Chapter : 8. SYSTEM OF LINEAR EQUATIONS

Exercise : 8A

Question: 1

Solution:

To prove: Set of given lines are inconsistent.

Given set of lines are : -

x + 2y = 9

2x + 4y = 7

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -11 \end{bmatrix}$$

Again converting into equation form, we get

x + 2y = 9

0x + 0y = - 11

∴ 0 = - 11

which is not true

∴ x + 2y = 9

2x + 4y = 7 are inconsistent.

Question: 2

Solution:

To prove: Set of given lines are inconsistent.

Given set of lines are : -

2x + 3y = 5

6x + 9y = 10

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$R_2 - 3R_1$

$$\begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$$

Again converting into equation form, we get

2x + 3y = 5

0x + 0y = - 5

∴ 0 = - 5

which is not true

∴ 2x + 3y = 5

6x + 9y = 10 are inconsistent.

Question: 3

Solution:

To prove: Set of given lines are inconsistent.

Given set of lines are : -

4x – 2y = 3

6x – 3y = 5

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$4R_2 - 6R_1$

$$\begin{bmatrix} 4 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Again converting into equation form, we get

4x – 2y = 3

0x + 0y = 2

∴ 0 = 2

which is not true

∴ 4x – 2y = 3

6x – 3y = 5 are inconsistent.

Question: 4

Solution:

To prove: Set of given lines are inconsistent.

Given set of lines are : -

6x + 4y = 5

9x + 6y = 8

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$2R_2 - 3R_1$

$$\begin{bmatrix} 6 & 4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Again converting into equation form, we get

$6x + 4y = 5$

$0x + 0y = 3$

$\therefore 0 = 3$

which is not true

$\therefore 6x + 4y = 5$

$9x + 6y = 8$ are inconsistent.

Question: 5

Solution:

To prove: Set of given lines are inconsistent.

Given set of lines are : -

$x + y - 2z = 5;$

$x - 2y + z = -2;$

$-2x + y + z = 4$

Converting the following equations in matrix form,

$AX = B$

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$R_2 - R_1$

$R_3 + 2R_1$

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ 14 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ 7 \end{bmatrix}$$

Converting back into equation form we get,

$x + y - 2z = 5;$

$0x - 3y + 3z = -7;$

$0x + 0y + 0z = 7$

$\therefore 0 = 7$

Which is not true.

$\therefore x + y - 2z = 5;$

$x - 2y + z = -2;$

$-2x + y + z = 4$

are inconsistent.

Question: 6

Solution:

To prove: Set of given lines are inconsistent.

Given set of lines are : -

$2x - y + 3z = 1;$

$3x - 2y + 5z = -4;$

$5x - 4y + 9z = 14$

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -1 & 3 \\ 3 & -2 & 5 \\ 5 & -4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 14 \end{bmatrix}$$

$2R_2 - 3R_1$

$2R_3 - 5R_1$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & -1 & 1 \\ 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -11 \\ 23 \end{bmatrix}$$

$R_3 - 3R_2$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -11 \\ 56 \end{bmatrix}$$

Converting back into equation form we get,

$2x - y + 3z = 1;$

$0x - 1y + 1z = -11;$

$0x + 0y + 0z = 56$

$\therefore 0 = 56$

Which is not true.

$\therefore 2x - y + 3z = 1;$

$3x - 2y + 5z = -4;$

$5x - 4y + 9z = 14$

are inconsistent.

Question: 7

Solution:

To prove: Set of given lines are inconsistent.

Given set of lines are : -

$x + 2y + 4z = 12;$

$y + 2z = -1;$

$3x + 2y + 4z = 4$

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -1 \\ 4 \end{bmatrix}$$

$R_3 - 3R_1$

$$\begin{bmatrix}1 & 2 & 4\\0 & 1 & 2\\0 & -4 & -8\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}12\\-1\\-32\end{bmatrix}$$

$R_3 + 4R_2$

$$\begin{bmatrix}1 & 2 & 4\\0 & 1 & 2\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}12\\-1\\-36\end{bmatrix}$$

Converting back into equation form we get,

$x + 2y + 4z = 12;$

$y + 2z = -1;$

$0x + 0y + 0z = -36$

$\therefore 0 = -36$

Which is not true.

$\therefore 2x - y + 3z = 1;$

$3x - 2y + 5z = -4;$

$5x - 4y + 9z = 14$

are inconsistent.

Question: 8

Solution:

To prove: Set of given lines are inconsistent.

Given set of lines are : -

$3x - y - 2z = 2;$

$2y - z = -1;$

$3x - 5y = 3$

Converting the following equations in matrix form,

$AX = B$

$$\begin{bmatrix}3 & -1 & -2\\0 & 2 & -1\\3 & -5 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-1\\3\end{bmatrix}$$

$R_3 - R_1$

$$\begin{bmatrix}3 & -1 & -2\\0 & 2 & -1\\0 & -4 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-1\\1\end{bmatrix}$$

$R_3 + 2R_2$

$$\begin{bmatrix}3 & -1 & -2\\0 & 2 & -1\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-1\\-1\end{bmatrix}$$

Converting back into equation form we get,

$3x - y - 2z = 2;$

$2y - z = -1;$

$0x + 0y + 0z = -1$

$\therefore 0 = -1$

Which is not true.

$\therefore 3x - y - 2z = 2;$

$2y - z = -1;$

$3x - 5y = 3$

are inconsistent.

Question: 9

Solution:

To find: - x , y

Given set of lines are : -

$5x + 2y = 4;$

$7x + 3y = 5.$

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$5R_2 - 7R_1$

$$\begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

Again converting into equation form, we get

$5x + 2y = 4;$

$y = -3$

$5x + 2 \times -3 = 4$

$5x = 10$

$X = 2$

$\therefore x = 2 , y = -3$

Question: 10

Solution:

To find: - x , y

Given set of lines are : -

$3x + 4y - 5 = 0;$

$x - y + 3 = 0$

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$3R_2 - R_1$

$$\begin{bmatrix} 3 & 4 \\ 0 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -14 \end{bmatrix}$$

Again converting into equation form, we get

$3x + 4y = 5$

$-7y = -14$

$Y = 2$

$3x + 4y = 5$

$3x + 4 \times 2 = 5$

$3x = -3$

$X = -1$

$\therefore x = -1, y = 2$

Question: 11

Solution:

To find: - x , y

Given set of lines are : -

$x + 2y = 1$

$3x + y = 4$

Converting the following equations in matrix form,

$AX = B$

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$R_2 - 3R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Again converting into equation form, we get

$x + 2y = 1$

$-5y = 1$

$Y = -\frac{1}{5}$

$x + 2x - \frac{1}{5} = 1$

$x + -\frac{2}{5} = 1$

$x = 1 + \frac{2}{5}$

$X = \frac{7}{5}$

$\therefore x = \frac{7}{5}, y = -\frac{1}{5}$

Question: 12

Solution:

To find: - x , y

Given set of lines are : -

$5x + 7y + 2 = 0;$

$4x + 6y + 3 = 0.$

Converting the following equations in matrix form,

AX = B

$\begin{bmatrix}5 & 7\\4 & 6\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-2\\-3\end{bmatrix}$

$5R_2 - 4R_1$

$\begin{bmatrix}5 & 7\\0 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-2\\-7\end{bmatrix}$

