)A = |A|I$$

Hence Proved

$$\text{Ans. } \begin{bmatrix} -42 & -24 & 30 \\ 3 & 30 & -21 \\ 7 & -18 & -5 \end{bmatrix}$$

Question: 9

Solution:

Here,

$$A = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, we have to find $\text{adj } A$, and for that, we have to find co-factors:

$$a_{11} = \begin{vmatrix} \cos \alpha & 0 \\ 0 & 1 \end{vmatrix} = \cos \alpha$$

$$a_{12} = - \begin{vmatrix} \sin \alpha & 0 \\ 0 & 1 \end{vmatrix} = -\sin \alpha$$

$$a_{13} = \begin{vmatrix} \sin \alpha & \cos \alpha \\ 0 & 0 \end{vmatrix} = 0$$

$$a_{21} = - \begin{vmatrix} -\sin \alpha & 0 \\ 0 & 1 \end{vmatrix} = \sin \alpha$$

$$a_{22} = \begin{vmatrix} \cos \alpha & 0 \\ 0 & 1 \end{vmatrix} = \cos \alpha$$

$$a_{23} = - \begin{vmatrix} \cos \alpha & -\sin \alpha \\ 0 & 0 \end{vmatrix} = 0$$

$$a_{31} = \begin{vmatrix} -\sin \alpha & 0 \\ \cos \alpha & 0 \end{vmatrix} = 0$$

$$a_{32} = - \begin{vmatrix} \cos \alpha & 0 \\ \sin \alpha & 0 \end{vmatrix} = 0$$

$$a_{33} = \begin{vmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{vmatrix} = [\cos^2 \alpha - \{-\sin^2 \alpha\}] = [\cos^2 \alpha + \sin^2 \alpha] = 1$$

$$[\because \cos^2 \alpha + \sin^2 \alpha = 1]$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Calculating $A(\text{adj } A)$

$$\begin{aligned}
 A \cdot (\text{adj } A) &= \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \cos \alpha \sin \alpha - \sin \alpha \cos \alpha & 0 \\ \sin \alpha \cos \alpha - \cos \alpha \sin \alpha & \cos^2 \alpha + \sin^2 \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & 0 & 0 \\ 0 & \cos^2 \alpha + \sin^2 \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

$$[\because \cos^2 \alpha + \sin^2 \alpha = 1]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

Calculating $(\text{adj } A)A$

$$\begin{aligned}
 (\text{adj } A) \cdot A &= \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & -\sin \alpha \cos \alpha + \cos \alpha \sin \alpha & 0 \\ -\sin \alpha \cos \alpha + \cos \alpha \sin \alpha & \sin^2 \alpha + \cos^2 \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & 0 & 0 \\ 0 & \cos^2 \alpha + \sin^2 \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

$$[\because \cos^2 \alpha + \sin^2 \alpha = 1]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

Calculating $|A| \cdot I$

Expanding along C_1 , we get

$$\begin{aligned}
 |A| &= a_{11} (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21} (-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} \\
 &\quad + a_{31} (-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}
 \end{aligned}$$

$$|A| \cdot I = \begin{vmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} I$$

$$= [0 - 0 + 1(\cos^2 \alpha - (-\sin^2 \alpha))]I$$

$$= [\cos^2 \alpha + \sin^2 \alpha] I$$

$$= (1)I [\because \cos^2 \alpha + \sin^2 \alpha = 1]$$

$$= I$$

$$\text{Thus, } A(\text{adj } A) = (\text{adj } A)A = |A|I = I$$

$$\Rightarrow A(\text{adj } A) = (\text{adj } A)A = |A|I$$

Hence Proved

$$\text{Ans. } \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question: 10

Solution:

$$\text{Here, } A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

Now, we have to find adj A, and for that, we have to find co-factors:

$$a_{11} = \begin{vmatrix} 0 & 1 \\ 4 & 3 \end{vmatrix} = 0 - 4 = -4$$

$$a_{12} = - \begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = -(3 - 4) = -(-1) = 1$$

$$a_{13} = \begin{vmatrix} 1 & 0 \\ 4 & 4 \end{vmatrix} = 4 - 0 = 4$$

$$a_{21} = - \begin{vmatrix} -3 & -3 \\ 4 & 3 \end{vmatrix} = -(-9 + 12) = -3$$

$$a_{22} = \begin{vmatrix} -4 & -3 \\ 4 & 3 \end{vmatrix} = -12 + 12 = 0$$

$$a_{23} = - \begin{vmatrix} -4 & -3 \\ 4 & 4 \end{vmatrix} = -(-16 + 12) = 4$$

$$a_{31} = \begin{vmatrix} -3 & -3 \\ 0 & 1 \end{vmatrix} = -3 + 0 = -3$$

$$a_{32} = - \begin{vmatrix} -4 & -3 \\ 1 & 1 \end{vmatrix} = -(-4 + 3) = 1$$

$$a_{33} = \begin{vmatrix} -4 & -3 \\ 1 & 0 \end{vmatrix} = 0 + 3 = 3$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}^T = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} = A$$

Thus, adj A = A

Hence Proved

Question: 11

Solution:

$$\text{We have, } A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

To show: adj A = 3A'

Firstly, we find the Transpose of A i.e. A'

Transpose of $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

So,

$$A' = \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} \dots (i)$$

Now, we have to find adj A, and for that, we have to find co-factors:

$$a_{11} = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = 1 - 4 = -3$$

$$a_{12} = - \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -(2 + 4) = -6$$

$$a_{13} = \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = -4 - 2 = -6$$

$$a_{21} = - \begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -(-2 - 4) = 6$$

$$a_{22} = \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = -1 + 4 = 3$$

$$a_{23} = - \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2 + 4) = -6$$

$$a_{31} = \begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = 4 + 2 = 6$$

$$a_{32} = - \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2 + 4) = -6$$

$$a_{33} = \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = -1 + 4 = 3$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

Now, taking Adj A i.e.

$$\text{adj } A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= 3 \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

$$= 3A' \text{ [from eq. (i)]}$$

Hence Proved

Question: 12

Solution:

$$\text{Here, } A = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$\text{We have to find } A^{-1} \text{ and } A^{-1} = \frac{\text{adj } A}{|A|}$$

Firstly, we find the adj A and for that we have to find co-factors:

$$a_{11} \text{ (co - factor of 3)} = (-1)^{1+1}(2) = (-1)^2(2) = 2$$

$$a_{12} (\text{co - factor of } -5) = (-1)^{1+2}(-1) = (-1)^3(-1) = 1$$

$$a_{21} (\text{co - factor of } -1) = (-1)^{2+1}(-5) = (-1)^3(-5) = 5$$

$$a_{22} (\text{co - factor of } 2) = (-1)^{2+2}(3) = (-1)^4(3) = 3$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

Now, $\text{adj } A = \text{Transpose of co-factor Matrix}$

$$\therefore \text{adj } A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Calculating $|A|$

$$|A| = \begin{vmatrix} 3 & -5 \\ -1 & 2 \end{vmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A , is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [3 \times 2 - (-1) \times (-5)]$$

$$= (6 - 5)$$

$$= 1$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}}{1} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$\text{Ans. } \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Question: 13

We have to find A^{-1} and $A^{-1} = \frac{\text{adj } A}{|A|}$

Firstly, we find the $\text{adj } A$ and for that we have to find co-factors:

$$a_{11} (\text{co - factor of } 4) = (-1)^{1+1}(3) = (-1)^2(3) = 3$$

$$a_{12} (\text{co - factor of } 1) = (-1)^{1+2}(2) = (-1)^3(2) = -2$$

$$a_{21} (\text{co - factor of } 2) = (-1)^{2+1}(1) = (-1)^3(1) = -1$$

$$a_{22} (\text{co - factor of } 3) = (-1)^{2+2}(4) = (-1)^4(4) = 4$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$$

Now, $\text{adj } A = \text{Transpose of co-factor Matrix}$

$$\therefore \text{adj } A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}^T = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$$

Calculating $|A|$

$$|A| = \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A , is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [4 \times 3 - 1 \times 2]$$

$$= (12 - 2)$$

$$= 10$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}}{10} = \frac{1}{10} \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{10} & -\frac{1}{10} \\ -\frac{2}{10} & \frac{4}{10} \end{bmatrix} = \begin{bmatrix} \frac{3}{10} & -\frac{1}{10} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

$$\text{Ans. } \begin{bmatrix} \frac{3}{10} & -\frac{1}{10} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

Question: 14

We have to find A^{-1} and $A^{-1} = \frac{\text{adj } A}{|A|}$

Firstly, we find the adj A and for that we have to find co-factors:

$$a_{11} (\text{co - factor of } 2) = (-1)^{1+1}(6) = (-1)^2(6) = 6$$

$$a_{12} (\text{co - factor of } -3) = (-1)^{1+2}(4) = (-1)^3(4) = -4$$

$$a_{21} (\text{co - factor of } 4) = (-1)^{2+1}(-3) = (-1)^3(-3) = 3$$

$$a_{22} (\text{co - factor of } 6) = (-1)^{2+2}(2) = (-1)^4(2) = 2$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} 6 & -4 \\ 3 & 2 \end{bmatrix}$$

Now, adj A = Transpose of co-factor Matrix

$$\therefore \text{adj } A = \begin{bmatrix} 6 & -4 \\ 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix}$$

Calculating |A|

$$|A| = \begin{vmatrix} 2 & -3 \\ 4 & 6 \end{vmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A, is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [2 \times 6 - (-3) \times 4]$$

$$= (12 + 12)$$

$$= 24$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix}}{24} = \frac{1}{24} \begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{6}{24} & \frac{3}{24} \\ -\frac{4}{24} & \frac{2}{24} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{8} \\ -\frac{1}{6} & \frac{1}{12} \end{bmatrix}$$

$$\text{Ans. } \begin{bmatrix} \frac{1}{4} & \frac{1}{8} \\ -\frac{1}{6} & \frac{1}{12} \end{bmatrix}$$

Question: 15

We have to find A^{-1} and $A^{-1} = \frac{adj A}{|A|}$

Firstly, we find the $adj A$ and for that we have to find co-factors:

$$a_{11} (\text{co - factor of } a) = (-1)^{1+1}(d) = (-1)^2(d) = d$$

$$a_{12} (\text{co - factor of } b) = (-1)^{1+2}(c) = (-1)^3(c) = -c$$

$$a_{21} (\text{co - factor of } c) = (-1)^{2+1}(b) = (-1)^3(b) = -b$$

$$a_{22} (\text{co - factor of } d) = (-1)^{2+2}(a) = (-1)^4(a) = a$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$$

Now, $adj A = \text{Transpose of co-factor Matrix}$

$$\therefore adj A = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Calculating $|A|$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

If $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then determinant of A , is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [a \times d - c \times b]$$

$$= ad - bc$$

$$\therefore A^{-1} = \frac{adj A}{|A|} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{(ad - bc)} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\text{Ans. } \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Question: 16

We have to find A^{-1} and $A^{-1} = \frac{adj A}{|A|}$

Firstly, we find $|A|$

Expanding $|A|$ along C_1 , we get

$$|A| = a_{11} (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21} (-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} (-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A| = (1) \begin{vmatrix} -1 & -1 \\ 3 & -1 \end{vmatrix} - (1) \begin{vmatrix} 2 & 5 \\ 3 & -1 \end{vmatrix} + 2 \begin{vmatrix} 2 & 5 \\ -1 & -1 \end{vmatrix}$$

$$= 1(1 - (-3)) - 1(-2 - 15) + 2(-2 - (-5))$$

$$= (1 + 3) - 1(-17) + 2(-2 + 5)$$

$$= 4 + 17 + 2(3)$$

$$= 21 + 6$$

$$= 27$$

Now, we have to find $adj A$, and for that, we have to find co-factors:

$$a_{11} = \begin{vmatrix} -1 & -1 \\ 3 & -1 \end{vmatrix} = 1 - (-3) = 1 + 3 = 4$$

$$a_{12} = - \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -(-1 + 2) = -1$$

$$a_{13} = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 - (-2) = 3 + 2 = 5$$

$$a_{21} = - \begin{vmatrix} 2 & 5 \\ 3 & -1 \end{vmatrix} = -(-2 - 15) = 17$$

$$a_{22} = \begin{vmatrix} 1 & 5 \\ 2 & -1 \end{vmatrix} = -1 - 10 = -11$$

$$a_{23} = - \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -(3 - 4) = 1$$

$$a_{31} = \begin{vmatrix} 2 & 5 \\ -1 & -1 \end{vmatrix} = -2 - (-5) = -2 + 5 = 3$$

$$a_{32} = - \begin{vmatrix} 1 & 5 \\ 1 & -1 \end{vmatrix} = -(-1 - 5) = 6$$

$$a_{33} = \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = -1 - 2 = -3$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} 4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3 \end{bmatrix}^T = \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}}{27} = \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

$$\text{Ans. } \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

Question: 17

We have to find A^{-1} and $A^{-1} = \frac{\text{adj } A}{|A|}$

Firstly, we find $|A|$

Expanding $|A|$ along C_1 , we get

$$|A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A| = (2) \begin{vmatrix} 0 & -1 \\ 6 & 0 \end{vmatrix} - (3) \begin{vmatrix} -1 & 1 \\ 6 & 0 \end{vmatrix} + 2 \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix}$$

$$= 2(0 - (-6)) - 3(0 - 6) + 2(1 - 0)$$

$$= 2(6) - 3(-6) + 2(1)$$

$$= 12 + 18 + 2$$

$$= 32$$

Now, we have to find $\text{adj } A$, and for that, we have to find co-factors:

$$a_{11} = \begin{vmatrix} 0 & -1 \\ 6 & 0 \end{vmatrix} = 0 - (-6) = 6$$

$$a_{12} = - \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} = -(0 + 2) = -2$$

$$a_{13} = \begin{vmatrix} 3 & 0 \\ 2 & 6 \end{vmatrix} = 18 - 0 = 18$$

$$a_{21} = - \begin{vmatrix} -1 & 1 \\ 6 & 0 \end{vmatrix} = -(0 - 6) = 6$$

$$a_{22} = \begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix} = 0 - 2 = -2$$

$$a_{23} = - \begin{vmatrix} 2 & -1 \\ 2 & 6 \end{vmatrix} = -(12 - (-2)) = -(12 + 2) = -14$$

$$a_{31} = \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} = 1 - 0 = 1$$

$$a_{32} = - \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -(-2 - 3) = 5$$

$$a_{33} = \begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix} = 0 - (-3) = 3$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} 6 & -2 & 18 \\ 6 & -2 & -14 \\ 1 & 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}}{32} = \frac{1}{32} \begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix}$$

$$\text{Ans. } \frac{1}{32} \begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix}$$

Question: 18

We have to find A^{-1} and $A^{-1} = \frac{\text{adj } A}{|A|}$

Firstly, we find $|A|$

Expanding $|A|$ along C_1 , we get

$$|A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A| = (2) \begin{vmatrix} 2 & 3 \\ -2 & 2 \end{vmatrix} - (2) \begin{vmatrix} -3 & 3 \\ -2 & 2 \end{vmatrix} + 3 \begin{vmatrix} -3 & 3 \\ 2 & 3 \end{vmatrix}$$

$$= 2(4 - (-6)) - 2(-6 - (-6)) + 3(-9 - 6)$$

$$= 2(4 + 6) - 2(-6 + 6) + 3(-15)$$

$$= 2(10) - 2(0) - 45$$

$$= 20 - 45$$

$$= -25$$

Now, we have to find $\text{adj } A$ and for that we have to find co-factors:

$$a_{11} = \begin{vmatrix} 2 & 3 \\ -2 & 2 \end{vmatrix} = 4 - (-6) = 4 + 6 = 10$$

$$a_{12} = - \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -(4 - 9) = 5$$

$$a_{13} = \begin{vmatrix} 2 & 2 \\ 3 & -2 \end{vmatrix} = -4 - 6 = -10$$

$$a_{21} = - \begin{vmatrix} -3 & 3 \\ -2 & 2 \end{vmatrix} = -(-6 - (-6)) = -(-6 + 6) = 0$$

$$a_{22} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = 4 - 9 = -5$$

$$a_{23} = - \begin{vmatrix} 2 & -3 \\ 3 & -2 \end{vmatrix} = -(-4 - (-9)) = -(-4 + 9) = -5$$

$$a_{31} = \begin{vmatrix} -3 & 3 \\ 2 & 3 \end{vmatrix} = -9 - 6 = -15$$

$$a_{32} = - \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = -(6 - 6) = 0$$

$$a_{33} = \begin{vmatrix} 2 & -3 \\ 2 & 2 \end{vmatrix} = 4 - (-6) = 4 + 6 = 10$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} 10 & 5 & -10 \\ 0 & -5 & -5 \\ -15 & 0 & 10 \end{bmatrix}^T = \begin{bmatrix} 10 & 0 & -15 \\ 5 & -5 & 0 \\ -10 & -5 & 10 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 10 & 0 & -15 \\ 5 & -5 & 0 \\ -10 & -5 & 10 \end{bmatrix}}{(-25)} = -\frac{1}{25} \begin{bmatrix} 10 & 0 & -15 \\ 5 & -5 & 0 \\ -10 & -5 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{10}{-25} & 0 & -\frac{15}{-25} \\ \frac{5}{-25} & -\frac{-5}{-25} & \frac{0}{-25} \\ -\frac{10}{-25} & -\frac{-5}{-25} & \frac{10}{-25} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix}$$

$$\text{Ans. } \frac{1}{5} \begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix}$$

Question: 19

We have to find A^{-1} and $A^{-1} = \frac{\text{adj } A}{|A|}$

Firstly, we find $|A|$

Expanding $|A|$ along C_1 , we get

$$|A| = a_{11} (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21} (-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} (-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A| = (0) \begin{vmatrix} 4 & 5 \\ -4 & -7 \end{vmatrix} - (3) \begin{vmatrix} 0 & -1 \\ -4 & -7 \end{vmatrix} + (-2) \begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix}$$

$$= 0 - 3(0 - 4) - 2(0 - (-4))$$

$$= 12 - 2(4)$$

$$= 12 - 8$$

$$= 4$$

Now, we have to find $\text{adj } A$ and for that we have to find co-factors:

$$a_{11} = \begin{vmatrix} 4 & 5 \\ -4 & -7 \end{vmatrix} = -28 - (-20) = -28 + 20 = -8$$

$$a_{12} = - \begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} = -(-21 - (-10)) = -(-21 + 10) = 11$$

$$a_{13} = \begin{vmatrix} 3 & 4 \\ -2 & -4 \end{vmatrix} = -12 - (-8) = -12 + 8 = -4$$

$$a_{21} = - \begin{vmatrix} 0 & -1 \\ -4 & -7 \end{vmatrix} = -(0 - 4) = 4$$

$$a_{22} = \begin{vmatrix} 0 & -1 \\ -2 & -7 \end{vmatrix} = 0 - 2 = -2$$

$$a_{23} = - \begin{vmatrix} 0 & 0 \\ -2 & -4 \end{vmatrix} = -(0) = 0$$

$$a_{31} = \begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix} = 0 - (-4) = 4$$

$$a_{32} = - \begin{vmatrix} 0 & -1 \\ 3 & 5 \end{vmatrix} = -(0 - (-3)) = -3$$

$$a_{33} = \begin{vmatrix} 0 & 0 \\ 3 & 4 \end{vmatrix} = 0$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} -8 & 11 & -4 \\ 4 & -2 & 0 \\ 4 & -3 & 0 \end{bmatrix}^T = \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}}{4} = \frac{1}{4} \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

$$\text{Ans. } \frac{1}{4} \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