Again converting into equation form, we get

5x + 7y = - 2

2y = - 7

$Y = -\frac{7}{2}$

$5x + 7 \times -\frac{7}{2} = -2$

$5x = -2 + \frac{49}{2}$

$5x = \frac{45}{2}$

$X = \frac{9}{2}$

$\therefore x = \frac{9}{2}, y = -\frac{7}{2}$

Question: 13

Solution:

To find: - x , y

Given set of lines are : -

2x - 3y + 1 = 0;

x + 4y + 3 = 0

Converting the following equations in matrix form,

AX = B

$\begin{bmatrix}2 & -3\\1 & 4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-1\\-3\end{bmatrix}$

$2R_2 - R_1$

$\begin{bmatrix}2 & -3\\0 & 11\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-1\\-5\end{bmatrix}$

Again converting into equation form we get

2x - 3y = - 1

11 y = - 5

$Y = -\frac{5}{11}$

$2x - 3 \times -\frac{5}{11} = -1$

$2x = -1 - \frac{15}{11}$

$X = -\frac{13}{11}$

$\therefore x = -\frac{13}{11}, y = -\frac{5}{11}$

Question: 14

Solution:

To find: - x , y

Given set of lines are : -

4x - 3y = 3;

3x - 5y = 7

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$4R_2 - 3R_1$

$$\begin{bmatrix} 4 & -3 \\ 0 & -11 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 19 \end{bmatrix}$$

Again converting into equation form, we get

4x – 3y = 3

- 11y = 19

$Y = -\frac{19}{11}$

$4x - 3 \times -\frac{19}{11} = 3$

$4x = 3 - \frac{57}{11}$

$4x = -\frac{24}{11}$

$X = -\frac{6}{11}$

$\therefore x = -\frac{6}{11}, y = -\frac{19}{11}$

Question: 15

Solution:

To find: - x , y , z

Given set of lines are : -

2x + 8y + 5z = 5;

x + y + z = - 2;

x + 2y - z = 2

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & 8 & 5 \\ 1 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$$

$2R_2 - R_1$

$2R_3 - R_1$

$$\begin{bmatrix} 2 & 8 & 5 \\ 0 & -6 & -3 \\ 0 & -4 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \\ -1 \end{bmatrix}$$

$3R_3 - 2R_2$

$$\begin{bmatrix} 2 & 8 & 5 \\ 0 & -6 & -3 \\ 0 & 0 & -15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \\ 15 \end{bmatrix}$$

Again converting into equations, we get

$2x + 8y + 5z = 5$

$- 6y - 3z = - 9$

$- 15z = 15$

$Z = - 1$

$- 6y - 3 \times - 1 = - 9$

$- 6y = - 9 - 3$

$Y = 2$

$2x + 8\times2 + 5\times - 1 = 5$

$2x = 5 - 16 + 5$

$X = - 3$

$\therefore x = -3, y = 2, z = -1$

Question: 16

Solution:

To find: - x , y , z

Given set of lines are : -

$x - y + z = 1;$

$2x + y - z = 2;$

$X - 2y - z = 4$

Converting following equations in matrix form,

$AX = B$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -3 \\ 0 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

$3R_3 + R_2$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -3 \\ 0 & 0 & -9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 9 \end{bmatrix}$$

Again converting into equations we get

X – y + z = 1

3y - 3z = 0

- 9z = 9

Z = - 1

Y = z

Y = - 1

X + 1 - 1 = 1

X = 1

$\therefore x = 1, y = -1, z = -1$

Question: 17

Solution:

To find: - x , y , z

Given set of lines are : -

3X + 4y + 7z = 4;

2x – y + 3z = - 3;

x + 2y – 3z = 8

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 8 \end{bmatrix}$$

$3R_2 - 2R_1$

$3R_3 - R_1$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & -11 & -5 \\ 0 & 2 & -16 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -17 \\ 20 \end{bmatrix}$$

$11R_3 + 2R_2$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & -11 & -5 \\ 0 & 0 & -186 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -17 \\ 186 \end{bmatrix}$$

Again converting into equations we get

3x + 4y + 7z = 4

- 11y - 5z = - 17

- 186z = 186

Z = - 1

- 11y + 5 = - 17

- 11y = - 22

Y = 2

3x + 4×2 + 7× - 1 = 4

3x = 4 - 8 + 7

X = 1

$\therefore x = 1, y = 2, z = -1$

Question: 18

Solution:

To find: - x , y , z

Given set of lines are : -

x + 2y + z = 7;

x + 3z = 11;

2x – 3y = 1

Converting following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

$R_2 - R_1$

$R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & -7 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -13 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & -9 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -9 \end{bmatrix}$$

Again converting into equations we get

X + 2y + z = 7

- 2y + 2z = 4

- 9y = - 9

Y = 1

- 2×1 + 2z = 4

2z = 6

Z = 3

X + 2×1 + 3 = 7

X = 7 - 2 - 3

X = 2

$\therefore x = 2, y = 1, z = 3$

Question: 19

Solution:

To find: - x , y , z

Given set of lines are : -

2x - 3y + 5z = 16;

3x + 2y − 4z = - 4

x + y − 2z = - 3

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -4 \\ -3 \end{bmatrix}$$

$2R_2 - 3R_1$

$2R_3 - R_1$

$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 13 & -23 \\ 0 & 5 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -56 \\ -22 \end{bmatrix}$$

$13R_3 - 5R_2$

$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 13 & -23 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -56 \\ -6 \end{bmatrix}$$

Again converting into equations, we get

2x - 3y + 5z = 16

13y - 23z = - 56

- 2z = - 6

Z = 3

13y − 23×3 = - 56

13y = - 56 + 69

Y = 1

2x - 3×1 + 5×3 = 16

2x = 16 + 3 - 15

2x = 4

X = 2

$\therefore x = 2, y = 1, z = 3$

Question: 20

Solution:

To find: - x , y , z

Given set of lines are : -

x + y + z = 4;

2x − y + z = - 1;

2x + y − 3z = - 9.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 1\\ 2 & 1 & -3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}4\\ -1\\ -9\end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - 2R_1$

$$\begin{bmatrix}1 & 1 & 1\\ 0 & -3 & -1\\ 0 & -1 & -5\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}4\\ -9\\ -17\end{bmatrix}$$

$3R_3 - R_2$

$$\begin{bmatrix}1 & 1 & 1\\ 0 & -3 & -1\\ 0 & 0 & -14\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}4\\ -9\\ -42\end{bmatrix}$$

Again converting into equations, we get

X + y + z = 4

- 3y - z = - 9

- 14z = - 42

Z = 3

- 3y - 3 = - 9

- 3y = - 6

Y = 2

X + 2 + 3 = 4

X = 4 - 5

X = - 1

$\therefore x = -1, y = 2, z = 3$

Question: 21

Solution:

To find: - x , y , z

Given set of lines are : -

2x - 3y + 5z = 11;

3x + 2y - 4z = - 5;

x + y – 2z = - 3.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}11\\ -5\\ -3\end{bmatrix}$$

$2R_2 - 3R_1$

$2R_3 - R_1$

$$\begin{bmatrix}2 & -3 & 5\\ 0 & 13 & -23\\ 0 & 5 & -9\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}11\\ -43\\ -17\end{bmatrix}$$

$13R_3 - 5R_2$

$$\begin{bmatrix}2 & -3 & 5\\0 & 13 & -23\\0 & 0 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}11\\-43\\-6\end{bmatrix}$$