Question: 20

We have to find A^{-1} and $A^{-1} = \frac{\text{adj } A}{|A|}$

Firstly, we find $|A|$

Expanding $|A|$ along C_1 , we get

$$|A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A| = (2) \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} - (-3) \begin{vmatrix} -1 & 4 \\ 1 & 2 \end{vmatrix} + (-1) \begin{vmatrix} -1 & 4 \\ 0 & 1 \end{vmatrix}$$

$$= 2(0 - 1) + 3(-2 - 4) - 1(-1 - 0)$$

$$= 2(-1) + 3(-6) - 1(-1)$$

$$= -2 - 18 + 1$$

$$= -19$$

$$A = \begin{bmatrix} 2 & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

Now, we have to find $\text{adj } A$ and for that we have to find co-factors:

$$a_{11} = \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = 0 - 1 = -1$$

$$a_{12} = - \begin{vmatrix} -3 & 1 \\ -1 & 2 \end{vmatrix} = -(-6 + 1) = 5$$

$$a_{13} = \begin{vmatrix} -3 & 0 \\ -1 & 1 \end{vmatrix} = -3 - 0 = -3$$

$$a_{21} = - \begin{vmatrix} -1 & 4 \\ 1 & 2 \end{vmatrix} = -(-2 - 4) = 6$$

$$a_{22} = \begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} = 4 + 4 = 8$$

$$a_{23} = - \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = -(2 - 1) = -1$$

$$a_{31} = \begin{vmatrix} -1 & 4 \\ 0 & 1 \end{vmatrix} = -1 - 0 = -1$$

$$a_{32} = - \begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix} = -(2 - (-12)) = -(2 + 12) = -14$$

$$a_{33} = \begin{vmatrix} 2 & -1 \\ -3 & 0 \end{vmatrix} = 0 - 3 = -3$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} -1 & 5 & -3 \\ 6 & 8 & -1 \\ -1 & -14 & -3 \end{bmatrix}^T = \begin{bmatrix} -1 & 6 & -1 \\ 5 & 8 & -14 \\ -3 & -1 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} -1 & 6 & -1 \\ 5 & 8 & -14 \\ -3 & 3 & -3 \end{bmatrix}}{(-19)} = -\frac{1}{19} \begin{bmatrix} -1 & 6 & -1 \\ 5 & 8 & -14 \\ -3 & -1 & -3 \end{bmatrix}$$

$$= \frac{1}{19} \begin{bmatrix} 1 & -6 & 1 \\ -5 & -8 & 14 \\ 3 & 1 & 3 \end{bmatrix}$$

$$\text{Ans. } \frac{1}{19} \begin{bmatrix} 1 & -6 & 1 \\ -5 & -8 & 14 \\ 3 & 1 & 3 \end{bmatrix}$$

Question: 21

Solution:

We have,

$$A = \begin{bmatrix} 8 & -4 & 1 \\ 10 & 0 & 6 \\ 8 & -1 & 6 \end{bmatrix}$$

We have to find A^{-1} and $A^{-1} = \frac{\text{adj } A}{|A|}$

Firstly, we find $|A|$

Expanding $|A|$ along C_1 , we get

$$|A| = a_{11} (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21} (-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} (-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$\begin{aligned}
 |A| &= 8 \begin{vmatrix} 0 & 6 \\ 1 & 6 \end{vmatrix} - (10) \begin{vmatrix} -4 & 1 \\ 1 & 6 \end{vmatrix} + 8 \begin{vmatrix} -4 & 1 \\ 0 & 6 \end{vmatrix} \\
 &= 8(0 - 6) - 10(-24 - 1) + 8(-24 - 0) \\
 &= 8(-6) - 10(-25) + 8(-24) \\
 &= -48 + 250 - 192 \\
 &= 250 - 240 \\
 &= 10
 \end{aligned}$$

Now, we have to find adj A and for that we have to find co-factors:

$$a_{11} = \begin{vmatrix} 0 & 6 \\ 1 & 6 \end{vmatrix} = 0 - 6 = -6$$

$$a_{12} = - \begin{vmatrix} 10 & 6 \\ 8 & 6 \end{vmatrix} = -(60 - 48) = -12$$

$$a_{13} = \begin{vmatrix} 10 & 0 \\ 8 & 1 \end{vmatrix} = 10 - 0 = 10$$

$$a_{21} = - \begin{vmatrix} -4 & 1 \\ 1 & 6 \end{vmatrix} = -(-24 - 1) = 25$$

$$a_{22} = \begin{vmatrix} 8 & 1 \\ 8 & 6 \end{vmatrix} = 48 - 8 = 40$$

$$a_{23} = - \begin{vmatrix} 8 & -4 \\ 8 & 1 \end{vmatrix} = -(8 - (-32)) = -(8 + 32) = -40$$

$$a_{31} = \begin{vmatrix} -4 & 1 \\ 0 & 6 \end{vmatrix} = -24 - 0 = -24$$

$$a_{32} = - \begin{vmatrix} 8 & 1 \\ 10 & 6 \end{vmatrix} = -(48 - 10) = -38$$

$$a_{33} = \begin{vmatrix} 8 & -4 \\ 10 & 0 \end{vmatrix} = 0 - (-40) = 40$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} -6 & -12 & 10 \\ 25 & 40 & -40 \\ -24 & -38 & 40 \end{bmatrix}^T = \begin{bmatrix} -6 & 25 & -24 \\ -12 & 40 & -38 \\ 10 & -40 & 40 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} -6 & 25 & -24 \\ -12 & 40 & -38 \\ 10 & -40 & 40 \end{bmatrix}}{10} = \frac{1}{10} \begin{bmatrix} -6 & 25 & -24 \\ -12 & 40 & -38 \\ 10 & -40 & 40 \end{bmatrix}$$

$$\text{Ans. } \frac{1}{10} \begin{bmatrix} -6 & 25 & -24 \\ -12 & 40 & -38 \\ 10 & -40 & 40 \end{bmatrix}$$

Question: 22

Solution:

$$\text{Here, } A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

$$\text{To show: } A^{-1} = \frac{1}{19} A$$

$$\text{We have to find } A^{-1} \text{ and } A^{-1} = \frac{\text{adj } A}{|A|}$$

Firstly, we find the adj A and for that we have to find co-factors:

$$a_{11} \text{ (co - factor of 2)} = (-1)^{1+1}(-2) = (-1)^2(-2) = -2$$

$$a_{12}(\text{co - factor of } 3) = (-1)^{1+2}(5) = (-1)^3(5) = -5$$

$$a_{21}(\text{co - factor of } 5) = (-1)^{2+1}(3) = (-1)^3(3) = -3$$

$$a_{22}(\text{co - factor of } -2) = (-1)^{2+2}(2) = (-1)^4(2) = 2$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} -2 & -5 \\ -3 & 2 \end{bmatrix}$$

Now, $\text{adj } A = \text{Transpose of co-factor Matrix}$

$$\therefore \text{adj } A = \begin{bmatrix} -2 & -5 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

Calculating $|A|$

$$|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A , is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [2 \times (-2) - 3 \times 5]$$

$$= (-4 - 15)$$

$$= -19$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}}{(-19)} = -\frac{1}{19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$= \frac{1}{19} \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

[Taking -1 common from the matrix]

$$A^{-1} = \frac{1}{19} A \left[\because A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} \right]$$

Hence Proved

Question: 23

Solution:

$$\text{We have, } A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

To show: $A^{-1} = A^2$

Firstly, we have to find A^{-1} and $A^{-1} = \frac{\text{adj } A}{|A|}$

Calculating $|A|$

Expanding $|A|$ along C_1 , we get

$$|A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A| = (1) \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} - (2) \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} + 1 \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix}$$

$$= 1(0) - 2(0) + 1(0 - (-1))$$

$$= 1(1)$$

$$= 1$$

Now, we have to find $\text{adj } A$ and for that we have to find co-factors:

$$a_{11} = \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$a_{12} = - \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$a_{13} = \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 0 - (-1) = 1$$

$$a_{21} = - \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} = -0 = 0$$

$$a_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 0 - 1 = -1$$

$$a_{23} = - \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -(0 - (-1)) = -(1) = -1$$

$$a_{31} = \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} = 0 - (-1) = 1$$

$$a_{32} = - \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = -(0 - 2) = 2$$

$$a_{33} = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1 - (-2) = -1 + 2 = 1$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}}{1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} \dots (i)$$

Calculating A^2

$$A^2 = A.A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+1 & -1+1+0 & 1+0+0 \\ 2-2+0 & -2+1+0 & 2+0+0 \\ 1+0+0 & -1+0+0 & 1+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= A^{-1} \text{ [from eq. (i)]}$$

$$\text{Thus, } A^2 = A^{-1}$$

Hence Proved

Question: 24

Solution:

$$\text{We have, } A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\text{To show: } A^{-1} = A^3$$