Again converting into equations we get

2x – 3y + 5z = 11

13y - 23 z = - 43

- 2z = - 6

Z = 3

13y - 23×3 = - 43

13y = - 43 + 69

13y = 26

Y = 2

2x – 3×2 + 5×3 = 11

2x = 11 + 6 - 15

X = 1

$\therefore x = 1, y = 2, z = 3$

Question: 22

Solution:

To find: - x , y , z

Given set of lines are : -

x + y + z = 1;

x – 2y + 3z = 2;

5x – 3y + z = 3.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 1 & 1\\1 & -2 & 3\\5 & -3 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$R_2 - R_1$

$R_3 - 5R_1$

$$\begin{bmatrix}1 & 1 & 1\\0 & -3 & 2\\0 & -8 & -4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\1\\-2\end{bmatrix}$$

$R_3 + 2R_2$

$$\begin{bmatrix}1 & 1 & 1\\0 & -3 & 2\\0 & -14 & -0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\1\\0\end{bmatrix}$$

Again converting into equations we get

X + y + z = 1

- 3y + 2z = 1

- 14 y = 0

Y = 0

- 3 × 0 + 2z = 1

$Z = \frac{1}{2}$

$X + 0 + \frac{1}{2} = 1$

$X = \frac{1}{2}$

$\therefore x = \frac{1}{2}, y = 0, z = \frac{1}{2}$

Question: 23

Solution:

To find: - x , y , z

Given set of lines are : -

x + y + z = 6;

x + 2z = 7;

3x + y + z = 12

Converting following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$R_2 - R_1$

$R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -6 \end{bmatrix}$$

$R_3 + 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -4 \end{bmatrix}$$

Again converting into equations we get

X + y + z = 6

- y + z = 1

- 4 y = - 4

Y = 1

- 1 + z = 1

Z = 2

X + 1 + 2 = 6

X = 6 - 3

X = 3

$\therefore x = 3, y = 1, z = 2$

Question: 24

Solution:

To find: - x , y , z

Given set of lines are : -

2x + 3y + 3z = 5;

x − 2y + z = - 4;

3x − y − 2z = 3

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$2R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 2 & 3 & 3 \\ 0 & -7 & -1 \\ 0 & -11 & -13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -13 \\ -9 \end{bmatrix}$$

R_3 - $13R_2$

$$\begin{bmatrix} 2 & 3 & 3 \\ 0 & -7 & -1 \\ 0 & 80 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -13 \\ 160 \end{bmatrix}$$

Again converting into equations we get

2X + 3y + 3z = 5

- 7y - z = - 13

80 y = 160

Y = 2

- 7×2 - z = - 13

Z = - 1

2x + 3×2 + 3× - 1 = 5

2x = 5 - 6 + 3

X = 1

$\therefore x = 1, y = 2, z = -1$

Question: 25

Solution:

To find: - x , y , z

Given set of lines are : -

4x - 5y - 11z = 12

X − 3y + z = 1;

2x + 3y − 7z = 2

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \\ 2 \end{bmatrix}$$

$4R_2 - R_1$

$2R_3 - R_1$

$$\begin{bmatrix} 4 & -5 & -11 \\ 0 & -7 & 15 \\ 0 & 11 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \\ -8 \end{bmatrix}$$

$5R_3 + R_2$

$$\begin{bmatrix} 4 & -5 & -11 \\ 0 & -7 & 15 \\ 0 & 48 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \\ -48 \end{bmatrix}$$

Again converting into equations we get

4x – 5y - 11z = 12

- 7y + 15z = - 8

48 y = - 48

Y = - 1

7 + 15z = - 8

15z = - 15

Z = - 1

4x + 5 + 11 = 12

4x = 12 - 5 - 11

4x = - 4

X = - 1

$\therefore x = -1, y = -1, z = -1$

Question: 26

Solution:

To find: - x , y , z

Given set of lines are : -

x - y + 2z = 7

3x + 4y – 5z = - 5

2x – y + 3z = 12

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$R_2 - 3R_1$

$R_3 - 2R_1$

$$\begin{bmatrix}1 & -1 & 2\\0 & 7 & -11\\0 & 1 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}7\\-26\\-2\end{bmatrix}$$

$7R_3 - R_2$

$$\begin{bmatrix}1 & -1 & 2\\0 & 7 & -11\\0 & 0 & 4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}7\\-26\\12\end{bmatrix}$$

Again converting into equations we get

$x - y + 2z = 7$

$7y - 11\,z = -26$

$4z = 12$

$Z = 3$

$7y - 11 \times 3 = -26$

$7y = -26 + 33$

$7y = 7$

$Y = 1$

$X - 1 + 2\times3 = 7$

$X = 7 + 1 - 6$

$X = 2$

$\therefore x = 2, y = 1, z = 3$

Question: 27

Solution:

To find: - x , y , z

Given set of lines are : -

$6x - 9y - 20z = -4$

$4x - 15y + 10z = -1$

$2x - 3y - 5z = -1$

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}6 & -9 & -20\\4 & -15 & 10\\2 & -3 & -5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-4\\-1\\-1\end{bmatrix}$$

$3R_2 - 2R_1$

$3R_3 - R_1$

$$\begin{bmatrix}6 & -9 & -20\\0 & -27 & 70\\0 & 0 & 5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-4\\5\\1\end{bmatrix}$$

Again converting into equations, we get

$6x - 9y - 20\,z = -4$

$-27y + 70z = 5$

$5z = 1$

$Z = \frac{1}{5}$

$-27y + 70 \times \frac{1}{5} = 5$

$-27y = 5 - 14$

$-27y = -9$

$Y = \frac{1}{3}$

$6x - 9 \times \frac{1}{3} - 20 \times \frac{1}{5} = -4$

$6x = -4 + 3 + 4$

$X = \frac{1}{2}$

$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

Question: 28

Solution:

To find: - x , y , z

Given set of lines are : -

$3x - 4y + 2z = -1$

$2x + 3y + 5z = 7;$

$x + z = 2$

Converting the following equations in matrix form,

$AX = B$

$$\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$3R_2 - 2R_1$

$3R_3 - R_1$

$$\begin{bmatrix} 3 & -4 & 2 \\ 0 & 17 & 11 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 23 \\ 7 \end{bmatrix}$$

$11R_3 - R_2$

$$\begin{bmatrix} 3 & -4 & 2 \\ 0 & 17 & 11 \\ 0 & 27 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 23 \\ 54 \end{bmatrix}$$

Again converting into equations, we get

$3x - 4y + 2z = -1$

$17y + 11z = 23$

$27y = 54$

$Y = 2$

$17 \times 2 + 11z = 23$

$11z = 23 - 34$

$Z = -1$

$3x - 4 \times 2 + 2 \times -1 = -1$

3x = - 1 + 8 + 2

3x = 9

X = 3

$\therefore x = 3, y = 2, z = -1$

Question: 29

Solution:

To find: - x , y , z

Given set of lines are : -

x + y - z = 1

3x + y - 2z = 3

x - y - z = - 1

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

$R_2 - 3R_1$

$R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

Again converting into equations we get

X + y – z = 1

- 2y + z = 0

- 2y = - 2

Y = 1

- 2 + z = 0

Z = 2

X + 1 - 2 = 1

X = 2

$\therefore x = 2, y = 1, z = 2$

Question: 30

Solution:

To find: - x , y , z

Given set of lines are : -

2x + y - z = 1

x - y + z = 2

3x + y - 2z = - 1

Converting the following equations in matrix form,

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AX = B

$$\begin{bmatrix}2 & 1 & -1\\1 & -1 & 1\\3 & 1 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\2\\-1\end{bmatrix}$$

$2R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix}2 & 1 & -1\\0 & -3 & 3\\0 & -1 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\3\\-5\end{bmatrix}$$

$3R_3 - R_2$

$$\begin{bmatrix}2 & 1 & -1\\0 & -3 & 3\\0 & 0 & -6\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\3\\-18\end{bmatrix}$$

Again converting into equations we get

2x + y – z = 1

- 3y + 3z = 3

- 6z = - 18

Z = 3

- 3y + 3×3 = 3

- 3y = 3 - 9

- 3y = - 6

Y = 2

2x + 2 - 3 = 1

2x = 1 + 1

X = 1

∴ x = 1 , y = 2 , z = 3

Question: 31

Solution:

To find: - x , y , z

Given set of lines are : -

x + 2y + z = 4

- x + y + z = 0

x - 3y + z = 4

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 2 & 1\\-1 & 1 & 1\\1 & -3 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}4\\0\\4\end{bmatrix}$$

$R_2 + R_1$

$R_3 - R_1$

$$\begin{bmatrix}1 & 2 & 1\\0 & 3 & 2\\0 & -5 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}4\\4\\0\end{bmatrix}$$

Again converting into equations we get

X + 2y + z = 4

3y + 2z = 4

- 5y = 0

Y = 0

0 + 2z = 4

Z = 2

X + 0 + 2 = 4

X = 2

∴ x = 2 , y = 0 , z = 2

Question: 32

Solution:

To find: - x , y , z

Given set of lines are : -

x - y - 2z = 3

x + y = 1

x + z = - 6

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & -1 & -2\\1 & 1 & 0\\1 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\1\\-6\end{bmatrix}$$

$R_2 - R_1$

$R_3 - R_1$

$$\begin{bmatrix}1 & -1 & -2\\0 & 2 & 2\\0 & 1 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\-2\\-9\end{bmatrix}$$

$2R_3 - R_2$

$$\begin{bmatrix}1 & -1 & -2\\0 & 2 & 2\\0 & 0 & 4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\-2\\-16\end{bmatrix}$$

Again converting into equations we get

X + y - 2z = 3

2y + 2z = - 2

4z = - 16

Z = - 4

2y – 8 = - 2

2y = - 2 + 8

2y = 6

Y = 3

X - 3 + 8 = 3

X = - 2

∴ x = - 2 , y = 3 , z = - 4

Question: 33

Solution:

To find: - x , y , z

Given set of lines are : -

5x - y = - 7

2x + 3z = 1

3y - z = 5

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}5 & -1 & 0\\2 & 0 & 3\\0 & 3 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-7\\1\\5\end{bmatrix}$$

$5R_2 - 2R_1$

$$\begin{bmatrix}5 & -1 & 0\\0 & 2 & 15\\0 & 3 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-7\\19\\5\end{bmatrix}$$

$2R_3 - 3R_2$

$$\begin{bmatrix}5 & -1 & 0\\0 & 2 & 15\\0 & 0 & -47\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-7\\19\\-47\end{bmatrix}$$

Again converting into equations we get

5x – y = - 7

2y + 15z = 19

- 47z = - 47

Z = 1

2y + 15 = 19

2y = 19 - 15

Y = 2

5x – 2 = - 7

5x = - 5

X = - 1

∴ x = - 1 , y = 2 , z = 1

Question: 34

Solution:

To find: - x , y , z

Given set of lines are : -

x - 2y + z = 0

y - z = 2

2x - 3z = 10

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & -2 & 1\\0 & 1 & -1\\2 & 0 & -3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\2\\10\end{bmatrix}$$

$R_3 - 2R_1$

$$\begin{bmatrix}1 & -2 & 1\\0 & 1 & -1\\0 & 4 & -5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\2\\10\end{bmatrix}$$

R_3 - $4R_2$

$$\begin{bmatrix}1 & -2 & 1\\0 & 1 & -1\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\2\\2\end{bmatrix}$$

Again converting into equations we get

X - 2y + z = 0

Y – z = 2

- z = 2

Z = - 2

Y + 2 = 2

Y = 0

X + 0 - 2 = 0

X = 2

∴ x = 2 , y = 0 , z = - 2

Question: 35

Solution:

To find: - x , y , z

Given set of lines are : -

x - y = 3

2x + 3y + 4z = 17

y + 2z = 7

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\17\\7\end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix}1 & -1 & 0\\0 & 5 & 4\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\11\\7\end{bmatrix}$$

$2R_3 - R_2$

$$\begin{bmatrix}1 & -1 & 0\\0 & 5 & 4\\0 & -3 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\11\\3\end{bmatrix}$$

Again converting into equations we get

X – y = 3

5y + 4z = 11

- 3y = 3

Y = - 1

5× - 1 + 4z = 11

4z = 16

Z = 4

X + 1 = 3

X = 2

∴ x = 2 , y = - 1 , z = 4

Question: 36

Solution:

To find: - x , y , z

Given set of lines are : -

4x + 3y + 2z = 60

x + 2y + 3z = 45

6x + 2y + 3z = 70

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}4 & 3 & 2\\1 & 2 & 3\\6 & 2 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}60\\45\\70\end{bmatrix}$$

$4R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix}4 & 3 & 2\\0 & 5 & 10\\0 & -5 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}60\\120\\-40\end{bmatrix}$$

Again converting into equations, we get

4x + 3y + 2z = 60

5y + 10z = 120

- 5y = - 40

Y = 8

5×8 + 10z = 120

10z = 120 - 40

10z = 80

Z = 8

$4x + 3\times 8 + 2\times 8 = 60$

$4x = 60 - 24 - 16$

4x = 20

X = 5

$\therefore x = 5 , y = 8 , z = 8$

Question: 37

If $A =$

Solution:

Given,

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj(A)$$

The determinant of matrix A is

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$

$= 2(2 \times - 2 - (- 4)\times 1) + 3(3\times - 2 - (- 4)\times 1) + 5(3\times 1 - 2\times 1)$

$= 2(- 4 + 4) + 3(- 6 + 4) + 5(3 - 2)$

$= 2(0) + 3(- 2) + 5(1)$

$= - 6 + 5$

$= - 1$

$|A| \neq 0$

$\therefore A^{-1}$ is possible.

$$A^T = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$$

$$Adj(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Given set of lines are : -

$2x - 3y + 5z = 11$

$3x + 2y - 4z = - 5$

$x + y - 2z = - 3$

Converting following equations in matrix form,

AX = B

Where $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Pre - multiplying by A^{-1}

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \times 11 - 5 \times 1 - 3 \times -2 \\ -2 \times 11 - 5 \times 9 - 3 \times -23 \\ -1 \times 11 - 5 \times 5 - 3 \times -13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

∴ x = 1 , y = 2 , z = 3

Question: 38

Solution:

Given,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|} adj(A)$

The determinant of matrix A is

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix}$$

$= 2(-2 \times -5 - (-1) \times 3) - (1 \times -5 - (-1) \times 0) + (1 \times 3 - (-2) \times 0)$