Firstly, we have to find A^{-1} and $A^{-1} = \frac{\text{adj } A}{|A|}$

Calculating $|A|$

Expanding $|A|$ along C_1 , we get

$$|A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A| = (3) \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} - (2) \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} + 0 \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix}$$

$$= 3(-3 - (-4)) - 2(-3 - (-4)) + 0$$

$$= 3(-3 + 4) - 2(-3 + 4)$$

$$= 3(1) - 2(1)$$

$$= 3 - 2$$

$$= 1$$

Now, we have to find $\text{adj } A$ and for that we have to find co-factors:

$$a_{11} = \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -3 - (-4) = -3 + 4 = 1$$

$$a_{12} = - \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} = -(2 - 0) = -2$$

$$a_{13} = \begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix} = -2 - 0 = -2$$

$$a_{21} = - \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -(-3 - (-4)) = -(-3 + 4) = -1$$

$$a_{22} = \begin{vmatrix} 3 & 4 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

$$a_{23} = - \begin{vmatrix} 3 & -3 \\ 0 & -1 \end{vmatrix} = -(-3 - 0) = 3$$

$$a_{31} = \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} = -12 - (-12) = 0$$

$$a_{32} = - \begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix} = -(12 - 8) = -4$$

$$a_{33} = \begin{vmatrix} 3 & -3 \\ 2 & -3 \end{vmatrix} = (-9 - (-6)) = -9 + 6 = -3$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}}{1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \dots (i)$$

Calculating A^3

$$A^2 = A.A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 - 6 + 0 & -9 + 9 - 4 & 12 - 12 + 4 \\ 6 - 6 + 0 & -6 + 9 - 4 & 8 - 12 + 4 \\ 0 - 2 + 0 & 0 + 3 - 1 & 0 - 4 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 - 8 + 0 & -9 + 12 - 4 & 12 - 16 + 4 \\ 0 - 2 + 0 & 0 + 3 + 0 & 0 - 4 + 0 \\ -6 + 4 + 0 & 6 - 6 + 3 & -8 + 8 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$= A^{-1} \text{ [from eq. (i)]}$$

$$\text{Thus, } A^3 = A^{-1}$$

Hence Proved

Question: 25

Solution:

$$\text{We have, } A = \frac{1}{9} \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix} = \begin{bmatrix} -\frac{8}{9} & \frac{1}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{4}{9} & \frac{7}{9} \\ \frac{1}{9} & -\frac{8}{9} & \frac{4}{9} \end{bmatrix}$$

$$\text{To show: } A^{-1} = A'$$

Firstly, we find the Transpose of A, i.e. A'.

$$\text{Transpose of } \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$$\text{Here, } A = \begin{bmatrix} -\frac{8}{9} & \frac{1}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{4}{9} & \frac{7}{9} \\ \frac{1}{9} & -\frac{8}{9} & \frac{4}{9} \end{bmatrix}$$

$$\text{So, } A' = \begin{bmatrix} -\frac{8}{9} & \frac{4}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{4}{9} & -\frac{8}{9} \\ \frac{4}{9} & \frac{7}{9} & \frac{4}{9} \end{bmatrix} \dots (i)$$

$$\text{Now, we have to find } A^{-1} \text{ and } A^{-1} = \frac{adj A}{|A|}$$

Calculating |A|

Expanding |A| along C₁, we get

$$|A| = a_{11} (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21} (-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} (-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A| = \left(-\frac{8}{9}\right) \begin{vmatrix} \frac{4}{9} & \frac{7}{9} \\ \frac{1}{9} & \frac{4}{9} \end{vmatrix} - \left(\frac{4}{9}\right) \begin{vmatrix} \frac{1}{9} & \frac{4}{9} \\ -\frac{8}{9} & \frac{4}{9} \end{vmatrix} + \left(\frac{1}{9}\right) \begin{vmatrix} \frac{1}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{7}{9} \end{vmatrix}$$

$$\begin{aligned}
 &= -\frac{8}{9}\left[\frac{4}{9} \times \frac{4}{9} - \left(\frac{7}{9} \times \left(-\frac{8}{9}\right)\right)\right] - \frac{4}{9}\left[\frac{1}{9} \times \frac{4}{9} - \frac{4}{9} \times \left(-\frac{8}{9}\right)\right] + \frac{1}{9}\left[\frac{1}{9} \times \frac{7}{9} - \frac{4}{9} \times \frac{4}{9}\right] \\
 &= -\frac{8}{9}\left(\frac{16}{81} + \frac{56}{81}\right) - \frac{4}{9}\left(\frac{4}{81} + \frac{32}{81}\right) + \frac{1}{9}\left(\frac{7}{81} - \frac{16}{81}\right) \\
 &= -\frac{8}{9} \times \frac{72}{81} - \frac{4}{9} \times \frac{36}{81} + \frac{1}{9}\left(-\frac{9}{81}\right) \\
 &= -\frac{8 \times 8}{81} - \frac{4 \times 4}{81} - \frac{1}{81} \\
 &= \frac{-64 - 1 - 16}{81} \\
 &= -\frac{81}{81} \\
 &= -1
 \end{aligned}$$

Now, we have to find adj A, and for that, we have to find co-factors:

$$a_{11} = \begin{vmatrix} \frac{4}{9} & \frac{7}{9} \\ \frac{8}{9} & \frac{4}{9} \end{vmatrix} = \frac{4}{9} \times \frac{4}{9} - \left(\frac{7}{9} \times \left(-\frac{8}{9}\right)\right) = \frac{16}{81} + \frac{56}{81} = \frac{72}{81} = \frac{8}{9}$$

$$a_{12} = -\begin{vmatrix} \frac{4}{9} & \frac{7}{9} \\ \frac{1}{9} & \frac{4}{9} \end{vmatrix} = -\left[\frac{4}{9} \times \frac{4}{9} - \left(\frac{7}{9} \times \frac{1}{9}\right)\right] = -\left[\frac{16}{81} - \frac{7}{81}\right] = -\frac{9}{81} = -\frac{1}{9}$$

$$a_{13} = \begin{vmatrix} \frac{4}{9} & \frac{4}{9} \\ \frac{1}{9} & -\frac{8}{9} \end{vmatrix} = \left[\frac{4}{9} \times \left(-\frac{8}{9}\right) - \left(\frac{4}{9} \times \frac{1}{9}\right)\right] = \left[-\frac{32}{81} - \frac{4}{81}\right] = -\frac{36}{81} = -\frac{4}{9}$$

$$a_{21} = -\begin{vmatrix} \frac{1}{9} & \frac{4}{9} \\ \frac{8}{9} & \frac{4}{9} \end{vmatrix} = -\left[\frac{1}{9} \times \frac{4}{9} - \left(-\frac{8}{9} \times \frac{4}{9}\right)\right] = -\left[\frac{4}{81} + \frac{32}{81}\right] = -\frac{36}{81} = -\frac{4}{9}$$

$$a_{22} = \begin{vmatrix} \frac{8}{9} & \frac{4}{9} \\ \frac{1}{9} & \frac{4}{9} \end{vmatrix} = \left[\frac{8}{9} \times \frac{4}{9} - \left(\frac{1}{9} \times \frac{4}{9}\right)\right] = \left[\frac{32}{81} - \frac{4}{81}\right] = \frac{28}{81} = \frac{4}{9}$$

$$a_{23} = -\begin{vmatrix} \frac{8}{9} & \frac{1}{9} \\ \frac{1}{9} & -\frac{8}{9} \end{vmatrix} = -\left[\frac{8}{9} \times \left(-\frac{8}{9}\right) - \left(\frac{1}{9} \times \frac{1}{9}\right)\right] = -\left[\frac{64}{81} - \frac{1}{81}\right] = -\frac{63}{81} = -\frac{7}{9}$$

$$a_{31} = \begin{vmatrix} \frac{1}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{7}{9} \end{vmatrix} = \left[\frac{1}{9} \times \frac{7}{9} - \left(\frac{4}{9} \times \frac{4}{9}\right)\right] = \left[\frac{7}{81} - \frac{16}{81}\right] = -\frac{9}{81} = -\frac{1}{9}$$

$$a_{32} = -\begin{vmatrix} \frac{8}{9} & \frac{4}{9} \\ \frac{4}{9} & \frac{7}{9} \end{vmatrix} = -\left[\frac{8}{9} \times \frac{7}{9} - \left(\frac{4}{9} \times \frac{4}{9}\right)\right] = -\left[\frac{56}{81} - \frac{16}{81}\right] = -\frac{40}{81} = -\frac{8}{9}$$

$$a_{33} = \begin{vmatrix} \frac{8}{9} & \frac{1}{9} \\ \frac{4}{9} & -\frac{8}{9} \end{vmatrix} = \left[\frac{8}{9} \times \left(-\frac{8}{9}\right) - \left(\frac{1}{9} \times \frac{4}{9}\right)\right] = \left[-\frac{64}{81} - \frac{4}{81}\right] = -\frac{68}{81} = -\frac{4}{9}$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} \frac{8}{9} & -\frac{1}{9} & -\frac{4}{9} \\ -\frac{4}{9} & \frac{4}{9} & \frac{7}{9} \\ -\frac{1}{9} & \frac{8}{9} & -\frac{4}{9} \end{bmatrix}^T = \begin{bmatrix} \frac{8}{9} & -\frac{4}{9} & -\frac{1}{9} \\ -\frac{1}{9} & -\frac{4}{9} & \frac{8}{9} \\ -\frac{4}{9} & -\frac{7}{9} & -\frac{4}{9} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} \frac{8}{9} & -\frac{4}{9} & -\frac{1}{9} \\ -\frac{1}{9} & -\frac{4}{9} & \frac{8}{9} \\ -\frac{4}{9} & -\frac{7}{9} & -\frac{4}{9} \end{bmatrix}}{-1} = - \begin{bmatrix} \frac{8}{9} & -\frac{4}{9} & -\frac{1}{9} \\ -\frac{1}{9} & -\frac{4}{9} & \frac{8}{9} \\ -\frac{4}{9} & -\frac{7}{9} & -\frac{4}{9} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{8}{9} & \frac{4}{9} & \frac{1}{9} \\ -\frac{1}{9} & \frac{4}{9} & \frac{8}{9} \\ \frac{4}{9} & \frac{7}{9} & \frac{4}{9} \end{bmatrix}$$

= A' [from eq. (i)]

Thus, $A^{-1} = A'$

Hence Proved

Question: 26

Solution:

Given: $D = \text{diag} [d_1, d_2, d_3]$

It is also given that $d_1 \neq 0, d_2 \neq 0, d_3 \neq 0$

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

A diagonal matrix $D = \text{diag}(d_1, d_2, \dots, d_n)$ is invertible iff all diagonal entries are non-zero, i.e. $d_i \neq 0$ for $1 \leq i \leq n$

If D is invertible then $D^{-1} = \text{diag}(d_1^{-1}, \dots, d_n^{-1})$

By the Inverting Diagonal Matrices Theorem, which states that

Here, it is given that $d_1 \neq 0, d_2 \neq 0, d_3 \neq 0$

$\therefore D$ is invertible

$$\Rightarrow D^{-1} = \text{diag} [d_1^{-1}, d_2^{-1}, d_3^{-1}]$$

Hence Proved.