$= 2(10 + 3) - (-5) + (3)$

$= 26 + 5 + 3$

$= 34$

$|A| \neq 0$

∴ A^{-1} is possible.

$$A^T = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 3 \\ 1 & -1 & -5 \end{bmatrix}$$

$$Adj(A) = \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|} adj(A)$

$A^{-1} = \frac{1}{34}\begin{bmatrix}13 & 8 & 1\\5 & -10 & 3\\3 & -6 & -5\end{bmatrix}$

Given set of lines are : -

$2x + y + z = 1$

$X - 2y - z = \frac{3}{2}$

$3y - 5z = 9$

Converting the following equations in matrix form,

$AX = B$

Where $A = \begin{bmatrix}2 & 1 & 1\\1 & -2 & -1\\0 & 3 & -5\end{bmatrix}$, $X = \begin{bmatrix}x\\y\\z\end{bmatrix}$, $B = \begin{bmatrix}1\\\frac{3}{2}\\9\end{bmatrix}$

Pre - multiplying by A^{-1}

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{34}\begin{bmatrix}13 & 8 & 1\\5 & -10 & 3\\3 & -6 & -5\end{bmatrix}\begin{bmatrix}1\\\frac{3}{2}\\9\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{34}\begin{bmatrix}1\times 13 + \frac{3}{2}\times 8 + 9\times 1\\1\times 5 + \frac{3}{2}\times -10 + 9\times 3\\1\times 3 + \frac{3}{2}\times -6 + 9\times -5\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{34}\begin{bmatrix}13 + 12 + 9\\5 - 15 + 27\\3 - 9 - 45\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{34}\begin{bmatrix}34\\17\\-51\end{bmatrix} = \begin{bmatrix}1\\\frac{1}{2}\\-\frac{3}{2}\end{bmatrix}$$

$\therefore x = 1, y = \frac{1}{2}, z = -\frac{3}{2}$

Question: 39 Using

Solution:

Given,

$A = \begin{bmatrix}1 & -2 & 0\\2 & 1 & 3\\0 & -2 & 1\end{bmatrix}$, $B = \begin{bmatrix}7 & 2 & -6\\-2 & 1 & -3\\-4 & 2 & 5\end{bmatrix}$

$AB = \begin{bmatrix}1 & -2 & 0\\2 & 1 & 3\\0 & -2 & 1\end{bmatrix}\begin{bmatrix}7 & 2 & -6\\-2 & 1 & -3\\-4 & 2 & 5\end{bmatrix}$

$$AB = \begin{bmatrix}7\times 1 - 2\times -2 - 4\times 0 & 2\times 1 + 1\times -2 + 2\times 0 & -6\times 1 - 3\times -2 + 5\times 0\\7\times 2 - 2\times 1 - 4\times 3 & 2\times 2 + 1\times 1 + 2\times 3 & -6\times 2 - 3\times 1 + 5\times 3\\7\times 0 - 2\times -2 - 4\times 1 & 2\times 0 + 1\times -2 + 2\times 1 & -6\times 0 - 3\times -2 + 5\times 1\end{bmatrix}$$

$$AB = \begin{bmatrix} 7+4+0 & 2-2+0 & -6+6+0 \\ 14-2-12 & 4+1+6 & -12-3+15 \\ 0+4-4 & 0-2+2 & 0+6+5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$AB = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

AB = 11I

Pre - multiplying by A^{-1}

$A^{-1}AB = 11 A^{-1}I$

$IB = 11 A^{-1}$

$B = 11 A^{-1}$

$A^{-1} = \frac{1}{11} B$

Given set of lines are : -

x – 2y = 10

x + y + 3z = 8

- 2y + z = 7

Converting following equations in matrix form,

AX = C

Where $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $C = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

Pre - multiplying by A^{-1}

$A^{-1}AX = A^{-1}C$

$IX = A^{-1}C$

$X = A^{-1}C$

$X = \frac{1}{11} BC$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 10\times 7 + 8\times 2 + 7\times -6 \\ 10\times -2 + 8\times 1 + 7\times -3 \\ 10\times -4 + 8\times 8 + 7\times 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + -16 + 35 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

$\therefore$ x = 4 , y = - 3 , z = 1

Question: 40

To find: - x , y , z

Given set of lines are : -

$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10,$

$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

$2R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & -1 \\ 0 & 7 & -5 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ -4 \end{bmatrix}$$

$R_3 - 5R_2$

$$\begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & -1 \\ 0 & -18 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ -54 \end{bmatrix}$$

Again converting into equations we get

$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$

$\frac{5}{y} - \frac{1}{z} = 10$

$-\frac{18}{y} = -54$

$y = \frac{1}{3}$

$5 \times 3 - \frac{1}{z} = 10$

$-\frac{1}{z} = 10 - 15$

$Z = \frac{1}{5}$

$\frac{2}{x} - 3 \times 3 + 3 \times 5 = 10$

$\frac{2}{x} = 10 + 9 - 15 = 4$

$X = \frac{1}{2}$

$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

Question: 41 VALUE

To find: - x , y , z

Given set of lines are : -

$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$

$\frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$

Converting following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -5 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \\ -2 \end{bmatrix}$$

Again converting into equations we get

$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$

$\frac{3}{y} - \frac{5}{z} = -8$

$\frac{2}{y} = -2$

$y = -1$

$3 \times -1 - \frac{5}{z} = -8$

$-\frac{5}{z} = -8 + 3$

Z = 1

$\frac{1}{x} - 1 \times -1 + 1 \times 1 = 4$

$\frac{1}{x} = 4 - 1 - 1$

$X = \frac{1}{2}$

$\therefore x = \frac{1}{2}, y = 1, z = 1$

Question: 42

Solution:

Let the three numbers be x, y and z.

According to the question,

X + y + z = 2

X + 2y + z = 1

5x + y + z = 6

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 1 & 1\\1 & 2 & 1\\5 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\1\\6\end{bmatrix}$$

$R_2 - R_1$

$R_3 - R_1$

$$\begin{bmatrix}1 & 1 & 1\\0 & 1 & 0\\4 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-1\\5\end{bmatrix}$$

Converting back into the equations we get

X + y + z = 2

Y = - 1

4x = 5

$X = \frac{5}{4}$

$\frac{5}{4} - 1 + z = 2$

$Z = 2 - \frac{5}{4} + 1$

$Z = \frac{7}{4}$

∴ The numbers are $\frac{5}{4}, \frac{7}{4}, -1$.

Question: 43

Solution:

Let the price of 1kg potato, wheat and rice be x, y and z respectively.

According to the question,

4x + 3y + 2z = 60

X + 2y + 3z = 45

6x + 2y + 3z = 70

Converting into matrix form

AX = B

$$\begin{bmatrix}4 & 3 & 2\\1 & 2 & 3\\6 & 2 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}60\\45\\70\end{bmatrix}$$

$4R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 5 & 10 \\ 0 & -5 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 120 \\ -40 \end{bmatrix}$$

Converting back into the equations we get

$4x + 3y + 2z = 60$

$5y + 10z = 120$

$-5y = -40$

$Y = 8$

$5\times8 + 10z = 120$

$10z = 120 - 40$

$Z = 8$

$4x + 3\times8 + 2\times8 = 60$

$4x = 60 - 24 - 16$

$4x = 20$

$X = 5$

∴ The cost of 1 kg potatoes, wheat and rice is Rs.5, Rs.8 and Rs. 8 respectively.