Question: 27

Solution:

$$\text{Given: } A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \text{ \& } B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$$

To Verify: $(AB)^{-1} = B^{-1}A^{-1}$

Firstly, we find the $(AB)^{-1}$

$$\begin{aligned}
 AB &= \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} \\
 &= \begin{bmatrix} 18 + 16 & 21 + 18 \\ 42 + 40 & 49 + 45 \end{bmatrix} \\
 &= \begin{bmatrix} 34 & 39 \\ 82 & 94 \end{bmatrix}
 \end{aligned}$$

We have to find $(AB)^{-1}$ and $(AB)^{-1} = \frac{\text{adj}(AB)}{|AB|}$

Firstly, we find the adj AB and for that we have to find co-factors:

$$a_{11} \text{ (co - factor of 34)} = (-1)^{1+1}(94) = (-1)^2(94) = 94$$

$$a_{12} \text{ (co - factor of 39)} = (-1)^{1+2}(82) = (-1)^3(82) = -82$$

$$a_{21} \text{ (co - factor of 82)} = (-1)^{2+1}(39) = (-1)^3(39) = -39$$

$$a_{22} \text{ (co - factor of 94)} = (-1)^{2+2}(34) = (-1)^4(34) = 34$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} 94 & -82 \\ -39 & 34 \end{bmatrix}$$

Now, adj AB = Transpose of co-factor Matrix

$$\therefore \text{adj AB} = \begin{bmatrix} 94 & -82 \\ -39 & 34 \end{bmatrix}^T = \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$$

Calculating |AB|

$$|AB| = \begin{vmatrix} 34 & 39 \\ 82 & 94 \end{vmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A, is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [34 \times 94 - (82) \times (39)]$$

$$= (3196 - 3198)$$

$$= -2$$

$$\therefore (AB)^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}}{-2} = -\frac{1}{2} \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$$

Now, we have to find $B^{-1}A^{-1}$

Calculating B^{-1}

$$\text{Here, } B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$$

We have to find A^{-1} and $B^{-1} = \frac{\text{adj } B}{|B|}$

Firstly, we find the adj B and for that we have to find co-factors:

$$a_{11} \text{ (co - factor of 6)} = (-1)^{1+1}(9) = (-1)^2(9) = 9$$

$$a_{12} \text{ (co - factor of 7)} = (-1)^{1+2}(8) = (-1)^3(8) = -8$$

$$a_{21} \text{ (co - factor of 8)} = (-1)^{2+1}(7) = (-1)^3(7) = -7$$

$$a_{22} \text{ (co - factor of 9)} = (-1)^{2+2}(6) = (-1)^4(6) = 6$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

Now, $\text{adj } B = \text{Transpose of co-factor Matrix}$

$$\therefore \text{adj } B = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}^T = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

Calculating $|B|$

$$|B| = \begin{vmatrix} 6 & 7 \\ 8 & 9 \end{vmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A, is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [6 \times 9 - 7 \times 8]$$

$$= (54 - 56)$$

$$= -2$$

$$\therefore B^{-1} = \frac{\text{adj } B}{|B|} = \frac{\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}}{-2} = -\frac{1}{2} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

Calculating A^{-1}

$$\text{Here, } A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$$

We have to find A^{-1} and $A^{-1} = \frac{\text{adj } A}{|A|}$

Firstly, we find the $\text{adj } A$ and for that we have to find co-factors:

$$a_{11} (\text{co - factor of } 3) = (-1)^{1+1}(5) = (-1)^2(5) = 5$$

$$a_{12} (\text{co - factor of } 2) = (-1)^{1+2}(7) = (-1)^3(7) = -7$$

$$a_{21} (\text{co - factor of } 7) = (-1)^{2+1}(2) = (-1)^3(2) = -2$$

$$a_{22} (\text{co - factor of } 5) = (-1)^{2+2}(3) = (-1)^4(3) = 3$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

Now, $\text{adj } A = \text{Transpose of co-factor Matrix}$

$$\therefore \text{adj } A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^T = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

Calculating $|A|$

$$|A| = \begin{vmatrix} 3 & 2 \\ 7 & 5 \end{vmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A, is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [3 \times 5 - 2 \times 7]$$

$$= (15 - 14)$$

$$= 1$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}}{1} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

Calculating $B^{-1}A^{-1}$

$$\text{Here, } B^{-1} = -\frac{1}{2} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix} \& A^{-1} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

So,

$$\begin{aligned} B^{-1}A^{-1} &= \left(-\frac{1}{2} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}\right) \left(\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}\right) \\ &= \frac{1}{-2} \begin{bmatrix} 45 + 49 & -18 - 21 \\ -40 - 42 & 16 + 18 \end{bmatrix} \\ &= \frac{1}{-2} \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix} \end{aligned}$$

So, we get

$$(AB)^{-1} = -\frac{1}{2} \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix} \text{ and } B^{-1}A^{-1} = -\frac{1}{2} \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1}$$

Hence verified

Question: 28

Solution:

$$\text{Given: } A = \begin{bmatrix} 9 & -1 \\ 6 & -2 \end{bmatrix} \& B = \begin{bmatrix} -4 & 3 \\ 5 & -4 \end{bmatrix}$$

To Verify: $(AB)^{-1} = B^{-1}A^{-1}$

Firstly, we find the $(AB)^{-1}$

Calculating AB

$$\begin{aligned} AB &= \begin{bmatrix} 9 & -1 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -4 & 3 \\ 5 & -4 \end{bmatrix} \\ &= \begin{bmatrix} -36 - 5 & 27 + 4 \\ -24 - 10 & 18 + 8 \end{bmatrix} \\ &= \begin{bmatrix} -41 & 31 \\ -34 & 26 \end{bmatrix} \end{aligned}$$

We have to find $(AB)^{-1}$ and $(AB)^{-1} = \frac{\text{adj } (AB)}{|AB|}$

Firstly, we find the adj AB and for that we have to find co-factors:

$$a_{11} \text{ (co - factor of -41)} = (-1)^{1+1}(26) = (-1)^2(26) = 26$$

$$a_{12} \text{ (co - factor of 31)} = (-1)^{1+2}(-34) = (-1)^3(-34) = 34$$

$$a_{21} \text{ (co - factor of -34)} = (-1)^{2+1}(31) = (-1)^3(31) = -31$$

$$a_{22} \text{ (co - factor of 26)} = (-1)^{2+2}(-41) = (-1)^4(-41) = -41$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} 26 & 34 \\ -31 & -41 \end{bmatrix}$$

Now, adj AB = Transpose of co-factor Matrix

$$\therefore \text{adj } AB = \begin{bmatrix} 26 & 34 \\ -31 & -41 \end{bmatrix}^T = \begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix}$$

Calculating $|AB|$

$$|AB| = \begin{vmatrix} -41 & 31 \\ -34 & 26 \end{vmatrix}$$

If $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then determinant of A , is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [-41 \times 26 - (-34) \times (31)]$$

$$= (-1066 + 1054)$$

$$= -12$$

$$\therefore (AB)^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix}}{-12} = -\frac{1}{12} \begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix}$$

Now, we have to find $B^{-1}A^{-1}$

Calculating B^{-1}

$$\text{Here, } B = \begin{bmatrix} -4 & 3 \\ 5 & -4 \end{bmatrix}$$

We have to find A^{-1} and $B^{-1} = \frac{\text{adj } B}{|B|}$

Firstly, we find the adj B and for that we have to find co-factors:

$$a_{11} (\text{co - factor of } -4) = (-1)^{1+1}(-4) = (-1)^2(-4) = -4$$

$$a_{12} (\text{co - factor of } 3) = (-1)^{1+2}(5) = (-1)^3(5) = -5$$

$$a_{21} (\text{co - factor of } 5) = (-1)^{2+1}(3) = (-1)^3(3) = -3$$

$$a_{22} (\text{co - factor of } -4) = (-1)^{2+2}(-4) = (-1)^4(-4) = -4$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} -4 & -5 \\ -3 & -4 \end{bmatrix}$$

Now, adj B = Transpose of co-factor Matrix

$$\therefore \text{adj } B = \begin{bmatrix} -4 & -5 \\ -3 & -4 \end{bmatrix}^T = \begin{bmatrix} -4 & -3 \\ -5 & -4 \end{bmatrix}$$

Calculating $|B|$

$$|B| = \begin{vmatrix} -4 & 3 \\ 5 & -4 \end{vmatrix}$$

If $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then determinant of A , is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [(-4) \times (-4) - 3 \times 5]$$

$$= (16 - 15)$$

$$= 1$$

$$\therefore B^{-1} = \frac{\text{adj } B}{|B|} = \frac{\begin{bmatrix} -4 & -3 \\ -5 & -4 \end{bmatrix}}{1} = \begin{bmatrix} -4 & -3 \\ -5 & -4 \end{bmatrix}$$

Calculating A^{-1}

$$\text{Here, } A = \begin{bmatrix} 9 & -1 \\ 6 & -2 \end{bmatrix}$$

We have to find A^{-1} and $A^{-1} = \frac{adj A}{|A|}$

Firstly, we find the $adj A$ and for that we have to find co-factors:

$$a_{11} (\text{co - factor of } 9) = (-1)^{1+1}(-2) = (-1)^2(-2) = -2$$

$$a_{12} (\text{co - factor of } -1) = (-1)^{1+2}(6) = (-1)^3(6) = -6$$

$$a_{21} (\text{co - factor of } 6) = (-1)^{2+1}(-1) = (-1)^3(-1) = 1$$

$$a_{22} (\text{co - factor of } -2) = (-1)^{2+2}(9) = (-1)^4(9) = 9$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} -2 & -6 \\ 1 & 9 \end{bmatrix}$$

Now, $adj A = \text{Transpose of co-factor Matrix}$

$$\therefore adj A = \begin{bmatrix} -2 & -6 \\ 1 & 9 \end{bmatrix}^T = \begin{bmatrix} -2 & 1 \\ -6 & 9 \end{bmatrix}$$

Calculating $|A|$

$$|A| = \begin{vmatrix} 9 & -1 \\ 6 & -2 \end{vmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A , is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [9 \times (-2) - (-1) \times 6]$$

$$= (-18 + 6)$$

$$= -12$$

$$\therefore A^{-1} = \frac{adj A}{|A|} = \frac{\begin{bmatrix} -2 & 1 \\ -6 & 9 \end{bmatrix}}{-12} = -\frac{1}{12} \begin{bmatrix} -2 & 1 \\ -6 & 9 \end{bmatrix}$$

Calculating $B^{-1}A^{-1}$

$$\text{Here, } B^{-1} = \begin{bmatrix} -4 & -3 \\ -5 & -4 \end{bmatrix} \text{ \& } A^{-1} = -\frac{1}{12} \begin{bmatrix} -2 & 1 \\ -6 & 9 \end{bmatrix}$$

So,

$$B^{-1}A^{-1} = \left(\begin{bmatrix} -4 & -3 \\ -5 & -4 \end{bmatrix} \right) \left(-\frac{1}{12} \begin{bmatrix} -2 & 1 \\ -6 & 9 \end{bmatrix} \right)$$

$$= -\frac{1}{12} \begin{bmatrix} 8 + 18 & -4 - 27 \\ 10 + 24 & -5 - 36 \end{bmatrix}$$

$$= -\frac{1}{12} \begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix}$$

So, we get

$$(AB)^{-1} = -\frac{1}{12} \begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix} \text{ and } B^{-1}A^{-1} = -\frac{1}{12} \begin{bmatrix} 26 & -31 \\ 34 & -41 \end{bmatrix}$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1}$$

Hence verified

Question: 29

Solution:

We have, $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

To find: $(AB)^{-1}$

We know that,

$$(AB)^{-1} = B^{-1}A^{-1}$$

and here, B^{-1} is given but we have to find A^{-1} and $A^{-1} = \frac{adj A}{|A|}$

Firstly, we find $|A|$

Expanding $|A|$ along C_1 , we get

$$|A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A| = (1) \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} - (0) \begin{vmatrix} 1 & 2 \\ -2 & 4 \end{vmatrix} + (3) \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$$= 1(8 - 6) - 0 + 3(-3 - 4)$$

$$= 1(2) + 3(-7)$$

$$= 2 - 21$$

$$= -19$$

Now, we have to find $adj A$ and for that we have to find co-factors:

$$a_{11} = \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = 8 - 6 = 2$$

$$a_{12} = - \begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = -(0 + 9) = -9$$

$$a_{13} = \begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = 0 - 6 = -6$$

$$a_{21} = - \begin{vmatrix} 1 & 2 \\ -2 & 4 \end{vmatrix} = -(4 + 4) = -8$$

$$a_{22} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2$$

$$a_{23} = - \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = -(-2 - 3) = 5$$

$$a_{31} = \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = -3 - 4 = -7$$

$$a_{32} = - \begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = -(-3 - 0) = 3$$

$$a_{33} = \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$$

$$\therefore adj A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} 2 & -9 & -6 \\ -8 & -2 & 5 \\ -7 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & -8 & -7 \\ -9 & -2 & 3 \\ -6 & 5 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{adj A}{|A|} = \frac{\begin{bmatrix} -1 & 6 & -1 \\ 5 & 8 & -14 \\ -3 & 3 & -3 \end{bmatrix}}{(-19)} = -\frac{1}{19} \begin{bmatrix} 2 & -8 & -7 \\ -9 & -2 & 3 \\ -6 & 5 & 2 \end{bmatrix} = \frac{1}{19} \begin{bmatrix} -2 & 8 & 7 \\ 9 & 2 & -3 \\ 6 & -5 & -2 \end{bmatrix}$$

Now, we have

$$B^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix} \& A^{-1} = \frac{1}{19} \begin{bmatrix} -2 & 8 & 7 \\ 9 & 2 & -3 \\ 6 & -5 & -2 \end{bmatrix}$$

So,

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix} \left\{ \frac{1}{19} \begin{bmatrix} -2 & 8 & 7 \\ 9 & 2 & -3 \\ 6 & -5 & -2 \end{bmatrix} \right\}$$

$$= \frac{1}{19} \begin{bmatrix} -2+18+0 & 8+4+0 & 7-6+0 \\ 0+27-6 & 0+6+5 & 0-9+2 \\ -2+0+12 & 8+0-10 & 7+0-4 \end{bmatrix}$$

$$= \frac{1}{19} \begin{bmatrix} 16 & 12 & 1 \\ 21 & 11 & -7 \\ 10 & -2 & 3 \end{bmatrix}$$

$$\text{Ans. } \frac{1}{19} \begin{bmatrix} 16 & 12 & 1 \\ 21 & 11 & -7 \\ 10 & -2 & 3 \end{bmatrix}$$

Question: 30

Solution:

Let

$$A = \begin{bmatrix} 1 & p & 0 \\ 0 & 1 & p \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ q & 1 & 0 \\ 0 & q & 1 \end{bmatrix} \& C = \begin{bmatrix} 1+pq & p & 0 \\ q & 1+pq & p \\ 0 & q & 1 \end{bmatrix}$$

To find: A^{-1} , B^{-1} and C^{-1}

Calculating A^{-1}

$$\text{We have, } A = \begin{bmatrix} 1 & p & 0 \\ 0 & 1 & p \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{We have to find } A^{-1} \text{ and } A^{-1} = \frac{\text{adj } A}{|A|}$$

Firstly, we find $|A|$

Expanding $|A|$ along C_1 , we get

$$|A| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A| = (1) \begin{vmatrix} 1 & p \\ 0 & 1 \end{vmatrix} - (0) + (0)$$

$$= 1(1 - 0)$$

$$= 1$$

Now, we have to find $\text{adj } A$ and for that we have to find co-factors:

$$a_{11} = \begin{vmatrix} 1 & p \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$a_{12} = - \begin{vmatrix} 0 & p \\ 0 & 1 \end{vmatrix} = 0$$

$$a_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

$$a_{21} = - \begin{vmatrix} p & 0 \\ 0 & 1 \end{vmatrix} = -(p - 0) = -p$$

$$a_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$a_{23} = - \begin{vmatrix} 1 & p \\ 0 & 0 \end{vmatrix} = -(0) = 0$$

$$a_{31} = \begin{vmatrix} p & 0 \\ 1 & p \end{vmatrix} = p^2 - 0 = p^2$$

$$a_{32} = - \begin{vmatrix} 1 & 0 \\ 0 & p \end{vmatrix} = -(p - 0) = -p$$

$$a_{33} = \begin{vmatrix} 1 & p \\ 0 & 1 \end{vmatrix} = 1$$

$$\therefore \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 0 \\ -p & 1 & 0 \\ p^2 & -p & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & -p & p^2 \\ 0 & 1 & -p \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 1 & -p & p^2 \\ 0 & 1 & -p \\ 0 & 0 & 1 \end{bmatrix}}{1} = \begin{bmatrix} 1 & -p & p^2 \\ 0 & 1 & -p \\ 0 & 0 & 1 \end{bmatrix} \dots (i)$$

Calculating B^{-1}

$$\text{We have, } B = \begin{bmatrix} 1 & 0 & 0 \\ q & 1 & 0 \\ 0 & q & 1 \end{bmatrix}$$

$$\text{We have to find } B^{-1} \text{ and } B^{-1} = \frac{\text{adj } B}{|B|}$$