Question: 44

Solution:

Let these investments be ₹x, ₹y and ₹z, respectively.

Then, $x + y + z = 5000$

$$\frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} = 358$$

$6x + 7y + 8z = 35800$

And, $\frac{6x}{100} + \frac{7y}{100} = \frac{8z}{100} + 70$

$6x + 7y - 8z = 7000.$

Representing in the matrix form,

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 0 & 0 & -16 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35800 \\ -28800 \end{bmatrix}$$

$R_2 - 6R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -16 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 5800 \\ -28800 \end{bmatrix}$$

Converting back into the equations we get

X + y + z = 5000

Y + 2z = 5800

- 16z = - 28800

Z = 1800

Y + 2×1800 = 5800

Y = 5800 - 3600

Y = 2200x + 2200 + 1800 = 5000

X = 5000 − 4000

X = 1000

Amount of 1000 , 2200 , 1800 were invested in the investments of 6% , 7%, 8% respectively.

Question: 45

Solution:

Let the amount x, y and z be considered for sincerity, truthfulness and helpfulness.

According to the questions,

3x + 2y + z = 1600

4x + y + 3z = 2300

X + y + z = 900

Converting into the matrix form

AX = B

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$R_1 - 3R_3$

$R_2 - 4R_3$

$$\begin{bmatrix} 0 & -1 & -2 \\ 0 & -3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1100 \\ -1300 \\ 900 \end{bmatrix}$$

$2R_2 - R_1$

$$\begin{bmatrix} 0 & -1 & -2 \\ 0 & -5 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1100 \\ -1500 \\ 900 \end{bmatrix}$$

Converting back into the equations we get

- y - 2z = - 1100

- 5y = - 1500

X + y + z = 900

Y = 300

- 300 - 2z = - 1100

- 2z = - 800

Z = 400

X + 300 + 400 = 900

X = 900 - 700

$X = 200$

₹ 200 for sincerity, ₹ 300 for truthfulness and ₹ 400 for helpfulness. One more value may be like honesty, kindness, etc.

Exercise : OBJECTIVE QUESTIONS

Question: 1

$(A+B) = \begin{pmatrix} 4 & -3 \\ 1 & 6 \end{pmatrix} - - - - - - -1$

$(A-B) = \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix} - - - - - - -2$

$1+2 \Rightarrow 2A = \begin{pmatrix} 4 & -3 \\ 1 & 6 \end{pmatrix} + \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix}$

$\Rightarrow 2A = \begin{pmatrix} 2 & -4 \\ 6 & 8 \end{pmatrix}$

Dividing the matrix by 2

$\Rightarrow A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$

$1-2 \Rightarrow 2B = \begin{pmatrix} 4 & -3 \\ 1 & 6 \end{pmatrix} - \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix}$

$\Rightarrow 2B = \begin{pmatrix} 6 & -2 \\ -4 & 4 \end{pmatrix}$

Dividing the matrix by 2

$\Rightarrow B = \begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$

$A \times B = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$

$= \begin{pmatrix} 1 \times 3 + (-2) \times (-2) & (1) \times (-1) + (-2) \times (2) \\ 3 \times 3 + 4 \times (-2) & 3 \times (-1) + 4 \times 2 \end{pmatrix}$

$= \begin{pmatrix} 7 & -5 \\ 1 & 5 \end{pmatrix}$

Question: 2

$\begin{pmatrix} 3 & -2 \\ 5 & 6 \end{pmatrix} + 2A = \begin{pmatrix} 5 & 6 \\ -7 & 10 \end{pmatrix}$

$\Rightarrow 2A = \begin{pmatrix} 5 & 6 \\ -7 & 10 \end{pmatrix} - \begin{pmatrix} 3 & -2 \\ 5 & 6 \end{pmatrix}$

$\Rightarrow 2A = \begin{pmatrix} 2 & 8 \\ -12 & 4 \end{pmatrix}$

Dividing the matrix by 2

$\Rightarrow A = \begin{pmatrix} 1 & 4 \\ -6 & 2 \end{pmatrix}$

Question: 3

$4A + 3X = 5B$

$\Rightarrow 4\begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix} + 3X = 5\begin{pmatrix} 4 & -3 \\ -6 & 2 \end{pmatrix}$

$\Rightarrow 3X = 5\begin{pmatrix} 4 & -3 \\ -6 & 2 \end{pmatrix} - 4\begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}$

$\Rightarrow 3X = \begin{pmatrix} 20 & -15 \\ -30 & 10 \end{pmatrix} - \begin{pmatrix} 8 & 0 \\ -12 & 4 \end{pmatrix}$

$\Rightarrow 3X = \begin{pmatrix} 12 & -15 \\ -18 & 6 \end{pmatrix}$

Dividing by 3

$\Rightarrow X = \begin{pmatrix} 4 & -5 \\ -6 & 2 \end{pmatrix}$

Question: 4

$(A\text{-}2B) = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$

Multiplying equation by 2

$2A\text{-}4B = \begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix}$ ———— (i)

$2A\text{-}3B = \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix}$ ——— (ii)

(ii)-(i)

$B = \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix} - \begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix}$

$= \begin{pmatrix} -4 & 6 \\ 3 & -3 \end{pmatrix}$

Question: 5

$(2A - B) = \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$

Multiplying by 2

$4A - 2B = \begin{pmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{pmatrix}$ ——(i)

$2B + A = \begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$ ——(ii)

(i)+ (ii)

$5A = \begin{pmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$

$= \begin{pmatrix} 15 & 10 & 5 \\ -10 & 5 & -5 \end{pmatrix}$

Dividing each element of the matrix by 5

$A = \begin{pmatrix} 3 & 2 & 1 \\ -2 & 1 & -1 \end{pmatrix}$

Question: 6

$2\begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

To solve this problem we will use the comparison that is we will use that all the elements of L.H.S are equal to R.H.S .

$= \begin{pmatrix} 6 & 8 \\ 10 & 2x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 7 & 8 + y \\ 10 & 2x + 1 \end{pmatrix}$

Comparing with R.H.S

$8+y = 0$

$y = -8$

$2x+1 = 5$

$2x = 4$

$x=2$

Question: 7

By comparing L.H.S and R.H.S

$x - y = -1$--------- i

$2x - y = 0$ -------- ii

$2x+ z = 5$ -------- iii

$3z + w =13$ -------iv

Using i in equation ii

$x = -1 + y$

ii becomes, $-2 + 2y - y = 0$

$y = 2$

$x = 1$

Putting x in iii

$2 + z =5$

$z = 3$

Putting z in iv

$9 + w = 13$

$w = 4$

Question: 8

$\begin{pmatrix} x & y \\ 3y & x \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} x \times 1 + y \times 2 \\ 3y \times 1 + x \times 2 \end{pmatrix}$

$= \begin{pmatrix} x + 2y \\ 3y + 2x \end{pmatrix}$

Comparing with R.H.S

$x + 2y = 3$ -------(i)

$2x + 3y = 5$--------(ii)

(i) x 2 – (ii)

$2x + 4y - 2x + 3y = 6 - 5$

$y = 1$

Putting y in (i)

$x + 2(1) = 3$

$x= 1$

Question: 9

When a given matrix is singular then the given matrix determinant is 0.

$|A| = 0$

Given, $A = \begin{pmatrix} 3-2x & x+1 \\ 2 & 4 \end{pmatrix}$

$|A| = 0$

$4(3-2x) - 2(x+1) = 0$

$12 - 8x - 2x - 2 = 0$

$10 - 10x = 0$

$10x = 0$

$x = 1$

Question: 10

Given, $A_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

$A_\alpha^2 = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos\alpha \times \cos\alpha - \sin\alpha \times \sin\alpha & \cos\alpha \times \sin\alpha + \sin\alpha \times \cos\alpha \\ -\sin\alpha \times \cos\alpha - \cos\alpha \times \sin\alpha & -\sin\alpha \times \sin\alpha + \cos\alpha \times \cos\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos^2\alpha - \sin^2\alpha & \cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha - \cos\alpha\sin\alpha & -\sin^2\alpha + \cos^2\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos2\alpha & \sin2\alpha \\ -\sin2\alpha & \cos2\alpha \end{pmatrix}$

Question: 11

L.H.S: $A + A' = \begin{pmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{pmatrix} + \begin{pmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{pmatrix}$

$= \begin{pmatrix} \cos a + \cos a & \sin a - \sin a \\ -\sin a + \sin a & \cos a + \cos a \end{pmatrix}$

$= \begin{pmatrix} 2\cos a & 0 \\ 0 & 2\cos a \end{pmatrix}$

This will be equal to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

When $2\cos a = 1$

$\cos a = \frac{1}{2}$

$a = \frac{\pi}{3}$

Question: 12

When a given matrix is singular then the given matrix determinant is 0.

$|A| = 0$

Given,

$A = \begin{pmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{pmatrix}$

$|A| = 0$

$1(-4k + 6) - k(-12 + 4) + 3(9 - 2k) = 0$

$-4k + 6 + 12k - 4k + 27 - 6k = 0$

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$-2k + 33 = 0$

$k = \frac{33}{2}$.

Question: 13

To find adj A we will first find the cofactor matrix

$C_{11} = d$ $C_{12} = -c$

$C_{21} = -b$ $C_{22} = a$

Cofactor matrix $A = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

$Adj\, A = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}'$

$= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Question: 14

We know that $A \times A^{-1} = I$

$\begin{pmatrix} 2x & 0 \\ x & x \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 2x \times 1 + 0 \times (-1) & 2x \times 0 + 0 \times 2 \\ x \times 1 + x \times (-1) & x \times 0 + x \times 2x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 2x & 0 \\ 0 & 2x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

To satisfy the above condition $2x = 1$

$x = \frac{1}{2}$

Question: 15

Since A and B are square matrices of same order.

$(A+B)(A-B) = A^2 - AB + BA - B$

Question: 16

Since A and B are square matrices of same order.

$(A + B)^2 = (A + B)(A + B)$

$= A^2 + AB + BA + B^2$

Question: 17

Since A and B are square matrices of same order.

$(A - B)^2 = (A - B)(A - B)$

$= A^2 - AB - BA + B^2$

Question: 18

Given A and B are symmetric matrices

$A' = A$ --- 1

$B' = B$----2

Now $(AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B'$

[$\because (AB)' = B'A'$]

$= BA - AB$ [Using 1 and 2]

∴ $(AB - BA)' = -(AB - BA)$

AB-BA is a skew symmetric matrix.

Question: 19

$A = B^{-1}$

$B=A^{-1}$

We know that

$AA^{-1}= I$

(Given $B=A^{-1}$)

$AB= I$------ 1

We know that

$BB^{-1}= I$

(Given $A=B^{-1}$)

$BA= I$ ------- 2

From 1 and 2

$AB= BA = I$

Question: 20

We know that $(AB)^{-1} = adj(AB)/ |AB|$

$adj (AB)= (AB)^{-1}. |AB|$

We also know that $(AB)^{-1} = B^{-1}. A^{-1}$

$|AB| = |A| |B|$

Putting them in 1

$Adj (AB) = B^{-1}. A^{-1}. |A|.|B|$

$= (A^{-1}. |A|) (B^{-1}|B|)$

$= adj(A) adj(B)$

Since, $adj (A)= (A)^{-1}. |A|$

$adj (B)= (B)^{-1}. |B|$

Question: 21

The property states that

$adj(adj A) = |A|^{n-2} . A$

Here n=2

$adj(adj A) = |4|_3^{-2} . A$

$= 4A$

Question: 22

The property states that $|adj A| = |A|^{n-1}$

Here n= 3 and $|A| =5$

$|adj A| = |5|^{3-1}$

$= |5|^2$

= 25.

Question: 23

If the two matrices A and B are of same order it is not necessary that in every situation AB= BA

$AB = BA = I$ only when $A = B^{-1}$

$B=A^{-1}$

Other time $AB \neq BA$

Question: 24

The matrix on the R.H.S of the given matrix is of order 2 x 2 and the one given on left side is 2 x 2 . Therefore A has to be a 2 x 2 matrix.

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

$\begin{pmatrix} 3a + b & 2a - b \\ 3c + d & 2c - d \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

$3a+b = 4$ ----1

$2a-b = 1$------2

$3c+d= 2$------ 3

$2c-d=3$------- 4

Using 1 and 2

$a=1$

$b=1$

Using 3 and 4

$c=1$

$d = -1$

So A becomes $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

Question: 25

We know that $AA^{-1} = \mathbf{I}$

Taking determinant both sides

$|AA^{-1}| = |I|$

$|A||A^{-1}| = |I|$ $(|AB|=|A||B|)$

$|A||A^{-1}| = 1$ $(|I| = 1)$

$|A^{-1}| = \frac{1}{|A|}$

Question: 26

$A^{-1}(AB)(AB)^{-1} = IA^{-1}$

$(A^{-1}A)B(AB)^{-1}=A^{-1}$

$IB(AB)^{-1} = A^{-1}$

$B(AB)^{-1} = A^{-1}$

$B^{-1}B(AB)^{-1} = B^{-1}A^{-1}$

$I(AB)^{-1} = B^{-1}A^{-1}$

$(AB)^{-1} = B^{-1}A^{-1}$

Question: 27

So $|AB| = |A|\ |B| = 0$

So $|A| = |B| = 0$

Question: 28

Multiplying by A^{-1}

$A^{-1}A^2 - A^{-1}A + 2I\,A^{-1} = 0$

$A - I + 2\,A^{-1} = 0$

$A^{-1} = \frac{1}{}(I - A)$

Question: 29

$$\begin{pmatrix} 1 & \lambda & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{pmatrix}$$

$|A| = 0$

$1(2 \times 1 - 5 \times 1) - \lambda(1 \times 1 - 5 \times 2) + 2(1 \times 1 - 2 \times 2) = 0$

$-3 + 9\lambda - 6 = 0$

$9\lambda = 9$

$\lambda = 1$

Question: 30

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$|A| = \cos^2\theta - (-\sin^2\theta)$

$= \cos^2\theta + (\sin^2\theta)$

$= 1$ -------- (I)

We know that $A^{-1} = \frac{1}{|A|}$ *adj* A

$=$ *adj* A [From I]

Question: 31

Solution:

Here the diagonal value is 2+3-3= 1

So the given matrix is idempotent.

Question: 33

$A(adjA) = A(|A|\ x\,A^{-1})$

Since determinant of singular matrix is always 0

$A(adjA) = 0$

So, it is a null matrix.