Firstly, we find $|B|$

Expanding $|B|$ along C_1 , we get

$$|B| = a_{11}(-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|B| = (1) \begin{vmatrix} 1 & 0 \\ q & 1 \end{vmatrix} - (q) \begin{vmatrix} 0 & 0 \\ q & 1 \end{vmatrix} + (0)$$

$$= 1(1 - 0) - q(0)$$

$$= 1$$

Now, we have to find $\text{adj } B$ and for that we have to find co-factors:

$$B = \begin{bmatrix} 1 & 0 & 0 \\ q & 1 & 0 \\ 0 & q & 1 \end{bmatrix}$$

$$a_{11} = \begin{vmatrix} 1 & 0 \\ q & 1 \end{vmatrix} = 1 - 0 = 1$$

$$a_{12} = - \begin{vmatrix} q & 0 \\ 0 & 1 \end{vmatrix} = -q$$

$$a_{13} = \begin{vmatrix} q & 1 \\ 0 & q \end{vmatrix} = q^2$$

$$a_{21} = - \begin{vmatrix} 0 & 0 \\ q & 1 \end{vmatrix} = 0$$

$$a_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$a_{23} = - \begin{vmatrix} 1 & 0 \\ 0 & q \end{vmatrix} = -(q - 0) = -q$$

$$a_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$a_{32} = -\begin{vmatrix} 1 & 0 \\ q & 0 \end{vmatrix} = 0$$

$$a_{33} = \begin{vmatrix} 1 & 0 \\ q & 1 \end{vmatrix} = 1$$

$$\therefore \text{adj } B = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} 1 & -q & q^2 \\ 0 & 1 & -q \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 0 \\ -q & 1 & 0 \\ q^2 & -q & 1 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{\text{adj } B}{|B|} = \frac{\begin{bmatrix} 1 & 0 & 0 \\ -q & 1 & 0 \\ q^2 & -q & 1 \end{bmatrix}}{1} = \begin{bmatrix} 1 & 0 & 0 \\ -q & 1 & 0 \\ q^2 & -q & 1 \end{bmatrix} \dots \text{(ii)}$$

Calculating C^{-1}

$$\text{Here, } C = \begin{bmatrix} 1+pq & p & 0 \\ q & 1+pq & p \\ 0 & q & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & p & 0 \\ 0 & 1 & p \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ q & 1 & 0 \\ 0 & q & 1 \end{bmatrix}$$

$$\Rightarrow C = AB \quad \because A = \begin{bmatrix} 1 & p & 0 \\ 0 & 1 & p \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ q & 1 & 0 \\ 0 & q & 1 \end{bmatrix}$$

$$\Rightarrow C^{-1} = (AB)^{-1}$$

We know that,

$$(AB)^{-1} = B^{-1}A^{-1}$$

Substitute the values, we get

$$C^{-1} = (AB)^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -q & 1 & 0 \\ q^2 & -q & 1 \end{bmatrix} \begin{bmatrix} 1 & -p & p^2 \\ 0 & 1 & -p \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -p & p^2 \\ -q & pq+1 & -p^2q-p \\ q^2 & -q^2p-q & p^2q^2+pq+1 \end{bmatrix}$$

$$\text{Ans. } \begin{bmatrix} 1 & -p & p^2 \\ 0 & 1 & -p \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ -q & 1 & 1 \\ q^2 & -q & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & -p & p^2 \\ -q & pq+1 & -p^2q-p \\ q^2 & -p^2q-p & p^2q^2+pq+1 \end{bmatrix}.$$

Question: 31

Solution:

$$\text{Given: } A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

To verify: $A^2 - 4A - I = 0$

Firstly, we find the A^2

$$A^2 = A.A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+4 & 6+2 \\ 6+2 & 4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix}$$

Taking LHS of the given equation .i.e.

$$A^2 - 4A - I$$

$$\Rightarrow \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix} - 4 \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix} - \begin{bmatrix} 12 & 8 \\ 8 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix} - \left\{ \begin{bmatrix} 12 & 8 \\ 8 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\Rightarrow \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix} - \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence verified

Now, we have to find A^{-1}

Finding A^{-1} using given equation

$$A^2 - 4A - I = 0$$

Post multiplying by A^{-1} both sides, we get

$$(A^2 - 4A - I)A^{-1} = 0A^{-1}$$

$$\Rightarrow A^2.A^{-1} - 4A.A^{-1} - I.A^{-1} = 0 \quad [0A^{-1} = 0]$$

$$\Rightarrow A.(AA^{-1}) - 4I - A^{-1} = 0 \quad [AA^{-1} = I]$$

$$\Rightarrow A(I) - 4I - A^{-1} = 0$$

$$\Rightarrow A - 4I - A^{-1} = 0$$

$$\Rightarrow A - 4I - 0 = A^{-1}$$

$$\Rightarrow A - 4I = A^{-1}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3-4 & 2-0 \\ 2-0 & 1-4 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$$

$$\text{Ans.} \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$$

Question: 32

Solution:

Given: $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$

To show: Matrix A satisfies the equation $x^2 + 4x - 42 = 0$

If Matrix A satisfies the given equation then

$$A^2 + 4A - 42 = 0$$

Firstly, we find the A^2

$$\begin{aligned} A^2 &= A.A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 64 + 10 & -40 + 20 \\ -16 + 8 & 10 + 16 \end{bmatrix} \\ &= \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} \end{aligned}$$

Taking LHS of the given equation .i.e.

$$\begin{aligned} &A^2 + 4A - 42 \\ &\Rightarrow \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + 4 \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} - 42 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &\Rightarrow \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix} \\ &\Rightarrow \begin{bmatrix} 74 - 32 & -20 + 20 \\ -8 + 8 & 26 + 16 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix} \\ &\Rightarrow \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix} \\ &\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= O \\ &= \text{RHS} \end{aligned}$$

$\therefore \text{LHS} = \text{RHS}$

Hence matrix A satisfies the given equation $x^2 + 4x - 42 = 0$

Now, we have to find A^{-1}

Finding A^{-1} using given equation

$$A^2 + 4A - 42 = O$$

Post multiplying by A^{-1} both sides, we get

$$\begin{aligned} &(A^2 + 4A - 42)A^{-1} = OA^{-1} \\ &\Rightarrow A^2.A^{-1} + 4A.A^{-1} - 42.A^{-1} = O \quad [OA^{-1} = O] \\ &\Rightarrow A.(AA^{-1}) + 4I - 42A^{-1} = O \quad [AA^{-1} = I] \\ &\Rightarrow A(I) + 4I - 42A^{-1} = O \\ &\Rightarrow A + 4I - 42A^{-1} = O \\ &\Rightarrow A + 4I - O = 42A^{-1} \\ &\Rightarrow A^{-1} = \frac{1}{42}(A + 4I) \\ &\Rightarrow A^{-1} = \frac{1}{42} \left\{ \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \end{aligned}$$

$$\Rightarrow A^{-1} = \frac{1}{42} \left\{ \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \right\}$$

$$\Rightarrow A^{-1} = \frac{1}{42} \begin{bmatrix} -8+4 & 5+0 \\ 2+0 & 4+4 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{42} \begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}$$

$$\text{Ans. } \frac{1}{42} \cdot \begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix} :$$

Question: 33

Solution:

$$\text{Given: } A = \begin{bmatrix} -1 & -1 \\ 2 & -2 \end{bmatrix}$$

To verify: $A^2 + 3A + 4I = O$

Firstly, we find the A^2

$$A^2 = A.A = \begin{bmatrix} -1 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 1+2 \\ -2-4 & -2+4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ -6 & 2 \end{bmatrix}$$

Taking LHS of the given equation i.e.

$$A^2 + 3A + 4I$$

$$\Rightarrow \begin{bmatrix} -1 & 3 \\ -6 & 2 \end{bmatrix} + 3 \begin{bmatrix} -1 & -1 \\ 2 & -2 \end{bmatrix} + 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 3 \\ -6 & 2 \end{bmatrix} + \begin{bmatrix} -3 & -3 \\ 6 & -6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1+(-3) & 3+(-3) \\ -6+6 & 2+(-6) \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= O$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence verified

Now, we have to find A^{-1}

Finding A^{-1} using given equation

$$A^2 + 3A + 4I = O$$

Post multiplying by A^{-1} both sides, we get

$$(A^2 + 3A + 4I)A^{-1} = OA^{-1}$$

$$\Rightarrow A^2.A^{-1} + 3A.A^{-1} + 4I.A^{-1} = O \quad [OA^{-1} = O]$$

$$\Rightarrow A(AA^{-1}) + 3I + 4A^{-1} = O \quad [AA^{-1} = I]$$

$$\Rightarrow A(I) + 3I + 4A^{-1} = O$$

$$\Rightarrow A + 3I + 4A^{-1} = O$$

$$\Rightarrow 4A^{-1} = -A - 3I + O$$

$$\Rightarrow A^{-1} = \frac{1}{4}[-A - 3I]$$

$$\Rightarrow A^{-1} = \frac{1}{4}\left\{-\begin{bmatrix} -1 & -1 \\ 2 & -2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

$$\Rightarrow A^{-1} = \frac{1}{4}\left\{\begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}\right\}$$

$$\Rightarrow A^{-1} = \frac{1}{4}\left\{\begin{bmatrix} 1+(-3) & 1+0 \\ -2+0 & 2+(-3) \end{bmatrix}\right\}$$

$$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 1 \\ -2 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -\frac{2}{4} & \frac{1}{4} \\ -\frac{2}{4} & -\frac{1}{4} \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & -\frac{1}{4} \end{bmatrix}$$

$$\text{Ans. } A^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & -\frac{1}{4} \end{bmatrix}$$

Question: 34

Solution:

$$\text{Given: } A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

To find: value of x and y

$$\text{Given equation: } A^2 + xI = yA$$

Firstly, we find the A^2

$$A^2 = A.A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$$

Putting the values in given equation

$$A^2 + xI = yA$$

$$\Rightarrow \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = y \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 16+x & 8+0 \\ 56+0 & 32+x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

On Comparing, we get

$$16 + x = 3y \dots(i)$$

$$y = 8 \dots(ii)$$

$$56 = 7y \dots(iii)$$

$$32 + x = 5y \dots(iv)$$

Putting the value of $y = 8$ in eq. (i), we get

$$16 + x = 3(8)$$

$$\Rightarrow 16 + x = 24$$

$$\Rightarrow x = 8$$

Hence, the value of $x = 8$ and $y = 8$

So, the given equation become $A^2 + 8I = 8A$

Now, we have to find A^{-1}

Finding A^{-1} using given equation

$$A^2 + 8I = 8A$$

Post multiplying by A^{-1} both sides, we get

$$(A^2 + 8I)A^{-1} = 8AA^{-1}$$

$$\Rightarrow A^2.A^{-1} + 8I.A^{-1} = 8AA^{-1}$$

$$\Rightarrow A.(AA^{-1}) + 8A^{-1} = 8I [AA^{-1} = I]$$

$$\Rightarrow A(I) + 8A^{-1} = 8I$$

$$\Rightarrow A + 8A^{-1} = 8I$$

$$\Rightarrow 8A^{-1} = -A + 8I$$

$$\Rightarrow A^{-1} = \frac{1}{8}[-A + 8I]$$

$$\Rightarrow A^{-1} = \frac{1}{8} \left\{ -\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} + 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\Rightarrow A^{-1} = \frac{1}{8} \left\{ \begin{bmatrix} -3 & -1 \\ -7 & -5 \end{bmatrix} + \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} \right\}$$

$$\Rightarrow A^{-1} = \frac{1}{8} \left\{ \begin{bmatrix} -3+8 & -1+0 \\ -7+0 & -5+8 \end{bmatrix} \right\}$$

$$\Rightarrow A^{-1} = \frac{1}{8} \begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$$

$$\text{Ans. } V = 8, M = 8 \text{ and } A^{-1} = \frac{1}{8} \cdot \begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix};$$

Question: 35

Solution:

Given: $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

To find: value of λ

Given equation: $A^2 = \lambda A - 2I$

Firstly, we find the A^2

$$\begin{aligned} A^2 &= A.A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \end{aligned}$$

Putting the values in given equation

$$\begin{aligned} A^2 &= \lambda A - 2I \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= \lambda \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= \begin{bmatrix} 3\lambda & -2\lambda \\ 4\lambda & -2\lambda \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= \begin{bmatrix} 3\lambda-2 & -2\lambda-0 \\ 4\lambda-0 & -2\lambda-2 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= \begin{bmatrix} 3\lambda-2 & -2\lambda \\ 4\lambda & -2\lambda-2 \end{bmatrix} \end{aligned}$$

On Comparing, we get

$$3\lambda - 2 = 1 \dots(i)$$

$$-2\lambda = -2 \dots(ii)$$

$$4\lambda = 4 \dots(iii)$$

$$-2\lambda - 2 = -4 \dots(iv)$$

Solving eq. (iii), we get

$$4\lambda = 4$$

$$\Rightarrow \lambda = 1$$

Hence, the value of $\lambda = 1$

So, the given equation become $A^2 = A - 2I$

Now, we have to find A^{-1}

Finding A^{-1} using given equation

$$A^2 = A - 2I$$

Post multiplying by A^{-1} both sides, we get

$$(A^2)A^{-1} = (A - 2I)A^{-1}$$

$$\Rightarrow A^2.A^{-1} = AA^{-1} - 2IA^{-1}$$

$$\Rightarrow A.(AA^{-1}) = I - 2A^{-1} [AA^{-1} = I]$$

$$\Rightarrow A(I) = I - 2A^{-1}$$

$$\Rightarrow A + 2A^{-1} = I$$

$$\Rightarrow 2A^{-1} = -A + I$$

$$\Rightarrow A^{-1} = \frac{1}{2}[-A + I]$$

$$\Rightarrow A^{-1} = \frac{1}{2}\left\{-\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

$$\Rightarrow A^{-1} = \frac{1}{2}\left\{\begin{bmatrix} -3 & 2 \\ -4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

$$\Rightarrow A^{-1} = \frac{1}{2}\left\{\begin{bmatrix} -3+1 & 2+0 \\ -4+0 & 2+1 \end{bmatrix}\right\}$$

$$\Rightarrow A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

$$\text{Ans. } \lambda = 1, A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix},$$

Question: 36

Solution:

Given:

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

We have to show that matrix A satisfies the equation $A^3 - A^2 - 3A - I = 0$

Firstly, we find the A^2

$$A^2 = A.A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0-6 & 0+0-8 & -2+0-2 \\ -2+2+6 & 0+1+8 & 4-2+2 \\ 3-8+3 & 0-4+4 & -6+8+1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}$$

Now, we have to calculate A^3

$$A^3 = A^2.A = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5+16-12 & 0+8-16 & 10-16-4 \\ 6-18+12 & 0-9+16 & -12+18+4 \\ -2+0+9 & 0+0+12 & 4+0+3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & -10 \\ 7 & 12 & 7 \end{bmatrix}$$

Taking LHS of the given equation i.e.

$$A^3 - A^2 - 3A - I$$

Putting the values, we get

$$\Rightarrow \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & -10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 - (-5) & -8 - (-8) & -10 - (-4) \\ 0 - 6 & 7 - 9 & -10 - 4 \\ 7 - (-2) & 12 - 0 & 7 - 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & -6 \\ -6 & -3 & 6 \\ 9 & 12 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 + 5 & -8 + 8 & -10 + 4 \\ -6 & -2 & -14 \\ 7 + 2 & 12 & 4 \end{bmatrix} - \left\{ \begin{bmatrix} 3 & 0 & -6 \\ -6 & -3 & 6 \\ 9 & 12 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 & -6 \\ -6 & -2 & -14 \\ 9 & 12 & 4 \end{bmatrix} - \left\{ \begin{bmatrix} 3 + 1 & 0 + 0 & -6 + 0 \\ -6 + 0 & -3 + 1 & 6 + 0 \\ 9 + 0 & 12 + 0 & 3 + 1 \end{bmatrix} \right\}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 & -6 \\ -6 & -2 & -14 \\ 9 & 12 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 0 & -6 \\ -6 & -2 & -14 \\ 9 & 12 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= O$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence, the given matrix A satisfies the equation $A^3 - A^2 - 3A - I$

Now, we have to find A^{-1}

Finding A^{-1} using given equation

$$A^3 - A^2 - 3A - I$$

Post multiplying by A^{-1} both sides, we get

$$(A^3 - A^2 - 3A - I)A^{-1} = OA^{-1}$$

$$\Rightarrow A^3.A^{-1} - A^2.A^{-1} - 3A.A^{-1} - I.A^{-1} = O \quad [OA^{-1} = O]$$

$$\Rightarrow A^2.(AA^{-1}) - A.(AA^{-1}) - 3I.A^{-1} = O$$

$$\Rightarrow A^2(I) - A(I) - 3I.A^{-1} = O \quad [AA^{-1} = I]$$

$$\Rightarrow A^2 - A - 3I - A^{-1} = O$$

$$\Rightarrow O + A^{-1} = A^2 - A - 3I$$

$$\Rightarrow A^{-1} = A^2 - A - 3I$$

$$\Rightarrow A^{-1} = [A^2 - A - 3I]$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -5 - 1 & -8 - 0 & -4 - (-2) \\ 6 - (-2) & 9 - (-1) & 4 - 2 \\ -2 - 3 & 0 - 4 & 3 - 1 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -6 & -8 & -4 + 2 \\ 6 + 2 & 9 + 1 & 2 \\ -5 & -4 & 2 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -6 - 3 & -8 + 0 & -2 + 0 \\ 8 + 0 & 10 - 3 & 2 + 0 \\ -5 + 0 & -4 + 0 & 2 - 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$\text{Ans. } A^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix},$$

Question: 37

Solution:

(i) To Prove: $\text{adj } I = I$

We know that, I means the Identity matrix

Let I is a 2×2 matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now, we have to find $\text{adj } I$ and for that we have to find co-factors:

$$a_{11} (\text{co - factor of } 1) = (-1)^{1+1}(1) = (-1)^2(1) = 1$$

$$a_{12} (\text{co - factor of } 0) = (-1)^{1+2}(0) = (-1)^3(0) = 0$$

$$a_{21} (\text{co - factor of } 0) = (-1)^{2+1}(0) = (-1)^3(0) = 0$$

$$a_{22} (\text{co - factor of } 1) = (-1)^{2+2}(1) = (-1)^4(1) = 1$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now, $\text{adj } I = \text{Transpose of co-factor Matrix}$

$$\therefore \text{adj } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Thus, $\text{adj } I = I$

Hence Proved

(ii) To Prove: $\text{adj } O = O$

We know that, O means Zero matrix where all the elements of matrix are 0

Let O is a 2×2 matrix

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Calculating $\text{adj } O$

Now, we have to find $\text{adj } O$ and for that we have to find co-factors:

$$a_{11} (\text{co - factor of } 0) = (-1)^{1+1}(0) = 0$$

$$a_{12} (\text{co - factor of } 0) = (-1)^{1+2}(0) = 0$$

$$a_{21} (\text{co - factor of } 0) = (-1)^{2+1}(0) = 0$$

$$a_{22} (\text{co - factor of } 0) = (-1)^{2+2}(0) = 0$$

$$\therefore \text{The co - factor matrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now, $\text{adj } O = \text{Transpose of co-factor Matrix}$

$$\therefore \text{adj } O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Thus, $\text{adj } O = O$

Hence Proved

(iii) To Prove: $I^{-1} = I$

We know that,

$$I^{-1} = \frac{\text{adj } I}{|I|}$$

From the part(i), we get adj I

So, we have to find |I|

Calculating |I|

$$|I| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A, is given by

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$= [1 \times 1 - 0]$$

$$= 1$$

$$\therefore I^{-1} = \frac{\text{adj } I}{|I|} = \frac{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Thus, $I^{-1} = I$

Hence Proved