Question: 34

$\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$

$= 8\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= |A| I$

$|A| = 8.$

Question: 35

$A = \begin{pmatrix} -2 & 3 \\ 1 & 1 \end{pmatrix}$

$|A| = -2-3 = -5$

We know that $|A^{-1}| = \frac{1}{|A|}$

$= \frac{1}{-5}$

Question: 36

$2 + xI = yA$

$\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix} + x\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$

$\begin{pmatrix} 16 & 8 \\ 56 & 32 \end{pmatrix} + x\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$

$8\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix} + x\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$

Comparing L.H.S and R.H.S

$x=8$ $y=8$

Question: 37

Solution:

To find adj A we will first find the cofactor matrix

$C_{11} = 3$ $C_{12} = -1$

$C_{21} = -5$ $C_{22} = 2$

Cofactor matrix $A = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

Adj $A = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}'$

$= \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$

Question: 39

$B = A^{-1} I$-------- 1

$A^{-1} = \frac{1}{|A|}$ adj A -------- 2

$|A| = 3 \times 2 - (-4) \times (-1)$

$= 2$

$C_{11} = 2$ $C_{12} = 1$

$C_{21} = 4$ $C_{22} = 3$

Cofactor matrix $A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

$Adj\, A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}'$

$= \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$

Putting in 2

$A^{-1} = \frac{1}{|2|} \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$

Putting in 1

$B = A^{-1} I$

$= A^{-1}$

$= \begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$

Question: 40

$A^{-1}(AB)(AB)^{-1} = IA^{-1}$

$(A^{-1}A)B\,(AB)^{-1} = A^{-1}$

$IB(AB)^{-1} = A^{-1}$

$B(AB)^{-1} = A^{-1}$

$B^{-1}B(AB)^{-1} = B^{-1} A^{-1}$

$I\,(AB)^{-1} = B^{-1}A^{-1}$

$(AB)^{-1} = B^{-1}A^{-1}$

Question: 41

$\frac{1}{|A|}$ adj A --------1

$|A| = 3 \times 2 - (1) \times (-1)$

$= 7$

$C_{11} = 3$ $C_{12} = -1$

$C_{21} = 1$ $C_{22} = 2$

Cofactor matrix $A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

$Adj\, A = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}'$

$= \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

Putting in 1

$A^{-1} = \frac{1}{|7|} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{pmatrix}$

Question: 42

$\frac{1}{|A|}$ *adj A*

adj A = $|A| \times A^{-1}$

$= 3 \times \begin{pmatrix} 3 & -1 \\ \frac{-5}{3} & \frac{2}{3} \end{pmatrix}$

$= \begin{pmatrix} 9 & -3 \\ -5 & 2 \end{pmatrix}$

Question: 43

$(A^{-1})^{-1} = A$

$(A^{-1})^{-1} = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}^{-1}$

$A = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}^{-1}$ ———— *1*

$|A|^{-1} = 3 \times 6 - 4 \times 5$

$= -2$

$C_{11} = 6$ $C_{12} = -5$

$C_{21} = -4$ $C_{22} = 3$

Cofactor matrix A = $\begin{pmatrix} 6 & -5 \\ -4 & 3 \end{pmatrix}$

Adj A = $\begin{pmatrix} 6 & -4 \\ -5 & 3 \end{pmatrix}$

$\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}^{-1} = \frac{1}{-2}\begin{pmatrix} 6 & -4 \\ -5 & 3 \end{pmatrix}$

$= \begin{pmatrix} -3 & 2 \\ \frac{5}{2} & \frac{-3}{2} \end{pmatrix}$

Putting in 1

$A = \begin{pmatrix} -3 & 2 \\ \frac{5}{2} & \frac{-3}{2} \end{pmatrix}$

Question: 44

$f(A) = 2A^2 - 4A + 5$

$A^2 = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$

$= \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}$

$f(A) = 2A^2 - 4A + 5I$

$= 2\begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix} - 4\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} + 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 18 & -8 \\ -16 & 34 \end{pmatrix} - \begin{pmatrix} 4 & 8 \\ 16 & -12 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

$= \begin{pmatrix} 19 & -16 \\ -32 & 51 \end{pmatrix}$

Question: 45

$A^2 = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix}$

$A^2 - 4A = \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix} - 4\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix} - \begin{pmatrix} 4 & 16 \\ 8 & 12 \end{pmatrix}$

$= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

$= 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= 5I$

Question: 46

$|A|I$

$= 6\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

Question: 47

$(AB)^{-1} = B^{-1}A^{-1}$

$(KA)^{-1} = A^{-1}K^{-1}$

$= \frac{1}{K}A^{-1}$

Question: 48

$|A| = 3 \times (0 - 2) - 4 \times (2-4) + 1 \times (-1)$

$= -6+8-1$

$= 1$

$C_{11} = -2$ $C_{12} = 2$ $C_{13} = -1$

$C_{21} = -9$ $C_{22} = 8$ $C_{23} = -5$

$C_{31} = -8$ $C_{32} = 7$ $C_{33} = -4$

Cofactor $(A) = \begin{bmatrix} -2 & 2 & -1 \\ -9 & 8 & -5 \\ -8 & 7 & -4 \end{bmatrix}$

$Adj\,A = \begin{bmatrix} -2 & 2 & -1 \\ -9 & 8 & -5 \\ -8 & 7 & -4 \end{bmatrix}'$

$= \begin{bmatrix} -2 & -9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & -4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\,adj\,A$

$= \frac{1}{1}\begin{bmatrix} -2 & -9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & -4 \end{bmatrix}$

$$\begin{pmatrix} -2 & -9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & -4 \end{pmatrix}$$

Question: 49

Let X = A+A'

X' = (A+A')'

= A' + (A')'

=A + A'

= X

Therefore (A+A') is symmetric matrix.

Question: 50

Let X = A-A'

X' = (A-A')'

= A' - (A')'

=A' – A

= -(A – A')

= -X

Therefore (A-A') is skew symmetric matrix.

Question: 51

So, k= 27

Tagging

Question: 52

$$= \begin{pmatrix} -8 & 0 \\ 0 & -8 \end{pmatrix}$$

$$= -8\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Since -8 could be taken common from each row or column. Hence C is a scalar matrix.

Question: 53

$$\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} B = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$$

$$A+B = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}$$

$$(A+B)^2 = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}$$

$$= \begin{pmatrix} (1+a)^2 & 0 \\ (2+b)(1+a) - 4 - 2b & -4 \end{pmatrix}$$

$$= \begin{pmatrix} (1+a)^2 & 0 \\ 2 + 2a + b + ab - 4 - 2b & 4 \end{pmatrix}$$

$$= \begin{pmatrix} (1+a)^2 & 0 \\ 2a + ab - b - 2 & 4 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$$

$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$B^2 = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix} \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$

$= \begin{pmatrix} a^2 + b & a - 1 \\ ab - b & b + 1 \end{pmatrix}$

$(A + B)^2 = (A^2 + B^2)$

$\begin{pmatrix} (1 + a)^2 & 0 \\ 2a + ab - b - 2 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} a^2 + b & a - 1 \\ ab - b & b + 1 \end{pmatrix}$

$= \begin{pmatrix} -1 + a^2 + b & a - 1 \\ ab - b & b \end{pmatrix}$

By comparison,

$a-1 = 0$

$a=1$

$b=4$